Composition of Functions

$(f \circ g)(x) = f[g(x)]$

$(g \circ f)(x) = g[f(x)]$

$f(x) = x^2 + x$

$g(x) = (4 - x)$

$f(3) = (3)^2 + (3)$

Find $(f \circ g)(x) = f[g(x)]$

$f[4-x]$

$(4-x)^2 + (4-x)$

$(4-x)(4-x) + (4-x)$

$16 - 4x - 4x + x^2 + 4 - x$

$x^2 - 8x - x + 16 + 4$

$f(g(x)) = \boxed{x^2 - 7x + 20}$

$(f \circ g)(x) = f[g(x)]$
$(g \circ f)(x) = g[f(x)]$

$f(x) = x^2 + x$
$g(x) = 4 - x$

Find $(g \circ f)(x) = g[f(x)]$

$g[x^2 + x] \to g(x)$

$(g \circ f)(x) = g[4 - (x^2 + x)] \to 4 - x^2 - x$

$-x^2 - x + 4$

$(g \circ f)(x) = g[-x^2 - x + 4]$

or

$\boxed{g(f(x)) = -x^2 - x + 4}$

Find $g(f(3)) \to g(f(x)) = -x^2 - x + 4$

$g(f(3)) = -(3)^2 - (3) + 4$

$-9 - 3 + 4$

$-12 + 4$

$\boxed{g(f(3)) = -8}$

You are doing two domain problems when finding the domain of a composition.

① Find the domain of the input / inside function.

② Find the domain of the new function after performing the composition.

Find the domain of the following composition of functions, $g(f(x))$.

Let $f(x) = \dfrac{1}{x}$ and let $g(x) = \dfrac{1}{(x+2)(x-3)}$

$\downarrow$

Find Domain of Input Function

$g(\underbrace{f(x)}_{\text{input}})$ Domain: $f(x) = \dfrac{1}{x} \rightarrow$ Exclude $\dfrac{1}{x} \ldots \boxed{x = 0}$

$\downarrow$

$g(f(x)) \rightarrow \overset{g(x)}{g\left(\dfrac{1}{x}\right)} \rightarrow \dfrac{1}{\left(\frac{1}{x}+2\right)\left(\frac{1}{x}-3\right)}$

$\downarrow$ what values would make 0 appear.

Cannot divide by 0

$$\frac{1}{x} + 2 = 0 \qquad\qquad \frac{1}{x} - 3 = 0$$

$$-2 \quad -2 \qquad\qquad +3 \quad +3$$

$$(x)\frac{1}{\cancel{x}} = -2(x) \qquad\qquad \frac{1}{x} = \frac{3}{1}$$

$$\frac{1 = -2x}{-2 \quad -2} \qquad\qquad \frac{1}{3} = \frac{\cancel{3}x}{\cancel{3}}$$

$$\boxed{-\frac{1}{2} = x} \qquad\qquad \boxed{\frac{1}{3} = x}$$

Exclude $-1/2$ Exclude $1/3$

Domain: All reals except $x = 0$, $x = -1/2$, $x = 1/3$

$-\infty$ ∞

$-1/2$ 0 $1/3$

$\downarrow$

Domain: $(-\infty, -1/2) \cup (-1/2, 0) \cup (0, 1/3) \cup (1/3, \infty)$

$\cup \rightarrow$ OR

$\cap \rightarrow$ AND

Given $f(x) = \frac{1}{x}$ and $g(x) = \sqrt{x+4}$,
find the domain of $g(f(x))$

$g(f(x))$; Domain $f(x) = \frac{1}{x} \rightarrow$ Exclude: $x = 0$

$\downarrow$

$g(x)$

$g\left(\frac{1}{x}\right) \rightarrow \sqrt{\frac{1}{x}+4}$

Domain? $\frac{1}{x} + 4 \geq 0$

$\downarrow$

$\frac{1}{x} + \frac{4 \cdot x}{1 \cdot x} \geq 0$

$\downarrow$

$\frac{1}{x} + \frac{4x}{x} \geq 0$

$\downarrow$

$\frac{1+4x}{x} \geq 0$

$\downarrow$

$1 + 4x = 0$

$-1 \quad -1$

$\frac{4x}{4} = \frac{-1}{4}$

$\boxed{x = -1/4}$ $\boxed{x = 0}$

Domain
$(-\infty, -1/4] \cup (0, \infty)$

$\downarrow$

Test $\frac{1+4x}{x} \geq 0$

$\frac{1 + \frac{4}{1}\left(-\frac{1}{4}\right)}{\left(-\frac{1}{4}\right)} \geq 0$

$\frac{0}{-\frac{1}{4}}$

$\downarrow$

$\boxed{0 \geq 0}$

TRUE

$\frac{1 + 4(0)}{0} \geq 0$

$\downarrow$

$\boxed{\text{Undef}}$

Test $-1/10$

$\frac{1 + 4\left(-\frac{1}{10}\right)}{\left(-\frac{1}{10}\right)} \geq 0$

$\downarrow$

$\boxed{-6 \geq 0}$

FALSE

Test -10

$\frac{1+4(-10)}{(-10)} \geq 0 \rightarrow \frac{1-40}{-10} \geq 0$

$\downarrow$

$\frac{-39}{-10} \geq 0$

$\boxed{\frac{39}{10} \geq 0}$ TRUE

$\frac{1+4(1)}{1} \geq 0$

$\downarrow$

$\frac{1+4}{1}$

$\boxed{5 \geq 0}$ TRUE

Given $f(x) = \sqrt{x-8}$ and $g(x) = x^2$

Find the domain of $g(f(x))$.

$$\overset{g(x)}{}$$

$$g(f(x)) = g(\sqrt{x-8}) =$$

$$\downarrow$$

$$(\sqrt{x-8})^2$$

$$\downarrow$$

$$g(f(x)) = x - 8$$

MISTAKE. Domain All real numbers.

Correct: $f(x) = \sqrt{x-8}$

$$x - 8 \geq 0$$

$$+8 \quad +8$$

Domain $\rightarrow$ $\boxed{x = 8}$

$x \geq 8$, $[8, \infty)$