Example 1

Derive an iterative formula for Newton's Method.

① Set x_0 to an intial guess solution for $f(x) = 0$.

② Find equation of tangent line to the curve $y = f(x)$ at $x = x_0$.

Assume $f'(x_0)$ exists

$$m = f'(x_0)$$

$y = mx + b$, where $m = f'(x_0)$

Tangent lines passes through point $(x_0, f(x_0))$

$y = f'(x_0) x + b$, where $m = f'(x_0)$

$f(x_0) = f'(x_0) x_0 + b$, where $f(x_0) = y$ and $x_0 = x$

③ Solve for b

$$f(x_0) = f'(x_0) x_0 + b$$
$$\underline{-f'(x_0) x_0 \qquad + b}$$

$$f(x_0) - f'(x_0) x_0 = b$$

or

$$b = f(x_0) - f'(x_0) x_0$$

④ Equation of Tangent line

$$y = f'(x_0) x + f(x_0) - f'(x_0) x_0$$

⑤ Find where tangent line crosses the x-axis.

$$y = f'(x_0)x + f(x_0) - f'(x_0)x_0$$

$$f'(x_0)x + f(x_0) - f'(x_0)x_0 = 0$$

Solve for x

$$f'(x_0)x + f(x_0) - f'(x_0)x_0 = 0$$
$$\underline{\qquad - f(x_0) \qquad\qquad - f(x_0)}$$

$$f'(x_0)x - f'(x_0)x_0 = -f(x_0)$$
$$\underline{\quad + f'(x_0)x_0 \qquad + f'(x_0)x_0}$$

$$\frac{f'(x_0)x}{f'(x_0)} = -\frac{f(x_0)}{f'(x_0)} + \frac{f'(x_0)x_0}{f'(x_0)}$$

$$\text{||}$$

$$x = -\frac{f(x_0)}{f'(x_0)} + 1 \cdot x_0$$

$$\text{||}$$

$$\boxed{x = x_0 - \frac{f(x_0)}{f'(x_0}}$$

Iterative Formula for
newton's method