

① Get Derivative

$$f'(x) = \frac{d}{dx} [x^2]$$

$$2x^{2-1}$$

$$2x^1$$

$$2x$$

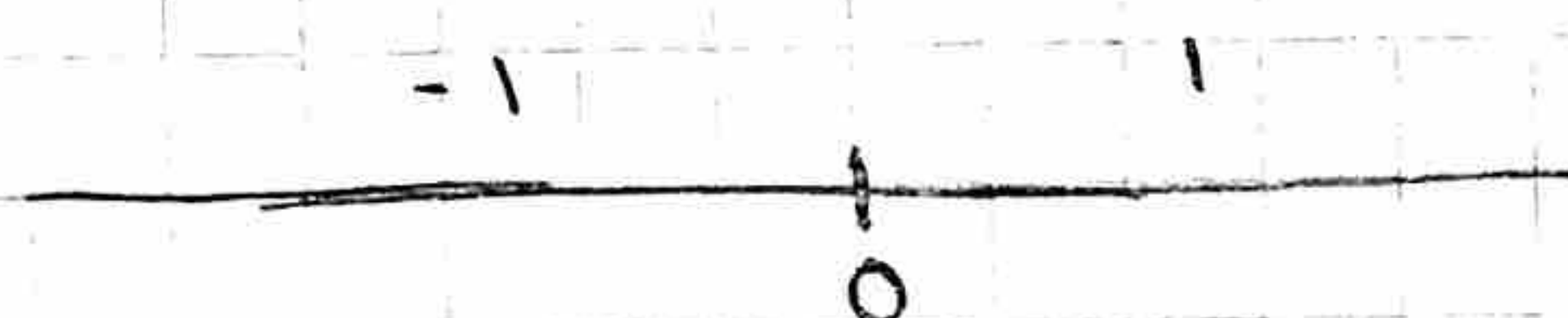
② Set $f'(x) = 0$

$$\frac{2x}{2} = \frac{0}{2}$$

$$x = 0$$

③ Set intervals

$$(-\infty, 0) \cup (0, \infty)$$



$$f'(x) = 2x$$

$$f'(-1) = 2(-1)$$

$$f'(-1) = -2$$



$$\frac{2x}{2} < \frac{0}{2}$$

$$x < 0$$

or

$$f'(x) = 2x$$

$$f'(1) = 2(1)$$

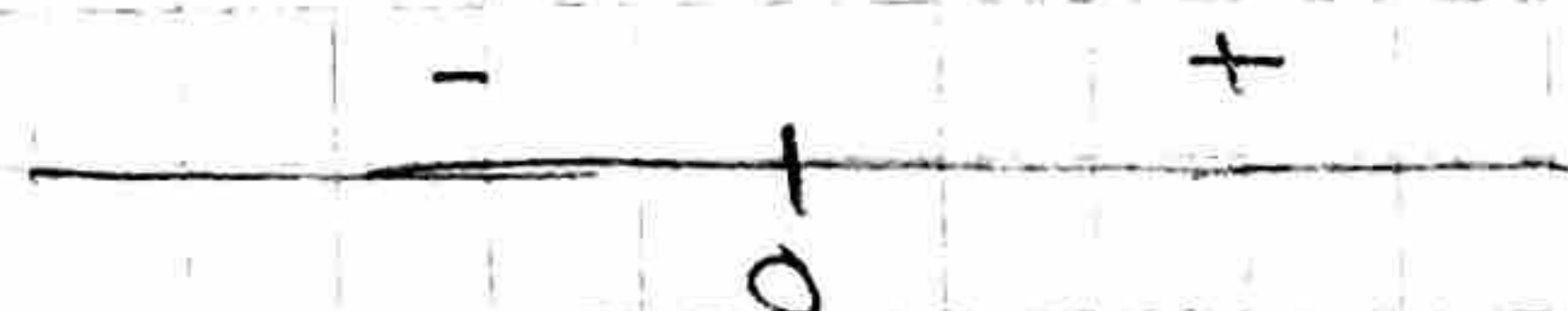
$$f'(1) = 2$$



$$\frac{2x}{2} > \frac{0}{2}$$

$$x > 0$$

$$f'(x)$$



⑤ Summary

The function f is decreasing on $(-\infty, 0)$ and increasing on $(0, \infty)$.

Example 1

Find where the function $f(x) = x^2$ is increasing and where it is decreasing.

① Get Derivative

Example 2

$$f'(x) = \frac{d}{dx} \left[\frac{f(x)}{g(x)} \right]$$

↓

$$(x-1) \frac{d}{dx} [1] - 1 \frac{d}{dx} [x-1]$$

↓

$$(x-1)^2$$

$$(x-1) \cdot 0 - 1 \cdot \frac{d}{dx} [x] - \frac{d}{dx} [1]$$

$$(x-1)^2$$

↓

$$0 - 1 \cdot 1x^{1-1} - 0$$

$$(x-1)^2$$

$$0 - 1 \cdot 1 - 0$$

$$(x-1)^2$$

↓

$$f'(x) = \frac{-1}{(x-1)^2}$$

$$\textcircled{2} \text{ Set } f'(x) = 0$$

$$\frac{-1}{(x-1)^2} = 0$$

$$(x-1) \cdot \frac{-1}{(x-1)^2} = 0 \cdot (x-1)^2$$

$$\begin{aligned} -1 &= 0 \\ \text{False, no solutions} \end{aligned}$$

$$\textcircled{3} \text{ Set intervals } x=1 \text{ causes } \frac{-1}{(x-1)^2} \text{ to be under}$$

Use $x=1$ as an endpoint

$$(-\infty, 1) \cup (1, \infty)$$



④ Set arbitrary values

$$f'(x) = -1/(x-1)^2$$

$$f'(0) = -1/(0-1)^2$$

"

$$-1/(-1)^2$$

"

$$-1/1$$

"

$$f'(0) = -1$$

$$f'(x) = -1/(x-1)^2$$

$$f'(2) = -1/(2-1)^2$$

$$-1/(1)^2$$

"

$$-1/1$$

"

$$f'(2) = -1$$

$$f'(x)$$

④ Summary

$f'(x)$ is negative over entire interval $(1, \infty)$

Example 3

Find where $f(x) = x^4 - 2x^2$ is increasing and where it is decreasing.

① Get Derivative

$$f'(x) = \frac{d}{dx} [x^4 - 2x^2]$$

$$\frac{d}{dx} [x^4] - \frac{d}{dx} [2x^2]$$

$$4x^{4-1} - 2 \cdot \frac{d}{dx} [x^2]$$

$$4x^3 - 2 \cdot 2x^{2-1}$$

$$4x^3 - 2 \cdot 2x^1$$

$$f'(x) = 4x^3 - 4x$$

② Set $f'(x) = 0$

$$4x^3 - 4x = 0$$

③ Set intervals

$$4x(x^2 - 1)$$

$$4x(x+1)(x-1)$$

$$\frac{4x}{4} = \frac{0}{4}$$

$$x = 0$$

$$x+1 = 0$$

$$-1 \quad -1$$

$$x = -1$$

$$x-1 = 0$$

$$+1 \quad +1$$

$$x = 1$$