Ex 2.9(b) Multiply $1+x+2x^2$ and $2+3x$.

Degree of product will be 3, so choose $n=4$, $w=i$

$$a = \begin{pmatrix} 1 \\ 1 \\ 2 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} 3+1 \\ -1+1i \\ 3-1 \\ -1-1i \end{pmatrix} = \begin{pmatrix} 4 \\ -1+i \\ 2 \\ -1-i \end{pmatrix}$$

fft(a, w) = fft((1, 1, 2, 0), i)

$\begin{pmatrix} 1+2 \\ 1-2 \end{pmatrix} = \begin{pmatrix} 3 \\ -1 \end{pmatrix}$

$\begin{pmatrix} 1+0 \\ 1-0 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

fft((1, 2), -1)

fft((1, 0), -1)

(1) (2)

(1) (0)

fft((1), 1) fft((2), 1)

fft((1), 1) fft((0), 1)

2.9(b) cont'd

$b = \begin{pmatrix} 2 \\ 3 \\ 0 \\ 0 \end{pmatrix}$ $\uparrow$ $\begin{pmatrix} 2+3 \\ 2+3i \\ 2-3 \\ 2-3i \end{pmatrix} = \begin{pmatrix} 5 \\ 2+3i \\ -1 \\ 2-3i \end{pmatrix}$

fft(b, w) = fft((2,3,0,0), i)

$\begin{pmatrix} 2+0 \\ 2-0 \end{pmatrix} = \begin{pmatrix} 2 \\ 2 \end{pmatrix}$ $\uparrow$ $\begin{pmatrix} 3+0 \\ 3-0 \end{pmatrix} = \begin{pmatrix} 3 \\ 3 \end{pmatrix}$

fft((2,0), -1) fft((3,0), -1)

(2) (0) (3) (0)

fft((2), 1) fft((0), 1) fft((3), 1) fft((0), 1)

$c = \begin{pmatrix} 4 \\ -1+i \\ 2 \\ -1-i \end{pmatrix} \begin{pmatrix} 5 \\ 2+3i \\ -1 \\ 2-3i \end{pmatrix}$ —component-wise mult→ $\begin{pmatrix} 20 \\ -2-3i+2i-3 \\ -2 \\ -2+3i-2i-3 \end{pmatrix} = \begin{pmatrix} 20 \\ -5-i \\ -2 \\ -5+i \end{pmatrix}$

So if $C(x) = A(x)\,B(x) = [1+x+2x^2][2+3x]$

We know that $C(w^0) = 20$

$C(w^1) = -5-i$

$C(w^2) = -2$

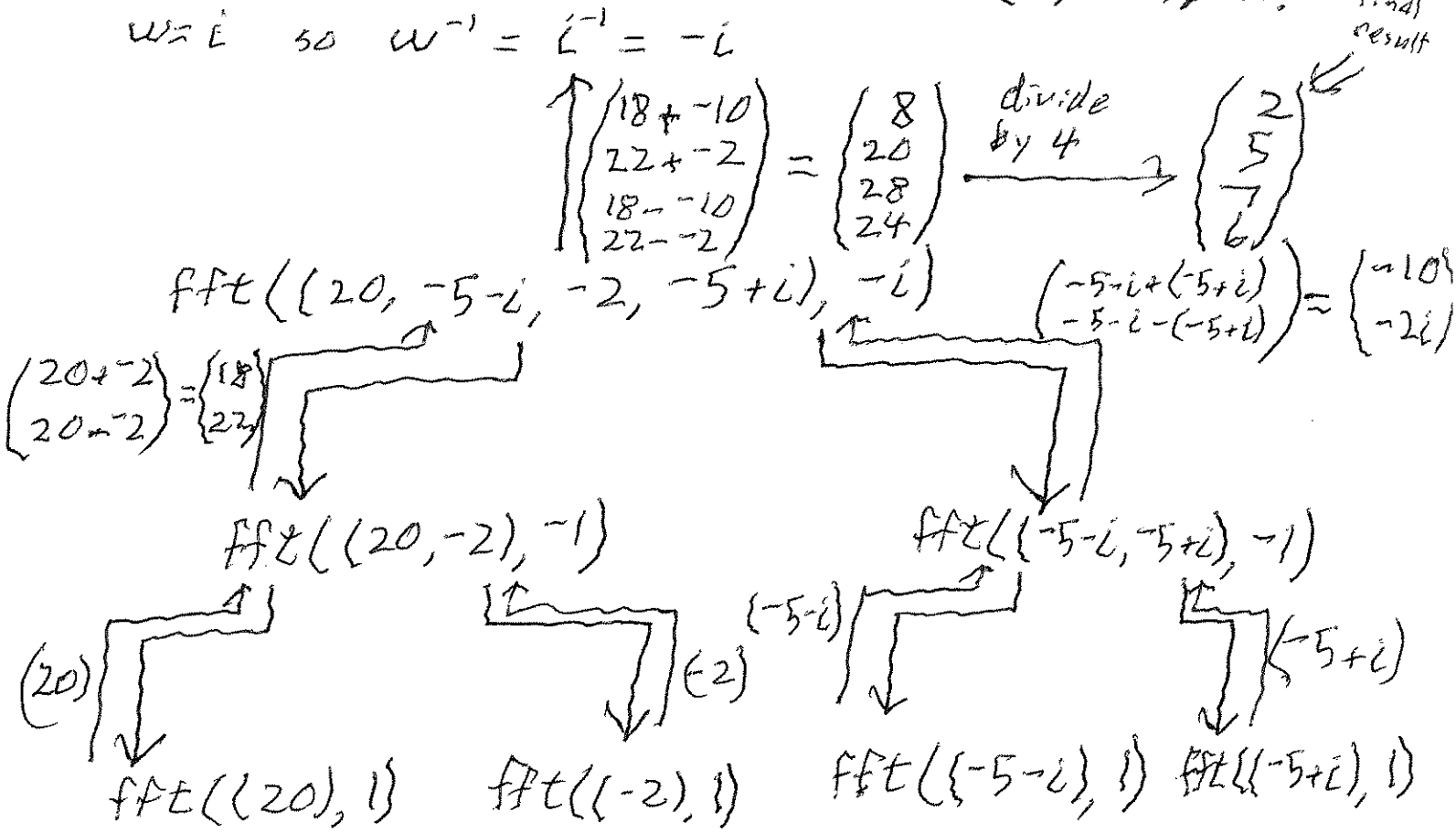
$C(w^3) = -5+i$

~~We find the coefficients of C~~

2.4(b) cont'd –

Now we find the coefficients of C by using the interpolation formula $C(x) = fft(c, w^{-1})/n$.

$w = i$ so $w^{-1} = i^{-1} = -i$



So $C(x) = 2 + 5x + 7x^2 + 6x^3$

Check:

$$\begin{array}{r} 2x^2 + x + 1 \\ \hline 3x + 2 \\ \hline 4x^2 + 2x + 2 \\ 6x^3 + 3x^2 + 3x \\ \hline 6x^3 + 7x^2 + 5x + 2 \end{array}$$

✓