1. Prove the following using proof by contradiction:
   Show $\exists\ x \in R_+ \ni x^2 \leq 1$ (Note: R_+ is: $x > 0$)

Proof: Assume $\forall\ x \in R_+,\ x^2 > 1$

if $x = 1$, then $1^2 > 1$

$1 > 1$

$1 \not> 1$

$\therefore \forall\ x \in R_+,\ x^2 > 1$ is false.

$\therefore$ By TT, $\exists\ x \in R_+ \ni x^2 \leq 1$ is true.

4. Prove the following using proof by contradiction:
Show If x + 2 is an even Integer, then x is even.

Proof: Assume $\exists\, x \in Z \ni x+2$ is an even Integer AND x is odd.

$\exists\, x \in Z \ni x+2$ is an even Z AND $x = 2k+1$ $\exists\, k \in Z$

$\exists\, x \in Z \ni x+2$ is an even Z AND $x+2 = 2k+1+2$,

$\exists\, x \in Z \ni x+2$ is an even Z AND $x+2 = 2k+2+1$

$\exists\, x \in Z \ni x+2$ is an even Z AND $x+2 = 2(k+1)+1$, $p = k+1$ for some p in Z.

$\exists\, x \in Z \ni x+2$ is an even Z AND $x+2 = 2p+1$

$\rightarrow \leftarrow$

$\nexists\, x \in Z \ni x+2$ is an even Z AND $x+2 = 2p+1$

$\therefore$ $\exists\, x \in Z \ni x+2$ is an even Integer AND x is odd is false.

$\therefore$ By TT, If $x+2$ is an even Integer, then x is even is true.

5. Prove the following using proof by contradiction:
Show If $x \neq 0$, then $10^x \neq 1$ (Hint: This is a for all case)

Proof ∴ Assume $\exists\ x \in R \ni x \neq 0$ AND $10^x = 1$

$\exists\ x \in R \ni x \neq 0$ AND $\log 10^x = \log(1)$

$\exists\ x \in R \ni x \neq 0$ AND $x \log 10 = 0$

$\exists\ x \in R \ni x \neq 0$ AND $x = \frac{0}{\log 10}$, $\log_b(b) = 1$, $\log_{10}(10) = 1$

$\exists\ x \in R \ni x \neq 0$ AND $x = 0$

$\longrightarrow \longleftarrow$

$\nexists\ x \in n \ni x \neq 0$ AND $x = 0$

∴ $\exists\ x \in N \ni x^2 < 1$ is false

∴ By T.T $\forall\ x \in N$, $x^2 \geq 1$ is true