Interpret the notation $\int_0^2 x\,dx$, identify Key terms, and express it as a limit. Then, find the value of the limit directly and make a geometric argument.

① Interpret Notation

$\int_0^2 x\,dx$ is the definite integral of $f(x) = x$ over interval $[0,2]$

Integrand of $f(x) = x$
Lower Limit $= 0$
Upper Limit $= 2$

② Express $\int_0^2 x\,dx$ as a limit

$$[0,2]$$
$$\downarrow$$
$$[0, 2/n], [2/n, 4/n], \ldots, [(n-1)(2/n), 2]$$

These are n subintervals of equal width

$\Delta x = (2-0)/n = 2/n$

Represents Range of $[0,2]$

increment translated to $2i/n$

Get one arbitrary point x_i^* to use to determine the height of the rectangle whose base spans the subinterval.

$x_i^* = 2i/n$

$\downarrow$

$f(x) = x$ for any vale of x

$\downarrow$

$f(x_i^*) = 2i/n$

$$\boxed{\int_0^2 x\,dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i^*)\,\Delta x = \lim_{n\to\infty} \sum_{i=1}^{n} \frac{4}{n^2}\,i}$$

$f(x_i^*) = 2i/n$
$\Delta x = 2/n$
$\downarrow$
$f(x_i^*)\,\Delta x$
$=$
$\frac{2i}{n} \cdot \frac{2}{n}$
$=$
$\boxed{\frac{4i}{n^2} \text{ or } \frac{4}{n^2}\,i}$

$x_i^* = 2i/n$
$f(x) = x$
$=$
$f(2i/n) = 2i/n$

Im inputing the x here which outputs to $2i/n$

③ Get Limit

$$\int_0^2 x\,dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i^*)\Delta x = \lim_{n\to\infty} \sum_{i=1}^{n} \frac{4}{n^2} i$$

Summation Proof

$$\boxed{\sum_{i=1}^{n} i = \frac{n(n+1)}{2}}$$

Set $\sum_{i=1}^{n} i$ as a geometric argument

$$= \lim_{n\to\infty} \frac{4}{n^2} \cdot \sum_{i=1}^{n} i$$

$$= \lim_{n\to\infty} \frac{4}{n^2} \cdot \frac{n(n+1)}{2}$$

$$\frac{\cancel{4}^{2}}{\cancel{n^2}_{n}} \cdot \frac{\cancel{n}^{1}(n+1)}{\cancel{2}_{1}}$$

$$= \frac{2(n+1)}{n}$$

$$\frac{2}{1} \cdot \frac{n+1}{n}$$

$$= \lim_{n\to\infty} 2 \cdot \lim_{n\to\infty} \frac{n+1}{n}$$

$$= 2 \cdot \frac{\lim_{n\to\infty} n + \lim_{n\to\infty} 1}{\lim_{n\to\infty} n}$$

$$= \frac{\infty + 1}{\infty}$$

$$= \frac{\infty}{\infty} + \frac{1}{1}$$

$$= \text{Indet} + 1$$

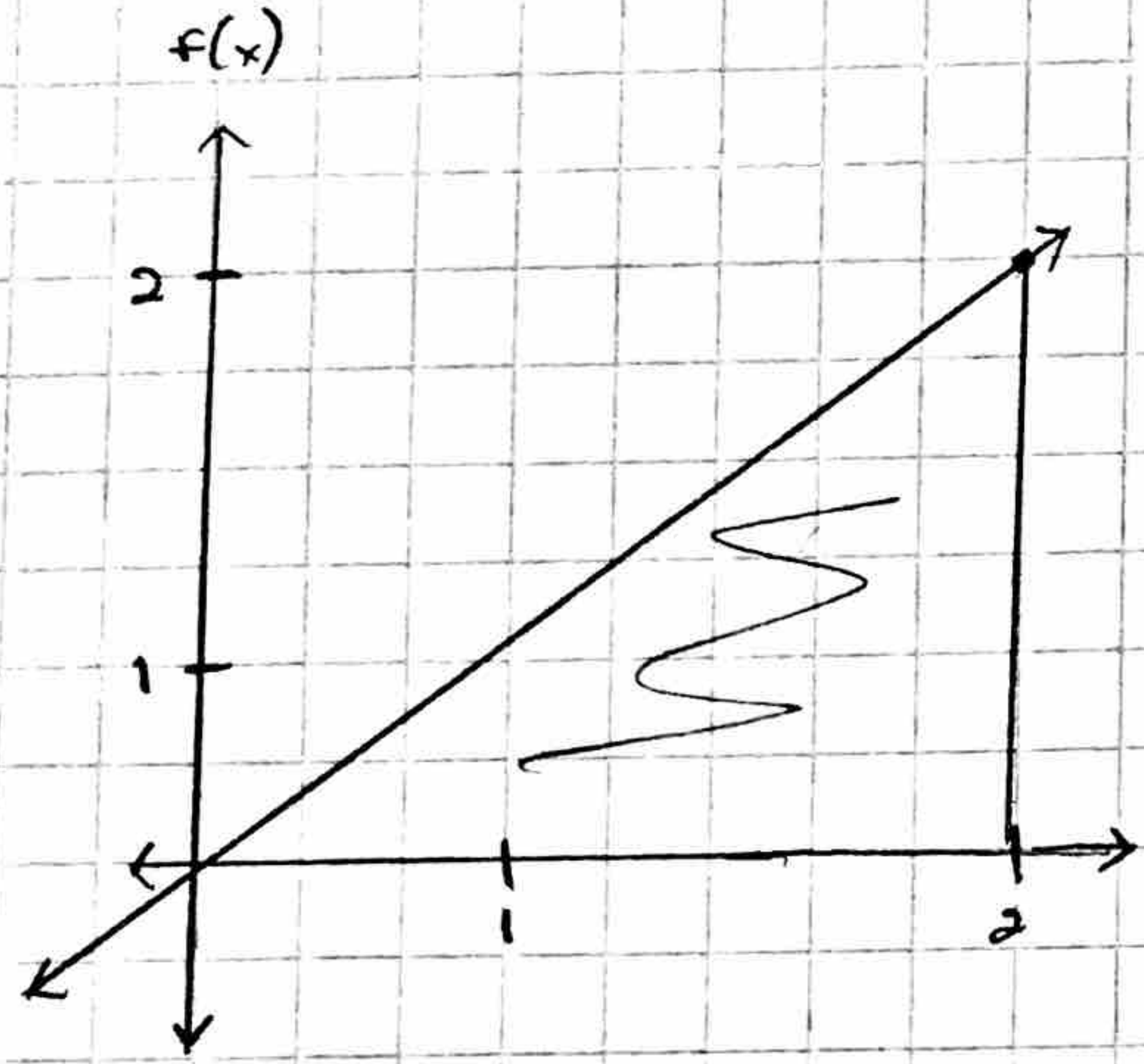
$$= 2 \cdot 1$$

$$= 2$$

$$\boxed{\int_0^2 x\,dx = 2}$$

(4) Get Geometric Interpretation

Set graph



Area of Triangle

$\frac{1}{2} \cdot \text{base} \cdot \text{height}$

triangle base = 2
triangle height = 2

$\frac{1}{2}(2)(2)$

$=$

$1 \cdot 2$

$=$

2

$\boxed{\int_0^1 2 \, dx = 2\text{, is area of triangle}}$

Interpret the notation $\int_{-2}^{2} x\,dx$, identify key terms, and express it as a limit. Then, find the value of the limit directly and make a geometric argument.

① Interpret Notation

$\int_{-2}^{2} x\,dx$ is the definite integral of $f(x) = x$ over interval $[-2, 2]$

Integrand of $f(x) = x$
Lower Limit $= -2$
Upper Limit $= 2$

② Express $\int_{-2}^{2} x\,dx$ as a limit

$[-2, 2]$

$[-2, -2 + 4/n], [-2 + 4/n, -2 + 8/n], \ldots, [-2 + (n-1)(4/n), 2]$

increment translated to $-2 + 4i/n$

$\Delta x = (2 - (-2))/n = 4/n$

Get one arbitrary point x_i^* to use to determine height of rectangle whose base spans the subinterval.

$x_i^* = -2 + 4i/n$

$\downarrow$

$f(x) = x$, for any value of x

$\downarrow$

$f(x_i^*)$

$\downarrow$

$f(-2 + 4i/n)$

$\parallel$

$f(x_i^*) = -2 + 4i/n$

$$\boxed{\int_{-2}^{2} x\,dx = \lim_{n \to \infty} \sum_{i=1}^{n} f(x_i^*)\Delta x = \lim_{n \to \infty} \sum_{i=1}^{n} \left(-2 + \frac{4i}{n}\right)\frac{4}{n}}$$

③ Get Limit

$$\int_{-2}^{2} x\,dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i^*)\,\Delta x = \lim_{n\to\infty} \sum_{i=1}^{n} \left(-2 + \frac{4i}{n}\right) \frac{4}{n}$$

$$= \lim_{n\to\infty} \sum_{i=1}^{n} \frac{4}{n}\left(-2 + \frac{4i}{n}\right)$$

$$= \lim_{n\to\infty} \frac{4}{n}\left[-2 \cdot \sum_{i=1}^{n} + \frac{4}{n} \cdot i \cdot \sum_{i=1}^{n}\right]$$

$$= \lim_{n\to\infty} \frac{4}{n}\left[-2n + \frac{\cancel{4}^{2}}{n} \cdot \frac{n(n+1)}{\cancel{2}_{1}}\right]$$

$$= \lim_{n\to\infty} \frac{4}{n}\left[-2n + \frac{2}{\cancel{n}} \cdot \cancel{n}(n+1)\right]$$

$$= \lim_{n\to\infty} \frac{4}{n}\left[-2n + 2 \cdot (n+1)\right]$$

$$= \lim_{n\to\infty} \frac{4}{n}\left[-2n + 2n + 2\right]$$

$$= \lim_{n\to\infty} \left[\frac{4}{n} \cdot -2n + \frac{4}{n} \cdot 2n + \frac{4}{n} \cdot 2\right]$$

$$= \lim_{n\to\infty} \left[\frac{-8\cancel{n}}{\cancel{n}} + \frac{8\cancel{n}}{\cancel{n}} + \frac{8}{n}\right]$$

$$= \lim_{n\to\infty} \left[-8 + 8 + \frac{8}{n}\right]$$

$$= \lim_{n\to\infty} \frac{8}{n}$$

$$= \frac{\lim_{n\to\infty} 8}{\lim_{n\to\infty} n}$$

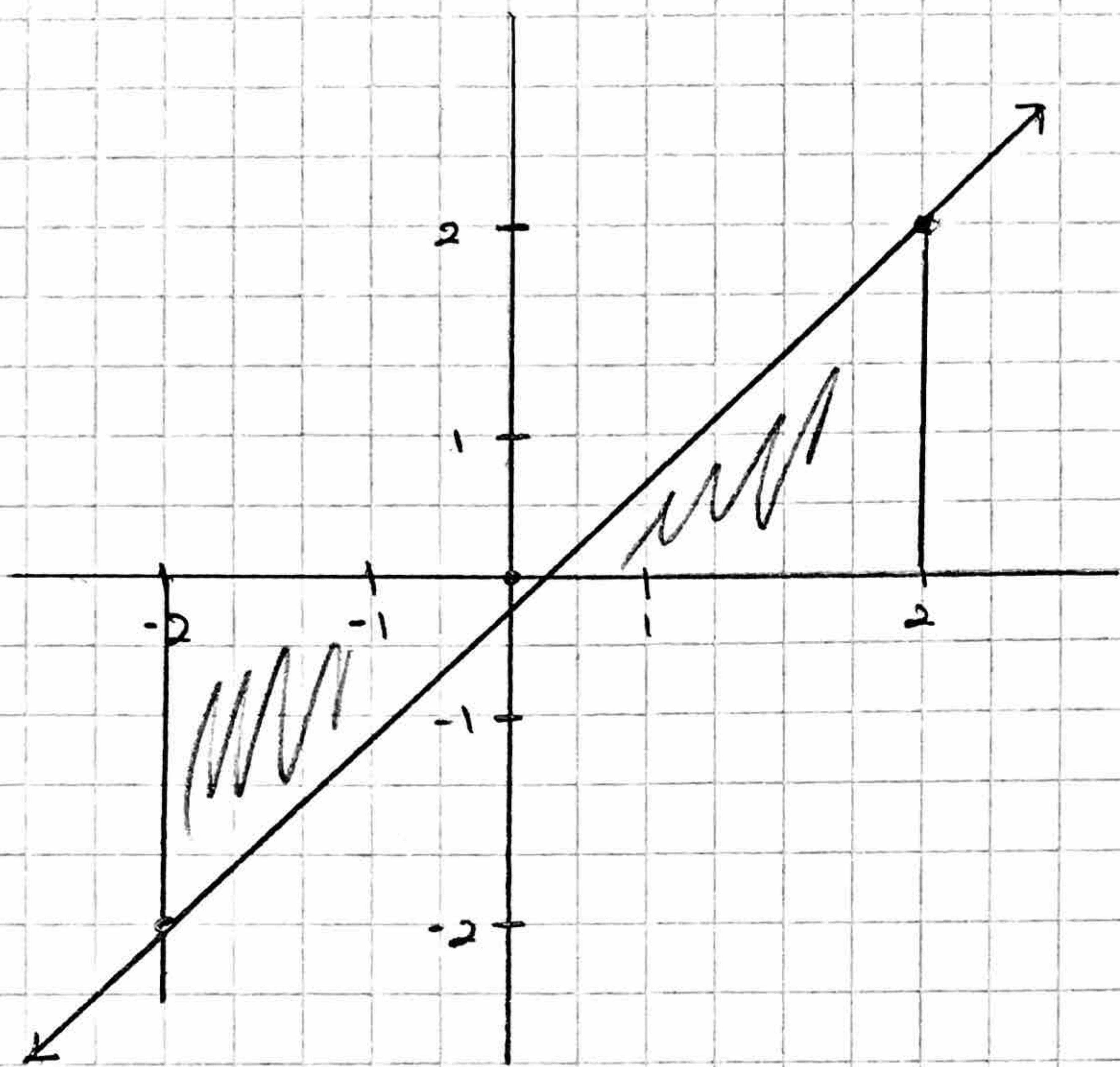
$$= \frac{8}{\infty}$$

$$= 0$$

$$\boxed{\int_{-2}^{2} x\,dx = 0}$$

④ Get Geometric Interpretation

Set graph



Area of Triangle for [-2, 0]

triangle base = 2
triangle height = -2

$\frac{1}{\cancel{2}_1}(\cancel{2}^1)(-2)$
$=$
-2

Area of Triangle for [0, 2]

triangle base = 2
triangle height = 2

$\frac{1}{\cancel{2}_1}(\cancel{2}_1)(2)$
$=$
2

$$-2 + 2$$
$$=$$
$$\boxed{\int_{-2}^{2} x\,dx = 0}$$

$\int_{-2}^{2} x\,dx = 0$ is the sum of areas between the two triangles

Interpret the notation $\int_0^1 2\,dx$, identify key terms, and express it as a limit. Then, find the value of the limit directly and make a geometric argument.

① Interpret Notation

$\int_0^1 2\,dx$ is the definite integral of $f(x) = 2$ over interval $[1, 0]$

Integrand of $f(x) = 2$

Lower Limit $= 0$

Upper Limit $= 1$

② Express $\int_0^1 2\,dx$ as a limit

$[0, 1]$

$\downarrow$

$[0, 1/n], [1/n, 2/n], \ldots, [(n-1)(1/n), 1]$

$\uparrow$

These are n subintervals of equal width

$\Delta x = (1-0)/n = 1/n$

$\uparrow$

This represents the range of $[1, 0]$

increment translated to $1i/n$

Get one arbitrary point x_i^* to use to determine the height of the rectangle whose base spans the subinterval.

$x_i^* = 1i/n$

$\downarrow$

$f(x) = 2$ for any value of x

$\downarrow$

$f(x_i^*) = 2$

$$\int_0^1 2\,dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i^*)\Delta x = \lim_{n\to\infty} \sum_{i=1}^{n} \frac{2}{n}$$

$f(x_i^*) = 2$

$\Delta x = 1/n$

$\downarrow$

$f(x_i^*)\Delta x$

$=$

$2 \cdot \frac{1}{n}$

$=$

$\frac{2}{n}$

$x_i^* = 1i/n$

$f(x) = 2$

$=$

$f(1i/n) = 2$

This is will equal 2 since there is no x value to leverage

③ Get Limit

$$\int_0^1 2\,dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i^*)\Delta x = \lim_{n\to\infty} \sum \frac{2}{n}$$

$$= \lim_{n\to\infty} \frac{2}{n} \cdot \sum_{i=1}^{n} 1$$

$$= \lim_{n\to\infty} \frac{2}{n} \cdot (1 \cdot 1 \ldots \cdot n)$$

$$= \lim_{n\to\infty} \frac{2}{\cancel{n}} \cdot \frac{\cancel{n}}{1}$$

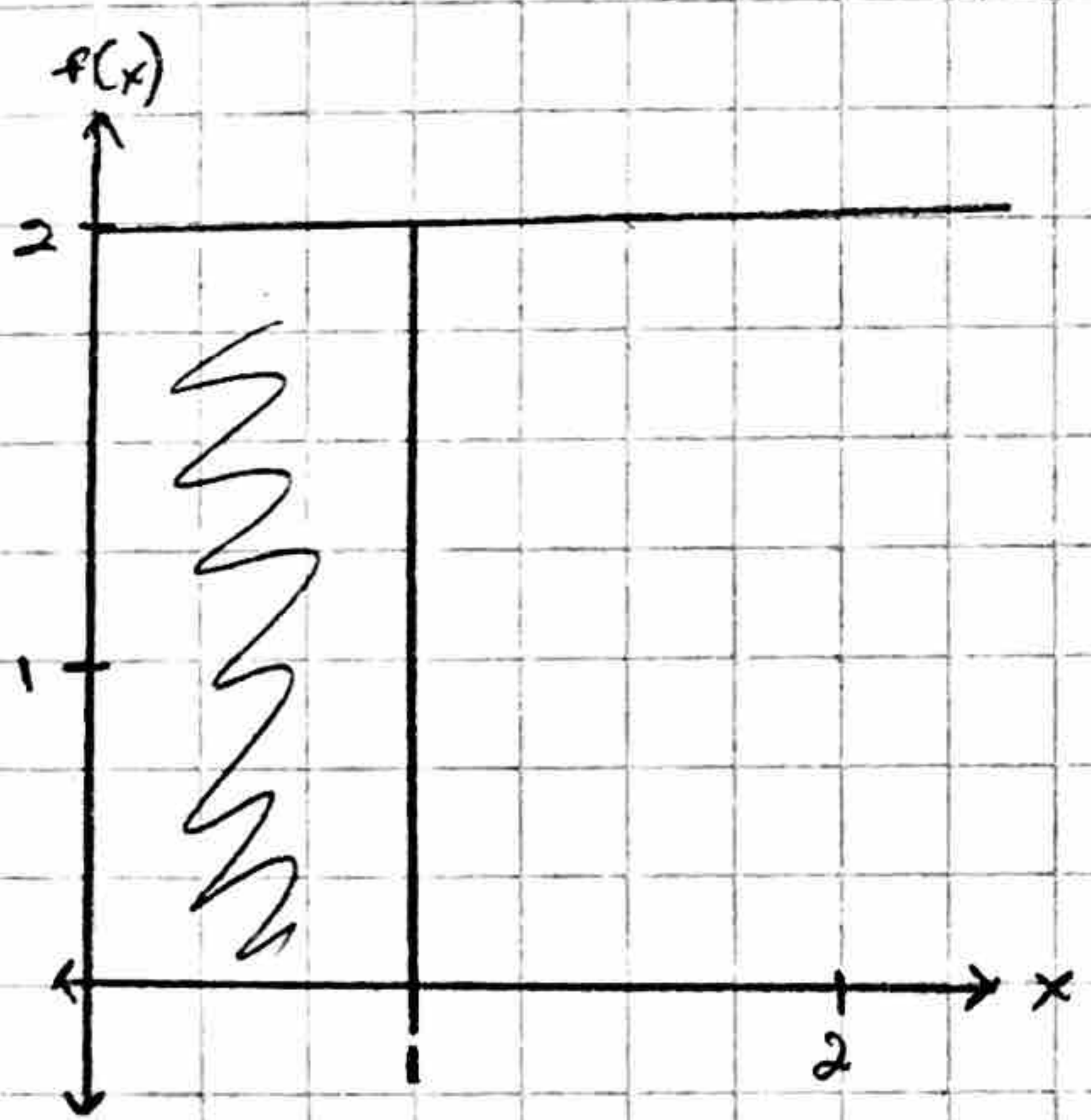
$$= \lim_{n\to\infty} 2$$

$$= 2$$

$$\boxed{\int_0^1 2\,dx = 2}$$

④ Get Geometric Interpretation

Set graph



rectangle width = 1

rectangle height = 2

area of rectangle = 2

$\int_2^1 2\,dx = 2$, represents the area of rectangle

Interpret the notation $\int_{-4}^{0} x\,dx$, identify key terms, and express it as a limit. Then, find the value of the limit directly and make a geometric argument.

① Interpret Notation

$\int_{-4}^{0} x\,dx$ is the definite integral of $f(x) = x$ over interval $[-4, 0]$

Integrand of $f(x) = x$
Lower Limit $= -4$
Upper Limit $= 0$

② Express $\int_{-4}^{0} x\,dx$ as a limit

$[-4, 0]$

$[-4, -4+4/n], [-4+4/n, -4+8/n], \ldots, [-4+(n-1)(4/n), 0]$

increment translated to $-4+4i/n$

$\Delta x = (0-(-4))/n = 4/n$

Get one arbitrary point x_i^* to use to determine height of rectangle whose base spans the subinterval.

$x_i^* = -4 + 4i/n$

$\downarrow$

$f(x) = x$, for any value of x

$\downarrow$

$f(x_i^*)$

$\parallel$

$f(-4+4i/n)$

$\parallel$

$f(x_i^*) = -4 + 4i/n$

$$\boxed{\int_{-4}^{0} x\,dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i^*)\,\Delta x = \lim_{n\to\infty} \sum_{i=1}^{n} \left(-4 + \frac{4i}{n}\right)\frac{4}{n}}$$

③ Get Limit

$$\int_{-4}^{0} x \, dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i^*)\Delta x = \lim_{n\to\infty} \sum_{i=1}^{n} \left(-4 + \frac{4i}{n}\right)\frac{4}{n}$$

$$= \lim_{n\to\infty} \sum_{i=1}^{n} \left(-4 + \frac{4i}{n}\right)\frac{4}{n}$$

$$= \lim_{n\to\infty} \sum_{i=1}^{n} \left(-\frac{16}{n} + \frac{16i}{n^2}\right)$$

$$= \lim_{n\to\infty} \sum_{i=1}^{n} \left(-\frac{16}{n} \cdot 1 + \frac{16}{n^2} \cdot i\right)$$

$$= \lim_{n\to\infty} \left(-\frac{16}{n} \cdot 1 \cdot \sum_{i=1}^{n} + \frac{16}{n^2} \cdot i \cdot \sum_{i=1}^{n}\right)$$

$$= \lim_{n\to\infty} \left(-\frac{16}{n} \sum_{i=1}^{n} 1 + \frac{16}{n^2} \sum_{i=1}^{n} i\right)$$

$$= \lim_{n\to\infty} \left(\frac{-16}{n} \cdot n + \frac{\cancel{16}^{8}}{\cancel{n^2}_{n}} \cdot \frac{\cancel{n}(n+1)}{\cancel{2}_{1}}\right)$$

$$= \lim_{n\to\infty} \left(-16 + \frac{8}{1} \cdot \frac{n+1}{n}\right)$$

$$= \lim_{n\to\infty} \left(-16 + 8 \cdot \frac{n+1}{n}\right)$$

Group n's together since were dealing with infinity

$$= \lim_{n\to\infty} -16 + \lim_{n\to\infty} 8 \cdot \lim_{n\to\infty} \frac{n+1}{n}$$

$$= \frac{\lim_{n\to\infty} n + \lim_{n\to\infty} 1}{\lim_{n\to\infty} n} = \frac{\infty + 1}{\infty} = \frac{\infty}{\infty} + 1 = \text{Indet } 1 = 1$$

$$-16 + 8 \cdot 1$$

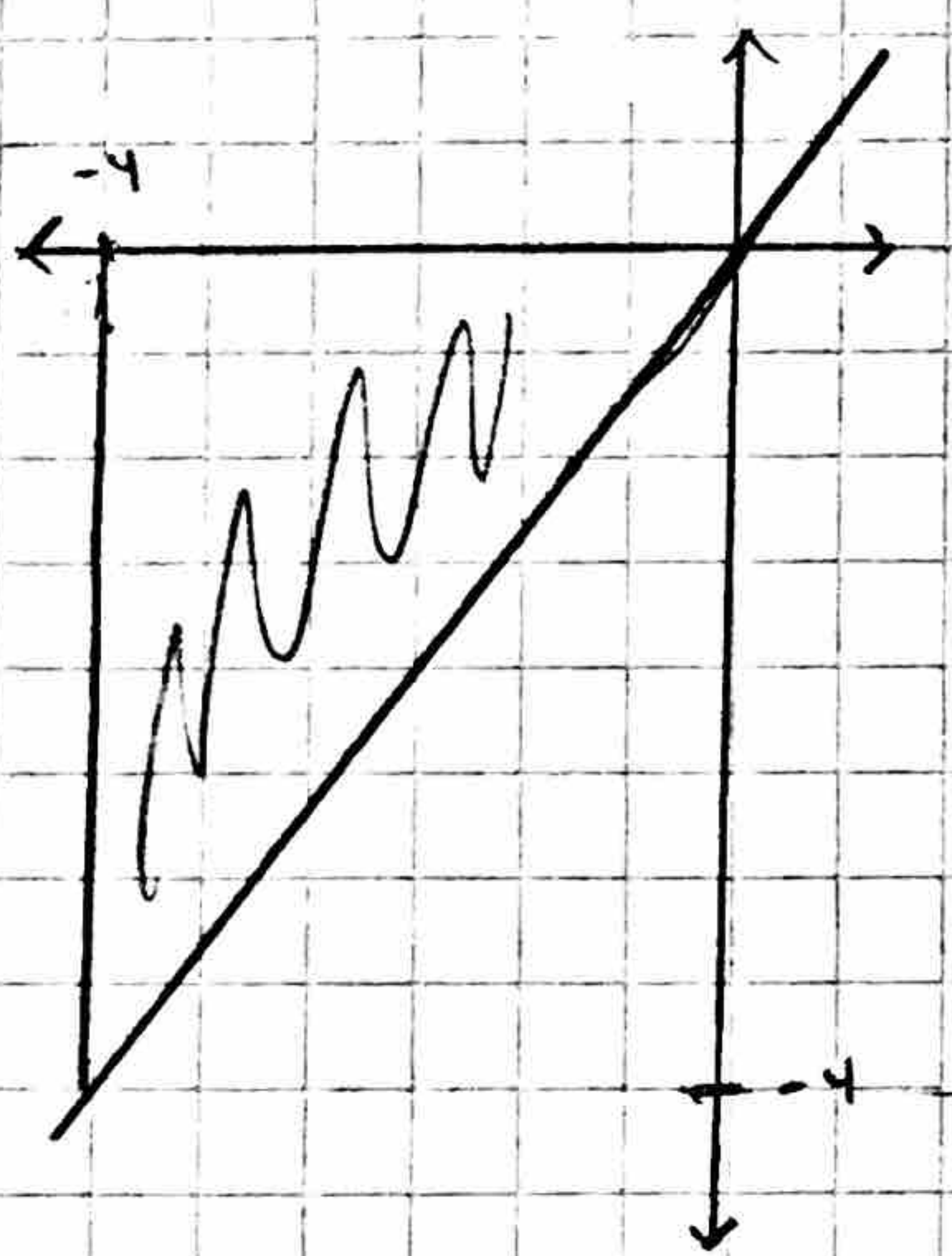
$$= -16 + 8$$

$$= -8$$

$$\boxed{\int_{-4}^{0} x \, dx = -8}$$

(4) Get Geometric Interpretation

Set graph



Area of Triangle

triangle base = 4

triangle height = 4

$\frac{1}{\cancel{2}_1}(\cancel{4}^2)(4)$

$=$

$2 \cdot 4$

$=$

8

Negative Area of Triangle is -8, $\int_{-4}^{0} x\,dx = -8$

$\int_{-4}^{0} x\,dx$ is negative area of triangle