1. Prove the following using the format of the class and induction:
Show $\forall\, n \in W,\ 1 + 5 + 9 + \ldots + (4n + 1) = (n + 1)(2n + 1)$

Proof: A) Show $n=0$ is true. $(4(0)+1) = (0+1)(2(0)+1)$

$1 = (1)(1)$

$1 = 1$ is True

B) Let k be a fixed but generic element of W.

If $\underbrace{1+5+9+\ldots+(4k+1)}^{P(k)} = (k+1)(2k+1)$ is false, then

$[1+5+9+\ldots(\overbrace{4k+1}^{P(k)}) = (k+1)(2k+1)] \rightarrow [\overbrace{1+5+9+\ldots(4(k+1)+1) = ((k+1)+1)(2(k+1)+1)}^{P(k+1)}]$

is true by T.T

If $1+5+9+\ldots+(4k+1) = (k+1)(2k+1)$ is true, then show

$[1+5+9+\ldots+(4k+1)=(k+1)(2k+1)] \rightarrow [1+5+9+\ldots(4k+5) = (k+2)(2k+3)]$ is true

C) Since $1+5+9+\ldots+(4k+1) = (k+1)(2k+1)$ is true

$1+5+9+\ldots+(4k+1)+4(k+1)+1 = (k+1)(2k+1) + 4(k+1)+1$

$1+5+9+\ldots+(4k+1)+4(k+1)+1 = 2k^2+k+2k+1 + 4k+4+1$

$1+5+9+\ldots+(4k+1)+4(k+1)+1 = 2k^2+k+2k+4k+1+4+1$

$1+5+9+\ldots+(4k+1)+4(k+1)+1 = 2k^2+7k+6$

$1+5+9+\ldots+(4k+1)+4(k+1)+1 = (k+2)(2k+3)$

D) $\therefore$ By M.I $\forall\, n \in W$, $1+5+9+\ldots+(4n+1) = (n+1)(2n+1)$ is true.

Perfect!

2. Prove the following using the format of the class and induction:
Show $\forall\ n \in N,\ 2^n < 3^n$

Proof : A) Show n = 1 is true. $2^1 < 3^1$

$2 < 3$ is true

B) Let K be a fixed but generic element of N.

If $2^K < 3^K$ is false, then $2^K < 3^K \rightarrow 2^{K+1} < 3^{K+1}$ is true by the T.T.

If $2^K < 3^K$ is true, then show $2^K < 3^K \rightarrow 2^{K+1} < 3^{K+1}$ is true

C) Since $2^K < 3^K$ is true

$2^K \cdot 2 < 3^K \cdot 2$

$2^{K+1} < 3^K \cdot 2 < 3^K \cdot 3 \quad \forall\ K \in N$

$2^{K+1} < 3^K \cdot 3$ By Transitive Property

$2^{K+1} < 3^{K+1}$ Same Base Exponent Addition Rule

$2^{K+1} < 3^{K+1}$

D) $\therefore$ By M.I. $\forall\ n \in N,\ 2^n < 3^n$ is true

Perfect!

3. Prove the following using the format of the class and induction:

Show $\forall\ n \geq 4,\ 2n < n!$

Proof: A) Show $n = 4$ is true. $2(4) < 4!$

$8 < 24$ is True

B) Let k be a fixed but generic element of $\mathbb{R}$.

If $2k < k!$ is false, then $2k < k! \rightarrow 2k+1 < (k+1)!$ is true by the T.T.

If $2k < k!$ is true, then show $2k < k! \rightarrow 2k+1 < (k+1)!$ is true.

c) Since $2k < k!$ is true

$2k(k+1) < k!(k+1)$

k >=4

$2k + 1 < 2k(k+1) < (k+1)!\quad \forall\ k \in \mathbb{N}$, Inductive Step

$2k+1 < (k+1)!$ Transitive Property

D) $\therefore$ By M.I. $\forall\ n \geq 4$, $2n < n!$ is true.

2k and 1 are attached with the operation of addition.

2k and (k+1) are attached with the operation of multiplication. Therefore, you cannot say these quantities are less than. Distribute and then you will have the operation of addition, and then you can compare.

4. Prove the following using the format of the class and induction:

Show $\forall\ n \geq 5, \quad 2^{n+3} < (n+1)!$

Proof: A) Show $n=5$ is true. $2^{5+3} < (5+1)!$

$$2^{8} < (6)!$$

$$256 < 720 \text{ is true.}$$

B) Let k be a fixed but generic element of $\mathbb{N}$.

If $2^{k+3} < (k+1)!$ is false, then $2^{k+3} < (k+1)! \rightarrow 2^{(k+1)+3} < ((k+1)+1)!$ is true by T.T.

If $2^{k+3} < (k+1)!$ is true, then $2^{k+3} < (k+1)! \rightarrow 2^{k+4} < (k+2)!$ is true

C) Since $2^{k+3} < (k+1)!$ is true.

$$2^{k+3} \cdot 2^{1} < (k+1) \cdot 2$$

$2^{k+4} < 2(k+1)$ Exponent Rule

$2^{k+4} < 2(k+1) < (k+1)!(k+2)$ Inductive ste

For all k > = 5.

$$2^{k+4} < 2(k+1) < (k+2)!$$

$2^{k+4} < (k+2)!$ Transitive Property

$$2^{k+4} < (k+2)!$$

D) $\therefore$ By M.I $\forall\ n \geq 5,\ 2^{n+3} < (n+1)!$ is true

5. Prove the following using the format of the class and induction:

Show $\forall\ n \in N,\quad 2n-1 < 2^n$

Proof: A) Show $n=1$ is true. $2(1)-1 < 2^1$

$2-1 < 2$

$1 < 2$ is true.

B) Let k be a fixed but generic element of N.

If $2k-1 < 2^k$ is false, then $2k-1 < 2^k \rightarrow 2(k+1)-1 < 2^{k+1}$ is true by T.T.

If $2k-1 < 2^k$ is true, then $2k-1 < 2^k \rightarrow 2k-1 < 2^{k+1}$ is true

Should be 2k + 1
So the rest of this
proof needs to be
reworked.

C) Since $2k-1 < 2^k$ is true

$2k-1(2) < 2^k \cdot 2$

$2(2k-1) < 2^{k+1}$ Exponent Rule

$2k-1 < 2(2k-1) < 2^{k+1}$ Inductive Step

$2k-1 < 2^{k+1}$ Transitive Property

$2k-1 < 2^{k+1}$

D) $\therefore$ By M.I. $\forall\ n \in N$, $2n-1 < 2^n$ is true.

6. Prove the following using the format of the class and induction:
Show $\forall\ n \in N$, $3^n - 3$ is divisible by 3.

Proof: A) Show $n=1$ is true. $3^1 - 3 = 0$

0 is divisible by 3 is true

B) Let k be a fixed but generic element of N.

If $3^k - 3$ is div by 3 is false, then

($3^k - 3$ is div by 3) $\rightarrow$ ($3^{k+1} - 3$ is div by 3) is true by T.T.

If $3^k - 3$ is div by 3 is true, then

($3^k - 3$ is div by 3) $\rightarrow$ ($3^{k+1} - 3$ is div by 3) is true.

C) Since ($3^k - 3$ is div by 3) is true.

$3k - 3 = 3j \quad \exists\, j \in N$ Definition of div by 3

$3(3k-3) = 3(3j)$

$3^{k+1} - 9 = 9j$

$3^{k+1} - 9 + 6 = 9j + 6$

$3^{k+1} - 3 = 9j + 6$ Verify div by 3

$3^{k+1} - 3 = 3(3j+2)$, $3j+2 \in N$, $p = 3j+2 \quad \exists\, p \in N$

$3^{k+1} - 3 = 3p$

$\rightarrow 3^{k+1} - 3$ is div by 3

this proof is perfect!

D) $\therefore$ By M.I $\forall\ n \in N$, $3^n - 3$ is divisible by 3 is true.

7. Prove the following using the format of the class and induction:

Show $\forall\ n \in \mathbb{N},\quad 1 + 4 + 7 + \ldots + (3n - 2) = \dfrac{3n^2 - n}{2}$

Proof: A) Show $n = 1$ is true. $(3(1)-2) = \dfrac{3(1^2) - 1}{2}$

$$1 = \frac{3 - 1}{2}$$

$$1 = \frac{2}{2}$$

$1 = 1$ is true

B) Let k be a fixed but generic element of N.

If $\underbrace{1 + 4 + 7 + \ldots + (3k-2) = \dfrac{3k^2 - k}{2}}_{P(k)}$ is false, then $\underbrace{}_{P(k+1)}$

$1 + 4 + 7 + \ldots + (3k-2) = \dfrac{3k^2 - k}{2} \rightarrow 1 + 4 + 7 + \ldots (3(k+1) - 2) = \dfrac{3(k+1)^2 - (k+1)}{2}$ is true by T.T.

If $1 + 4 + 7 + \ldots + (3k-2) = \dfrac{3k^2 - k}{2}$ is true, then show

$1 + 4 + 7 + \ldots + (3k-2) = \dfrac{3k^2 - k}{2} \rightarrow 1 + 4 + 7 + \ldots 3k+1 = \dfrac{3(k+1)^2 - (k+1)}{2}$

Work on this a bit more until you have [3k^2+5k+2]2

C) Since $1 + 4 + 7 + \ldots + (3k-2) = \dfrac{3k^2 - k}{2}$

$1 + 4 + 7 + \ldots + (3k-2) + 3k+1 = \dfrac{3k^2 - k}{2} + 3k+1$

6k + 2

$1 + 4 + 7 \ldots + (3k-2) + 3k+1 = \dfrac{3k^2 - k + \cancel{3k+1}}{2}$ ← LOST HERE

Fix the error I have to the left, and then work on it until it equals [3k^2+5k+2]/2