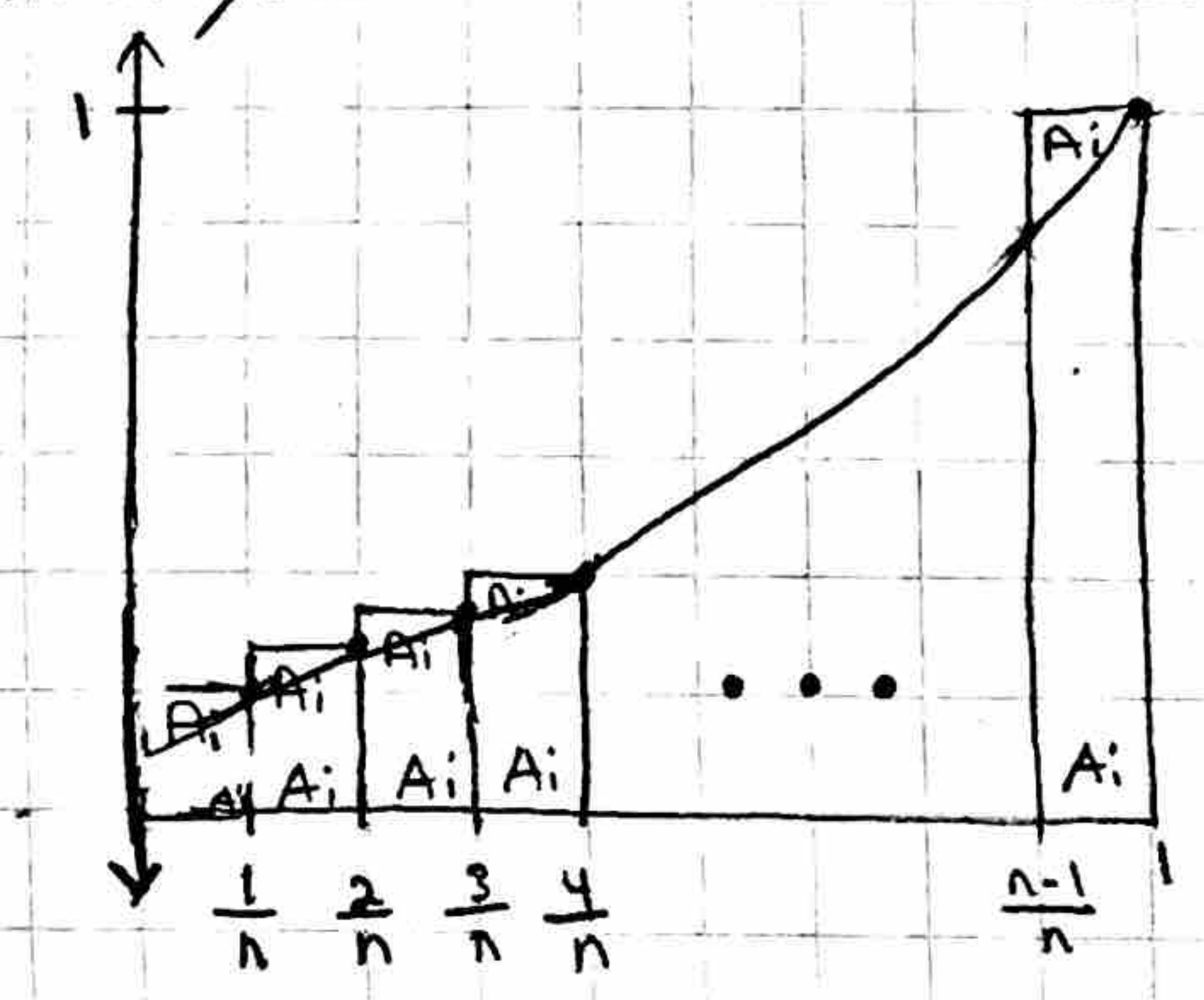
Right Endpoint Rule

Use n rectangles to estimate the area A between the parabola $f(x)=x^2$ and the x-axis over the interval $[0, 1]$.

Find the limit of the sum of the areas of the n rectangles as n goes to infinity.



Not An Artist with Right Endpoint Rule

$i = 1, 2, 3, \ldots n$, where i is the rectangle position along x-axis and n is the total number of rectangles.

$i=1$

Width of rectangle 1 is $\frac{1}{n}$

Height of rectangle 1 $f(i/n) = f(1/n) = f(1/n)^2$

$i=2$

width of rectangle 2 is $\frac{1}{n}$

Height of rectangle 2 is $f(2/n) = (2/n)^2$

$i=n$

Width of rectangle n is $\frac{1}{n}$

Height of rectangle n is $f(1) = 1^2 = 1$, where n is the last rectangle.

$R_n = A_1 + A_2 + \ldots + A_n$, where R_n is the sum of rectangle areas

$$\frac{1}{n}\left(\frac{1}{n}\right)^2 + \frac{1}{n}\left(\frac{2}{n}\right)^2 + \ldots + \frac{1}{n}\left(\frac{n}{n}\right)^2$$

$$\frac{1}{n}\left[\frac{1^2 + 2^2 + n^2}{n^2}\right]$$

$$\downarrow$$

Set $1^2 + 2^2 + \ldots + n^2 = \frac{n(n+1)(2n+1)}{6}$

$$\downarrow$$

$$R_n = \frac{1}{n}\left[\frac{1^2 + 2^2 + \ldots + n^2}{n^2}\right]$$

$$R_n = \frac{1}{n}\left[\frac{\frac{n(n+1)(2n+1)}{6}}{n^2}\right]$$

$$R_n = \frac{1}{n}\left[\frac{n(n+1)(2n+1)}{6} \div \frac{n^2}{1}\right]$$

$$R_n = \frac{1}{n}\left[\frac{n(n+1)(2n+1)}{6} \cdot \frac{1}{n^2}\right]$$

$$R_n = \frac{1}{\cancel{n}}\left[\frac{\cancel{n}(n+1)(2n+1)}{6n^2}\right]$$

$$\boxed{R_n = \frac{(n+1)(2n+1)}{6n^2}}$$

$n = 10$, where n is the number of rectangles

$$R_{10} = \frac{(10+1)(2(10)+1)}{6(10)^2}$$

$$\boxed{R_{10} = 0.385}$$

$$\lim_{n\to\infty} R_n = \lim_{n\to\infty} \frac{(n+1)(2n+1)}{6n^2} \rightarrow (n+1)(2n+1) = 2n^2+n+2n+1 = \boxed{2n^2+3n+1}$$

$$= \lim_{n\to\infty} \frac{2n^2+3n+1}{6n^2}$$

$$= \lim_{n\to\infty} \frac{1}{6}\left(\frac{2n^2+3n+1}{n^2}\right)$$

$$= \frac{1}{6} \cdot \lim_{n\to\infty}\left(\frac{2n^2+3n+1}{n^2}\right)$$

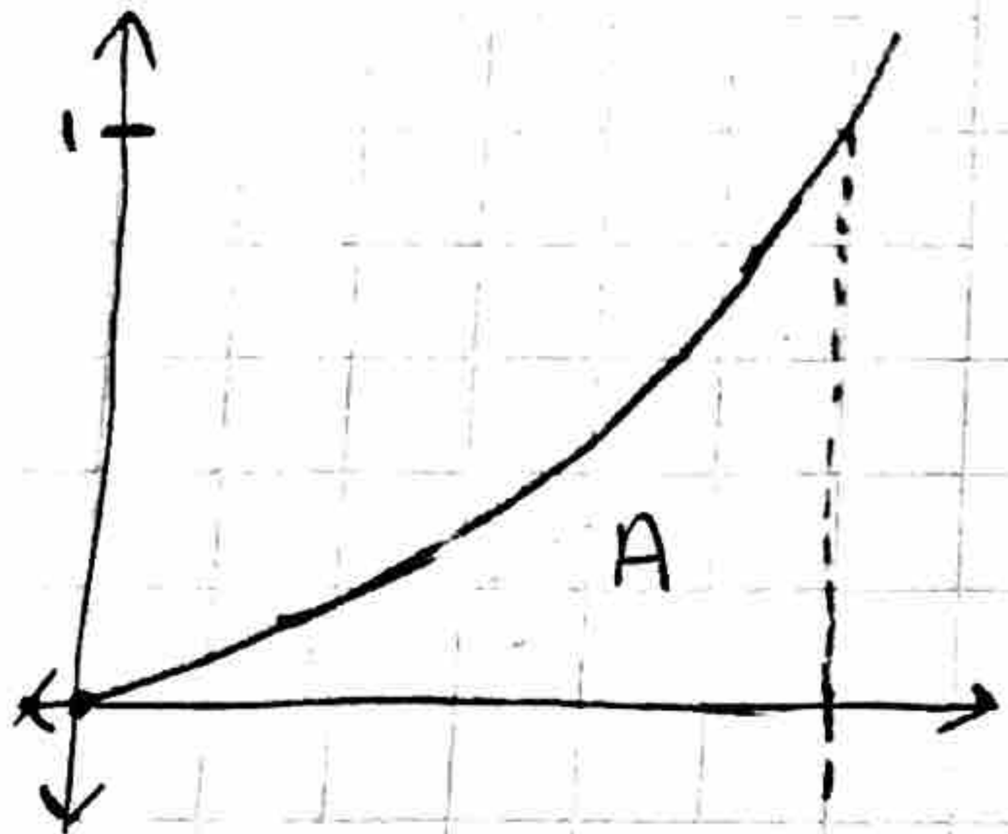
$$= \frac{1}{6} \cdot \lim_{n\to\infty}\left(2+\frac{3}{n}+\frac{1}{n^2}\right)$$

$$\lim_{n\to\infty} 2 + \lim_{n\to\infty}\left(\frac{3}{n}\right) + \lim_{n\to\infty}\left(\frac{1}{n^2}\right)$$

$$2 \qquad = \frac{\lim_{n\to\infty} 3}{\lim_{n\to\infty} n} = \frac{3}{\infty} = 0 \qquad = \frac{\lim_{n\to\infty} 1}{\lim_{n\to\infty} n^2} = \frac{1}{\infty} = 0$$

$$\frac{c}{\infty} = 0$$

$$= \frac{1}{6} \cdot 2 = \frac{2}{6} = \frac{1}{3}$$



Area A between parabola $f(x)=x^2$ and x-axis over $[0,1]$ is $1/3$

$A = 1/3 \approx 0.33\overline{3}$