Find Solutions for each of the following in the interval $[0, 2\pi]$

$$\cos^2(2\theta) = \frac{1}{4}$$

$$\downarrow$$

$$(\cos(2\theta))^2 = \frac{1}{4}$$

① $$\sqrt{(\cos(2\theta))^2} = \pm\sqrt{\frac{1}{4}}$$

$$\downarrow$$

$$\cos(2\theta) = \pm\frac{1}{2}$$

$$\downarrow$$

$$\cos(2\theta) = \pm\frac{1}{2}$$

② let $y = 2\theta$ ③ θ in $[0, 2\pi]$ → 2θ in $[0, 4\pi]$

$$\downarrow$$

$$\cos(y) = \pm\frac{1}{2}$$

④ Solve: $\cos(y) = \pm\frac{1}{2}$, in $[0, 4\pi]$

Where will cosine $= \pm\frac{1}{2}$ within interval of $[0, 4\pi]$

$\cos(y) = \frac{1}{2}$ ⑤ $\cos(y) = -\frac{1}{2}$

$$\downarrow$$

$\frac{\pi}{3}$, $\frac{5\pi}{3}$, $\frac{7\pi}{3}$, $\frac{11\pi}{3}$ ← Add 2π to $\pi/3$ and $5\pi/3$

1 rev 2π — 2 rev $4\pi \to 2\pi + 2\pi$

$$\downarrow$$

$\frac{2\pi}{3}$, $\frac{4\pi}{3}$, $\frac{8\pi}{3}$, $\frac{10\pi}{3}$ ← Add 2π to $2\pi/3$ and $4\pi/3$

1 rev 2π — 2 rev $4\pi \to 2\pi + 2\pi$

$\cos y = \frac{1}{2}$

$2\theta = \frac{\pi}{3}$

$2\theta = \frac{5\pi}{3}$

$2\theta = \frac{7\pi}{3}$

$2\theta = \frac{11\pi}{3}$

$\boxed{y = 2\theta}$

⑥

⑦

Solve for θ (Isolate θ)

$\left(\frac{1}{2}\right)2\theta = \frac{\pi}{3}\left(\frac{1}{2}\right)$

$\downarrow$

$\boxed{\theta = \frac{\pi}{6}}$

$\left(\frac{1}{2}\right)2\theta = \frac{5\pi}{3}\left(\frac{1}{2}\right)$

$\downarrow$

$\boxed{\theta = \frac{5\pi}{6}}$

$\left(\frac{1}{2}\right)2\theta = \frac{7\pi}{3}\left(\frac{1}{2}\right)$

$\downarrow$

$\boxed{\theta = \frac{7\pi}{6}}$

$\left(\frac{1}{2}\right)2\theta = \frac{11\pi}{6}\left(\frac{1}{2}\right)$

$\downarrow$

$\boxed{\theta = \frac{11\pi}{12}}$

$\cos y = -\frac{1}{2}$

$2\theta = \frac{2\pi}{3}$

$2\theta = \frac{4\pi}{3}$

$2\theta = \frac{8\pi}{3}$

$2\theta = \frac{10\pi}{3}$

$\left(\frac{1}{2}\right)2\theta = \frac{2\pi}{3}\left(\frac{1}{2}\right)$

$\theta = \frac{2\pi}{6} = \boxed{\frac{\pi}{3}}$

$\left(\frac{1}{2}\right)2\theta = \frac{4\pi}{3}\left(\frac{1}{2}\right)$

$\theta = \frac{4\pi}{6} = \boxed{\frac{2\pi}{3}}$

$\left(\frac{1}{2}\right)2\theta = \frac{8\pi}{3}\left(\frac{1}{2}\right)$

$\downarrow$

$\theta = \frac{8\pi}{6} = \boxed{\frac{4\pi}{3}}$

$\left(\frac{1}{2}\right)2\theta = \frac{10\pi}{3}\left(\frac{1}{2}\right)$

$\downarrow$

$\theta = \frac{10\pi}{6} = \boxed{\frac{5\pi}{3}}$

Solutions that fall $[0, 2\pi]$

$\frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{12}, \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$

Find Solutions for each of the following in the interval $[0, 2\pi]$

Identity

$\sin^2(\theta) = 1 - \cos^2(\theta)$

$2\sin^2 x = 1 + \cos x$

$2(1-\cos^2 x) = 1 + \cos x$

$2 - 2\cos^2 x = 1 + \cos x$

$+2\cos^2 x \quad +2\cos^2 x$

$2 = 1 + \cos x + 2\cos^2 x$

$-2 \quad -2$

$0 = 1 - 2 + \cos x + 2\cos^2 x$

$0 = -1 + \cos x + 2\cos^2 x \rightarrow 0 = 2\cos^2 x + \cos x - 1$

$0 = (2\cos x - 1)(\cos x + 1)$ — Test

$2\cos^2 x + 2\cos x - \cos x - 1$

$2\cos^2 x + \cos x - 1$

$2\cos x - 1 = 0$

$+1 \quad +1$

$\frac{2\cos x}{2} = \frac{1}{2}$

$\cos x = \frac{1}{2}$

$\cos x = \frac{1}{2}$ at $\pi/3$

$x = \frac{\pi}{3}$

$\cos x + 1 = 0$

$-1 \quad -1$

$\cos x = -1$

$\cos x = -1$ at π

$x = \pi$

$[0, 2\pi)$

$(\sin x)(2\cos x + 1) = 0$

$x = \theta$

$\boxed{\sin\theta = 0}$

$\boxed{0, \pi}$

$2\cos\theta + 1 = 0$

$-1 \quad = -1$

$\frac{2\cos\theta}{2} = \frac{-1}{2}$

$\downarrow$

$\boxed{\cos\theta = -1/2}$

$\boxed{\frac{2\pi}{3}, \frac{4\pi}{3}}$

Solutions with $[0, 2\pi)$

$0, \pi, \frac{2\pi}{3}, \frac{4\pi}{3}$

Algebraic Checks

$\sin x = 0$

$\boxed{0 = 0}$

$2\left(-\frac{1}{2}\right) + 1 = 0$

$-1 \quad + 1 = 0$

$\boxed{0 = 0}$