Real Analysis

learning note For reading translation

我真的不懂忧郁

Delft University of Technology

Real Analysis

learning note For reading translation

by

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Project Duration: Month, Year - Month, Year

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Preface

A preface...

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Summary

A summary...

目录

Nomenclature

If a nomenclature is required, a simple template can be found below for convenience. Feel free to use, adapt or completely remove.

Abbreviations

Abbreviation	Definition
ISA	International Standard Atmosphere
...	

Symbols

Symbol	Definition	Unit
V	Velocity	[m/s]
...		
ρ	Density	[kg/m^3]
...		

Chapter 1

Folland. Measure

1.1. Exercises.1

Question 1: . *A family of sets $\mathcal{R} \subset \mathcal{P}(X)$ is call **Ring**, if it is closed under unions and differences(i.e. if $E_1, \cdots, E_n \in \mathcal{R}$,then $\bigcup_{j=1}^{n} E_j \in \mathcal{R}$, if $E, F \in \mathcal{R}$, then $E/F \in \mathcal{R}$)*[1]

1. *Rings(resp. σ-rings) are closed under finite(resp. countable) intersections;*
2. *If $\mathcal{R}$ is a Ring, then $\mathcal{R}$ is a **Algebra**[2] iff $X \in \mathcal{R}$;*
3. *If $\mathcal{R}$ is a $\sigma - ring$, then $\{E \subset X : E \in \mathcal{R}$ or $E^c \in \mathcal{R}\}$ is a $\sigma - algebra$;*
4. *If $\mathcal{R}$ is a $\sigma - ring$, then $\{E \subset X : E \cap F \in \mathcal{R}\ for\ all\ F \in \mathcal{R}\}$ is a $\sigma - algebra$;*

***proof*:** 1. $\forall E_1, E_2 \in \mathcal{R}$, $E_1 \cap E_2 = E_1/(E_1/E_2) \in \mathcal{R}$ *by closed under differences ;*

2. *if $X \in \mathcal{R}$, $\forall E \subset X, E \in \mathcal{R}$, $X/E = E^c \in \mathcal{R}$ by closed under differences;If $\mathcal{R}$ is a Algebra, the conclusion natually.*
3. *Let's $\mathcal{S} := \{E \subset X : E \in \mathcal{R}$ or $E^c \in \mathcal{R}\}$, if $E \in \mathcal{S}$, $(E^c)^c \in \mathcal{S} \Rightarrow E^c \in \mathcal{S}$, and $X = E \cup E^c \in \mathcal{S}$ by closed under unions;*
4. *Let's $\mathcal{S} := \{E \subset X : E \cap F \in \mathcal{R}\ for\ all\ F \in \mathcal{R}\}$,*
   (a) *Assume $E \in \mathcal{S}$, we prove $E^c \in \mathcal{S}$.*
   if $E^c \in \mathcal{S}$,then $\forall F \in \mathcal{R}$, $E^c \cap F \in \mathcal{R} \Rightarrow F/(E \cap F) = \mathcal{R}$, then $E \cap F \in \mathcal{R}$ by closed under differences,so $E \in \mathcal{S}$;
   (b) *prove closed of unions;*

[1] If a Ring countable for unions,then be calledσ-**Ring**

[2] A family set $\mathcal{A} \in \mathcal{P}(X)$ has proporty as : $(1) X \in \mathcal{A}$, $(2) E \in \mathcal{A}, E^c \in \mathcal{A}$, $(3) \forall E_i \in \mathcal{R}, \bigcup_{i=0}^{n} E_i \in \mathcal{R}$

$\forall E_1, E_2, \cdots, E_n \in \mathcal{S}$, If $\bigcup_{i=1}^n E_i \in \mathcal{S}$,then

$$(\bigcup_{i=1}^{n} E_i) \cap F = \bigcup_{i=1}^{n} (E_i \cap F) \in \mathcal{R}, \ \ \forall F \in \mathcal{R} \tag{1.1}$$

So $(\bigcup_{i=1}^n E_i) \in \mathcal{S}$

It is trivial to verify that the intersection of any family of σ-algebra on X is again a σ-algebra. It follows that if $\mathcal{E}$ is any subset of $\mathcal{P}(X)$, there is a **unique smallest σ-algebra $\mathcal{M}(\mathcal{E})$ contains** $\mathcal{E}$, $\mathcal{M}(\mathcal{E})$ is called the **σ-algebra generated by** $\mathcal{E}$.

If X is any metric space, or more generally any topological space, the σ-algebra generated by the family of open sets (or, equivalently, by the family of closed sets in X) in X is called the ***Borel σ-Algebra*** amd is denoted by $\mathcal{B}_X$.Its members are called ***Borel sets***.

Question 2: *$\mathcal{B}_{\mathbb{R}}$ is generated by each of the following:*

1. *the open intervals: $\mathcal{E}_1 = \{(a,b) : a < b\}$,*
2. *the closed intervals: $\mathcal{E}_2 = \{[a,b] : a < b\}$,*
3. *the half-open intervals: $\mathcal{E}_3 = \{(a,b] : a < b\}$ or $\mathcal{E}_4 = \{[a,b) : a < b\}$,*
4. *the open rays: $\mathcal{E}_5 = \{(a,\infty) : a \in \mathbb{R}\}$ or $\mathcal{E}_6 = \{(-\infty,a) : a \in \mathbb{R}\}$,*
5. *the closed rays: $\mathcal{E}_7 = \{[a,\infty) : a \in \mathbb{R}\}$ or $\mathcal{E}_8 = \{(-\infty,a] : a \in \mathbb{R}\}$,*

***proof*:** *Just prove*

1. *prove $M(\mathcal{E}_j) \subset \mathcal{B}_{\mathbb{R}}$;*

   $\mathcal{M}(\mathcal{E}_j) \subset \mathcal{B}_{\mathbb{R}}$ is natually for $j = 1,2$ by definition. for $j = 3,4$,the element of $\mathcal{E}_3$ and $\mathcal{E}_4$ are G_δ sets[a] , for example,

   $$(a,b] = \bigcap_{1}^{\infty} (a, b + \frac{1}{n}) \tag{1.2}$$

   So all there are Borel sets, so $\mathcal{M}(\mathcal{E}) \subset \mathcal{B}_{\mathbb{R}}$.

2. *prove $\mathcal{B}_{\mathbb{R}} \subset M(\mathcal{E}_j)$;*

   Every open set in $\mathbb{R}$ is a countable union of open intervals, so $\mathcal{B}_{\mathbb{R}} \subset \mathcal{M}(\mathcal{E}_1)$, for $j \geqslant 2$ can now be established by showing that all open intervals lie in $\mathcal{M}(\mathcal{E}_j)$, note that $\mathcal{M}(\mathcal{E}_|)$ is σ-algebra, for example

   $$(a,b) = \bigcup_{1}^{\infty} [a + \frac{1}{n}, b - \frac{1}{n}] \in \mathcal{M}(\mathcal{E}_2) \tag{1.3}$$

   $$(a,b) = \bigcap_{1}^{\infty} (a, b + \frac{1}{n}] \in \mathcal{M}(\mathcal{E}_3) \tag{1.4}$$

[a] A countable intersection of open sets is called a G_δ **sets**;a countable unions of closed sets is called an F_δ **sets**.

Question 3: *Let $\mathcal{M}$ be an infinite σ-algebra.*

1. *$\mathcal{M}$ contains an infinite sequence disjoint sets,*
2. *$card(\mathcal{M}) \geqslant c$.*

***proof*:** *Assume $\mathcal{M} \subset \mathcal{P}(X)$ is a σ-algebra*

1.

Question 4: *An algebra $\mathcal{A}$ is a σ-algebra iff $\mathcal{A}$ is closed under countable increasing unions(i.e. if $\{E_j\}_1^\infty \subset \mathcal{A}$ and $E_1 \subset E_2 \subset \cdots$, $then \bigcup_1^\infty E_j \in \mathcal{A}$)*

***proof*:**

Question 5: *If $\mathcal{M}$ is the σ-algebra generated by $\mathcal{E}$, then $\mathcal{M}$ is the union of the σ-algebras generated by $\mathcal{F}$ as $\mathcal{F}$ ranges over all countable subsets of $\mathcal{E}$.(Hint: Show that the latter object is a σ-algebra).*

***proof*:**

References

[1] I. Surname, I. Surname, and I. Surname. “The Title of the Article”. In: *The Title of the Journal* 1.2 (2000), pp. 123–456.

Chapter A

Source Code Example

Adding source code to your report/thesis is supported with the package `listings`*. An example can be found below. Files can be added using* `\lstinputlisting[language=<language>]{<filename>}`.

```python
"""
ISA Calculator: import the function, specify the height and it will return a
list in the following format: [Temperature,Density,Pressure,Speed of Sound].
Note that there is no check to see if the maximum altitude is reached.
"""

import math
g0 = 9.80665
R = 287.0
layer1 = [0, 288.15, 101325.0]
alt = [0,11000,20000,32000,47000,51000,71000,86000]
a = [-.0065,0,.0010,.0028,0,-.0028,-.0020]

def atmosphere(h):
    for i in range(0,len(alt)-1):
        if h >= alt[i]:
            layer0 = layer1[:]
            layer1[0] = min(h,alt[i+1])
            if a[i] != 0:
                layer1[1] = layer0[1] + a[i]*(layer1[0]-layer0[0])
                layer1[2] = layer0[2] * (layer1[1]/layer0[1])**(-g0/(a[i]*R))
            else:
                layer1[2] = layer0[2]*math.exp((-g0/(R*layer1[1]))*(layer1[0]-layer0[0]))
    return [layer1[1],layer1[2]/(R*layer1[1]),layer1[2],math.sqrt(1.4*R*layer1[1])]
```

Chapter B

Task Division Example

If a task division is required, a simple template can be found below for convenience. Feel free to use, adapt or completely remove.

表 **B.1:** Distribution of the workload

	Task	Student Name(s)
	Summary	
Chapter 1	Introduction	
Chapter 2		
Chapter 3		
Chapter *		
Chapter *	Conclusion	
	Editors	
	CAD and Figures	
	Document Design and Layout	