$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$, where x_n is the nth approximation of the root.

Derive a sequence of approximations to a root of the equation $x^2 - 1 = 0$ using a starting value of 3, then -2 and 0.

$f(x) = x^2 - 1$

① Get $f'(x)$

$$f'(x) = \frac{d}{dx}[x^2 - 1]$$

$$\frac{d}{dx}[x^2] - \frac{d}{dx}[1]$$

$$2x^{2-1} - 0$$

$$2x^1 -$$

$$\boxed{f'(x) = 2x}$$

Subscript the x

② $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$ $\quad f(x_n) = x_n^2 - 1 \quad f'(x) = 2x_n$

$$\boxed{x_{n+1} = x_n - \frac{x_n^2 - 1}{2x_n}}$$

③ Iteration 1

$n=0$, $x_0 = 3$

$$x_{n+1} = x_n - \frac{x^2_n - 1}{2x_n}$$

$$x_1 = x_0 - \frac{x_0^2 - 1}{2x_0}$$

$$x_1 = 3 - \frac{3^2 - 1}{2 \cdot 3}$$

$$x_1 = 3 - \frac{9-1}{6}$$

$$x_1 = 3 - \frac{8}{6}$$

$$\frac{3}{1} = \frac{18}{6}$$

$$x_1 = \frac{-8}{6} = \frac{8}{6}$$

$$x_1 = \frac{10}{6}$$

$$\boxed{x_1 = \frac{5}{3}}$$

④ Iteration 2

$$x_2 = x_1 - \frac{x_1^2 - 1}{2x_1}$$

$$x_2 = \frac{5}{3} - \frac{\left(\frac{5}{3}\right)^2 - 1}{2\left(\frac{5}{3}\right)}$$

$$x_2 = \frac{5}{3} - \frac{\frac{25}{9} - 1}{\frac{10}{3}} \rightarrow \frac{25}{9} = \frac{25}{9} \cdot \frac{1}{1} = \frac{9}{9}$$

$$\frac{16}{9}$$

$$\frac{16}{9} \div \frac{10}{3}$$

$$\frac{\cancel{16}^{8}}{\cancel{9}_{3}} \cdot \frac{\cancel{3}^{1}}{\cancel{10}_{5}}$$

$$\frac{5}{3} - \frac{8}{15} \rightarrow \frac{5}{3} \quad \frac{25}{15}$$

$$-\frac{8}{15} = -\frac{8}{15}$$

$$\frac{17}{15}$$

$$\boxed{x_2 = \frac{17}{15}}$$

Iteration 3

$$x_3 = x_2 - \frac{x_2^2 - 1}{2x_2} =$$

$$x_3 = \frac{17}{15} - \frac{\left(\frac{17}{15}\right)^2 - 1}{2\left(\frac{17}{15}\right)}$$

$$\boxed{x_3 = \frac{257}{255}}$$

$x_0 = -2$ (Different Root)

① Iteration 1

$$x_1 = x_0 - \frac{x_0^2 - 1}{2x_0}$$

$$x_1 = -2 - \frac{(-2)^2 - 1}{2(-2)}$$

$$\boxed{x_1 = -\frac{5}{4}}$$

Iteration 2

$$x_2 = x_1 - \frac{x_1^2 - 1}{2x_n}$$

$$x_2 = -\frac{5}{4} - \frac{\left(-\frac{5}{3}\right)^2 - 1}{2\left(-\frac{5}{4}\right)}$$

$$\boxed{x_2 = -\frac{41}{40}}$$

Iteration 3

$$x_3 = x_2 - \frac{x_2^2 - 1}{2x_2} = -\frac{41}{40} - \frac{\left(-\frac{41}{40}\right)^2 - 1}{2\left(-\frac{41}{40}\right)}$$

$$\boxed{x_3 = -\frac{3281}{3280}}$$