Inverse Basics

Functions UNDO Each Other

$f(x) = \sqrt[3]{x} + 1$ $\qquad$ $g(x) = (x-1)^3$ or $f^{-1}(x) = (x-1)^3$

$x = 8 \longrightarrow f(8) = \sqrt[3]{8} + 1 = 2 + 1 = \boxed{3}$

$\underline{8} = (2)^3 = (3-1)^3 = g(3)$

Point on Graph $\rightarrow (8,3)$

$x = 3 \longrightarrow g(3) = (3-1)^3 = (2)^3 = \boxed{8} \rightarrow (3,8)$ Point on Graph

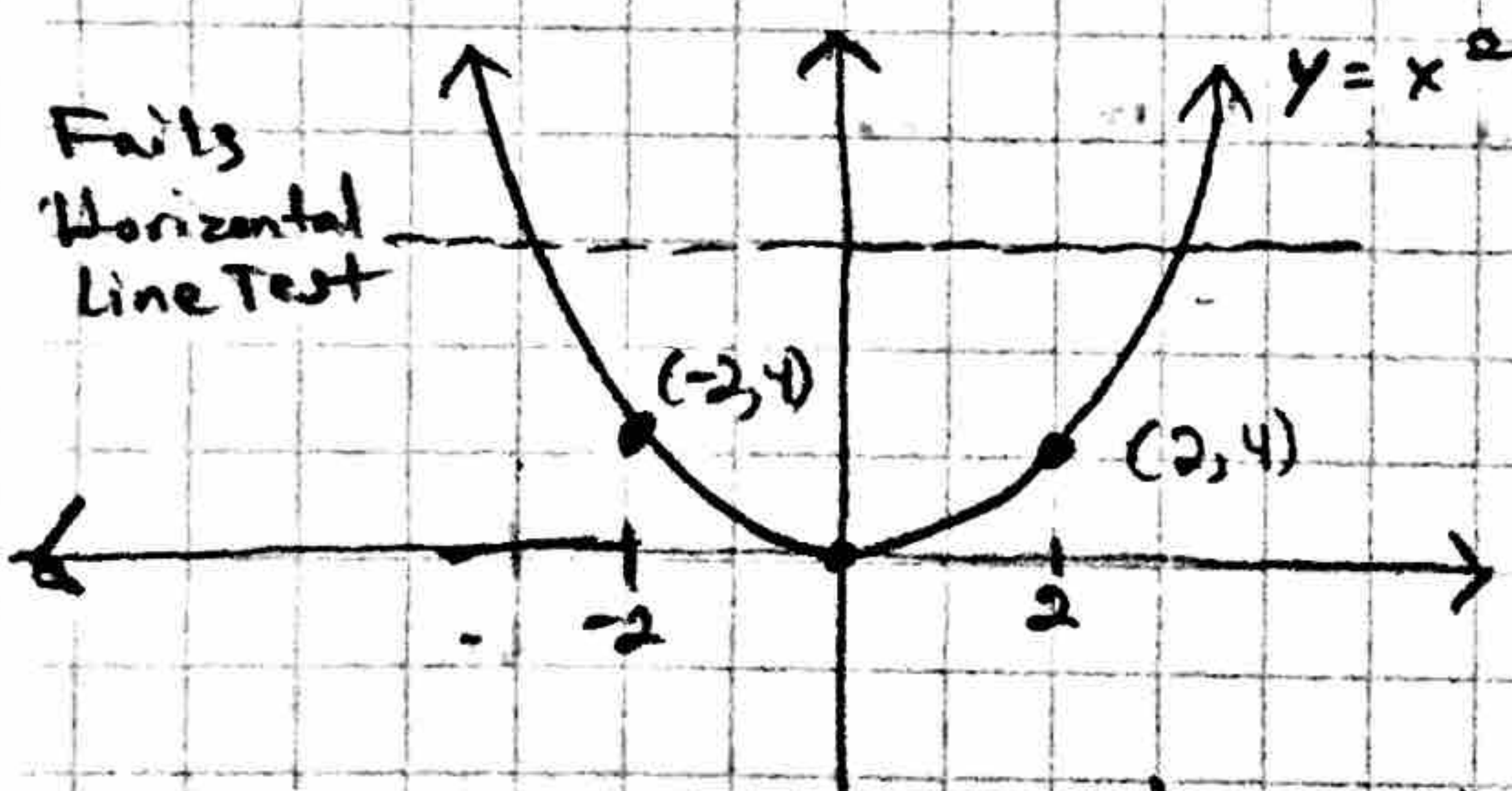
$\underline{3} = 2 + 1 = \sqrt[3]{8} + 1 = f(8)$

no

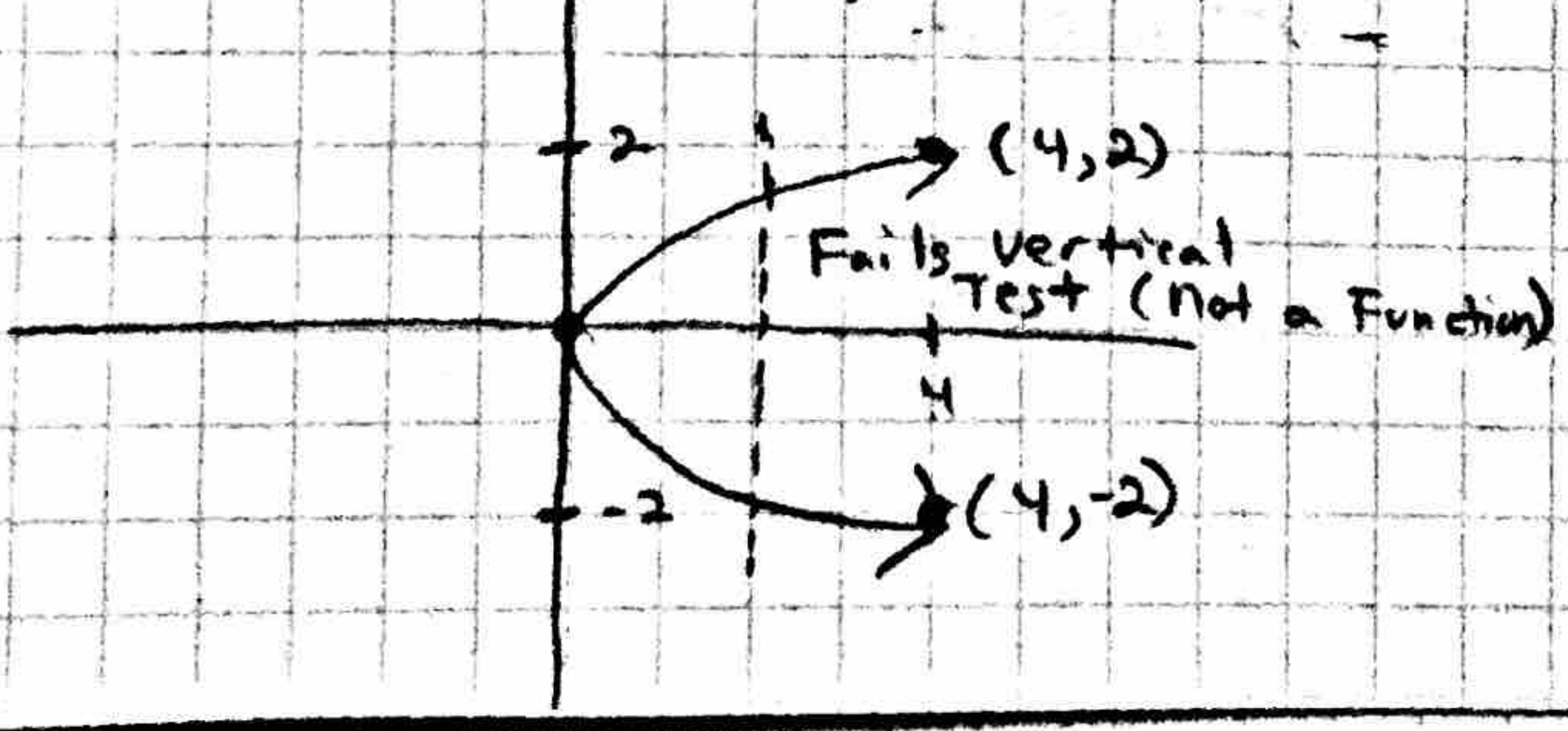
$\boxed{f^{-1}(x) \neq \frac{1}{f(x)}}$

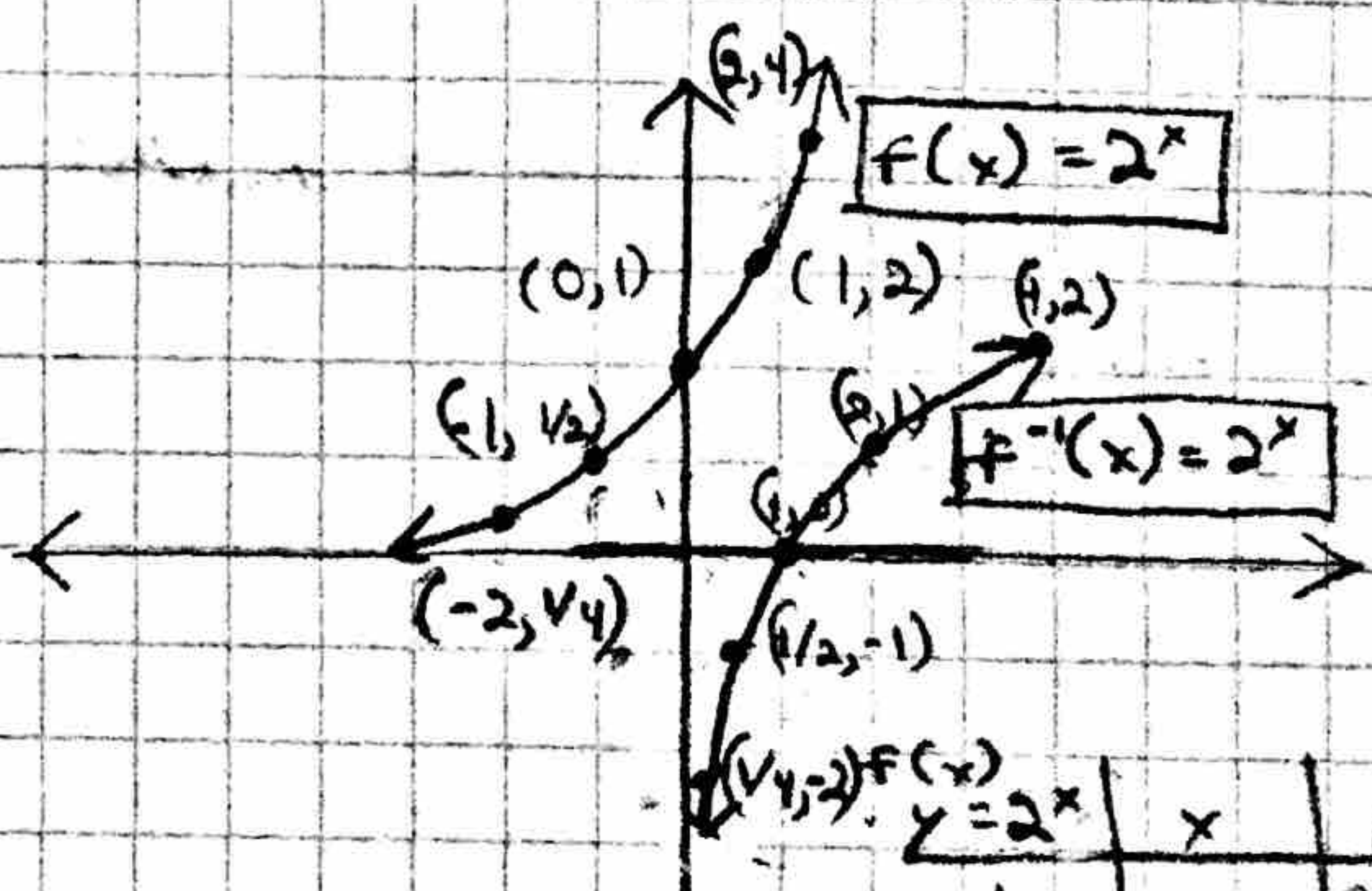
Do all functions have inverses? No

Why Not?

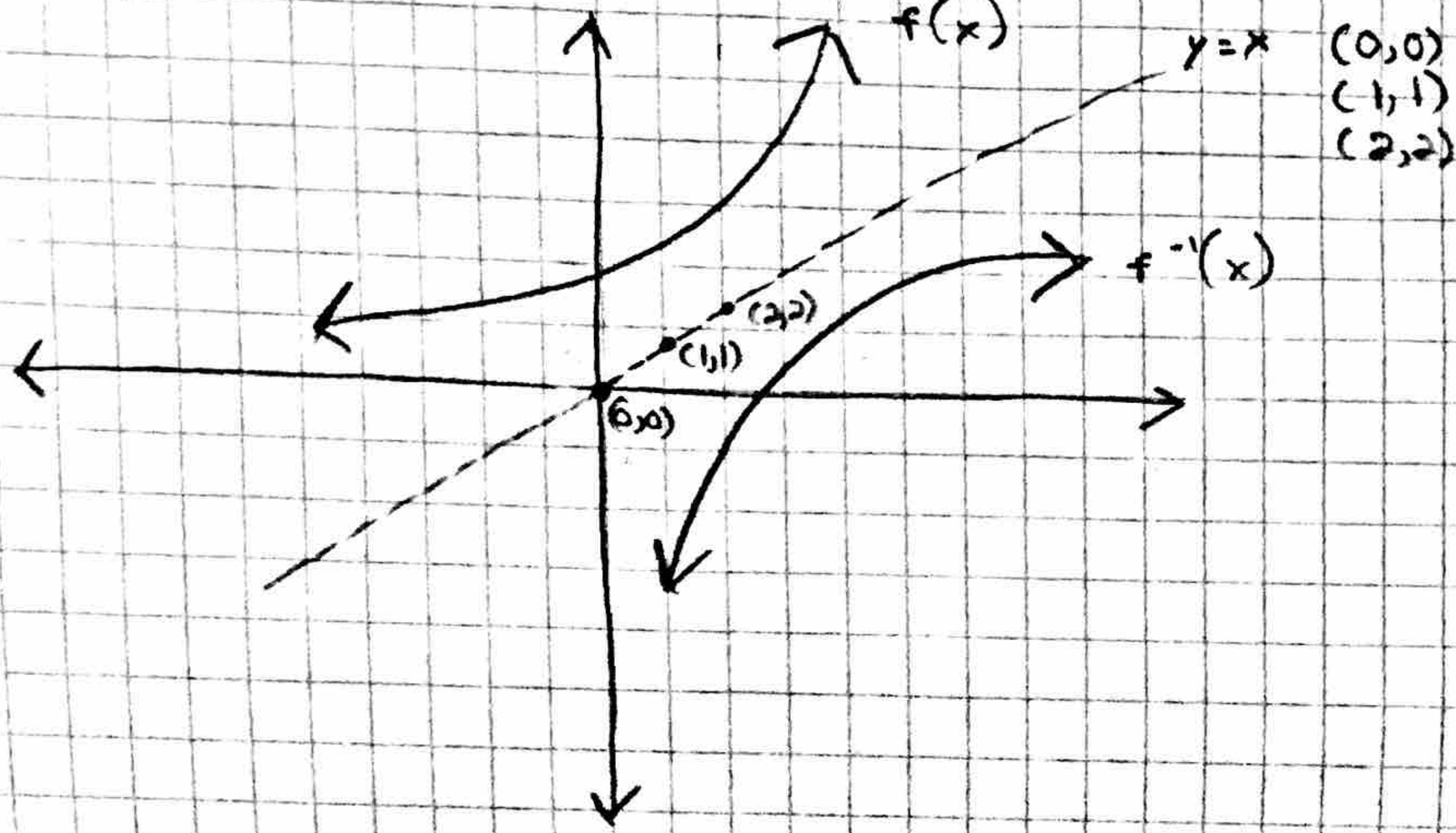


$y=x^2$	x	Point
4	2	(2,4)
4	-2	(-2,4)





$y=2^x$	x	Point
1	0	(0,1)
2	1	(1,2)
4	2	(2,4)
1/2	-1	(-1,1/2)
1/4	-2	(-2,1/4)



Functions that Pass Horizontal Line Test Have Inverse Function

f(x) and its Inverse "UNDO" Each Other

If (a,b) is on f(x), Then (b,a) is on $f^{-1}(x)$
Domain of f(x) Is The Range of $f^{-1}(x)$
x y
Range of f(x) Is The Domain of $f^{-1}(x)$
y x

Points Flip Flop
(1,2) (2,1)
f(x) $f^{-1}(x)$

To find the inverse of f(x) reflect across line y=x

Finding The Inverse of A Function

1. Replace $f(x)$ with y
2. Switch x's and y's
3. Solve for y
4. Replace 'y' with $f^{-1}(x)$

$$f(x) = \sqrt{x+4} - 3$$

$$\downarrow$$

$$y = \sqrt{x+4} - 3$$

$$\downarrow$$

$$x = \sqrt{y+4} - \cancel{3}$$

$$+3 \qquad\qquad +\cancel{3}$$

$$x + 3 = \sqrt{y+4}$$

$$\downarrow$$

$$(x+3)^2 = (\sqrt{y+4})^2$$

$$\downarrow$$

$$(x+3)(x+3) = y + 4$$

$$\downarrow$$

$$x^2 + 3x + 3x + 9 = y + 4$$

$$\downarrow$$

$$x^2 + 6x + 9 = y + 4$$

$$-4 \qquad -4$$

$$x^2 + 6x + 9 - 4 = y$$

$$\downarrow$$

$$\boxed{x^2 + 6x + 5 = y} \rightarrow f^{-1}(x)$$

$$\downarrow \quad \text{replace}$$

$$x^2 + 6x + 5 = f^{-1}(x)$$

$$y = \frac{5x-3}{2x+1}$$

$$\downarrow$$

$$\frac{x}{1} = \frac{5y-3}{2y+1}$$

$$\downarrow$$

$$x(2y+1) = 1(5y-3)$$

$$\downarrow$$

$$2xy + x = \cancel{5y} - 3$$
$$-5y \quad -\cancel{5y}$$

$$\downarrow$$

$$2xy - 5y + x = -3$$
$$-x \quad -x$$

$$2xy - 5y = -3 - x$$

$$\downarrow$$

$$\frac{y\cancel{(2x-5)}}{\cancel{(2x-5)}} = \frac{-3-x}{(2x-5)}$$

$$\downarrow$$

$$\boxed{y = \frac{-3-x}{(2x-5)}}$$

$$\downarrow$$

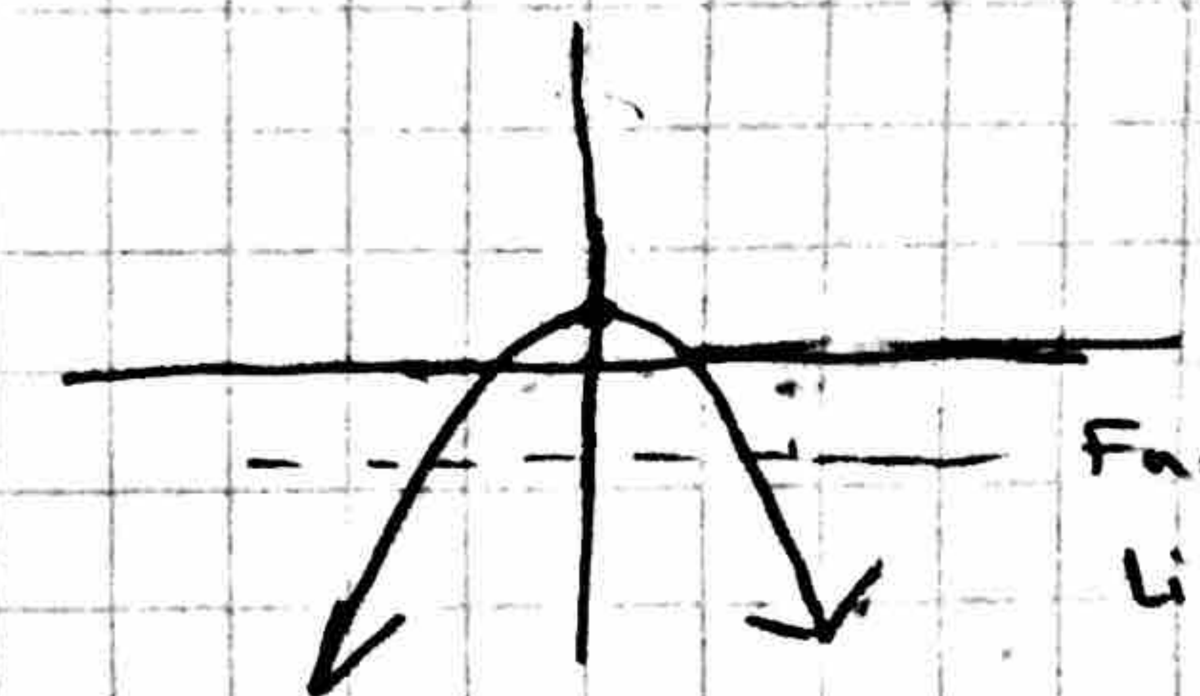
$$\boxed{f^{-1}(x) = \frac{-3-x}{(2x-5)}}$$

Find the inverse of each function, if one exists.

$$f(x) = \frac{8-x^2}{5} \longrightarrow y = \frac{8}{5} - \frac{1}{5}x^2$$

$$\downarrow$$

$$-\frac{1}{5}x^2$$



Fails Horizontal Line Test

$$y = \frac{8-x^2}{5}$$

$$\frac{x}{1} = \frac{8-y^2}{5}$$

$$5x = 8 - y^2$$
$$+y^2 \quad +y^2$$

$$y^2 + 5x = 8$$
$$-5x \quad -5x$$

$$\sqrt{y^2} = \pm\sqrt{8-5x}$$

$$y = \pm\sqrt{8-5x}$$

No Inverse Function
For One Input, There is Multiple Outputs

$$g(x) = \frac{-5+5x}{-3-3x}$$

$$y = \frac{-5+5x}{-3-3x}$$

$$\frac{x}{1} = \frac{-5+5y}{-3-3y}$$

$$-3x - 3xy = -5 + 5y$$
$$+3xy \quad +3xy$$

$$-3x = -5 + 5y + 3xy$$
$$+5 \quad +5$$

$$-3x + 5 = 5y + 3xy$$
$$-3x + 5 = y(5+3x)$$

$$\frac{-3x+5}{(5+3x)} = \frac{y\cancel{(5+3x)}}{\cancel{(5+3x)}}$$

$$\frac{-3x+5}{(5+3x)} = y$$

$$\frac{5-3x}{5+3x} = y = f^{-1}(x)$$

$$\boxed{f^{-1}(x) = \frac{5-3x}{5+3x}}$$

$h(x) = 8\sqrt{8x-3} - 1, \quad x \geq \frac{3}{8}$ Domain

$\downarrow$

$8\sqrt{\cancel{8}\left(\frac{3}{\cancel{8}}\right) - 3} - 1$

$\downarrow$

$8\sqrt{3 - 3} - 1$

$\downarrow$

$8\sqrt{0} - 1$

$\downarrow$

$8(0) - 1$

$\downarrow$

$0 - 1$

$\downarrow$

$h(x) = -1$

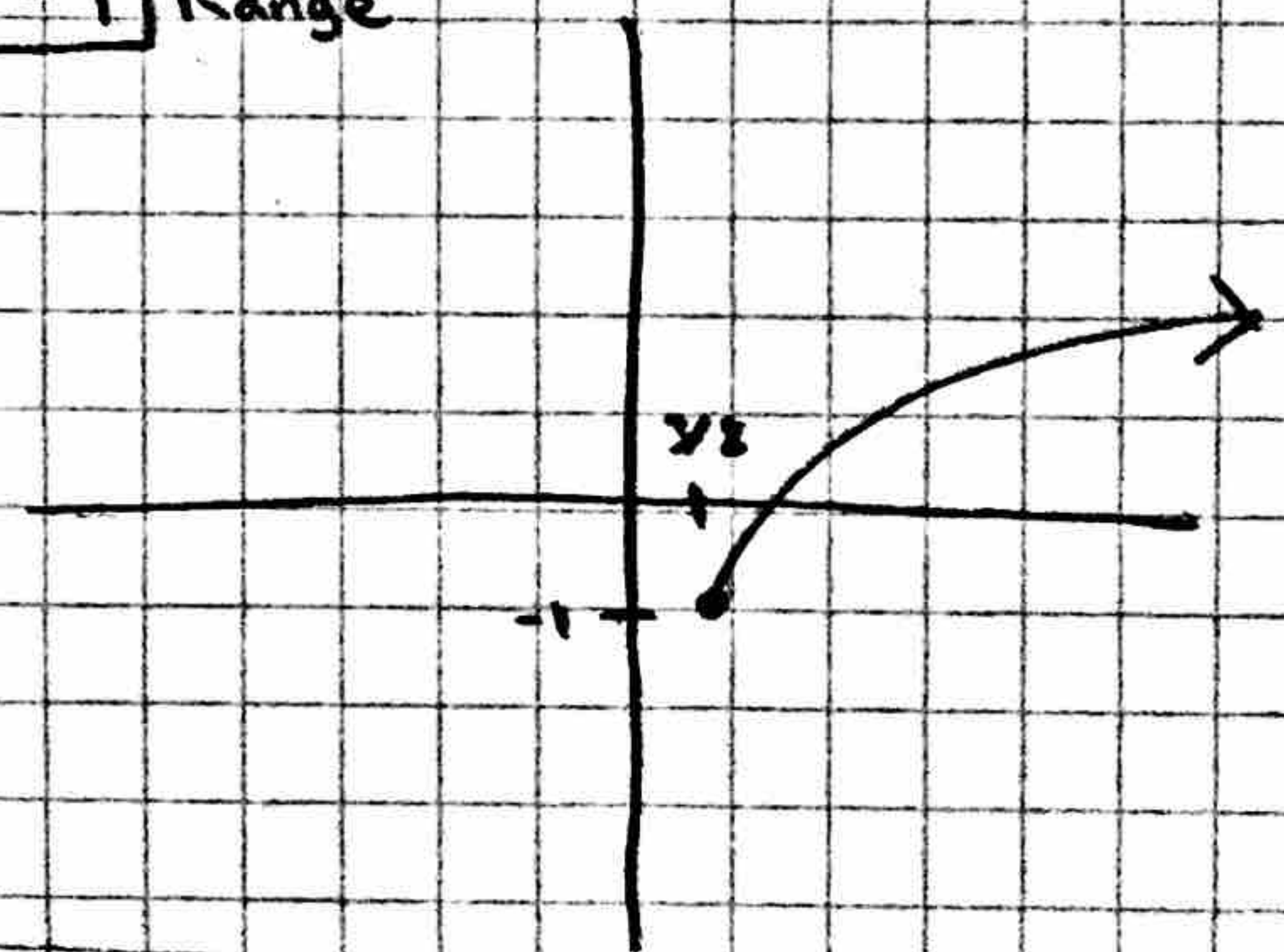
$\boxed{y = -1}$ Range

Domain : $[3/8, \infty)$

Range : $[-1, \infty)$

Smallest y value when $x \geq 3/8$

P: $\left(\frac{3}{8}, -1\right)$



Find Inverse

$h(x) = 8\sqrt{8x-3} - 1$

$\downarrow$

$y = 8\sqrt{8x-3} - 1$

$\downarrow$

$x = 8\sqrt{8y-3} - \cancel{1}$

$+1 \qquad + \cancel{1}$

$(x+1)^2 = \left(8\sqrt{8y-3}\right)^2$

$\downarrow$

$\frac{(x+1)^2}{64} = \frac{64(8y-3)}{\cancel{64}}$

$\frac{(x+1)^2}{64} = 8y - 3$

$\frac{(x+1)^2}{64} = 8y - 3$

$+3 \qquad +3$

$\left(\frac{1}{8}\right)\left(\frac{x+1}{64}\right)^2 + 3 = 8y\left(\frac{1}{8}\right)$

$\downarrow$

$\left(\frac{1}{8}\right)\left(\frac{x+1}{64}\right)^2 + \left(\frac{1}{8}\right)\frac{3}{1} = y$

$\downarrow$

$\frac{(x+1)^2}{512} + \frac{3}{8} = y = f^{-1}(x)$, where domain $[-1, \infty)$

Think about your restrictions. The range range of $h(x)$ is the domain of $h^{-1}(x)$.