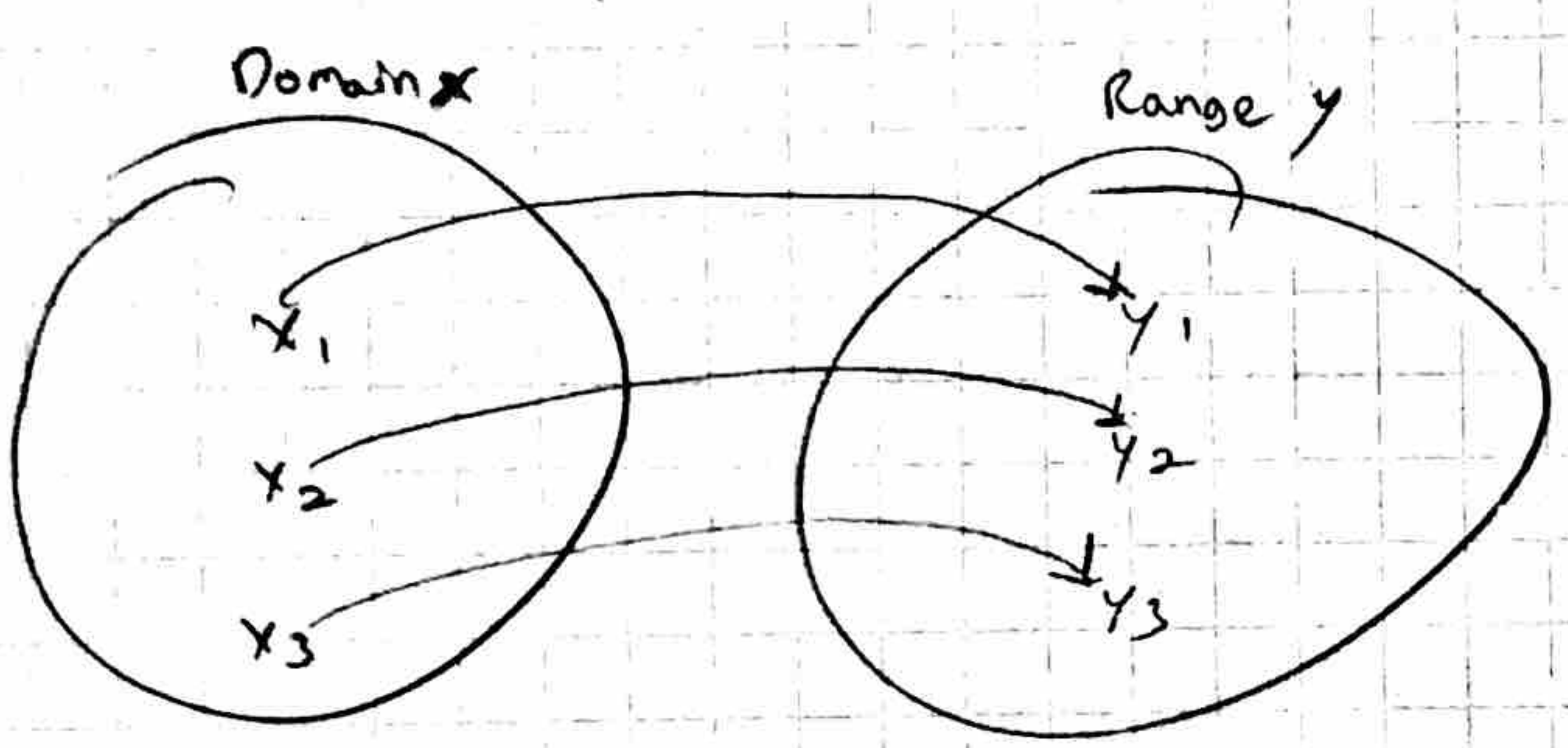
A relation is a set of ordered pairs.

A relation is a function

1. There are no unassigned domain elements.
2. Every domain element is assigned exactly one range element

Vertical Line Test: Draw a vertical line through a function. If line intersects function more than once, it is not a function.

<u>One-to-One Function</u>



* Each x-value in the domain has a unique corresponding y-value in the range.

$f(x) = 3x - 5$ $\quad$ $f(2)$, $f(-2)$

$f(2) = 3(2) - 5$
$= 6 - 5$
$= \boxed{f(2) = 1}$

$f(-2) = 3(-2) - 5$
$= -6 - 5$
$= \boxed{f(-2) = -11}$

$f(-3) = -2(-3)^3 + 3(-3) - 4$
$= -2(-27) - 9 - 4$
$= 54 - 9 - 4$
$\boxed{f(-3) = 41}$

$g(-2) = -3(-2)^2 - (-2)$
$= -3(4) + 2$
$= -12 + 2$
$= \boxed{f(-2) = -10}$

Given $f(x) = 2x - 7$: $f(a)$, $f(x+h)$, $-f(x)$

$f(a)$

$f(\ ) = 2(\ ) - 7$

$f(a) = 2(a) - 7$

$\boxed{f(a) = 2a - 7}$

$f(x+h)$

$f(\ ) = 2(\ ) - 7$

$f(x+h) = 2(x+h) - 7$

$\boxed{f(x+h) = 2x + 2h - 7}$

$-f(x)$: Take negative of entire function

$-f(x) = -(2x - 7)$

$\boxed{-f(x) = -2x + 7}$

The Difference Quotient

$$\frac{f(x+h)-f(x)}{h}, \quad f(x)=x^2+1$$

$f(x+h)$

$(x+h)^2+1$

$=$

$(x+h)(x+h)+1$

$=$

$x^2+xh+xh+h^2+1$

$=$

$\boxed{x^2+2xh+h^2+1}$

$f(x)$

$\boxed{x^2+1}$

h

$\boxed{h}$

Replace

$$\frac{f(x+h)-f(x)}{h}$$

$=$

$$\frac{(x^2+2xh+h^2+1)-(x^2+1)}{h}$$

$=$

$$\frac{\cancel{x^2}+2xh+h^2\cancel{+1}\cancel{-x^2}\cancel{-1}}{h}$$

$=$

$$\frac{2xh+h^2}{h}$$

$=$

$$\frac{\cancel{h}(2x+h)}{\cancel{h}}$$

$=$

$$\boxed{2x+h}$$

$X = \{1, 2, 3, 4, 5\}$ $Y = \{1, 2, 3, 4, 5\}$

Relations and Functions

1. $R = \{(1,3), (2,4), (3,5), (4,1)\}$

Domain: $\{1, 2, 3, 4\}$
Range: $\{3, 4, 5, 1\}$

a. 5 is unassigned in domain X
b. R is not a function, so it's not a one-to-one function

3. $R = \{(1,4), (2,3), (3,2), (4,3), (5,4)\}$

Domain: $\{1, 2, 3, 4, 5\}$
Range: $\{4, 3, 2, 3, 4\}$

a. All domain elements are assigned
b. All domain elements are assigned to exactly one range element
R is a function

Not one-to-one, Each domain element are not assigned to a unique range element
(1, 4), (5, 4)
(2,3), (4,3)

5. $R = \{(1,3), (2,4), (3,5), (2,1), (4,2), (5,1)\}$

X: $\{1, 2, 3, 2, 4, 5\}$
Y: $\{3, 4, 5, 1, 2, 1\}$

a. All domain elements are assigned
b. All domain elements are not assigned exactly one range element
R is not a function (2,4); (2,1)

Not one-to-one, Since R is not a function it is not a one-to-one function

7. $R = \{(1,5), (2,4), (3,3), (4,2), (5,1)\}$

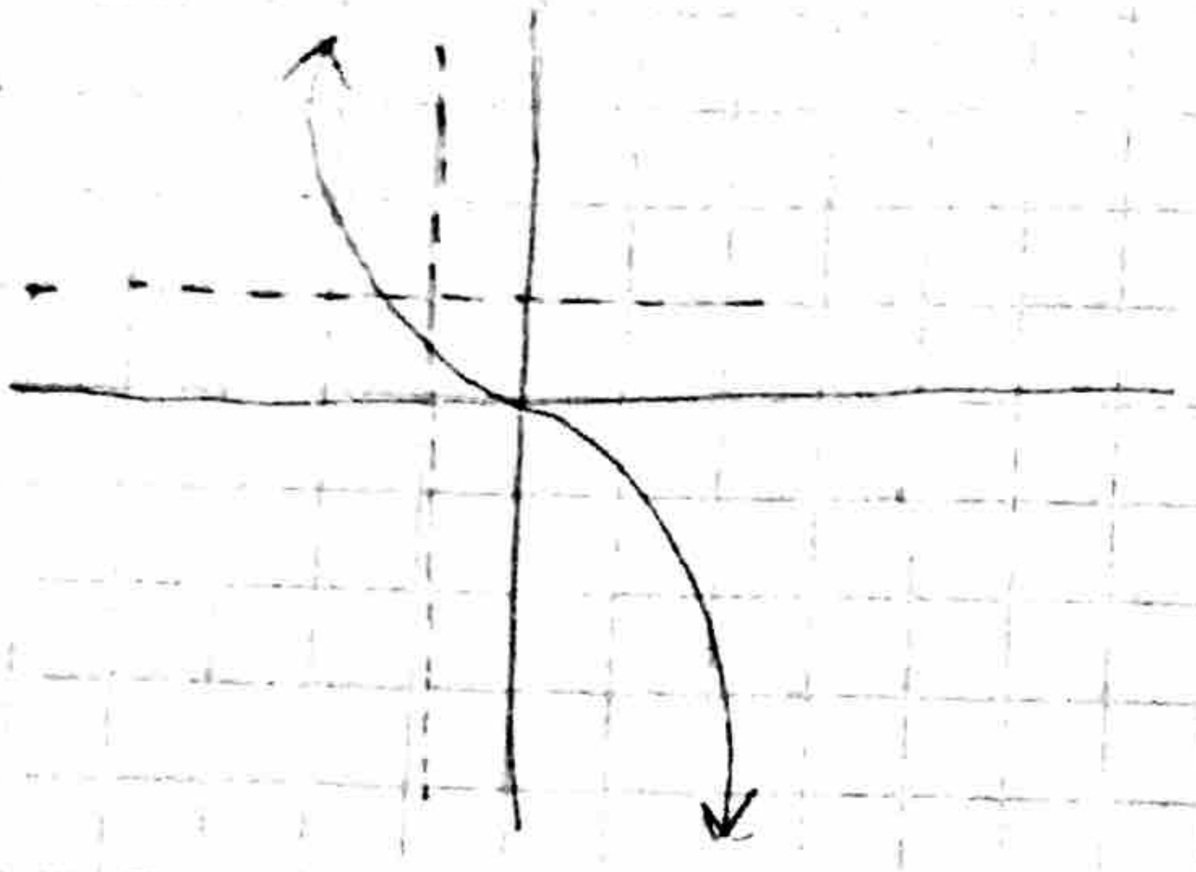
X: $\{1, 2, 3, 4, 5\}$
Y: $\{5, 4, 3, 2, 1\}$

a. All domain elements are assigned
b. All domain elements are assigned to exactly one range element.
R is a function

One-to-one, all x values map to a unique y-value.

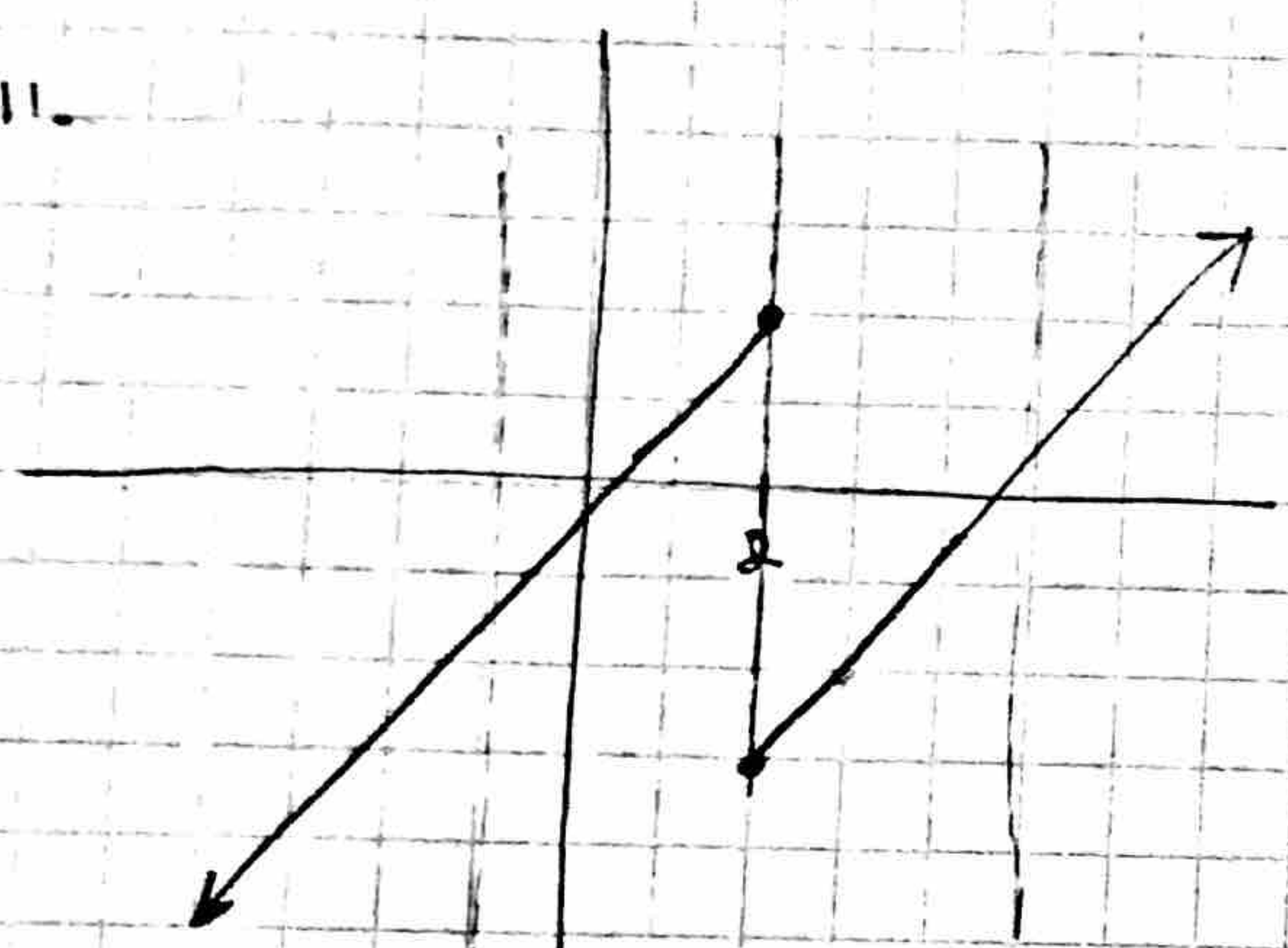
Vertical line test: Validate Function
Horizontal line test: Validate One-to-One Function

9.



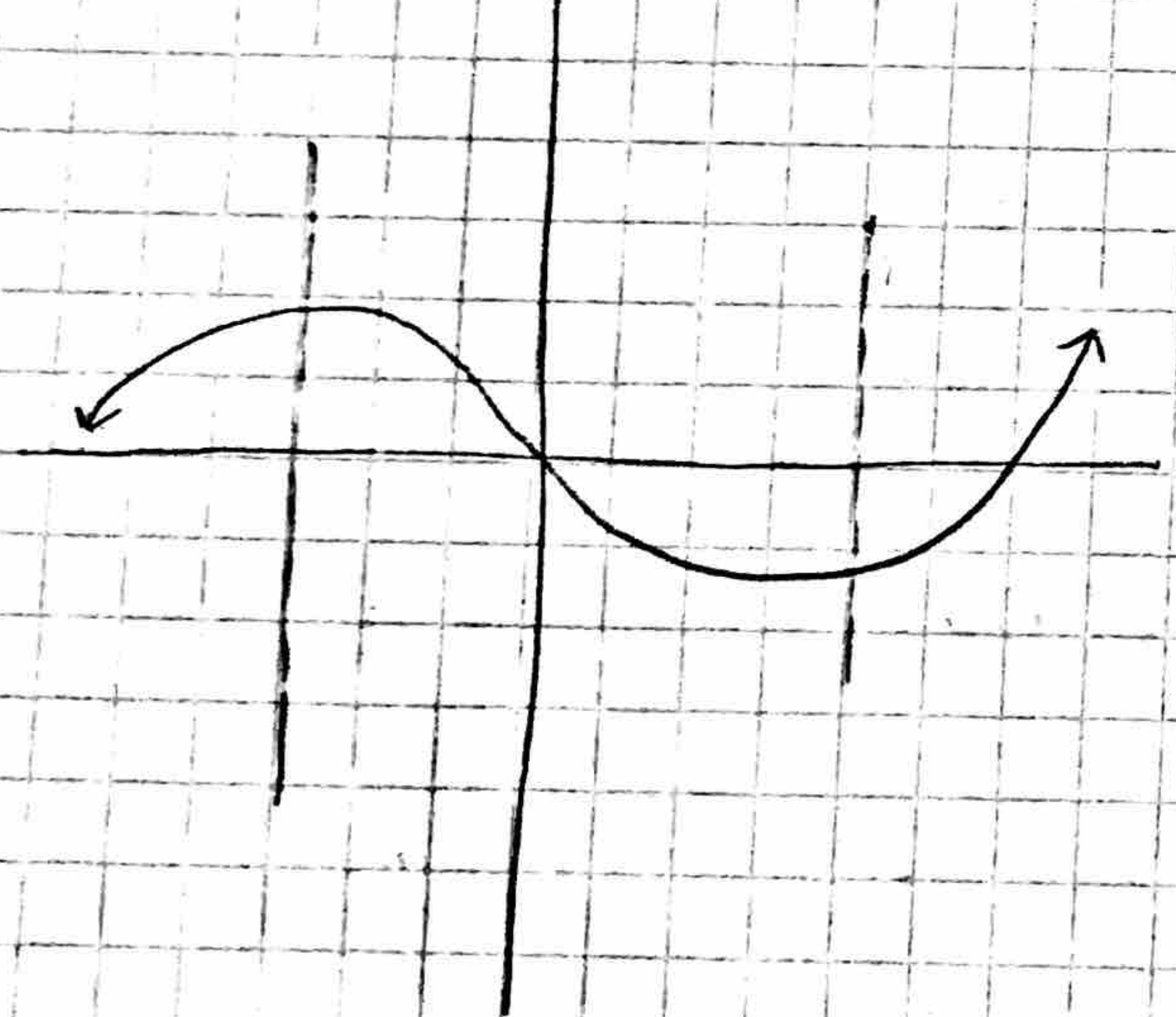
1. Pass Vertical Line Test: Function
2. Pass Horizontal Line Test: One-to-One

11.



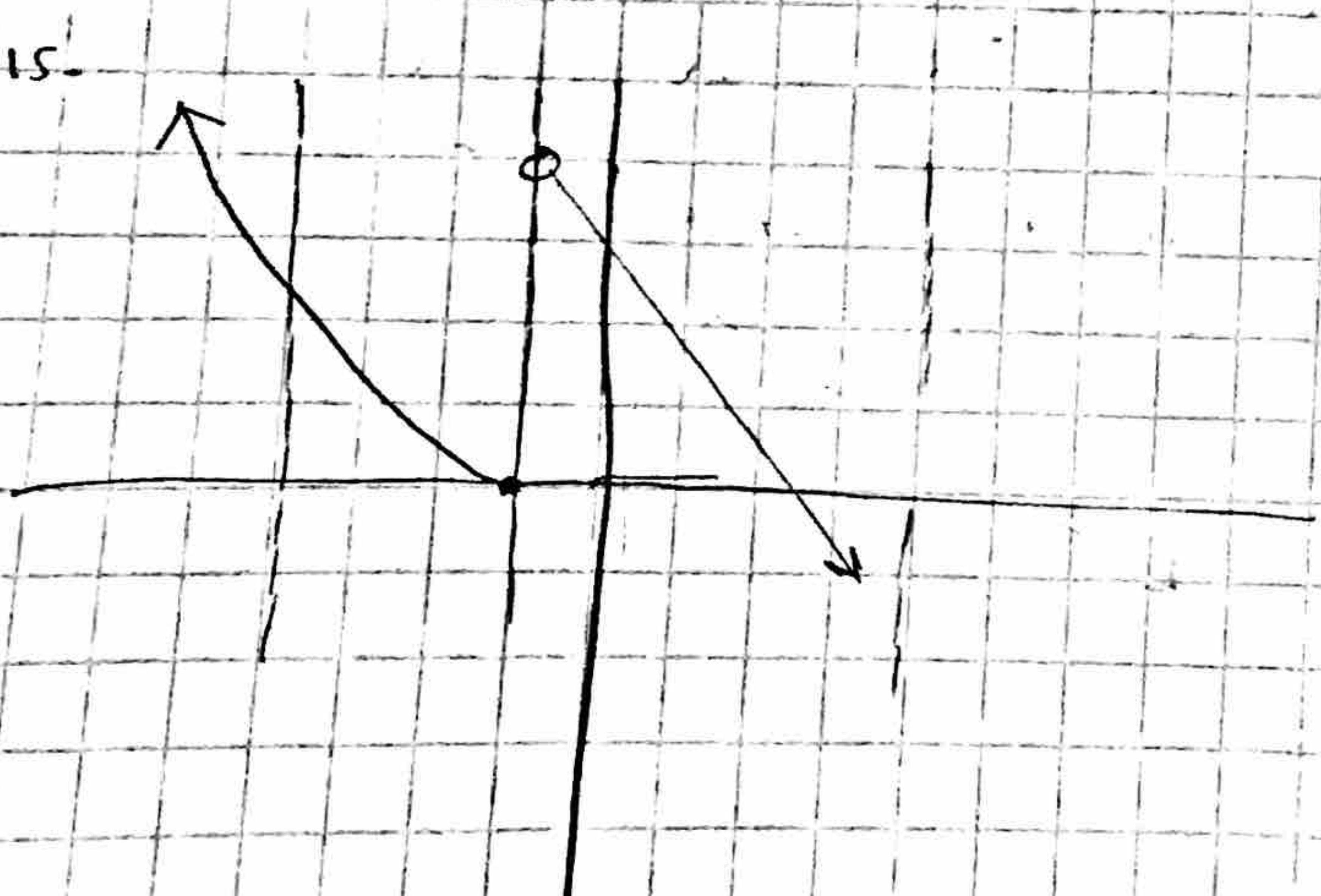
1. Fail Vertical Line Test: Not a Function at $x = 2$

13.



1. Fail Vertical Line Test: Not a Function

15.



1. Fail Vertical Line Test at $y = 0$: Not a Function

Graph in Desmos

17. $y = -9x + 2$

1. Pass Vertical Line Test : Function
2. Pass Horizontal Line Test : One-to-One
3. $y = -9x + 2$ is a linear equation.

All linear equations are both functions and one-to-one functions

19. $x = y + 4$

$$\begin{array}{r} x = y + 4 \\ \underline{-4 \quad\ -4} \end{array}$$

$x - 4 = y$

$\boxed{y = x - 4}$

1. Pass Vertical Line Test : Function
2. Pass Horizontal Line Test : One-to-One

All linear equations are both functions and one-to-one functions.

21. $y = -x^2 - 20$

1. Pass Vertical Line Test : Function
2. Fails Horizontal Line Test : Not One-to-One

23. $x^2 + y^2 = 16$ 1. Fails Vertical Line Test : Not a Function

25. $x = y^2 + 2$ 1. Fails Vertical Line Test : Not a Function

27. $y = -x^3 + 4$
1. Pass Vertical Line Test : Function
2. Pass Horizontal Line Test : One-to-One

29. $y = 4x$
1. Pass Vertical Line Test : Function
2. Pass Horizontal Line Test : One-to-One

31. $y = \pm\sqrt{x}$

$x = 1$

$y = \pm\sqrt{1}$

$y = 1$ and $y = -1$ → x maps to both 1 and -1, not a function

33. $x = |y| - 4$ ∴ See Below

35. $y = \frac{1}{2}x^2$

1. Pass Vertical Line Test : Function
2. Fails Horizontal Line Test : Not one-to-one

37. $y = x^4$

1. Pass Vertical Line Test : Function
2. Fails Horizontal Line Test : Not One-to-One

39. $y = |x+3|$

1. Pass Vertical Line Test : Function
2. Fails Horizontal Line Test : Not One-to-One

41. $y = 2(0.5x + 6)$

1. Pass Vertical Line Test : Function
2. Pass Horizontal Line Test : One-to-One

43. $y = \sqrt{x+2}$

1. Pass Vertical Line Test : Function
2. Pass Horizontal Line Test : One-to-one

33. $x = |y| - 4$

$\underline{+4 \qquad\quad +4}$

$x + 4 = |y|$

$\|$

$|y| = x + 4$

$\|$

$x = -3$

$\|$

$|y| = -3 + 4$

$\|$

$|y| = \pm 1$

$\|$

$y = 1$ and $y = -1$

$x = -3$ maps to both $y = 1$ and $y = -1$

$x = |y| - 4$ is not a function

45. widget : x , cost : y

widget	cost
1	5
2	7
3	7
4	8

{1, 2, 3, 4}
{5, 7, 7, 8}

Function ✓
Not One-to-One
(2,7), (3,7)

47. time : x , height : y

time	height
1	20
2	40
3	60
4	80

Balloon rises 80 feet after 4 minutes.
80/4 = 20 ft

1 min = 20 ft

{1, 2, 3, 4}
{20, 40, 60, 80}

Function ✓
One-to-One

49. time : x rate : y

time	rate
1	60
2	60
3	70
4	80
5	90

{1, 2, 3, 4, 5}
{60, 60, 70, 80, 90}

Function ✓
Not one-to-One
(1, 60), (2, 60)

51. X = {1, 2, 3, 4}

Y = {1, 2, 3, 4}

Function : R = { (1, 1), (2, 2), (3, 2), (4, 3) }

One-to-one : R = { (1, 1), (2, 2), (3, 3), (4, 4) }

Not a Function : R = { (1, 1), (1, 2), (2, 2), (3, 3) }

53. a. A Function is always a relation because a function needs to be a set of ordered pairs.

b. A relation is not always a function because of the below requirements:

1. No unassigned domain elements
2. Every domain element is assigned exactly one range element.

You can have a relation that don't fit the above requirements.

c. A function is not always a one-to-one function.

d. I think a one-to-one function is alway a relation.

y "is a function of" x

55. a. The year is not a function of the US debt, in trillions of dollars. The year do not depend on the amount of debt in trillions of dollars.

b. The U.S debt, in trillions of dollars is a function of the year. The amount of debt in trillions depend on the year.

57. x: price
y: demand

The demand is a function of price.

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59. $f(x) = 2x^2 - 5x + 1$

$f(0) = 2(0)^2 - 5(0) + 1$

$=$

$2(0) - 0 + 1$

$=$

$0 - 0 + 1$

$=$

$\boxed{f(0) = 1}$

$f(-1) = 2(-1)^2 - 5(-1) + 1$

$=$

$2(1) + 5 + 1$

$=$

$2 + 5 + 1$

$=$

$\boxed{f(-1) = 8}$

$f(-2) = 2(-2)^2 - 5(-2) + 1$

$=$

$2(4) + 10 + 1$

$=$

$8 + 10 + 1$

$=$

$\boxed{f(-2) = 19}$

61. $f(x) = -x^3 - 3x^2 - 5x + 2$

$$f(-4) = -(-4)^3 - 3(-4)^2 - 5(-4) + 2$$

$$= -(-64) - 3(16) + 20 + 2$$

$$= 64 - 48 + 20 + 2$$

$$= 16 + 22$$

$$= \boxed{f(-4) = 38}$$

$$f(3) = -(3)^3 - 3(3)^2 - 5(3) + 2$$

$$= -(27) - 3(9) - 15 + 2$$

$$= -27 - 27 - 15 + 2$$

$$= -54 - 15 + 2$$

$$= -69 + 2$$

$$\boxed{f(3) = -67}$$

$$f(-2) = -(-2)^3 - 3(-2)^2 - 5(-2) + 2$$

$$= -(-8) - 3(4) + 10 + 2$$

$$= 8 - 12 + 10 + 2$$

$$= -4 + 10 + 2$$

$$= 6 + 2$$

$$= \boxed{f(-2) = 8}$$

63. $f(x) = \sqrt{x+1} - 2$

$f(0) = \sqrt{0+1} - 2$
$= \sqrt{1} - 2$
$= 1 - 2$
$\boxed{f(0) = -1}$

$f(3) = \sqrt{3+1} - 2$
$= \sqrt{4} - 2$
$= 2 - 2$
$\boxed{f(3) = 0}$

$f(8) = \sqrt{8+1} - 2$
$= \sqrt{9} - 2$
$= 3 - 2$
$\boxed{f(8) = 1}$

65. $f(x) = 9x^2 + 2x - 1$

$f(-2) = 9(-2)^2 + 2(-2) - 1$
$= 9(4) - 4 - 1$
$= 36 - 4 - 1$
$= 36 - 5$
$\boxed{f(-2) = 31}$

67. $f(x) = 3x\sqrt{x^2 - 16} + 2$

$f(5) = 3(5)\sqrt{(5)^2 - 16} + 2$
$= 15\sqrt{25 - 16} + 2$
$= 15\sqrt{9} + 2$
$= 15(3) + 2$
$= 45 + 2$
$\boxed{f(5) = 47}$

69. $f\left(-\frac{2}{5}x+5\right)$, $f(x)=5x+1$

$$f\left(-\frac{2}{5}x+5\right) = \cancel{5}\left(-\frac{2}{\cancel{5}}x+5\right)+1$$

$$= -2x+25+1$$

$$\boxed{f\left(-\frac{2}{5}x+5\right) = -2x+26}$$

$f\left(-\frac{2}{5}x+5\right)$
Set $5x+1$ as template
Plug $-\frac{2}{5}x+5$ in x

71. $f(x+y)$, $f(x)=2x^2+4x+5$

$$f(x+y) = 2(x+y)^2+4(x+y)+5$$

$$= 2(x^2+2xy+y^2)+4(x+y)+5$$

$$= 2(x^2+2xy+y^2)+4x+4y+5$$

$$\boxed{f(x+y)=2x^2+4xy+2y^2+4x+4y+5}$$

$(x+y)(x+y)$
$= x^2+xy+xy+y^2$
$= \boxed{x^2+2xy+y^2}$

$f(x+y)$
Set $2x^2+4x+5$ as template.
Plug $x+y$ in x

73. $g(x)=2x+7$, $-g(x^2+3x-2)$

$$-g(x^2+3x-2) = -(2(x^2+3x-2)+7)$$

$$= -(2x^2+6x-4+7)$$

$$= -(2x^2+6x+3)$$

$$\boxed{-g(x^2+3x-2) = -2x^2-6x-3}$$

$g(x)$
Set $-g(x^2+3x-2)$ as outer template
Set $2x+7$ as inner template
Plug x^2+3x-2 in x

75. $f(x) = x+2$, $\frac{f(x+h)-f(x)}{h}$

Set $\frac{f(x+h)-f(x)}{h}$ as outer template

Set $x+2$ as inner template

Plug $x+2$ in x

$$\frac{f(x+h)-f(x)}{h} = \frac{f([\,]+h)-f([\,])}{h} = \frac{(x+2+h)-(x+2)}{h} = \frac{x+2+h-x-2}{h} = \frac{\cancel{x}-x+h+2-2}{h} = \frac{h}{h}$$

$$\boxed{\frac{f(x+h)-f(x)}{h} = 1}$$

70. $f(x) = \frac{1}{2}x^2+3x+2$, $f(-3x)$

Set $\frac{1}{2}x^2+3x+2$ as temp

Plug $-3x$ in x

$$f(-3x) = \frac{1}{2}(-3x)^2+3(-3x)+2 = \frac{1}{2}(9x^2) - 9x + 2$$

$$\boxed{f(-3x) = \frac{9x^2}{2} - 9x + 2}$$

71. $f(x+y)$, $f(x) = 2x^2 + 4x + 5$

$f(x+y) = 2(x+y)^2 + 4(x+y) + 5$

$2(x^2 + 2xy + y^2) + 4x + 4y + 5$

$\boxed{f(x+y) = 2x^2 + 4xy + 2y^2 + 4x + 4y + 5}$

$(x+y)(x+y)$

$x^2 + xy + xy + y^2$

$\boxed{x^2 + 2xy + y^2}$

72. $f(x+y)$, $f(x) = 9x^2 - 2x + 10$

$f(x+y) = 9(x+y)^2 - 2(x+y) + 10$

$9(x^2 + 2xy + y^2) - 2x - 2y + 10$

$\boxed{f(x+y) = 9x^2 + 18xy + 9y^2 - 2x - 2y + 10}$

76. $f(x) = x^2 + 2$, $\dfrac{f(x+h) - f(x)}{h}$

$$\frac{f(x+h) - f(x)}{h} = \frac{([\;]^2 + 2) - (x^2 + 2)}{h}$$

$$= \frac{((x+h)^2 + 2) - (x^2 + 2)}{h}$$

$(x+h)(x+h)$

$x^2 + xh + xh + h^2$

$x^2 + 2xh + h^2$

$$= \frac{x^2 + 2xh + h^2 + 2 - (x^2 + 2)}{h}$$

$$= \frac{\boxed{x^2} + 2xh + h^2 \boxed{+2} \boxed{-x^2} \boxed{-2}}{h}$$

$$= \frac{\cancel{x^2 - x^2} + \cancel{2 - 2} + 2xh + h^2}{h}$$

$$= \frac{2xh + h^2}{h}$$

$$= \frac{\cancel{h}(2x + h)}{\cancel{h}}$$

$$\boxed{\frac{f(x+h) - f(x)}{h} = 2x + h}$$

77. $\dfrac{f(x-h) - f(x)}{h}$, $f(x) = (x+2)^2$

Set $f(x-h) - f(x)$ as outer term

$$\frac{f(x-h) - f(x)}{h} = \frac{(\boxed{}+2)^2 - (x+2)^2}{h}$$

Set $(x+2)^2$ as inner term

$$\frac{(x-h+2)^2 - (x+2)^2}{h}$$

$f(x) = (x+2)^2$

Plug in $(x-h)$ inside $\boxed{}$

$$(x-h+2)(x-h+2)$$
$$=$$
$$\boxed{x^2} \boxed{-xh} \boxed{+2x} \boxed{-xh} \boxed{+h^2} \boxed{-2h} \boxed{+2x} \boxed{-2h} \boxed{+4}$$
$$=$$
$$x^2 - xh - xh + 2x + 2x - 2h - 2h + h^2 + 4$$
$$\boxed{x^2 - 2xh + 4x + h^2 - 4h + 4}$$

$$-(x+2)^2$$
$$=$$
$$(x+2)(x+2)$$
$$=$$
$$x^2 + 2x + 2x + 4$$
$$=$$
$$\boxed{x^2 + 4x + 4}$$

$$\frac{f(x-h) - f(x)}{h} = \frac{x^2 - 2xh + 4x + h^2 - 4h + 4 - (x^2 + 4x + 4)}{h}$$
$$=$$
$$\frac{\boxed{x^2} - 2xh \boxed{+4x} + h^2 - 4h + 4 - x^2 - 4x \boxed{-4}}{h}$$
$$=$$
$$\frac{\cancel{x^2} \cancel{-x^2} \cancel{+4x} \cancel{-4x} \cancel{+4} \cancel{-4} - 2xh + h^2 - 4h}{h}$$
$$=$$
$$\frac{-2xh + h^2 - 4h}{h}$$
$$=$$
$$\frac{\cancel{h}(-2x + h - 4)}{\cancel{h}}$$
$$=$$
$$\boxed{\frac{f(x-h) - f(x)}{h} = -2x + h - 4}$$

78. $\dfrac{-f(x+h)+f(x)}{h}$, $f(x) = 3x + 10$

$$= \frac{-([\;]) + (3x+10)}{h}$$

$$= \frac{-(3(x+h)+10) + (3x+10)}{h}$$

$$= \frac{-(3x+3h+10) + 3x + 10}{h}$$

$$= \frac{\boxed{-3x} - 3h \boxed{-10} \boxed{+3x} \boxed{+10}}{h}$$

$$\frac{\cancel{-3x+3x} \cancel{-10+10} - 3h}{h}$$

$$= \frac{-3\cancel{h}}{\cancel{h}}$$

$$\boxed{\frac{-f(x+h)+f(x)}{h} = -3}$$

Set $-\dfrac{f(x+h)+f(x)}{h}$ as outer template

Set $3x+10$ as inner template

Plug $x+h$ inside inner template

79. $f(x) = \frac{4}{3}x^2 + \frac{2}{3}x + 1$, $\frac{3}{4}f(x)$

$$\frac{3}{4}\left(\frac{4}{3}x^2 + \frac{2}{3}x + 1\right)$$

$$=$$

$$\left(\frac{\cancel{3}}{\cancel{4}}\right)\frac{\cancel{4}}{\cancel{3}}x^2 + \left(\frac{3}{4}\right)\frac{2}{3}x + \left(\frac{3}{4}\right)1$$

$$=$$

$$x^2 + \frac{2}{4}x + \frac{3}{4}$$

$$=$$

$$\boxed{\frac{3}{4}f(x) = x^2 + \frac{1}{2}x + \frac{3}{4}}$$

80. $2f(x)$, $f(x) = \sqrt{x^2+5x+10}$

$$=$$

$$\boxed{2f(x) = 2\sqrt{x^2+5x+10}}$$

---

81. $h(2) = -2$ 83. $g(5) = 6$ 85. $f(-5) = 2$

87. $g(0) = 1$