Case 1

Show $\overbrace{\exists\ x \in C \ni P(x)}^{r}$

Proof: Assume $\overbrace{\forall\ x \in C,\ \overline{P(x)}}^{\bar{r}}$ Assume $\bar{r}$ is true

$\forall\ x \in C\ \ \overline{P(x)}$ [False]

$\overline{P(x)}$: $\{\ \}$ True

note: $1 = 1 + 0i$

$\overline{P(x)}$: $\{\ldots, 1, \ldots\}$ False

$\therefore\ \forall\ x \in C,\ \overline{P(x)}$ is false. $\bar{r}$ is false

$\therefore$ By TT, $\exists\ x \in C \ni$ r is true by T.T

Case 1

Show $\overbrace{\exists\, x, y \in Q \ni \frac{x}{y} \notin Q}^{r}$ → equivalent statement:

$\exists\ x \in Q\ \ \exists\ y \in Q \ni \frac{x}{y} \notin Q$

Proof: Assume $\overbrace{\forall\, x, y \in Q\ \ \frac{x}{y} \in Q}^{\bar{r}}$ Assume $\bar{r}$ is true

if $x = 1$, $y = 0$, then

$\frac{1}{0} \in Q$

$\rightarrow\leftarrow$

$\frac{1}{0} \notin Q$

$\therefore\ \forall\ x, y \in Q,\ \frac{x}{y} \in Q$ is false. $\bar{r}$ is false

$\therefore$ By T.T. $\exists\ x, y \in Q \ni \frac{x}{y} \notin Q$ is true. r is true by T.T

Case 2

Show $\overbrace{\forall \quad x \in C, \; P(x)}^{r}$

Proof: $\overbrace{\text{Assume } \exists \; x \in C \ni \overline{P(x)}}^{T}$ Assume $\overline{T}$ is true

$\exists \; x \in C \ni \overline{P(x)}$ [False]

$\overline{P(x)} : \{ \; \}$ (True)

$\overline{P(x)} : \{\ldots, 1, \ldots\}$ (False) note: $1 = 1 + 0i$

$\rightarrow \leftarrow$

$\nexists \; x \in C \ni \overline{P(x)}$

$\therefore \exists \, x \in C \ni \overline{P(x)}$ is false. $\overline{T}$ is false

$\therefore$ By TT, $\forall \; x \in C, \; P(x)$ is true. r is true by T.T

Case 1

Show $\overbrace{\exists\ y \in N \ni y^3 > y^2}^{r}$

Proof: Assume $\overbrace{\forall\ y \in N,\ y^3 \leq y^2}^{\overline{r}}$ Assume $\overline{r}$ is true

if $y = 2$, then
$$(2)^3 \leq (2)^2$$
$$8 \leq 4$$
$$\rightarrow \leftarrow$$
$$8 \not\leq 4$$

$\therefore \forall\ y \in N,\ y^3 \leq y^2$ is false. $\overline{r}$ is false

$\therefore$ By T.T $\exists\ y \in N \ni y^3 > y^2$ is true. r is true by T.T

Case 2

Show $\overbrace{\forall\ x \in N,\ x - 1 \geq 0}^{r}$ Proposition r

Proof: Assume $\overbrace{\exists\ x \in N \ni x - 1 < 0}^{\overline{r}}$ Assume $\overline{r}$ is true

$$\exists\ x \in N \ni x - 1 < 0$$
$$\underline{\quad +1 \quad +1}$$
$$x < 1$$

$\exists\ x \in N \ni x < 1$ $\quad x = 0,\ 0 \notin N$

$\rightarrow \leftarrow$

$\nexists\ x \in N \ni x < 1$

$\therefore \exists\ x \in N \ni x - 1 < 0$ is false. $\overline{r}$ is false

$\therefore$ By T.T $\forall\ x \in N,\ x - 1 \geq 0$ is true. By T.T. r is true

Case 2

Show $\overbrace{\forall\ x \in R,\ x^2 \geq 0}^{r}$

Proof: $\overbrace{\text{Assume } \exists\ x \in R \ni x^2 < 0}^{T}$

$\exists\ x \in R \ni x^2 < 0$ $+ \mid +$ 0

$\exists\ x \in R \ni$ on $\{\ \}$

$\longrightarrow$ $\longleftarrow$

$\exists\ x \in R$ on $\{\ \}$

$\therefore \exists\ x \in R \ni x^2 < 0$ is false

$\therefore$ By T.T $\forall\ x \in R,\ x^2 \geq 0$ is true

Understand whats Going On Here First

$x^2 < 0$

$\sqrt{x^2} < \sqrt{0}$

$x < 0$ $x = 0$

$+$ $+$

0

$(-1): (-1)^2 < 0$ $(+)$

$(1): (1)^2 < 0$ $(+)$

$(-\infty, 0) \cup (0, \infty)$

$x < 0$ or $x > 0$

0 is not included in the solution. So we can't use $x^2 < 0 = (0)^2 < 0$

$0 < 0$

Invalid

Case 1

Show $\exists\ x, y \in N \ni \frac{x}{y} \notin n$

Proof: Assume $\forall\ x,y \in N\ \ \frac{x}{y} \in N$

if $x = 1, y = 2$, then $\frac{1}{2} \in N$

$\rightarrow \leftarrow$

$\frac{1}{2} \notin N$

$\therefore \forall\ x, y \in N, \frac{x}{y} \in N$ is false

$\therefore$ By TT. $\exists\ x, y \in N \ni \frac{x}{y} \notin N$ is true

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Case 1

Show $\exists\ x \in R$ if $\overset{p}{x > 1}$, then $\overset{\rightarrow}{}$ $\overset{q}{\frac{x}{x^2+1} < \frac{1}{3}}$

Proof: Assume $\forall\ x \in R$ $\overset{p}{x > 1}$ $\overset{\wedge}{\text{and}}$ $\overset{\overline{q}}{\frac{x}{x^2+1} \geq \frac{1}{3}}$

If $x = 0$, then $(0 > 1)$ and $\frac{0}{(0)^2+1} \geq \frac{1}{3}$

$(0 > 1)$ and $\left(0 \geq \frac{1}{3}\right)$

$\rightarrow \leftarrow$

$(0 \leq 1)$ and $\left(0 \leq \frac{1}{3}\right)$

$\therefore \forall\ x \in R$ $x > 1$ and $\frac{x}{x^2+1} \geq \frac{1}{3}$ is false.

$\therefore$ By TT. $\exists\ x \in R$ if $x > 1$, then $\frac{x}{x^2+1} < \frac{1}{3}$ is true.

Case 1

$$\overset{p}{\text{Show If } \underset{x}{x+1} \text{ is an even } \underset{\mathbb{Z}}{\text{Integer}}}, \overset{\rightarrow}{\text{then}} \overset{q}{x \text{ is odd.}}$$ ← This is a "for all" statement: $\forall$

Negate to "there exists" $\exists$.

Negate $p \to q$ to $p \wedge \bar{q}$

Proof: Assume $\exists\, x$ in $\mathbb{Z} \ni \overset{p}{\overline{x+1 \text{ is an even Integer}}}$ $\overset{\wedge}{\text{AND}}$ $\overset{\bar{q}}{\overline{x \text{ is even.}}}$

$\exists\, x$ in $\mathbb{Z} \ni \overset{p}{\overline{x+1 \text{ is an even } \mathbb{Z}}}$ $\overset{\wedge}{\text{(AND)}}$ $\overset{\bar{q}}{\overline{x = 2k}}$ $\exists\, k \in \mathbb{Z}$ Definition of an even integer

$\exists\, x$ in $\mathbb{Z} \ni \overset{p}{\overline{x+1 \text{ is an even } \mathbb{Z}}}$ $\overset{\wedge}{\text{(AND)}}$ $\overset{\bar{q}}{\overline{x+1 = 2k+1}}$ $x+1$ can't be both even and odd

$\rightarrow\leftarrow$

$\nexists\, x \in \mathbb{Z} \ni x+1$ is even and $x+1$ is odd — There is no x in the Integers such that, $x+1$ is even and odd.

$\therefore\ \exists\, x$ in $\mathbb{Z} \ni x+1$ is an even Integer AND x is even is false.

$\therefore$ By TT If $x+1$ is an even Integer, then x is odd is true.

Case 2: Show ∀

Show ∀ $x, y \in \mathbb{R}$ if $x + y \geq 2$, then [$x \geq 1$ or $y \geq 1$]

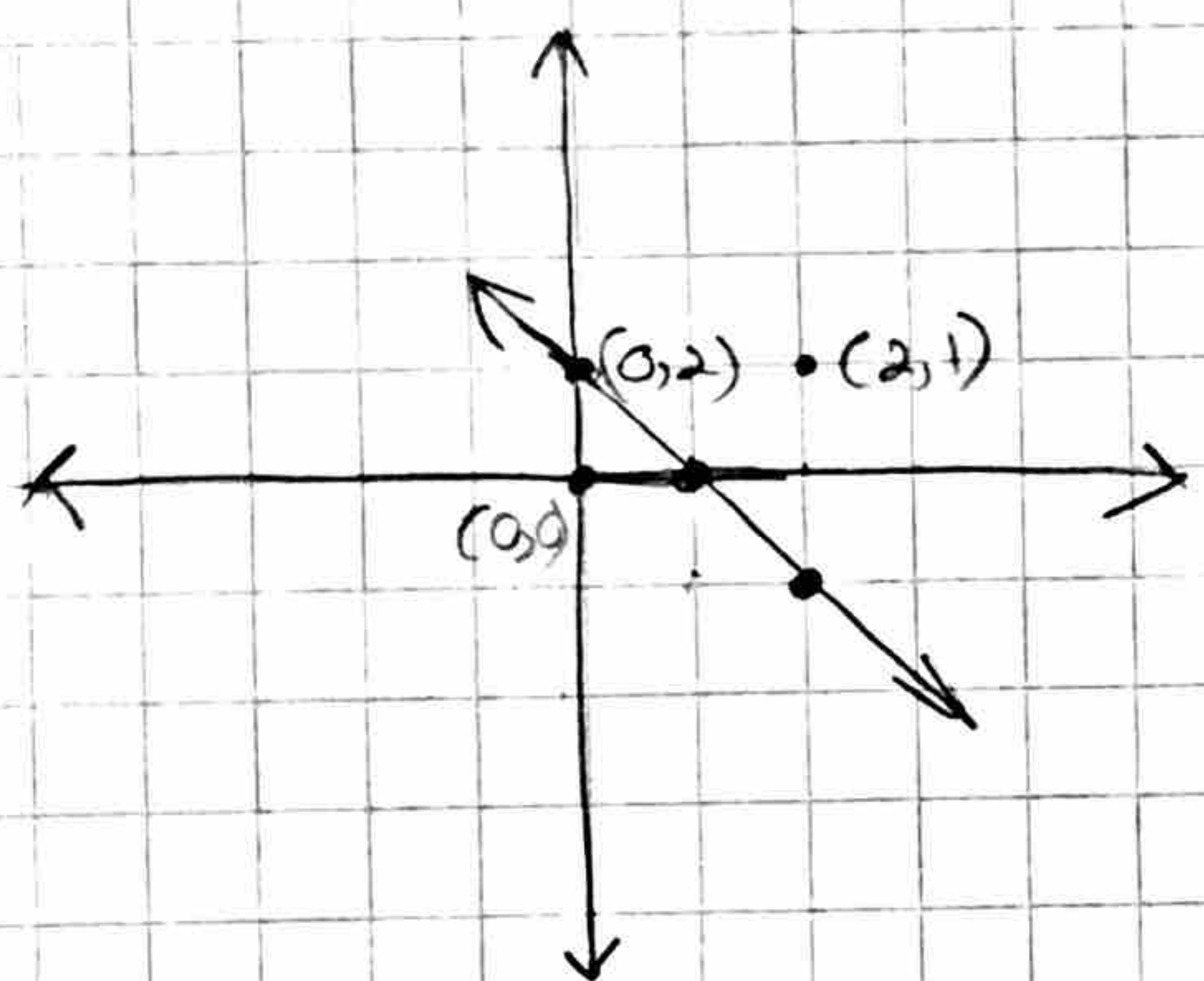
Proof: Assume ∃ $x, y \in \mathbb{R}$ ∋ $x + y \geq 2$ and [$x < 1$ and $y < 1$]

Verify.

$$x + y \geq 2$$
$$-x \quad -x$$

$y \geq -x + 2$ or $x + y \geq 2$

$$y = -x + 2$$



Test (0,0)

$x + y \geq 2$

$0 + 0 \geq 2$

$0 \geq 2$

False

Test (2,1)

$x + y \geq 2$ → $x + y > 2$ Use negation to substitute [$x < 1$ and $y < 1$]

$2 + 1 \geq 2$

$3 \geq 2$

True

(Case 2) Show $\forall$

$$\overset{p}{}\quad \rightarrow \quad \overset{q}{}$$

Show $\forall$ $x, y \in \mathbb{R}$ if $x+y \geq 2$, then $[x \geq 1$ or $y \geq 1]$

$$\overset{p}{}\quad \wedge$$

Proof: Assume $\exists$ $x, y \in \mathbb{R}$ $x+2 \geq 2$ AND $[x < 1$ or $y < 1]$

$\exists$ $x, y \in \mathbb{R}$ $x+2 \geq 2$ and $x+y < 2$ negation of $x+y \geq 2$
from verification

$\rightarrow\leftarrow$

$\nexists$ $x, y \in \mathbb{R}$ $x+2 \geq 2$ and $x+y < 2$

$\therefore$ $\exists$ $x, y \in \mathbb{R}$ $x+2 \geq 2$ and $[x<1$ or $y<1]$ is false

$\therefore$ By T.T. $\forall$ $x, y \in \mathbb{R}$ if $x+y \geq 2$, then $[x \geq 1$ or $y \geq 1]$ is true

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(Case 2) Show $\forall$

Show $\forall$ $x \in \mathbb{R}$ if $x > 1$, then $x^2 > x$

Proof: Assume $\exists x \in \mathbb{R}$ $\ni$ $x > 1$ and $x^2 \leq x$

Verification

$$x^2 \leq x$$
$$-x \quad -x$$
$$x^2 - x \leq 0$$

$$x(x-1) \leq 0$$

$x = 0$ $\qquad$ $x - 1 = 0$
$\qquad\qquad\quad +1 \quad +1$

$+ \qquad - \qquad +$

$0 \qquad\quad 1$

$0 \leq x \leq 1$

(-1): $-1(-1-1) \leq 0$
$(-)(-)$
$(+)$

$(.5)$: $.5(.5-1) \leq 0$
$(+)(-)$
$(-)$

(2): $2(2-1) \leq 0$
$(+)(+)$
$(+)$

Show $\forall\ x \in \mathbb{R}$ if $x > 1$, then $x^2 > x$

Proof: Assume $\exists\ x \in \mathbb{R} \ni x > 1$ and $x^2 \leq x$

$\exists\ x \in \mathbb{R} \ni x > 1$ and $x^2 - x \leq 0$

$\exists\ x \in \mathbb{R} \ni x > 1$ and $x(x-1) \leq 0$

$\exists\ x \in \mathbb{R} \ni \underbrace{x > 1 \text{ and } 0 \leq x \leq 1}$,

$x > 1$ and $0 \leq x \leq 1$

$\rightarrow \leftarrow$

$\nexists\ x \in \mathbb{R} \ni x > 1$ and $0 \leq x \leq 1$

$\therefore\ \exists\ x \in \mathbb{R} \ni x > 1$ and $x^2 \leq x$ is false

$\therefore$ By T.T. $\forall\ x \in \mathbb{R}$ if $x > 1$, then $x^2 > x$ is true.

(Case 2) Show 4

Show $\forall$ $x \in \mathbb{R}$, $\frac{1}{x^2+1} \leq 1$

Proof: Assume $\exists$ $x \in \mathbb{R}$ $\ni$ $\frac{1}{x^2+1} > 1$

Verification

$$\frac{1}{x^2+1} > 1$$
$$-1 \quad -1$$

$$\frac{1}{x^2+1} - 1 > 0$$
$$=$$
$$\frac{1}{x^2+1} - \frac{1(x^2+1)}{1(x^2+1)} > 0$$
$$=$$
$$\frac{1}{x^2+1} - \frac{x^2+1}{x^2+1} > 0$$
$$=$$
$$\frac{1-x^2+1}{x^2+1} > 0$$
$$=$$
$$(-1)\frac{-x^2}{x^2+1} > 0(-1)$$

multiply by -1 Change Sign

$$=$$
$$\frac{-x^2}{x^2+1} < 0$$

$$\frac{-x^2}{x^2+1} > 0$$

x^2+1 ← produce $\sqrt{-1}$ or i

$$\frac{-x^2}{-1} \geq \frac{0}{-1}$$
$$=$$
$$x^2 \leq 0$$

← Sign will flip

$$\boxed{\frac{-x^2}{x^2+1} < 0}$$

$$x^2 < 0 \qquad\qquad x^2+1 < 0$$
$$-1 \quad -1$$
$$\sqrt{x^2} = \sqrt{0} \qquad\qquad \sqrt{x^2} < \sqrt{-1}$$
$$= \qquad\qquad =$$
$$x = 0 \qquad\qquad x < i$$

$\sqrt{-1} = i$

$$+ \quad 0 \quad +$$

$$(-1): \frac{(-1)^2}{(-1)^2+1} = \frac{(+)}{(+)} = (+)$$

$$(1): \frac{(1)^2}{(1)^2+1} = \frac{(+)}{(+)} = (+)$$

$$\boxed{\frac{x^2}{x^2+1} < 0 \quad \text{Solution Set } \{\}}$$

(Case 2) Show $\forall$

Show $\forall \; x \in \mathbb{R}, \; \frac{1}{x^2+1} \leq 1$

Proof: Assume $\exists \; x \in \mathbb{R} \ni \frac{1}{x^2+1} > 1$

$\exists \; x \in \mathbb{R} \ni \frac{1}{x^2+1} - 1 > 0$

$\exists \; x \in \mathbb{R} \ni \frac{1}{x^2+1} - \frac{1(x^2+1)}{x^2+1} > 0$

$\exists \; x \in \mathbb{R} \ni \frac{1 - x^2 - 1}{x^2+1} > 0$

$\exists \; x \in \mathbb{R} \ni (-1) \frac{-x^2}{x^2+1} > 0(-1)$

$\exists \; x \in \mathbb{R} \ni \frac{x^2}{x^2+1} < 0$

$\exists \; x \in \mathbb{R}$ on $\{\}$

$\rightarrow \leftarrow$

$\nexists \; x \in \mathbb{R}$ on $\{\}$

$\therefore \; \exists \; x \in \mathbb{R} \ni \frac{1}{x^2+1} > 1$ is false.

$\therefore$ By T.T $\forall \; x \in \mathbb{R}, \; \frac{1}{x^2+1} \leq 1$ is true