

Limits and Vertical Asymptotes

$$f(x) = \frac{-3x^2}{x^2 - 9}$$

Find:

$$\lim_{x \rightarrow -3^-} \frac{-3x^2}{x^2 - 9}$$

$$\lim_{x \rightarrow -3^+} \frac{-3x^2}{x^2 - 9}$$

$$\lim_{x \rightarrow 3^-} \frac{-3x^2}{x^2 - 9}$$

$$\lim_{x \rightarrow 3^+} \frac{-3x^2}{x^2 - 9}$$

Im riding on this piece
function approaching
-3 from the right!

$$\lim_{x \rightarrow -3^+} \frac{-3x^2}{x^2 - 9} = \infty$$

Im riding on this
piece approaching 3
from the left!

$$\lim_{x \rightarrow 3^-} \frac{-3x^2}{x^2 - 9} = \infty$$

$x = 3$ vertical asymptote

Im riding on this piece
function approaching -3

So!

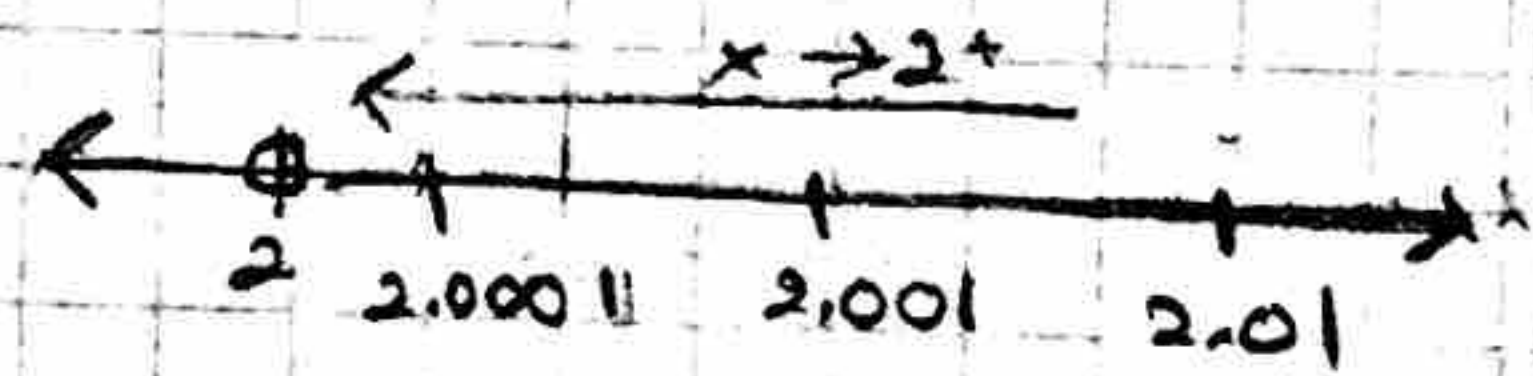
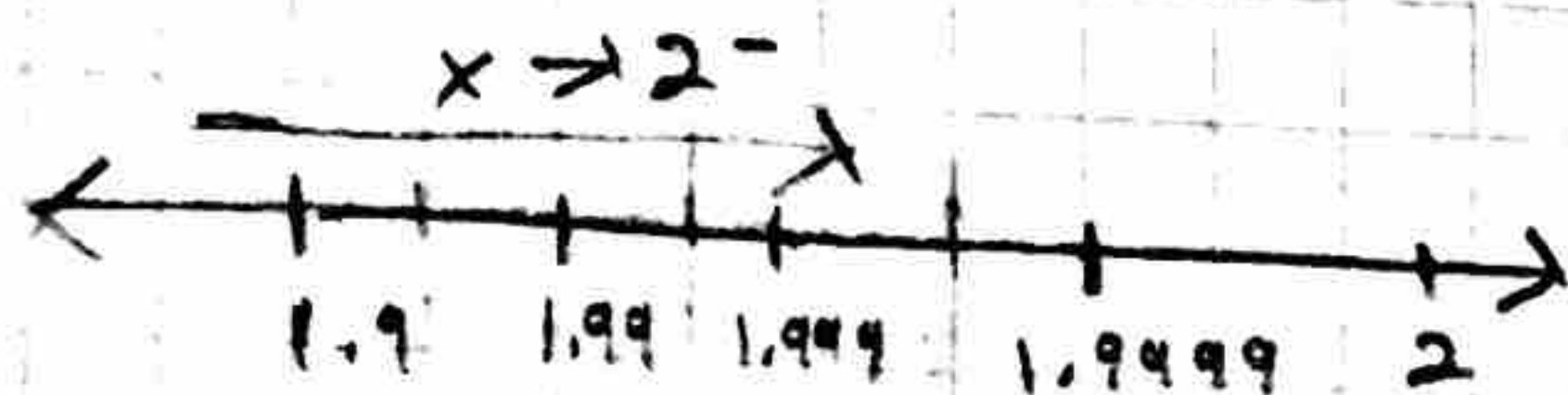
$$\lim_{x \rightarrow -3^-} \frac{-3x^2}{x^2 - 9} = -\infty$$

Im riding on this piece
as I approach 3 from the
right.

$$\lim_{x \rightarrow 3^+} \frac{-3x^2}{x^2 - 9} = -\infty$$

I can approach -3 all the way from positive ∞ . Riding
~~But it's~~ two pieces, But it is the last piece to
ride that matters. This is my stupid way of
understanding whats going on here.

$$\lim_{x \rightarrow 2} \frac{x^2 + 2x - 8}{x^2 - 4}$$



x	$x^2 + 2x - 8 / x^2 - 4$
1.9	1.51282
1.99	1.50125
1.999	1.50013
1.9999	1.50001

x	$x^2 + 2x - 8 / x^2 - 4$
2.01	1.49875
2.001	1.49988
2.0001	1.49999

$$\lim_{x \rightarrow 2} \frac{x^2 + 2x - 8}{x^2 - 4} = 1.5$$

$$\begin{aligned} x^2 - 4 &= 0 \\ +4 \quad +4 \\ \hline \sqrt{x^2 - 4} &= \pm \sqrt{4} \\ &= \pm 2 \\ x &= \pm 2 \end{aligned}$$

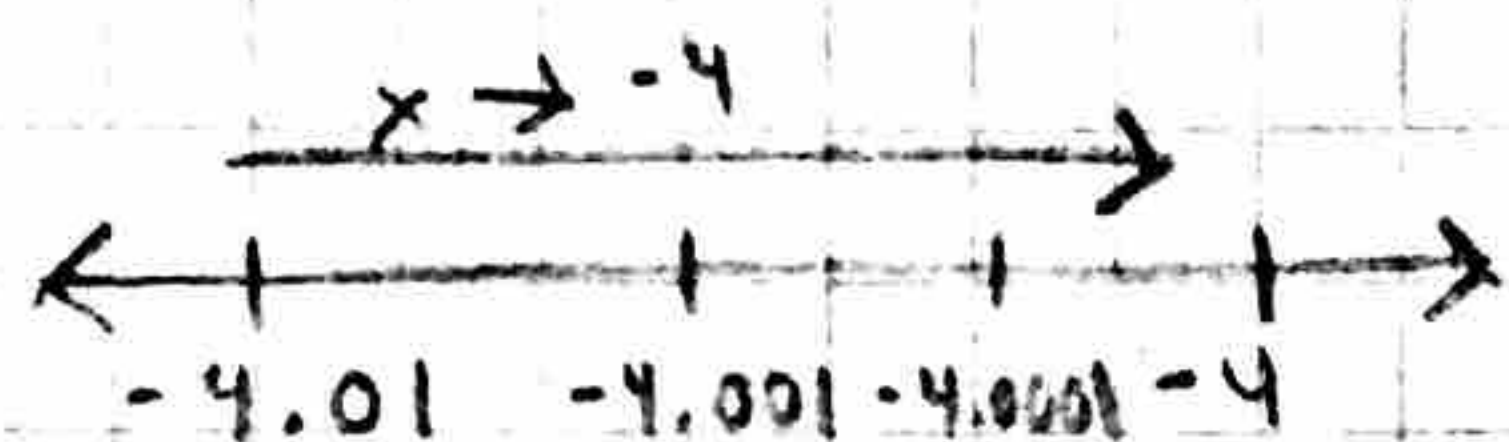
Asymptote Form
 $\frac{x}{0}$

$$\begin{aligned} \frac{(4)^2 + 2(4) - 8}{(4)^2 - 4} \\ \frac{16 + 8 - 8}{16 - 4} \\ \frac{16}{12} \\ \frac{4}{3} \end{aligned}$$

No Asymptote at $x = 2$

$$\begin{aligned} \frac{(-4)^2 + 2(-4) - 8}{(-4)^2 - 4} \\ \frac{16 - 8 - 8}{16 - 4} \\ \frac{16 - 16}{12} \\ \frac{0}{12} = 0 \end{aligned}$$

43. $\lim_{x \rightarrow -4} \frac{x^2 + 6x + 8}{x^2 - 16}$



x	$\frac{x^2 + 6x + 8}{x^2 - 16}$
-4.01	.250936
-4.001	.250094
-4.0001	.250009

$$\lim_{x \rightarrow -4} \frac{x^2 + 6x + 8}{x^2 - 16} = .25$$

$$\begin{aligned} x^2 - 16 &= 0 \\ +16 \quad +16 \\ \hline \sqrt{x^2 - 16} &= \pm 4 \\ x &= \pm 4 \end{aligned}$$

$$\frac{124}{16}$$

$$\begin{aligned} \frac{(4)^2 + 6(4) + 8}{(4)^2 - 16} \\ \frac{16 + 24 + 8}{16 - 16} \\ \frac{48}{0} = \text{undefined} \end{aligned}$$

Vertical Asymptote
at $x = 4$

$$\begin{aligned} \frac{(-4)^2 + 6(-4) + 8}{(-4)^2 - 16} \\ \frac{16 - 24 + 8}{16 - 16} \end{aligned}$$

$$\frac{0}{0}$$

no Vertical
Asymptote
at $x = -4$