Section 2.9 #3

Implicit

$x^2 - 3xy + 7y = 5$

$\frac{d}{dx}[x^2] - \frac{d}{dx}[3xy] + \frac{d}{dx}[7y] = \frac{d}{dx}[5]$

$=$

$2x^{2-1} - 3 \cdot \frac{d}{dx}[xy] + 7 \cdot \frac{d}{dx}[y] = 0$

$2x - 3 \cdot \frac{d}{dx}[xy] + 7 \cdot 1y^{1-1} \cdot y' = 0$

$2x - \underbrace{3 \cdot x \cdot \frac{d}{dx}[y] + y \cdot \frac{d}{dx}[x]} + 7y' = 0$

$=$

$$\boxed{\begin{aligned} &3x \cdot 1y^{1-1} \cdot y' + y \cdot 1x^{1-1} \\ &= \\ &3x \cdot 1 \cdot y' + y \cdot 1 \\ &= \\ &3(x \cdot y' + y) \\ &= \\ &3(y + xy') \end{aligned}}$$

$2x - 3(y + xy') + 7y' = 0$

$2x - 3y - 3xy' + 7y' = 0$

$-2x \qquad -2x$

$-3y - 3xy' + 7y' = -2x$

$+3y \qquad +3y$

$-3xy' + 7y' = -2x + 3y$

$=$

$\frac{y'(-3x+7)}{-3x+7} = \frac{-2x+3y}{-3x+7} = \frac{3y-2x}{7-3x}$

$$\boxed{y' = \frac{3y-2x}{7-3x}}$$

At (2,1)

$$y' = \frac{3(1) - 2(2)}{7 - 3(2)}$$

$$= \frac{3 - 4}{7 - 6}$$

$$= \frac{-1}{1}$$

$$= -1$$

$$\boxed{y' = -1}$$

Explicit

$$x^2 - 3xy + 7y = 5$$
$$-x^2 \qquad\qquad -x^2$$

$$-3xy + 7y = 5 - x^2$$

$$\frac{y(-3x+7)}{-3x+7} = \frac{5-x^2}{-3x+7}$$

$$y = \frac{5-x^2}{-3x+7} = \frac{5-x^2}{7-3x}$$

$$\boxed{y = \frac{5-x^2}{7-3x}}$$

$$\frac{d}{dx}[y] = \frac{d}{dx}\left[\frac{\overset{f(x)}{5-x^2}}{\underset{g(x)}{7-3x}}\right]$$

$$y' = \frac{7-3x \cdot \frac{d}{dx}[5-x^2] - 5-x^2 \cdot \frac{d}{dx}[7-3x]}{(7-3x)^2}$$

$$y' = \frac{(7-3x)\left(\frac{d}{dx}[5] - \frac{d}{dx}[x^2]\right) - (5-x^2)\left(\frac{d}{dx}[7] - \frac{d}{dx}[3x]\right)}{(7-3x)^2}$$

$$y' = \frac{(7-3x)(0-2x^{2-1}) - (5-x^2)\left(0 - 3 \cdot \frac{d}{dx}[x]\right)}{(7-3x)^2}$$

$$y' = \frac{(7-3x)(-2x) - (5-x^2)(0-3 \cdot 1)}{(7-3x)^2}$$

$$\boxed{y' = \frac{(7-3x)(-2x) - (5-x^2)(-3)}{(7-3x)^2}}$$

At $(2,1)$

$$y' = \frac{(7-3(2))(-2(2)) - (5-(2)^2)(-3)}{(7-3(2))^2}$$

$$y' = \frac{(7-6)(-4) - (5-4)(-3)}{(7-6)^2}$$

$$y' = \frac{(1)(-4) - (1)(-3)}{(1)^2}$$

$$y' = \frac{(-4) - (-3)}{1}$$

$$y' = (-4) + 3$$

$$= -1$$

$$\boxed{y' = -1}$$