

MATH E-3  
PRACTICE FINAL EXAMINATION SOLUTIONS  
With SOLUTIONS  
Spring 2009

You have approximately one hour and 45 minutes to complete this exam. Please write all your answers in the spaces provided. If you use your own graph paper, please write your name on it and make sure it is inserted in your exam. We will not be held responsible for the loss of loose sheets of paper.

ROUNDING: Round appropriately. The criteria for rounding are stated in the instructions for *some* of the problems; otherwise it is your choice. Remember to round after other calculations have been completed. Deductions will be made for incorrect rounding.

SHOW YOUR WORK.

**Question 1: CONFIDENCE INTERVALS**

An automobile insurance company wants to determine from a sample what proportion of its thousands of policyholders intend to buy a new car within the next year. The company asks a random sample of 250 of its policyholders and finds that 42 of them intend to buy a new car within the next year.

- a) Find a 95% Confidence Interval for the true proportion of those who intend to buy a new car within the next year.

Do the calculations: Find the s.d.  $P = \frac{42}{250} = .168 = 16.8\%$

$$\sigma = \sqrt{\frac{.168(1 - .168)}{250}} = .02364... = 2.4\% \quad \text{MOE} = 2 \times \text{s.d.} = 2 \times 2.4\% = 4.8\%$$

The 95% Confidence Interval for the true proportion of those who intend to buy a new card is

$16.8\% \pm 4.8\% \quad \text{or} \quad 12\% \text{ <-----> } 21.6\%$
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- b) What is your margin of error? Margin of error = 4.8%

- c) What is the standard deviation for the above 95% Confidence Interval you chose?

s.d. = 2.4%
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**d) extra credit** If the insurance company wanted their 95% confidence interval to be no wider than plus or minus 3%, how big would their sample size have to be?

**Since we want the moe = 3%, the s.d. must be 1.5% which is .015 as a decimal. Put this into the sd formula and guess and check the number of people needed to get .015**

$.015 = \sqrt{\frac{.168(1 - .168)}{n}}$  You can take an educated guess for n. The standard deviation shrinks as the sample size increases. Thus since we want a s.d. less than 2.4% we can guess perhaps n = 500. That was a bit small, so you may then try 600. Algebraically, this may be solved by squaring both sides and solving for n.

$$.015^2 = \frac{.139776}{n} \quad \text{thus } .000225n = .139776 \quad \text{divide by } .000225 \quad \frac{.139776}{.000225} = 621.22$$

**Answer:** Sample size must be about 621

## Question 2: EXPONENTIAL GROWTH

A few years ago, *The Boston Globe* reported that in his will, Benjamin Franklin had bequeathed about \$4,400 each to Boston and Philadelphia. However, this money was to be given to the cities 200 years after his death, that is, in 1990.

a) Ignoring factors such as inflation, changing interest rates, and assuming an annual interest rate of 3.5% how much should Boston's \$4,400 be worth at the end of the 200 year period? Give a figure rounded to dollars and cents.

**ANSWER:**  $y = 4,400(1.035)^{200} = \$ 4,280,777.30$

Years	Amount of Money
0	\$4,400
1	$4,400(1.035)$
2	$4,400(1.035)^2$
n	$4,400(1.035)^n$

b) By what **OVERALL percent** did the \$4,400 increase in the 200 year period. (Round to one decimal place.)

**ANSWER**  $\frac{4,280,777.30 - 4,400}{4,400} * 100 = 971.90393 * 100 = \mathbf{97190.4\%}$

c) How much would Boston have after 200 years if the interest is compounded quarterly? (Round to dollars and cents.)

$$\text{ANSWER: } 4,400\left(1 + \frac{.035}{4}\right)^{4*200} = \boxed{\$4,680,485.03}$$

d) If instead of earning 3.5% the entire time, how much would Boston have if the \$4,400 earned the following:

3.5% annually for the first 100 years **and then** 4% compounded monthly for the next 100 years?  
(Round to dollars and cents.)

$$y = 4,400(1.035)^{100} = \$137,242.20 \text{ For the first 100 years}$$

Then compound this amount monthly for another 100 years

$$\text{ANSWER: } \$137,242.20(1 + 4\%/12)^{12*100} = \boxed{\$7,443,491.90}$$

**Question 3: EXPONENTIAL DECAY**

Because of a congenital problem (a slight leakage in a heart valve), a cardiologist recently took some x-ray type pictures of a patient's heart function. A radioactive drug, Technetium, was injected and a short time later, the images were taken. (Technetium is not only used for the detection of coronary artery disease, but it is also used as an oncological tracer.\*)

The physical half-life of technetium is about 6 hours. That means, after 6 hours, one-half of the drug is left in one's system. Assume the doctor initially injected 20 mCi into the patient's system. *Note: Ci is the measurement used for radioactive isotopes, called curies. Do not worry about the units as no changes need to be made. Simply think of them as say, milligrams.* Answer the following questions.

- a) How much of the drug is left in the patient's system after 12 hours?

Time	Amount
0	20
6	$20(.5)$
12	$20(.5)^2$

**ANSWER: 5 mCi**

- b) Write a general formula for this type of exponential decay. Use specific numbers relating to this particular problem in your answer.

**ANSWER:**

$$Y = 20 (.5)^{t/6}$$

- c) How much of the drug is left in my system after  $\frac{1}{2}$  hour? Hint: the formula will help you here.

$$Y = 20 (.5)^{t/6} \quad \text{replace the t with .5 for } \frac{1}{2} \text{ hour}$$

**ANSWER:**  $Y = 20 (.5)^{.5/6} = 20 (.5)^{.0833333333} = \mathbf{18.9 \text{ mCi}}$

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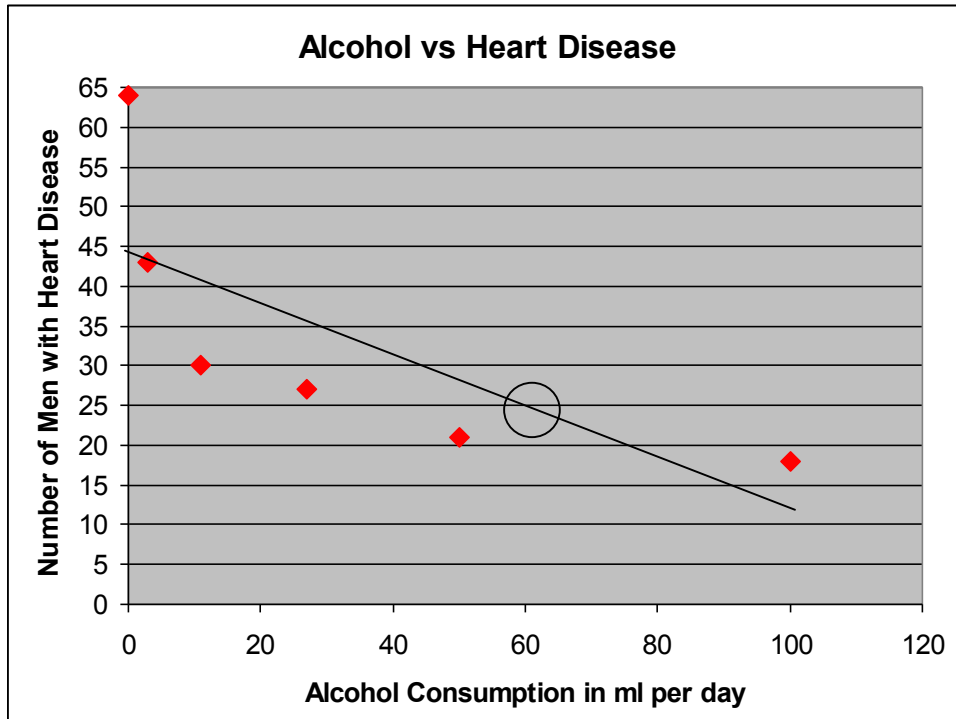
\* Micromedex Health Care Series, Children's Hospital Boston Clinician Web Site. For more information on Curies, go to <http://en.wikipedia.org/wiki/Curie>.

**Question 4: REGRESSION**

The following is a table showing the incidence of coronary heart disease among men who consume alcohol. The table gives an average amount of alcohol (in milliliters) per day over a certain period of several years.

Alcohol Consumption (in ml per day*)	Number of men with heart disease
0	64
3	43
11	30
27	27
50	21
100	18

- a) Using the sheet of graph paper attached or your own graph paper (Put your name on it), draw a scatterplot using these data. Make sure you put the independent variable on the horizontal axis.



- b) Describe in appropriate statistical language using (type of correlation) the trend you see.  
There is a moderately strong negative correlation between drinking alcohol and the incidence of heart disease.
- c) Choose (circle) which of the following numbers is most likely to be “r,” the *correlation coefficient*. (I made it bigger and bolder. . .)

-2   -1   **-.7**   -.3   0   .1   .7   1   2

d) Assuming that the relationship is linear, draw a regression line on graph paper (or grid given on previous page) that best represents the data. Then **FIND THE EQUATION OF YOUR REGRESSION LINE using the techniques learned in class.**

I used my y-intercept (0,45) and the point (60, 25) because (60, 25) is on my line and my line did not go through any data points.

$$\text{Slope} = \frac{45 - 25}{0 - 60} = \frac{20}{-60} = -.3333 \quad \boxed{\text{Equation } y = -.33X + 45}$$

The actual computer generated equation is  $Y = -.33X + 44.435$

e) Explain in words the meaning of YOUR particular slope and y-intercept for this problem.

i) Slope: the slope =  $-.33$  or  $-33/100$  This means there is an additional drop in the number of men with heart disease for each additional 100 ml/day of alcohol consumed.

ii) y-intercept: y-intercept = 45 which means that even if no alcohol is consumed, there will be 45 men with heart disease.

f) Use your regression **equation** to predict the incidence of coronary heart disease that corresponds to the following amounts of consumption of alcohol Answer the two questions that follow.

i) **10 ml All answers will differ according to equation. My answer is  $\boxed{41.7}$**

What is the name of this type of estimation? Interpolation

How confident are you about your prediction and why? Pretty confident because we are predicting within the known data values

ii) **60 ml My answer is  $\boxed{25.2}$  This is also interpolation**

How confident are you about your prediction and why? Still confident but as you can see there is a big gap between 50 ml and 100 ml. But it is still a pretty good prediction based on what we have.

iii) **200 ml My answer is  $\boxed{-21}$  This is Extrapolation**

How confident are you about your prediction and why? We are not confident at all of this prediction because it is so far from the given data that we cannot know if the linear assumption holds this far from the data. **BUT more importantly, the answer is negative, which makes no sense!!**

g) Based on the data in the table above, can you conclude that drinking alcohol causes (a reduction) in coronary artery disease? Do not just answer 'yes' or 'no.'

We cannot assume there is a cause and effect because of a few reasons.

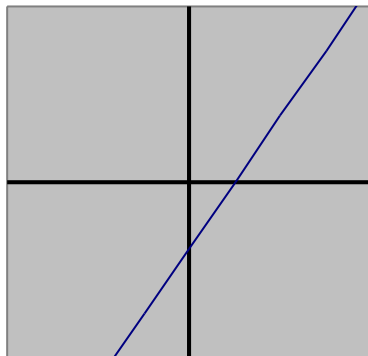
1. The sample size is rather small.
2. The data does not look linear to begin with. Actually a curve would fit the data better.
3. Even if we had more data, there are still other factors involved in determining the reasons why there would be less heart disease. For example, diet, exercise, physical condition- weight,, heredity, etc.

**Question 5: MISCELLANEOUS**

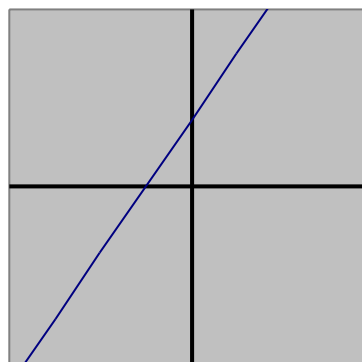
For **PARTS I - III**, look at the graphics. Match the graph with the appropriate expression on the left side of this sheet for each part of the problem. Notice the expressions are labeled with letters. Place the letter of your answer in the space provided under the graphic.

**PART I:**

- A.  $Y = 3x - 3$
- B.  $Y = -3X + 3$
- C.  $Y = 3X + 3$



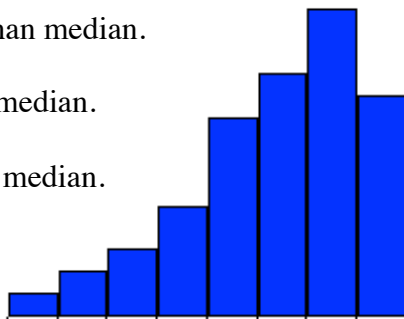
Answer:   A  



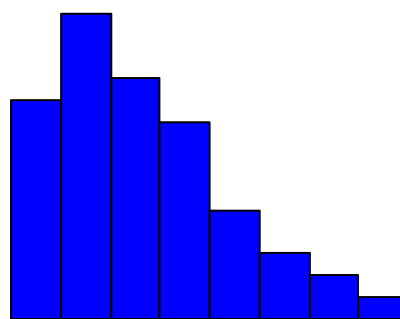
Answer:   C  

**PART II:**

- D. Mean is greater than median.
- E. Mean is equal to median.
- F. Mean is less than median.



Answer:   F  



Answer:   D

**PART III:**

**SET Q:** 3, 7, 5, 14, 9, 4, 11, 3

**SET R:** 14, 5, 3, 3, 7, 14, 14

G: Median = 7

First, put each set in numerical order.

3, 3, 5, 7, 14, 14, 14

3, 3, 4, 5, 7, 9, 11, 14

H: Median = 3

Average 5 and 7 Med = 6

Med = 7 the middle data value

I: Median = 14

**Answer: \_\_\_\_J\_\_\_\_**

**Answer: \_\_\_\_G\_\_\_\_**

J: Median = 6

**PART IV:** The populations of six different cities are growing or declining based on the equations below.

City A:  $Y = 10(1.056)^t$

City B:  $Y = 250(0.935)^t$

City C:  $Y = 63(1.052)^{t/10}$

City D:  $Y = 398(1.024)^t$

City E:  $Y = 35t + 125$

Y = the population in thousands and 't' is the number of years since 1980. Answer the following questions by placing the letter of the appropriate City in the space provided. Read the questions carefully.

Note: there is only one answer for each.

a.) The population of this city is increasing by 5.2% every 10 years? \_\_\_\_C\_\_\_\_

b) The population of this city is decreasing by 6.5% annually. \_\_\_\_B\_\_\_\_

c) The population of this city is growing linearly. \_\_\_\_E\_\_\_\_

d) This city has the highest initial population. \_\_\_\_D\_\_\_\_

e) This city is growing at the fastest rate. \_\_\_\_A\_\_\_\_