1. Prove the following using proof by contradiction:

Show $\exists\ x \in R_+ \ni x^2 \leq 1$ (Note: R_+ is: $x > 0$)

Proof: Assume $\forall\ x \in R_+,\ x^2 > 1$

if $x = 1$, then $1^2 > 1$

$1 > 1$

$\rightarrow \leftarrow$

Make a true statement here like 1 not > to 1.

$\therefore \forall\ x \in R_+,\ x^2 > 1$ is false.

$\therefore$ By TT. $\exists\ x \in R_+ \ni x^2 \leq 1$ is true.

2. Prove the following using proof by contradiction:
Show $\forall\ x \in N \ni x^2 \geq 1$

Proof: Assume $\exists\ x \in N \ni x^2 < 1$

$\exists\ x \in N \ni x^2 - 1 < 0$

$\exists\ x \in N \ni (x+1)(x-1) < 0$

Move this up to this line. Otherwise this proof is perfect!

$\exists\ x \in N \ni -1 < x < 1$

$\longrightarrow \longleftarrow$

$\nexists\ x \in N \ni -1 < x < 1$

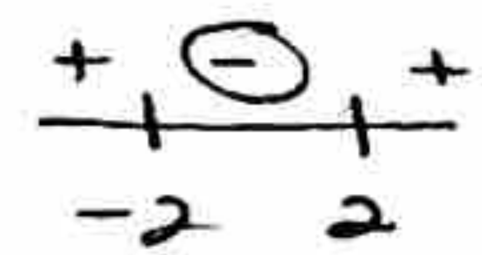
$\therefore\ \exists\ x \in N \ni x^2 < 1$ is false.

$\therefore$ By TT, $\forall\ x \in N, x^2 \geq 1$

3. Prove the following using proof by contradiction:
Show $\forall\ x \in R$ if $x > 2$, then $x^2 - 4 \geq 0$

Proof: Assume $\exists\ x \in R \ni x > 2$ AND $x^2 - 4 < 0$

$\exists\ x \in R \ni x > 2$ AND $(x+2)(x-2) < 0$



$\exists\ x \in R \ni x > 2$ AND $-2 < x < 2$

$\longrightarrow \longleftarrow$

$\nexists\ x \in R \ni x > 2$ AND $-2 < x < 2$

This proof is perfect!

$\therefore \exists\ x \in R \ni x > 2$ AND $x^2 - 4 < 0$ is false.

$\therefore$ By TT. $\forall x \in R \ni$ if $x > 2$, then $x^2 - 4 \geq 0$ is true.

4. Prove the following using proof by contradiction:
Show If $x + 2$ is an even Integer, then x is even.

Proof: Assume $\exists\ x \in \mathbb{Z} \ni x+2$ is an even Integer AND x is odd.

$\exists\ x \in \mathbb{Z} \ni x+2$ is an even $\mathbb{Z}$ AND $x = 2k+1 \quad \exists\ k \in \mathbb{Z}$

$\exists\ x \in \mathbb{Z} \ni x+2$ is an even $\mathbb{Z}$ AND $x+2 = (2k+1)+2$

$\longrightarrow \longleftarrow$

$\exists\ x \in \mathbb{Z} \ni x+2$ is even and $x+2$ is odd

$\therefore \exists\ x \in \mathbb{Z} \ni x+2$ is an even Integer AND x is odd is false.

$\therefore$ By TT If $x+2$ is an even Integer, then x is even is true.

You are not done yet. You can only say x+2 is odd for 2k+1 or 2k-1, and not 2k+1 + 2.
So either work on 2k+2 + 1, as in
x+2 = 2(k+1) + 1, k+1 is an Integer, so let p = k+1 for some p in Z
x+2 = 2p+1
OR use 2k-1 here instead of 2k+1

5. Prove the following using proof by contradiction:
Show If $x \neq 0$, then $10^x \neq 1$ (Hint: This is a for all case)

Proof: Assume $\exists\, x \in \mathbb{R} \ni x \neq 0$ AND $10^x = 1$

~~let $x = 1$, then $10^1 \neq 1$~~

$\rightarrow\leftarrow$

$\nexists\, x \in \mathbb{R} \ni x \neq 0$ AND $10^x = 1$

$\therefore\ \exists\, x \in \mathbb{R} \ni x \neq 0$ AND $10^x = 1$ is false.

$\therefore$ By TT if $x \neq 0$, then $10^x \neq 1$ is true.

This is a there exist statement, and you just showed one case does not work. YOu want to show this statement is false, so you have to show all elements in the domain produce a false statement.
Set this up like the problem #4, and then work on 10^x = 1. Log both sides, and solve for x. You will then have a contradiction to x !=0. You can resend me your work if you like.