MATH E-3: Lecture 9

Quantitative Reasoning: Practical Math

https://c2.staticflickr.com/4/3473/3819185169_8287e0f62c_b.jpg

Homework

- **Assignment 8** Excel extra credit – grades will be available April 16

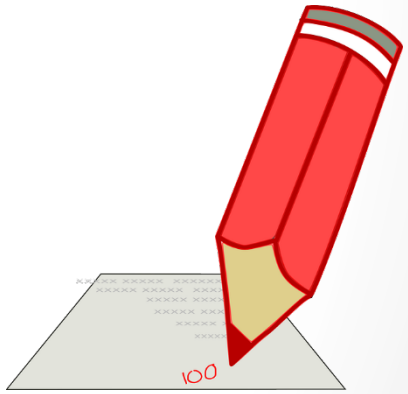
- **Assignment 9** will be posted tomorrow, April 13

2 quizzes down….

1 final exam to go……..

….and help is here…….if you

need it……..



RESOURCES

SECTIONS:

On campus – Tuesday, 5:30 pm, Sever 104

Online – Wednesday, 7:30 pm (ET),
"Conferences"

Math Question Center:

https://www.extension.harvard.edu/resources-policies/resources/math-question-center

And, a reminder that law enforcement is not a resource (at least not for this course):

http://www.wimp.com/kidmath

ANNOUNCEMENTS

November 3, 2015

Harvard Kennedy School Littauer Building

https://commons.wikimedia.org/wiki/File:Harvard_Kennedy_School_Littauer_Building.jpg

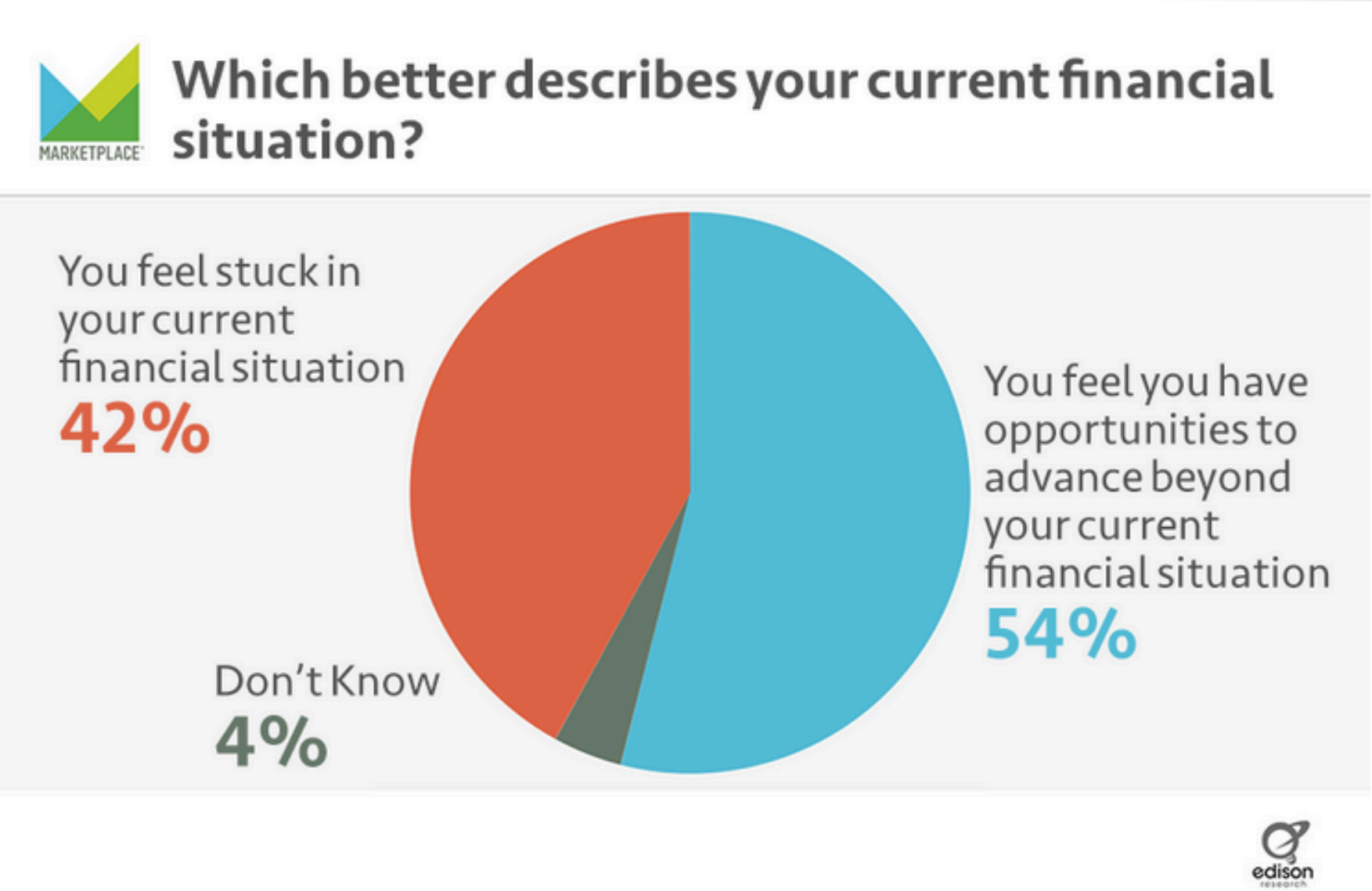
Math in the news . . .

“42 percent”

That's the percentage of people who said they feel stuck in their current financial situation, according to the first Marketplace-Edison Research Poll. (fall 2015)

By most measures, the American economy is improving. The unemployment rate has fallen, as have initial jobless claims. But wages seem to be stuck, and the economic recovery doesn’t feel like one to many Americans. Here are results from one of the questions asked:

http://www.marketplace.org



So with those findings in mind, it's no wonder people feel stuck – only a slim majority in the survey said they had the opportunity to advance financially.

The raw data . . . 79 pages in total!!

Marketplace Survey - Banner 1 - September 2015
Question 11A: Which better describes your current financial situation?
Edison Research

				Sex		Age						Race			Employment status	
			TOTAL	Men	Women	18-24	25-34	35-44	45-54	55-64	65+	White/Other	African-American	Hispanic	Employed full-time or part-time	Not currently employed
TOTAL		# of resp	1016	495	521	134	178	178	195	159	173	745	127	144	541	475
		Row %	100.0%	48.7%	51.3%	13.2%	17.5%	17.5%	19.2%	15.6%	17.0%	73.4%	12.5%	14.2%	53.2%	46.8%
Which better describes your current financial situation?	You feel stuck in your current financial situation	# of resp	427	179	248	50	76	77	68	69	87	308	46	73	170	257
		Col %	42.0%	36.1%	47.6%	37.2%	42.6%	43.1%	35.1%	43.3%	50.5%	41.3%	35.9%	50.9%	31.5%	54.0%
		Row %	100.0%	41.9%	58.1%	11.7%	17.7%	18.0%	16.0%	16.1%	20.5%	72.2%	10.7%	17.1%	39.9%	60.1%
	You feel you have opportunities to advance beyond your current financial situation	# of resp	548	295	253	81	101	99	123	78	66	401	78	69	362	186
		Col %	53.9%	59.6%	48.5%	60.3%	56.9%	55.9%	63.0%	49.3%	37.9%	53.8%	61.6%	47.8%	67.0%	39.1%
		Row %	100.0%	53.9%	46.1%	14.7%	18.5%	18.2%	22.4%	14.3%	12.0%	73.2%	14.3%	12.5%	66.1%	33.9%
	DK/NA	# of resp	41	21	20	3	1	2	4	12	20	36	3	2	9	33
		Col %	4.1%	4.3%	3.9%	2.4%	.5%	1.0%	1.9%	7.4%	11.6%	4.9%	2.5%	1.3%	1.6%	6.9%
		Row %	100.0%	50.7%	49.3%	7.9%	2.0%	4.2%	9.1%	28.3%	48.5%	87.6%	7.7%	4.6%	20.5%	79.5%

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So many numbers, and so many percents! Let's take a closer look . . .

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What is the 47.6%?

What is the 58.1%?

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What is the 47.6%? The percentage of women who feel “stuck”: 248/521

What is the 58.1%? The percentage of those “stuck” who are women: 248/427

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What is the 47.6%? The percentage of women who feel “stuck”: 248/521

What is the 58.1%? The percentage of those “stuck” who are women: 248/427

Why are they different??

Consider this more "extreme example . . .

Small survey on job satisfaction

	men	women	total
unsatisfied	5	1	6
satisfied	14	0	14
total	19	1	20

Linear Growth

This is seriously something completely different . . .

From probability to linear growth

Probability:

Uncertainty, possibilities, likelihoods, polls, surveys, margins of error, normal distributions . . .

Linear growth:

Observation of trends, calculation of growth or decline, slopes, intercepts, graphs . . .

Some things in common . . . Uncertainty of future predictions

Some new terminology . . .

- Coordinates
- Intercepts
- Axes
- Slopes – positive, negative, and zero

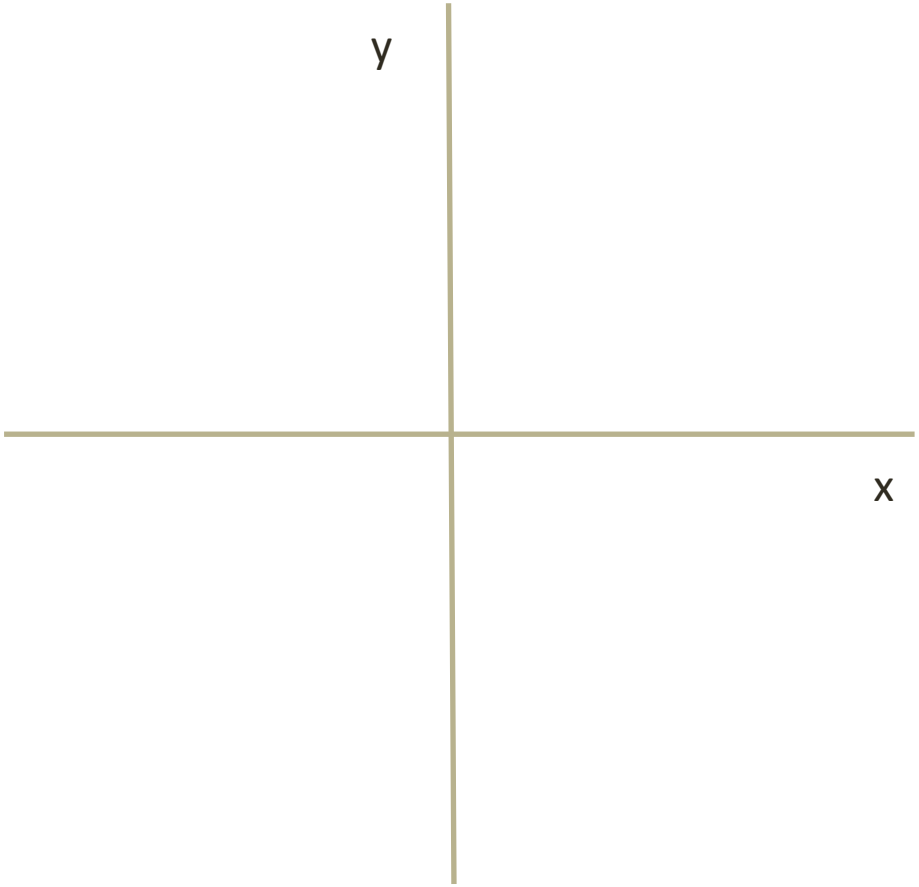
A step back . . .

A point has	0 dimensions	+ location	
A (straight) line has	1 dimension	+ location	+ direction
A (flat) surface has	2 dimensions	+ location	+ direction
A solid has	3 dimensions	+ location	+ direction
??? has	4 dimensions . . .		

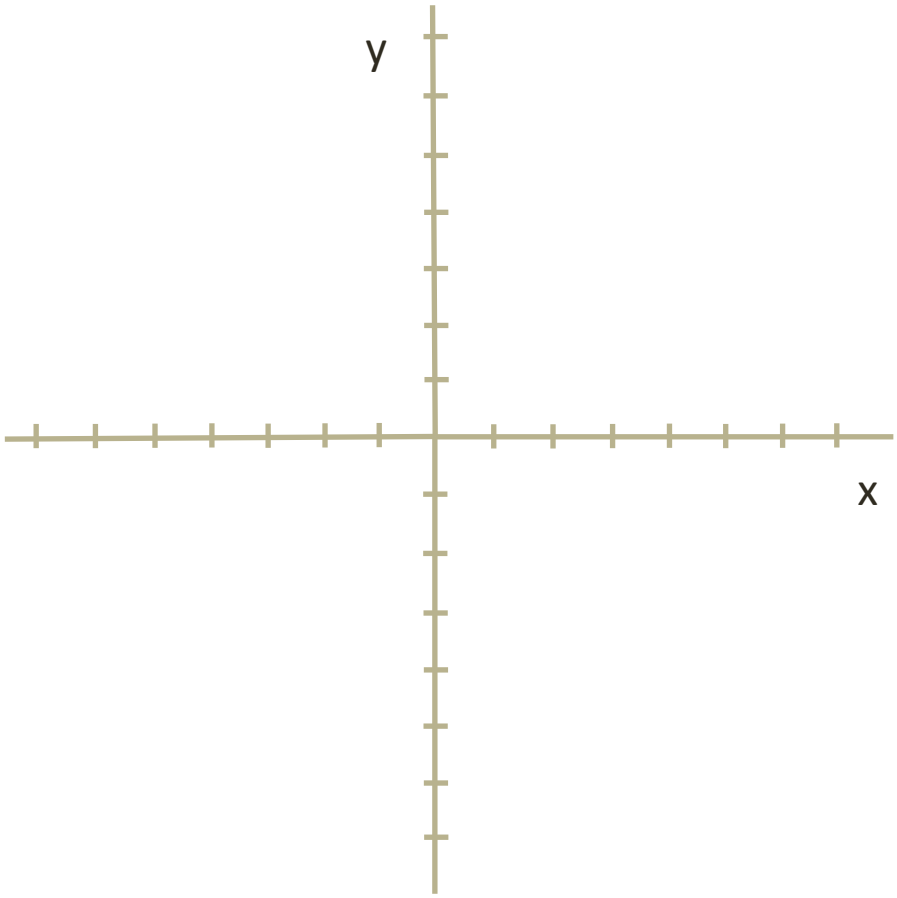
Our focus for now:

- Straight lines – they have:
- a location, which we'll call the "y-intercept,"
- and a direction, which we'll call the "slope."

The x-y axes



The x-y axes



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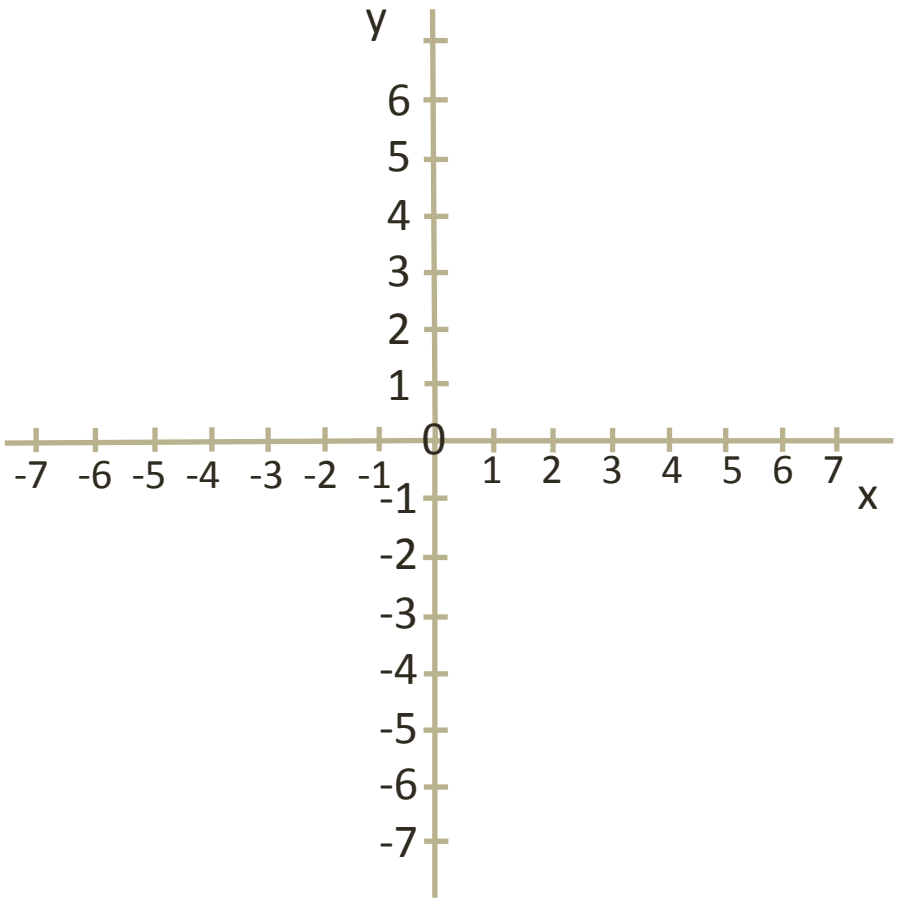
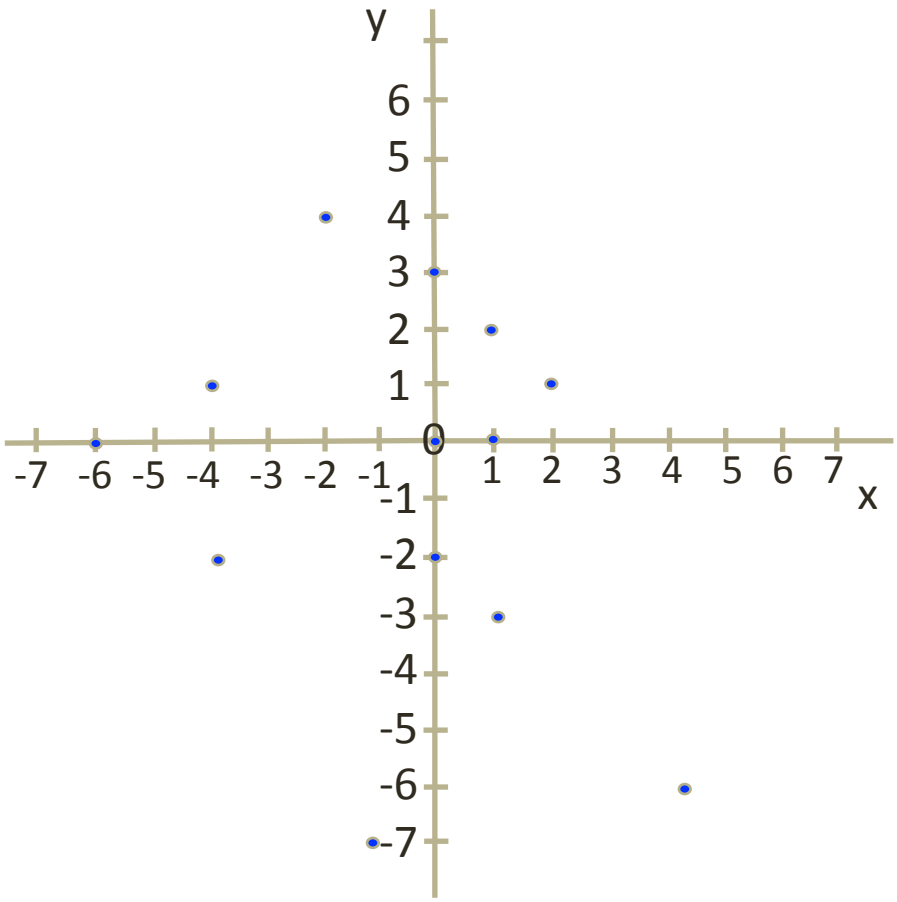
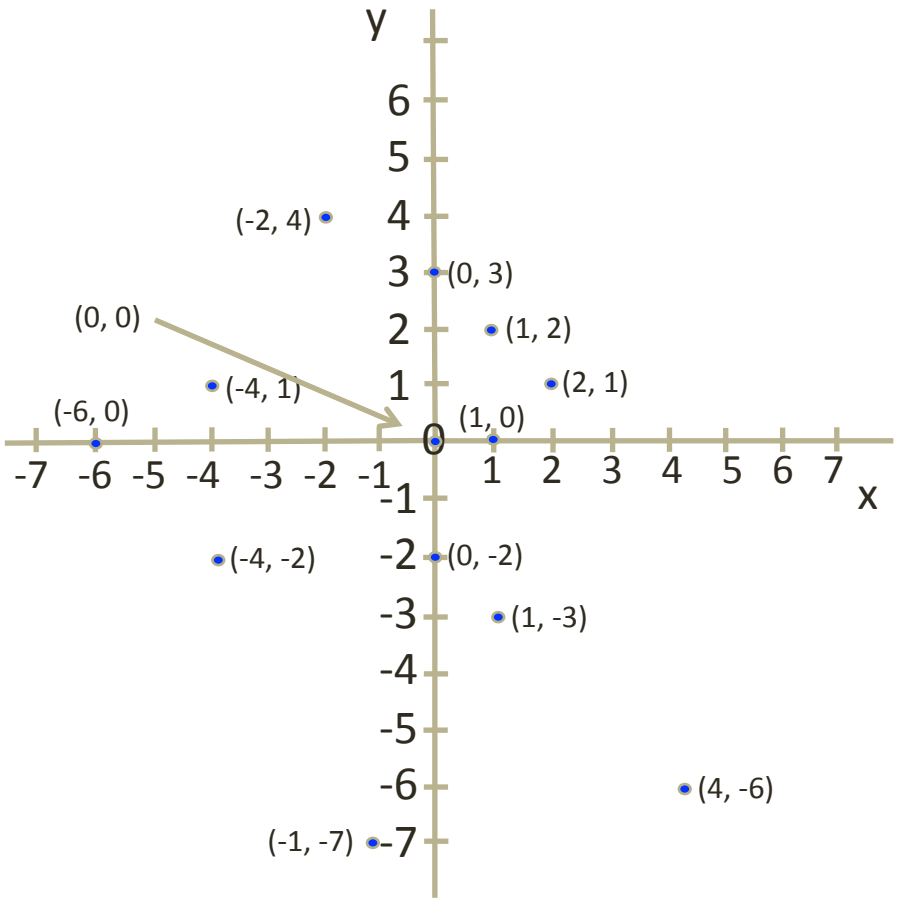


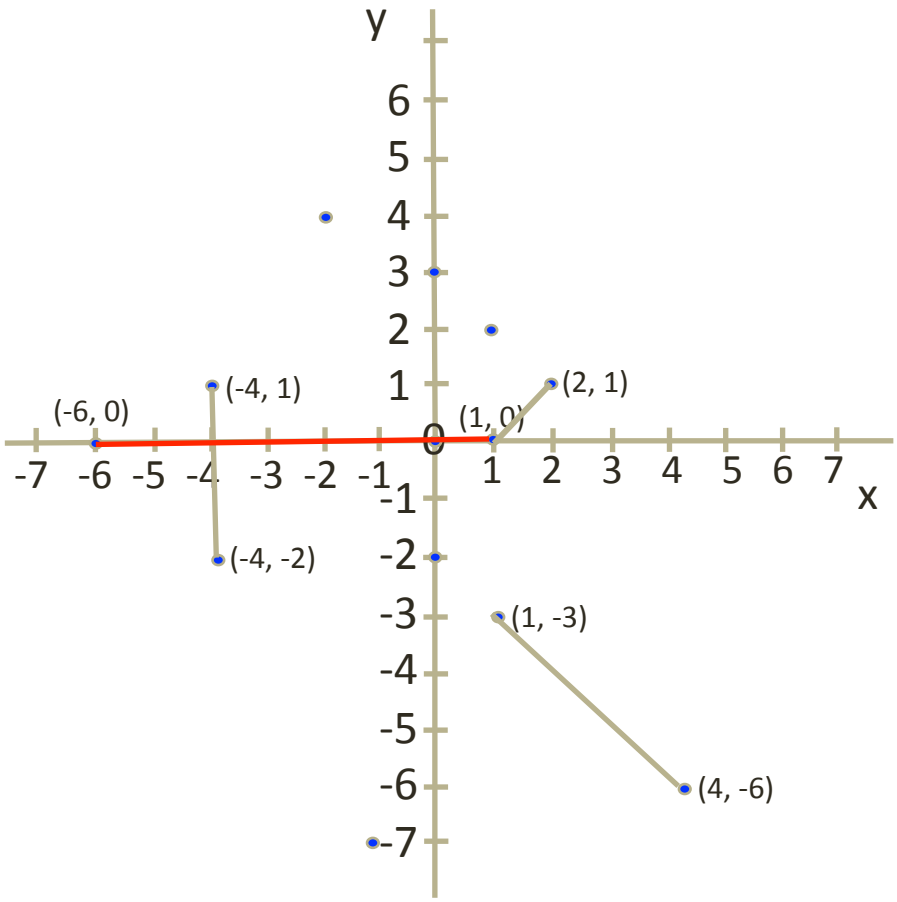
Points



Coordinates



Joining points to find the slope . . .



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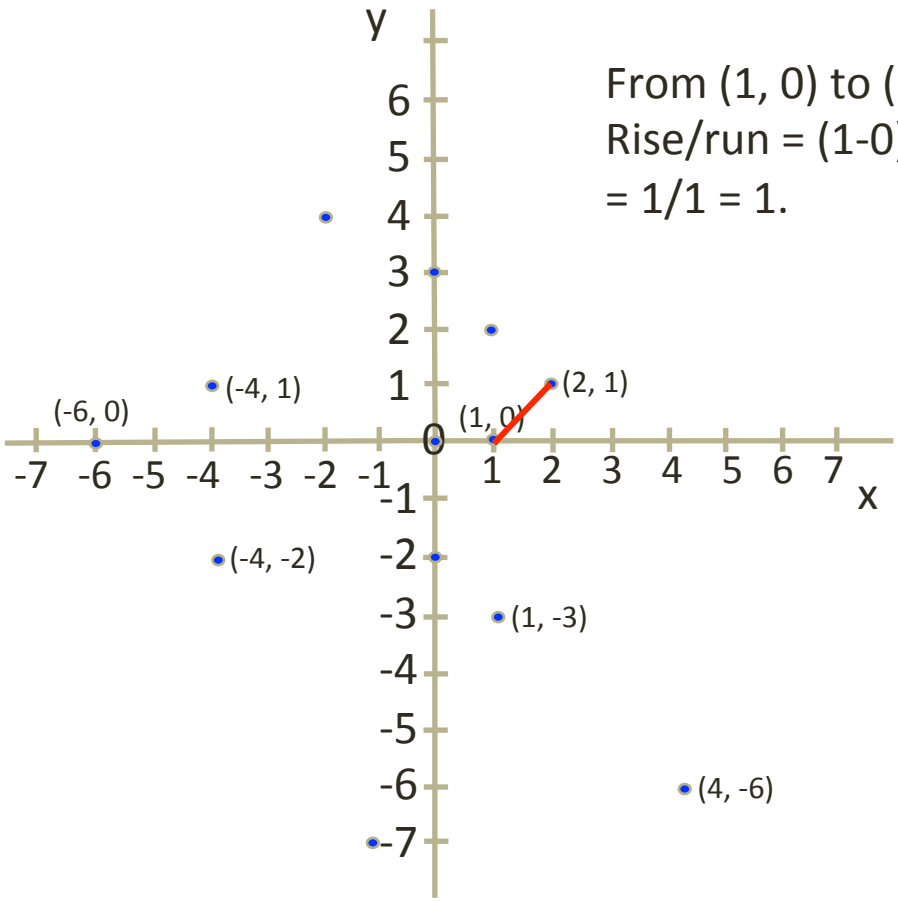
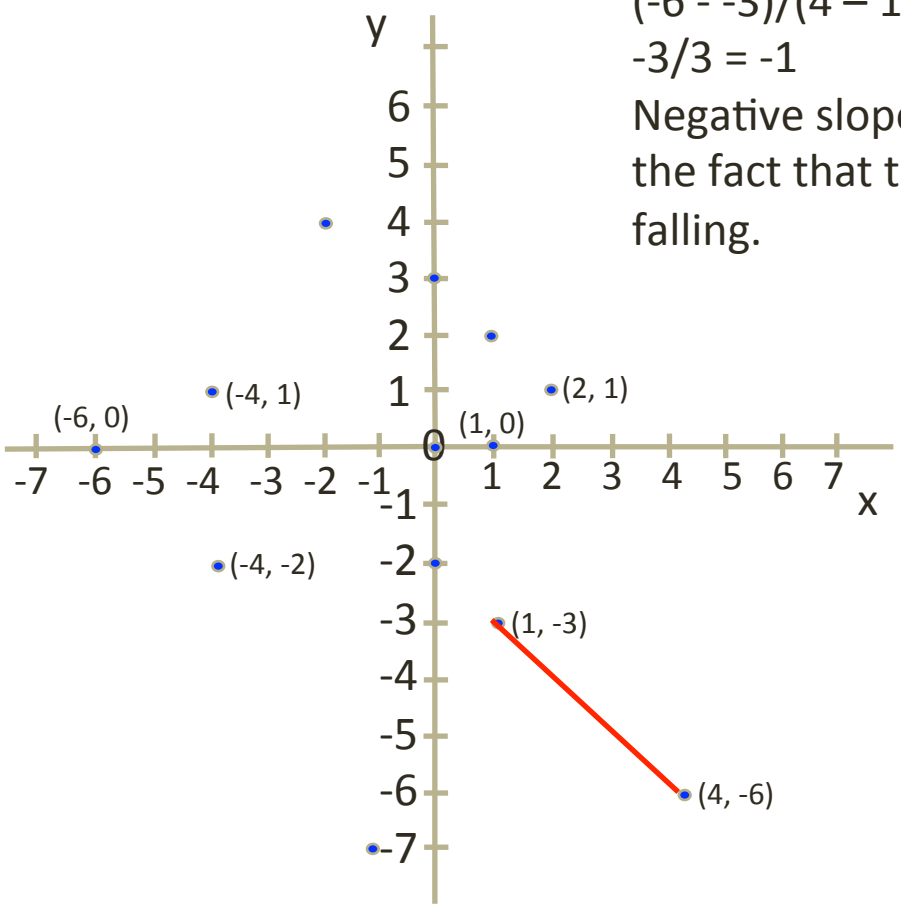


From (1, 0) to (2, 1)
Rise/run = (1-0)/(2-1)
= 1/1 = 1.

What is the slope?

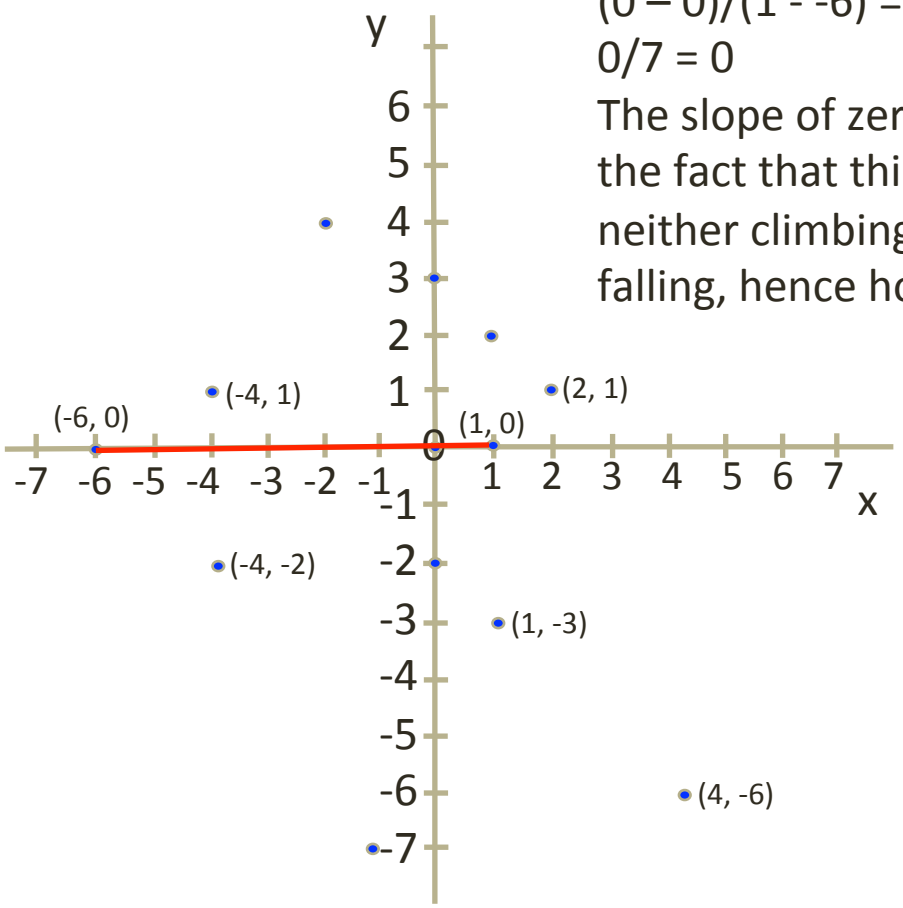
- Slope, steepness, gradient, direction, angle . . .
- “Rise over run” – also known as $\Delta y/\Delta x$
- Remember the first number is x and the second is y (x, y).
- So from (1, 0) to (2, 1) we have “change in y”/”change in x”
- I.e. $(1 – 0)/(2 – 1) = 1/1 = 1$.
- This positive, reflecting the fact that the line is climbing.

Joining points to find the slope . . .



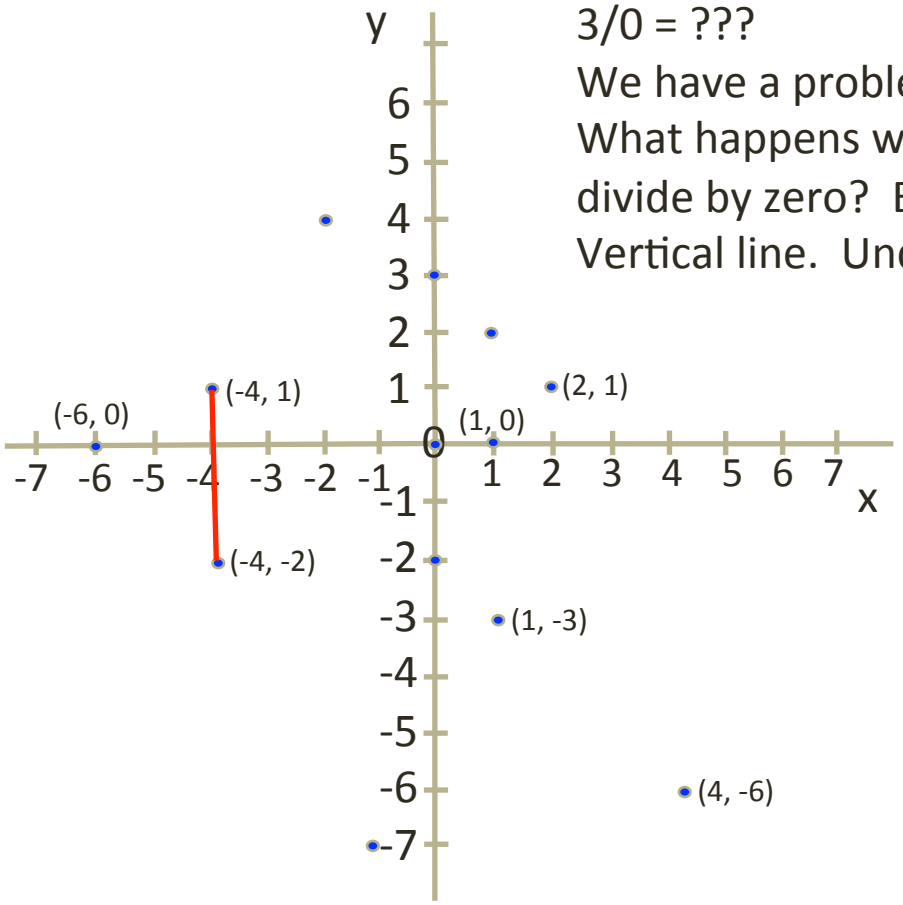
(-6 - -3)/(4 – 1) =
-3/3 = -1
Negative slope, reflecting the fact that the line is falling.

Joining points to find the slope . . .



(0 – 0)/(1 - -6) =
0/7 = 0
The slope of zero reflects the fact that this line is neither climbing nor falling, hence horizontal.

Joining points to find the slope . . .



(1 - -2)/(-4 - -4) =
3/0 = ???
We have a problem!
What happens when you divide by zero? Error!
Vertical line. Undefined slope.

The general equation of a line:

- $y = mx + b$

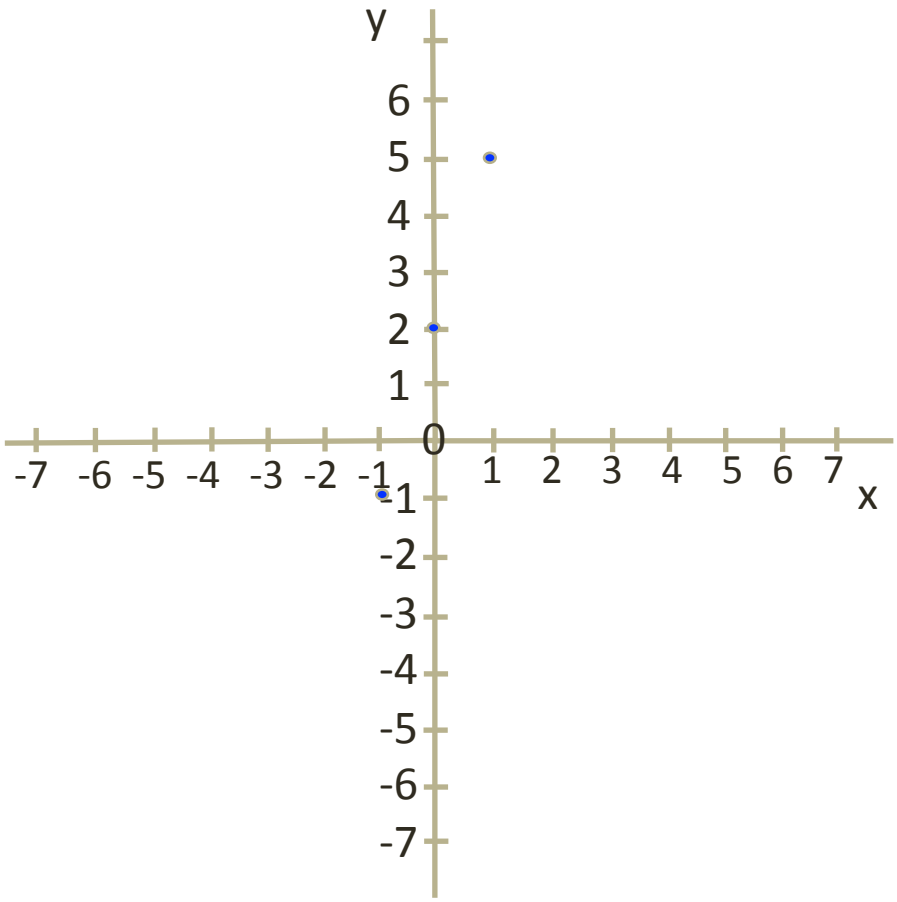
- m is the slope
- b is the y-intercept.

Example 1:

$$y = 3x + 2$$

- The slope is 3
- The y-intercept is 2.
- You can use the equation to “generate” and plot points:
- If x = 0, y = 2; if x = 1, y = 5; if x = -1, y = -1. This gives you 3 points:
- (0, 2), (1, 5), (-1, -1).

$y = 3x + 2$



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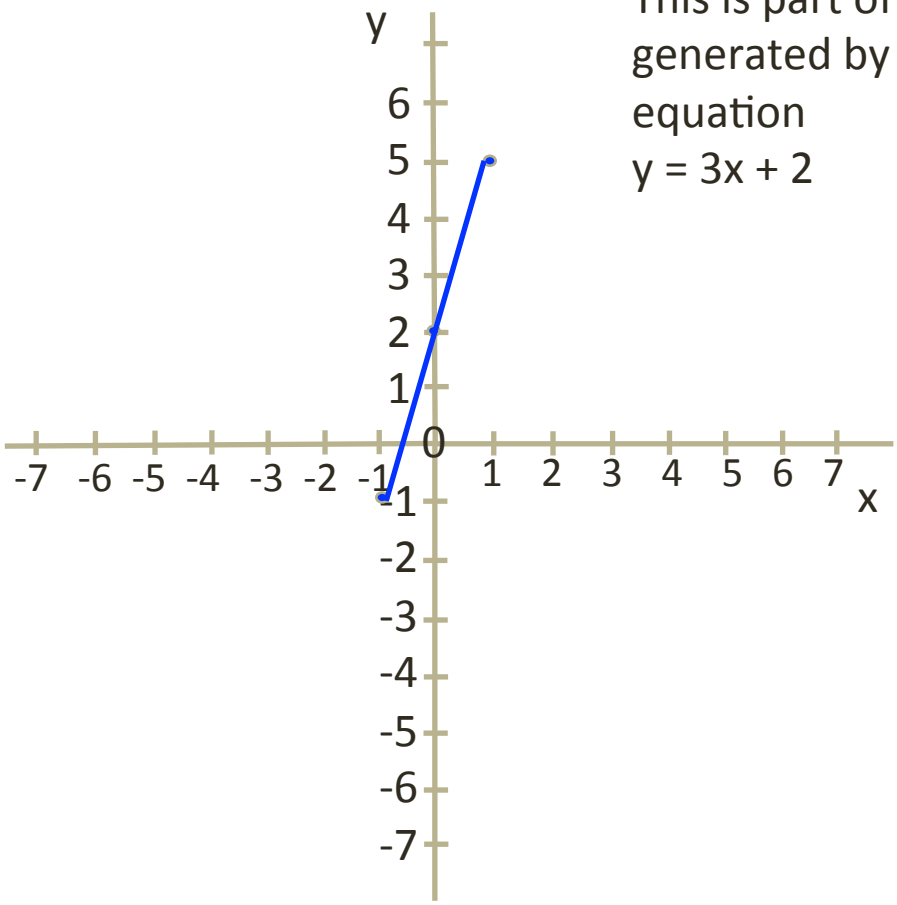


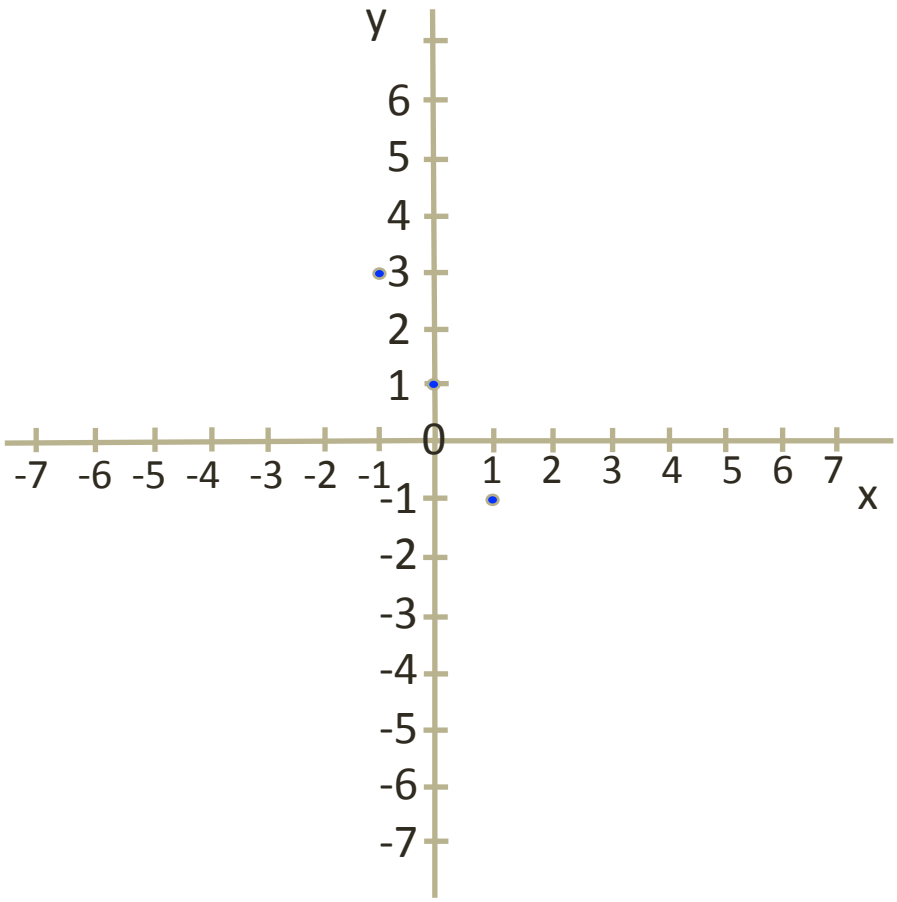
This is part of the line generated by the equation
$y = 3x + 2$

Example 2:

$$y = -2x + 1$$

- The slope is -2
- The y-intercept is 1.
- You can use the equation to “generate” and plot points:
- If x = 0, y = 1; if x = 1, y = -1; if x = -1, y = 3. This gives you 3 points:
- (0, 1), (1, -1), (-1, 3).

$y = -2x + 1$



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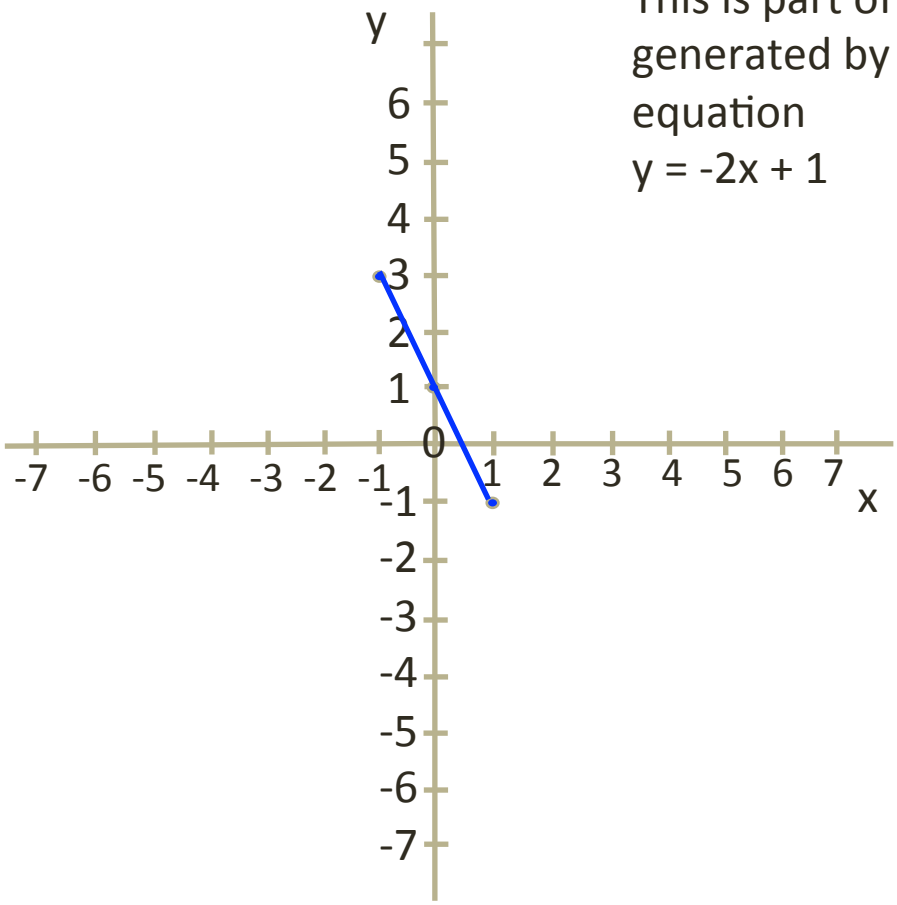


This is part of the line generated by the equation
$y = -2x + 1$

An application of linear growth:

Rental car company A charges an initial fee of $15 plus $0.24 per mile for renting its cars. Company B charges $35 plus $0.18 per mile.

a) Write equations for each rental car company.

b) Graph each equation

c) Under what circumstances is it less expensive to deal with Company A? You can do this by "guestimation," and also more precisely, by using algebra.

Things to consider:

Which is x and which is y? And why?

“Dependent and independent variables.”

Which is slope and which is y-intercept, and why?

Axes with different scales.

Finding an exact point of intersection.

Consider the following:

price charged	quantity demanded
$30	100
$15	400

This relates to a “demand curve”

Even though in this case we assume it will be a straight line.

Can we find the slope and y-intercept, and hence the linear equation?

Instead of using x and y, let’s use p and q.

Tasks . . .

1. Decide which is x and which is y.

2. Find the slope

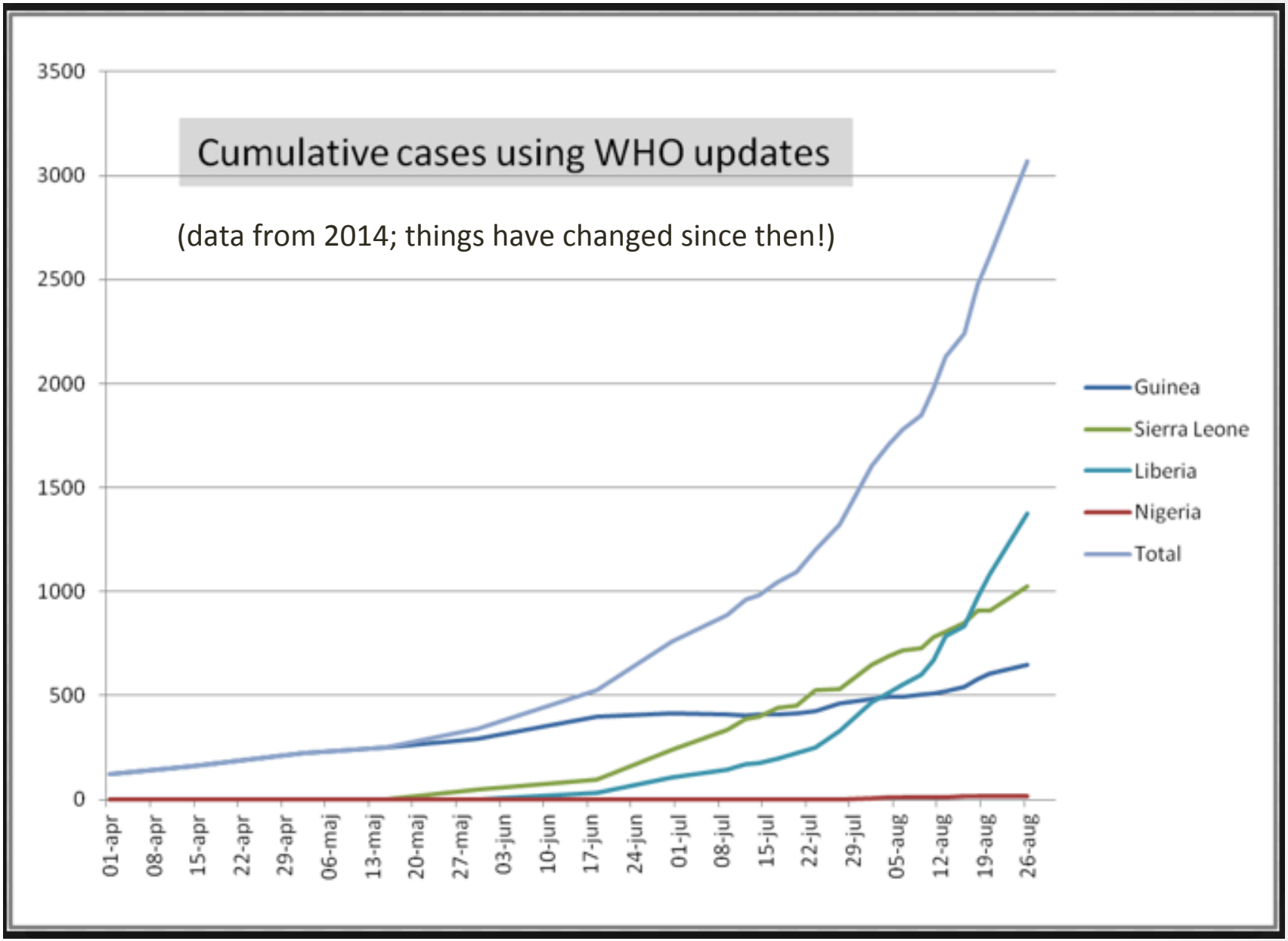
3. Find the “y-intercept”

4. Create a chart

5. Note: in economics, they “switch the axes.” What effect does this have?

Another application of Linear Growth

The next slide contains a graph of Ebola data from West African countries, based on WHO data. Four countries are included: Sierra Leone, Liberia, Guinea, and Nigeria. The data end in late August of last year.


Cumulative cases using WHO updates
(data from 2014; things have changed since then!)
3500
3000
2500
2000
1500
1000
500
0
01-apr
08-apr
15-apr
22-apr
29-apr
06-maj
13-maj
20-maj
27-maj
03-jun
10-jun
17-jun
24-jun
01-jul
08-jul
15-jul
22-jul
29-jul
05-aug
12-aug
19-aug
26-aug
Guinea
Sierra Leone
Liberia
Nigeria
Total

Another application of Linear Growth

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Which country's graph looks the most linear?

Another application of Linear Growth

The previous slide contained a graph of Ebola data from West African countries, based on WHO data. Four countries are included: Sierra Leone, Liberia, Guinea, and Nigeria. The data end in late August of this year.

Which country's graph looks the most linear?

Guinea; starting at approximately 120 on April 1st, and ending with about 620 on August 26, 2014.

Using Linear Growth to make a projection

If we assume linear growth, we should be able to come up with a linear equation with a slope and a y-intercept, and use it to make projections into the future.

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If we take x as the time in days since April 1, and y as the cumulative number of Ebola cases, let's find the slope and y-intercept.

Using Linear Growth to make a projection

If we assume linear growth, we should be able to come up with a linear equation with a slope and a y-intercept, and use it to make projections into the future.

If we take x as the time in days since April 1, and y as the cumulative number of Ebola cases, let's find the slope and y-intercept.

On April 1, x = 0 and y = 120

On August 26, x = 148* and y = 620

* Check this 148 figure!

Finding the slope

On April 1, $x = 0$, and $y = 120$

On August 26, $x = 148$, and $y = 620$

Finding the slope

On April 1, $x = 0$, and $y = 120$

On August 26, $x = 148$, and $y = 620$

Slope $= \Delta y/\Delta x$ (change in y divided by change in x)

$= (620 – 120)/(148 – 0)$

$= 500/148$

$= 3.378378378 \ldots$

$= 3.38$ (rounded to 2 d. p.)

Finding the y-intercept

Since we are using April 1 as our "starting point," i.e. where x = 0, then our y-intercept is (0, 120).

So the equation is:

Finding the y-intercept

Since we are using April 1 as our "starting point," i.e. where x = 0, then our y-intercept is (0, 120).

So the equation is:

$$Y = 3.38x + 120$$

Using our equation

$$Y = 3.38x + 120$$

Let's make a projection for November 4, 2014.

So x will be 217 (no. of days from April 1 to November 4)

Y =

Using our equation

$$Y = 3.38x + 120$$

Let's make a projection for November 4.

So x will be 217 (no. of days from April 1 to November 4)

Y = 3.38*217 + 120

= 853.46, rounded to 853.

Compare to the the actual figure . . .

According to the BBC, based on WHO reports, in November 4, 2014, the cumulative number of Ebola deaths in Guinea was 1,054.

So our projection of 853 was significantly lower than the actual figure.

http://www.bbc.com/news/world-africa-28755033

A couple of things to remember . . .

a) Our linear formula was based on the assumption that the number of Ebola deaths would continue to grow linearly; for Sierra Leone and Liberia it looks more like exponential rather than linear growth.

b) Our projection didn't take into account that by some accounts the number of new cases is slowing in some areas.

All this to say – any projection, linear or otherwise, can never be a guarantee of what will actually happen. More on this next week!

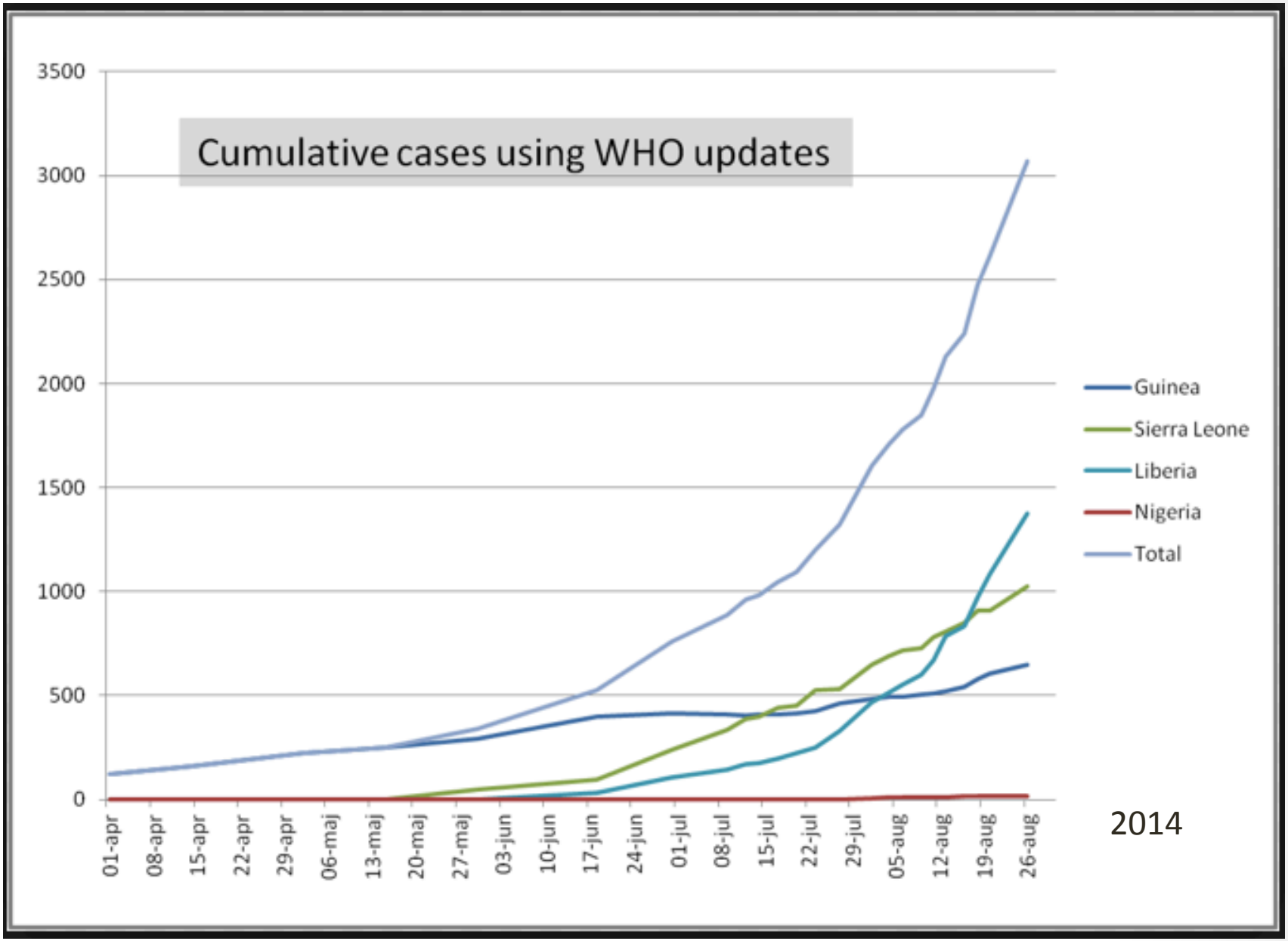
Usefulness of Linear Growth Functions

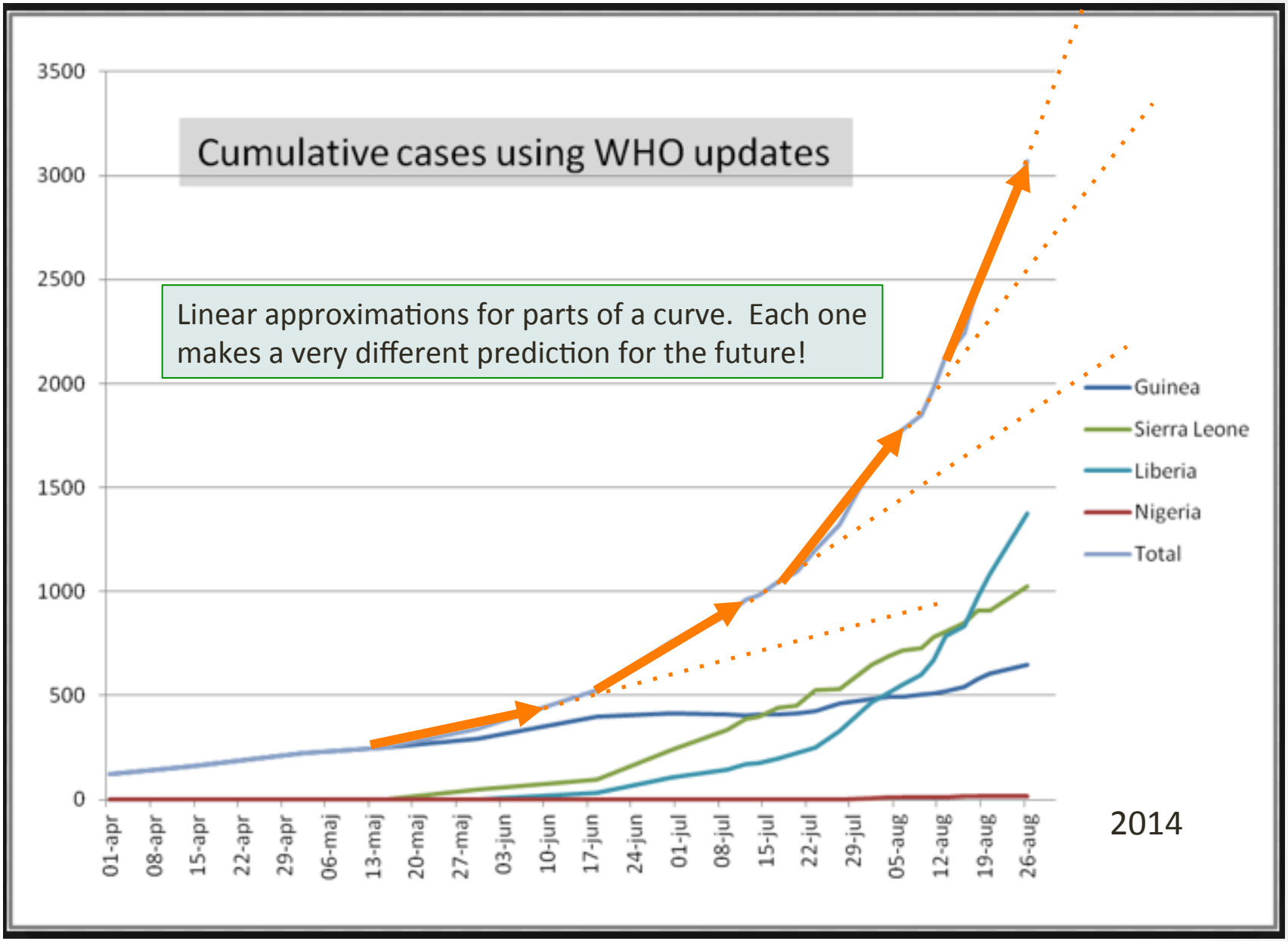
As we have seen, the “quintessential” linear equation is:

$$y = mx + b$$

This is fairly simple to understand and to work with; and in addition it very often resembles or approximates other more complicated types of growth, such as exponential growth.

This approximation generally is quite close, as long as you don’t “push it” too far!


Cumulative cases using WHO updates
3500
3000
2500
2000
1500
1000
500
0
01-apr
08-apr
15-apr
22-apr
29-apr
06-maj
13-maj
20-maj
27-maj
03-jun
10-jun
17-jun
24-jun
01-jul
08-jul
15-jul
22-jul
29-jul
05-aug
12-aug
19-aug
26-aug
Guinea
Sierra Leone
Liberia
Nigeria
Total
2014


Cumulative cases using WHO updates
Linear approximations for parts of a curve. Each one makes a very different prediction for the future!
3500
3000
2500
2000
1500
1000
500
0
01-apr
08-apr
15-apr
22-apr
29-apr
06-maj
13-maj
20-maj
27-maj
03-jun
10-jun
17-jun
24-jun
01-jul
08-jul
15-jul
22-jul
29-jul
05-aug
12-aug
19-aug
26-aug
Guinea
Sierra Leone
Liberia
Nigeria
Total
2014