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PENS-Wheel (One-Wheeled Self Balancing Vehicle) Balancing Control using PID Controller

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Abstract— **Pens-Wheel is an electric vehicle which uses one wheel that able to balance itself so the rider does not fall forward or backward while riding it. This vehicle uses one brushless DC motor as actuator which capable to rotate in both directions symmetrically. The vehicle uses a combination of accelerometer and gyroscope contained in IMU (Inertial Measurement Unit) for balancing sensor. The controlled motion on the vehicle occurs only on the x-axis (pitch angle), in forward and backward directions. The PID (Proportional Integral Derivative) control algorithm is used to maintain the balance and movement of the vehicle. From the simulation and application in the real vehicle, the use of PID control is capable driving the vehicle in maintaining the balance condition within ±10° tilt angle boundary on flat surface, bumpy road, and inclining road up to 15° slope.**

Keyword: IMU, accelerometer, gyroscope, balancing control, PID controller, BLDC motor.

I. INTRODUCTION

Transportation is a crucial means in citizen activities, especially the people of metropolitan area in Indonesia who needs more mobility. The increasing mobility of citizens must be balanced with sufficient transportation tools and facilities. Citizens are also required to move quickly with a short time to be able to finish the job on time. Improved technology in the field of transportation is necessary to help the mobility of people. A lot of innovation in transportation technologies, for example Segway Personal Transporter that works on the principle of two-wheel inverted pendulum on a sliding carriage that has only one degree of freedom [1]. However, balancing robot with two wheels has some shortcomings such as: requires more drive wheels, have a larger physical width, and more heavy [2]. So an alternative vehicle which is simple in operation and easy to carry so that people do not have to bother to learn deeply in how to operate and doesn't need to be confused to take the vehicle is needed.

Therefore in this study, we propose a vehicle that is able to help the mobility of people in that manner. The proposed electric vehicle is practical and also friendly to the environment because it uses the electricity from battery as the power source. The challenge in this vehicle is in how to implement its self-balancing capability. In this study the PID (Proportional Integral Derivative) control algorithm is used to maintain the balance and movement of the vehicle. IMU sensor is used as the tilt sensor in maintain the balance condition. There is a limit of the tilt angle on the vehicle within ±10° maximum boundaries. For the safety reason, when the vehicle tilt angle exceeds ±8° boundary or the vehicle speed is exceeding 20km/h the alarm will be triggered to warn the rider to slowdown the vehicle.

II. RELATED WORKS

A. *Modelling of Pens-Wheel*

It is important to understand the dynamics of the Inverted Pendulum in order to model the proposed vehicle system. Fig. 1 is an illustration of the inverted pendulum [3].

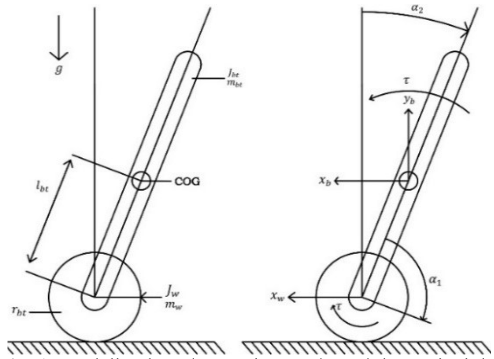

Fig. 1. Modeling based on an inverted pendulum principle

Where:

α_1 = Angle between wheel axis and vehicle
α_2 = Angle between vehicle and vertical axis
m_w = Mass of the wheel
m_{bt} = Vehicle weight
τ = Motor torque
x_w = Horizontal position of the wheel
x_b = Horizontal position of the rider's body
y_b = Vertical position of rider's body
J_w = Inertia moment of the wheel
J_{bt} = Inertia moment of the rider
l_{bt} = Distance between the wheel axis to the center of mass of the vehicle
r_{bt} = Wheel radius
g = Earth's gravitational acceleration

The first step is to develop the Lagrange equations of the inverted pendulum by paying attention to energy work on the system. There are three energy work on the vehicle body and the wheels, namely:

- The kinetic energy
  - Translational kinetic energy of the vehicle
  - Rotational kinetic energy from the wheels and the body of the vehicle
- Potential energy
- Dissipation energy

To get dynamics of the system we use Lagrange equation as follows [4]:

$$\frac{d}{dt}\left(\frac{\partial E}{\partial \dot{q}_r}\right) - \frac{\partial E}{\partial q_r} + \frac{\partial F}{\partial q_r} + \frac{\partial U}{\partial q_r} = \tau_r \quad (1)$$

Lagrange Equation 1 is an equation of the inverted pendulum where, E is the kinetic energy, F is dissipation energy, U is the potential energy, and τ_r is the torque generated by the BLDC motor for the vehicle driving wheel. To simplify the model, the issue of energy wasted as a result of frictional forces axle with wheel bearing and the friction force with the road surface is neglected. Lagrange equation thus becomes the following:

$$\frac{d}{dt}\left(\frac{\partial E}{\partial \dot{q}_r}\right) - \frac{\partial E}{\partial q_r} + \frac{\partial U}{\partial q_r} = \tau_r \quad (2)$$

The kinetic energy is obtained from the summation of the translational energy of the rider's body, the rotational energy of the rider's body, the translational energy of the wheels and the rotational energy of the wheels. So we get the following equation:

$$E = \frac{1}{2}(m_{bt}v^2 + l_{bt}^2\dot{\alpha}_2^2) + m_{bt}vl_{bt}cos\alpha_2\dot{\alpha}_2 + \frac{1}{2}J_{bt}\dot{\alpha}_2^2 + \frac{1}{2}m_w v^2 + \frac{1}{2}J_w\frac{v^2}{r_{bt}^2} \quad (3)$$

This system potential energy turns on the vehicle wheel when it moves, so the potential energy U on the system become:

$$U = m_{bt}gl_{bt}cos\alpha_2 \quad (4)$$

By using the lagrange equation, dynamic equations on the pitch angle of the rider obtained the following:

$$\left(m_{bt} + m_w + \frac{J_w}{r_{bt}^2}\right)\ddot{x} + m_{bt}l_{bt}(cos\alpha_2)\ddot{\alpha}_2 = 0 \quad (5)$$

$$m_{bt}l_{bt}(cos\alpha_2)\ddot{x} + (m_{bt}l_{bt}^2 + J_{bt})\ddot{\alpha}_2 + (m_{bt}v\dot{\alpha_2} - m_{bt}g)l_{bt}(sin\alpha_2) = \tau \quad (6)$$

From the above two equations can be sought equation for $\ddot{x}$ and $\ddot{\alpha}_2$ as follow:

$$\ddot{x} = \left[-\frac{(m_{bt}l_{bt})^2 g}{den}\right]\alpha_2 + \left[-\frac{m_{bt}l_{bt}}{den}\right]\tau \quad (7)$$

Equations 7 and 8 show a non-linear equations on the pitch angle of the rider. Before doing further analysis, the differential equations are needed to be linearized first. The linearization is done by using an assumption that there is only one point of equilibrium in which vehicle stands balanced. For that, it is assumed that $\alpha 2 = 0$.

$$\ddot{\alpha}_2 = \left[\frac{\left(m_{bt}+m_w+\frac{J_w}{r_{bt}^2}\right)m_{bt}gl_{bt}}{den}\right]\alpha_2 + \left[\frac{m_{bt}+m_w+\frac{J_w}{r_{bt}^2}}{den}\right]\tau \quad (8)$$

$$sin\alpha_2 \approx 0 \quad (9)$$

$$cos\alpha_2 \approx 1 \quad (10)$$

Linear equations on the pitch movement can obtained by inserting equations 9 and 10 into the non-linear equations.

$$\left(m_{bt} + m_w + \frac{J_w}{r_{bt}^2}\right)\ddot{x} + m_{bt}l_{bt}\ddot{\alpha}_2 = 0 \quad (11)$$

$$m_{bt}l_{bt}\ddot{x} + (m_{bt}l_{bt}^2 + J_{bt})\ddot{\alpha}_2 - m_{bt}gl_{bt}\alpha_2 = \tau \quad (12)$$

From equations 11 and 12, the equation $\ddot{x}$ and $\ddot{\alpha}_2$ can be derived as follows:

$$\ddot{x} = \left[-\frac{(m_{bt}l_{bt})^2 g}{den}\right]\alpha_2 + \left[-\frac{m_{bt}l_{bt}}{den}\right]\tau \quad (13)$$

$$\ddot{\alpha}_2 = \left[\frac{\left(m_{bt} + m_w + \frac{J_w}{r_{bt}^2}\right)m_{bt}gl_{bt}}{den}\right]\alpha_2 + \left[\frac{m_{bt}+m_w+\frac{J_w}{r_{bt}^2}}{den}\right]\tau \quad (14)$$

Where:

$$den = \left(m_{bt} + m_w + \frac{J_w}{r_{bt}^2}\right)(m_{bt}l_{bt}^2 + J_{bt}) - (m_{bt}l_{bt})^2 \quad (15)$$

B. *PID Controller*

The balance control of the vehicle uses the descrete PID control since PID is a simple yet dependable controller. This controller is used to minimized error value to obtain the actual value close to the reference value [6]. We modify the rate of error data by using data from gyroscope sensor G_x directly to observe the the rate of error instead of deriving error data with the time dt.

$$u(k) = K_p e(k) + K_i\left(ie(k-1) + \frac{e(k)+e(k-1)}{2}dt\right) + K_d G_x(k) \quad (16)$$

Where

u = Controller output.
e = Error
ie = Integral of error
G_x = Rate of error (from gyroscope sensor)
K_p = Proportional constant gain.
K_i = Integral constant gain.
K_d = Derivative constant gain.

III. SYSTEM DESIGN AND METHODOLOGY

Mechanical systems used in this study is a form of vehicle that has pedals on the right and left side of the wheel. Both of these pedals are located slightly lower than the axle and resting on a holder which is connected directly to the wheel axle. The Pens-Wheel will be enclosed with a casing made of fiberglass. The actuator used in here is 14”, 500 Watt, 60V BLDC hub motor. Fig. 2 shows the design of design PENS-Wheel.

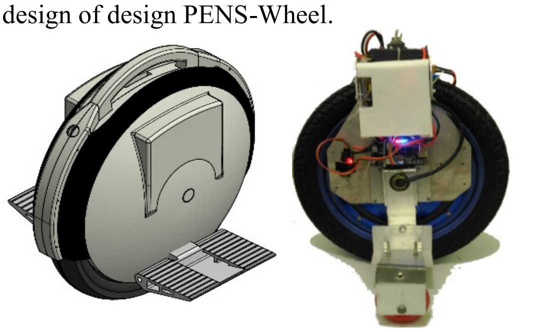

Fig. 2. Mechanical design (left) and realization without the case with support wheels (right) of PENS-Wheel

Fig. 3 shows the diagram of the existing electronics system on the vehicle. Data from the gyroscope and accelerometer sensors contained in the MPU IMU 6050. IMU sensor data is processed using a microcontroller ARM Cortex-M4 STM32F407. Before the data from the gyroscope and accelerometer used in PID control, they need to be filtered by digital filter to reduce the effect of bias and noise, and then fused together to obtain an accurate vehicle’s tilt data. The tilt data thus used by PID controller implemented in the microcontroller to control the motor movement so that the vehicle is able to balance itself in accordance to the given reference angle.

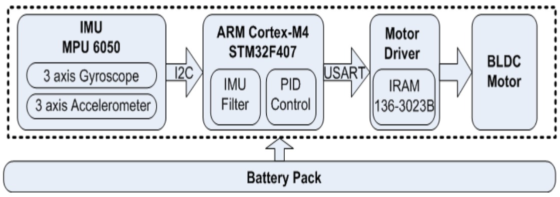

Fig. 3. Diagram Block of Electronics System

A. Balancing PID Control

The PID controller uses data reading from the actual tilt angle of the vehicle as the feedback and then provide a response in the form of the motor movement to reach the balanced condition.

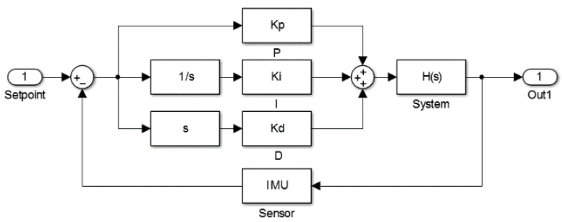

Fig. 4. PID system control

Tilt angle is compared to the reference angle to produce error angle. The parameters used in the PID controller are error value or the difference value output system (process variable) with the expected value (set point), integrated error through time, and rate of error. PID control is used to minimize error value to obtain the actual value close to the reference value [11]. Fig. 4 shows the diagram of PID control used on vehicle.

B. Balancing Control on Simulation

Based on mechanics has made, vehicle has the following specifications:

TABLE 1.
One Wheeled Self Balancing Vehicle Parameters

Symbol	Parameter	Value
m_w	Wheel weight	6 Kg
m_b	Chassis weight	4 Kg
m_m	Average rider’s mass	60 Kg
m_{bt}	Total weight of vehicle	70 Kg
r_{bt}	Wheel radius	0.1778 m
r_c	Chassis radius	0.25 m
h_m	Average human height	1.6 m
l_{bt}	Distance between the wheel axis to the center of mass of the vehicle	0.76 m
g	Earth's gravitational acceleration	9.8 m/s^2

By using the parameters of Table 1, the moment of inertia of vehicle, center of mass, and the matrix state space of the vehicle can be calculated.

- **Inertial Moment**

The moment of inertia on this vehicle is a total inertia of vehicle body, rider, and wheel.

$$J_w = \frac{1}{2} m_w r_{bt}^2 \tag{17}$$

$$J_w = \frac{1}{2}\ x6\ x\ 01778^2 = 0.094\ kg.m^2 \tag{18}$$

Where:

J_w = Inertial moment of wheel (Kg.m^2)
m_w = Wheel weight (Kg)
r_{bt} = Wheel radius (m)

The moment of inertia of the vehicle is the total moment of inertia of the rider and the moment of inertia of the chassis is reduced by inertia of the wheel, since the wheel is inside vehicle. The rider is assumed as a solid rectangular and circular shape to simplify the calculation. The following is a calculation of the inertia of vehicle along with

$$J_{bt} = \frac{1}{12} m_m (h_m^2 + d_m^2) + \frac{1}{2} m_b r_c^2 - J_w \tag{19}$$

$$J_{bt} = \frac{1}{12}\ x\ 60\ x\ (1.6^2 + 0.2^2) + \left(\frac{1}{2}\ x\ 4\ x\ 0.25^2\right) -0.094\ = 13.031\ kg.m^2 \tag{20}$$

Where:

J_{bt} = Total of inertial moment (rider and vehicle) (Kg.m^2)
m_m = Rider mass (Kg)
m_b = Vehicle mass (Kg)
h_m = Rider height (m)
d_m = The width of the rider from the side (m)
r_c = Vehicle radius (m)

- **Center of Mass**

Vertical center of mass is calculated as an equilibrium point of the vehicle measured from the vehicle's bottom, which can be obtained as:

$$COG = \frac{\left(m*\frac{h}{2}+lp*0.55*mp\right)}{mtot} = \frac{(10*0.049 + 1.6*0.55*60)}{70}$$

$$= 0.7612\text{ m} \quad (21)$$

- **State Space Model**

State space model of the dynamic system is a mathematical model that consists of input, output, and state variables of the system. Standard state space model of the linear system consists of four matrices in the form of two equations shown in equations 22 and 23 below.

$$\dot{x} = Ax + Bu \quad (22)$$

$$y = Cx + Du \quad (23)$$

Where:

A = State matrices C = output matrices

B = Input matrices D =direct transmission matrices

To get the state space models of dynamic systems of the vehicle, four state variables of linear equations have been determined. Four vector elements are

$$X = [x \ \ \alpha_2 \ \ \dot{x} \ \ \dot{\alpha}_2]^T \quad (24)$$

Variable of state velocity and the angular velocity are respectively derived from the traveling distance and pitch angle of the vehicle.

$$X_1 = x \quad X_2 = \alpha_2 \quad X_3 = \dot{x} \quad X_4 = \dot{\alpha}_2 \quad (25)$$

The matrices A, B, C, D of the state space are shown in equation 26, 27, 28, and 29.

$$A = \begin{bmatrix} A_{11} & A_{12} & A_{13} & A_{14} \\ A_{21} & A_{22} & A_{23} & A_{24} \\ A_{31} & A_{32} & A_{33} & A_{34} \\ A_{41} & A_{42} & A_{43} & A_{44} \end{bmatrix} \quad (26)$$

$$B = \begin{bmatrix} B_1 \\ B_2 \\ B_3 \\ B_4 \end{bmatrix} \quad (27)$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \quad (28)$$

$$D = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad (29)$$

$$A = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & -\frac{(m_{bt}l_{bt})^2 g}{den} & 0 & 0 \\ 0 & \frac{\left(m_{bt}+m_w+\frac{J_w}{r_{bt}^2}\right)m_{bt}g l_{bt}}{den} & 0 & 0 \end{bmatrix} \quad (30)$$

$$B = \begin{bmatrix} 0 \\ 0 \\ -\frac{m_{bt}l_{bt}}{den} \\ \frac{m_{bt}+m_w+\frac{J_w}{r_{bt}^2}}{den} \end{bmatrix} C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \quad (31)$$

$$D = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad (32)$$

$$den = \left(m_{bt} + m_w + \frac{J_w}{r_{bt}^2}\right)\left(m_{bt}l_{bt}^2 + J_{bt}\right) - (m_{bt}l_{bt})^2$$

$$= 957.54 \quad (33)$$

$$A_{32} = -\frac{(m_{bt}l_{bt})^2 g}{den} = -24.22 \quad (34)$$

$$A_{42} = \frac{\left(m_{bt}+m_w+\frac{J_w}{r_{bt}^2}\right)m_{bt}g l_{bt}}{den} = 36.26 \quad (35)$$

$$B_3 = -\frac{m_{bt}l_{bt}}{den} = -0.05 \quad (36)$$

$$B_4 = \frac{m_{bt}+m_w+\frac{J_w}{r_{bt}^2}}{den} = 0.076 \quad (37)$$

State space model of the vehicle is determined by substituting numerical parameters presented in Table 1 into equation 22 and 23. Equations 38 and 39 are the form of complete state space model of the vehicle that will be used in the simulation.

$$\dot{X} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & -24.22 & 0 & 0 \\ 0 & 36.26 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ \alpha_2 \\ \dot{x} \\ \dot{\alpha}_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ -0.05 \\ 0.076 \end{bmatrix} u \quad (38)$$

$$Y = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ \alpha_2 \\ \dot{x} \\ \dot{\alpha}_2 \end{bmatrix} \quad (39)$$

IV. RESULTS

A. Balancing Response on Simulation

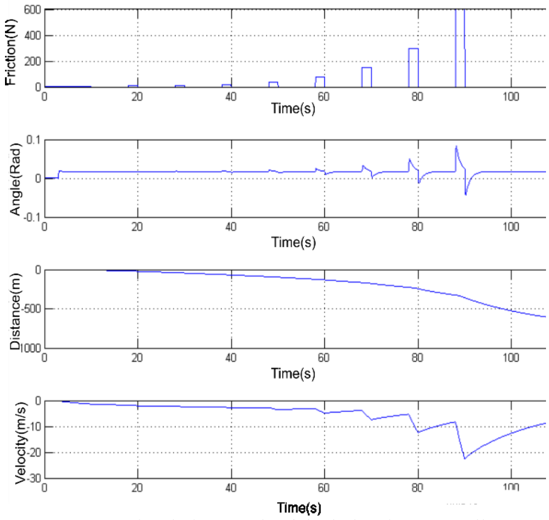

Fig. 5. Simulation result of the balancing controller

The balancing control performance is evaluated by adding the predefined disturbance in the form of frictional forces between the wheels and the floor. In this simulation the tilt setpont is set to 1°, thus the vehicle moves forward. During its movement, a periodically increased frictional forces for 2 seconds long every 8 seconds interval are given, and the system response can be seen from Fig. 5. Vehicle is able to move towards its balancing condition for every given disturbance values. The controller able to withstand disturbance up to 600N, but the tilt angle of the vehicle has risen up to 0.8 radian. The PID constant gain used in this simulation are Kp = 90, Ki = 8, and Kd = 6 respectively.

B. Balancing Response on PENS-Wheel

PID control applied to the vehicle in discrete with time sampling at 5ms, running on STM32F407 microcontroller. Shock disruptions are applied to vehicle to observe the its response when receives disturbances. Because the vehicle is only working on the pitch axis, so vehicle need a support to keep vehicle from falling left or to the right. To get the appropriate PID parameter, in this vehicle use heuristic method. We get the PID parameter values corresponding to the vehicle by the following steps:

- Set the value parameter Kp until the vehicle is able to balanced or able to reach steady state condition. In this experiment with the value of Kp = 100, vehicle start capable to be balanced but the response is slow. After Kp is increased to 115, it capable to stay balanced but still has up to -5° steady state error with small oscillation as depicted at Fig. 6. The higher Kp value causes the higher oscillation magnitude.

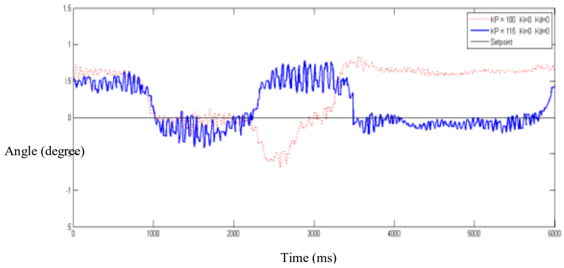

Fig. 6. Proportional controller response

- Kd parameter is added to reduce the effects of overshoot and oscillation in the vehicle. By giving value Kp = 115 and Kd = 20 has been able to reduce oscillations that occur. But due to too large values of Kd, the vehicle unable to reach the setpoint. By giving value Kp = 115 and Kd = 8, it has been able to reduce oscillation and able to reach the setpoint, but the vehicle is still moving. Fig. 7 shows the the response of the PD controller.

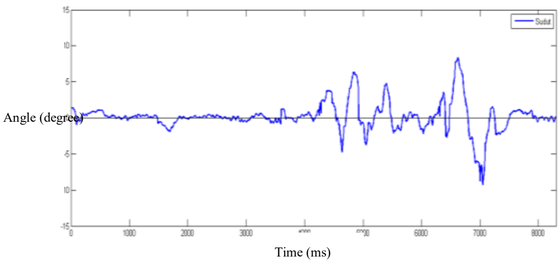

Fig. 7. PD controller response

- For eliminating motion in balanced condition or making the vehicle in stationary, it necessary to add a Ki parameter. Higher the Ki value, the smaller the steady state error, but if it too high it will introduce oscillation. From the experiment we found the optimal Ki value is 6.7. After that, a disturbance is given to the vehicle to evaluate its response. Fig. 8 shows the result of a response using PID control. In the 3rd second, a disturbance is given. The vehicle still capable to balance itself, but it has overshoot of up to ±2.1°, and takes 1 second to reach steady state condition with steady state error at 0.01°.

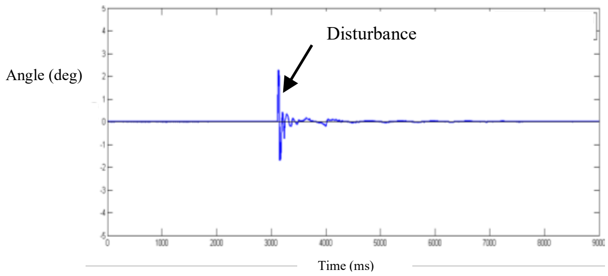

Fig. 8. PID controller response

C. PENS-Wheel Balancing Response, Driven on A Flat Road

This experiment is conducted to observe the response of the balance control on the vehicle with the rider. In this experiment PID control is used with parameter values obtained from the tuning process when vehicle is tested without the rider. Fig. 9 shows the result of experiment on a flat surface with a rider mass at 50kg. Based on Fig. 9, at 4^{th} second the rider starts to ride on the vehicle. Vehicle capable to maintain the position at the setpoint in its upright position, but an oscillation occurs up to -9° when the rider starts to thrust the vehicle forward from the stop condition.

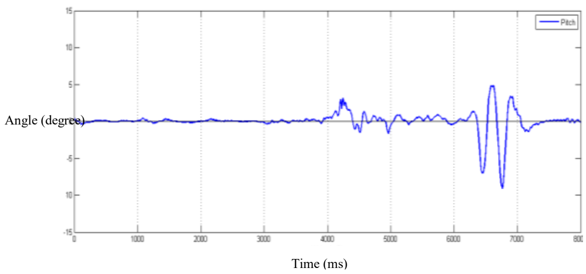

Fig. 9. Response with rider mass at 50kg

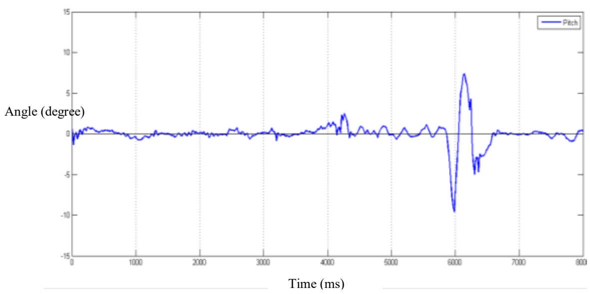

Fig. 10. Response with rider mass at 72kg

Fig. 10 shows the result of experiment on a flat surface with the mass of rider is 72kg. The effect of the change in the rider mass resulting more tilt angle overshoot up to -10°, but in a very short time (less than 200ms) that occur when the rider starts to thrust the vehicle. But after 500ms, the vehicle capable to stay balanced again with a small oscillation.

D. PENS-Wheel Balancing Response, Driven on The Inclined Road

This experiment aims to test the strength of a vehicle to run on an inclined road. Fig. 11 is a response to vehicle on an incline road at 15° slope with the rider mass at 72kg.

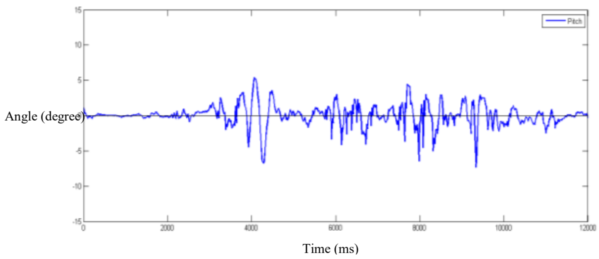

Fig. 11. Response with rider mass at 72kg, on the inclined road

From Fig. 11, it can be seen that the vehicle still capable to balance itself during the inclining road. An oscillation on the tilt angle is occur up to -6° which is affected by the oscillation of the slower torque response generated by the motor in the increase of the load.

E. PENS-Wheel Balancing Response, Driven on A Bumpy Road

This experiment aims to observe the response of the PID control on the vehicle as it passes through an non flat surface.

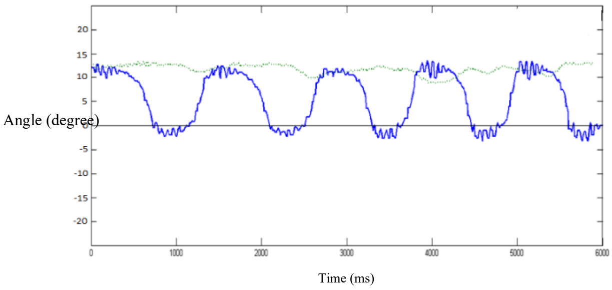

Fig. 12. Response with rider mass at 50kg, on a bumpy road

During this experiment, the vehicle still capable to balance itself even though it has gone through a bumpy road as depicted at Fig 12. In this experiment, overshoots that occur up to 12.5° due to the road bumps at around 5cm height.

V. CONCLUSIONS

From the experiments, we conclude that:

1. The proposed PID control running on STM32F407 microcontroller (in discrete model with 5ms sampling time) is capable to make the vehicle to stay balanced without and with rider using the same PID parameters at Kp = 115, Ki = 6.7, and Kd = 8.
2. The proposed controller has an acceptable result to make the vehicle stay balanced while running on a flat surface, bumpy roads and the inclining road with a maximum slope of 15° with up to 72kg rider's mass.
3. Rider's mass variation still has impact on the response of the proposed PID controller, thus it will be considered on the future work.

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