Determining Whether a Trigonometric Function is ODD, Even, OR Neither

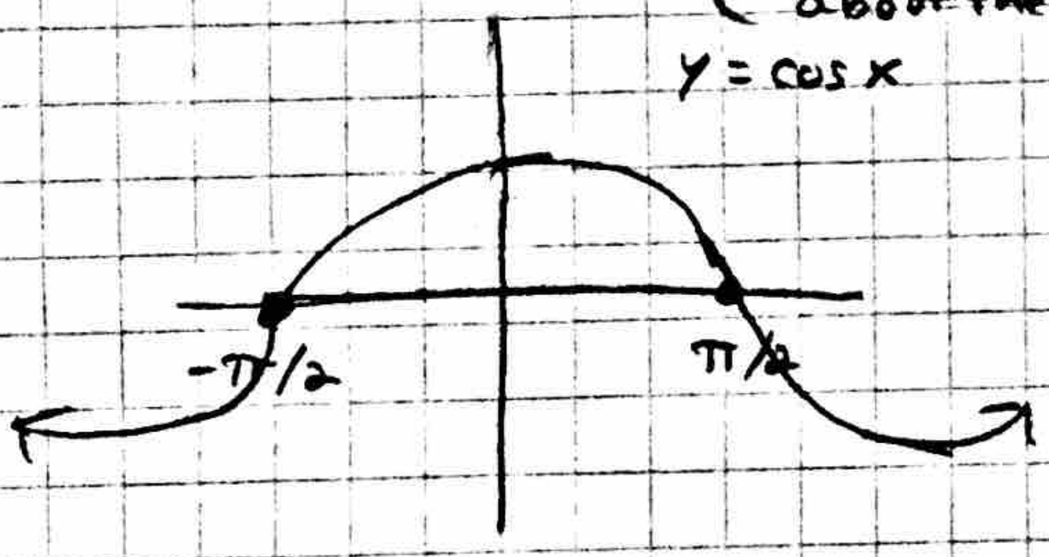
Even: Symmetric about the Y-axis

$f(-x) = f(x)$

Odd: Symmetric about the origin

$f(-x) = -f(x)$

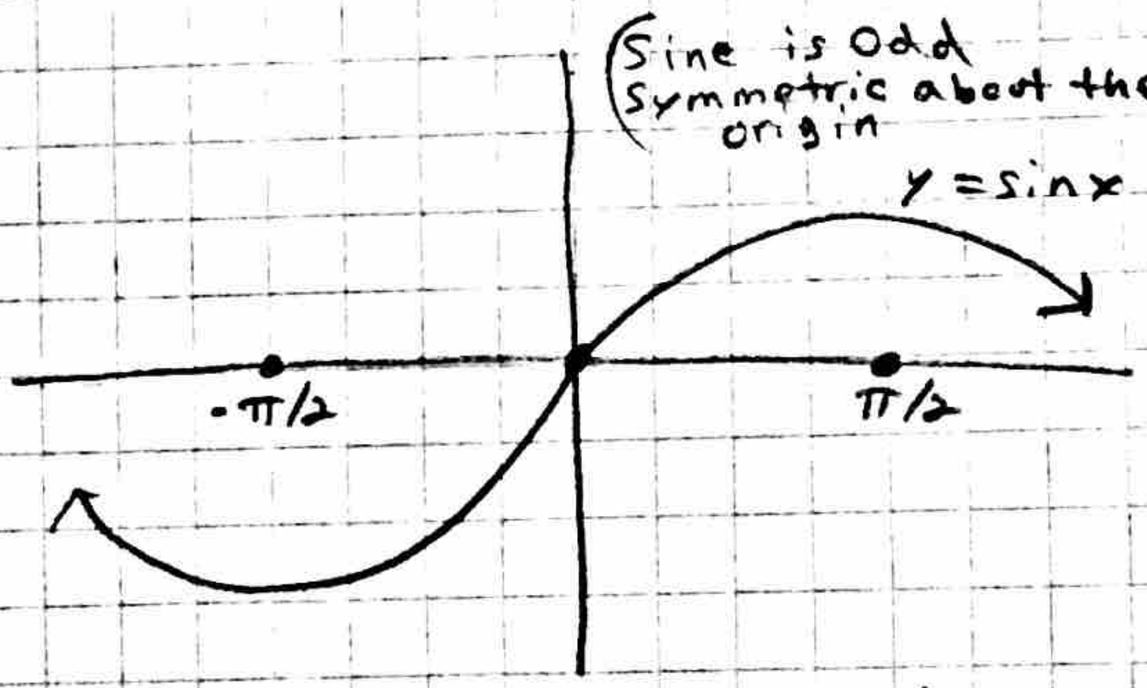
Cos is Even (Symmetric about the y-axis)



$\cos(-x) = \cos x$

even

(Sine is Odd Symmetric about the origin)



$\sin(-x) = -\sin(x)$

odd

$f(x) = \tan x = \frac{\sin x}{\cos x}$

$f(-x) = \tan(-x) = \frac{\sin(-x)}{\cos(-x)} = \frac{-\sin x}{\cos x} = -\tan x$

$\| \quad \| \quad \|$

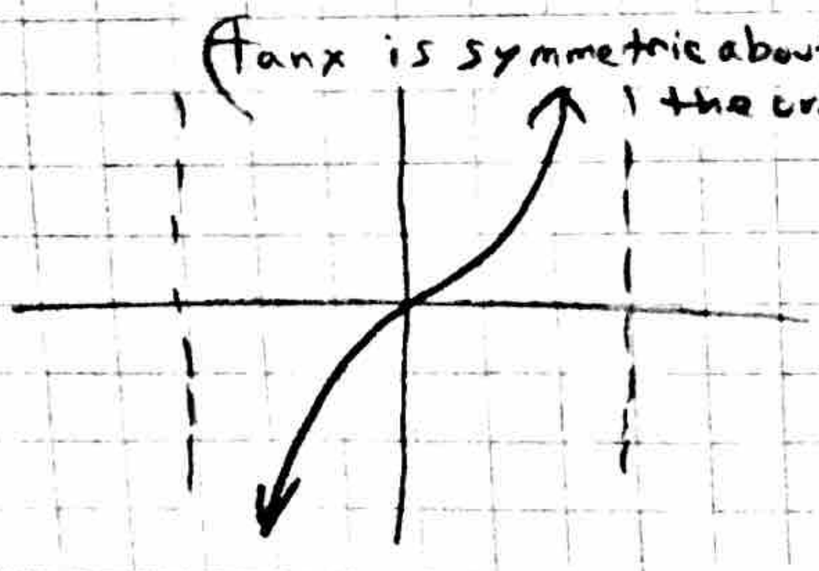
$f(-x) = -\tan x$

$\|$

$f(-x) = -f(x)$

odd

(tan x is symmetric about the origin)



$$f(x) = \cot x = \frac{\cos x}{\sin x}$$

$$\| \quad f(x)$$

$$f(-x) = \cot(-x) = \frac{\cos(-x)}{\sin(-x)} = \frac{\cos(x)}{-\sin(x)} = -\cot(x)$$

$$\| \quad f(x)$$

$$f(-x) = -\cot(x) \Rightarrow f(-x) = -f(x)$$

odd

$$f(x) = \sec(x) = \frac{1}{\cos(x)}$$

$$\| \quad f(x)$$

$$f(-x) = \frac{1}{\cos(-x)} = \frac{1}{\cos(x)} = \sec(x)$$

$$\| \quad f(x)$$

$$f(-x) = f(x)$$

even

$$f(x) = \csc(x) = \frac{1}{\sin(x)}$$

$$\| \quad -f(x)$$

$$f(-x) = \csc(-x) = \frac{1}{\sin(-x)} = \frac{1}{-\sin(x)} = -\csc(x)$$

$$f(-x) = -f(x)$$

odd

TABLE

$\cos(-x) = \cos x$	$\sec(-x) = \sec(x)$	EVEN
$\sin(-x) = -\sin x$	$\csc(-x) = -\csc(x)$	ODD
$\tan(-x) = -\tan x$	$\cot(-x) = -\cot(x)$	ODD

a. $f(x) = \sec x \cdot \tan x$

$f(-x) = \sec(-x) \cdot \tan(-x)$

$= \sec(x) \cdot (-\tan(x))$

$f(-x) = -\sec(x) \cdot \tan(x)$

$f(-x) = -(\sec x \cdot \tan x) \rightarrow f(-x) = -f(x)$

odd

$g(x) = x^4 \sin x \cdot \cos^2 x$

$g(x) = (x)^4 \sin(x) [\cos(x)]^2$

$g(-x) = (-x)^4 \sin(-x) [\cos(-x)]^2$

$= x^4 - \sin x (\cos(x))^2$

$g(-x) = -(x^4 \sin x \cos^2 x) \rightarrow f(-x) = -f(x)$

odd

$$h(x) = \cos x + \sin x$$

$$h(-x) = \cos(-x) + \sin(-x)$$

$$h(-x) = \cos x + -(\sin(x))$$

$$h(-x) = \cos x - \sin(x)$$

neither even or odd

$f(-x) \neq f(x)$ or $f(-x) \neq -f(x)$