

1. Prove the following using proof by contrapositive:  
Show If  $x + 2$  is an even  $\mathbb{Z}$ , then  $x$  is even.

$$P \rightarrow Q = \bar{Q} \rightarrow \bar{P}$$

Show if  $x$  is odd, then  $x + 2$  is an odd  $\mathbb{Z}$

Proof: If  $x$  is odd is true

$$x = 2k + 1, \exists k \in \mathbb{Z}$$

$$x + 2 = 2k + 1 + 2$$

$$x + 2 = 2k + 2 + 1$$

$$x + 2 = 2(k+1) + 1, \quad \text{for some } k+1 \in \mathbb{Z}, \text{ so let } p = k+1, \quad p \in \mathbb{Z}$$

$$x + 2 = 2p + 1$$

$\Rightarrow x + 2$  is an odd integer

$\therefore$  If  $x$  is odd, then  $x + 2$  is an odd  $\mathbb{Z}$  is true.

$\therefore$  By T.T. If  $x + 2$  is an even  $\mathbb{Z}$ , then  $x$  is even.

2. Prove the following using proof by contrapositive:  
(Hint: you will need to log both sides of the equation.)  
Show If  $x \neq 0$ , then  $2^x \neq 1$

$$P \rightarrow Q = \overline{Q} \rightarrow \overline{P}$$

Show If  $2^x = 1$ , then  $x = 0$

Proof: If  $2^x = 1$  is true

$$2^x = 1$$

$$\log 2^x = \log 1$$

$$x \log 2 = \log 1$$

$$\frac{x \log 2}{\log 2} = \frac{0}{\log 2}$$

$$\rightarrow x = 0$$

$\therefore$  If  $2^x = 1$ , then  $x = 0$

$\therefore$  If  $x \neq 0$ , then  $2^x \neq 1$

This proof is perfect!