Solution 1.13

Using Fermat's little theorem.

$5^{30} \bmod 31 = 1$.

Since 30000 is a multiple of 30.

we can say $5^{30000} \bmod 31 = 1$

Now

Using long division to find 123456 mod 30

```
      411
30 ) 123456
    -120
       34
     - 30
        45
      - 30
        156
      - 150
          6
```

Similarly,

$6^{30} \bmod 31 = 1$. since 123456 mod 30 is 6

Which is same as

$5^{30} = 1 \bmod 31$

$6^{30} = 1 \bmod 31$

The difference between two number is $0 \bmod 31$

Therefore the difference is divisible by 31

Q.1 solution

| | $f(n)$ | | $g(n)$ |
|---|---|---|---|
| a) | $n-100$ | $=\Theta$ | $n-200$ |
| b) | $n^{1/2}$ | $=O$ | $n^{2/3}$ |
| c) | $100n+\log n$ | $=O$ | $n+(\log n)^2$ |
| d) | $n\log n$ | $=\Theta$ | $10n\log 10n$ |
| e) | $\log 2n$ | $=\Theta$ | $\log 3n$ |
| f) | $10\log n$ | $=\Theta$ | $\log(n^2)$ |
| g) | $n^{1.01}$ | $=\Omega$ | $n\log^2 n$ |
| h) | $n^2/\log n$ | $=\Omega$ | $n(\log n)^2$ |
| i) | $n^{0.1}$ | $=O$ | $(\log n)^{10}$ |
| j) | $(\log n)^{\log n}$ | $=O$ | $n/\log n$ |
| k) | $\sqrt{n}$ | $=\Omega$ | $(\log n)^3$ |
| l) | $n^{1/2}$ | $=O$ | $5^{\log_2 n}$ |
| m) | $n2^n$ | $=O$ | $3^n$ |
| n) | $2^n$ | $=\Theta$ | $2^{n+1}$ |
| o) | $n!$ | $=\Omega$ | $2^n$ |
| p) | $(\log n)^{\log n}$ | $=O$ | $2^{(\log_2 n)^2}$ |
| q) | $\sum_{i=1}^{n} i^k$ | $=\Omega$ | $n^{k+1}$ |

Scanned by CamScanner

~~Q2~~

Problem No 3

Ans 8