Example 9 — Get Inflection Point

Determine if $f(x) = x^3 + x$ has any inflection points.

① Get Derivative for $f(x) = x^3 + x$

$$f'(x) = \frac{d}{dx}[x^3 + x]$$
$$= \frac{d}{dx}[x^3] + \frac{d}{dx}[x]$$
$$= 3x^{3-1} + 1x^{1-1}$$
$$= 3x^2 + 1$$
$$\boxed{f'(x) = 3x^2 + 1}$$

② Get $f''(x)$

$$f''(x) = \frac{d}{dx}[3x^2 + 1]$$
$$= 3 \cdot \frac{d}{dx}[x^2] + \frac{d}{dx}[1]$$
$$= 3 \cdot 2x^{2-1} + 0$$
$$= 3 \cdot 2x^1$$
$$\boxed{f''(x) = 6x}$$

③ Set $f''(x) = 0$

$$\frac{6x}{6} = \frac{0}{6}$$
$$\boxed{x = 0}$$

④ Check $f(x) = x^3 + x$ is continuous at 0.

$f(x) = x^3 + x$ is a polynomial

Polynomial functions are continuous everywhere on their domains

$x = 0$ is in the domain of $f(x) = x^3 + x$

$f(x) = x^3 + x$ is continuous at $x = 0$

⑤ Test Concavity

⑥ Determine Intervals

$f''(x) = 6x$ $\quad$ $x = 0$ is an endpoint

$-$ $\quad\quad$ $+$

0

(-1): $6(-1)$ $\quad$ -6

(1): $6(1)$ $\quad$ $+6$

⑦ Concavity Summary

From $(-\infty, 0)$ $f(x) = x^3 + x$ concaves downward.

From $(0, \infty)$ $f(x) = x^3 + x$ concaves upward

⑧ Wrap Up

$$\boxed{\text{Point of inflection for } f(x) = x^3 + x \text{ is at } x = 0}$$

Example 10

Determine if $f(x) = x^4$ has any points of inflection.

① Get Derivative of $f(x) = x^4$

$$f'(x) = \frac{d}{dx}[x^4]$$
$$=$$
$$4x^{4-1}$$
$$=$$
$$\boxed{f'(x) = 4x^3}$$

② Get Second Derivative $f''(x)$

$$f''(x) = \frac{d}{dx}[4x^3]$$
$$=$$
$$4 \cdot \frac{d}{dx}[x^3]$$
$$=$$
$$4 \cdot 3x^{3-1}$$
$$=$$
$$4 \cdot 3x^2$$
$$=$$
$$\boxed{f''(x) = 12x^2}$$

③ Set $f''(x) = 0$

$$\frac{12x^2}{12} = \frac{0}{12}$$
$$=$$
$$\sqrt{x^2} = \sqrt{0}$$
$$=$$
$$\boxed{x = 0}$$

④ Check $f(x) = x^4$ is continuous at 0.

$f(x) = x^4$ is a polynomial

Polynomial functions are continuous everywhere on their domains

$x = 0$ is in the domain of $f(x) = x^4$

$f(x) = x^4$ is continuous at $x = 0$

5. Test Convavity

6. Determine Intervals

$f''(x) = 12x^2$ $\quad x = 0$ is an endpoint

$+$ $\qquad$ $+$

0

$(-1): 12(-1)^2 = 12(1)$ $\quad +12$

$(1): 12(1)^2 = 12(1)$ $\quad +12$

7. Concavity Summary

No change in convavity because both signs are positive at intervals $(-\infty, 0)$ and $(0, \infty)$

8. Wrap Up

$f(x) = x^4$ have no inflection points