19.
From the unit circle:
$\theta_1 = -45^o$

$\theta_2 = 360^o + (-45^o) = 315^o$

$\theta_3 = 180^o - (-45^o) = 225^o$

21.
From the unit circle:
$\theta_1 = 60^o$

$\theta_2 = 180^o - (60^o) = 120^o$

23.
Using your calculator:
$\theta_1 = -40.54^o$

$\theta_2 = 360^o + (-40.54^o) = 319.46^o$

$\theta_3 = 180^o - (-40.54^o) = 220.54^o$

25.
Using your calculator:
$\theta_1 = 14.48^o$

$\theta_2 = 180^o - (14.48^o) = 119.54^o$

27.
From the unit circle:
$\theta_1 = 45^o$

$\theta_2 = 360^o - (45^o) = 315^o$

29.
From the unit circle:
$\theta_1 = 150^o$

$\theta_2 = 360^o - (150^o) = 210^o$

31.
Using your calculator:
$\theta_1 = 28.36^o$

$\theta_2 = 360^o - (28.36^o) = 331.63^o$

33.
Using your calculator:
$\theta_1 = 132.84^o$

$\theta_2 = 360^o - (132.84^o) = 227.16^o$

35.
From the unit circle:
$\theta_1 = 30^o$

$\theta_2 = 180^o + (30^o) = 210^o$

37.
From the unit circle:
$\theta_1 = -60^o$

$\theta_2 = 180^o + (-60^o) = 120^o$

$\theta_3 = 360^o + (-60^o) = 300^o$

39.
Using your calculator:
$\theta_1 = 70.35^o$

$\theta_2 = 180^o + (70.35^o) = 250.35^o$

41.
Using your calculator:
$\theta_1 = -60.95^o$

$\theta_2 = 180^o + (-60.95^o) = 119.05^o$

$\theta_3 = 360^o + (-60.95^o) = 299.05^o$

49.

$$\cos\left(\sin^{-1}\left(\frac{1}{2}\right)\right)$$

$$\tan\left(\frac{\pi}{6}\right)$$

$$\frac{\sqrt{3}}{2}$$

51.

$$\sin^{-1}\left(\cos\left(\frac{2\pi}{3}\right)\right)$$

$$\sin^{-1}\left(\frac{-1}{2}\right)$$

$$\frac{-\pi}{6}$$

53.

$$\tan\left(\sin^{-1}\left(\frac{\sqrt{2}}{2}\right)\right)$$

$$\tan\left(\frac{\pi}{4}\right)$$

1

55.

$\cos(\tan^{-1}(-1))$

$$\cos\left(\frac{-\pi}{4}\right)$$

$$\frac{\sqrt{2}}{2}$$

57.

$$\cos^{-1}\left(\sin\left(\frac{-\pi}{6}\right)\right)$$

$$\cos^{-1}\left(\frac{-1}{2}\right)$$

$$\frac{2\pi}{3}$$

59.

$$\cot^{-1}\left(\tan\left(\frac{\pi}{3}\right)\right)$$

$$\cot^{-1}\left(\sqrt{3}\right)$$

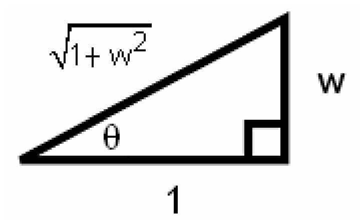
$$\frac{\pi}{6}$$

61.

Show: $\sec\left(\tan^{-1}(w)\right) = \sqrt{1+w^2}$

Let $\theta = \tan^{-1}(w)$ then
$\tan(\theta) = \tan(\tan^{-1}(w))$
thus $\tan(\theta) = w$

We then get the following triangle:



Now find $\sec(\theta)$:

$$\sec(\theta) = \frac{\sqrt{1+w^2}}{1} = \sqrt{1+w^2}$$

Therefore:

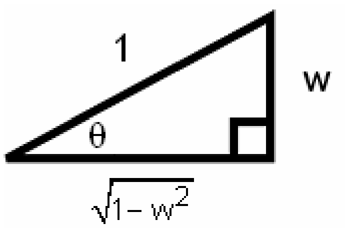
$$\sec(\theta) = \sec\left(\tan^{-1}(w)\right) = \sqrt{1+w^2}$$

63.

Show: $\tan\left(\sin^{-1}(w)\right) = \dfrac{w\sqrt{1-w^2}}{1-w^2}$

Let $\theta = \sin^{-1}(w)$, then
$\sin(\theta) = \sin(\sin^{-1}(w))$
thus $\sin(\theta) = w$

We then get the following triangle:



Now find $\tan(\theta)$:

$$\tan(\theta) = \frac{w}{\sqrt{1-w^2}}$$

Rationalize the denominator:

$$\frac{w}{\sqrt{1-w^2}} = \frac{w}{\sqrt{1-w^2}} \bullet \frac{\sqrt{1-w^2}}{\sqrt{1-w^2}}$$

$$= \frac{w\sqrt{1-w^2}}{1-w^2}$$

Therefore:

$$\tan(\theta) = \tan\left(\sin^{-1}(w)\right) = \frac{w\sqrt{1-w^2}}{1-w^2}$$

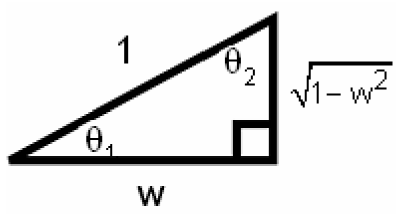
65.

Show: $\cos^{-1}(w) + \sin^{-1}(w) = \dfrac{\pi}{2}$

let $\theta_1 = \cos^{-1}(w)$ and $\theta_2 = \sin^{-1}(w)$
$\cos(\theta_1) = \cos(\cos^{-1}(w))$
thus $\cos(\theta_1) = w$

also from $\theta_2 = \sin^{-1}(w)$
$\sin(\theta_2) = \sin(\sin^{-1}(w))$
thus $\sin(\theta_2) = w$

Knowing $\cos(\theta_1) = w$ and $\sin(\theta_2) = w$, we get the following triangle:



Since θ_1 and θ_2 are part of the same triangle, the sum of the three angles equals 180^o. Since this is a right triangle, one of the angles is a 90^o angle.

Therefore: $\theta_1 + \theta_2 = \dfrac{\pi}{2}$

Therefore: $\cos^{-1}(w) + \sin^{-1}(w) = \dfrac{\pi}{2}$

67.

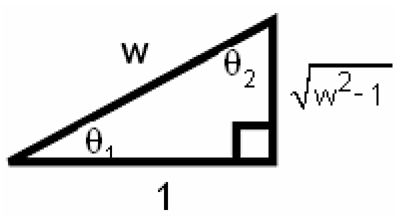
Show: $\sec^{-1}(w) + \csc^{-1}(w) = \frac{\pi}{2}$

$\sec^{-1}(w) + \csc^{-1}(w) = \frac{\pi}{2}$

let $\theta_1 = \sec^{-1}(w)$ and $\theta_2 = \csc^{-1}(w)$
$\sec(\theta_1) = \sec(\sec^{-1}(w))$
thus $\sec(\theta_1) = w$

also from $\theta_2 = \csc^{-1}(w)$
$\csc(\theta_2) = \csc(\csc^{-1}(w))$
thus $\csc(\theta_2) = w$

Knowing $\sec(\theta_1) = w$ and $\csc(\theta_2) = w$, we get the following triangle:



Since θ_1 and θ_2 are part of the same triangle, the sum of the three angles must equal 180^o. Since this is a right triangle, one angle is a 90^o angle.

Therefore: $\theta_1 + \theta_2 = 90^o = \frac{\pi}{2}$

Therefore: $\sec^{-1}(w) + \csc^{-1}(w) = \frac{\pi}{2}$