1. Show if a is an even integer, then 5a is even

Proof: since a is even,

$a = 2k \ \exists k \in \mathbb{Z}$ by definition of even Z

$5a = 5(2k)$

$5a = 2(5k)$, let $p = 5k$, $\exists\ p \in \mathbb{Z}$

$5a = 2p$

=> 5a is even

$\therefore$ By T.T. if a is even, then 5a is even is true

3. Show if $x = 5$, then $x^2 = 25$

Proof: Since $x = 5$

$x^2 = 5^2$

$x^2 = 25$

$\therefore$ By T.T. if $x = 5$ then $x^2 = 25$ is true

5. Let $x \in \mathbb{Z}$, Show if x is even, then $x + 2$ is even.

Proof: Since x is even

$x = 2k$ for $\exists k \in \mathbb{Z}$ by definition of even Z

$x + 2 = 2k + 2$

$x + 2 = 2(k + 1)$, let $p = k + 1$, $\exists\ p \in \mathbb{Z}$

$x + 2 = 2p$

=> $x + 2$ is even

$\therefore$ By T.T. if x is even, then $x + 2$ is even is true

7. Show if a and b are odd integers, then ab is odd

Proof: Since a and b are odd integers, then

$a = 2k + 1 \quad \exists k \in \mathbb{Z}$ By definition of odd Z.

$b = 2p + 1 \quad \exists p \in \mathbb{Z}$ By definition of odd Z.

Consider 'a':

$a = 2p + 1$

$ab = (2k + 1)(2p + 1)$

$ab = 4kp + 2k + 2p + 1$

$ab = 2(2kp + k + p) + 1$, let $r = 2kp + k + p$, $\exists r \in \mathbb{Z}$

$ab = 2r + 1$

=> ab is odd

$\therefore$ By T.T. if a and b are odd integers, then ab is odd is true

9. Show if n is an odd integer, then n^2 is odd.

Proof: Since n is an odd integer

$n = 2k + 1 \quad \exists k \in \mathbb{Z}$ by definition of odd Z

$n^2 = (2k + 1)(2k + 1)$

$n^2 = 4k^2 + 2k + 2k + 1$

$n^2 = 4k^2 + 4k + 1$

$n^2 = 2(2k^2 + 2k) + 1$, let $p = 2k^2 + 2k$, $\exists p \in \mathbb{Z}$

$n^2 = 2p + 1$

=> n^2 is odd

$\therefore$ By T.T. if n is an odd integer, then n^2 is odd is true

11. Show if n is an even integer, then $(-1)^n = 1$

Proof: Since n is even, then

$n = 2k \quad \exists k \in \mathbb{Z}$ by definition of even Z

$(-1)^n = (-1)^{2k}$

$(-1)^n = ((-1)^2)^k$

$(-1)^n = (1)^k$

$(-1)^n = 1$

$\therefore$ By T.T. if n is an even integer, then $(-1)^n = 1$ is true

13. Show if $10^x = 1$, then $x = 0$.
Proof: Since $10^x = 1$
$\log(10^x) = \log(1)$
$x\log(10) = 0$
$x(1) = 0$
$x = 0$

$\therefore$ By T.T. if $10^x = 1$, then $x = 0$ is true.

15. Show if a and b are odd integers, then a + b is even.
Proof: Since a and b are odd integers.
$a = 2k + 1 \quad \exists k \in \mathbb{Z}$ By definition of odd Z.
$b = 2p + 1 \quad \exists p \in \mathbb{Z}$ By definition of odd Z.
Consider a + b:
$a + b = (2k + 1) + (2p + 1)$
$a + b = 2k + 2p + 2$
$a + b = 2(k + p + 1)$, let $r = k + p + 1$, $\exists\ p \in \mathbb{Z}$
$a + b = 2r$
=> a + b is even

$\therefore$ By T.T. if a and b are odd integers, then a + b is even.

17. Note the following statement says:
    If a goes into b evenly and b goes into c evenly, then show that a goes into c evenly.

Show if a|b and b|c, then a|c
Proof: since a| b and b|c, then

$b = ak \quad \exists k \in \mathbb{Z}$ By definition of divisible by a.

$c = bp \quad \exists p \in \mathbb{Z}$ By definition of divisible by b.

Consider 'c':

$c = bp$

$c = (ak)p$

=> a divides c

$\therefore$ By T.T. if a|b and b|c, then a|c is true

This is an example using random numbers to demonstrate how this problem works.

Let a = 3, b = 12, c = 24, then:

3|12 and 12|24

12 = 3(k) where k = 4

24 = 12(p) where p = 2

24 = (3k)p At this point, you can see that 3 goes into 24 evenly.

3 divides 24