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↗ Vectors ↙

Quiz 5

1a. Pic 3

1b. $\bar{v} = \langle 3, 1 \rangle$ $\quad \bar{w} = \langle 2, -4 \rangle$

$5\bar{v} + 6\bar{w}$

$5\langle 3, 1 \rangle + 6\langle 2, -4 \rangle$

$\langle 15, 5 \rangle + \langle 12, -24 \rangle$

$\langle 15 + 12, \; 5 + (-24) \rangle$

$5\bar{v} + 6\bar{w} = \boxed{\langle 27, -19 \rangle}$

1c. $|\bar{v}|$ and $|\bar{w}|$

$|\bar{v}| = \sqrt{3^2 + 1^2}$

$|\bar{v}| = \sqrt{9 + 1}$

$|\bar{v}| = \boxed{\sqrt{10}}$

$|\bar{w}| = \sqrt{2^2 + (-4)^2}$

$|\bar{w}| = \sqrt{4 + 16}$

$|\bar{w}| = \sqrt{20}$

$|\bar{w}| = \sqrt{4 \cdot 5}$

$|\bar{w}| = \boxed{2\sqrt{5}}$

1d. $\bar{v} \cdot \bar{w}$ $\quad \bar{v} = \langle 3, 1 \rangle$ $\quad \bar{w} = \langle 2, -4 \rangle$

$(3 \cdot 2) + (1 \cdot -4)$

$6 + (-4)$

$\textcircled{2}$

$\boxed{\bar{v} \cdot \bar{w} = 2}$

1e.

$\bar{v} = \langle 3, 1 \rangle$
$\bar{w} = \langle 2, -4 \rangle$

$$\text{Proj}_{\bar{w}} \bar{v} = \frac{\bar{w} \cdot \bar{v}}{|\bar{w}|^2} \bar{w}$$

$\bar{w} \cdot \bar{v}$
$(2 \cdot 3) + (-4 \cdot 1)$
$6 + (-4)$
(2)

$|\bar{w}| = \sqrt{2^2 + (-4)^2}$
$|\bar{w}| = \sqrt{4 + 16}$
$|\bar{w}| = \sqrt{20}$
$|\bar{w}| = \sqrt{4 \cdot 5} = \boxed{2\sqrt{5}}$

$$\text{Proj}_{\bar{w}} \bar{v} = \frac{2}{(2\sqrt{5})^2} \langle 2, -4 \rangle$$

$(2\sqrt{5})(2\sqrt{5})$
$(2)(2)(\sqrt{5})(\sqrt{5})$
$(4)(\sqrt{25})$
$(4)(5)$
(20)

$\frac{2}{20}$
$\frac{1}{10} \langle 2, -4 \rangle$
$\left\langle \frac{1}{10} \cdot \frac{2}{1}, \frac{1}{10} \cdot \frac{-4}{1} \right\rangle$
$\left\langle \frac{1}{5}, -\frac{2}{5} \right\rangle$

$$\boxed{\text{Proj}_{\bar{w}} \bar{v} = \left\langle \frac{1}{5}, -\frac{2}{5} \right\rangle}$$

1f.

$\overline{v} = \langle 3, 1 \rangle$ $\quad$ $\overline{w} = \langle 2, -4 \rangle$

Get dot product and magnitudes

$\cos\theta = \dfrac{\overline{v} \cdot \overline{w}}{|\overline{v}| \cdot |\overline{w}|}$ Exact Value

$\overline{v} \cdot \overline{w} = (3 \cdot 2) + (1 \cdot -4)$

$6 + (-4)$

$\boxed{2}$

$|\overline{v}| = \sqrt{3^2 + 1^2}$

$|\overline{v}| = \sqrt{9+1}$

$|\overline{v}| = \boxed{\sqrt{10}}$

$|\overline{w}| = \sqrt{2^2 + (-4)^2}$

$|\overline{w}| = \sqrt{4+16}$

$|\overline{w}| = \sqrt{20}$

$|\overline{w}| = \sqrt{4 \cdot 5}$

$|\overline{w}| = \boxed{2\sqrt{5}}$

Get $\cos\theta$ Exact Value

$\cos\theta = \dfrac{2}{\sqrt{10} \cdot 2\sqrt{5}}$

$\cos\theta = \dfrac{2}{2\sqrt{50}} \rightarrow \dfrac{2}{2\sqrt{25} \cdot 2} \rightarrow \dfrac{2}{2 \cdot 5\sqrt{2}} \rightarrow \dfrac{2}{10\sqrt{2}} \cdot \dfrac{\sqrt{2}}{\sqrt{2}} = \dfrac{2\sqrt{2}}{10\sqrt{4}} = \dfrac{\sqrt{2}}{5 \cdot 2}$

$\cos\theta = \boxed{\dfrac{\sqrt{2}}{10}}$

Get Angle

$\cos^{-1}(\cos\theta) = \cos^{-1}\left(\dfrac{\sqrt{2}}{10}\right)$

$\theta \approx 81.8699^\circ = \boxed{82^\circ}$