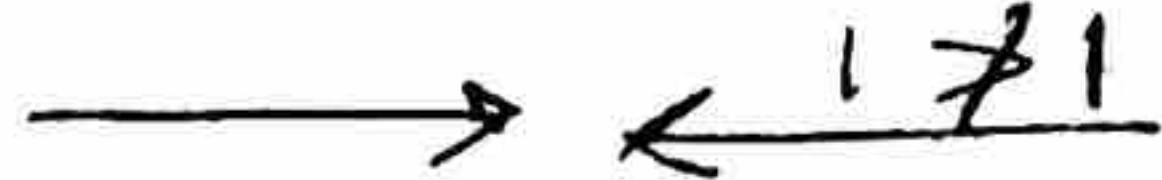


1. Prove the following using proof by contradiction:
Show $\exists x \in R_+ \ni x^2 \leq 1$ (Note: R_+ is: $x > 0$)

Proof: Assume $\forall x \in R_+, x^2 > 1$

if $x = 1$, then $1^2 > 1$

$$1 > 1$$



$\therefore \forall x \in R_+, x^2 > 1$ is false.

\therefore By TT, $\exists x \in R_+ \ni x^2 \leq 1$ is true.

4. Prove the following using proof by contradiction:
Show If $x + 2$ is an even Integer, then x is even.

Proof: Assume $\exists x \in \mathbb{Z} \rightarrow x+2$ is an even Integer AND x is odd.

$$\exists x \in \mathbb{Z} \rightarrow x+2 \text{ is an even } \mathbb{Z} \text{ AND } x = 2k+1 \quad \exists k \in \mathbb{Z}$$

$$\exists x \in \mathbb{Z} \rightarrow x+2 \text{ is an even } \mathbb{Z} \text{ AND } x+2 = 2k+1+2,$$

$$\exists x \in \mathbb{Z} \rightarrow x+2 \text{ is an even } \mathbb{Z} \text{ AND } x+2 = 2k+2+1$$

$$\exists x \in \mathbb{Z} \rightarrow x+2 \text{ is an even } \mathbb{Z} \text{ AND } x+2 = 2(k+1)+1, \quad p = k+1 \text{ for some } p \text{ in } \mathbb{Z}.$$

$$\exists x \in \mathbb{Z} \rightarrow x+2 \text{ is an even } \mathbb{Z} \text{ AND } x+2 = 2p+1$$



$$\nexists x \in \mathbb{Z} \rightarrow x+2 \text{ is an even } \mathbb{Z} \text{ AND } x+2 = 2p+1$$

$\therefore \exists x \in \mathbb{Z} \rightarrow x+2$ is an even Integer AND x is odd is false.

\therefore By TT, If $x+2$ is an even Integer, then x is even is true.

5. Prove the following using proof by contradiction:
Show If $x \neq 0$, then $10^x \neq 1$ (Hint: This is a for all case)

Proof: Assume $\exists x \in \mathbb{R} \rightarrow x \neq 0$ AND $10^x = 1$

$$\exists x \in \mathbb{R} \rightarrow x \neq 0 \text{ AND } \log 10^x = \log(1)$$

$$\exists x \in \mathbb{R} \rightarrow x \neq 0 \text{ AND } x \log 10 = 0$$

$$\exists x \in \mathbb{R} \rightarrow x \neq 0 \text{ AND } x = \frac{0}{\log 10}, \log_b(b) = 1, \log_{10}(10) = 1$$

$$\exists x \in \mathbb{R} \rightarrow x \neq 0 \text{ AND } x = 0$$

$$\xrightarrow{\hspace{1cm}} \xleftarrow{\hspace{1cm}} \\ \nexists x \in \mathbb{N} \rightarrow x \neq 0 \text{ AND } x = 0$$

$\therefore \exists x \in \mathbb{N} \rightarrow x^2 < 1$ is false

\therefore By T.T $\forall x \in \mathbb{N}, x^2 \geq 1$ is true