

$f(x) = x^2 - x + 1$ , Find the average rate of change of the function  $f$  with respect to  $x$  over the interval  $[0, 4]$ .

① Average Speed  
 $\frac{\text{Distance}}{\text{Time}}$

Average Rate of Change

$$\frac{\Delta y}{\Delta x} = \frac{f(b) - f(a)}{b - a} = \frac{f(a+h) - f(a)}{h}$$

where  $b = a+h$   
 and  $h > 0$

②  $\frac{f(b) - f(a)}{b - a}$   $[0, 4]$   
 $a, b$

$$\frac{f(4) - f(0)}{4 - 0}$$

$$f(4) = (4)^2 - (4) + 1$$

$$f(0) = (0)^2 - (0) + 1$$

$$\frac{(4^2 - 4 + 1) - (0^2 - 0 + 1)}{4 - 0}$$

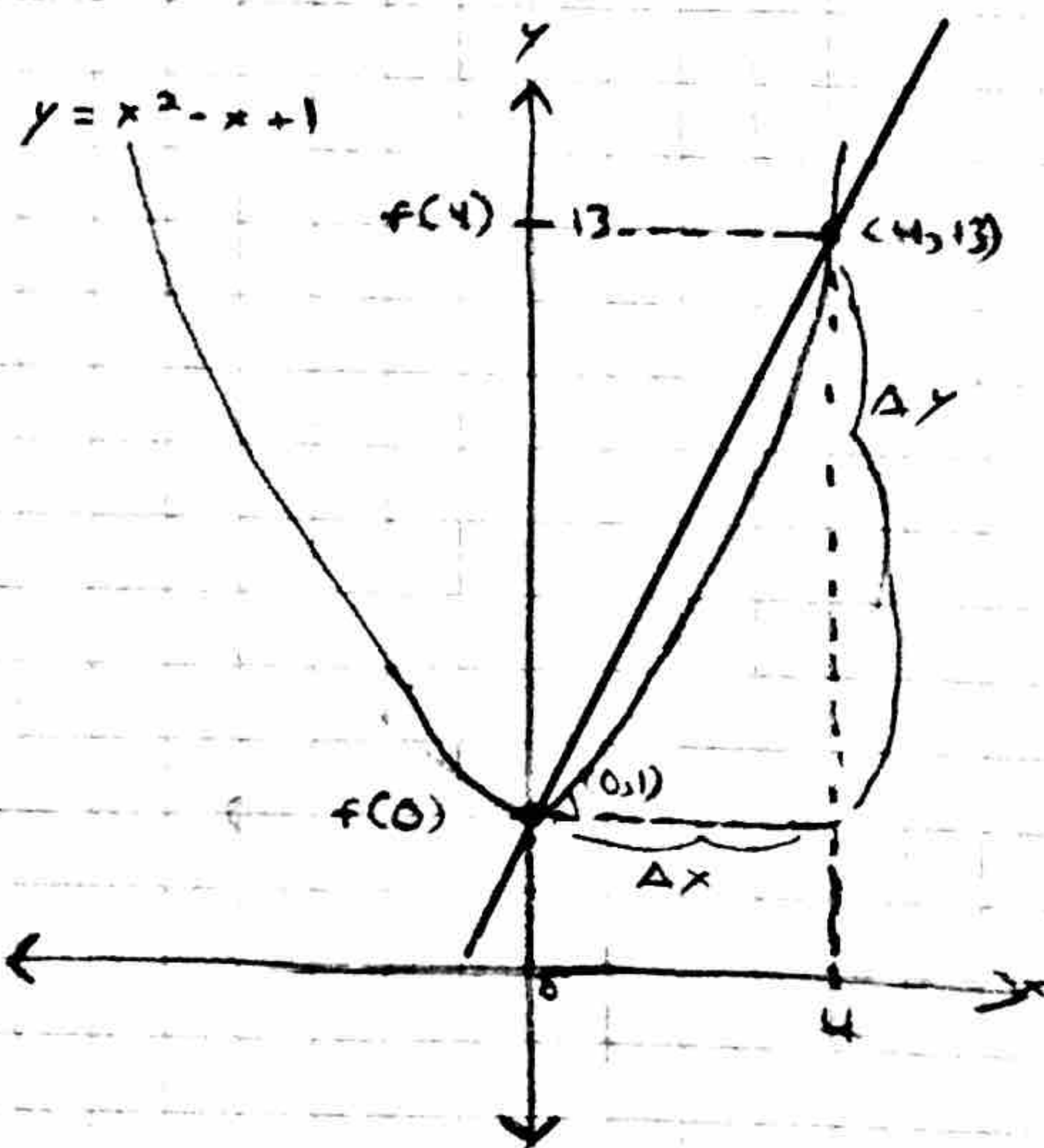
$$\frac{(16 - 4 + 1) - (1)}{4}$$

$$\frac{13 - 1}{4}$$

$$\frac{12}{4}$$

$$3$$

Average Rate of Change of the function  $f$  with respect to  $x$  over the interval  $[0, 4]$  is 3.



$$m = \frac{13 - 1}{4 - 0} = \frac{12}{4} = 3$$

$$m = 13$$

Slope of secant line passing through  $(0, 1)$  and  $(4, 13)$  is 3.



$f(x) = 4x^2$ , Find Average rate of change of function  $f$  over  $[1, 2]$   
 $a, b$

$$\frac{\Delta y}{\Delta x} = \frac{f(b) - f(a)}{b - a}$$

$$\frac{f(2) - f(1)}{2 - 1}$$

$$\frac{4(2)^2 - 4(1)^2}{2 - 1}$$

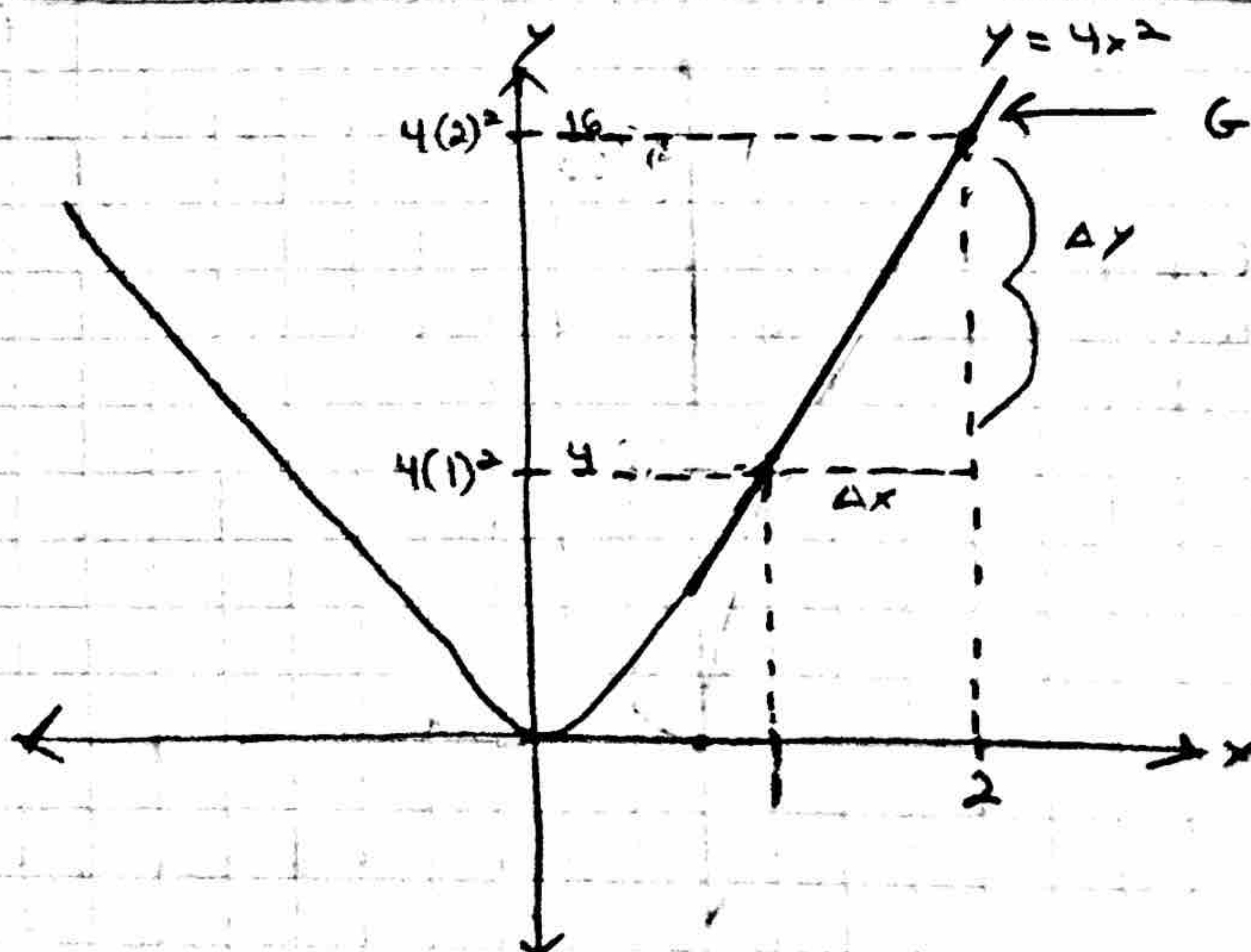
$$\frac{4 \cdot 4 - 4 \cdot 1}{2 - 1}$$

$$\frac{16 - 4}{1}$$

$$\frac{12}{1}$$

$$\frac{12}{1}$$

Average rate of change of the function  $f$  with respect to  $x$  over the interval  $[1, 2]$  is 12.



Geometrically, the slope of the secant line passing through the points  $(1, 4)$  and  $(2, 16)$  is 12.

$$m = \frac{16 - 4}{2 - 1} = \frac{12}{1} = 12$$



Find the average rate of change of the function  $f(x) = \sqrt{x} + 1$  over the interval  $[1, 4]$ .

Average Rate of Change

$$\frac{\Delta y}{\Delta x} = \frac{f(b) - f(a)}{b - a}$$

$$\frac{f(b) - f(a)}{b - a} \quad \begin{matrix} [1, 4] \\ a, b \end{matrix}$$

$$\begin{aligned} f(x) &= \sqrt{x} + 1 \\ f(1) &= \sqrt{1} + 1 \\ f(4) &= \sqrt{4} + 1 \end{aligned}$$

$$\begin{aligned} f(a) &= f(1) \\ f(b) &= f(4) \end{aligned}$$

$$\frac{(\sqrt{4} + 1) - (\sqrt{1} + 1)}{4 - 1}$$

$$\frac{(2 + 1) - (1 + 1)}{3}$$

$$\frac{3 - 1 - 1}{3}$$

$$\frac{3 - 1 + (-1)}{3}$$

$$\frac{2 + (-1)}{3}$$

$$\frac{1}{3} \approx 0.33$$

The average rate of change of the function  $f$  over the interval  $[1, 4]$  is approximately 0.33