MATH E-3: Lecture 7

Quantitative Reasoning: Practical Math

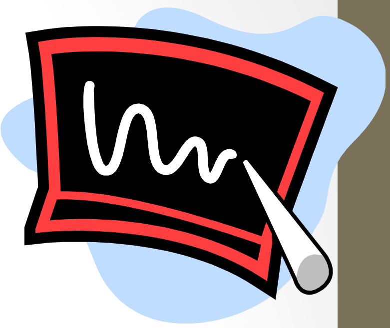
WELCOME BACK!
March 22, 2016

https://upload.wikimedia.org/wikipedia/commons/9/92/Colorful_spring_garden.jpg

Quiz 1

- Quiz 1 grades are now available

Homework



- Assignment 6 is due Saturday, March 26
- Assignment 7 will be posted tomorrow

HOMEWORK RESOURCES

“Homework Help Center”

- Sections (online and on campus)
- Math Question Center
- Khan Academy link (practice problems)
- Homework Schedule
- Assignment Submission checklist
- PDF Tips, including PDF compressor tool, merging PDFs

Quizzes . . .

Quiz 1 grades are available on Canvas

Quiz 2 will be April 5

Use quiz 1 as a learning tool for quiz 2!

Interpreting your score from quiz #1

Your raw score is out of 60.

To convert into a score out of 100, divide it by 0.6 (why is this?).

Grades: numbers and letters

93 ↑	A
90-92	A-
87-89	B+
83-86	B
80-82	B-
77-79	C+
73-76	C
70-72	C-

Etc.

Grades – a necessary evil . . .

Grading: The following is the grading scheme for the course. It is set up so that if you miss a quiz, you will not be penalized with a zero grade. Your final exam will account for the missed quiz. You MUST take the final exam even if all your other grades are 100%. Otherwise, the final exam will be counted as a zero.

Homework	-	25%; the two lowest grades will be dropped.
Quiz #1	-	25% if better than the final grade.
Quiz #2	-	25% if better than the final grade.
Final Exam	-	Minimum of 25%, Maximum 75% depending on quizzes.

A possible scenario . . .

Homework	Midterm 1	Midterm 2	Final Exam
90%	68%	78%	75%
.25*90%	not counted	.25*78%	.50*75%
22.5%		19.5%	37.5%
Grade =	79.5%		
Rounded =	80%		
Letter =	B-		

Another scenario . . .

Homework	Midterm 1	Midterm 2	Final Exam
0%	85%	91%	83%
0*90%	.25*85%	.25*91%	.25*83%
0.0%	21.25%	22.75%	20.75%
Grade =	64.8%		
Rounded =	65%		
Letter =	D		

Moral of the story??

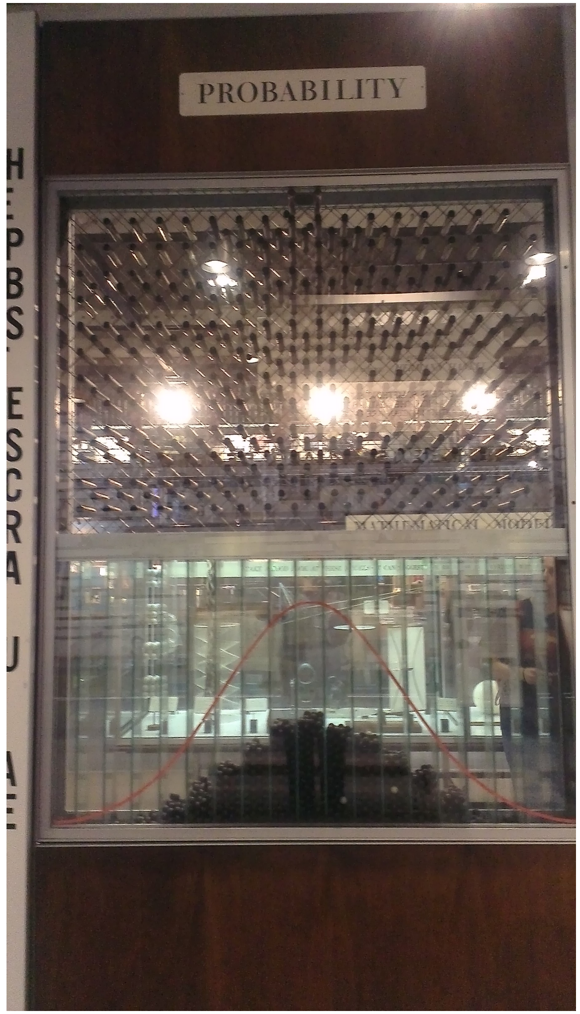
Something about doing homework . . .

Probability at the Museum of Science

Probability at the Museum of Science

Probability at the Museum of Science

Probability at the Museum of Science



Probability at the Museum of Science

Do the math? Only if I agree with it!

Better arithmetic is no match for partisan politics

By Keith O'Brien
Globe Correspondent
October 20, 2013

It's comforting, in these polarized times, to blame the idiots. Our failure to come together on issues like health care and gun control, climate change and economic policy, isn't our fault, we like to tell ourselves. It's a result of the talk-radio blowhards, the cable TV know-nothings, and the blind partisans—on the left and right—too dumb to look at the data and do the math.

http://www.bostonglobe.com/ideas/2013/10/20/math-only-agree-with/dNXiuubRILEUqtQ8IzUqEP/story.html

Do the math? Only if I agree with it!

But now new research is suggesting something different. The problem is not people who can't do the math. The problem is the ones who can.

In a new working paper, Yale Law School's Dan Kahan and three other researchers make the case that those more skilled at math are less likely to come to the correct conclusion on controversial matters—even when the numbers to support that conclusion are clear, empirical, and staring them in the face.

http://www.bostonglobe.com/ideas/2013/10/20/math-only-agree-with/dNXiuubRILEUqtQ8IzUqEP/story.html

Do the math? Only if I agree with it!

In the study, conducted this past spring, over 1,100 participants were asked to use numbers to assess whether a particular intervention had worked—either a skin rash treatment or a gun ban. In assessing the rash treatment, which had no political implications, those with lower "numeracy," or math skills, were, as you might guess, far less successful than their "high-numerate" peers: They were likely to get the question right about one-third of the time, compared to two-thirds of the time for the group with better math skills.

http://www.bostonglobe.com/ideas/2013/10/20/math-only-agree-with/dNXiuubRILEUqtQ8IzUqEP/story.html

Do the math? Only if I agree with it!

But the question about the gun ban revealed something that Kahan called "shocking and really disturbing." When doing the math yielded an answer that contradicted a participant's politics, the high-numerates were about as likely to get it wrong as their low-numerate counterparts.

High-numerate conservative Republicans were far more likely to come to the right, data-based conclusion in the gun ban question if they had been told that crime increased—a result that squares with their ideology. High-numerate liberal Democrats were far more likely to get it right if crime decreased. Perhaps most troubling, Kahan found, the numerate people were more polarized than those who struggled with the math: They were 45 percentage points more likely to get the right answer if the data backed up their views than if it didn't. Low-numerate people, by comparison, were far less polarized.

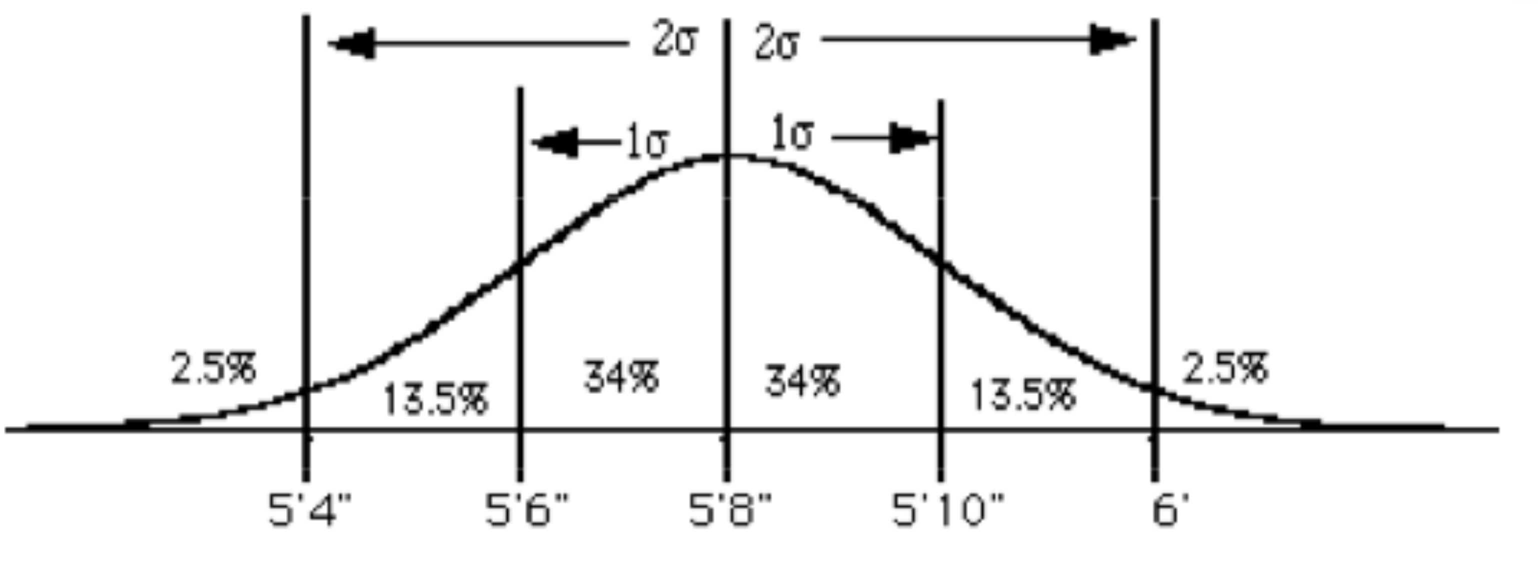
http://www.bostonglobe.com/ideas/2013/10/20/math-only-agree-with/dNXiuubRILEUqtQ8IzUqEP/story.html

Z-scores and Hypothesis Testing

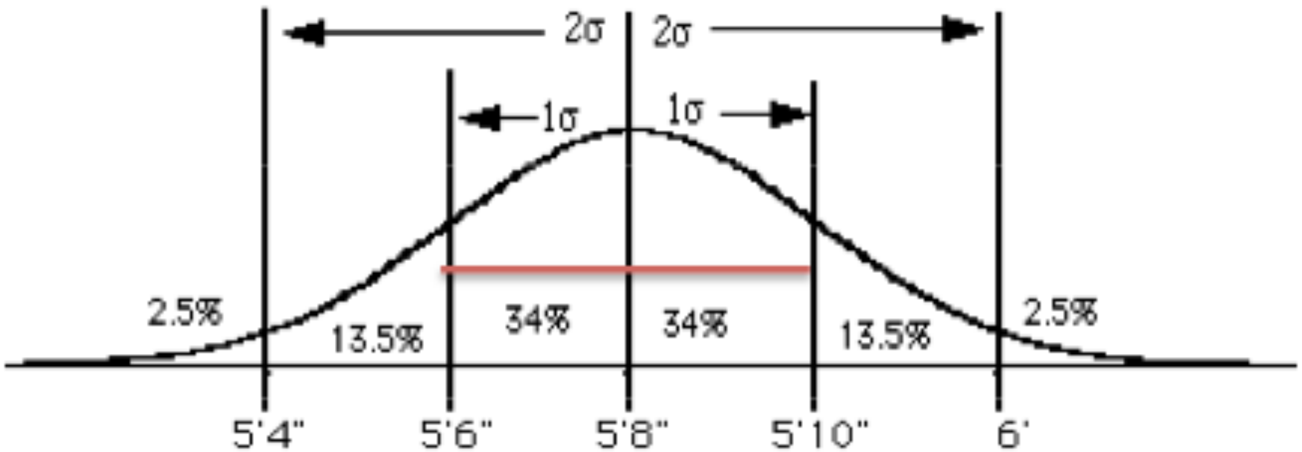
First, “z-scores”

- Remember when we looked at the heights of US adult males?
- We were given that the mean was 5’8”, and the standard deviation was 2”.
- We were able to calculate the percentage of US adult males who were between 5’6” and 5’10” etc.

Normal distribution of heights of US adult males



What percent have heights:



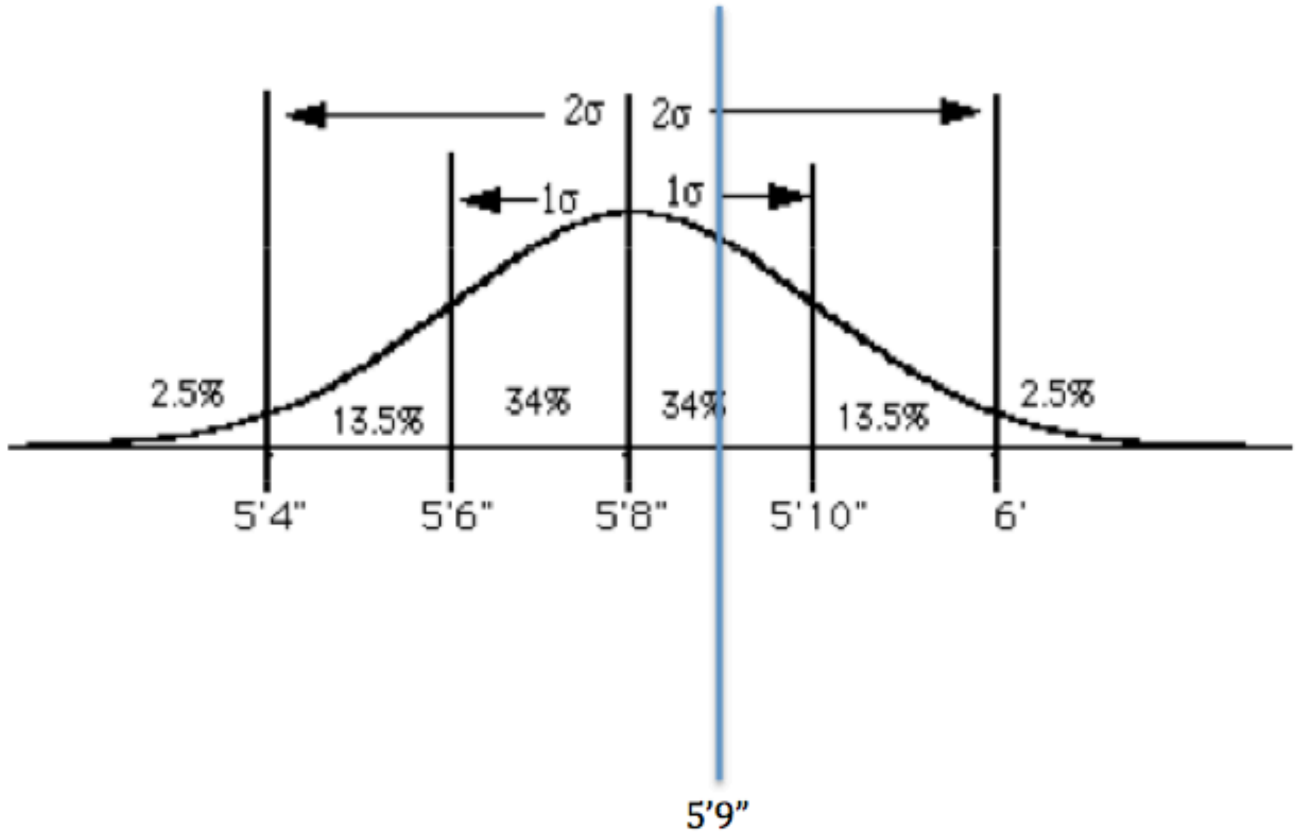
- a) between 5'6" and 5'10"? 34% + 34% = **68%**
- b) greater than 5'10"?
- c) less than 5'4"?
- d) between 5'4" and 6'?
- e) greater than 6'?

But we could <u>not</u> find, using this method:

- What percent have heights between 5’8” and 5’9”?
- *We can only calculate ranges that are multiples of the standard deviation (at least for “now” . . .)*
- Well, “now” has arrived . . .

- Why can’t we just say 5’9” is half way between 5’8”and 5’10,” and so say that the percentage is half of 34%, or 17%?

5’9” *is* half-way between 5’8” and 5’10,” but…



Why things are not quite so simple . . .

- The *distance* between 5’8” and 5’9” may be the same as the distance between 5’9” and 5’10”

- However, the *area* of the two pieces is not the same, because the bell curve is “sloping down . . .”

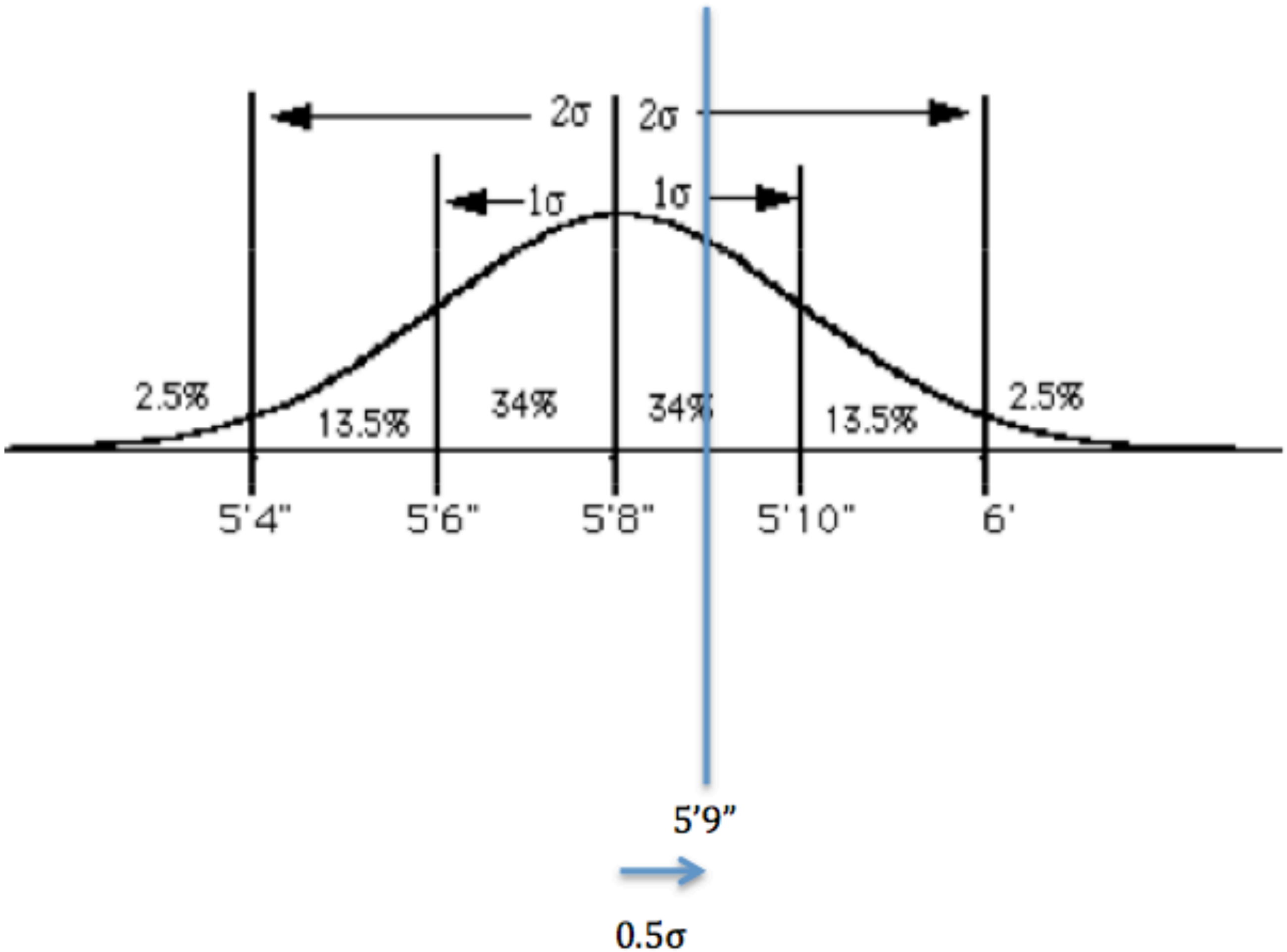
- The area between 5’8” and 5’9” is actually <u>more than</u> the area between 5’9” and 5’10”; so we expect the percentage to be more than half of the 34%.

That's why we need z scores!

- If we know how many standard deviations apart two numbers are, we can use z scores to calculate the percentage within that range.

- So in this case, what percentage of US adult males have heights between 5'8" and 5'9"?

5’9” is half-way between 5’8” and 5’10,” which is one half of a standard deviation above the mean.



And now, the moment you've all been waiting for . . .

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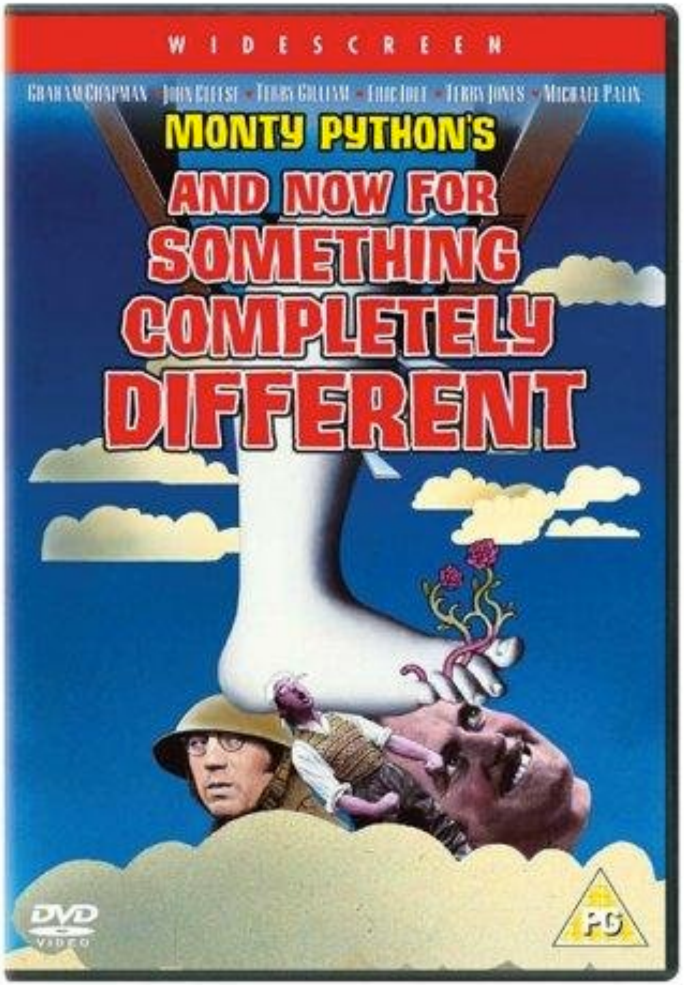
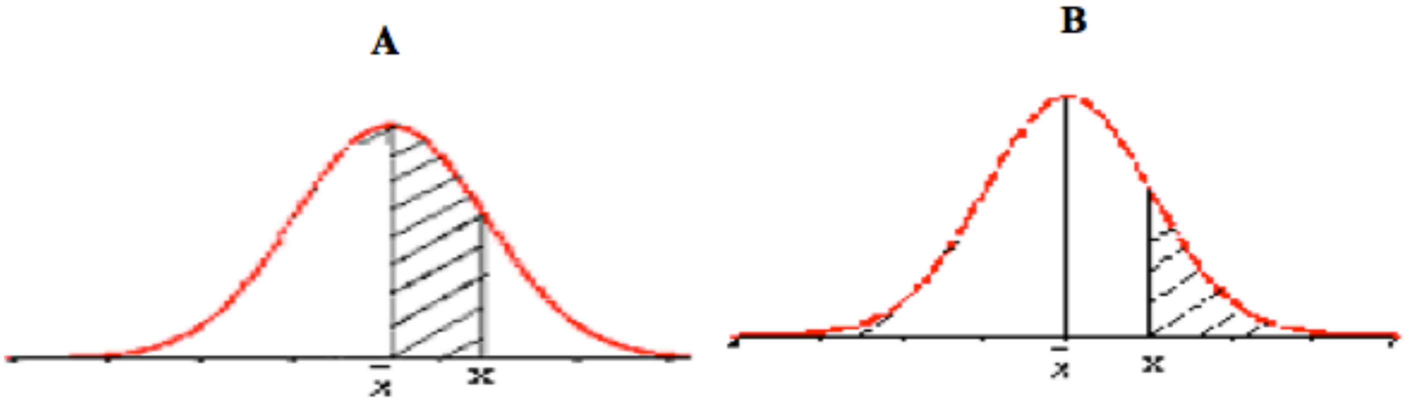


Oops!! Wrong moment

Z scores . . .

Z-Scores*

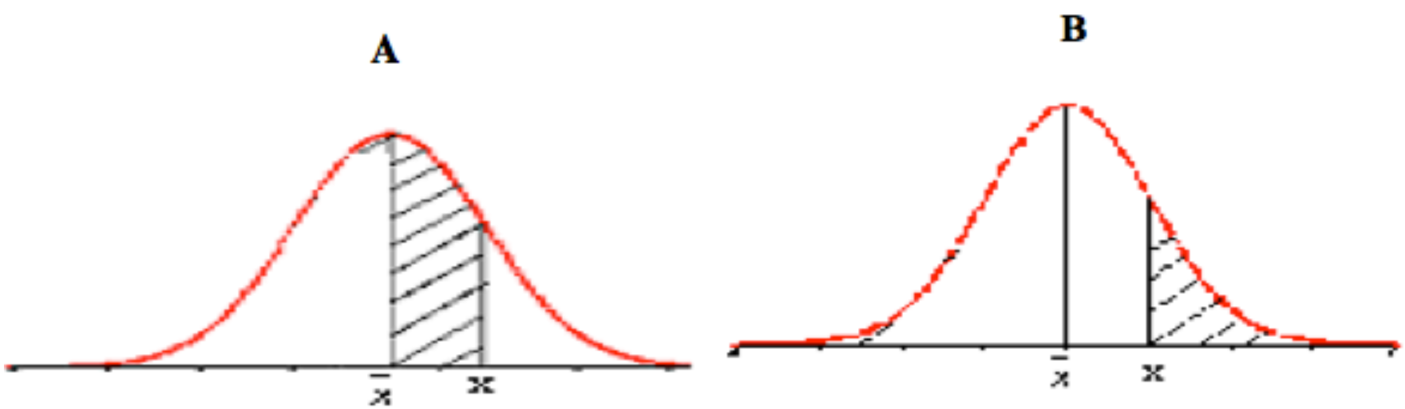


Z	Area between the Mean and x (curve A)	Area beyond x (curve B)
0	0.0000	0.0000
0.1	0.0398	0.4602
0.2	0.0793	0.4207
0.3	0.1179	0.3821
0.4	0.1554	0.3446
0.5	0.1915	0.3085
0.6	0.2257	0.2743
0.7	0.2580	0.2420
0.8	0.2881	0.2119
0.9	0.3159	0.1841
1	0.3413	0.1587

* Adapted from "*Understanding Social Statistics,*" by Jane Fielding and Nigel Gilbert.

Z scores, using “curve A”

Z-Scores*



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[illegible]	0.2257	0.2743
0.7	0.2580	0.2420
0.8	0.2881	0.2119
0.9	0.3159	0.1841
1	0.3413	0.1587

What does this mean?

The “0.5” means that we are looking at a number one half of a standard deviation away from the mean (*above* the mean in this case).

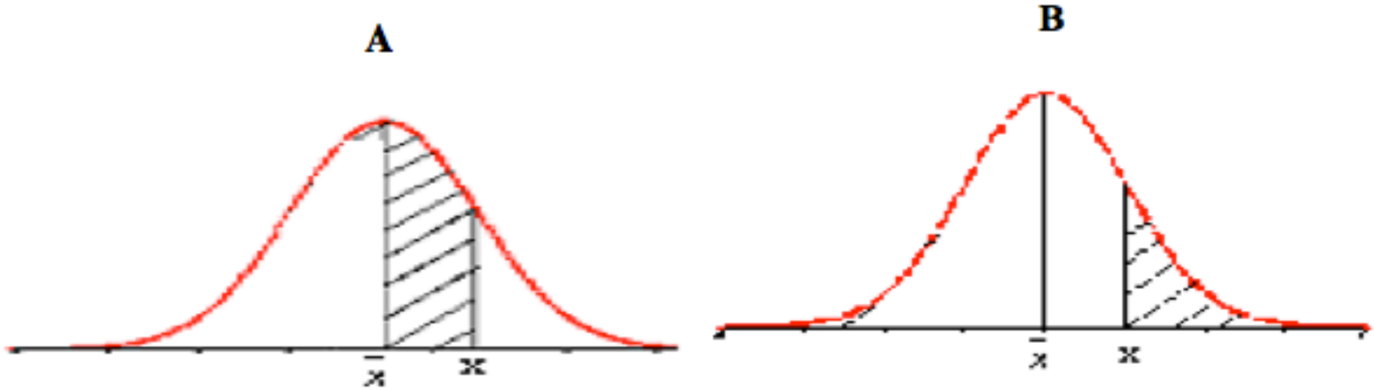
The “0.1915” means this is how much of the data is between the mean and ½ a standard deviation above (or below) the mean.

We convert this to a percent, and get 19.2% (1 d.p.).

As we expected, this is a bit higher than what we would get from just taking half of 34%, which would be 17%.

What about "curve B"?

Z-Scores*



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Another question . . .

- What percentage of US adult males are shorter than 5 feet 5 ½ inches?
- First, how many standard deviations is this away from the mean?
- There is a formula for this: $\boxed{z = \dfrac{x - \bar{x}}{\sigma}}$
- This means, to find the number of standard deviations (z), you first do the number (x) minus the mean (x-bar), then divide that by the standard deviation (σ). If it's negative you may ignore the negative sign!

Calculating z . . .

What percentage of US adult males are shorter than 5 feet 5 ½ inches?

X = 5' 5 ½"; x-bar = 5'8"; σ = 2".

So we first do 5' 5 ½" – 5'8" and get -2.5" (we ignore the negative sign and so just say 2.5").

Next we divide this by the standard deviation which is 2".

2.5"/2" = 1.25. This is "z" which is the number of standard deviations that 5 feet 5 ½ inches is away from the mean.

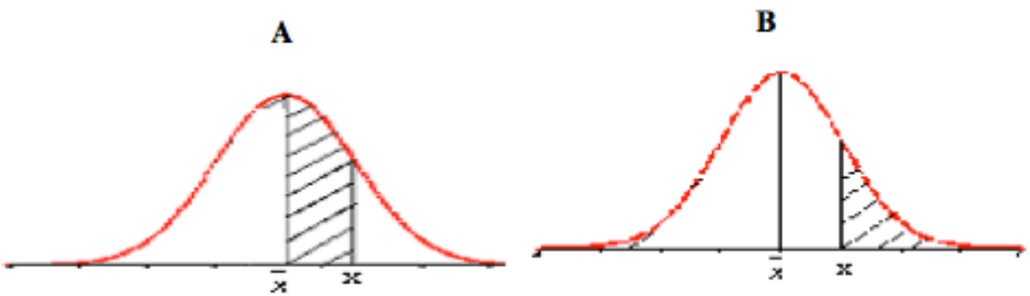
Back to the z tables . . .

- In this case, since our z-tables only go to one decimal place, we'll round 1.25 up to 1.3.

- Also, we'll need a bit more of the actual tables than we saw previously. The assignment will include enough of the z tables to do the questions (seemed like a sensible idea . . .).

A bit more of the z tables

Z-Scores*



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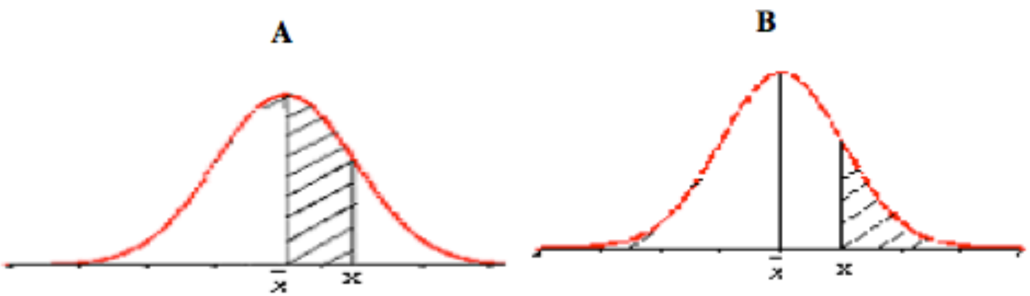
0.9	0.3159	0.1841
1	0.3413	0.1587
1.1	0.3643	0.1357
1.2	0.3849	0.1151
1.3	0.4032	0.0968
1.4	0.4192	0.0808
1.5	0.4332	0.0668
1.6	0.4452	0.0548
1.7	0.4554	0.0446
1.8	0.4641	0.0359
1.9	0.4713	0.0287
2	0.4772	0.0228

A bit more of the z tables

Z-Scores*



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Conclusion…

According to the tables, when z = 1.3, we find that 0.0968 or 9.68% of the data is further than 1.3σ away from the mean.

In other words, 9.7% (1 d.p.) of US adult men should be shorter than 5 feet 5 ½ inches (and also another 9.7% should be *taller* than 5 feet 10 ½ inches, since the normal distribution is symmetrical).

Note: even though "curves A and B" have "x" on the *right hand side* of the diagram, the z-tables can be used for values both above and below the mean.

Hypothesis Testing

What's a hypothesis?

- *Synonyms (sort of . . .)*:
- Theory
- Claim
- (Pre-)supposition
- Belief
- Opinion

So far we have:

- Done our own sampling and tried to draw conclusions from our results . . .
- i.e. given a range of values within which we believe the true figure actually lies (the “margin of error” relates to this)
- We call this a 95% confidence interval, because:
- a) we are 95% confident that we are correct, and
- b) it is an interval or range, rather than one precise figure.

But now . . .

- We will first identify a hypothesis, generally one held by someone other than ourselves,
- We'll be naturally suspicious about this hypothesis,
- We'll actually attempt to disprove or reject the hypothesis
- We'll come to a conclusion about the hypothesis, and about the situation in general.
- NOTE: The math involved will be pretty much what we have been doing already.

For example . . .

Suppose Hillary Clinton's organization claims that Clinton has 53% popularity amongst registered Democratic voters.

You, working for Bernie Sanders, are suspicious of this claim, and decide to do your own research.

You take a random sample of 100 likely Democratic voters, and find that 46 of them are in favor of Clinton.

What can you conclude, "at a 5% level of significance"?

Follow these steps!

- 1. Set up a “Null Hypothesis.”
- In this case, you use the claim as your Null Hypothesis. You are generally trying to disprove or reject the NH (also the symbol H_0 is used); however you set it up as a supposedly true figure.
- So the NH is: “Clinton’s true figure of support is 53%.”

Next step . . .

- 2. Calculate the standard deviation, using the NH figure for "p," and your sample size for "n."

- You MUST use the NH figure for "p," because you are assuming that it is the true figure, for the sake of argument.

- So calculate $\sqrt{(.53*(1-.53)/100)}$

Next step . . .

- 2. Calculate the standard deviation, using the NH figure for "p," and your sample size for "n."
- You MUST use the NH figure for "p," because you are assuming that it is the true figure, for the sake of argument.
- So calculate $\sqrt{(.53*(1-.53)/100)}$
- You should get 0.0499, which equals 5.0% (rounded).

And the next step . . .

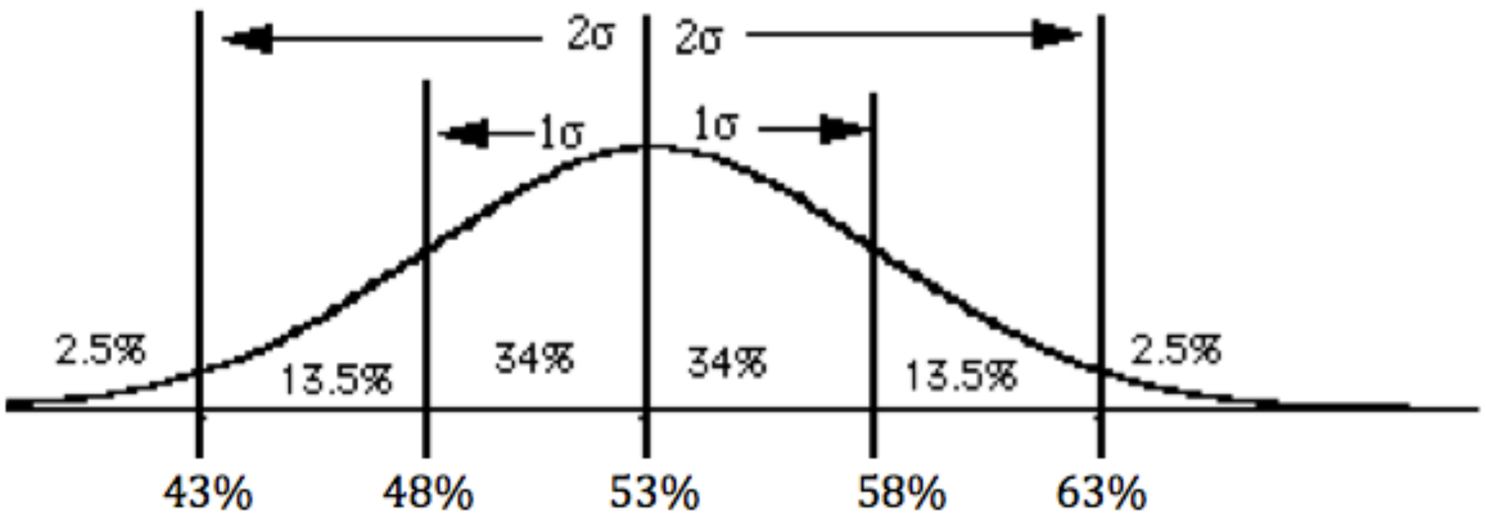
- 3. We decided that we are testing the hypothesis "at a 5% level of significance." This means that we want to have no more than a 5% chance of being wrong, i.e. we want to be 95% sure of being right.

- Now where have we heard that number before?

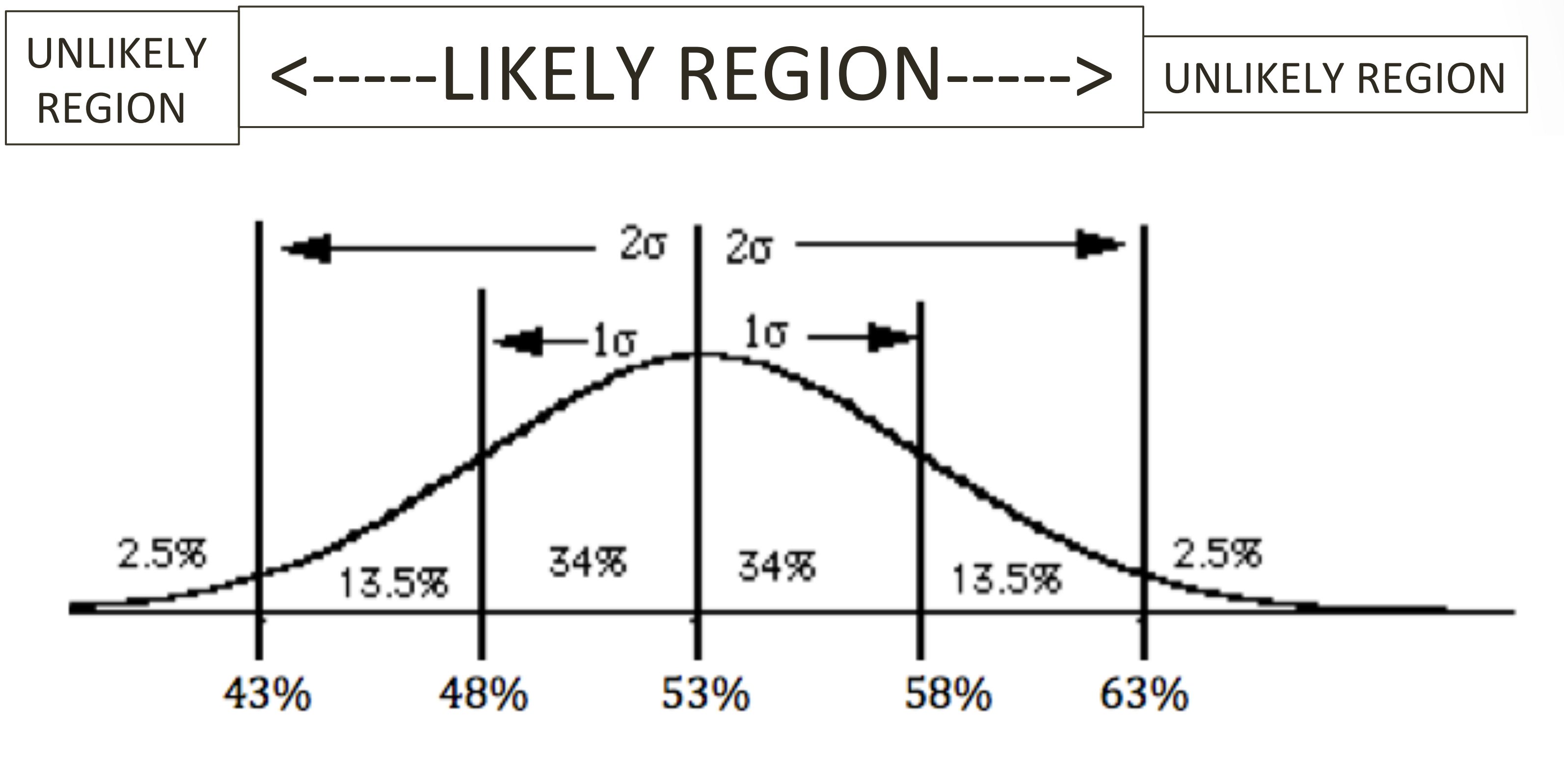
- It is associated with the ± 2 standard deviations, along with the normal distribution.

- So we draw the diagram . . .

Step 3 continued…



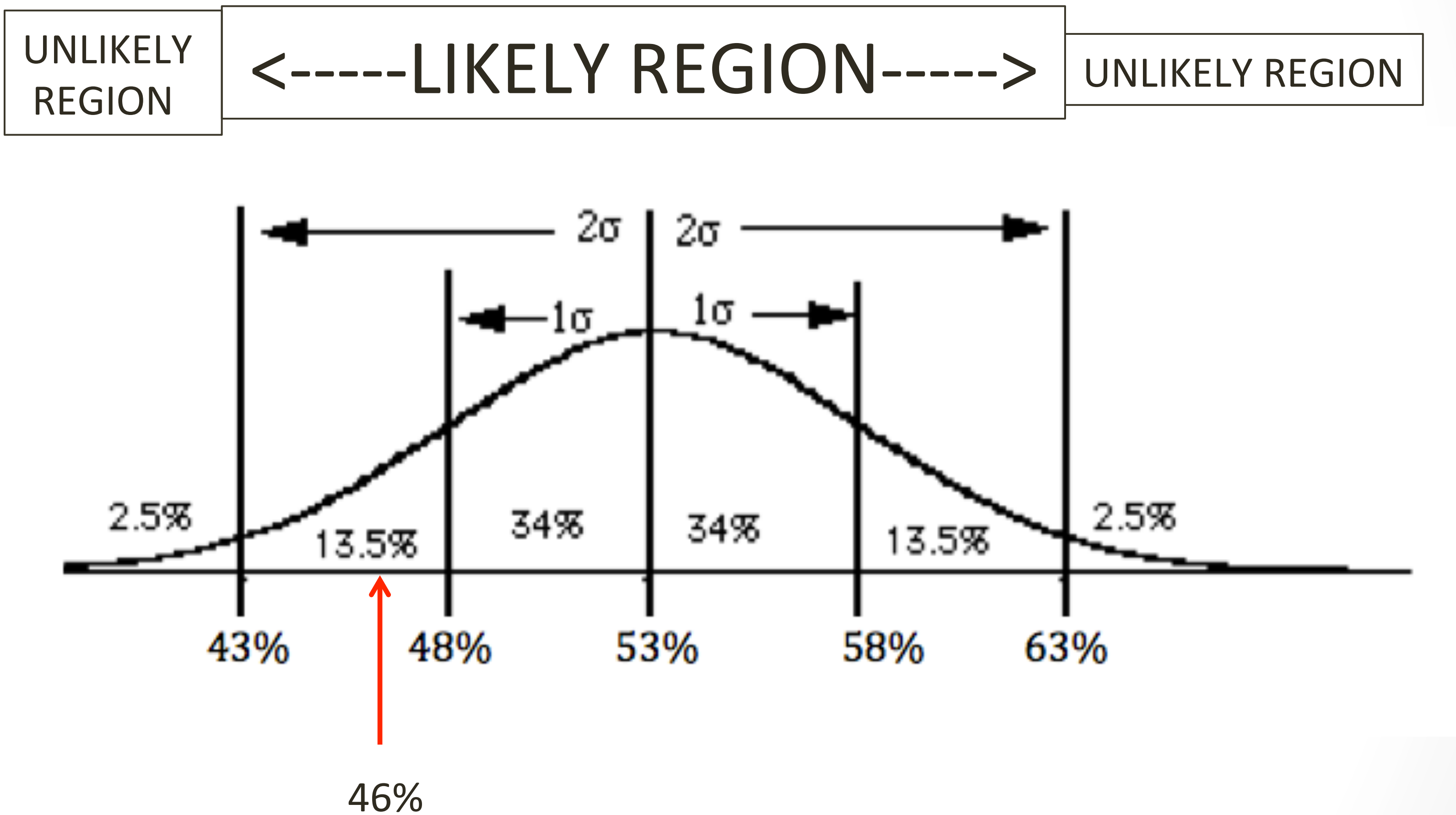
We can divide the diagram as follows:



Step 4 . . .

- We see where our sample poll result fits into the picture we have just drawn.
- Does it fall into the LIKELY or the UNLIKELY region?
- We found that 46 out of our 100 people polled were in favor of Clinton, i.e. 46%.

Where does our result fit into the picture?



Our result:

- Step 5:

- Our 46% result falls within $p \pm 2\sigma$, or inside the LIKELY region, meaning that we are unable to reject the Null Hypothesis, "at a 5% level of significance."

- This is because, if the 53% claim is true (as we are allowing for it to be), then our result of 46% is not particularly unusual, and certainly doesn't seem so far out of the ordinary as to cast significant doubt on the Null Hypothesis.

- One reason we are unable to reject the Null Hypothesis in this case is because our sample size was rather small ($n = 100$).

Step 6: conclusion

6. In this case, we have to allow that perhaps Clinton's claim of 53% is correct after all.

We DO NOT "accept" her claim; rather we say i) we cannot reject it and ii) we admit it *may be* correct.

But we have another trick up our metaphorical sleeve . . .

If at first you don't succeed . . .

Sometimes (in this course, as also in real life), you may be inclined (and able) to re-test the Null Hypothesis.

In this case, we will ask a larger sample poll, this time of 1000 likely Democratic voters (this would cost quite a bit more).

This time, we find that 480 of them are in favor of Clinton, i.e. 48%. This actually seems worse than last time, when we got 46%.

BUT, this time the standard deviation should be smaller . . .

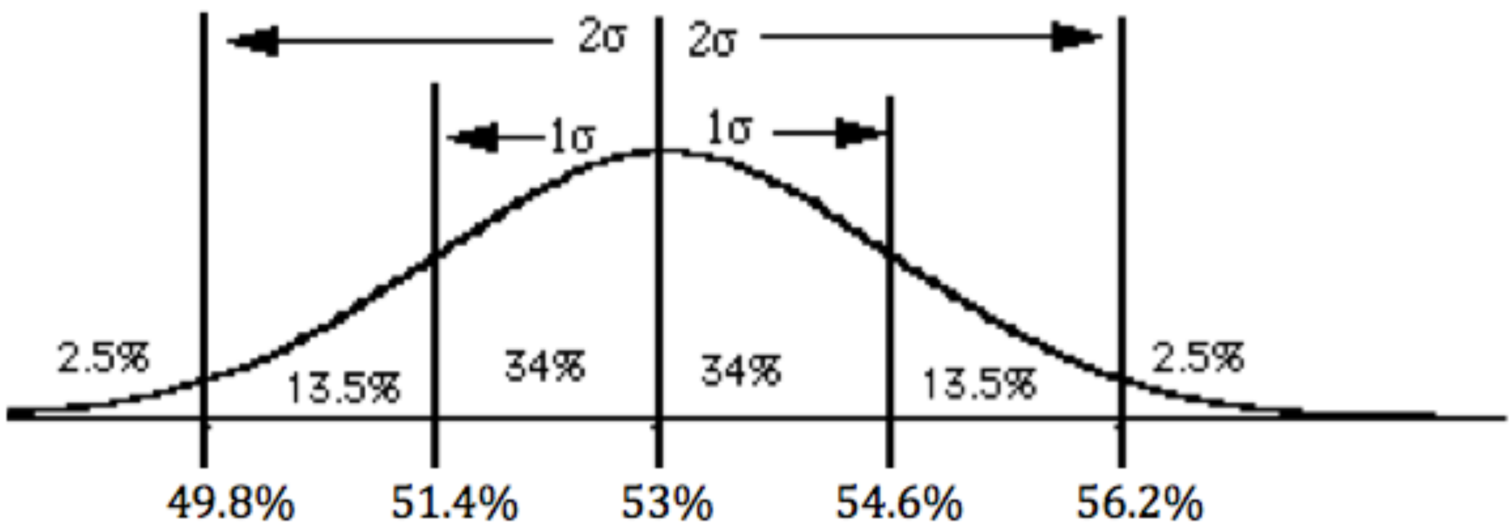
Recalculate the standard deviation

- Step 1 will be the same as last time (same Null Hypothesis).
- Step 2. Calculate the standard deviation, with $p = 0.53$ as before, but this time with $n = 1000$.

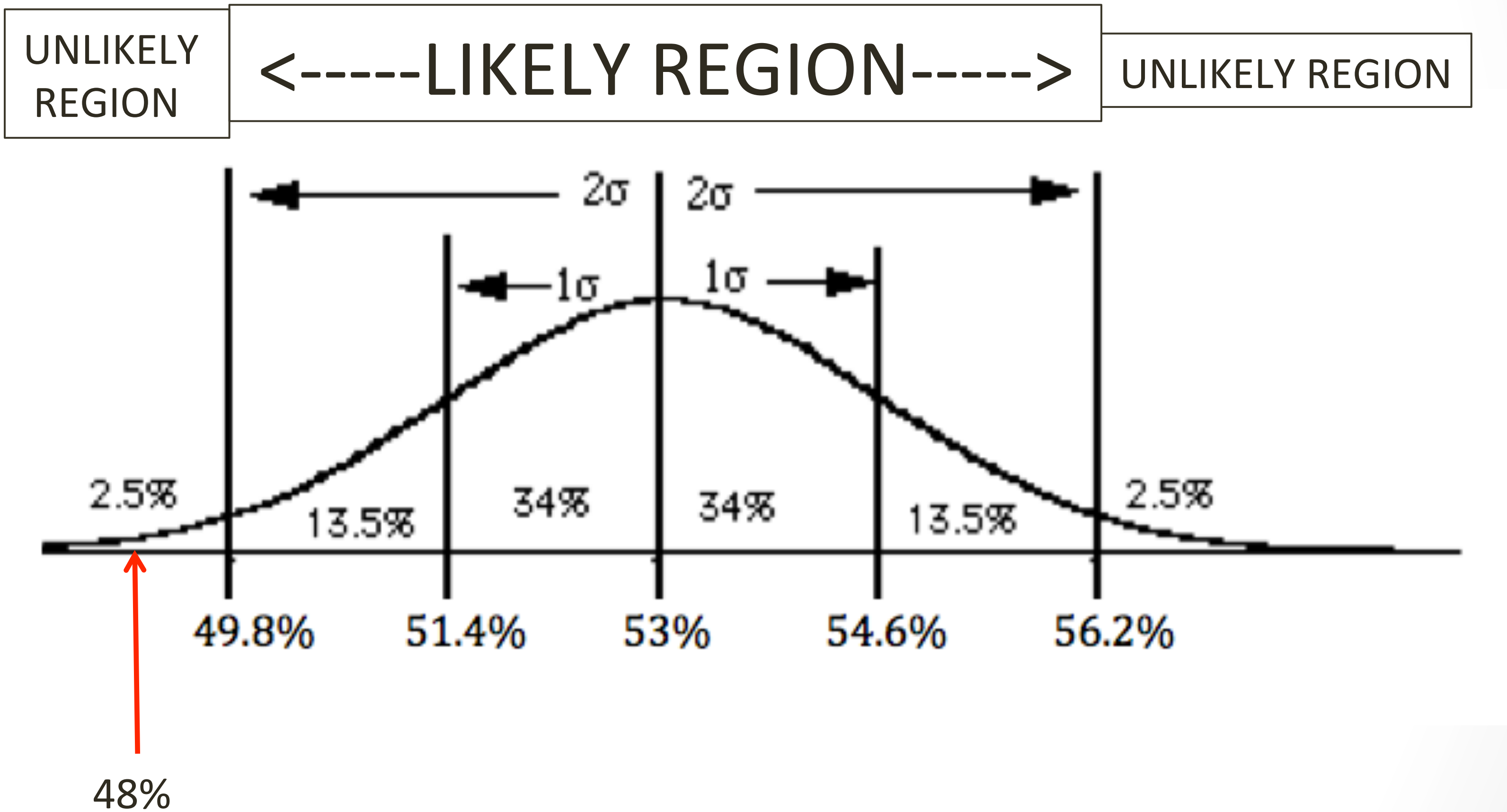
Recalculate the standard deviation

- Step 1 will be the same as last time (same Null Hypothesis).
- Step 2. Calculate the standard deviation, with $p = 0.53$ as before, but this time with $n = 1000$.
- You should get $\sigma = 1.6\%$.

Step 3: re-draw the picture
What is different this time?



Step 3: re-draw the picture
What is different this time?



This time where does our result fall?

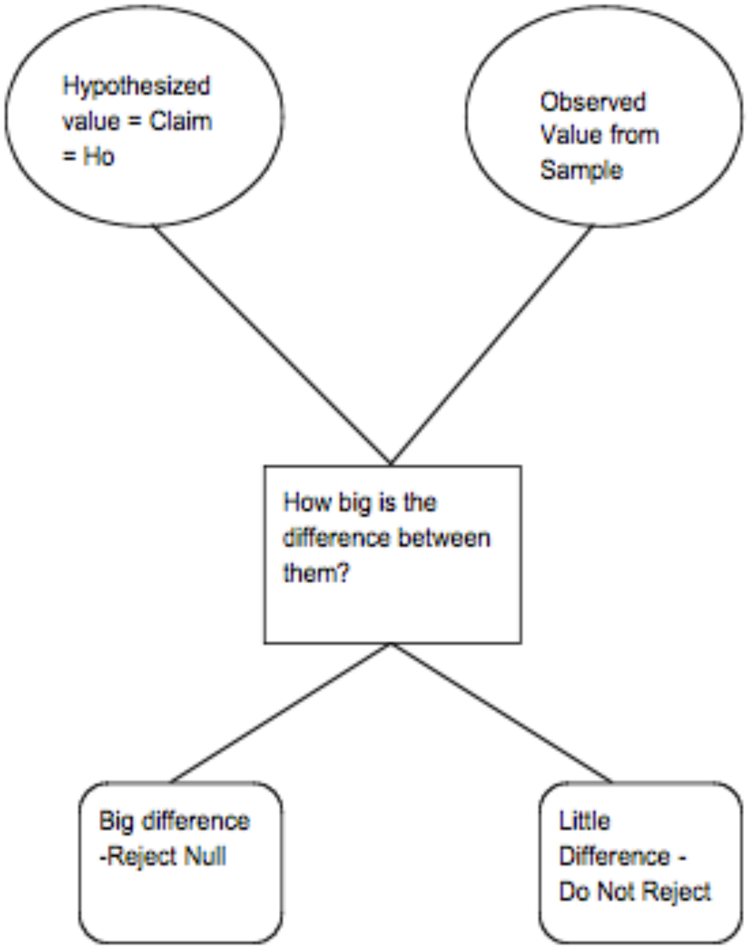
- Step 4:
- This time our 48% result falls <u>outside</u> $p \pm 2\sigma$, in the UNLIKELY region.

- Step 5:
- Therefore we REJECT the Null Hypothesis, “at a 5% level of significance.”

- Step 6:
- We conclude that Clinton’s claim of 53% is most likely to be incorrect.

A visual perspective

Taken from Applied Statistics for the Behavioral Sciences, by Hinkle, Wiersma, and Jurs.



Read chapter 7 carefully!

Chapter 7 has a lot of clear explanation of the rationale behind hypothesis testing, as well as some good examples.

So be sure to read chapter 7 carefully.

Hypothesis testing has practical application in many situations. In a previous edition of the Lowell Sun (Sunday March 25, 2012), there was a story about increases in high school test scores in Ohio that seemed "suspicious."

Details from the web . . .

- "The analysis found that more than 500 districts and charter schools in Ohio had at least one school with an improbably large score change from 2005-2011. In 42 districts and charters, the probability of so many big swings happening by chance in any one year was less than one in 1,000, or less than one-tenth of 1 percent."
- http://www.journal-news.com/news/hamilton-news/suspect-test-scores-found-1349558.html
- An example of a result being in the UNLIKELY region!!
- What was the Null Hypothesis?