Example 1

$$\text{Velocity} = \frac{\text{meters}}{\text{Seconds}} = \frac{\Delta y}{\Delta x} = \frac{\Delta s}{\Delta t} = \frac{ds}{dt}$$

$$\frac{d}{dt}\left[t^3 - 4t^2 + 4t\right]$$

$$\frac{d}{dt}\left[t^3\right] - \frac{d}{dt}\left[4t^2\right] + \frac{d}{dt}\left[4t\right]$$

$$3t^{3-1} - 4 \cdot \frac{d}{dt}\left[t^2\right] + 4\frac{d}{dt}\left[t\right]$$

$$3t^2 - 4 \cdot 2t^{2-1} + 4 \cdot \frac{dt}{dt}$$

$$\boxed{3t^2 - 8t + 4}$$

Velocity of the particle at time t, $v(t) = 3t^2 - 8t + 4$ for $t > 0$

At $t = 0.5$ second, the velocity of the particle is

$$v(0.5) = 3(0.5)^2 - 8(0.5) + 4$$

0.75 meters/second along straight line at $t = 0.5$ seconds

At $t = 1$ second, the velocity of the particle is

$$v(1) = 3(1)^2 - 8(1) + 4$$

-1 meters/second, particle is moving 1 meter per second backwards along the straight line at $t = 1$ second

Example 2

Find when the particle is at rest, moving toward and moving backwards.

$v(t) = 3t^2 - 8t + 4$, where $t > 0$

Particle is at rest when $v(t) = 0$

$$3t^2 - 8t + 4 = v(t)$$
$$ax^2 + bx + c = 0$$

$$t = \frac{8 \pm \sqrt{(-8)^2 - 4(3)(4)}}{6}$$

$$t = \frac{8 \pm \sqrt{64 - 4(12)}}{6}$$

$$t = \frac{8 \pm \sqrt{64 - 48}}{6}$$

$$t = \frac{8 \pm \sqrt{16}}{6}$$

$$t = \frac{8 \pm 4}{6}$$

$t = \frac{8 - 4}{6}$, $t = \frac{8 + 4}{6}$

$t = \frac{4}{6}$, $t = \frac{12}{6}$

$\boxed{t = \frac{2}{3}}$, $\boxed{t = 2}$

$t = 2/3$ and $t = 2$ are the zeros for $3t^2 - 8t + 4$

$v(t) = 0$ when $t = 2/3$ and $t = 2$

The particle is at rest when $t = 2/3$ seconds and $t = 2$ seconds

Particle is moving forward when $v(t) > 0$

Get all values of t when $3t^2 - 8t + 4 > 0$

$3t^2 - 8t + 4 > 0$

$1 \cdot 3 \qquad 1 \cdot 4$

$2 \cdot 2$

$(t - 2)(3t - 2) > 0$

-6

-2

$1 \quad 3$

$\frac{2 \quad 2}{-6 \quad -2}$

Check

$(t-2)(3t-2)$

$3t^2 - 2t - 6t + 4$

$3t^2 - 8t + 4$ ✓

$(t-2)(3t-2) > 0$

$t - 2 = 0$

$+2 \quad +2$

$t = 2$

$3t - 2 = 0$

$+2 \quad +2$

$\frac{3t}{3} = \frac{2}{3}$

$t = 2/3$

(+) $\quad$ − $\quad$ (+)

$2/3 \qquad 2$

$v(0) : (0-2)(3(0)-2)$

$(-2)(-2)$

$v(0) = 4$

$v(1) : (1-2)(3(1)-2)$

$(-1)(1)$

$v(1) = -1$

$v(3) : (3-2)(3(3)-2)$

$(1)(9-2)$

$(1)(7)$

$v(3) = 7$

The particle is moving forward on $(0, 2/3) \cup (2, \infty)$ and moving backwards on $(2/3, 2)$