Limits Involving Absolute Value

$$\lim_{x \to 1} \frac{|x-1|}{x-1}$$

Recall $|x| = \begin{cases} -x \text{ or } -(x) & , x < 0 \\ x & , x \geq 0 \end{cases}$

$$|x-1| = \begin{cases} -(x-1) & , x < 1 \\ (x-1) & , x \geq 1 \end{cases}$$

① ② ③

Want to use 1 for x since $|1-1| = 0$

Use this from Piecewise function →

$$\lim_{x \to 1^-} \frac{-(x-1)}{(x-1)}$$

$$\lim_{x \to 1^-} -1$$

-1

$$\lim_{x \to 1^+} = \frac{(x-1)}{(x-1)}$$

$$\lim_{x \to 1^+} 1$$

1

This limit do not exist for

$$\lim_{x \to 1} \frac{|x-1|}{x-1}$$

$$\lim_{x\to 0^-} \frac{1}{x} - \frac{1}{|x|} \rightarrow$$

① $|x| = \begin{cases} -x & x<0 \\ x & x\geq 0 \end{cases}$

② x is approaching 0 from the left

③ Replace $|x|$ with $-x$

$$\lim_{x\to 0^-} \frac{1}{x} - \frac{1}{-x}$$

$$\lim_{x\to 0^-} \frac{1}{x} - \left(-\frac{1}{x}\right)$$

$$\lim_{x\to 0^-} \frac{1}{x} + \frac{1}{x}$$

④ Replace

$$\frac{1}{x} + \frac{1}{x}$$

$$\lim_{x\to 0^-} \frac{2}{x}$$

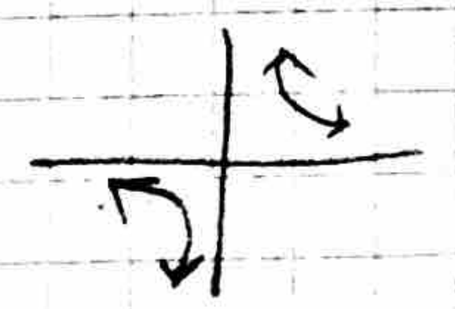
$$\lim_{x\to 0^-} \frac{2}{x} = \frac{2}{-.01} \quad \frac{2}{-.0001} \quad \frac{2}{-.00001}$$

$$\lim_{x\to 0^-} x$$

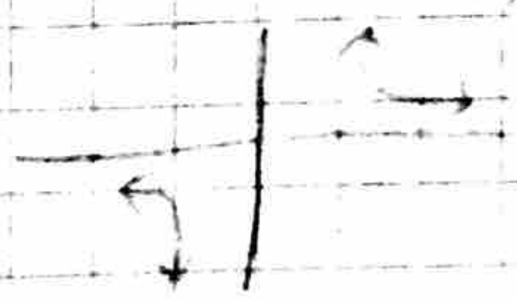
x will not be 0 and I am approaching 0 from the left, taking me to $-\infty$

$$= -\infty$$

Graph of $f(x) = \frac{1}{x}$



Graph of $f(x) = \frac{2}{x}$ Vertical Shift



$$\lim_{x\to 0} |x| \sin\left(\frac{1}{x}\right)$$

① Domain of $\sin$ $[-1, 1]$

② $$-1 \le |x| \sin\left(\frac{1}{x}\right) \le 1$$

$$-|x| \le |x| \sin\left(\frac{1}{x}\right) \le |x|$$

$$\lim_{x\to 0} -|x| \le \lim_{x\to 0} |x| \sin\left(\frac{1}{x}\right) \le \lim_{x\to 0} |x|$$

③ $$|x| = \begin{cases} -x, & x < 0 \\ x, & x \ge 0 \end{cases}$$

④ $$\lim_{x\to 0} -(-x) \le \lim_{x\to 0} |x| \sin\left(\frac{1}{x}\right) \le \lim_{x\to 0} (x)$$

$$\lim_{x\to 0} x \le \lim_{x\to 0} |x| \sin\left(\frac{1}{x}\right) \le \lim_{x\to 0} x$$

$$0 \le \lim_{x\to 0} |x| \sin\left(\frac{1}{x}\right) \le 0$$