CSC 464/564 Homework #10 — Instructor: Jeff Ward
Assigned: Wednesday, November 7, 2018 — Due: 11:59pm, Wednesday, December 5
Covers: NP-Completeness — Worth 25 points

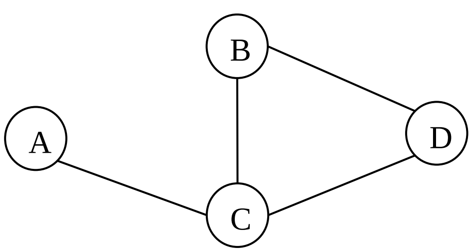
Problem 1 (5 points): In Ch8.ppt review the reduction of CIRCUIT-SAT to 3-SAT. (It is the slide titled "3-SAT is NP-Complete", around slide 17.) Apply this reduction to the CIRCUIT-SAT instance on slide 14, and show the resulting 3-SAT problem instance. Is the instance an element of 3-SAT? (I.e., Is it satisfiable?)

Problem 2 (5 points): In Ch8.ppt review the reduction of 3-SAT to INDEPENDENT-SAT. ("Independent Set is NP-Complete", around slide 19.) Apply this reduction to the formula:

$$(\neg x \lor \neg y \lor z) \land (x \lor y \lor \neg z) \land (\neg x \lor z) \land (x \lor \neg z)$$

Show the resulting INDEPENDENT-SET instance. (Be sure to specify both the graph and the natural number g.) Is the instance an element of INDEPENDENT-SET? (I.e., Does it have an independent set of the size specified by the transformation?)

Problem 3 (5 points): Review the reduction of INDEPENDENT-SET to CLIQUE from the Ch8 notes. ("CLIQUE is NP-Complete", around slide 21.) Apply the reduction to the instance (G,3) where G is this graph:



Show the resulting CLIQUE problem instance. (Be sure to specify both the graph and the natural number g.) Is the instance an element of CLIQUE? (I.e., Does it have a clique of the size specified by the transformation?)

Problem 4 (10 points): Exercise 8.4(a-c) in the textbook.
Hints:
Part (a) Hint: Review Ch8.ppt, slides 5-8. Give a polynomial time algorithm that works as a certifier for CLIQUE-3. You do not need to specify a lot of detail. One or two clearly expressed sentences may suffice. You should specify: (1) what form a certificate takes (e.g. "A certificate is a set of ________ from the graph.") and (2) explain what the certifier algorithm does with the certificate (e.g. "The certifier returns true if ___________ and otherwise returns false.").
Part (b) Hint: Review the "recipe" on Slide 16. How is the given argument failing to follow the recipe?
Part (c) The part that is incorrect is the sentence that starts "Now, a subset C ...". Give a counterexample showing that this sentence is false.

You are not asked to solve Part (d), but here is a solution for your edification:
Note that the largest clique in a CLIQUE-3 problem can have size at most 4. So if $g > 4$ the answer is 'false': the instance is not in CLIQUE-3. If $g \leq 4$ then the following $O(|V|^4)$ algorithm will determine whether the instance is in CLIQUE-3:

```
for u1 in V:
    for u2 in V:
        for u3 in V:
            for u4 in V:
                let S = {u1,u2,u3,u4}
                if |S| = g and each pair of elements in S has an edge between them:
                    return true
return false
```