MATH E-3: Lecture 2

Quantitative Reasoning: Practical Math

Spring 2016

https://www.google.com/search?q=harvard+maxwell+dworkin&source=lnms&tbm=isch&sa=X&ved=0ahUKEwi4zdidjdfKAhWDcT4KHUiBDVEQ_AUICSgD&biw=1280&bih=564#q=harvard+maxwell+dworkin&tbs=sur:f&tbm=isch&imgrc=vPxYB1176r1SPM%3A

ASSIGNMENTS

- Assignments will be posted on Wednesday
- All assignments are due by 11:59 am (Eastern Time) Saturday of the following week, unless otherwise stated. **Assignment upload closes at noon on Saturday, Eastern Time**
  - **TIP:** Do not wait until the last minute to upload your assignment
- Late assignments will not be accepted. **Do not email your assignment**; only assignments uploaded to the course website by the assignment due date will be graded
- We will drop your 2 lowest assignment grades

ASSIGNMENTS

- The homework assignment schedule, listing lecture topics, due dates and grade release dates, is posted on the course website under the **Homework Help Center** module.

- Solutions will be posted to the course website shortly after the upload deadline. Compare solutions with your answers and, if you have questions, attend a section.

ASSIGNMENTS

- Assignment 1 (introduction) is due Saturday, 2/6 by 11:59 a.m. (Eastern Time)
- Upload closes at noon (Eastern Time)
- Comments from your grader by 2/13.

Homework Assignment Submission

- Access the assignment in the weekly module, print, complete by hand, scan (using a desktop scanner or smartphone), and upload
  - **No photos saved as a PDF file, no editing tools**

- See "PDF Help" on the **Homework Help Center** page for help

Homework Assignment Submission

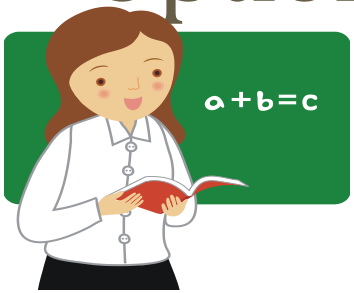
"Homework Help Center"

CHECKLIST

- ☑ single PDF file
- ☑ 4mb maximum
- ☑ file name (example: albrigo.assign1)
- ☑ work must be neat and legible (flag your final answer) and scanned to the appropriate drop box (pages in order and no upside down or sideways pages)
- ☑ review your assignment after submitting
- ☑ upload by Saturday, 11:59 a.m. (ET) deadline

Review carefully the Homework Policies section outlined in the syllabus.

Optional TA Sections/Resources



- Online (via Conferences on the course website): Wednesdays 7:30-9:00 pm (Eastern Time) with Jessica.
  - First online section will be February 10
- On campus: Tuesdays from 5:30-7:00 pm, Sever 104, with Sue
  - First on campus section will be February 16
- The Math Question Center: http://www.extension.harvard.edu/resources-policies/resources/math-question-center

Video Course Guidelines and Technical help

- http://www.extension.harvard.edu/academics/courses/types-courses/video-course-guidelines

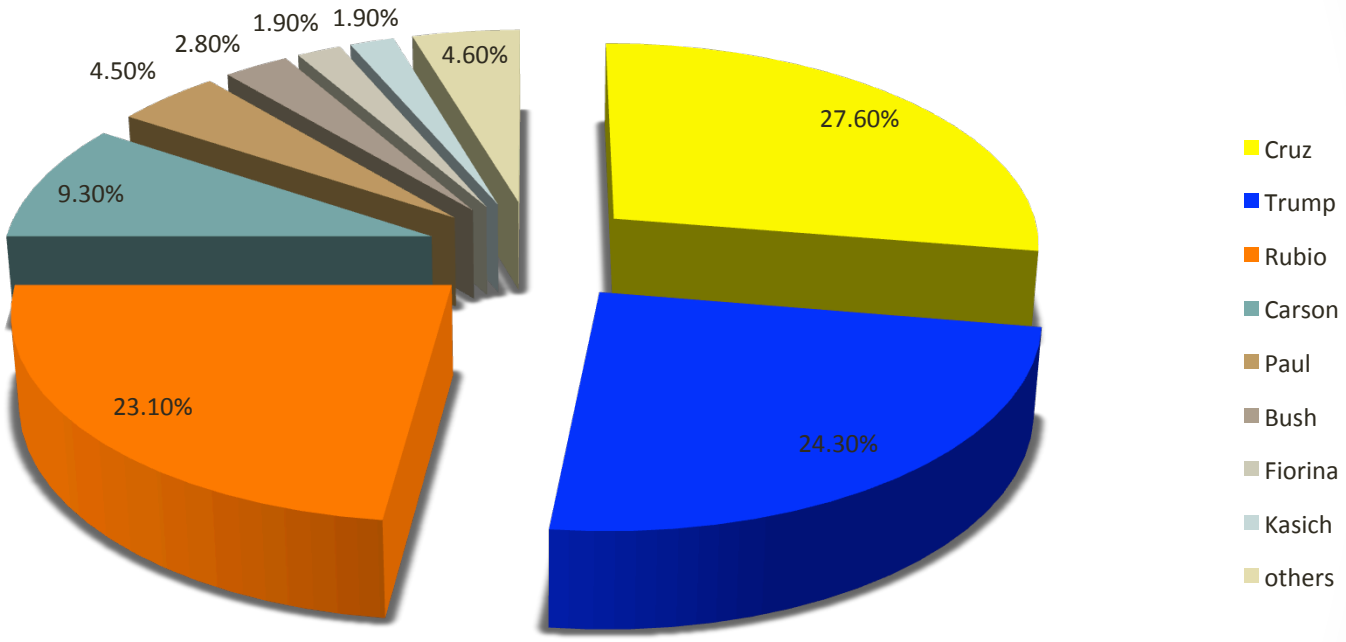
Resources

For technical help with Canvas, you may refer to the **"Help"** button on the course website, which includes the **CANVAS STUDENT GUIDE:**
https://community.canvaslms.com/community/answers/guides/

MATH E-3 COMMUNITY

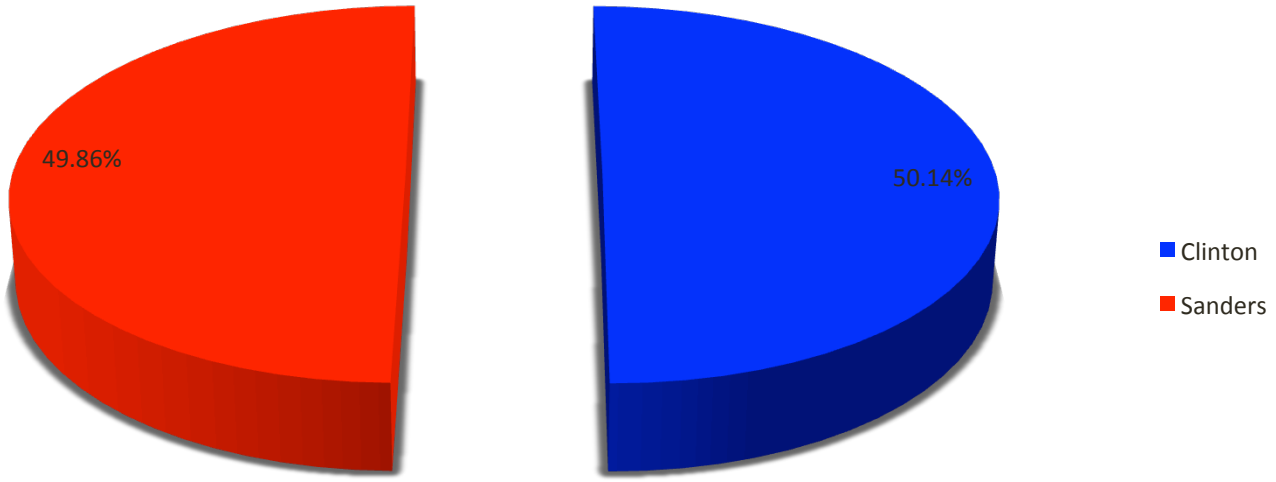
- **Say Hello!**
- **Student Locations**
- **Discussion Board (optional)** – **Get to know your fellow classmates!** If you do collaborate on assignments, you must follow the Collaboration Policy outlined in the syllabus.

Iowa Caucus results:
Republican candidates



http://www.foxnews.com/politics/elections/2016/primary-caucus-results/iowa

Iowa Caucus results:
Democratic candidates



http://www.foxnews.com/politics/elections/2016/primary-caucus-results/iowa

Why would anyone take a math course??

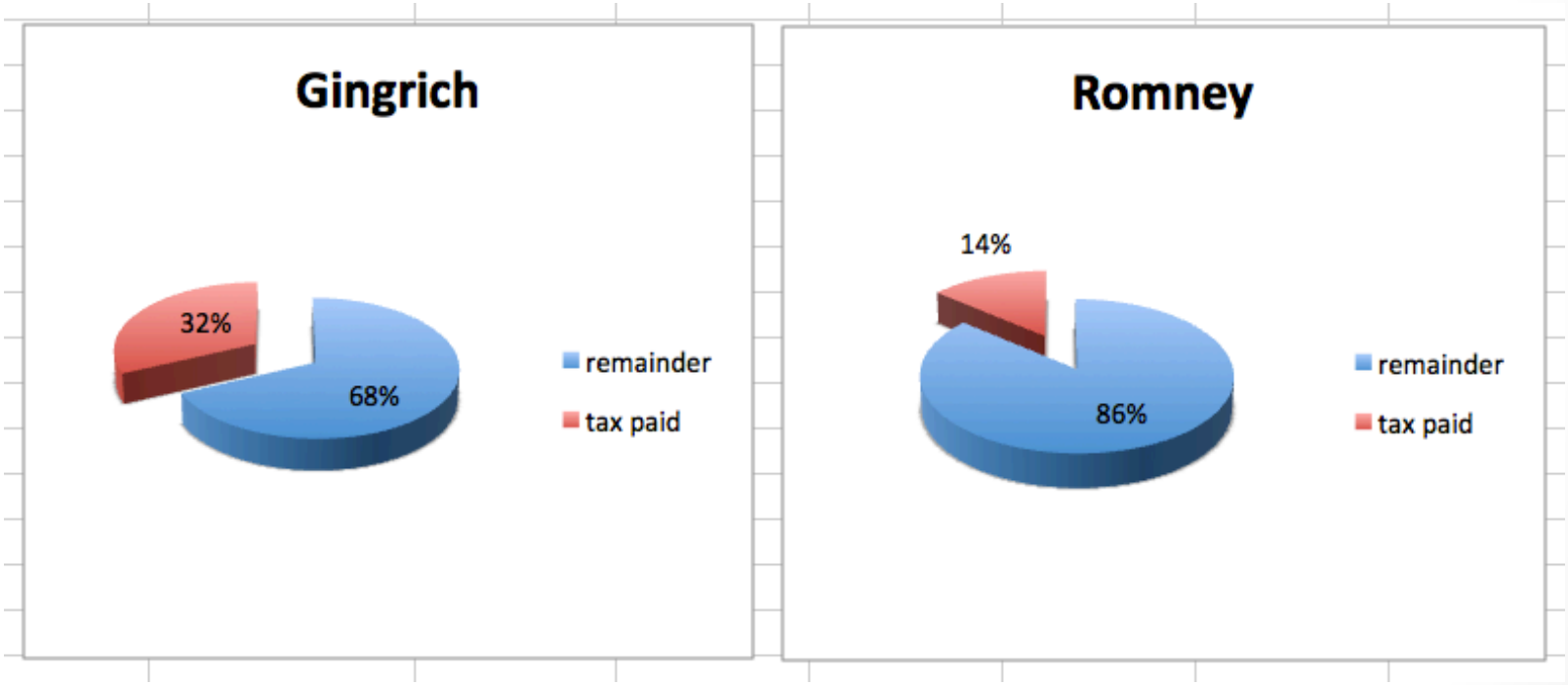
- Math is useful
- Math is interesting
- Math is fun
- The requirement . . .

Recap . . .

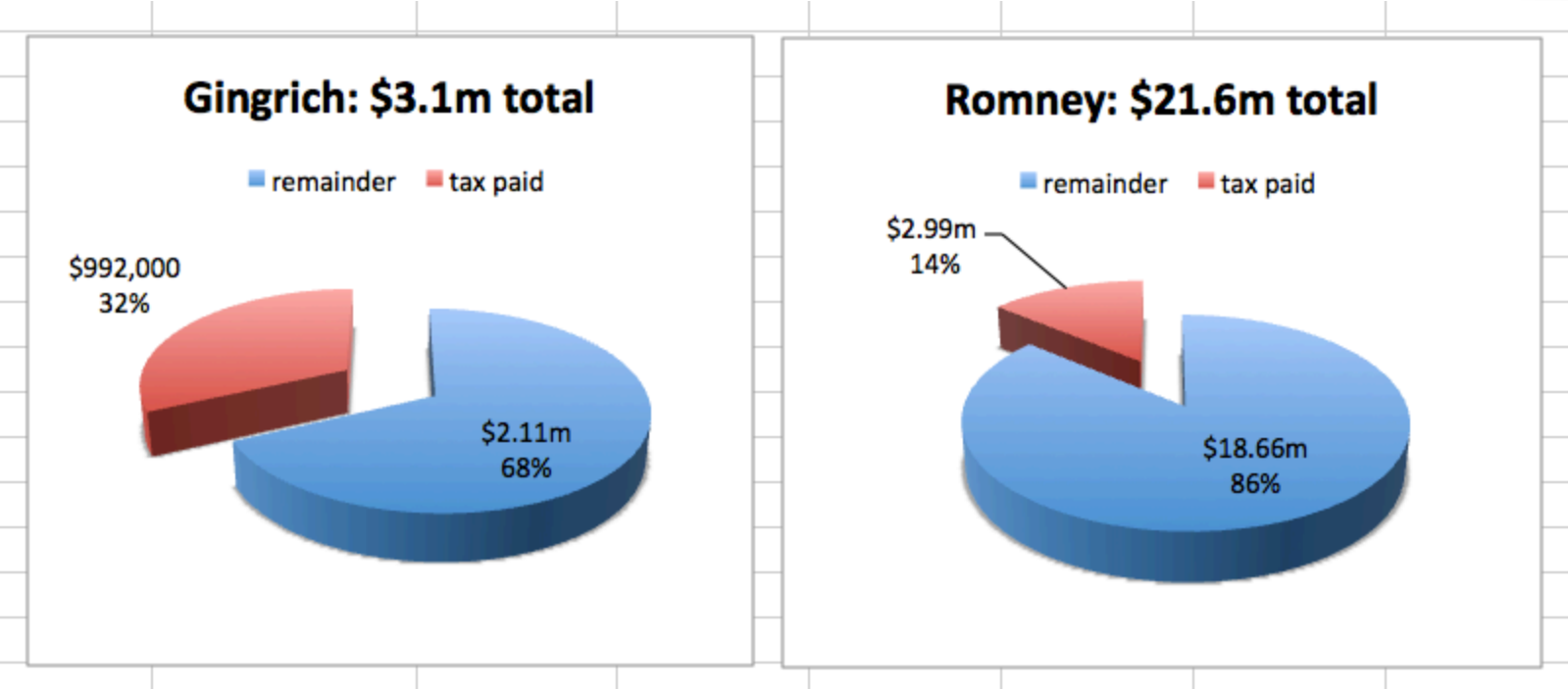
- Displaying of data
- Organizing of data
- Looking for trends, patterns
- Types of charts
- Types of numbers

2 interesting pie charts . . .

From a few years back…

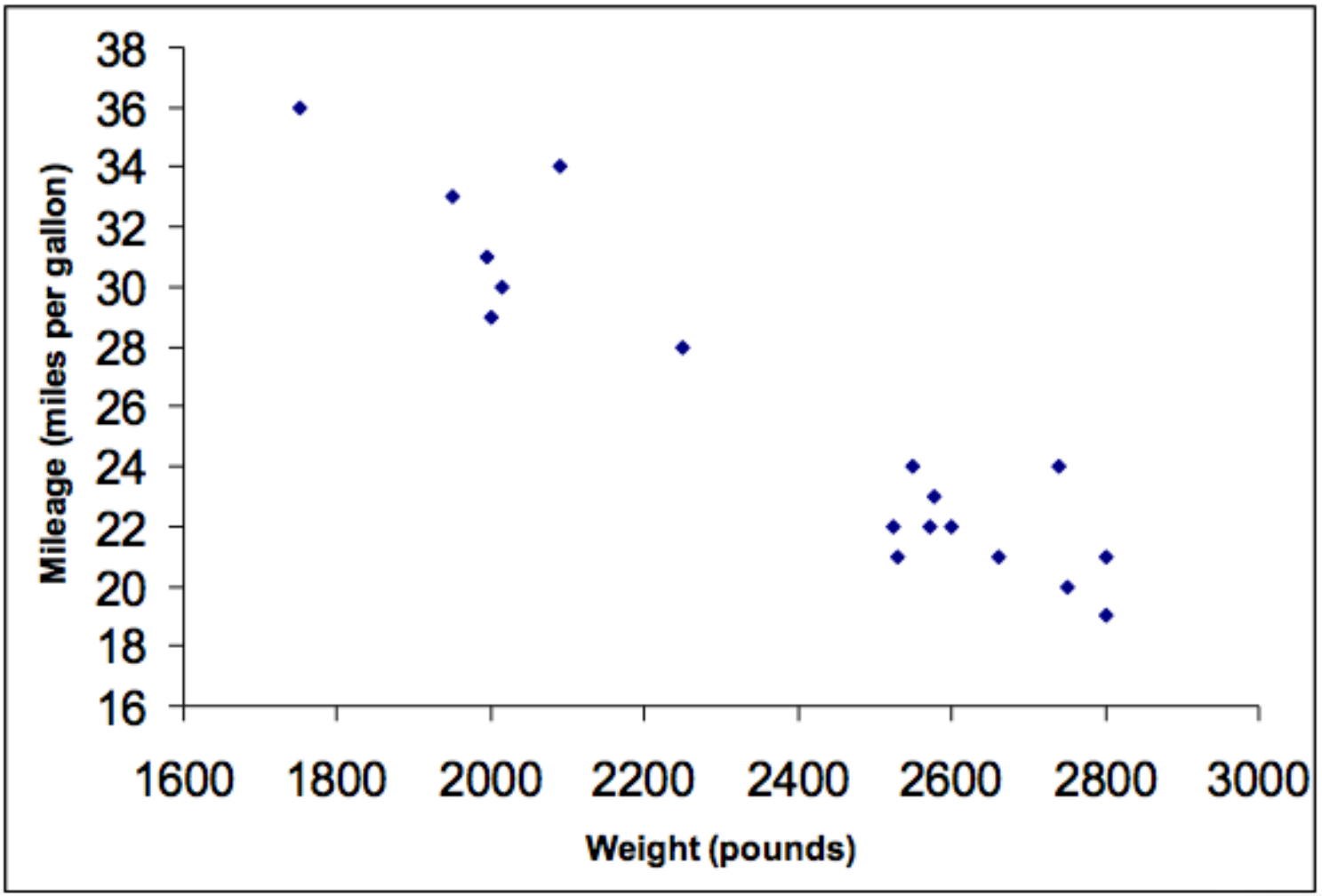


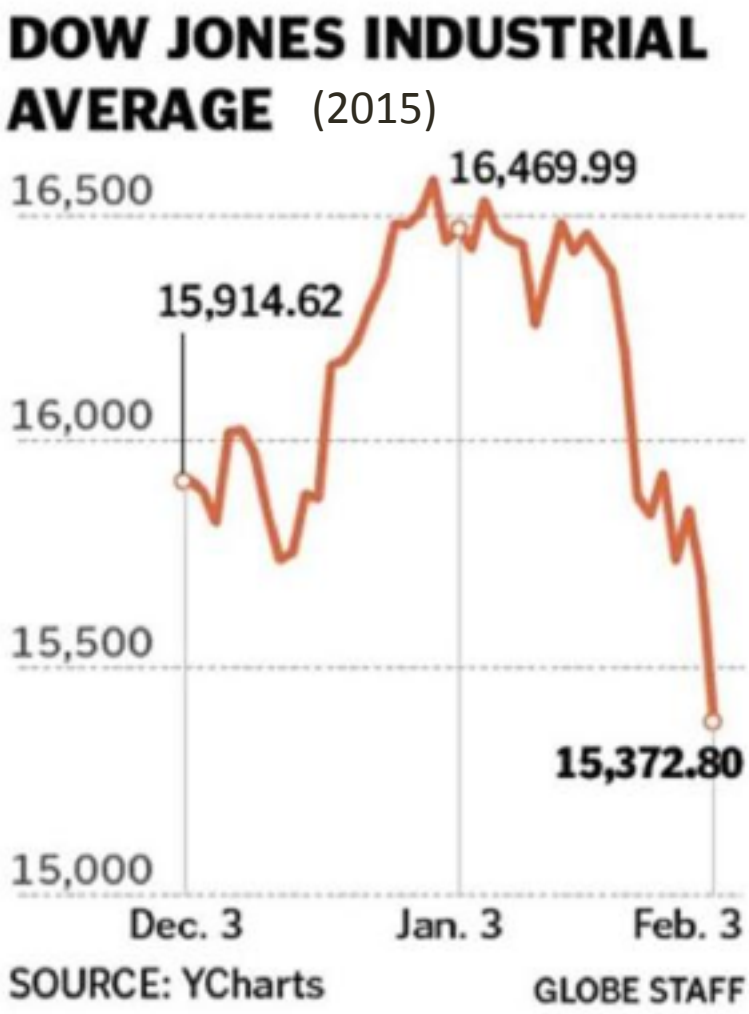
Important information



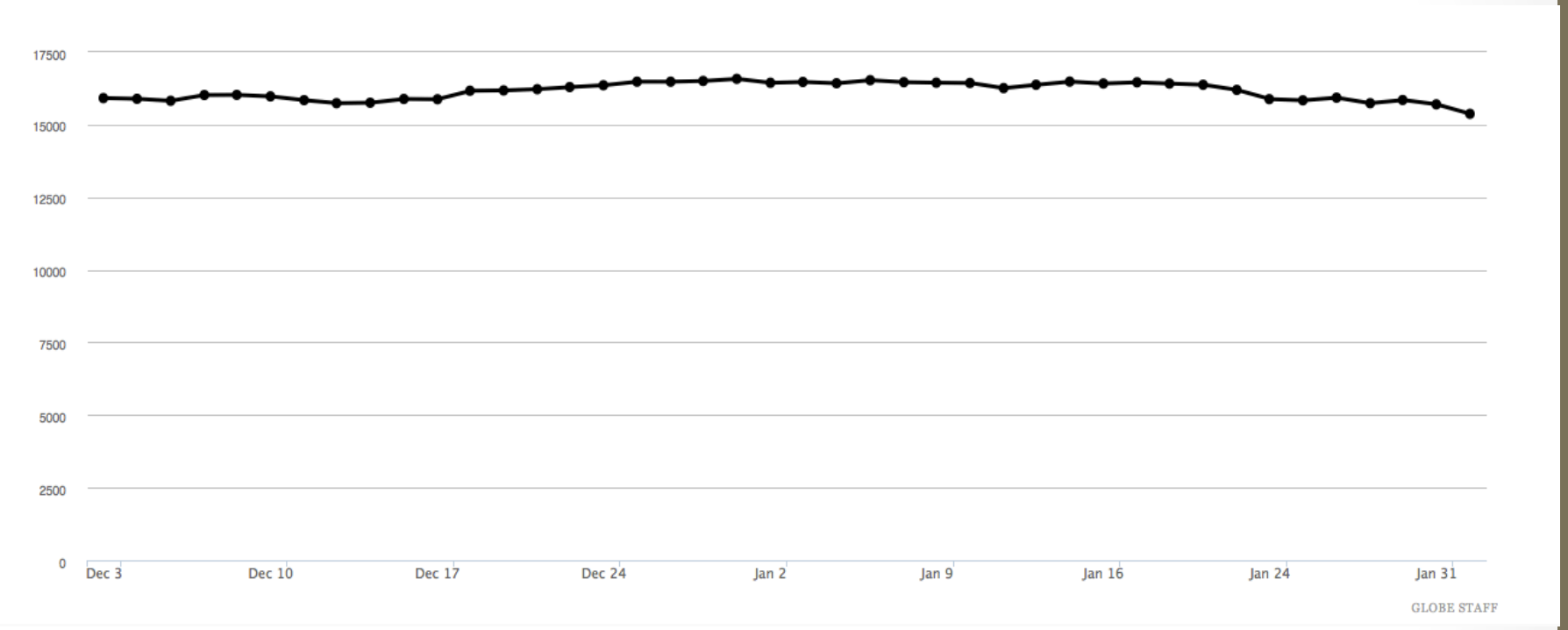
http://www.forbes.com/sites/beltway/2012/01/26/what-the-romney-and-gingrich-1040s-tell-us-about-how-we-tax-the-rich/

Scatterplot




DOW JONES INDUSTRIAL AVERAGE (2015)
16,469.99
16,500
15,914.62
16,000
15,500
15,372.80
15,000
Dec. 3
Jan. 3
Feb. 3
SOURCE: YCharts
GLOBE STAFF

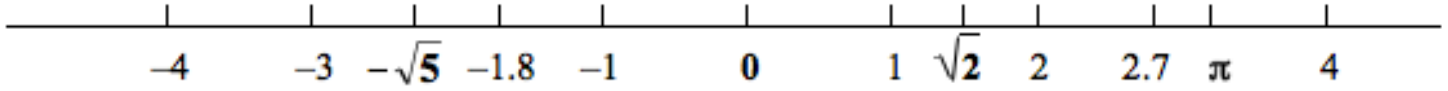
The same data, presented “differently”



Types of numbers

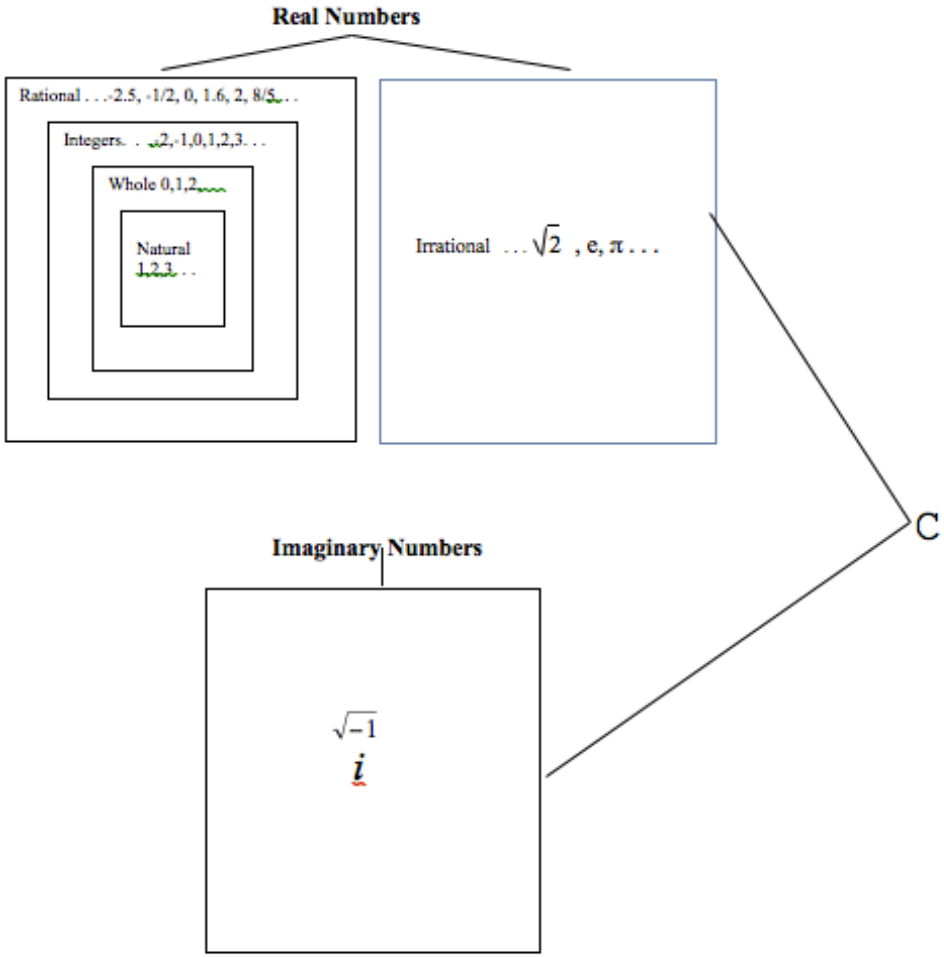
- **The Real Number System (R) consists of 'SETS' of numbers as follows:**
- **(N)** **Natural** Numbers – Counting No's
- **(W)** **Whole** Numbers – Natural No's and Zero
- **(Z)** **Integers** – Whole No's and Negatives
- **(Q)** **Rational** Numbers–Integers and Fractions
- **(H)** **Irrational** Numbers
- **Thus, the real numbers consists of all the Rational and Irrational numbers.**
- **(R) Real Numbers** = **Rational and Irrationals**

The Real Number Line:



Note: Negative numbers are to the left of zero on the number line and positive numbers are to the right of zero.

The Complex Number System



Important Mathematical Notations and Symbols #1

Inequalities

"less than"

$5 < 8$ $-4 < 0$ watch the negative sign!

"greater than"

$7 > 3$ $-2 > -6$

more examples:

$2/3 > \frac{1}{2}$ $-2.75 < 0.1$ $1.25\% > 0.3\%$ $\sqrt{2} < \pi$

"Less than or equal to" **English: "AT MOST"**

a number, x, that is less than or equal to 5 is written $x \leq 5$

"Greater than or equal to" **English: "AT LEAST"**

a number, x, that is greater than or equal to 3 is written $x \geq 3$

Important Mathematical Notations and Symbols #2

Absolute Value

a. $|-7| = 7$

b. $|99| = 99$

c. $|11 - 2| = |9| = 9$

d. $|15 - 20| = |-5| = 5.$

Important Mathematical Notations and Symbols #3

- **Interval Notation and Infinity**
-
- Open and closed intervals:

- (2, 5) "open": $2 < x < 5$ [2, 5] "closed": $2 \leq x \leq 5$
-
- How about (-4, 4]? This is called a 'half –OPEN' (or half-closed) interval. It is OPEN on the left, CLOSED on the right. $-4 < x \leq 4$
-
- **What is infinity?? (beyond the scope of this course . . .)**
-

"All positive numbers": (0, ∞)

"All numbers greater than or equal to -3": [-3, ∞ **)**

Important Mathematical Notations and Symbols #4

Summation Notation

The symbol, capital Greek sigma or Σ, means to 'add up.'

For example, $\sum_{k=1}^{4} k$ means to take the number 1 and add it to the number 2, then add that sum to the number 3, etc.

The 'k' indicates what the beginning and ending numbers should be.

Written out we get, $1 + 2 + 3 + 4$. The sum = 10.

Often, we only see the summation sign itself, Σ .

Important Mathematical Notations and Symbols #5

Radicals

The $\sqrt{\ }$ is called a 'radical' sign.
A radical indicates a 'root' of something. e.g. the square root of 2 is written $\sqrt{2}$.

We can have other roots: e.g. the cube root of 8, written $\sqrt[3]{8} = 2$, or
the cube root of negative 8, $\sqrt[3]{-8} = -2$ because $-2 \times -2 \times -2 = -8$

A root is also a power, but the power is a fraction.

For example, the square root of a number may be written as a number raised to the ½ power.

The cube root is a number raised to the $\frac{1}{3}$ power.

e.g. $\sqrt{36} = 36^{1/2} = 6$ since $6 \times 6 = 36$

e.g. $\sqrt[4]{256} = 256^{1/4} = 4$ since $4 \times 4 \times 4 \times 4 = 256$

Important Mathematical Notations and Symbols #6

Rounding Numbers

If the number to the right of where you wish to round is 5 or larger,

then add one to the rounded place and truncate the remaining numbers.

e.g. round to 3 decimal places, 0.078249 = 0.078 because the number after the 8 is 2.

e.g. round to 1 decimal place 2.962 = 3.0 since there is a 6 after the 9.

Do not start way over to the right and keep rounding one number after the other.

Only look at the one spot after the number of places to which you wish to round.

Important Mathematical Notations and Symbols #7

What's wrong with this slide?

Scientific Notation

Rule 1: Only one (1) digit appears to the left of the decimal point. There can be as many numbers to the right of the decimal as you like, depending on the desired accuracy

e.g. $68.214 = 6.82147 \times 10^1$ The '6' is to the left of the decimal point.

Rule 2: Following the numbers, we multiply by a power of 10. e.g. $4500 = 4.5 \times 10^3$
If the power is positive, we have a larger number. To get to the expanded form, move the decimal point to the RIGHT the number of places indicated by the magnitude of the power. Empty spaces are filled in with zeroes.

e.g. 4.5×10^3 means move the decimal point 3 places to the right, i.e. 4500.

If the power is negative, move the decimal point that number of places to the left.

e.g. $7.2583 \times 10^{-4} = .00072583$

Example: The distance from the earth to the sun is 9.3×10^7 miles. Write out in expanded form.
Answer: Since the exponent of 10 is 7, move the decimal point 7 places to the right. Fill in those empty places with zeroes. Thus:
$9.3 \times 10^7 = 93{,}000{,}000$ miles. (The decimal point was moved 7 places to the right.)

Important Mathematical Notations and Symbols #7a

Improved version . . .

Scientific Notation

Rule 1: Only one (1) digit appears to the left of the decimal point.

There can be as many numbers to the right of the decimal as you like, depending on the desired accuracy

e.g. $68.214 = 6.82147 \times 10^1$

The '6' is to the left of the decimal point.

Important Mathematical Notations and Symbols #7b

Improved version . . .

Rule 2: Following the numbers, we multiply by a power of 10.

e.g. $4500 = 4.5 \times 10^3$

If the power is positive, we have a larger number. To get to the expanded form, move the decimal point to the RIGHT the number of places indicated by the magnitude of the power. Empty spaces are filled in with zeroes.

e.g. 4.5×10^3 means move the decimal point 3 places to the right, i.e. 4500.

If the power is negative, move the decimal point that number of places to the left.

e.g. $7.2583 \times 10^{-4} = .00072583$

Important Mathematical Notations and Symbols #7c

Improved version . . .

Example 1: The distance from the earth to the sun is 9.3×10^7 miles. Write out in expanded form.

Answer: Since the exponent of 10 is 7, move the decimal point 7 places to the right.

Fill in those empty places with zeroes. Thus:

$9.3 \times 10^7 = 93{,}000{,}000$ miles. (The decimal point was moved 7 places to the right.)

Example 2: A hydrogen atom has a diameter of approximately 1.06×10^{-10} meters. Write this in expanded form.

Answer: Move the decimal place 10 places to the left, filling in the empty spaces with zeroes:

$1.06 \times 10^{-10} = 0.000000000106$ meters.

Calculators and Scientific Notation

Most calculators display a number like

7.2583×10^{-4}

by putting the exponent as a two digit number a few blank spaces to the right of the digits,

7.2583 –04

or 93 million as **9.30000 07**.

Check your calculators!

Factorials

Take the number and multiply it by each preceding number, i.e. by that number less 1 and so on. Look at the examples for a better understanding.

ex. 1. 6 factorial is written as 6! It means to multiply the following:

$6 \times 5 \times 4 \times 3 \times 2 \times 1 = 720$

ex. 2. $48! = 48 \times 47 \times 46 \times 45 \times 44 \times 43 \times 42 \times \ldots = 1.241391 \times 10^{61}$ (a very large number)

Factorials are used in the study of probability.

Addition and Subtraction in the Real Number System

- In order to add or subtract numbers as we do everyday, we must first start by looking at two numbers to check their signs and the ***operation*** between them. For example:
- ex. a. $7 + -3$ We have a positive 7 and a negative 3 and the operation is addition.
- ex. b. $-8 - 2$ We have a negative 8 and a positive 2 and the operation is subtraction.

(the sign on the 2 is invisible. When a number is positive we do not need to show the plus sign. This could have been written as $-8 - +2$)

THE RULES FOR ADDITION

(1) If the signs on the two numbers are the same, just add the two numerals*, use the sign that's there.

(2) If the signs are different, find the difference of the numerals*,

use the sign of the larger one.

***Note: The numeral is simply the number *without* the sign attached. This is often designated as the 'absolute' value or the 'magnitude' of the number.**

THE RULES FOR SUBTRACTION

1. CHANGE the *subtraction* operation sign (a minus sign) to an addition (plus) sign.

2. CHANGE the sign of the number **after** the operation sign to its opposite sign. Say aloud,

"ADD THE OPPOSITE"

Examples . . .

ex. 1. **13 – – 8** (the ‘numerals’ are 13 and 8)

Step One: "add the opposite" **13 – – 8 = 13 + + 8**

Step Two: Since the signs are the same just add

13 + 8 = 21

Examples . . .

- ex. 2. **– 8 – 2** (ex. b. from previous slide)
-
- Step One: "add the opposite" **– 8 – 2 = – 8 + – 2**
-
- Step Two: Signs are the same on both numerals so add numerals, use sign that's there
-
- **– 8 + – 2 = – 10**

Examples . . .

- ex. 3. **10 – 6** this is a normal subtraction as we know it, but let's see if the rules work.
-
- Step One: "add the opposite" 10 + – 6
-
- Step Two: Look at the signs. Signs are different so find the difference of just the numbers themselves, i.e. the 10 and the 6. Now use the sign of the larger numeral.
-
- **10 – 6 = 10 + – 6 = 4**

- Since 10 is positive and largest numeral.

Examples . . .

- ex. 5. **– 7 – 1**
-
- Step One: "add the opposite" **– 7 – 1 = – 7 + – 1**
-
- Step two: Signs same so add the numerals and use sign that's there.
-
- **– 7 + – 1 = – 8**
-

Examples . . .

- ex. 6. **4 – 11**
-
- Step One: "add the opposite" **4 – 11 = 4 + – 11**
-
- Step two: Signs different so find the difference between 4 and 11. Use sign of larger.
-
- **4 + – 11 = – 7**

PEMDAS (what??)

P*arentheses* (innermost first)

E*xponents*

M*ultiplication*
D*ivision* } left to right whichever comes first

A*ddition*
S*ubtraction* } left to right whichever comes first

Doing it wrong . . .

- ex. 1: This is an example of an **INCORRECT method with the correction**.
-
- (10 – 2) – (6 + 10)
-
- 8 – 6 + 10 = 2 + 10 = 12 WRONG!!
- You CANNOT take off parentheses arbitrarily.
-
- Always do what's inside the **Parentheses** first.
- (10 – 2) – (6 + 10)
- 8 – 16 = – 8 **CORRECT**

A more involved example

- ex. 2: **$18 - 5 * 2 (6 - 9) - 3^2 - 24 \div 2 * 4$**
-
- Step 1: Do what's in **Parentheses first** rewriting entire problem to avoid confusion
-
- $= 18 - 5 * 2 (- 3) - 3^2 - 24 \div 2 * 4$

- Note: the parentheses $(6-9) = -3$

A more involved example

- Step 2: **Exponents**
-
- = 18 – 5 * 2(– 3) – 9 – 24 ÷ 2 * 4
-
- Step 3: **Multiplication** and **Division** going left to right doing whichever **comes first**. (You may need to repeat this step)
-
- = 18 – 10(– 3) – 9 – 12 * 4 Notice: we multiplied 5 times 2 instead of subtracting 5 from 18.
- Also, we divided 24 by 2 instead of multiplying 2 * 4 first since the division came first! We repeated the multiplication step.
- = 18 – – 30 – 9 – 48

A more involved example

- Step 4: Addition and Subtraction, again going left to right doing whichever comes first.
- $= 48 - 9 - 48$
- $= 39 - 48$

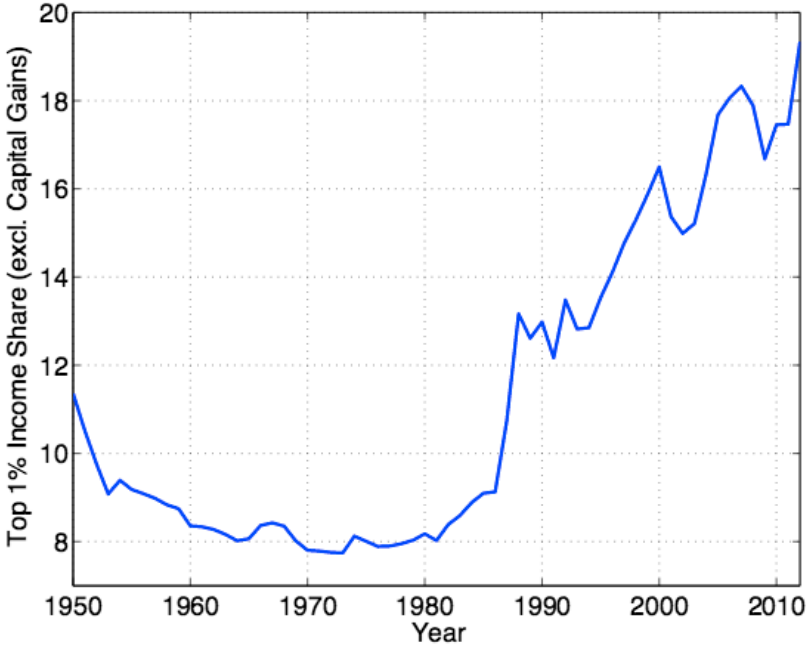
- Of course, this problem could be done using fewer steps, but if you do combine steps, you must be careful!

- $= -9$

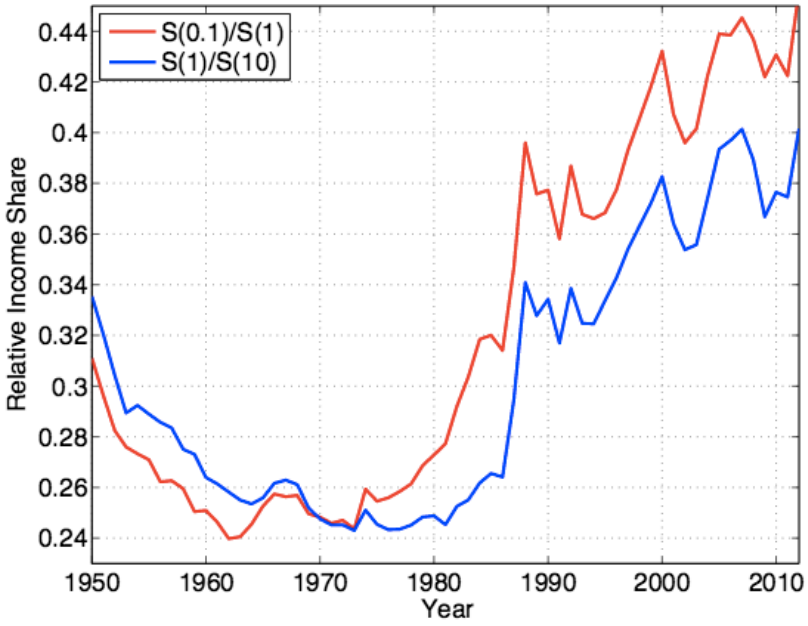
The Dynamics of Inequality*

Xavier Gabaix, Jean-Michel Lasry, Pierre-Louis Lions, Benjamin Moll

June 25, 2015



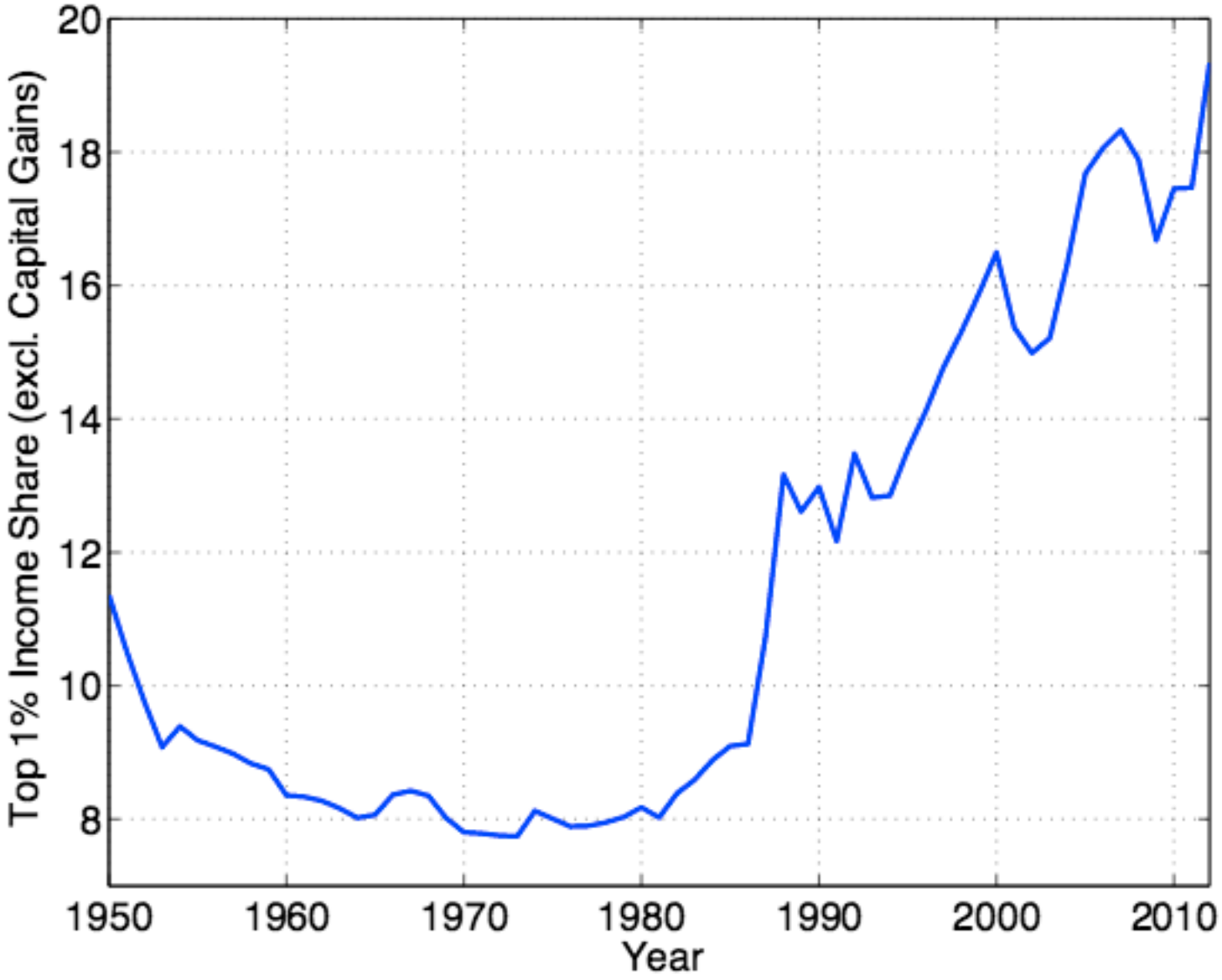
(a) Top Income Inequality



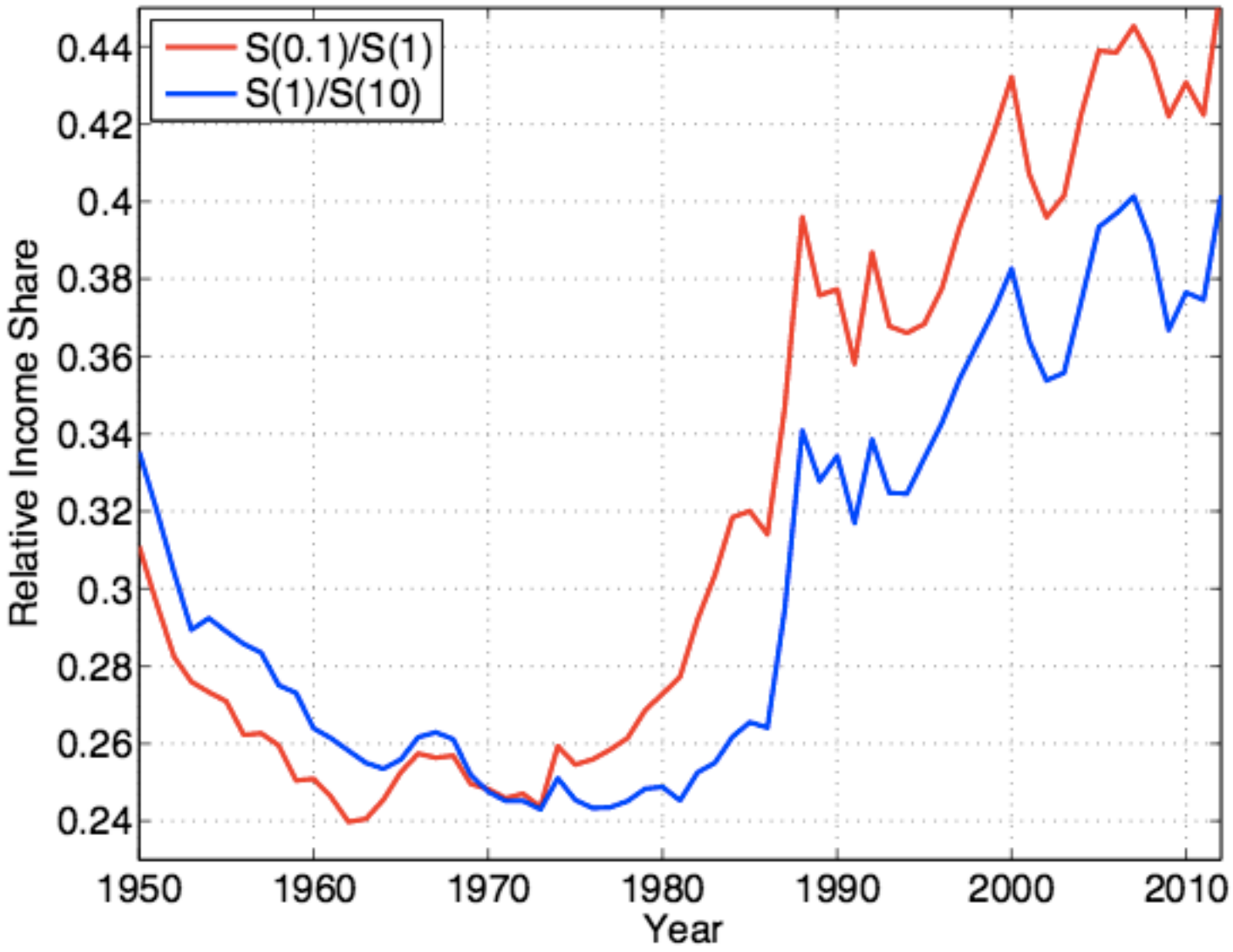
(b) Relative Income Shares

Figure 1: Evolution of Top 1% Income Share and "Fractal Inequality" in U.S.

http://www.princeton.edu/~moll/dynamics.pdf



(a) Top Income Inequality



(b) Relative Income Shares

Lots more good stuff to come . . .