MATH E-3
Assignment 6 Solutions TOTAL POSSIBLE POINTS = 83

SHOW ALL YOUR WORK TO RECEIVE FULL CREDIT.

Problems 6.1 and 6.2
The Normal Distribution Curve

Draw your distribution curves by hand, and please be neat. You will need to show the mean, the values for plus or minus 1 standard deviation and 2 standard deviations, along with the rough and ready percentages that apply. Be sure to indicate, with label arrows, 1 and 2 standard deviations (see readings and lecture slides for example). There is no need to show 3 standard deviations.

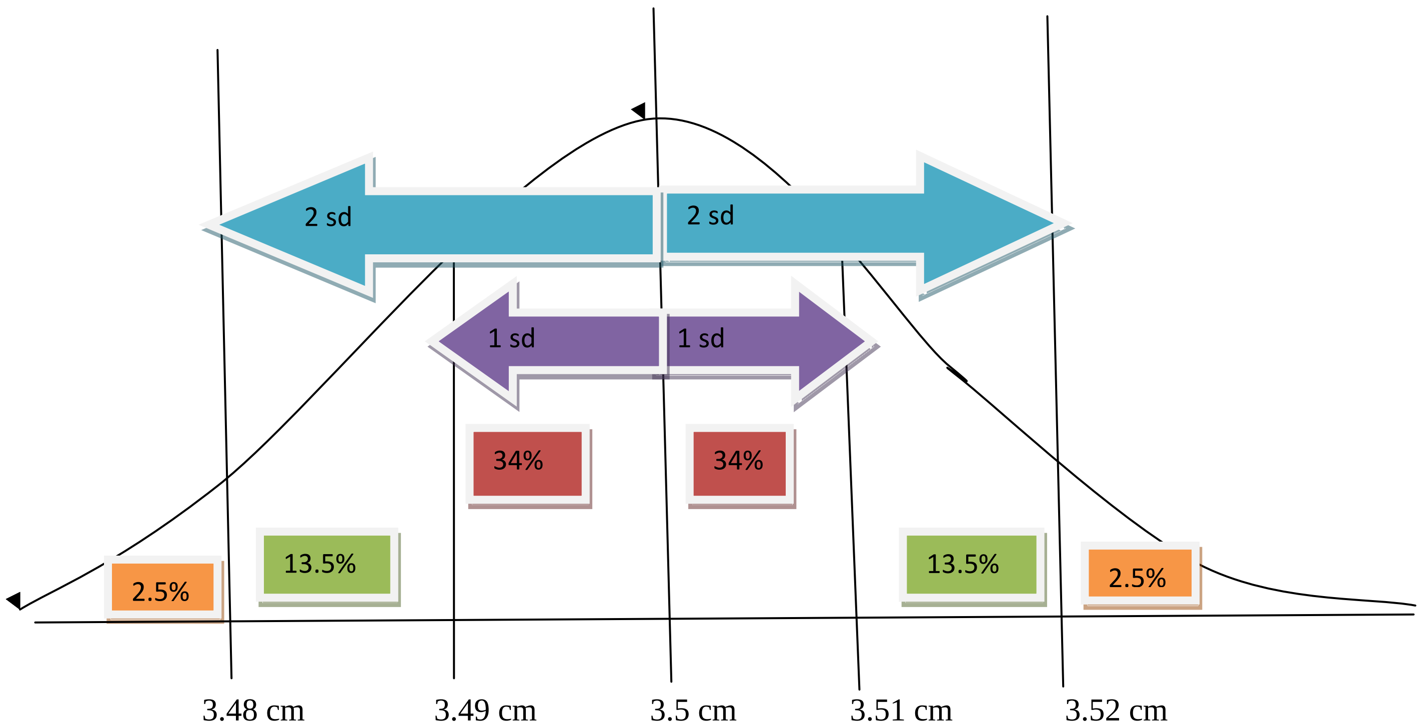
Problems 6.1 (6 points)

A quality control engineer for a major toy company recently performed some statistical analysis on the length of plastic screws produced by its 'Plastics Division.' These particular screws are used in the construction of certain children's toys, so their length is important in the structural integrity of the toys. Thus, a check is periodically performed. He found that the mean length of the screws is **3.5** centimeters and the standard deviation is **0.01** cm.

Answer the following. Make sure you draw the appropriate diagram for the normal distribution of these screws.

Draw Diagram here:

Plastics Division Length of Screws



Assume the quality control engineer chooses one screw at random. What percentage of the time will the length of the screw be:

a) More than 3.51 cm? 13.5% + 2.5% =**16%** (2 points)

b) Less than 3.48 cm? **2.5%** (2 points)

c) More than 3.49 cm? 34% + 34% + 13.5% + 2.5% = **84%** (2 points)

d) Less than 3.52 cm? 13.5% +34% + 34% + 13.5% + 2.5% = 97.5%,

or 100% - 2.5% = **97.5%** (2 points)

e) Less than 3.5 cm?   34% + 13.5% + 2.5% =**50%**   (2 points)

f) Between 3.48 and 3.51 cm?   13.5% + 34% + 34% = **81.5%**   (2 points)

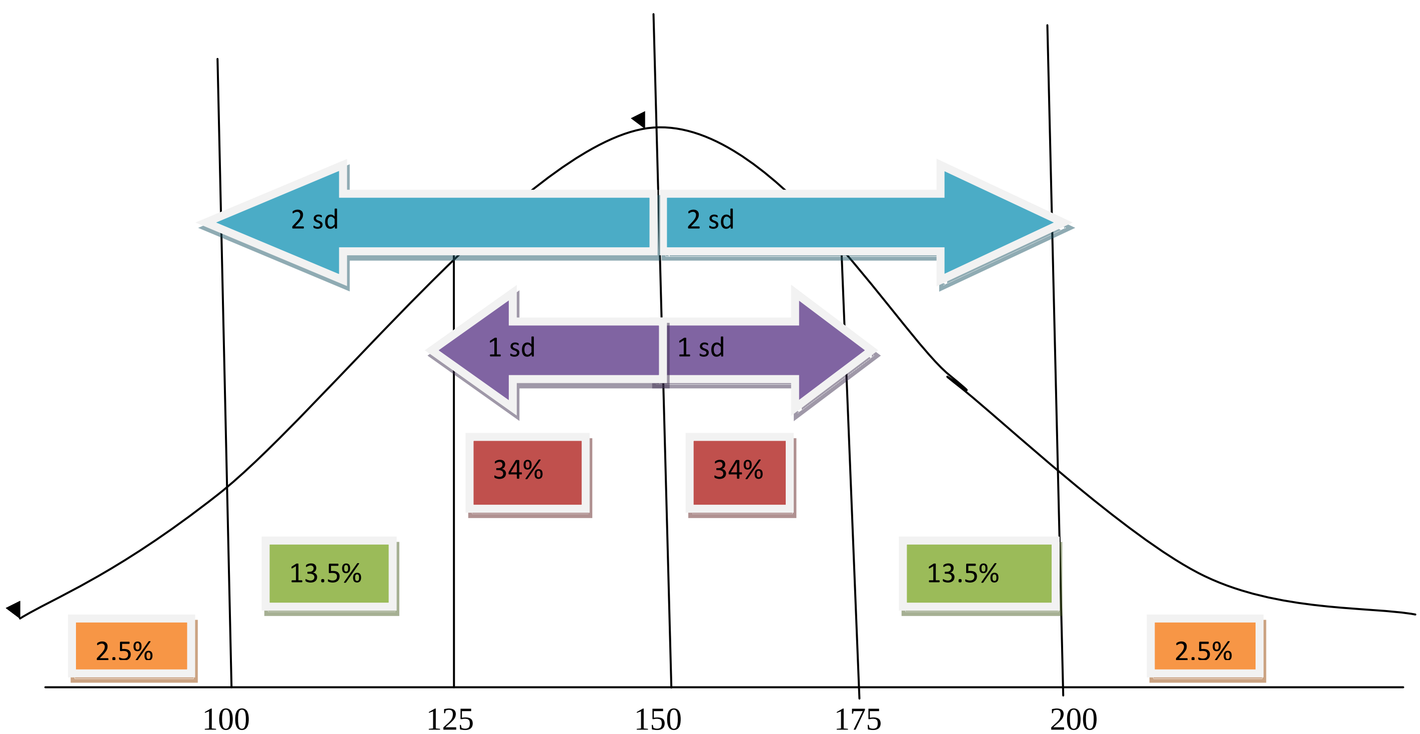
g) Between 3.51 cm and 3.52 cm?   **13.5%**   (2 points)

Problem 6.2   (6 points)

The 911 emergency center in a certain city receives various numbers of calls on different days. A record of the center shows that the number of daily calls is normally distributed with mean of 150 and standard deviation of 25. Remember to sketch a diagram.

Draw diagram here:

911 Emergency Calls



a) What percent of the days in this record have more than 100 calls? **97.5%** (2 points)

b) What percent of the days in this record have fewer 175 calls? (2 points)

34% + 34% + 13.5% + 2.5% = **84%**

c) What percent of the days in this record have between 125 and 200 calls? (2 points)

34% + 34% + 13.5% = **81.5%**

d) What percent of the days in this record have more than 200 calls? **2.5%** (2 points)

Problems 6.3-6.7
The new formula for standard deviation, and 95% Confidence Intervals.

Please give standard deviations as a <u>percent</u>, rounded to one decimal point. Round <u>after</u> turning into a percent.

Problems 6.3

For the following problems, calculate the **standard deviation** using our new formula. Show all your work as done in class. No diagram necessary for these questions.

a) Assume p = 54% and our sample size, n = 375. (4 points)

$$\sigma = \sqrt{\frac{.54(1-.54)}{375}} = \sqrt{\frac{.54(.46)}{375}} = \sqrt{\frac{.2484}{375}} = \sqrt{.0006624} = .025737133 = \mathbf{2.6\%}$$

b) Assume p = 68% and our sample size, n = 1105. (4 points)

$$\sigma = \sqrt{\frac{.68(1-.68)}{1105}} = \sqrt{\frac{.68(.32)}{1105}} = \sqrt{\frac{.2176}{1105}} = \sqrt{.000196923} = .014032928 = \mathbf{1.4\%}$$

c) Assume p = 4% and our sample size, n = 250 (4 points)

$$\sigma = \sqrt{\frac{.04(1-.04)}{250}} = \sqrt{\frac{.04\ (.96)}{250}} = \sqrt{\frac{.0384}{250}} = \sqrt{.0001536} = .012393547 = \mathbf{1.2\%}$$

Problem 6.4

The University of Michigan Survey Research Center conducted a survey of 2000 people in 1976. 20% of those surveyed said they felt unhappy about their jobs.

Find a 95% Confidence Interval for the true proportion of people who felt unhappy about their jobs. (6 points)

p=20%=.20 n=2000

Standard deviation = $\sqrt{\frac{(.2)(1-.2)}{2000}} = \sqrt{\frac{(.2)(.8)}{2000}} = \sqrt{\frac{.16}{2000}} = \sqrt{.00008}$ =.008944272 =

0.89442% = .9%

2 * (.9%) = 1.8%

The 95% confidence interval can be stated two ways (either is correct):

20% $\pm$ 1.8%, or

18.2% ⟵————⟶ **21.8%**

Problem 6.5

In July, 1998, the National Science Foundation commissioned a survey which revealed that vast numbers of American adults are scientifically illiterate. A Boston Globe article describing the results of this study reported that of 2041 adults questioned, only 72% knew the earth revolved around the sun and not the other way around!!!

a) Assuming this was a random sample of all American adults, find a 95% confidence interval for the true proportion of scientifically knowledgeable American adults. (6 points)

p=72%= .72 n=2041

Standard deviation = $\sqrt{\frac{(.72)(1-.72)}{2041}} = \sqrt{\frac{(.72)(.28)}{2041}} = \sqrt{\frac{.2016}{2041}} = \sqrt{.000098775}$ = .009938567

=0.9938567% = 1.0%

2 * (1.0%) =2.0%

The 95% confidence interval can be stated two ways (either is correct):

72% $\pm$ 2.0%, or

70% ⟵————⟶ 74%

b) How would the size of the Confidence Interval change if four times as many American adults were surveyed? Explain why your answer is different (if it is) and <u>show</u> your calculations to prove your answer. (6 points)

p=72%=.72 n= 4*2041 = 8164

Standard deviation = $\sqrt{\frac{(.72)(1-.72)}{8164}} = \sqrt{\frac{(.72)(.28)}{8164}} = \sqrt{\frac{.2016}{8164}} = \sqrt{.000024694}$ =.004969283

=0.4969283% = 0.5%

2 * (0.5%) =1.0%

The 95% confidence interval can be stated two ways (either is correct):

72% $\pm$ 1.0%, or

71% ⟷ 73%

The confidence interval is narrower because the margin of error is cut in half when you quadruple the sample size.

c) What if the size of the survey was one-fourth as large? i.e. 1/4 of 2041 (with rounding)? Again, explain and <u>show</u> your calculations. (6 points)

p=72%=.72 n=1/4 * 2041 = 510

Standard deviation = $\sqrt{\frac{(.72)(1-.72)}{510}} = \sqrt{\frac{(.72)(.28)}{510}} = \sqrt{\frac{.2016}{510}} = \sqrt{.000395294}$ =.019882005

=1.9882005% = 2.0%

2 * (2.0%) =4.0%

The 95% confidence interval can be stated two ways (either is correct):

72% $\pm$ 4.0%, or

68% ⟷ 76%

The confidence interval is wider because the margin of error doubles when you cut the sample size to ¼ of its original size.

Problem 6.6

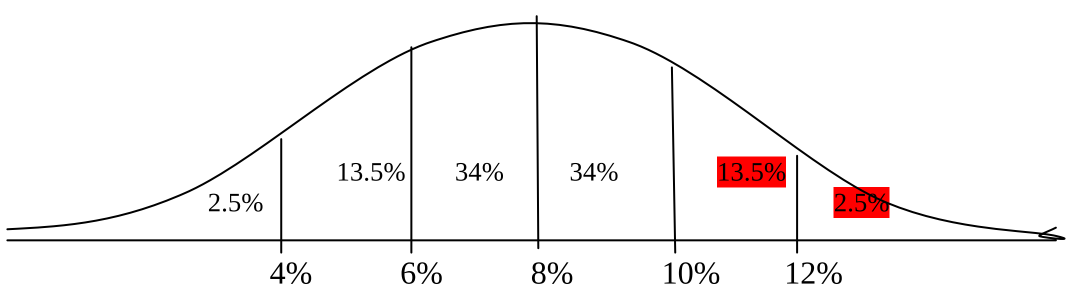
The *Encyclopaedia Britannica* reports that 8% (use this value for your mean=p) of all North American males have some form of red-green color blindness. Imagine that you take a random sample of 184 North American males. What is the probability that in this sample over 10% of these people will be color blind? **Hint:** You should draw a normal distribution curve to answer this question.

(6 points)

p=8%=.08 n=184

Standard deviation = $\sqrt{\frac{(.08)(1-.08)}{184}} = \sqrt{\frac{(.08)(.92)}{184}} = \sqrt{\frac{.0736}{184}} = \sqrt{.0004} = .02 = 2\%$

North American Males' Red-Green Color Blindness



Probability that over 10% are color blind in this sample: 13.5% +2.5% =**16%**

Problem 6.7

In yet another article in the Boston Globe (back in May of 1989) it was reported that in a study of 854 young women (ages 12-23) about 67% of them were dissatisfied with their weight.

a) Assuming once again that this was a random sample, find a 95% Confidence Interval for the true proportion of young women (ages 12-23) who were dissatisfied with their weight.

(6 points)

p = 67% = .67 n = 854

Standard deviation = $\sqrt{\frac{(.67)(1-.67)}{854}} = \sqrt{\frac{(.67)(.33)}{854}} = \sqrt{\frac{.2211}{854}} = \sqrt{.000258899}$

= .016090348 = 1.6090348% = 1.6%

2 * (1.6%) = 3.2%

The 95% confidence interval can be stated two ways (either is correct):

67% $\pm$ 3.2%, or

63.8% $\longleftrightarrow$ **70.2%**

b) How much smaller a group could you have surveyed if you were willing to accept a 95% Confidence Interval of 67% $\pm$ 5%? (1 pt extra credit)

Hint: You can use trial and error or algebra. There is nothing wrong with trial and error! In either case, we will give you 1 point extra credit if you get within 5 of the true answer. Round your answer to a whole number since we can't survey parts of people! To solve the problem algebraically, you will have to use the new formula "in reverse." I.e. instead of knowing p and n, and then solving for the standard deviation, you will start by knowing the standard deviation and p, and then solving for n. That's one of the great things about formulas – you can generally use them in more than one way.

If margin of error =5%, and the margin of error =2 * the standard deviation, then standard deviation =2.5% or stated as a decimal .025.

We have our standard deviation, and our mean, but not our sample size:

p = 67% = .67 standard deviation = .025 n = unknown

So use the formula with *n* as the variable.

$.025 = \sqrt{\frac{(.67)(1-.67)}{n}}$

You either guess what n is and do multiple calculations to see how close you can get to a 5% margin of error, or solve algebraically.

Guessing:

If a sample size of 854 above gave you a margin of error of 3.2%, our sample size would be smaller than 854 to give the larger margin of error of 5%.

Start with 500 (you could start with something else but this is for example).

$$\text{Standard deviation} = \sqrt{\frac{(.67)(1-.67)}{500}} = \sqrt{\frac{(.67)(.33)}{500}} = \sqrt{\frac{.2211}{500}} = \sqrt{.0004422} = .021028552 =$$

2.1028552 % = 2.1%

Since the standard deviation is 2.1, our margin of error is 2*2.1=4.2.

This is not low enough of a sample size, so try 400:

$$\text{Standard deviation} = \sqrt{\frac{(.67)(1-.67)}{400}} = \sqrt{\frac{(.67)(.33)}{400}} = \sqrt{\frac{.2211}{400}} = \sqrt{.00055275} = .023510636=$$

2.3510636% = 2.4%

Since the standard deviation is 2.4%, the margin of error is 4.8%.
This is close but we can do better.

Try a bit lower at 350:

$$\text{Standard deviation} = \sqrt{\frac{(.67)(1-.67)}{350}} = \sqrt{\frac{(.67)(.33)}{350}} = \sqrt{\frac{.2211}{350}} = \sqrt{.000631714}$$

= .025133927 =

2.5133927= 2.5%

Since the standard deviation is approximately 2.5%, the margin of error for a sample size of 350 is approximately 5%. We are really close here. **The answer would be that a sample size of approximately 350 will produce a 5% margin of error.**

Now let's see how to do this algebraically.

Algebraic Solution

Start with the formula for standard deviation filled in for the known values and use n or some other variable to represent the unknown sample size:

$$.025 = \sqrt{\frac{(.67)(1-.67)}{n}}$$

Simplify for any operations that can be immediately be done:

Perform the subtraction inside the parenthesis:

$$.025 = \sqrt{\frac{(.67)(.33)}{n}}$$

Now do the multiplication:

$$.025 = \sqrt{\frac{.2211}{n}}$$

Now, begin to isolate or uncover *n* by eliminating the square root sign. You do this by squaring both sides of the equation. Squaring is the opposite of taking a square root. It "undoes" the square root:

$$(.025)^2 = \left(\sqrt{\frac{.2211)}{n}}\right)^2$$

When you square both sides, you are left with:

$$.000625 = \frac{.2211}{n}$$

Our variable *n* is in the denominator so in order to bring it up into the numerator, we would multiply both sides of the equation by *n*:

$$.000625(n) = \left(\frac{.2211}{n}\right)(n)$$

The *n*'s on the right hand side cancel:

$$.000625(n) = \left(\frac{.2211}{\cancel{n}}\right)(\cancel{n})$$

We are left with:

$.000625n = .2211$

Now we can finally isolate *n* completely and solve the equation. Since *n* is being multiplied by .000625, we need to do the opposite operation, which is division, to both sides of the equation:

$$\frac{.000625n}{.000625} = \frac{.2211}{.000625}$$

The .000625 cancels on the left side, and dividing .2211 by .000625 leaves us with 353.76.

$$\frac{\cancel{.000625}n}{\cancel{.000625}} = \frac{.2211}{.000625}$$

$n = 353.76$

Round up to the nearest whole person since it wouldn't make sense to survey .76 of a person!

The answer is 354, very close to our guess!