Using Definition of Continuity and Limit Laws

Show that the function $f(x) = \frac{1}{x} + \sqrt{x+1}$ is continuous at $x = 1$

① $a = x$, $a = 1$

$f(a) = \frac{1}{a} + \sqrt{a+1}$, $a = 1$, $g(a) = 1/a$, $h(a) = \sqrt{a+1}$

Domain of $g(a)$: $(-\infty, 0) \cup (0, \infty)$

Domain of $h(a)$: $[-1, \infty)$ ← $a = 1$ falls within these intervals

Both $g(a)$ and $h(a)$ are continuous at $a = 1$,
then $f(a)$ is also continuous at $a = 1$.

② $\lim_{x \to 1} \underbrace{\frac{1}{x}}_{g(a)} + \underbrace{\sqrt{x+1}}_{h(a)}$

$=$

$\lim_{x \to 1} \frac{1}{x} + \lim_{x \to 1} \sqrt{x+1}$

$=$

$\frac{\lim_{x \to 1} 1}{\lim_{x \to 1} x} + \sqrt{\lim_{x \to 1} x+1}$

Domain of $g(a)$: $(-\infty, 0) \cup (0, \infty)$
Domain of $h(a)$: $[-1, \infty)$
→ Same intervals as $f(a)$

③ $f(a)$ and $\lim_{x \to 1} \frac{1}{x} + \sqrt{x+1}$ have same domains,
f is continuous at $x = 1$