LECTURE 2: Additional Material on Operations with Everyday Numbers

Order of Operations

When working with expressions with parentheses, we need to be careful of the order in which operations are performed. By convention, we follow the rules in the order given below. PEMDAS (Please Excuse My Dear Aunt Sally!) is a helpful mnemonic.

P*arentheses* (innermost first)
E*xponents*
$\left.\begin{array}{l}\textbf{M}\textit{ultiplication}\\ \textbf{D}\textit{ivision}\end{array}\right\}$ left to right whichever comes first
$\left.\begin{array}{l}\textbf{A}\textit{ddition}\\ \textbf{S}\textit{ubtraction}\end{array}\right\}$ left to right whichever comes first

Examples follow:

ex. 1: This is an example of an **INCORRECT method with the correction**.

(10 – 2) – (6 + 10)

8 – 6 + 10 = 2 + 10 = 12 You CANNOT take off parentheses arbitrarily. Always do what's inside the **Parentheses** first.

(10 – 2) – (6 + 10)
8 – 16 = – 8 **CORRECT**

ex. 2: **18 – 5 * 2 (6 – 9) – 3^2 – 24 ÷ 2 * 4**

Step 1: Do what's in **Parentheses first** rewriting entire problem to avoid confusion

= 18 – 5 * 2 (– 3) – 3^2 – 24 ÷ 2 * 4 Note: the parentheses (6−9) = −3

Step 2: **Exponents**

= 18 – 5 * 2(– 3) – 9 – 24 ÷ 2 * 4

Step 3: **Multiplication** and **Division** going left to right doing whichever **comes first**. You may need to repeat this step.

= 18 – 10(– 3) – 9 – 12 * 4
= 18 – – 30 – 9 – 48

Notice: we multiplied 5 times 2 instead of subtracting 5 from 18. Also, we divided 24 by 2 instead of multiplying 2 * 4 first since the division came first! We repeated the multiplication step.

Step 4: Addition and Subtraction, again going left to right doing whichever comes first.

= 48 – 9 – 48
= 39 – 48
= – 9

Of course, this problem could be done using fewer steps, but if you do combine steps, you must be careful!

ex. 3: $\mathbf{-7(6-9) * 2^3 - 40 \div 4*5}$ Here's one where we can do a few steps at the same time, but you must still follow the rules and be careful.

$-7(-3) * 8 - 10*5$ We divided 40 by 4 first since division came before the multiplication. (if you had multiplied first, you would have had $40 \div 20$ -- incorrect.)

$= 21*8 - 50 = 168 - 50 = 118$

<u>Fractions</u>

The 'Golden Rule' of Fractions: The value of a fraction will not change if you multiply or divide the numerator of a fraction by some number, <u>as long as you do the same thing</u> to the denominator and vice-versa. (This excludes zero.) The division line in a fraction is similar to the parentheses in the order of operations. That is, do what is in the numerator and/or denominator of the fraction FIRST before doing anything else.

ex 1: $\frac{5-8}{5+6}$ You must do the operations first. DO NOT CANCEL THE 5's. Thus

$$\frac{5-8}{5+6} = \frac{-3}{11}$$

ex 2: $\frac{(3+2)-6}{-4+3} = \frac{5-6}{-1} = \frac{-1}{-1} = 1$

<u>Multiplication of Fractions:</u> to multiply two fractions together, just multiply the numerators and then the denominators. **Do <u>NOT</u> find a lowest common denominator (LCD) when multiplying fractions.**

ex. 3. $\frac{2}{3} * \frac{4}{7} = \frac{8}{21}$

<u>Division of Fractions:</u> (Note: You do not need to find a LCD.) In order to divide one fraction by another, multiply the numerator by the <u>reciprocal</u> of the denominator. A reciprocal is simply the fraction turned upside down. $\frac{7}{4}$ is the reciprocal of $\frac{4}{7}$

ex. 4. $\dfrac{\frac{2}{3}}{\frac{4}{7}}$ This is the same as $\frac{2}{3} \div \frac{4}{7} = \frac{2}{3} * \frac{7}{4} = \frac{14}{12}$ Reduce $\frac{14}{12} = \frac{7}{6}$ or $1\frac{1}{6}$

<u>Adding and Subtracting Fractions:</u> You <u>must</u> **find a common denominator to add or subtract fractions. (It does NOT need to be the LCD as you can reduce later.)**

ex. 5. $\frac{5}{6} + \frac{1}{4}$ What is the lowest common denominator of 6 and 4? **12**

Therefore, change each fraction to its equivalent fraction with 12 as the denominator and simply *add the numerators only*. Put the answer over the common denominator.

$\frac{5}{6} = \frac{10}{12}$ and $\frac{1}{4} = \frac{3}{12}$ so, we get

$\frac{10}{12} + \frac{3}{12} = \frac{13}{12}$

Subtraction of fractions follows the same pattern, but you must watch your signs.

ex. 6. $\frac{3}{8} - \frac{6}{8} = -\frac{3}{8}$ This example has the same denominator. Simply, f**ollow the rules for signed numbers**

ex. 7. $\frac{3}{234} - \frac{1}{9}$ First find LCD. In this case, it's 234 since 9 'goes into' 234, 26 times.

$\frac{3}{234} - \frac{26}{234}$ Subtract numerators. (Denominators stay the same.)

So 3 – 26 = –23 Therefore answer is

$\frac{3}{234} - \frac{26}{234} = -\frac{23}{234}$

Reducing fractions

(Use the Golden Rule of Fractions when reducing.) We always wish to keep a fraction in its lowest terms if possible. So look closely at the fraction to see if it contains any other factors. Both the numerator and the denominator must have the same factor. Divide both by this number to get the fraction 'reduced' to a lower form.

ex. 1. $\frac{18}{24}$ both 18 and 24 have a 'common' factor = 6. Thus dividing both the numerator and denominator by 6 we get $\frac{3}{4}$

ex. 2. $\frac{75}{125}$ both have 25 as a factor. We get $\frac{3}{5}$

The Negative Sign in Fractions

The fractions below are all equal although the negative sign is in *three* different positions.

In the Numerator $\frac{-5}{8}$

In the Denominator $\frac{5}{-8}$

Next to the dividing line $-\frac{5}{8}$

Decimals

Every number has a decimal point. If it does not show, then the number is a whole number.

e.g. 7 = 7. or 7.0 For numbers that begin with a decimal point, we often put a leading zero before the decimal point, but it is optional. e.g. 0.25 = .25

Conversions:

To convert a fraction to a decimal, divide the numerator by the denominator.

e.g. $\frac{2}{3}$ means 2 divided by 3 = 0.6666666… a repeating decimal

To convert a terminating decimal to a fraction, write the number without the decimal point over the power of 10 given by the number of decimal places and reduce if possible. For example, 0.60 has two decimal places, thus write 60 over 10^2.

e.g. $.60 = \frac{60}{100} = \frac{3}{5}$ $\qquad$ $0.125 = \frac{125}{1000} = \frac{1}{8}$

To convert a repeating decimal to a fraction requires a little bit of algebra. We will not attempt it in this course. However, if you are interested, we will be happy to work with you outside of class.

Adding and subtracting decimals: line up the decimal points

e.g.
```
   60.58
+  0.0163
```
decimal points not lined up so rewrite as
```
  60.58
+  0.0163
  60.5963
```

Multiplying decimals: multiply the numbers forgetting about the decimal point until the multiplication is complete. Then, count the number of decimal places in both factors and put into the product.

e.g. 0.93 * 1.2 is the same as
```
   0.93
 x  1.2
   1 8 6
   9 3
 1.1 1 6
```
three decimal places in the both factors.

Dividing decimals: Move the decimal place of the **divisor** to the right however many places you need to make it a whole number, then move the decimal place of the numerator the same number of places. What you are actually doing in this case is multiplying both the numerator and denominator by a power of ten to get a whole number in the divisor. Now divide normally.

e.g. $\frac{.8}{.004} = \frac{.8 \times 10^3}{.004 \times 10^3} = \frac{800}{4} = 200$ $\qquad$ e.g. $\frac{0.3451}{.07} = \frac{34.51}{7} = 4.93$

Percents

Per Cent means per 100 or out of 100. e.g. 35% means 35 out of 100. Percent also means 'portion' or 'part of.'

To change from a **percent to a decimal,** divide by 100 or move the decimal point two places to the left. Don't forget to remove the percent sign.

ex. 1. $35\% = \frac{35}{100} = 0.35$ ex. 2. $7.5\% = \frac{7.5}{100} = 0.075$

To change from a **decimal to a percent**, multiply by 100 or move the decimal point two places to the right and attach the % sign.

ex. 1. 0.125 = 12.5% ex. 2. 2.12 = 212% ex. 3. 93.15 = 9315%

Four general types of percent problems

1)	**WHAT IS**	35	%	of	$250	?	(MULTIPLY)
2)	$25.00 IS	**WHAT**	**%**	of	$120	?	(DIVIDE)
3)	$30.00 IS	15	%	**of**	**WHAT**	?	(DIVIDE)
4)	20% of	**WHAT**		**IS**	500	?	(DIVIDE)

Question 1) says that some number is 35% of $250. When we are looking for the actual portion in numbers OF something and we have the percent, multiply that percent times the amount given (this is the WHOLE amount) to find the portion. First change 35% to a decimal and multiply by 250. i.e.

0.35*250 = 87.5 checks $87.50 is 35% of $250. (Hint: 'OF' means 'multiply.')

Question 2) tells us that $25.00 is some 'portion' or some 'part' of $120. We need to find the percent. Think of the percent as 'part' over the 'whole' or

$$\text{Percent} = \frac{\text{part}}{\text{whole}} \times 100$$ (The fraction gives a number in decimal form. You need to multiply by 100 to change to a percent.)

In this case 25 is the 'part' and 120 is the 'whole.' Thus,

$$\text{Percent} = \frac{25}{120} \times 100 = 0.2083 * 100 = 20.83\%$$ (rounded to 2 decimal places)

Question 3) tells us that 30 is the portion of some whole. We know what portion (15%), but we need to find the whole, i.e. we write:

$$15\% = \frac{30}{\text{whole}}$$ In this case, we need to divide 30 by 15% (as demonstrated in class) to find the Whole.

$$\frac{30}{0.15} = 200$$ always check your answer to see if it makes sense.

Question 4) is worded a bit differently, but is very similar to Question 3. We still need to find the 'whole.' Again, we divide by the percent as above.

$$\frac{500}{0.20} = 2500$$ Checking: 500 is indeed 20% of 2500

Exponents

a^n In general, '**a**' is the base and '**n**' is the exponent.

6^2 read, 'six squared,' means $6 \times 6 = 36$.
Here, 6 is the base and 2 is the exponent.
It is also read as "6 to the power of 2."

$2^3 = 2*2*2 = 8$ be careful here. We multiply 2 by itself three times. The answer is 8 not 6.

Negative Numbers raised to a power.

$(-3)^4 = (-3)(-3)(-3)(-3) = 81$ Here, the number (–3) is raised to the 4th power.
BUT
$-(3)^4 = -\ 3x3x3x3 = -81$. The negative sign is outside the parentheses. The exponent only applies to what's inside the parentheses. Thus, the 3 is raised to the 4th power. The entire answer is negative due to the negative sign.

Using letters with exponents instead of numbers works the same way.

x^4 is read as "x to the power of 4" "x" is the base and "4" is the exponent

x^4 means multiply x by itself 4 times i.e. $x*x*x*x$

***ZERO - AGAIN.** **ANY NUMBER RAISED TO THE ZERO POWER = 1.**

examples:

$5^0 = 1$

$397^0 = 1$

$(q^7xyz)^0 = 1$

$\mathbf{-(42)^0 = -1}$ **Look closely at this example**

The rules for exponents are summarized on the next page.

RULES FOR MANIPULATION OF EXPONENTS

Multiplying numbers with the same base that have an exponent - add the exponents.

ex. $5^4 \text{ x } 5^2 = 5^{(4+2)} = 5^6$

ex. $a^b \times a^c = a^{b+c}$

Dividing numbers with the same base that have an exponent - subtract the exponents.

ex. $5^4 \div 5^2 = \dfrac{5^4}{5^2} = 5^{4-2} = 5^2$

ex. $\dfrac{5^3}{5^5} = 5^{3-5} = 5^{-2}$

ex. $a^b \div a^c = \dfrac{a^b}{a^c} = a^{b-c}$

Raising a number with an exponent to a power - multiply exponents.

ex. $(5^2)^4 = 5^8$

A number with a negative exponent means: 1 divided by the number with the positive of the exponent.

ex. $5^{-2} = \dfrac{1}{5^2} = \dfrac{1}{25}$

You can have fractional powers.

ex. $16^{1/2}$ means the square root of 16. The denominator indicates the 'root.'

ex. $16^{3/2}$ this mean the square root of 16 all raised to the 3rd power. It can be written as: $(16^{1/2})^3 = 4^3$