Heuristic analysis

This brief report describes what kinf of heuristics I used in this project, their evaluation results in `tournament.py` and overall desicion of taking one for final submission.

Heuristics used

`score_moves`

First and very obvious heuristic is `score_moves` which just counts available moves of the player and the opponent. This one is used already in `improved_score` function by default, so I took an idea from lections to not just use raw differenct, but add some bigger multiplier for opponent moves: `score = player_moves - opponent_scaling_factor * opponent_moves`. Values used for tests are `1`, `1.5` and `2`.

`score_center`

Second heuristic `score_center` is based on the idea that cells closer to center are more preferable than edge ones -- there is potentially more space to go since no board-limits exist. I used manhattan distance from the very central cell to count the distance. Because best position gives 0 distance and worst gives 6 and we want heuristic to return higher values for better moves, I substract distance from maximum value. Thus, worst value is 0 and the best is 6. As before, I added scaling factor for opponent moves. Noteworthy is that ***I used scaling for opponent moves for every custom heuristics***.

`score_freecells`

Third idea is to count all available free cells in a radius of 2 (since knight move range). If given cell has more open spaces around, consider it better. This is a bit correlated with number of moves, but is different. Consider a cell which has only 1 move available (e.g. one near to the corner). But there is about 5 other cells available so within this little space there might be more possible moves. Another cell can have 2 moves and 2 free cells so after moving once game might be also over. This seems better to combine number of moves and number of space. I make it below.

`score_moves_center`

This one is a combination of `score_moves` and `score_center`. Scaling factor applied for both heuristics independently and with same value, i.e.

```
def score_moves_center(game, player, opponent_scaling_factor=1):
    if game.is_loser(player):
```

```
        return float("-inf")

    if game.is_winner(player):
        return float("inf")

    move_score = score_moves(game, player, opponent_scaling_factor)
    center_score = score_center(game, player, opponent_scaling_factor)
    score = move_score + center_score
    return score
```

score_moves_freecells

Combination of `score_moves` and `score_freecells`. Scaling factor policy is the same.

score_moves_center_freecells

Ultimate combination of `score_moves`, `score_center` and `score_freecells` Scaling factor policy is the same.

score_combined

Ultimate combination of `score_moves`, `score_center` and `score_freecells`. Scaling factor policy is based on previous evaluations of listed 3 heuristics. Best values used:

- x1.5 for `score_moves` -- 82.86% win ratio (see table below)
- x2 for `score_center` -- 75.00% win ratio (see table below)
- default for `score_freecells` -- 77.86% win ratio (see table below)

Evaluation results

Default column means `player_score - opponent_score` for custom heuristics. Scaling factors 1.5 and 2 noted in corresponding columns ***x1.5*** and ***x2***. Raw results of `tournament.py` represented in Appendix section.

Heuristic	Default	x1.5	x2	Comments
`ID_improved` -- baseline	70.00%			
`score_moves`	72.86%	82.86%	77.14%	82.86%: minimal is 14-6 vs AB_Improved
`score_center`	72.14%	72.14%	75.00%	Not that much good heuristic
`score_freecells`	77.86%	75.71%	75.71%	Almost no difference with scaling

Heuristic	Default	x1.5	x2	Comments
`score_moves_center`	70.00%	75.00%	80.71%	80.71%: minimal is 12-8 vs AB_Improved
`score_moves_freecells`	72.86%	73.57%	72.14%	Kinda useless combination :(
`score_moves_center_freecells`	80.00%	82.14%	80.71%	Stablest heuristic -- always about 80%!
`score_combined`	72.86%			Much worse than separate ones

Every custom heuristic is better than baseline, but because of stochastic nature I consider results less than 80% as poor ones. In only 5 cases 80% theshold was passed, 3 times of them by last heuristic -- `score_moves_center_freecells`. It seems to be best one because of its stability.

Resulting heuristic and recommendation

Final heuristic `score_moves_center_freecells` with scaling factor 1.5 is choosen for submission based on its results. Highest value is given by simple `score_moves`, but it is much less stable with different scaling factors. This high score might be caused by randomness, but `score_moves_center_freecells` seems to be more stable.

Generally, here are specific reasons for taking `score_moves_center_freecells` as resulting heuristic:

- Best average winning rate among other heuristics -- more than 80%
- Results of `score_moves_center_freecells` show that underlying idea of focusing on central cells with more open space and more moves around gives better results almost always, regardless to scaling factor. Other heuristic seem to be less robust and dependent on scaling factor.
- Underlying calculations of `score_moves_center_freecells` include only simple operations, which makes calculations easy and allow deeper tree search
- Independent internal sub-heuristics usage (see code snippet above) allows easy parallelization for modern multi-core systems.

Recomendations about evaluation functions usage:

- combine different approaches together, this provides better results as for `score_moves_center_freecells`;
- not implemented here, but is good to test differerent weighting coefficients for such heuristics within a combination;
- based on results, it seems that pushing to choose central cells (as with `score_center`) is useful, other evaluation functions are likely to try.

- without scaling factors, `score_freecells` is performing better than `score_moves` -- so counting open area is also important. More sophisticated functions, which for example can count open cells within Γ-distance from each other, should be tried.

Appendix -- Raw evaluation results

```
**************************
 Evaluating: ID_Improved
**************************

Playing Matches:
----------
  Match 1: ID_Improved vs   Random        Result: 16 to 4
  Match 2: ID_Improved vs   MM_Null       Result: 16 to 4
  Match 3: ID_Improved vs   MM_Open       Result: 17 to 3
  Match 4: ID_Improved vs MM_Improved     Result: 11 to 9
  Match 5: ID_Improved vs   AB_Null       Result: 15 to 5
  Match 6: ID_Improved vs   AB_Open       Result: 13 to 7
  Match 7: ID_Improved vs AB_Improved     Result: 10 to 10


Results:
----------
ID_Improved         70.00%

**************************
Evaluating: Student: score_moves
**************************

Playing Matches:
----------
  Match 1: Student: score_moves vs   Random        Result: 19 to 1
  Match 2: Student: score_moves vs   MM_Null       Result: 16 to 4
  Match 3: Student: score_moves vs   MM_Open       Result: 14 to 6
  Match 4: Student: score_moves vs MM_Improved     Result: 11 to 9
  Match 5: Student: score_moves vs   AB_Null       Result: 14 to 6
  Match 6: Student: score_moves vs   AB_Open       Result: 15 to 5
  Match 7: Student: score_moves vs AB_Improved     Result: 13 to 7


Results:
----------
Student: score_moves     72.86%

**************************
Evaluating: Student: score_moves p-1.5o
**************************

Playing Matches:
----------
  Match 1: Student: score_moves p-1.5o vs   Random        Result: 17 to 3
```

```
  Match 2: Student: score_moves p-1.5o vs   MM_Null       Result: 16 to 4
  Match 3: Student: score_moves p-1.5o vs   MM_Open       Result: 19 to 1
  Match 4: Student: score_moves p-1.5o vs MM_Improved     Result: 16 to 4
  Match 5: Student: score_moves p-1.5o vs   AB_Null       Result: 17 to 3
  Match 6: Student: score_moves p-1.5o vs   AB_Open       Result: 17 to 3
  Match 7: Student: score_moves p-1.5o vs AB_Improved     Result: 14 to 6


Results:
----------
Student: score_moves p-1.5o     82.86%

*************************
Evaluating: Student: score_moves p-2o
*************************

Playing Matches:
----------
  Match 1: Student: score_moves p-2o vs   Random        Result: 20 to 0
  Match 2: Student: score_moves p-2o vs   MM_Null       Result: 16 to 4
  Match 3: Student: score_moves p-2o vs   MM_Open       Result: 16 to 4
  Match 4: Student: score_moves p-2o vs MM_Improved     Result: 14 to 6
  Match 5: Student: score_moves p-2o vs   AB_Null       Result: 16 to 4
  Match 6: Student: score_moves p-2o vs   AB_Open       Result: 13 to 7
  Match 7: Student: score_moves p-2o vs AB_Improved     Result: 13 to 7


Results:
----------
Student: score_moves p-2o     77.14%

*************************
Evaluating: Student: score_center
*************************

Playing Matches:
----------
  Match 1: Student: score_center vs   Random        Result: 19 to 1
  Match 2: Student: score_center vs   MM_Null       Result: 18 to 2
  Match 3: Student: score_center vs   MM_Open       Result: 13 to 7
  Match 4: Student: score_center vs MM_Improved     Result: 10 to 10
  Match 5: Student: score_center vs   AB_Null       Result: 18 to 2
  Match 6: Student: score_center vs   AB_Open       Result: 15 to 5
  Match 7: Student: score_center vs AB_Improved     Result: 8 to 12


Results:
----------
Student: score_center     72.14%

*************************
Evaluating: Student: score_center p-1.5o
```

```
*************************

Playing Matches:
----------
  Match 1: Student: score_center p-1.5o vs   Random        Result: 16 to 4
  Match 2: Student: score_center p-1.5o vs   MM_Null       Result: 17 to 3
  Match 3: Student: score_center p-1.5o vs   MM_Open       Result: 16 to 4
  Match 4: Student: score_center p-1.5o vs MM_Improved     Result: 12 to 8
  Match 5: Student: score_center p-1.5o vs   AB_Null       Result: 15 to 5
  Match 6: Student: score_center p-1.5o vs   AB_Open       Result: 15 to 5
  Match 7: Student: score_center p-1.5o vs AB_Improved     Result: 10 to 10


Results:
----------
Student: score_center p-1.5o     72.14%

*************************
Evaluating: Student: score_center p-2o
*************************

Playing Matches:
----------
  Match 1: Student: score_center p-2o vs   Random        Result: 17 to 3
  Match 2: Student: score_center p-2o vs   MM_Null       Result: 19 to 1
  Match 3: Student: score_center p-2o vs   MM_Open       Result: 16 to 4
  Match 4: Student: score_center p-2o vs MM_Improved     Result: 12 to 8
  Match 5: Student: score_center p-2o vs   AB_Null       Result: 15 to 5
  Match 6: Student: score_center p-2o vs   AB_Open       Result: 11 to 9
  Match 7: Student: score_center p-2o vs AB_Improved     Result: 15 to 5


Results:
----------
Student: score_center p-2o     75.00%

*************************
Evaluating: Student: score_freecells
*************************

Playing Matches:
----------
  Match 1: Student: score_freecells vs   Random        Result: 19 to 1
  Match 2: Student: score_freecells vs   MM_Null       Result: 19 to 1
  Match 3: Student: score_freecells vs   MM_Open       Result: 14 to 6
  Match 4: Student: score_freecells vs MM_Improved     Result: 14 to 6
  Match 5: Student: score_freecells vs   AB_Null       Result: 15 to 5
  Match 6: Student: score_freecells vs   AB_Open       Result: 16 to 4
  Match 7: Student: score_freecells vs AB_Improved     Result: 12 to 8


Results:
```

```
----------
Student: score_freecells     77.86%

*************************
Evaluating: Student: score_freecells p-1.5o
*************************

Playing Matches:
----------
  Match 1: Student: score_freecells p-1.5o vs   Random        Result: 19 to 1
  Match 2: Student: score_freecells p-1.5o vs   MM_Null       Result: 20 to 0
  Match 3: Student: score_freecells p-1.5o vs   MM_Open       Result: 13 to 7
  Match 4: Student: score_freecells p-1.5o vs MM_Improved     Result: 14 to 6
  Match 5: Student: score_freecells p-1.5o vs   AB_Null       Result: 17 to 3
  Match 6: Student: score_freecells p-1.5o vs   AB_Open       Result: 10 to 10
  Match 7: Student: score_freecells p-1.5o vs AB_Improved     Result: 13 to 7


Results:
----------
Student: score_freecells p-1.5o     75.71%

*************************
Evaluating: Student: score_freecells p-2o
*************************

Playing Matches:
----------
  Match 1: Student: score_freecells p-2o vs   Random        Result: 19 to 1
  Match 2: Student: score_freecells p-2o vs   MM_Null       Result: 19 to 1
  Match 3: Student: score_freecells p-2o vs   MM_Open       Result: 14 to 6
  Match 4: Student: score_freecells p-2o vs MM_Improved     Result: 12 to 8
  Match 5: Student: score_freecells p-2o vs   AB_Null       Result: 18 to 2
  Match 6: Student: score_freecells p-2o vs   AB_Open       Result: 13 to 7
  Match 7: Student: score_freecells p-2o vs AB_Improved     Result: 11 to 9


Results:
----------
Student: score_freecells p-2o     75.71%

*************************
Evaluating: Student: score_moves_center
*************************

Playing Matches:
----------
  Match 1: Student: score_moves_center vs   Random        Result: 19 to 1
  Match 2: Student: score_moves_center vs   MM_Null       Result: 17 to 3
  Match 3: Student: score_moves_center vs   MM_Open       Result: 12 to 8
  Match 4: Student: score_moves_center vs MM_Improved     Result: 14 to 6
  Match 5: Student: score_moves_center vs   AB_Null       Result: 14 to 6
```

```
  Match 6: Student: score_moves_center vs   AB_Open       Result: 10 to 10
  Match 7: Student: score_moves_center vs AB_Improved     Result: 12 to 8


Results:
----------
Student: score_moves_center     70.00%

*************************
Evaluating: Student: score_moves_center p-1.5o
*************************

Playing Matches:
----------
  Match 1: Student: score_moves_center p-1.5o vs   Random        Result: 18 to 2
  Match 2: Student: score_moves_center p-1.5o vs   MM_Null       Result: 17 to 3
  Match 3: Student: score_moves_center p-1.5o vs   MM_Open       Result: 14 to 6
  Match 4: Student: score_moves_center p-1.5o vs MM_Improved     Result: 13 to 7
  Match 5: Student: score_moves_center p-1.5o vs   AB_Null       Result: 18 to 2
  Match 6: Student: score_moves_center p-1.5o vs   AB_Open       Result: 14 to 6
  Match 7: Student: score_moves_center p-1.5o vs AB_Improved     Result: 11 to 9


Results:
----------
Student: score_moves_center p-1.5o     75.00%

*************************
Evaluating: Student: score_moves_center p-2o
*************************

Playing Matches:
----------
  Match 1: Student: score_moves_center p-2o vs   Random        Result: 20 to 0
  Match 2: Student: score_moves_center p-2o vs   MM_Null       Result: 16 to 4
  Match 3: Student: score_moves_center p-2o vs   MM_Open       Result: 17 to 3
  Match 4: Student: score_moves_center p-2o vs MM_Improved     Result: 14 to 6
  Match 5: Student: score_moves_center p-2o vs   AB_Null       Result: 18 to 2
  Match 6: Student: score_moves_center p-2o vs   AB_Open       Result: 16 to 4
  Match 7: Student: score_moves_center p-2o vs AB_Improved     Result: 12 to 8


Results:
----------
Student: score_moves_center p-2o     80.71%

*************************
Evaluating: Student: score_moves_freecells
*************************

Playing Matches:
----------
```

```
  Match 1: Student: score_moves_freecells vs   Random        Result: 19 to 1
  Match 2: Student: score_moves_freecells vs   MM_Null       Result: 16 to 4
  Match 3: Student: score_moves_freecells vs   MM_Open       Result: 13 to 7
  Match 4: Student: score_moves_freecells vs MM_Improved     Result: 17 to 3
  Match 5: Student: score_moves_freecells vs   AB_Null       Result: 15 to 5
  Match 6: Student: score_moves_freecells vs   AB_Open       Result: 12 to 8
  Match 7: Student: score_moves_freecells vs AB_Improved     Result: 10 to 10


Results:
----------
Student: score_moves_freecells     72.86%

*************************
Evaluating: Student: score_moves_freecells p-1.5o
*************************

Playing Matches:
----------
  Match 1: Student: score_moves_freecells p-1.5o vs   Random        Result: 17 to
3
  Match 2: Student: score_moves_freecells p-1.5o vs   MM_Null       Result: 16 to
4
  Match 3: Student: score_moves_freecells p-1.5o vs   MM_Open       Result: 17 to
3
  Match 4: Student: score_moves_freecells p-1.5o vs MM_Improved     Result: 14 to
6
  Match 5: Student: score_moves_freecells p-1.5o vs   AB_Null       Result: 14 to
6
  Match 6: Student: score_moves_freecells p-1.5o vs   AB_Open       Result: 13 to
7
  Match 7: Student: score_moves_freecells p-1.5o vs AB_Improved     Result: 12 to
8


Results:
----------
Student: score_moves_freecells p-1.5o     73.57%

*************************
Evaluating: Student: score_moves_freecells p-2o
*************************

Playing Matches:
----------
  Match 1: Student: score_moves_freecells p-2o vs   Random        Result: 18 to 2
  Match 2: Student: score_moves_freecells p-2o vs   MM_Null       Result: 16 to 4
  Match 3: Student: score_moves_freecells p-2o vs   MM_Open       Result: 13 to 7
  Match 4: Student: score_moves_freecells p-2o vs MM_Improved     Result: 13 to 7
  Match 5: Student: score_moves_freecells p-2o vs   AB_Null       Result: 16 to 4
  Match 6: Student: score_moves_freecells p-2o vs   AB_Open       Result: 13 to 7
  Match 7: Student: score_moves_freecells p-2o vs AB_Improved     Result: 12 to 8
```

```
Results:
----------
Student: score_moves_freecells p-2o     72.14%

*************************
Evaluating: Student: score_moves_center_freecells
*************************

Playing Matches:
----------
  Match 1: Student: score_moves_center_freecells vs   Random        Result: 19 to
1
  Match 2: Student: score_moves_center_freecells vs   MM_Null       Result: 20 to
0
  Match 3: Student: score_moves_center_freecells vs   MM_Open       Result: 12 to
8
  Match 4: Student: score_moves_center_freecells vs MM_Improved     Result: 15 to
5
  Match 5: Student: score_moves_center_freecells vs   AB_Null       Result: 18 to
2
  Match 6: Student: score_moves_center_freecells vs   AB_Open       Result: 15 to
5
  Match 7: Student: score_moves_center_freecells vs AB_Improved     Result: 13 to
7


Results:
----------
Student: score_moves_center_freecells     80.00%

*************************
Evaluating: Student: score_moves_center_freecells p-1.5o
*************************

Playing Matches:
----------
  Match 1: Student: score_moves_center_freecells p-1.5o vs   Random        Result:
 19 to 1
  Match 2: Student: score_moves_center_freecells p-1.5o vs   MM_Null       Result:
 18 to 2
  Match 3: Student: score_moves_center_freecells p-1.5o vs   MM_Open       Result:
 17 to 3
  Match 4: Student: score_moves_center_freecells p-1.5o vs MM_Improved     Result:
 13 to 7
  Match 5: Student: score_moves_center_freecells p-1.5o vs   AB_Null       Result:
 16 to 4
  Match 6: Student: score_moves_center_freecells p-1.5o vs   AB_Open       Result:
 17 to 3
  Match 7: Student: score_moves_center_freecells p-1.5o vs AB_Improved     Result:
 15 to 5
```

```
Results:
----------
Student: score_moves_center_freecells p-1.5o     82.14%

*************************
Evaluating: Student: score_moves_center_freecells p-2o
*************************

Playing Matches:
----------
  Match 1: Student: score_moves_center_freecells p-2o vs   Random        Result: 1
8 to 2
  Match 2: Student: score_moves_center_freecells p-2o vs   MM_Null       Result: 1
8 to 2
  Match 3: Student: score_moves_center_freecells p-2o vs   MM_Open       Result: 1
5 to 5
  Match 4: Student: score_moves_center_freecells p-2o vs MM_Improved     Result: 1
3 to 7
  Match 5: Student: score_moves_center_freecells p-2o vs   AB_Null       Result: 1
9 to 1
  Match 6: Student: score_moves_center_freecells p-2o vs   AB_Open       Result: 1
5 to 5
  Match 7: Student: score_moves_center_freecells p-2o vs AB_Improved     Result: 1
5 to 5


Results:
----------
Student: score_moves_center_freecells p-2o     80.71%

*************************
   Evaluating: Student: score_combined
*************************

Playing Matches:
----------
  Match 1:   Student   vs   Random        Result: 19 to 1
  Match 2:   Student   vs   MM_Null       Result: 17 to 3
  Match 3:   Student   vs   MM_Open       Result: 15 to 5
  Match 4:   Student   vs MM_Improved     Result: 12 to 8
  Match 5:   Student   vs   AB_Null       Result: 12 to 8
  Match 6:   Student   vs   AB_Open       Result: 15 to 5
  Match 7:   Student   vs AB_Improved     Result: 12 to 8


Results:
----------
Student: score_combined     72.86%
```