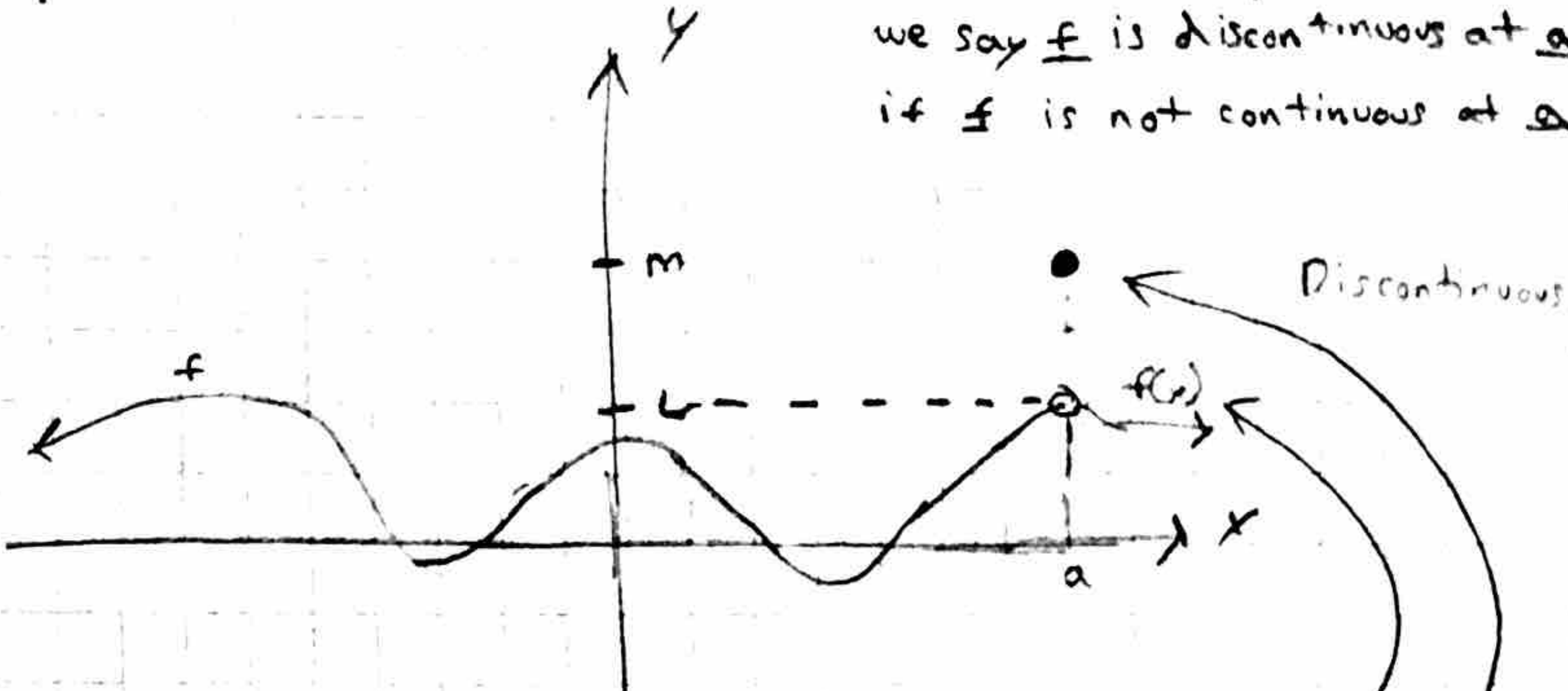
Continuity

Open Interval

Discontinuous at a Point

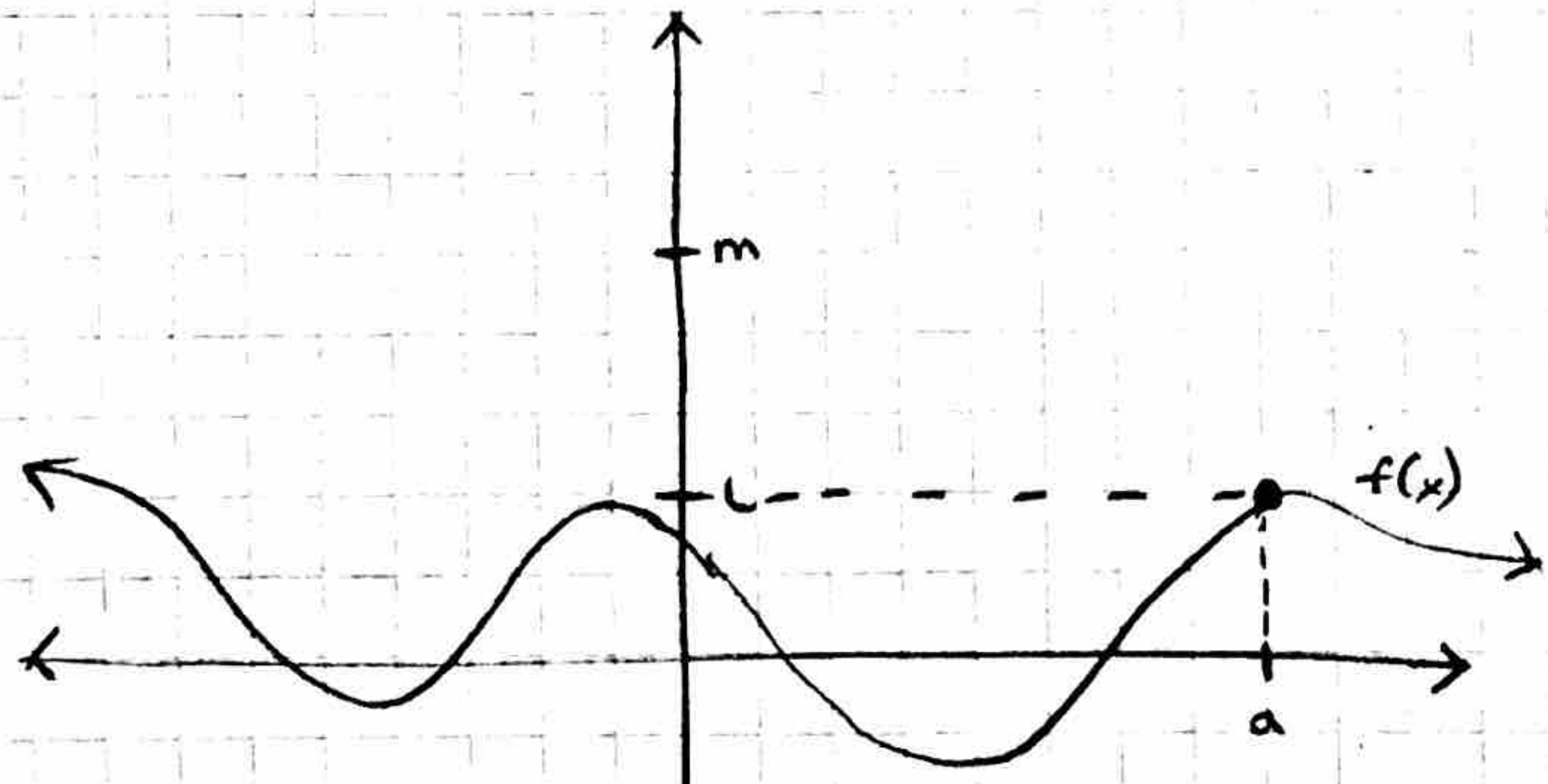
If f is defined on an open interval containing a, except perhaps at a, we say f is discontinuous at a, if f is not continuous at a



$$\lim_{x \to a} f(x) = L$$

$$f(a) = m$$

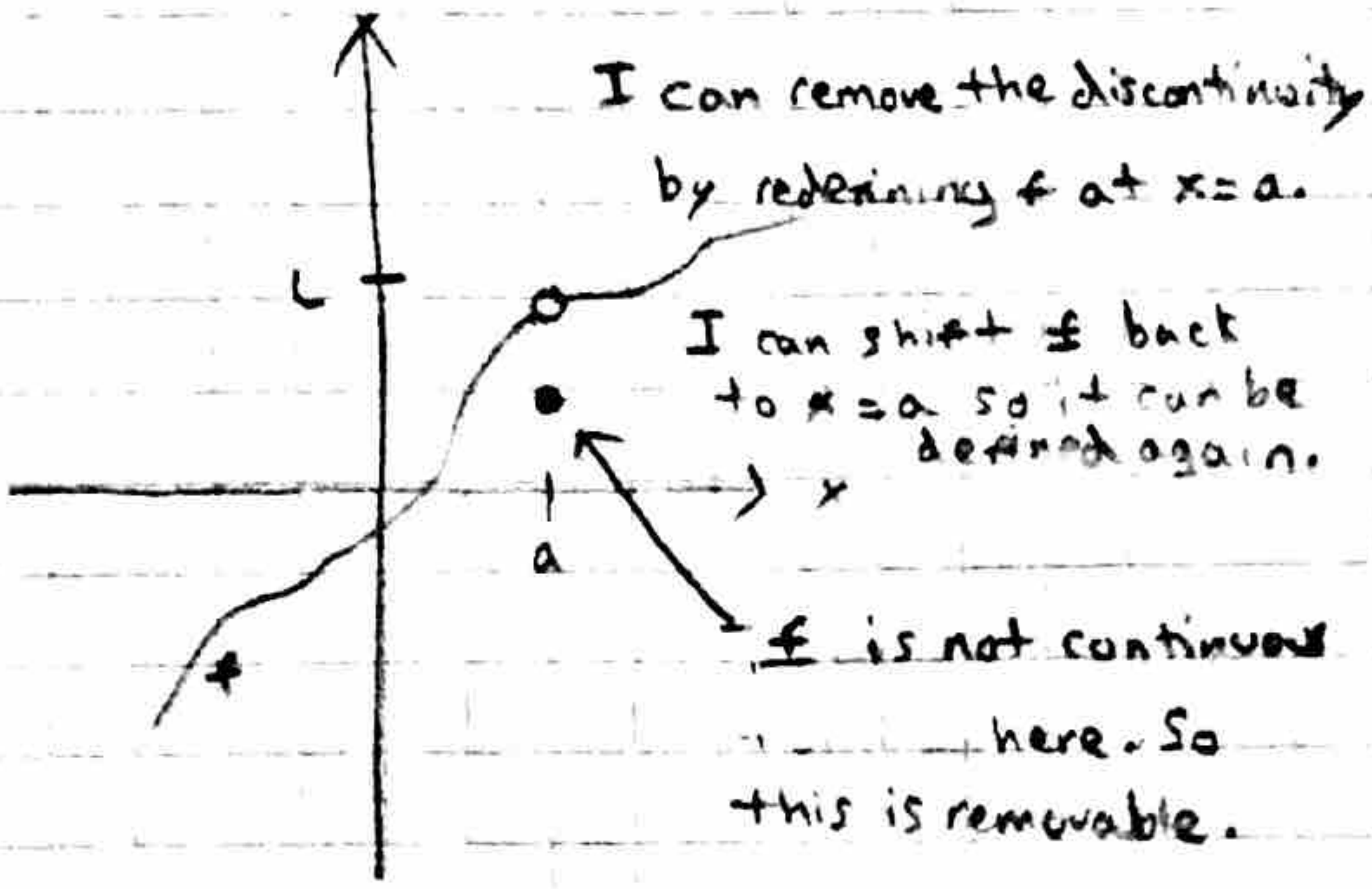
$$\lim_{x \to a} f(x) \neq f(a)$$

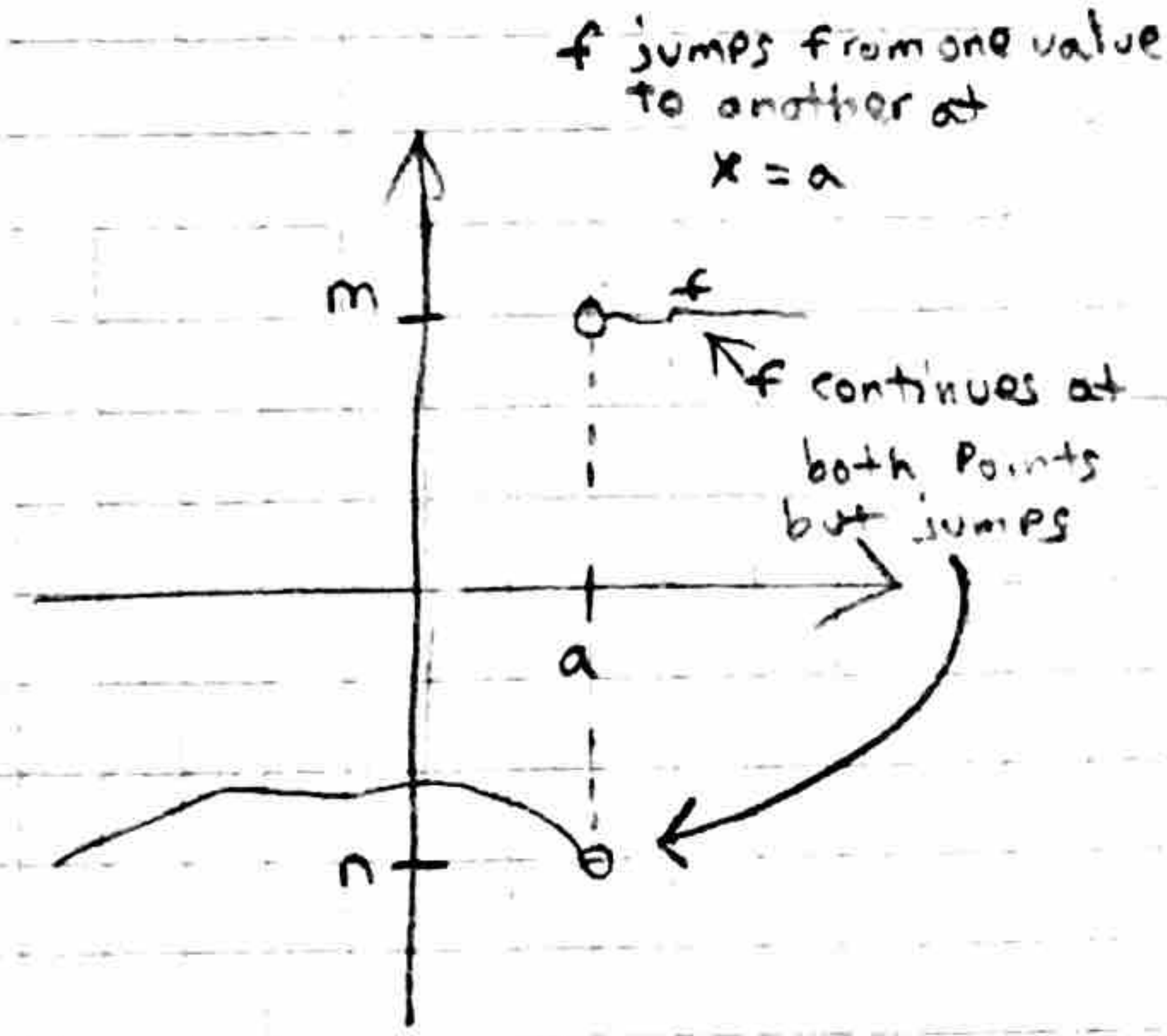


$$\lim_{x \to a} f(x) = f(a)$$

$f(x)$ is continuous at $x = a$

Types of Discontinuities



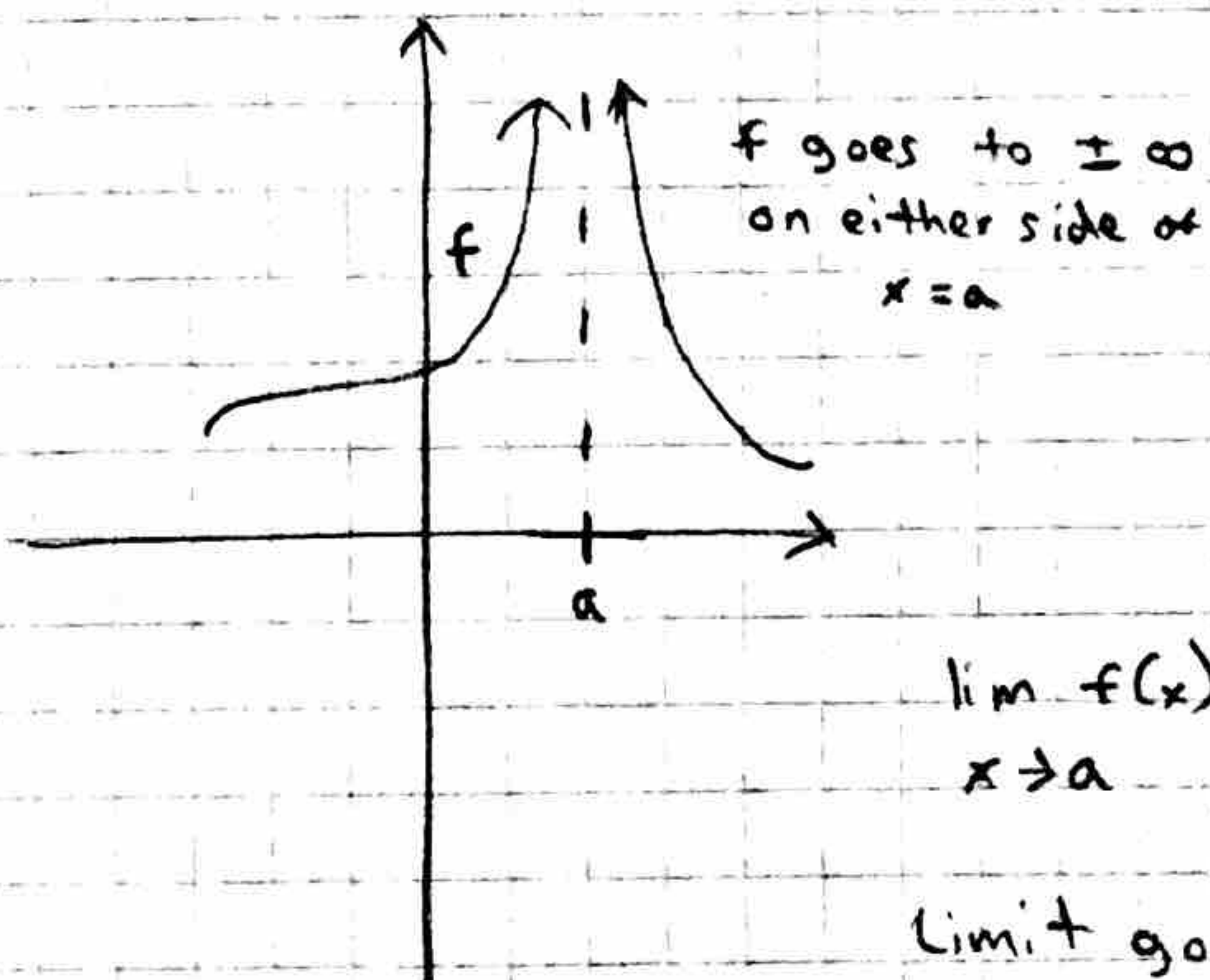


$\lim_{x \to a} f(x) = L$; $f(a) = m$

Limit exist but value is
different : <u>Removable</u>

$\lim_{x \to a^-} f(x) = n$ $\lim_{x \to a^+} f(x) = m$

Limit do not exist.
The graph looks like it
jumps. <u>Jump</u>



$\lim_{x \to a} f(x) = \infty$

Limit goes to Infinity.
<u>Infinite</u>

Discontinuous at a Point

If f is defined on an open interval containing a, except perhaps at a, we say f is discontinuous at a if f is not continuous at a.

$f(x) = 1/x$: Find all points of discontinuities

⓪ $a = x$

① Domain of $1/a$; $(-\infty, 0) \cup (0, \infty)$

② $f(a)$ is defined at all points except $a = 0$

③ f is discontinous at 0.

$f(x) = \dfrac{x^2 + 5x - 2}{x + 3}$: Find all points of discontinuities.

① $a = x$

② Domain of $\dfrac{a^2 + 5a - 2}{a + 3}$; $(-\infty, -3) \cup (-3, \infty)$

③ $f(a)$ is is defined at all points except $a = -3$

④ f is discontinuous at -3.

$$g(x) = \begin{cases} x-3 \ ; & x \leq -1 \\ x^2+1 \ ; & -1 < x \leq 2 \\ x^3+4 \ ; & x > 2 \end{cases}$$

There is a break at $x = 2$, notice the signs.

$\lim_{x \to -1^-} = x - 3$

$=$

$\lim_{x \to -1^-} = (-1) - 3$

$=$

$\lim_{x \to -1^-} = -4$

$=$

-4

$\lim_{x \to -1^+} = x^2 + 1$

$=$

$\lim_{x \to -1^+} = (-1)^2 + 1$

$=$

$\lim_{x \to -1^+} = 1 + 1$

$=$

2

Limit do not exist.
Not Continuous at $x = -1$

$\lim_{x \to 2^-} = x^2 + 1$

$=$

$\lim_{x \to 2^-} = (2)^2 + 1$

$=$

$\lim_{x \to 2^-} = 4 + 1$

$=$

5

$\lim_{x \to 2^+} = x^3 + 4$

$=$

$\lim_{x \to 2^+} = (2)^3 + 4$

$=$

$\lim_{x \to 2^+} = 8 + 4$

$=$

12

Limit do not exist.
Not Continuous at
$x = 2$

$$f(x) = \begin{cases} \dfrac{1}{x+2}, & x \neq -2 \\ 1, & x = -2 \end{cases}$$

There is a break at -2, notice the signs.

$$\lim_{x \to -2^-} = \frac{1}{x+2} \to$$

$$= \frac{\lim_{x \to -2^-} 1}{\lim_{x \to -2^-} x+2}$$

$$= \frac{1}{\lim_{x \to -2^-} x + \lim_{x \to -2^-} 2}$$

$$= \frac{1}{-2+2}$$

$$= \boxed{\text{Undef at } x = -2}$$

← This tells me that this is an <u>infinite discontinuity</u> since the limit is undef at $x = -2$

$$\lim_{x \to -2^+} = 1$$

$$= 1$$

$$\boxed{\begin{array}{c}\text{Limit do not exist} \\ \text{not continuous at} \\ x = -2\end{array}}$$

$$g(x) = \begin{cases} x-3; & x \le -1 \\ x^2+1; & -1 < x \le 2 \\ x^3+4; & x > 2 \end{cases}$$ ← There is a break at 2

$\lim_{x \to -1^-} = x-3$

$\lim_{x \to -1^-} = (-1)-3$

$\lim_{x \to -1^-} = -4$

$= -4$

$\lim_{x \to -1^+} = x^2+1$

$\lim_{x \to -1^+} = (-1)^2+1$

$\lim_{x \to -1^+} = 1+1$

$\lim_{x \to -1^+} = 2$

$= 2$

Limit Do Not Exist
Not Continuous at $x = -1$

$\lim_{x \to 2} f(x) = f(2)$?

$\lim_{x \to 2^-} = x^2+1$

$\lim_{x \to 2^-} = (2)^2+1$

$= 4+1$

$\lim_{x \to 2^-} = 5$

$= 5$

$\lim_{x \to 2^+} = x^3+4$

$\lim_{x \to 2^+} = (2)^3+4$

$\lim_{x \to 2^+} = 8+4$

$\lim_{x \to 2^+} = 12$

$= 12$

Limit Do Not Exist

Not Continuous at
$x = 2$

$$f(x) = \begin{cases} x^2 - 1, & x < 0 \\ x, & x \geq 0 \end{cases}$$

There is a break at 0

$$\lim_{x \to 0^-} = x^2 - 1$$

$$=$$

$$\lim_{x \to 0^-} = (0)^2 - 1$$

$$=$$

$$\lim_{x \to 0^-} = -1$$

-1

$$\lim_{x \to 0^+} = x$$

$$=$$

$$\lim_{x \to 0^+} = 0$$

$$=$$

0

$$\boxed{\text{Limit do not exist, not continuous at } x = 0}$$

These values jump from -1 to 0 and vice versa making this a <u>jump discontinuity</u>