Practice Problem related to Exercise 2.3 in Dasgupta et al.

Suppose $T(n) = 5\,T(\frac{n}{3}) + O(1)$.

We know from the Master Thm (pg 49) that $T(n) = O(n^{\log_3 5})$. (Check it.)

Try to analyze it using the technique from Exercise 2.3 instead:

$$T(n) \le 5\,T(\tfrac{n}{3}) + c$$

$$\le 5[5\,T(\tfrac{n}{9}) + c] + c = 5^2 T(\tfrac{n}{3^2}) + 5c + c$$

$$\le 5^2[5T(\tfrac{n}{3^3}) + c] + 5c + c = 5^3 T(\tfrac{n}{3^3}) + 5^2 c + 5c + c$$

$$\vdots$$

$$T(n) \le 5^k T(\tfrac{n}{3^k}) + \underbrace{5^{k-1}c + \cdots + 5c + c}_{\text{geometric series}}$$

$$= 5^k T(\tfrac{n}{3^k}) + \frac{5^k - 1}{5-1} c$$

Plugging in $k = \log_3 n$

$$T(n) \le 5^{\log_3 n} T(\frac{n}{3^{\log_3 n}}) + \frac{5^{\log_3 n} - 1}{5-1} c$$

$$= n^{\log_3 5} T(1) + \frac{n^{\log_3 5} - 1}{4} c$$

$$= O(n^{\log_3 5})$$ just as the Master Thm said. ✓