Prove that $\lim\limits_{x \to 0} \frac{|x|}{x}$ does not exist.

Approach 0 from the left

$$\lim_{x \to 0^-} \frac{|x|}{x}$$

$x = -x$ Since we are approaching 0 from the left.

$$=$$

$$\lim_{x \to 0^-} \frac{|0|}{0} \rightarrow \text{Undef}$$

Let Me Replace

Let Me Start Over

① Use Replacement

$$\frac{|x|}{-x} = \frac{x}{-x} = -1$$

$$\lim_{x \to 0^-} \frac{|x|}{x} = \lim_{x \to 0^-} -1$$

$$= -1$$

Approach 0 from the right

Im going to Replace

$$\lim_{x \to 0^+} \frac{|x|}{x}$$

Using x since Im approaching 0 from the right.

Use Replacement

$$\frac{|x|}{x} = \frac{x}{x} = 1$$

$$\lim_{x \to 0^+} \frac{|x|}{x} = \lim_{x \to 0^+} 1$$

$$= 1$$

$$\boxed{\lim_{x \to 0^+} \frac{|x|}{x} \neq \lim_{x \to 0^-} \frac{|x|}{x} \text{ ; } \lim_{x \to 0} \frac{|x|}{x} \text{ does not exist}}$$