1. Note R_+ means $x > 0$

Show $\exists x \in R_+ \ni x^2 \leq 1$

Proof: Assume $\forall x \in R_+ \ni x^2 > 1$

Let $x = 1$, then $(1)^2 > 1$

$$1 > 1$$
$$\rightarrow\leftarrow$$
$$1 \leq 1$$

$\therefore\ \forall x \in R_+,\ x^2 > 1$ is false

$\therefore$ By T.T. $\exists x \in R_+ \ni x^2 \leq 1$ is true

3. Show $\exists x \in N \ni \dfrac{4}{x} \in N$

Proof: Assume $\forall x \in N \ni \dfrac{4}{x} \notin N$

Let $x = 1$, then $\dfrac{4}{1} \notin N$

$$4 \notin N$$
$$\rightarrow\leftarrow$$
$$4 \in N$$

$\therefore\ \forall x \in N,\ \dfrac{4}{x} \notin N$ is false

$\therefore$ By T.T. $\exists x \in N \ni \dfrac{4}{x} \in N$ is true

5. Show $\forall\, x \in R \ni$ if $x > 2$, then $x^2 - 4 \geq 0$

Proof: Assume $\exists\, x \in R \ni x > 2$ and $x^2 - 4 < 0$

$\exists\, x \in R \ni x > 2$ and $(x-2)(x+2) < 0$ + | - | +

$\exists\, x \in R \ni x > 2$ and $-2 < x < 2$ -2   2

$\rightarrow\leftarrow$

$\nexists\, x \in R \ni x > 2$ and $-2 < x < 2$

$\therefore \exists\, x \in R \ni x > 2$ and $x^2 - 4 < 0$ is false

$\therefore$ By T.T. $\forall\, x \in R \ni$ if $x > 2$, then $x^2 - 4 \geq 0$ is true

7. Show $\forall\, x \in N,\ x^2 \geq 1$

Proof: Assume $\exists\, x \in N \ni x^2 < 1$

$\exists\, x \in N \ni x^2 - 1 < 0$

$\exists\, x \in N \ni (x-1)(x+1) < 0$ + | - | +

$\exists\, x \in N \ni -1 < x < 1$ -1   1

$\rightarrow\leftarrow$

$\nexists\, x \in N \ni -1 < x < 1$

$\therefore \exists\, x \in N \ni x^2 < 1$ is false

$\therefore$ By T.T. $\forall\, x \in N,\ x^2 \geq 1$ is true

9. Show $\exists x \in N \ni x^2 \geq 0$

Proof: Assume $\forall x \in N, x^2 < 0$

Let x = 1, then $(1)^2 < 0$

$1 < 0$

$\rightarrow\leftarrow$

$1 \geq 0$

$\therefore \forall x \in N, x^2 < 0$ is false

$\therefore$ By T.T. $\exists x \in N \ni x^2 \geq 0$ is true

11. Show if x + 2 is even, then x is even. (Note this is really a "for all" case)

Proof: Assume x + 2 is even and x is odd

x is odd and x + 2 is even (swap sides)

x is odd and x + 2 = 2k, $\exists k \in Z$ by definition of even

x is odd and x = 2k – 2

x is odd and x = 2(k – 1) $k - 1 \in Z$, so let $p = k - 1$ $\exists p \in Z$

x is odd and x = 2p

x is odd and x is even

$\rightarrow\leftarrow$

$\nexists x \in Z \ni$ x is odd and x is even

$\therefore$ x + 2 is even and x is odd is false

$\therefore$ By T.T. If x + 2 is even, then x is even is true

13. A year has 365 days. Show at least 4 of 22 days chosen at random are the same day of the week.

P: Show at least 4 of 22 days chosen at random are the same day of the week.

$\overline{P}$: 1,2 or 3 of 22 days chosen at random are the same day of the week.

A year has 365 days.
Show at least 4 of 22 days chosen at random are the same day of the week.

Proof: Assume 1,2 or 3 of 22 days chosen at random are the same day of the week.

Since there are 7 days a week, and at most three days are chosen, then at most 21 days can be chosen.

$\rightarrow\leftarrow$

To 22 days

$\therefore$ 1,2 or 3 of 22 days chosen at random are the same day of the Week is false

$\therefore$ By T.T at least 4 of 22 days chosen at random are the same day of the week is true

15.

Show If $x \neq 0$ then $10^x \neq 1$ (Note: This really a "for all" case)

Proof: Assume $x \neq 0$ and $10^x = 1$

$x \neq 0$ and $\log(10^x) = \log(1)$

$x \neq 0$ and $x\log(10) = \log(1)$

$x \neq 0$ and $x = 0$

$\rightarrow\leftarrow$

$\bar{\exists}\, x \in R \ni x \neq 0$ and $x = 0$

$\therefore$ $x \neq 0$ and $10^x = 1$ is false

$\therefore$ By T.T If $x \neq 0$ then $10^x \neq 1$ is true

17. Show $\exists\, x, y \in \mathbb{N} \ni \frac{x}{y} \in \mathbb{Q}$

Proof: Assume $\forall\, x, y \in \mathbb{N}, \frac{x}{y} \notin \mathbb{Q}$

Let x = 1, y = 1 then $\frac{1}{1} \notin \mathbb{Q}$

$1 \notin \mathbb{Q}$

$\rightarrow\leftarrow$

$1 \in \mathbb{Q}$

$\therefore$ $\forall\, x, y \in \mathbb{N}, \frac{x}{y} \notin \mathbb{Q}$ is false

$\therefore$ By T.T. $\exists\, x, y \in \mathbb{N} \ni \frac{x}{y} \in \mathbb{Q}$ is true

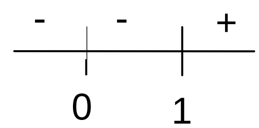
19.

Show $\forall x \in \mathbb{R}$, If $x > 1$, then $x^3 > x^2$

Proof: Assume $\exists x \in \mathbb{R},\ x > 1$ and $x^3 \le x^2$

$\exists x \in \mathbb{R},\ x > 1$ and $x^3 - x^2 \le 0$

$\exists x \in \mathbb{R},\ x > 1$ and $x^2(x-1) \le 0$



$\exists x \in \mathbb{R},\ x > 1$ and $x \le 1$

$\rightarrow\leftarrow$

$\overline{\exists}\, x \in \mathbb{R},\ x > 1$ and $x \le 1$

$\therefore\ \exists x \in \mathbb{R},\ x > 1$ and $x^3 \le x^2$ is false

$\therefore$ By T.T. $\forall x \in \mathbb{R}$, If $x > 1$, then $x^3 > x^2$ is true

21. Show $\forall x \in \mathbb{R}, 3x^2 + 2x + 4 \ne 0$

Proof: Assume $\exists\ x \in \mathbb{R} \ni 3x^2 + 2x + 4 = 0$

$$\exists\ x \in \mathbb{R} \ni\ x = \frac{-(2) \pm \sqrt{(2)^2 - 4(3)(4)}}{2(3)}$$

$$\exists\ x \in \mathbb{R} \ni\ x = \frac{-2 \pm \sqrt{-44}}{6}$$ (This is a complex number)

$\rightarrow\leftarrow$

$$\overline{\exists}\ x \in \mathbb{R} \ni\ x = \frac{-2 \pm \sqrt{-44}}{6}$$

(x can't be both real and complex)

$\therefore\ \exists\ x \in \mathbb{R} \ni 3x^2 + 2x + 4 = 0$ is false

$\therefore$ By T.T. $\forall x \in \mathbb{R}, 3x^2 + 2x + 4 \ne 0$ is true

23.

Show $\forall$ Sets A, $A \subseteq A$

Proof: Assume $\exists$ a set A $\ni$ $A \not\subseteq A$

Since $A \not\subseteq A$, then there is an x in A(left set A) which is not in A(right set A)

$\rightarrow\leftarrow$

All elements in A is also in A

$\therefore$ $\exists$ a set A $\ni$ $A \not\subseteq A$ is false

$\therefore$ By T.T $\forall$ Sets A, $A \subseteq A$ is true