1.

$$\frac{d}{dx}\left[\frac{x^3}{3}+\frac{x^2}{2}+2x+C\right] = \frac{1}{3}\frac{d}{dx}\left[x^3\right] + \frac{1}{2}\frac{d}{dx}\left[x^2\right] + 2\frac{d}{dx}[x] + \frac{d}{dx}[C]$$

$$=\frac{1}{3}\cdot 3x^2 + \frac{1}{2}\cdot 2x + 2\cdot 1 + 0$$

$$=x^2+x+2$$

2.

$$\frac{d}{dx}\left[\frac{x^4}{4}+x^2+x+C\right] = \frac{1}{4}\frac{d}{dx}\left[x^4\right] + \frac{d}{dx}\left[x^2\right] + \frac{d}{dx}[x] + \frac{d}{dx}[C]$$

$$=\frac{1}{4}\cdot 4x^3 + 2x + 1 + 0$$

$$=x^3 + 2x + 1$$

3.

$$\frac{d}{dx}[x\sin(x)+C] = \frac{d}{dx}[x\sin(x)] + \frac{d}{dx}[C]$$

$$=[1\cdot\sin(x) + x\cdot\cos(x)] + 0$$

$$=\sin(x) + x\cos(x)$$

4.

$$\frac{d}{dx}[x\cos(x)+C] = \frac{d}{dx}[x\cos(x)] + \frac{d}{dx}[C]$$

$$= 1\cdot\cos(x) + x\cdot(-\sin(x)) + 0$$

$$= \cos(x) - x\sin(x)$$

5.

$$\frac{d}{dx}\left[2\sqrt{x^2+x}+C\right] = 2\frac{d}{dx}\left[(x^2+x)^{1/2}\right] + \frac{d}{dx}[C]$$

$$= 2\cdot\frac{1}{2}(x^2+x)^{-1/2}(2x+1)+0$$

$$= \frac{2x+1}{\sqrt{x^2+x}}$$

6.

$$\frac{d}{dx}\left[\sqrt{x^2+1}+C\right] = \frac{d}{dx}\left[(x^2+1)^{1/2}\right] + \frac{d}{dx}[C]$$

$$= \frac{1}{2}(x^2+1)^{-1/2}(2x)+0$$

$$= \frac{x}{\sqrt{x^2+1}}$$

7.

$$\frac{d}{dx}\left[\frac{x}{x^2+1}+C\right] = \frac{d}{dx}\left[\frac{x}{x^2+1}\right]+\frac{d}{dx}[C]$$

$$=\left[\frac{(x^2+1)\cdot 1 - x\cdot(2x)}{(x^2+1)^2}\right]+0$$

$$=\frac{1-x^2}{(x^2+1)^2}$$

8.

$$\frac{d}{dx}\left[\frac{2x}{x+1}+C\right] = \frac{d}{dx}\left[\frac{2x}{x+1}\right]+\frac{d}{dx}[C]$$

$$=\left[\frac{(x+1)\cdot 2 - 2x\cdot 1}{(x+1)^2}\right]+0$$

$$=\frac{2}{(x+1)^2}$$

9. Answer: $\int 3x^2\,dx = x^3 + C$

Detailed Solution:

$$\int 3x^2\,dx = 3\int x^2\,dx$$

$$= 3\cdot\frac{x^3}{3} + C$$

$$= x^3 + C$$

10. $\int 2x^3\,dx = \frac{x^4}{2} + C$

11. Answer: $\int 2t + 1\,dt = t^2 + t + C$

Detailed Solution:

$$\int 2t + 1\,dt = 2\int t\,dt + \int 1\,dt$$

$$= 2\cdot\frac{t^2}{2} + t + C$$

$$= t^2 + t + C$$

12. $\int 2t^2 + 4\,dt = \frac{2}{3}t^3 + 4t + C$

13. Answer: $\int (t-3)(t+1)\ dt = \frac{1}{3}t^3 - t^2 - 3t + C$

Detailed Solution:

$$\int (t-3)(t+1)\ dt = \int t^2 - 2t - 3\ dt$$

$$= \int t^2\,dt - 2\int t\,dt - \int 3\,dt$$

$$= \frac{t^3}{3} - 2\cdot\frac{t^2}{2} - 3t + C$$

$$= \frac{1}{3}t^3 - t^2 - 3t + C$$

14. $\int (2t-1)(t+2)\ dt\ = \frac{2}{3}t^3 + \frac{3}{2}t^2 - 2t + C$

15. Answer: $\int \frac{x+\sqrt{x}}{x}dx \;\; = x-\sqrt{x}+C$

Detailed Solution:

$$\int \frac{x+\sqrt{x}}{x}dx = \int \frac{x}{x}+\frac{\sqrt{x}}{x}dx$$

$$= \int 1+x^{-1/2}\,dx$$

$$= \int 1\,dx \;\; + \;\; \int x^{-1/2}\,dx$$

$$= x \; - \; x^{1/2} \; + C$$

$$= x-\sqrt{x}+C$$

16. $\int \frac{x^2+x}{x}dx \;\; = \; \frac{1}{2}x^2+x+C$

17. Answer: $\int \frac{x+1}{\sqrt{x}}dx \;= \frac{2}{3}x^{3/2} + 2\sqrt{x} + C$

Detailed Solution:

$$\int \frac{x+1}{\sqrt{x}}dx \;= \int \frac{x}{\sqrt{x}} + \frac{1}{\sqrt{x}}dx$$

$$= \int x^{1/2} + x^{-1/2}\; dx$$

$$= \int x^{1/2}dx + \int x^{-1/2}dx$$

$$= \frac{x^{3/2}}{3/2} + \frac{x^{1/2}}{1/2} + C$$

$$= \frac{2}{3}x^{3/2} + 2\sqrt{x} + C$$

18. $\int \frac{x^2 + x}{\sqrt{x}}dx = \frac{2}{5}x^{5/2} + \frac{2}{3}x^{3/2} + C$

19. Answer: $\int \sin(u)+u\,du = -\cos(u)+\frac{1}{2}u^2+C$

Detailed Solution:

$$\int \sin(u)+u\,du = \int \sin(u)\,du + \int u\,du$$

$$= -\cos(u)+\frac{1}{2}u^2+C$$

20. $\int \cos(u)-\sin(u)\,du = \sin(u)+\cos(u)+C$

21. Answer: $\int |x+1|\ dx = \begin{cases} \frac{1}{2}x^2 + x + C_1 & \text{if } x \geq -1 \\ -\frac{1}{2}x^2 - x + C_2 & \text{if } x < -1 \end{cases}$

Detailed Solution:

We need to consider two cases: $x + 1 \geq 0$ and when $x + 1 < 0$.
In the case that $x + 1 \geq 0$:

$$\int |x+1|\ dx = \int x+1\ dx = \frac{1}{2}x^2 + x + C_1$$

In the case that $x + 1 < 0$:

$$\int |x+1|\,dx = \int -(x+1)\ dx = -\frac{1}{2}x^2 - x + C_2.$$

Thus:

$$\int |x+1|\,dx = \begin{cases} \frac{1}{2}x^2 + x + C_1 & \text{if } x \geq -1 \\ -\frac{1}{2}x^2 - x + C_2 & \text{if } x < -1 \end{cases}$$

22. $\int |2x-1|\,dx = \begin{cases} x^2 - x + C_1 & \text{if } x \geq \frac{1}{2} \\ -x^2 + x + C_2 & \text{if } x < \frac{1}{2} \end{cases}$

23. Answer: $\int \sqrt{2t}\, dt = \frac{2\sqrt{2}}{3} t^{3/2} + C$

Detailed Solution:

$$\int \sqrt{2t}\, dt = \sqrt{2}\int t^{1/2}\, dt$$

$$= \sqrt{2}\,\frac{t^{3/2}}{3/2} + C$$

$$= \frac{2\sqrt{2}}{3} t^{3/2} + C$$

24. $\int \sqrt{4t}\, dt = \frac{4}{3} t^{3/2} + C$

25. Answer: $\int_0^2 x^2 - x\, dx = \frac{2}{3}$

Detailed Solution:

$$\int_0^2 x^2 - x\, dx = \left[\frac{x^3}{3} - \frac{x^2}{2}\right]_0^2$$

$$= \left[\frac{2^3}{3} - \frac{2^2}{2}\right] - \left[\frac{0^3}{3} - \frac{0^2}{2}\right]$$

$$= \frac{2}{3}$$

26. $\int_1^2 x^3 - 2x\, dx = \frac{3}{4}$

27. Answer: $\int_0^1 \sin(x) + x dx = \frac{3}{2} - \cos(1)$

Detailed Solution:

$$\int_0^1 \sin(x) + x\, dx = \left[-\cos(x) + \frac{x^2}{2}\right]_0^1$$

$$= \left[-\cos(1) + \frac{1^2}{2}\right] - \left[-\cos(0) + \frac{0^2}{2}\right]$$

$$= \frac{3}{2} - \cos(1)$$

28. $\int_0^1 \cos(x) - x\, dx = -\frac{1}{2} + \sin(1)$

29. Answer: $\int_1^4 \left(x+\frac{1}{x}\right)^2 dx = \frac{111}{4}$

Detailed Solution:

$$\int_1^4 \left(x+\frac{1}{x}\right)^2 dx = \int_1^4 x^2+2+\frac{1}{x^2}dx$$

$$= \int_1^4 x^2+2+x^{-2}\ dx$$

$$= \left[\frac{x^3}{3}+2x-x^{-1}\right]_1^4$$

$$= \left[\frac{4^3}{3}+2(4)-4^{-1}\right]-\left[\frac{1^3}{3}+2(1)-1^{-1}\right]$$

$$= \frac{111}{4}$$

30. $\int_1^2 \left(2x-\frac{1}{x}\right)^2 dx = \frac{35}{6}$

31. Answer: $\int_1^4 \frac{u+1}{\sqrt{u}} du = \frac{20}{3}$

Detailed Solution:

$$\int_1^4 \frac{u+1}{\sqrt{u}} du = \int_1^4 \frac{u}{\sqrt{u}} + \frac{1}{\sqrt{u}} du$$

$$= \int_1^4 u^{1/2} + u^{-1/2} du$$

$$= \left[\frac{u^{3/2}}{3/2} + \frac{u^{1/2}}{1/2} \right]_1^4$$

$$= \left[\frac{4^{3/2}}{3/2} + \frac{4^{1/2}}{1/2} \right] - \left[\frac{1^{3/2}}{3/2} + \frac{1^{1/2}}{1/2} \right]$$

$$= \frac{20}{3}$$

32. $\int_4^9 \frac{u^2+u}{\sqrt{u}} du = \frac{1456}{15}$

33. Answer: $\int_{-2}^{-1} |v+1|\,dv = \dfrac{1}{2}$

Detailed Solution:

Over the interval [-2, -1], |v + 1| = -(v + 1). Thus:

$$\int_{-2}^{-1} |v+1|\,dv = \int_{-2}^{-1} -(v+1)\,dv$$

$$= \left[-\frac{v^2}{2} - v\right]_{-2}^{-1}$$

$$= \left[-\frac{(-1)^2}{2} - (-1)\right] - \left[-\frac{(-2)^2}{2} - (-2)\right]$$

$$= \frac{1}{2}$$

34. $\int_{1}^{2} |1-v|\,dv = \dfrac{1}{2}$

35. a) $\int_{0}^{5} h'(t)\,dt$ represents the change (total growth) in height of the individual after the first five years.

b) inches

36. a) $\int_{0}^{5} w'(t)\,dt$ represents the change (total increase) in weight of the individual after the first five years.

b) pounds

37. a) $\int_0^{10} r(t)\,dt$ represents the change in water volume (total volume of water leakage) after the first ten minutes.

    b) gallons

38. a) $\int_0^{1} v(t)\,dt$ represents the displacement (change in distance) of the car after the first hour.

    b) miles

39. a) $\int_{30}^{60} r(t)\,dt$ represents the change in temperature (total decrease) after 30 minutes starting from 30 minutes.

    b) Fahrenheit

40. a) $\int_0^{5} r(t)\,dt$ represents the change in temperature (total increase) after the first five seconds.

    b) Fahrenheit

41. Answer: Displacement of 48 meters, traveled a total of 50 meters in first four seconds. The particle traveled 1 meter in the negative direction, and then 49 meters in the positive direction.

Detailed Solution:

To calculate the displacement (overall change in position) of the particle, we apply the net change theorem. The displacement of the particle is given by:

$$\int_0^4 t^3 - 2t\, dt = \left[\frac{t^4}{4} - t^2\right]_0^4$$

$$= \left[\frac{4^4}{4} - 4^2\right] - \left[\frac{0^4}{4} - 0^2\right]$$

$$= [64 - 16] - [0]$$

$$= 48$$

We interpret this to mean that the displacement of the particle is 48 meters after four seconds.

To calculate the distance traveled, we need to calculate: $\int_0^4 \left|t^3 - 2t\right| dt$

To see why this is so, remember that the particle is traveling along a straight line. Thus, we associate positive velocity as moving forward along the line, and negative velocity as moving backwards along the line. Looking at |v(t)| takes into account the distance traveled whether or not the particle moves forward or backwards.

Setting $v(t) = t^3 - 2t$ equal to 0, we see that v(t) = 0 when t = 0 or t = $\sqrt{2}$, thus dividing the interval [0, 4] into two subintervals [0, $\sqrt{2}$] and [$\sqrt{2}$, 4].

On the first subinterval, v(t) is non-positive. On the second interval, v(t) is non-negative.

Thus, $|v(t)| = -(t^3 - 2t)$ on [0, $\sqrt{2}$] and $|v(t)| = t^3 - 2t$ on [$\sqrt{2}$, 4].

41. Continued:

$$\int_0^4 |v(t)|\,dt = \int_0^4 |t^3 - 2t|\,dt$$

$$= \int_0^{\sqrt{2}} |t^3 - 2t|\,dt + \int_{\sqrt{2}}^4 |t^3 - 2t|\,dt$$

$$= \int_0^{\sqrt{2}} -\left(t^3 - 2t\right)dt + \int_{\sqrt{2}}^4 t^3 - 2t\,dt$$

$$= \int_0^{\sqrt{2}} -t^3 + 2t\,dt + \int_{\sqrt{2}}^4 t^3 - 2t\,dt$$

$$= \left[-\frac{t^4}{4} + t^2\right]_0^{\sqrt{2}} + \left[\frac{t^4}{4} - t^2\right]_{\sqrt{2}}^4$$

$$= \left[\left(-\frac{\left(\sqrt{2}\right)^4}{4} + \left(\sqrt{2}\right)^2\right) - \left(-\frac{(0)^4}{4} + (0)^2\right)\right] + \left[\left(\frac{(4)^4}{4} - (4)^2\right) - \left(\frac{\left(\sqrt{2}\right)^4}{4} - \left(\sqrt{2}\right)^2\right)\right]$$

$$= \left[\left(-\frac{4}{4} + 2\right) - (0)\right] + \left[\left(\frac{256}{4} - 16\right) - \left(\frac{4}{4} - 2\right)\right]$$

$$= \left[(-1 + 2) - (0)\right] + \left[(64 - 16) - (1 - 2)\right]$$

$$= 1 + 49$$

The particle travels a total of 50 meters during the first four seconds.

The particle traveled 1 meter in the negative direction, and then 49 meters in the positive direction. This places its position on the number line, in meters, at 48 meters, which we calculated as the displacement.

42. Displacement of 0 meters
 Traveled a total of 21.33 meters

43. Answer: 2.4t ft^3
 Detailed Solution:
 By the net change theorem, the change in volume as a function of time is given by:

$$\Delta v(t) = \int_0^t v'(t)\, dt$$

$$= \int_0^t 2.4\, dt$$

$$= 2.4t\Big|_0^t$$

$$= 2.4t\ ft^3$$

44. 3.2t ft^3