Example 7 Get absolute maximum and absolute minimum values for $f(x) = x^3 - x$ on interval $[0, 3]$

① Get Domain

$f(x) = x^3 - x$

$=$

$x(x^2 - 1)$

$x(x+1)(x-1)$

$x = 0$ $\quad x + 1 = 0$ $\quad x - 1 = 0$

$\quad -1 = -1 \quad +1 \; +1$

$x = -1 \quad x = 1$

$-\quad +\quad -\quad +$

$-1 \quad 0 \quad 1$

(-2) : $-2(2+1)(2-1)$

$-2(3)(1)$

$-2(3)$

-3

(-0.5): $-0.5(-0.5+1)(-0.5-1)$

$-0.5(.5)(-1.5)$

$(-)(-) = .0375$

(0.5): $0.5(0.5+1)(0.5-1)$

$0.5(1.5)(-0.5)$

$-.375$

(2) : $2(2+1)(2-1)$

$2(3)(1)$

$2(3)$

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Where $x^3 - x \geq 0$

$[-1, 0] \cup [1, \infty)$

Where $x^3 - x \leq 0$

$(-\infty, -1] \cup [0, 1]$

② Get $f'(x)$

$f'(x) = \frac{d}{dx}[x^3 - x]$

$=$

$\frac{d}{dx}[x^3] - \frac{d}{dx}[x]$

$=$

$\boxed{f'(x) = 3x^2 - 1}$

Domain of $f'(x)$

$(-\infty, \infty)$

$\downarrow$

No critical values for $f'(x)$ in $[0, 3]$ which makes $f'(x)$ undefined.

$\downarrow$

Set $f'(x) = 0$

$3x^2 - 1 = 0$

$\quad +1 \quad +1$

$\frac{3x^2}{3} = \frac{1}{3}$

$=$

$\sqrt{x^2} = \pm\sqrt{\frac{1}{3}}$

$=$

$x = -\sqrt{\frac{1}{3}}$, $x = \sqrt{\frac{1}{3}}$

$-.57735 \qquad .57735$

$\downarrow$

$+\sqrt{\frac{1}{3}} = .57735$ is in the interval of $[0, 3]$

Compare Values

$f(0) = 0^3 - 0$
$= 0$

$f(3) = 3^3 - 3$
$27 - 3$
$= 24$

$f\left(\sqrt{\frac{1}{3}}\right) = \left(\sqrt{\frac{1}{3}}\right)^3 - \left(\sqrt{\frac{1}{3}}\right)$
$\approx -.3849$

Absolute Maximum on $[0, 3]$ is 24

Absolute minimum on $[0, 3]$ is -0.385

What is the number of widgets per order that will minimize cost?

SI #5

$$c(x) = 2x + \frac{30000}{x}, \quad 0 < x \leq 500$$

Domain of $c(x)$

$(-\infty, 0) \cup (0, \infty)$

$$c'(x) = \frac{d}{dx}\left[2x + \frac{30000}{x}\right]$$

$$\parallel$$

$$c'(x) = \frac{d}{dx}[2x] + \frac{d}{dx}\left[\frac{\overset{f(x)}{30000}}{\underset{g(x)}{x}}\right]$$

$$\parallel$$

$$c'(x) = 2 \cdot \frac{d}{dx}[x] + \frac{x \cdot \frac{d}{dx}[30000] - 30000 \cdot \frac{d}{dx}[x]}{x^2}$$

$$\parallel$$

$$c'(x) = 2 \cdot \frac{dx}{dx} + \frac{x \cdot 0 - 30000 \cdot \frac{dx}{dx}}{x^2}$$

$$\parallel$$

$$c'(x) = 2 \cdot 1 + \frac{0 - 30000 \cdot 1}{x^2}$$

$$\parallel$$

$$c'(x) = 2 + \frac{0 - 30000}{x^2}$$

$$\parallel$$

$$c'(x) = 2 - \frac{30000}{x^2}$$

$$\parallel$$

$$\boxed{c'(x) = 2 - \frac{30000}{x^2}}$$

Domain for $c'(x)$

$(-\infty, 0) \cup (0, \infty)$

0 cause $c'(x)$ to be undefined. This makes 0 a critical value for $c'(x)$.

SI #5 cont.

$$c'(x) = 2 - \frac{30000}{x^2} = 0$$
$$-2 \qquad\qquad\qquad -2$$

$$\frac{\cancel{x^2}}{1} \cdot -\frac{30000}{\cancel{x^2}} = -2 \cdot \frac{x^2}{1}$$

$$\frac{-30000}{-2} = \frac{-2x^2}{-2}$$

$$\sqrt{15000} = \sqrt{x^2}$$

$$122.474 = x$$

$$\boxed{x = 123 \text{ is a critical value}}$$

$$c(0) = 2(0) + \frac{30000}{0} = \text{undef}$$

$$c(123) = 2(123) + \frac{30000}{123} = \boxed{489.90}$$

$$c(500) = 2(500) + \frac{30000}{500} = 1060$$

$$\boxed{x = 123 \text{ is the number of widgets per order that will minimize cost}}$$

Get absolute minimum value of $f(x) = \sin(x)$ on interval $[0, \pi]$

SI #4

Domain of $f(x) = \sin(x)$
$(-\infty, \infty)$

$$f'(x) = \frac{d}{dx}[\sin(x)]$$
$$=$$
$$f'(x) = \cos(x)$$

Domain of $f'(x)$
$(-\infty, \infty)$

No critical values for $f'(x)$ in $[0, \pi]$
which makes $f'(x)$ undef.

$$f'(x) = \cos(x) = 0$$
$$=$$
$$\pi/2 + \pi K$$

$\pi/2$ is in interval
$[0, \pi]$

Compare Values

$f(0) = \sin(0) = \boxed{0}$

$f(\pi) = \sin(\pi) = \boxed{0}$

$f\left(\frac{\pi}{2}\right) = \sin\left(\frac{\pi}{2}\right) = 1$

Absolute Minimum Value of f on interval $[0, \pi]$
is 0