

# MATH E-3: Lecture 3

Quantitative Reasoning: Practical Math

# Announcements

February 9, 2016



# Homework



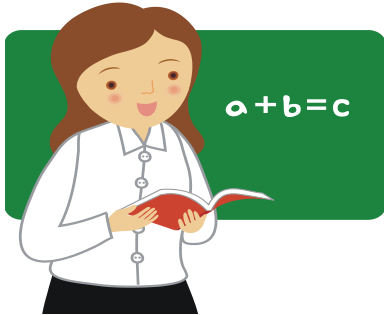
- Assignment 1, with grader comments, will be available on 2/13
- Assignment 2, the first graded assignment, is due Saturday, 2/13 by upload deadline. Solutions will be posted shortly thereafter
- Homework schedule, as well as other helpful homework resources, is posted in the Homework Help Center module

# Homework Submission

## CHECKLIST

- ☒ single PDF file
- ☒ 4mb maximum
- ☒ work must be neat and legible, scan appropriately (not upside down or sideways pages), and flag your final answer
- ☒ file name (example: albrigo.assign1)
- ☒ upload by Saturday, 11:59 a.m. (ET) deadline

# Optional TA Sections/Resources



- Online (via Conferences on the course website): Wednesdays from 7:30-9:00 pm (Eastern Time) with Jessica. Jessica's first section is February 10.
- On campus: Tuesdays from 5:30-7:00 pm, Sever 104, with Sue

Math Question Center:

<http://www.extension.harvard.edu/resources-policies/resources/math-question-center>



# Student Locations fall 2015

https://canvas.harvard.edu/courses/44742/external\_courses/1002

Sue Albrigo Inbox Settings Logout Help

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**MATH E-3 (12500)**

2015-2016 Fall

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World view:



The world map displays student locations across five continents. Red pins are clustered in North America (primarily the United States and Canada), with a few pins in South America, Europe, and Africa. The map includes labels for the continents and oceans, and a Google logo is visible in the bottom left corner.

Boston Area view:



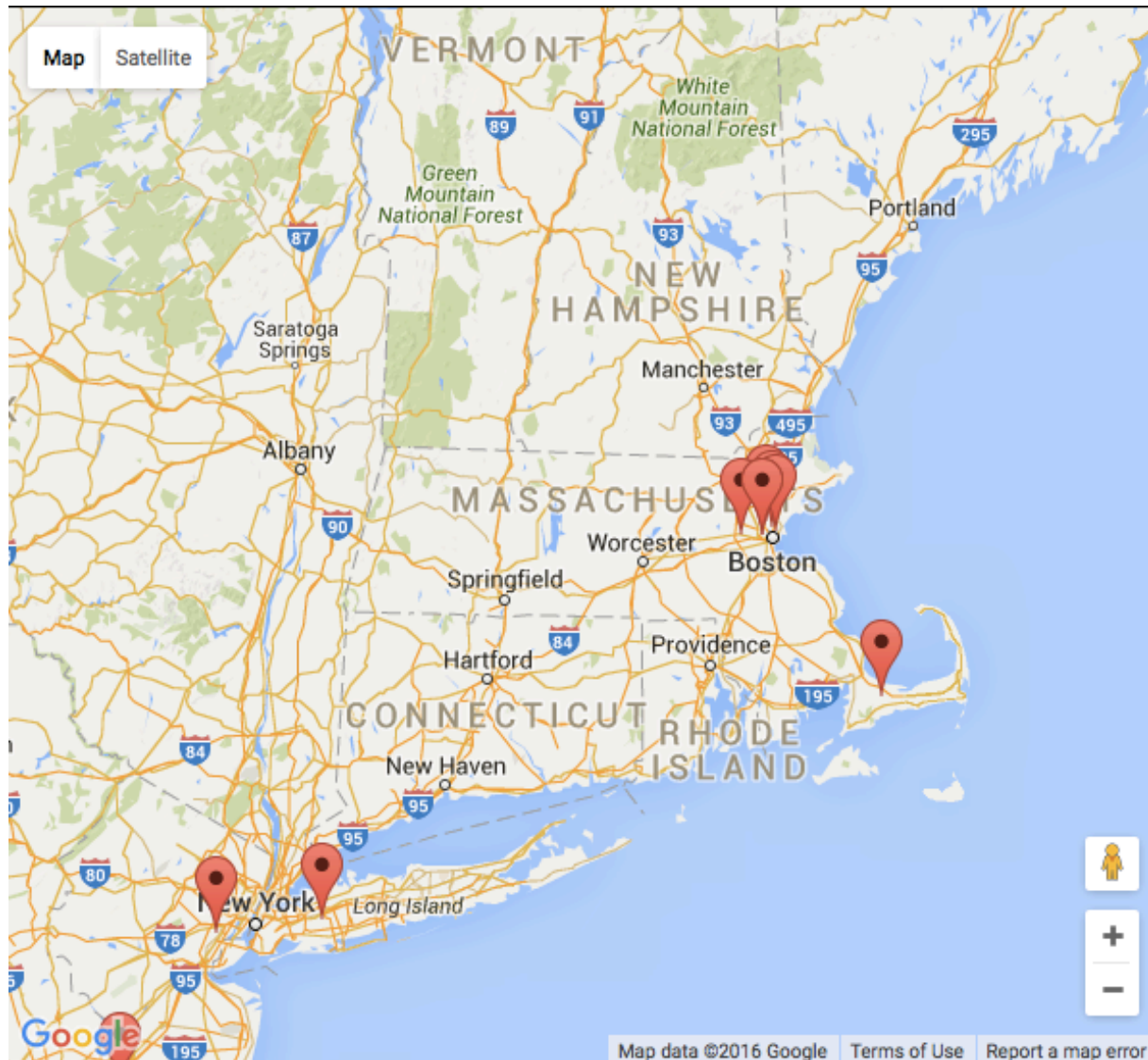
The Boston area map shows the Greater Boston region, including parts of Vermont, New Hampshire, and Massachusetts. Red pins are clustered in the Greater Boston area, with labels for White Mountain National Forest, Portland, and various highways (89, 93, 95, 295). The map includes a Google logo and a 'Terms of Use' link.

# Student Locations spring 2016





# And more locally ...



# Using the TI-30 IIS, part 1

Exponents or "raising to the power of"

E.g. squaring a number: what is  $5^2$  ( $5*5$ )?

Press: 5  $x^2$  enter (the enter key also says "=")

Answer = 25

E.g. raising to the third power: what is  $4^3$  ( $4*4*4$ )?

Press: 4  $^$  3 enter. Answer = 64.

# Using the TI-30 IIS, part 2

## **Square roots:**

To find the square root of 9, press:

2nd  $x^2$  9 enter. Answer = 3.

## **Other roots:**

To find the 3<sup>rd</sup> root (cube root) of 8, press:

3 2nd  $^{\wedge}$  8 enter. Answer = 2.

4<sup>th</sup> root of 16:

4 2<sup>nd</sup>  $^{\wedge}$  16 enter. Answer = 2.

# Using the TI-30 XA, part 1

Exponents or "raising to the power of"

E.g. squaring a number: what is  $5^2$  ( $5*5$ )?

Press: 5  $x^2$  (no need to hit "enter" or "=")

Answer = 25

E.g. raising to the third power: what is  $4^3$  ( $4*4*4$ )?

Press: 4  $y^x$  3 =. Answer = 64.

# Using the TI-30 XA, part 2

## **Square roots:**

E.g. To find the square root of 9, press:

$$9\sqrt{x} \quad \text{Answer} = 3.$$

## **Other roots:**

E.g. To find the 3<sup>rd</sup> root (cube root) of 8, press:

$$8 \text{ 2nd } \sqrt[3]{y} 3 =. \quad \text{Answer} = 2$$

E.g. To find the 4<sup>th</sup> root of 16:

$$16 \sqrt[4]{y} 4 =. \quad \text{Answer} = 2.$$

# Percents: do we really need them?

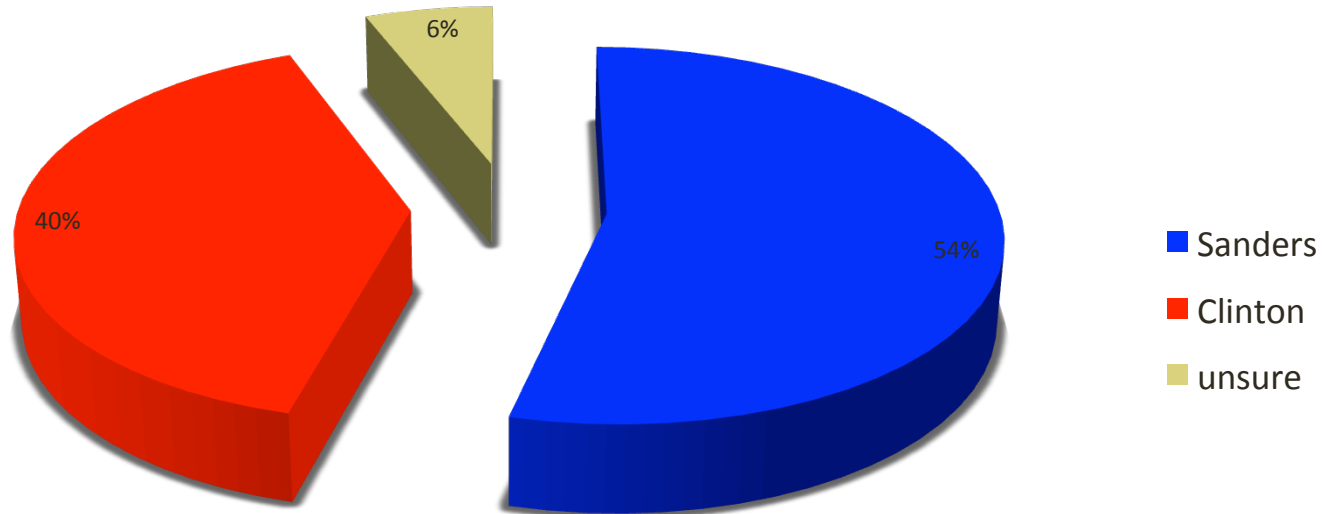
- Tipping
- Interest rates
- Taxes
- Unemployment
- Grades
- Political candidates
- Etc.

# Percents: do we really need them?

- Tipping
- Interest rates
- Taxes
- Unemployment
- Grades
- **Political candidates** (hint, hint)

# Speaking of which, going into NH...

## Democrats

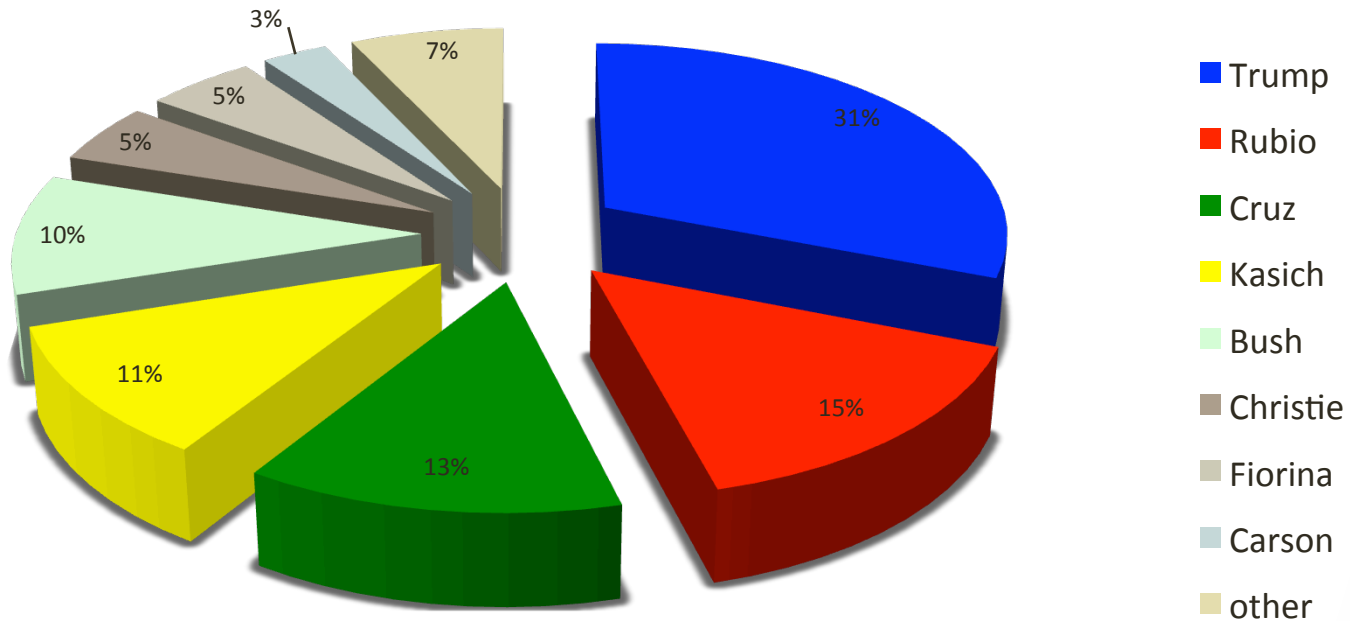


<http://www.cnn.com/2016/02/08/politics/new-hampshire-polling-snapshot/>



# And ...

## Republicans



<http://www.cnn.com/2016/02/08/politics/new-hampshire-polling-snapshot/>

# Sanders, Kasich Win First Votes in N.H.

Vermont Sen. Bernie Sanders and Ohio Gov. John Kasich received an early boost in the New Hampshire primary Tuesday as the first ballots were cast in the tiny town of Dixville Notch. Sanders beat rival Hillary Clinton 4-0 while Kasich beat Donald Trump 3-2 for a total of nine votes cast in a tradition that dates back to 1960. Under state law, New Hampshire communities with fewer than 100 voters can get permission to open their polls at midnight and close them as soon as all those registered have voted. Dixville Notch joined nearby towns Millsfield and Hart's Location in opening their polls at midnight. In Millsfield, Texas Sen. Ted Cruz won nine votes compared with Trump's three for the GOP vote. Several other Republican candidates received one vote apiece. Clinton received two votes to beat Sanders' one vote.

# Sanders, Kasich Win First Votes in N.H.

## **Dixville Notch:**

Sanders 4, Clinton 0.

Kasich 3, Trump 2.

## **Millsfield:**

Cruz 9, Trump 3.

Clinton 2, Sanders 1.

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*So let's just stop there . . . (jk)*

# What are percents?

- Per Cent means “out of 100.”
- E.g. 35% means “35 out of 100” or  $35/100$ .
- E.g.  $2\frac{1}{2}$  percent means “ $2\frac{1}{2}$  out of 100” or  $2.5/100$ .

## Changing a percent into a decimal

To change from a **percent to a decimal**, divide by 100 or move the decimal point two places to the left. Don't forget to remove the percent sign.

$$\text{ex. 1. } 35\% = \frac{35}{100} = \underline{0.35}$$

$$\text{ex. 2. } 7.5\% = \frac{7.5}{100} = 0.075$$

## Changing a decimal into a percent

To change from a **decimal to a percent**, multiply by 100 or move the decimal point two places to the right and attach the % sign.

ex. 1.  $0.125 = 12.5\%$

ex. 2.  $2.12 = 212\%$

ex. 3.  $93.15 = 9315\%$



## Four general types of percent problems

- 1) **WHAT IS** 35 % of \$250 ? (MULTIPLY)
- 2) \$25.00 IS **WHAT %** of \$120 ? (DIVIDE)
- 3) \$30.00 IS 15 % **of WHAT ?** (DIVIDE)
- 4) 20% of **WHAT IS** 500 ? (DIVIDE)

## 1<sup>st</sup> type of percent

- 1) **WHAT IS** 35 % of \$250 ?
- “Translate” into a math sentence – an equation.

$$X = 0.35 * 250$$

$$X = \$87.50$$

## 2<sup>nd</sup> type of percent

- 2) \$25.00 IS **WHAT %** of \$120 ?
- Again, translate into a math “sentence.”
- $25 = X/100 * 120$
- This time, the “X” is not “isolated,” so we need to use some algebra.

## 2<sup>nd</sup> type of percent, continued . . .

- $25 = X/100 * 120$
- We isolate the “X” by moving the 100 and the 120 to the other side (left hand side). We move things by “undoing” their operations – i.e. by doing the opposite. We have to do the same thing to both sides – to keep it as an equation.
- The 100 is being divided, so we multiply both sides by 100; the 120 is being multiplied, so we divide both sides by 120.

## 2<sup>nd</sup> type of percent, continued . . .

- $25 = X/100 * 120$
- $25 * 100 \div 120 = X/\cancel{100} * \cancel{100} * \cancel{120} \div \cancel{120}$
- $2500/120 = X$
- $20.833333 = X$
- Rounding . . .
- \$25 is 20.83% of \$120

### 3<sup>rd</sup> type of percent

- 3) \$30.00 IS 15 % **of WHAT ?**

- $30 = 0.15 * X$

- To isolate the “X,” divide both sides by 0.15

- $30 \div 0.15 = \cancel{0.15 \div 0.15} * X$

- $\$200 = X$

## 4<sup>th</sup> type of percent (similar to 3<sup>rd</sup>)

- 4) 20% of **WHAT** IS 500 ?

- $0.20 * X = 500$

- Divide both sides by 0.20

- $\cancel{0.20} \div \cancel{0.20} * X = 500 \div 0.20$

- $X = 2500$

# Percent Change

$$\% \text{ change} = \frac{\text{New} - \text{Old}}{\text{Old}} * 100\%$$



## Example of percent change

E.g. If the size of our class increases from 10 students to 18 students, then

$$\begin{aligned}\% \text{ increase } &= \frac{18 - 10}{10} * 100 = \frac{8}{10} * 100 = \\ &0.8 * 100 = 80\%\end{aligned}$$

# Example of percent change

**E.g. Find the percent change if the Dow Jones Average drops from 10,200 to 9,400**

$$\text{Percent change} = \frac{\text{New} - \text{Old}}{\text{Old}} * 100$$

$$\% \text{ Change} = \frac{9400 - 10200}{10200} * 100 = \frac{-800}{10200} * 100 = - .07843 * 100$$

= - 7.8% rounded to 1 dec.

You can also call this a 7.8% decrease. If you do not use the negative sign, make sure the reader knows it's a decrease.

## More examples ...

Your weekly salary of \$750 is going to be increased by 7%.  
What will your new salary be?

2 ways to do this:

i) What is 7% of \$750? Then add this to the original \$750.

ii) What is (100% + 7%) of \$750? (percent type 1)

$$X = 1.07 * 750 = \$802.50$$

## More examples . . .

This time, your monthly rent of \$1,200 is going to be reduced by 5% (yay!). What will your new rent be?

Again 2 ways:

i) What is 5% of \$1,200; then subtract this from \$1,200.

ii) What is (100% - 5%) of \$1,200?

$$X = 0.95 * 1200 = \$1,140.$$

# Converting fractions to decimals

Divide the numerator by the denominator:

E.g. convert  $5/8$  to a decimal.

$$5/8 = 0.625$$

E.g. convert  $2/3$  to a decimal.

$$2/3 = 0.6666666 \dots$$

Round to desired accuracy, e.g. 0.67 (2 dec. places).

# Converting decimals to fractions

E.g. What is 0.3 as a fraction?

$$0.3 = 3/10$$

E.g. What is 0.35 as a fraction?

$0.35 = 35/100$  This can be “reduced”: since 35 and 100 both have a “common factor” of 5, we can write  $(7*5)/(20*5)$ ; we can “cancel” the 5’s, leaving us with  $7/20$ .

E.g.  $0.25 = 25/100$  cancel common factor of 25, leaving  $1/4$ .

# Multiple discounts

I once read in the Boston Globe that AAA was offering two separate discounts, each of 10%, for a total discount of 19%.

That can't be right, I thought! Everyone knows that  $10\% + 10\% = 20\%$ , right?

Let's check it out . . .

First, choose a starting cost – it doesn't matter which number you choose.

## Multiple Discounts, continued . . .

Start with \$100, and decrease it by 10%.

Now take 10% off this new number . . .

And the final amount is . . .?



## Medieval population changes ... and medieval math?

During a period of intense wars, the population of Europe decreased by 15%, but soon afterwards it increased again by 15%, thus returning to its previous figure.

Shortly afterwards came the dreaded bubonic plague, during which the population dropped by nearly 400%.

Finally, there was a period of unparalleled prosperity and health, and the population increased from 20 million to 61 million, an increase of over 300%.

## A while ago in the news . . .

Back in August of 2012, I heard on NPR that the unemployment rate had risen from 8.2% to 8.3%. A bit later, someone took a closer look and discovered that it had gone from 8.22% to 8.25%.

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So . . . the bad news: an increase of 0.1%;

# A while ago in the news . . .

Back in August of 2012, I heard on NPR that the unemployment rate had risen from 8.2% to 8.3%. A bit later, someone took a closer look and discovered that it had gone from 8.22% to 8.25%.

So . . . the bad news: an increase of 0.1%;

The good news: an increase of 0.03%.

The power of Rounding!!

# Short term versus long term change

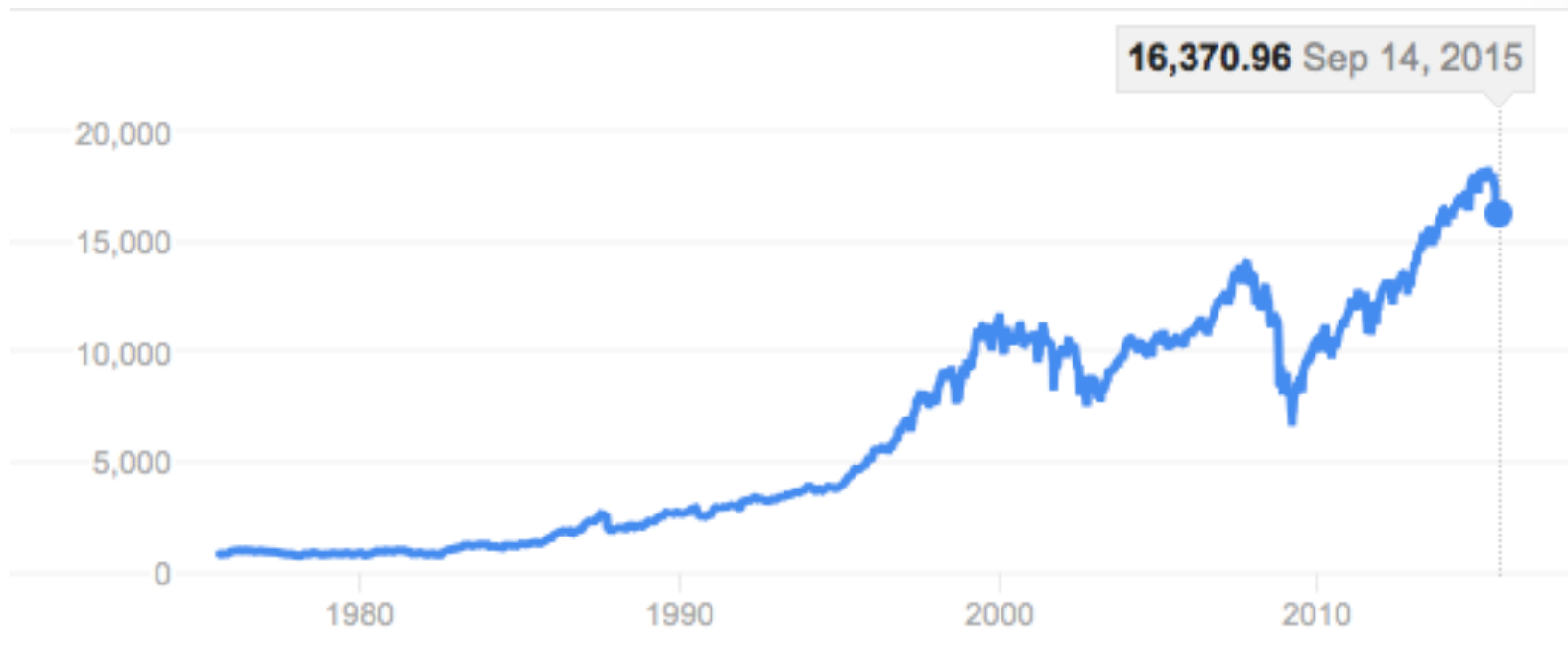
# Dow Jones Sept 19, 1975



# Dow Jones May 15, 2015



# Dow Jones Sept 14, 2015





# Let's do some percent calculations

date	DJIA
sept 19, 1975	829.79
may 15, 2015	18,272.56
sept 14, 2015	16,370.96
feb 8, 2016	16,027.05

# Dow Jones over 1 day



<http://money.cnn.com/data/markets/dow/>

# Dow Jones over 3 days



# Dow Jones over 5 days



# Dow Jones over 1 month



# Dow Jones over 3 months



# Dow Jones over 6 months



# Dow Jones year to date





# Dow Jones over 1 year



# Dow Jones over 3 years

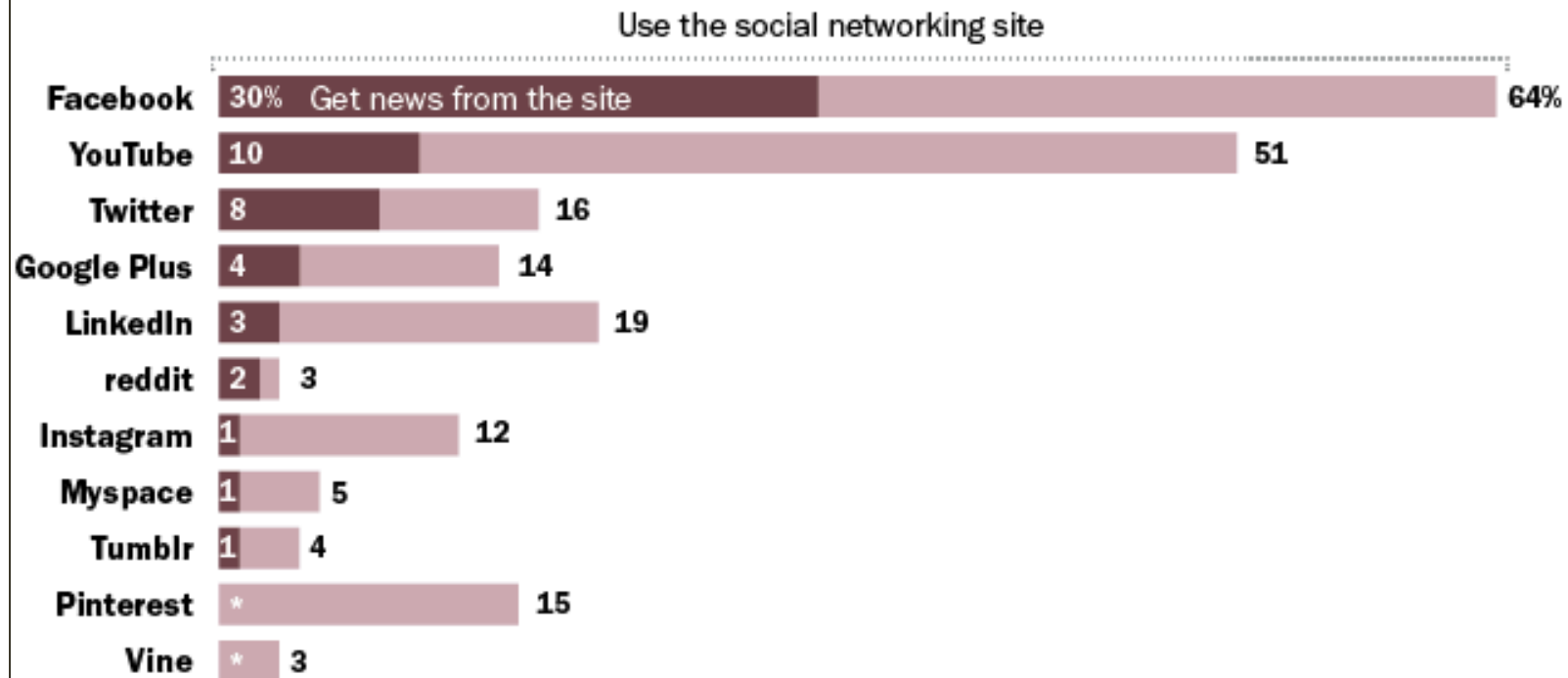


# Dow Jones over 5 years



# And one more percents example ...

*Percent of U.S. adults who use each social networking site and percent of U.S. adults who get news from each social networking site*



Note: The percent of U.S. adults who get news on Pinterest and Vine each account to less than one percent.  
Facebook News Survey, Aug. 21-Sept. 2, 2013 (N=5,173)

PEW RESEARCH CENTER

<http://www.digitaltrends.com/social-media/americans-get-news-facebook-feed/>