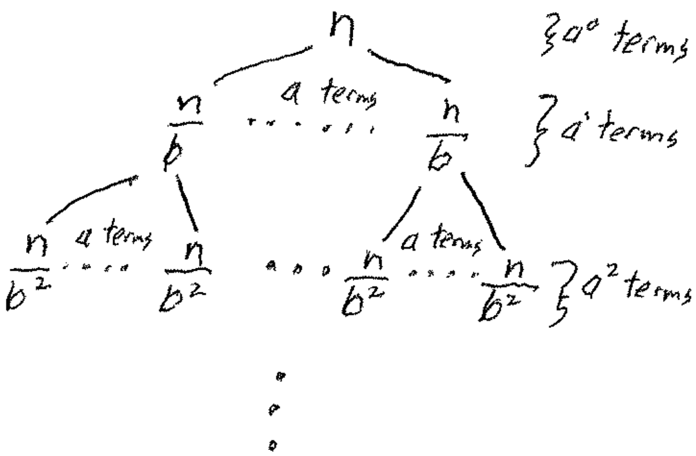
Sect 2.2 Proof sketch of Master Thm (pg 49)

Assume $T(n) = a\,T(\frac{n}{b}) + O(n^d) \quad a>0,\ b>1,\ d\geq 0$

To simplify the argument, assume n is a power of b.

Call tree:



Work done at each level:

cn^d

$a^1 c\left(\frac{n}{b}\right)^d = \frac{a}{b^d}\cdot c\,n^d$

$a^2 c\left(\frac{n}{b^2}\right)^d = \left(\frac{a}{b^d}\right)^2 c\,n^d$

$a^3 c\left(\frac{n}{b^3}\right)^d = \left(\frac{a}{b^d}\right)^3 c\,n^d$

$\vdots$

$a^k c\left(\frac{n}{b^k}\right)^d = \left(\frac{a}{b^d}\right)^k c\,n^d$

$\vdots$

$a^{\log_b n} c\left(\frac{n}{b^{\log_b n}}\right)^d = \left(\frac{a}{b^d}\right)^{\log_b n} c\,n^d$

$$\sum = \underbrace{\left[1 + \frac{a}{b^d} + \left(\frac{a}{b^d}\right)^2 + \left(\frac{a}{b^d}\right)^3 + \cdots + \left(\frac{a}{b^d}\right)^{\log_b n}\right]}_{\text{Geometric series w/ ratio } \frac{a}{b^d} \text{ Call it } G.} c\,n^d$$

Apply Exercise 0.2 result:

If $\frac{a}{b^d} < 1$ then $G = \Theta(1)$ so $\sum = O(n^d)$ case: $d > \log_b a$

If $\frac{a}{b^d} = 1$ then $G = \Theta(\log_b n)$ so $\sum = O(n^d \log n)$ case: $d = \log_b a$

If $\frac{a}{b^d} > 1$ then $G = \Theta\left(\underbrace{\left(\frac{a}{b^d}\right)^{\log_b n}}_{= \frac{n^{\log_b a}}{n^d}}\right)$ so $\sum = O(n^{\log_b a})$ case: $d < \log_b a$