Complete The Square

$$x^2 + 5x + 6 = 0$$
$$(x+2)(x+3) = 0$$

$$
\begin{array}{cc}
x + 2 = 0 & x + 3 = 0 \\
\underline{-2 \quad -2} & \underline{-3 \quad -3} \\
\boxed{x = -2} & \boxed{x = -3}
\end{array}
$$

$$(x^2 + 5x) + 6 = 0$$

$$\frac{1}{2}(5) = \boxed{\frac{5}{2}}^2 = \frac{25}{4}$$

$$(x^2 + 5x + 25/4) + 6 = 0 \longrightarrow (x^2 + 5x + 25/4) + 6 = 0 + 25/4$$

$$\left(x + \boxed{\frac{5}{2}}\right)^2 = \left(x^2 + 5x + \frac{25}{4}\right)$$

$$\left(x + \frac{5}{2}\right)^2 + 6 = \frac{25}{4}$$

$$\underline{\qquad -6 \quad -6 \qquad}$$

$$\left(x + \frac{5}{2}\right)^2 = \frac{25}{4} - 6$$

$$\left(x + \frac{5}{2}\right)^2 = \frac{25}{4} - \frac{24}{4} = \frac{1}{4}$$

$$\frac{25}{4} = \frac{25}{4}$$
$$\frac{-6}{1} = \frac{24}{4}$$
$$\underline{\qquad}$$
$$\frac{1}{4}$$

$$\left(x + \frac{5}{2}\right)^2 = \frac{1}{4} \longrightarrow \sqrt{\left(x + \frac{5}{2}\right)^2} = \pm\sqrt{\frac{1}{4}}$$

$$x + \frac{5}{2} = \pm\frac{\sqrt{1}}{\sqrt{4}} = \pm\frac{1}{2} \longrightarrow x + \frac{5}{2} = \pm\frac{1}{2}$$

$$
\begin{array}{cc}
x + \dfrac{5}{2} = \dfrac{1}{2} & x + \dfrac{5}{2} = -\dfrac{1}{2} \\[2mm]
\underline{-\dfrac{5}{2} = -\dfrac{5}{2}} & \underline{-\dfrac{5}{2} = -\dfrac{5}{2}} \\[2mm]
x = \dfrac{-4}{2} = -2 & x = \dfrac{-6}{2} = -3 \\[2mm]
\boxed{x = -2} & \boxed{x = -3}
\end{array}
$$

$$2x^2 + 8x - 10 = 0$$

$$(2x^2 + 8x) - 10 = 0$$

$$2(x^2 + 4x + 4) - 10 = 0 + 8$$

$2 \times 4 = 8$

$$\tfrac{1}{2}(4) = (2)^2$$

Add 8 here because coefficient 2 times 4 = 8.

If you are adding 8 to the left side then you need to add 8 to the right side.

$$2(x^2 + 4x + 4) = 2(x + 2)^2$$

$$2(x + 2)^2 - 10 = 8$$
$$ +10 \quad +10$$

$$\frac{2(x + 2)^2}{2} = \frac{18}{2}$$

$$\sqrt{(x + 2)^2} = \pm\sqrt{9}$$

$$x + 2 = \pm 3 \rightarrow$$

$$x + 2 = 3$$
$$-2 = -2$$
$$\boxed{x = 1}$$

$$x + 2 = -3$$
$$-2 = -2$$
$$\boxed{x = -5}$$

Different Technique

$$2x^2 - 6x + 3 = 0$$

$$\downarrow$$

$$2(x^2 - 3x) + 3 = 0$$

$$\downarrow$$

$$\tfrac{1}{2}(3) = \left(\tfrac{3}{2}\right) = \tfrac{9}{4}$$

$$\downarrow$$

$$\tfrac{2}{1}\left(x^2 - 3x + \tfrac{9}{4}\right) + 3 = 0$$

$$\tfrac{2}{1} \cdot \tfrac{9}{4} = \tfrac{18}{4} = \boxed{\tfrac{9}{2}}$$

$$\downarrow \qquad\qquad \downarrow$$

$$2\left(x^2 - 3x + \tfrac{9}{4}\right) - \tfrac{9}{2} + 3 = 0$$

$$\downarrow$$

$$2\left(x - \tfrac{3}{2}\right)^2 - \tfrac{9}{2} = 0$$

$$+ \tfrac{9}{2} + \tfrac{9}{2}$$

$$\overline{}$$

$$2\left(x - \tfrac{3}{2}\right)^2 = \tfrac{9}{2}$$

$$\downarrow$$

$$\tfrac{1}{2} \cdot \tfrac{2}{1}\left(x - \tfrac{3}{2}\right)^2 = \tfrac{9}{2} \cdot \tfrac{1}{2}$$

$$\downarrow$$

$$\left(x - \tfrac{3}{2}\right)^2 = \tfrac{9}{4}$$

$$\sqrt{\left(x - \tfrac{3}{2}\right)^2} = \pm\sqrt{\tfrac{9}{4}}$$

$$\downarrow$$

$$\left(x - \tfrac{3}{2}\right) = \pm\frac{\sqrt{3}}{\sqrt{4}} = \pm\frac{\sqrt{3}}{2}$$

$$\downarrow$$

$$x - \tfrac{3}{2} = \pm\frac{\sqrt{3}}{2}$$

$$+ \tfrac{3}{2} \qquad + \tfrac{3}{2}$$

$$\overline{}$$

Right side:

$$-\tfrac{9}{2} = -\tfrac{9}{2}$$

$$+3 = \tfrac{6}{2}$$

$$\overline{}$$

$$-\tfrac{3}{2}$$

$$\rightarrow \quad x = \tfrac{3}{2} \pm \frac{\sqrt{3}}{2}$$

$$\boxed{x = \tfrac{3}{2} + \frac{\sqrt{3}}{2} \;,\; x = \tfrac{3}{2} - \frac{\sqrt{3}}{2}}$$