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Problem # 1 Solution

$P(0) = 0$, $P(1) = p_1 = 4$

$P(2) = \max\{P(1), p_2\} = \max\{4, 2\} = 4$

$P(3) = \max\{P(2), p_3\} = \max\{4, 7\} = 7$

$P(4) = \max\{P(3), p_4, p_4 + P(1)\} = \max\{7, 6, 6+4\}$
$= 10$

$P(5) = \max\{P(4), p_5, p_5 + P(3)\} = \max\{10, 5, 5+7\}$
$= 12$

$P(6) = \max\{P(5), p_6, p_6 + P(4)\} = \max\{12, 9, 9+10\}$
$= 19$

$P(7) = \max\{P(6), p_7, p_7 + P(4)\} = \max(19, 9, 9+10)$
$= 19$

Selected Resturants ($i = 1, 4, 6$)

Problem #2 solution.

		S	H	A	R	K
	0	0	0	0	0	0
M	0	0	0	0	0	0
A	0	0	0	1	0	0
R	0	0	0	0	2	0
K	0	0	0	0	0	3
S	0	1	0	0	0	0

The length of longest common substring of MARKS and SHARK is 3 which is "ARK"

Problem #3 Solution.

let $L(i,j)$ = the length of the longest common subsequence of $x[1 \cdots i]$ and $y[1 \cdots j]$ that ends at i^{th} character in x and the j^{th} character in y (zero if there is no such subsequence)

Base Cases

$L(0,j) = 0$

$L(i,0) = 0$

~~$L(i,j) = 0$ if ith character is not equal to the jth character~~

$$L(i,j) = \max \begin{cases} L(i,j-1), L(i-1,j), \\ 1 + L(i-1, j-1) \end{cases} \text{ if } x_i = y_i$$

$$L(i,j) = \max \{ L(i,j-1), L(i-1,j) \} \text{ if } x_i \neq y_i$$

return $L(i,j)$

runtime = $O(mjn)$

Problem # 4 Solution.

$C(w)$ = true iff it is possible to make change for value w

Base Cases

$C(0)$ = true

$C(-w)$ = false

$C(w)$ = false if $w > 0$

$C(w) = \vee \{ C(w - w_i) : x_i \leq w \}$

where $x_i = x_1, x_2, x_3, \ldots, x_n$ denominations

return $C(w)$;

Runtime : $O(nw)$.

Problem # 5 Solution.

$C(w, K)$ = true iff it is possible to make change for value w

Base cases

$C(0, K)$ = true if $K \geq 0$

$C(w, 0)$ = false if $w > 0$

$C(-w, K)$ = false if $K \geq 0$

$C(w, K) = \vee \{ C(w - w_i^\circ, K-1) : x_i^\circ \leq w \}$

where $x_i^\circ = x_1, x_2, x_3 \ldots x_n$ denomination

return $C(W, K)$

runtime: $O(wK.n)$