1. $\lim_{x\to\infty} \frac{x+1}{x-1}$

$$\lim_{x\to\infty} \frac{x+1}{x-1} \cdot \frac{\frac{1}{x}}{\frac{1}{x}}$$

$$\lim_{x\to\infty} \frac{(x+1)\frac{1}{x}}{(x-1)\frac{1}{x}}$$

$$\lim_{x\to\infty} \frac{\frac{1}{x}(x) + \frac{1}{x}(1)}{\frac{1}{x}(x) - \frac{1}{x}(1)}$$

$$\lim_{x\to\infty} \frac{1 + \frac{1}{x}}{1 - \frac{1}{x}}$$

$$\frac{\lim_{x\to\infty}\left(1 + \frac{1}{x}\right)}{\lim_{x\to\infty}\left(1 - \frac{1}{x}\right)}$$

$$\frac{\lim_{x\to\infty} 1 + \lim_{x\to\infty} \frac{1}{x}}{\lim_{x\to\infty} 1 - \lim_{x\to\infty} \frac{1}{x}} \rightarrow \lim_{x\to\infty} \frac{1}{x^r} = 0$$

$$\frac{1 + 0}{1 - 0}$$

$$\frac{1}{1}$$

$$1 \rightarrow \lim_{x\to\infty} \frac{x+1}{x-1} = 1$$

2. $\lim_{x\to\infty} \frac{4x^2+1}{2x^2+x-1}$

$$\lim_{x\to\infty} \frac{4x^2+1}{2x^2+x-1} \cdot \frac{\frac{1}{x^2}}{\frac{1}{x^2}}$$

$$\lim_{x\to\infty} \frac{\left(\frac{1}{x^2}\right)4x^2 + \left(\frac{1}{x^2}\right)1}{\left(\frac{1}{x^2}\right)2x^2 + \left(\frac{1}{x^2}\right)x - \left(\frac{1}{x^2}\right)1}$$

$$\lim_{x\to\infty} \frac{\frac{4x^2}{x^2} + \frac{1}{x^2}}{\frac{2x^2}{x^2} + \frac{x}{x^2} - \frac{1}{x^2}}$$

$\frac{x^1}{x^2} = x^{1-2} = x^{-1} = \frac{1}{x}$

$$\lim_{x\to\infty} \frac{4 + \frac{1}{x^2}}{2 + \frac{1}{x} - \frac{1}{x^2}}$$

$$\lim_{x\to\infty} \left(4 + \frac{1}{x^2}\right)$$
$$\lim_{x\to\infty} \left(2 + \frac{1}{x} - \frac{1}{x^2}\right)$$

$$\lim_{x\to\infty} 4 + \lim_{x\to\infty} \frac{1}{x^2} \to \lim_{x\to\infty} \frac{1}{x^2} = 0$$

$$\lim_{x\to\infty} 2 + \lim_{x\to\infty} \frac{1}{x^1} - \lim_{x\to\infty} \frac{1}{x^2}$$

$$\frac{4 + 0}{2 + 0 - 0}$$

$$\frac{4}{2}$$

$$2$$

$$\lim_{x\to\infty} \frac{4x^2+1}{2x^2+x-1} = 0$$

3. $\lim_{x\to\infty} \frac{\sqrt{2x^2+1}}{x-1}$

$=$

$\lim_{x\to\infty} \frac{\sqrt{2x^2+1}}{x-1} \cdot \frac{\frac{1}{x}}{\frac{1}{x}}$

$=$

$\lim_{x\to\infty} \frac{\left(\sqrt{2x^2+1}\right)\frac{1}{x}}{(x-1)\frac{1}{x}}$

$=$

$\lim_{x\to\infty} \frac{\sqrt{2}+\frac{1}{x}}{1-\frac{1}{x}}$

$=$

$\frac{\lim_{x\to\infty}\sqrt{2} + \lim_{x\to\infty}\frac{1}{x}}{\lim_{x\to\infty} 1 - \lim_{x\to\infty}\frac{1}{x}}$ $\to \lim_{x\to\infty}\frac{1}{x^r} = 0$

$=$

$\frac{\sqrt{2}+0}{1-0}$

$=$

$\sqrt{2}$

$=$

$\sqrt{2}$

$\frac{1}{x}(x-1)$

$=$

$\frac{1}{x}(x) - \frac{1}{x}(1)$

$=$

$\frac{x}{x} - \frac{1}{x}$

$=$

$1 - \frac{1}{x}$

$\frac{1}{x}\cdot\sqrt{2x^2+1}$

$=$

$\frac{1}{x}\cdot\frac{\sqrt{2x^2}}{1} + \frac{1}{x}\cdot\frac{\sqrt{1}}{1}$

$=$

$\frac{\sqrt{2x^2}}{x} + \frac{\sqrt{1}}{x}$

$=$

$\frac{\sqrt{2}\cdot\sqrt{x^2}}{x} + \frac{1}{x}$

$=$

$\frac{\sqrt{2}\cdot x}{x} + \frac{1}{x}$

$=$

$\sqrt{2} + \frac{1}{x}$

$\lim_{x\to\infty}\frac{\sqrt{2x^2+1}}{x-1} = \sqrt{2}$

4. $\lim_{x \to -\infty} \frac{\sqrt{2x^2+1}}{x}$

$=$

$\lim_{x \to -\infty} \frac{\sqrt{2x^2+1}}{x} \cdot \frac{\frac{1}{x}}{\frac{1}{x}}$

$=$

$\lim_{x \to -\infty} \frac{\left(\sqrt{2x^2+1}\right)\frac{1}{x}}{x \cdot \frac{1}{x}}$

$=$

$\lim_{x \to -\infty} \frac{-\sqrt{2} - \frac{1}{x}}{1}$

$=$

$\lim_{x \to -\infty} -\sqrt{2} - \lim_{x \to -\infty} -\left(\frac{1}{x}\right) \to \lim_{x \to -\infty} \frac{1}{x^r} = 0$

$\lim_{x \to -\infty} 1$

$=$

$-\sqrt{2} - 0$

$=$

$-\sqrt{2}$

$\left(\frac{-1}{x}\right) \cdot \sqrt{2x^2} \cdot \left(\frac{-1}{x}\right) \cdot \sqrt{1}$

$=$

$-\frac{\sqrt{2x^2}}{x} + -\frac{\sqrt{1}}{x}$

$=$

$-\frac{\sqrt{2}\,x}{x} - \frac{1}{x}$

$=$

$-\sqrt{2} - \frac{1}{x}$

$=$

$-\sqrt{2} - \frac{1}{x}$

$x \cdot \frac{1}{x}$

$=$

$\frac{x}{x}$

$=$

1

$\frac{1}{x}(-1) \to$ Multiply by -1 since we are going to $-\infty$

$=$

$-\frac{1}{x}$

$\lim_{x \to -\infty} \frac{\sqrt{2x^2+1}}{x} = -\sqrt{2}$

5. $\lim_{x\to\infty} \frac{\sqrt{x}}{x}$

$\lim_{x\to\infty} \frac{\sqrt{x}}{x} \cdot \frac{\sqrt{x}}{\sqrt{x}} \longrightarrow$

$\frac{\sqrt{x}}{x} \cdot \frac{\sqrt{x}}{\sqrt{x}}$

$\frac{\sqrt{x^2}}{x\sqrt{x}}$

$\frac{x}{x\sqrt{x}}$

$\frac{1}{\sqrt{x}}$ or $\frac{1}{x^{1/2}}$

$\lim_{x\to\infty} \frac{1}{x^{1/2}}$ $\lim_{x\to\infty} \frac{1}{x^r} = 0$

1

$\lim_{x\to\infty} \frac{\sqrt{x}}{x} = 0$