1. Prove the following using an if and only if proof:
Show $x + 2$ is an even Z $\leftrightarrow$ x is even.

$p \leftrightarrow q = (p \to q) \wedge (q \to p)$

Show $x+2$ is an even Z iff $x$ is even

$\rightarrow$ Show if $x+2$ is an even Z, then $x$ is even

Proof: Since $x+2$ is an even Z

$x+2 = 2k \quad \exists\, k \in Z$, definition of an even integer

$x = 2k - 2$

k-1 is an integer, so let

$x = 2(k-1)$, $p = k-1$, $\exists\, p \in Z$

$x = 2p$

$\rightarrow$ $x$ is even

$\therefore$ By T.T, if $x+2$ is an even Z, then $x$ is even is true

$\leftarrow$ Show if $x$ is even, then $x+2$ is an even Z

Proof: Since $x$ is even

$x = 2k \quad \exists\, k \in Z$, definition of an even integer

$x+2 = 2k+2$

same comment as above

$x+2 = 2(k+1)$, $p = k+1$, $\exists\, p \in Z$

$x+2 = 2p$

$\rightarrow$ $x+2$ is even

$\therefore$ By T.T. if $x$ is even, then $x+2$ is an even Z

$\therefore$ By T.T. $x+2$ is an even Z iff $x$ is even