MATH E-3: Lecture 5

Quantitative Reasoning: Practical Math

MATH E -3

February 23, 2016

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Homework

- Assignment 4 is due 2/27
- Solutions to Assignment 3 have been posted
- Assignment 3 grades will be available 2/27

Quiz #1
(LECTURES 1-5)

Date: Tuesday, March 8

Time: 7:40 pm through March 9, 7:40 pm (Eastern time)

Location: Online, MATH E-3 Canvas course site

(no proctor needed)

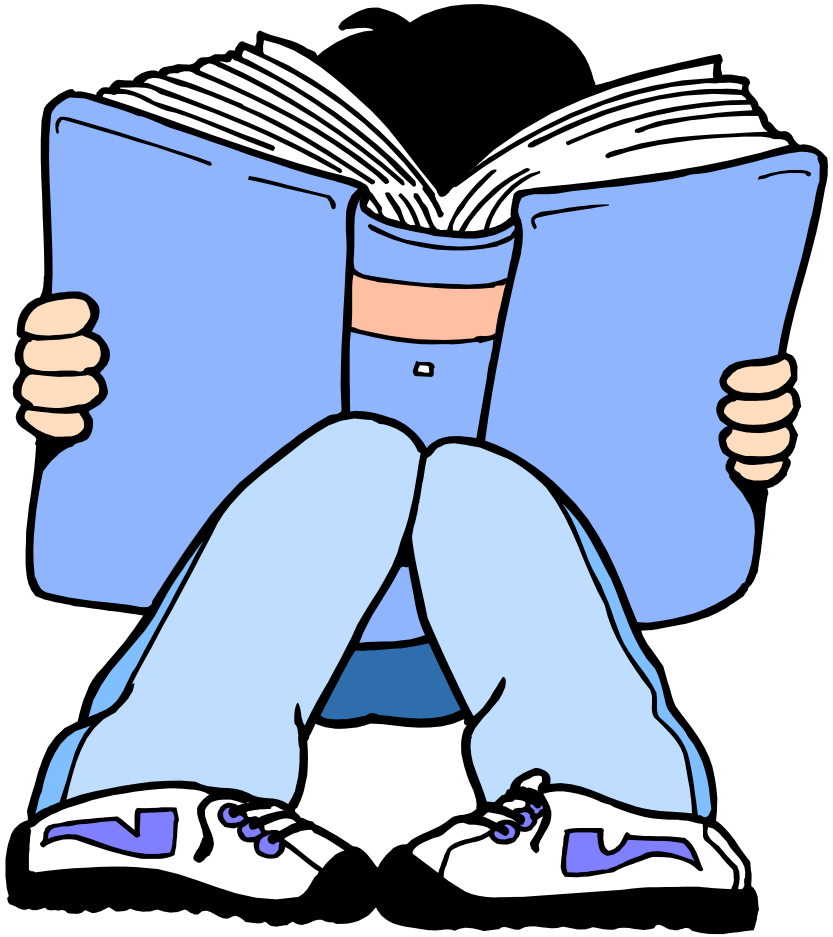
No class meeting

Quiz #1 is open book. You will have 75 minutes to complete the test

Quiz #1 Review

- **Review Section**: Friday, 3/4, beginning at 5:30 pm (ET), live, online via Canvas Conference; recorded session and slides will be posted shortly thereafter to the Quiz #1 module on the course site home page.

- **Practice Test** will be posted in Quiz #1 module




Math in the news . . .

A recent poll

This first is a Gravis poll that was released last Thursday. Trump is up significantly in this poll:

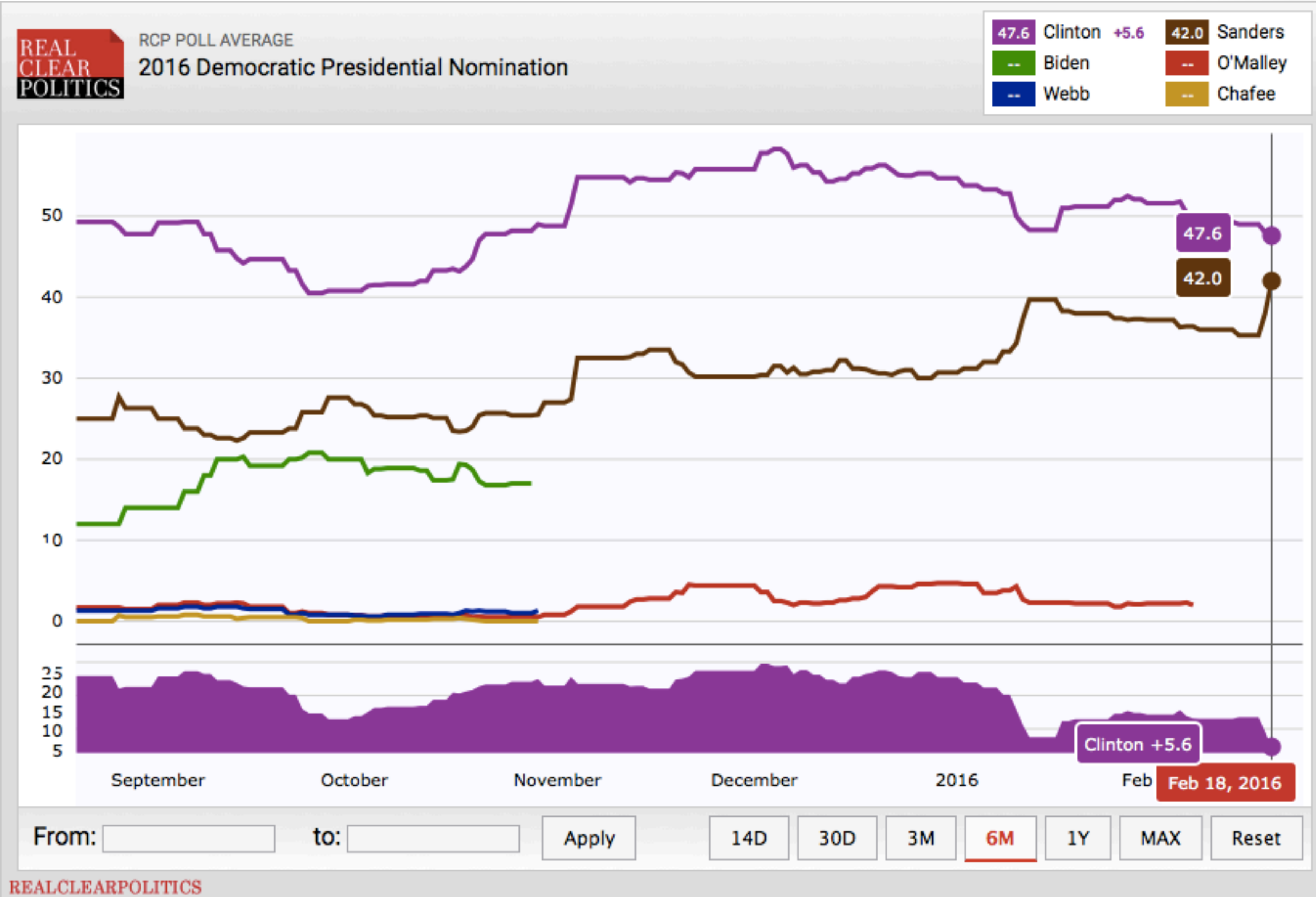
> “
>
> | | |
> |---|---|
> | Trump | 39 |
> | Cruz | 23 |
> | Rubio | 19 |
> | Kasich | 9 |
> | Carson | 5 |
> | Bush | 5 |

Here’s the lowdown:

> ‘ The poll was conducted from February 14th to the 15th and includes 687 Republican Caucus participants… The poll has a margin of error of ± 4% for Republican caucuses. The polls were conducted using automated telephone calls (IVR technology) and weighted separately for each population in the question presented.

http://therightscoop.com/here-are-the-two-latest-polls-from-nevada/

And another . . .



http://www.realclearpolitics.com/epolls/2016/president/us/2016_democratic_presidential_nomination-3824.html

But if you don’t like politics

Gas prices in the UK
(September 2014)

http://www.greencarreports.com/news/1055944_u-s-drivers-shut-up-stop-whining-you-dont-pay-8-gallon

Using the “conversion factors”

	A	B
1		
2		
3	1.309	pounds per litre
4		
5	1.62	pounds to dollars
6		
7	3.7854125	litres to gallons
8		

A few calculations, being careful with the units!

1.309 pounds/litre * 1.62 dollars/pound = 2.12058 dollars per litre

So should we be complaining…?

1.309 pounds/litre * 1.62 dollars/pound = 2.12058 dollars per litre.

2.12058 dollars per litre * 3.7854125 litres per gallon = 8.027270112 dollars per gallon.

Rounding, we get $8.03 per gallon!!

(a couple of days ago I bought gas for $2.09; this was October 2015)

Probability Type I

- What is probability? Synonyms . . .

Important terms:

- Event:
- Outcome:
- Equally Likely:
- Dichotomous Variable:

Applications of Probability type I

- Tossing coins
- Rolling dice
- Winning the lottery
- Picking a playing card from a deck of cards
- Others . . .

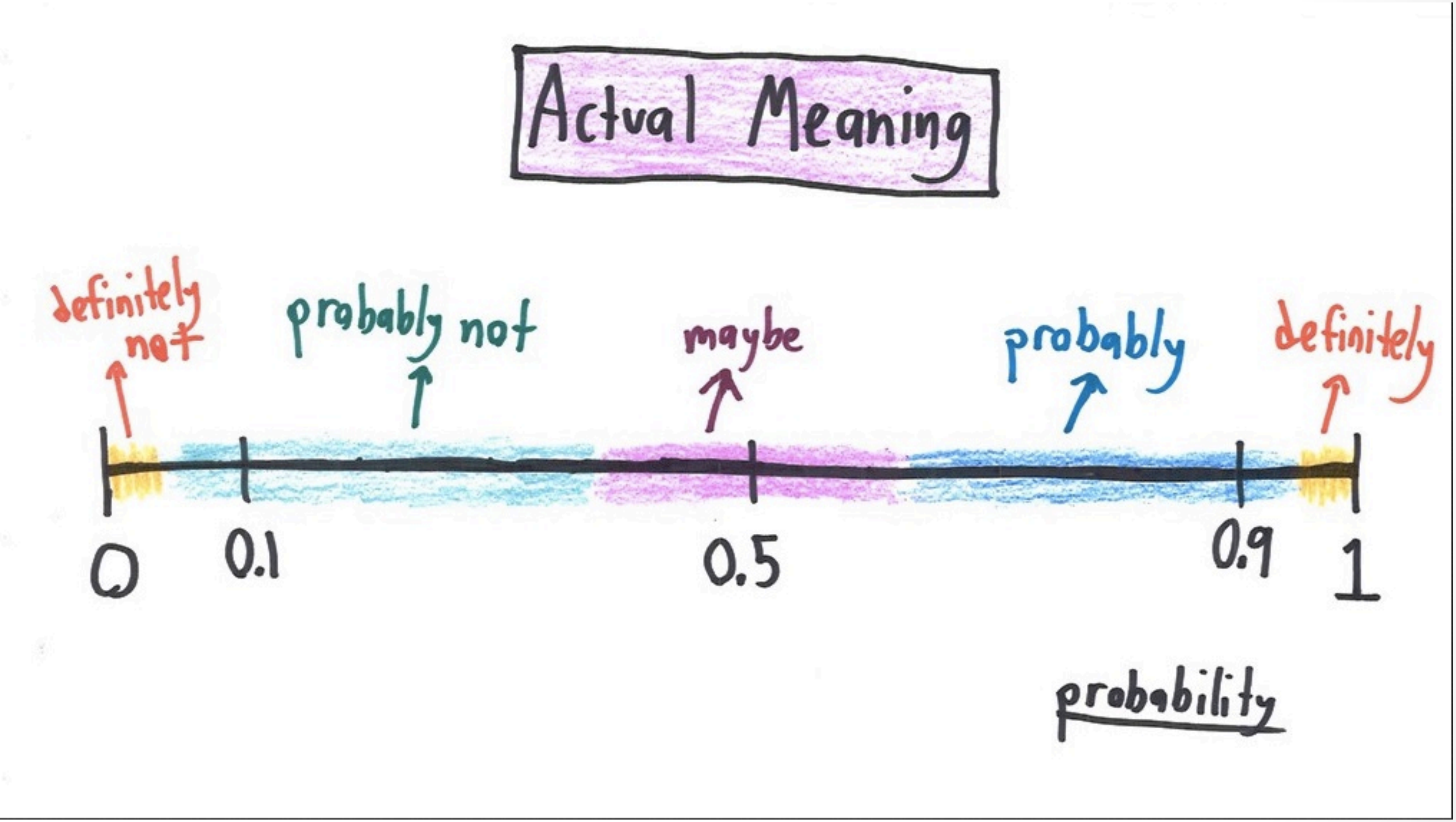
Things NOT applicable to Probability Type I

- Weather
- Political races
- Your future health
- Your score on a math test
- How do these differ from the previous ones?

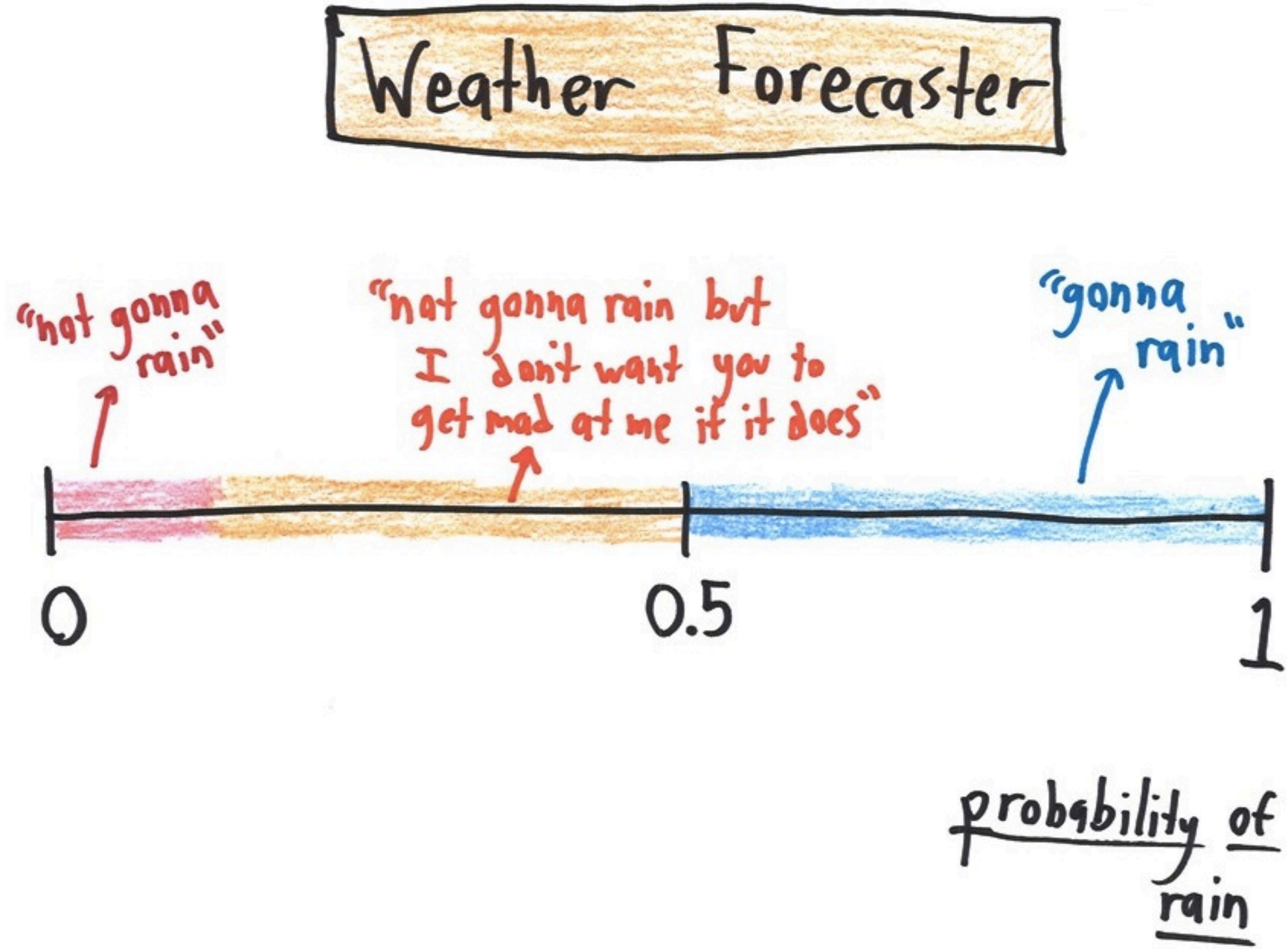
Probability in words and numbers

Words	decimal	percent
certain	1	100%
almost certain	0.95	95%
very likely	0.85	85%
likely	0.65	65%
toss-up	0.5	50%
unlikely	0.35	35%
very unlikely	0.15	15%
virtually impossible	0.05	5%
impossible	0	0%

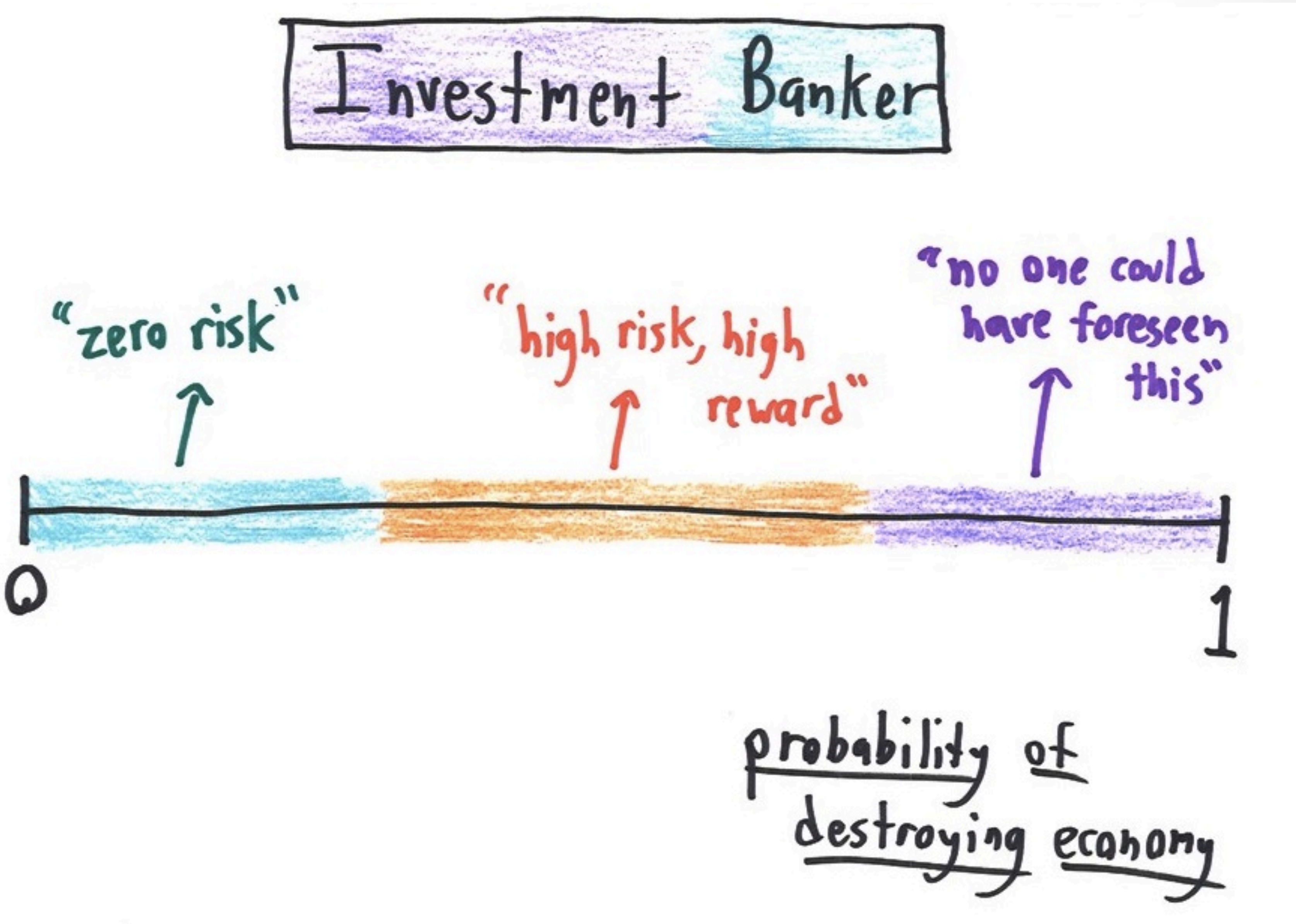
Concepts of probability . . .



http://mathwithbaddrawings.com/2015/09/23/what-does-probability-mean-in-your-profession/


Weather Forecaster
"not gonna rain"
"not gonna rain but I don't want you to get mad at me if it does"
"gonna rain"
0
0.5
1
probability of rain


Political Journalist
"the threat is real"
"far from certain"
"too close to call"
0
0.5
1
"a distinct possibility"
"still in doubt"
probability of election outcome


Investment Banker
"zero risk"
"high risk, high reward"
"no one could have foreseen this"
0
1
probability of destroying economy

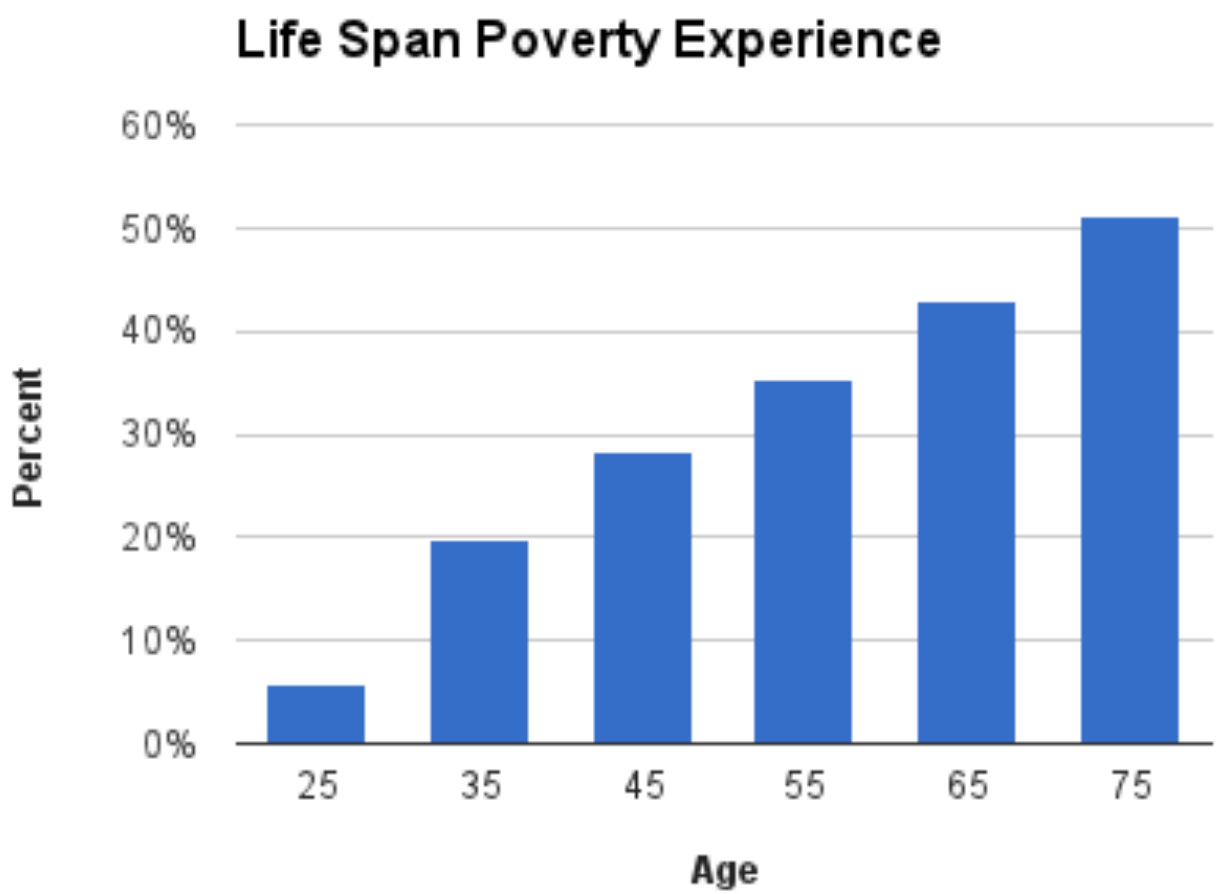

Local News Anchor
"justified panic"
"crisis situation"
"buy firearms and canned goods"
"your child is statistically a goner"
0
0.5
1
probability of harm to your child

New Climate change report

- **(CNN)** -- Human activity has caused at least half of climate change in the last half-century, hundreds of scientists say. They are 95% certain of this, the surest they've ever been, says a United Nations report published September 27 of last year.

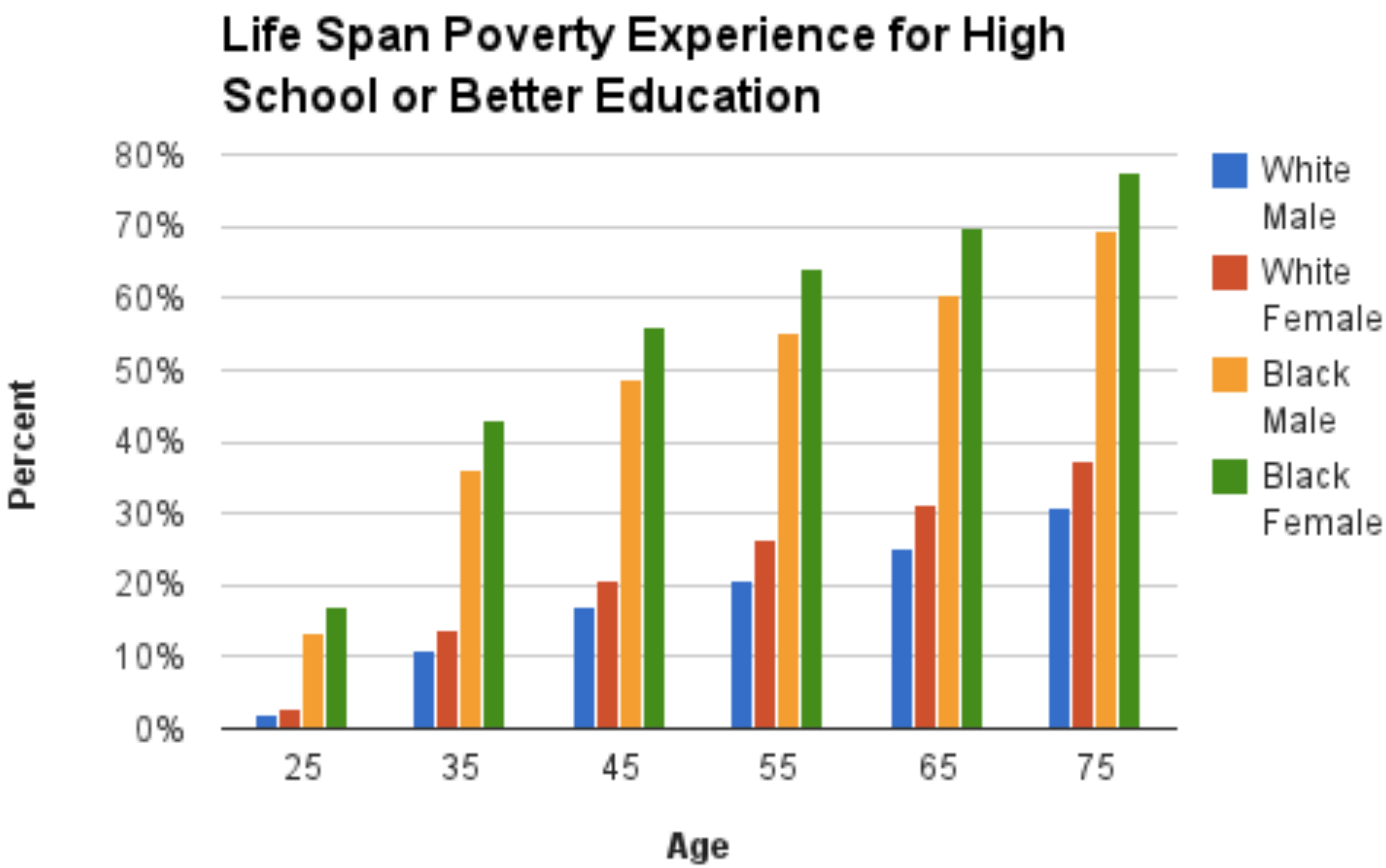
- While 90% constitutes a "very likely" degree of certainty, Friday's report stating scientists are now 95% sure indicates an "extremely likely" degree of certainty, which is considered the gold standard when discussing probability.

The probability of poverty

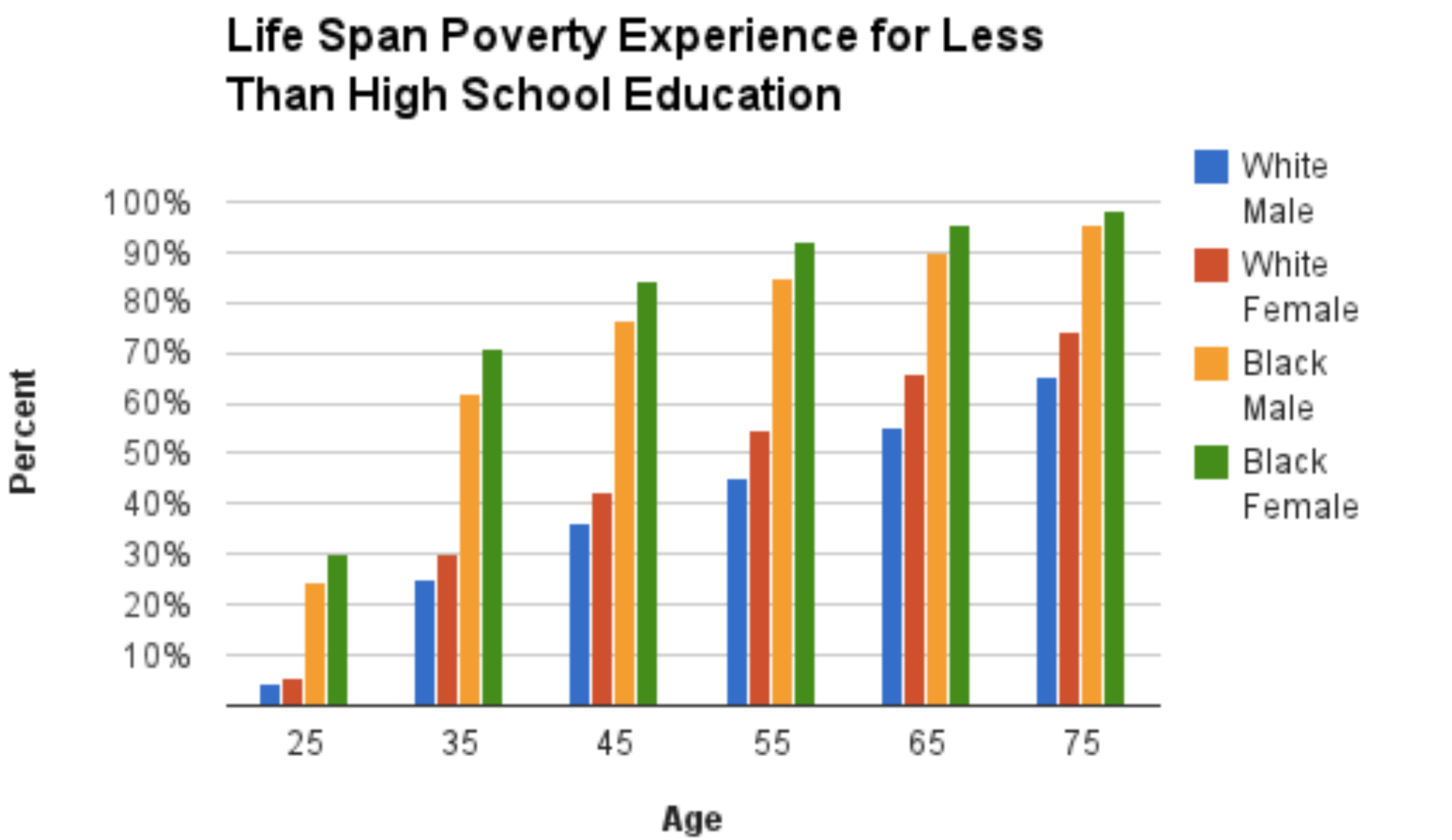


http://prospect.org/article/high-probability-being-poor

Looking more closely . . .



One more graph . . .



A Definition of Probability

In general, the probability of getting an outcome satisfying a particular condition can be computed as:

$$\textbf{Probability} = \frac{\text{number of outcomes satisfying the condition}}{\text{total number of possible outcomes}}$$

provided each outcome is <u>equally likely</u> to occur!

MATH E-3 and Gambling . . .

(this is not an endorsement)

- Tossing coins (only “fair” coins)
- Rolling dice
- Playing cards (maybe mentioned in class, but not on the assignment or test)
- Playing the lottery . . .

Tossing Coins . . .

- What are the possible outcomes, when you toss one fair coin?

Tossing Coins . . .

- What are the possible outcomes, when you toss one fair coin?

- H or T

Tossing Coins . . .

- What are the possible outcomes, when you toss one fair coin?
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- How about two fair coins?

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Tossing Coins . . .

- What are the possible outcomes, when you toss one fair coin?
- H or T
- How about two fair coins?
- HH HT TH TT
- Three fair coins?

Tossing Coins . . .

- What are the possible outcomes, when you toss one fair coin?
- H or T
- How about two fair coins?
- HH HT TH TT
- Three fair coins?
- HHH, HHT, HTH, THH, TTH, THT, HTT, TTT

Possibilities with up to three coins . . .

One Coin	Two Coins	Three Coins
H	H H	H H H
T	H T	H H T
	T H	H T H
	T T	H T T
		T H H
		T H T
		T T H
		T T T

Do you see a pattern?

# of Coins	# of Outcomes
1	2
2	4
3	8
4	?
5	?

Do you see a pattern?

# of Coins	# of Outcomes
1	2
2	4
3	8
4	16
5	32

Do you see a pattern?

# of Coins	# of Outcomes
1	2
2	4
3	8
4	16
5	32
…	…
n	?

Do you see a pattern?

- **# of Coins** **# of Outcomes**
  - 1 2
  - 2 4
  - 3 8
  - 4 16
  - 5 32
  - … …
  - n 2^n

A useful math diagram

(and it's quite esthetically pleasing too…)

PASCAL'S TRIANGLE

1 1

1 2 1

1 3 3 1

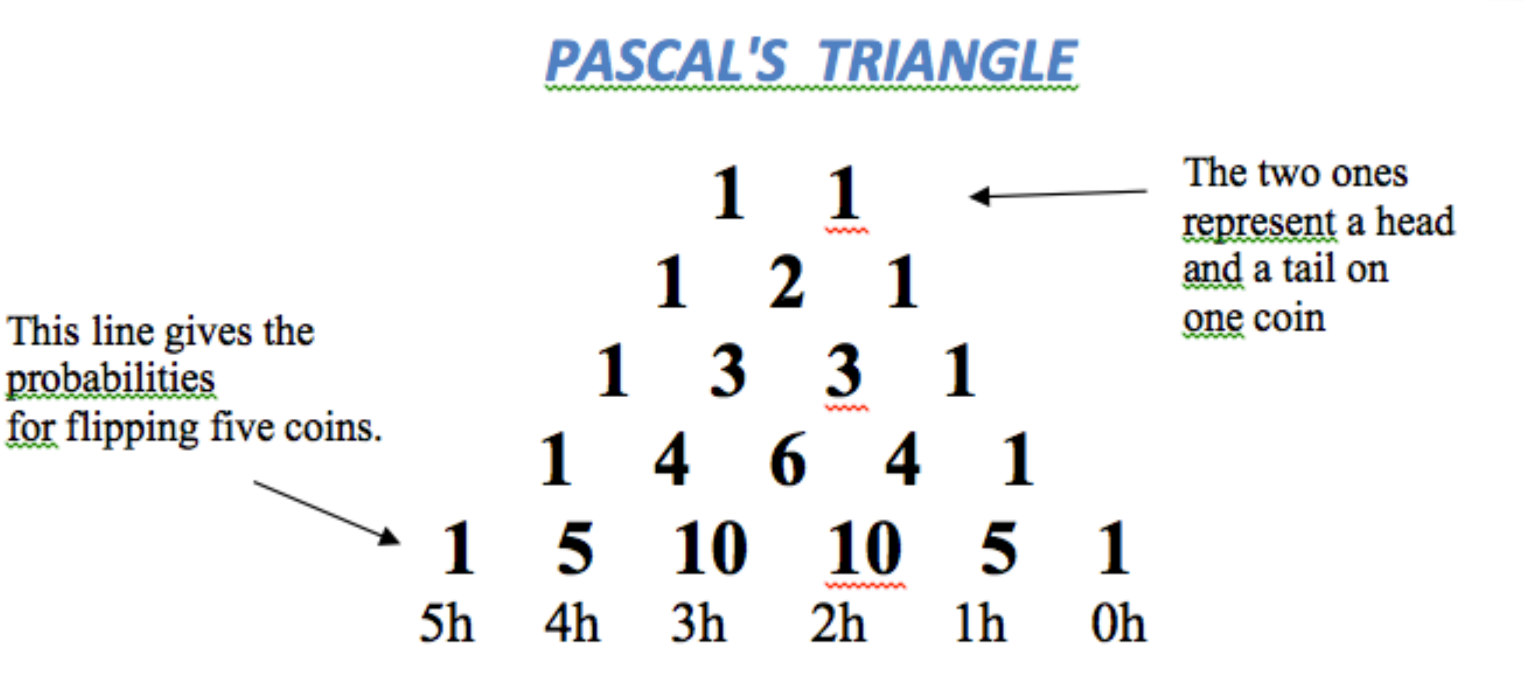
1 4 6 4 1

1 5 10 10 5 1

Using Pascal's Triangle

- If you toss 5 fair coins, what is the probability that exactly 3 of them will be Heads?
- We need to find out how many possible outcomes there are when you toss 5 coins.
- Actually we already found that it was 32, but let's check that again . . .
- Then how many of those outcomes are exactly 3 Heads.

Using Pascal's Triangle



Using Pascal's Triangle

This line gives the probabilities for flipping five coins.

				1		**1**				
			1		**2**		**1**			
		1		**3**		**3**		**1**		
	1		**4**		**6**		**4**		**1**	
1		**5**		**10**		**10**		**5**		**1**
5h		4h		3h		2h		1h		0h

There are 10 ways of getting 3 heads when you toss 5 coins.

So P(3H, 5 coins) = ??

P(3H, 5 coins) =

$$\frac{\text{Number of desired outcomes}}{\text{Number of possible outcomes}}$$

$$\frac{\text{Outcomes with 3 Heads}}{\text{All possible outcomes}}$$

$$\frac{10}{32} = 0.31 \text{ (rounded to 2 dec. pl.)}$$

Rolling Dice . . .

- What are the possible outcomes when you roll one "die"?

Rolling Dice . . .

- What are the possible outcomes when you roll one "die"?

- 1, 2, 3, 4, 5, 6

Rolling Dice . . .

- What are the possible outcomes when you roll one “die”?
- 1, 2, 3, 4, 5, 6
- How about two dice?

Rolling Dice . . .

- What are the possible outcomes when you roll one "die"?
- 1, 2, 3, 4, 5, 6
- How about two dice?
- 1-1, 1-2, 1-3, 1-4, 1-5, 1-6,
- 2-1, 2-2, 2-3, 2-4, 2-5, 2-6
- Etc. . . (how many total outcomes?)

A “Dice Table”

	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

Using the Dice Table

- What is the probability of getting a "9" when you roll two dice?

	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	**9**
4	5	6	7	8	**9**	10
5	6	7	8	**9**	10	11
6	7	8	**9**	10	11	12

Probability of getting a 9 with 2 dice . . .

- There are 36 total possible outcomes, of which 4 are 9's (the "desired" outcomes).
- So the probability is 4 out of 36, or 4/36
- 4/36 = 1/9, which = 0.111111 . . .
- Round to 2 decimal places, so you get **0.11** (or 11%)

Playing the Lottery

To play Megabucks (at least as I remember it, when they used to mail me free tickets . . .), you had to choose six different numbers between 1 and 48.

What is the probability of winning megabucks if you buy one ticket?

How about . . . since there are <u>two</u> possible outcomes, winning and losing, and you will be happy with <u>one</u> of those, your chance of winning is

$$\frac{1}{2} \qquad \text{or } 50\%!!!$$

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Too good to be true . . .

- Needless to say, if the chance of winning at Megabucks were actually 1/2 or 50%, the State Lottery Commission would soon be out of business!
- But what is wrong with our reasoning here?
- It’s the little word “provided”
- Remember this slide from earlier . . .

A Definition of Probability

In general, the probability of getting an outcome satisfying a particular condition can be computed as:

$$\textbf{Probability} = \frac{\text{number of outcomes satisfying the condition}}{\text{total number of possible outcomes}}$$

provided each outcome is equally likely to occur!

Too good to be true . . .

- So if in fact there are only two possible outcomes when playing the lottery, winning and losing, are they equally likely to occur?
- Not *exactly* . . . (actually not even close)
- What we need to do is to find outcomes that *are* equally likely. Such as: possible combinations of numbers – this will be some rather large number, on the denominator; and on the numerator: winning combinations of numbers. This will be a rather small number, like 1, if you buy one ticket.

Too good to be true . . .

- So the actual probability of me winning Megabucks with one ticket will be:

$$\frac{1}{\text{some very large number}}$$

- Which will equal:

Some very small number, with a decimal point and lots of zeros . . .

Winning Megabucks

- So we need to calculate how many possible combinations of 6 different numbers between 1 and 48 there are.

- 2 things to consider:

- a) no repeated numbers

- b) the order doesn't matter

Winning Megabucks . . .

- Imagine you are counting the ways you can choose your six numbers. You start with six empty boxes. In the first box you write how many ways there are to choose your first number:

Winning Megabucks . . .

- Imagine you are counting the ways you can choose your six numbers. You start with six empty boxes. In the first box you write how many ways there are to choose your first number:

$\boxed{48}\quad\boxed{}\quad\boxed{}\quad\boxed{}\quad\boxed{}\quad\boxed{}$

Winning Megabucks . . .

- How many ways to choose your second number:

48	47				

Winning Megabucks . . .

- How many ways to choose your third number:

$$\boxed{48}\qquad\boxed{47}\qquad\boxed{46}\qquad\boxed{}\qquad\boxed{}\qquad\boxed{}$$

Winning Megabucks . . .

- And so on . . . how many ways to choose all six numbers:

48	47	46	45	44	43

Winning Megabucks . . .

- And finally . . . What operation comes in between each of *these* numbers?

48	47	46	45	44	43

Winning Megabucks . . .

- And finally . . . What operation comes in between each of *these* numbers?

$\boxed{48}$ x $\boxed{47}$ x $\boxed{46}$ x $\boxed{45}$ x $\boxed{44}$ x $\boxed{43}$

Find this number on your calculator!

Winning Megabucks . . .

- And finally . . . What operation comes in between each of *these* numbers?

$\boxed{48} \times \boxed{47} \times \boxed{46} \times \boxed{45} \times \boxed{44} \times \boxed{43}$

Find this number on your calculator!

- 8,835,488,640

Winning Megabucks . . .

- But wait . . . remember that the order of your numbers doesn't matter;
- i.e. choosing e.g. 3, 15, 8, 28, 31, 22 is no different from choosing 28, 15, 3, 22, 31, 8.
- Which means our number 8,835,488,640 is too big as it is, as it includes combinations of the same numbers in different orders.
- So . . . we need to reduce the total number somehow.

Winning Megabucks . . .

- So the question is: when you choose your six numbers, how many different ways could you arrange them?

- And then only count *one* of those ways.

- In other words, how many ways are there of arranging 6 numbers?

Back to the boxes . . .

- Remember an earlier slide, where we imagined how many ways there are of *choosing* 6 numbers? Now it is how many ways of *arranging* 6 numbers, and the process is the same:

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Back to the boxes . . .

- And finally:

$\boxed{6} \quad \boxed{5} \quad \boxed{4} \quad \boxed{3} \quad \boxed{2} \quad \boxed{1}$

- Multiply:

$\boxed{6} \times \boxed{5} \times \boxed{4} \times \boxed{3} \times \boxed{2} \times \boxed{1}$

- Calculate to get:

Back to the boxes . . .

- And finally:

$\boxed{6} \quad \boxed{5} \quad \boxed{4} \quad \boxed{3} \quad \boxed{2} \quad \boxed{1}$

- Multiply:

$\boxed{6} \times \boxed{5} \times \boxed{4} \times \boxed{3} \times \boxed{2} \times \boxed{1}$

- Calculate to get: 720 ways to arrange 6 numbers

A note on factorials

- 6 x 5 x 4 x 3 x 2 x 1 = 720.
- This can be written as “6!” and is called “six factorial.” Your calculator should have a “!” button on it (or else buried somewhere inside it . . .).

Back to Megabucks . . .

- So we have the total number of possible combinations of six different numbers between 1 and 48, with no repeated numbers, and where the order doesn't matter, as:

$$\frac{\boxed{48} \times \boxed{47} \times \boxed{46} \times \boxed{45} \times \boxed{44} \times \boxed{43}}{\boxed{6} \times \boxed{5} \times \boxed{4} \times \boxed{3} \times \boxed{2} \times \boxed{1}}$$

Back to Megabucks . . .

- Two ways of calculating this:

$$\frac{\boxed{48} \times \boxed{47} \times \boxed{46} \times \boxed{45} \times \boxed{44} \times \boxed{43}}{\boxed{6} \times \boxed{5} \times \boxed{4} \times \boxed{3} \times \boxed{2} \times \boxed{1}}$$

Either using our previous calculations:

$$\frac{8,835,488,640}{720} = 12,271,512$$

Back to Megabucks . . .

- Or by employing “legal” canceling:

$$\frac{\boxed{\cancel{48}} \times \boxed{47} \times \boxed{46} \times \boxed{45} \times \boxed{44} \times \boxed{43}}{\boxed{\cancel{6}} \times \boxed{5} \times \boxed{\cancel{4}} \times \boxed{3} \times \boxed{\cancel{2}} \times \boxed{1}}$$

More “legal” canceling…

$$\frac{\boxed{47} \times \boxed{46} \times \overset{3}{\boxed{\cancel{45}}} \times \boxed{44} \times \boxed{43}}{\boxed{\cancel{5}} \times \boxed{\cancel{3}} \times \boxed{1}}$$

More "legal" canceling…

- Leaving:
- 47 x 46 x 3 x 44 x 43
- Which =   12,271,512 as with the other method.

Why are we here?

- So . . . where does this get us?
- Remember, we’re trying to calculate the probability of us winning Megabucks with one ticket.
- We have found that there are 12,271,512 equally likely ways of choosing six different numbers between 1 and 48, where the order does not matter.
- So where does this number go?

Why are we here?

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- Remember, we're trying to calculate the probability of us winning Megabucks with one ticket.
- We have found that there are 12,271,512 equally likely ways of choosing six different numbers between 1 and 48, where the order does not matter.
- So where does this number go? On the *denominator*.
- And what goes on the numerator?

Why are we here?

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- We have found that there are 12,271,512 equally likely ways of choosing six different numbers between 1 and 48, where the order does not matter.
- So where does this number go? On the *denominator*.
- And what goes on the numerator? The number 1.

And the results are:

- So . . . the probability of winning Megabucks (under the stated conditions) with one ticket is:

$$\frac{1}{12{,}271{,}512}$$

Calculate this number:

Final calculation . . .

$$\frac{1}{12{,}271{,}512}$$

- Is equal to: 0.0000000815 (depending on rounding; also your calculator may present this number differently.)

Note …

- How about if you bought 1000 tickets?

$$\frac{1000}{12,271,512}$$

- Probability of winning =

Note …

- How about if you bought 1000 tickets?

$$\frac{1000}{12{,}271{,}512}$$

- Probability of winning = 0.0000815 or 0.00815%

- How about if you bought 1 million tickets??

Note . . .

- If you bought 1,000,000 tickets . . .

$$\frac{1{,}000{,}000}{12{,}271{,}512}$$

- Probability of winning = 0.0815 or 8.15%

Moral of the story??

Note . . .

- If you bought 1,000,000 tickets . . .

$$\frac{1{,}000{,}000}{12{,}271{,}512}$$

- Probability of winning = 0.0815 or 8.15%

Moral of the story??

Psychology trumps statistics for most people

Probability in real life

- Very few things are either 100% certain to happen or 0% impossible. In fact we tend to make jokes about what is certain (death and taxes) and impossible (a snowball in hell, when pigs fly, etc.)

Probability in real life

Very few things are either 100% certain to happen or 0% impossible. In fact we tend to make jokes about what is certain (death and taxes) and impossible (a snowball in hell, when pigs fly, etc.)

And some folks tend to “mis-use” probability when they assure us that “I am certain that this car will last you for at least 2 years”; or “I guarantee you that Clinton (or perhaps Trump, or perhaps Sanders . . .) will win the Presidential election in November.” (This one still far from certain at this point . . .)

Humans may crave certainty, but it is rather elusive!

The lottery again . . .

How likely is it for:

Someone to win the lottery? (i.e. there to be a winner?)

Someone to win the lottery twice?

A specific person to win the lottery?

A specific person to win the lottery twice?