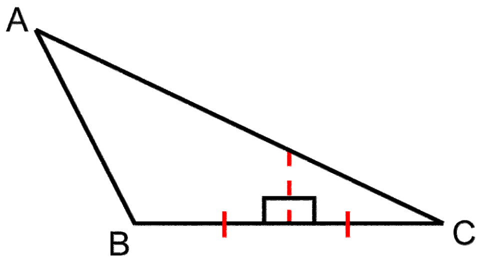
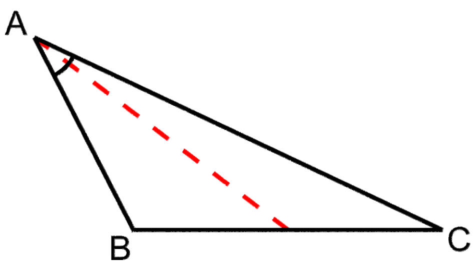
1.

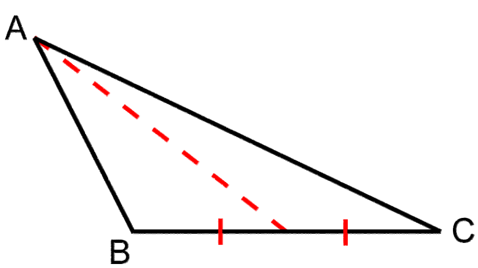
a)



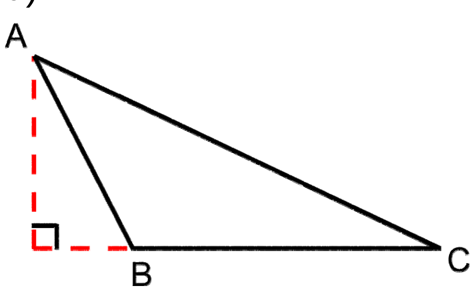
b)



c)

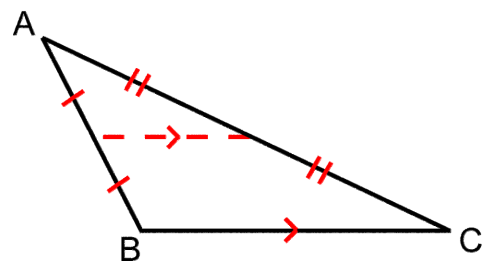


d)



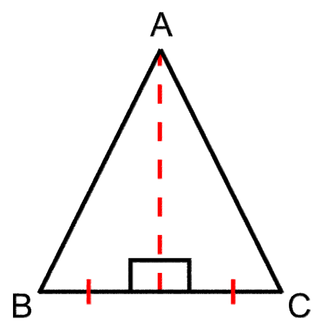
1. Continued:

e)

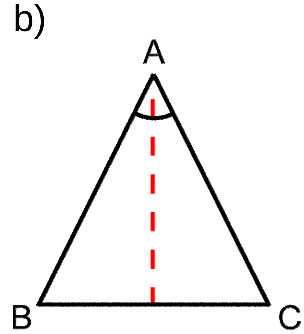


2.

a)

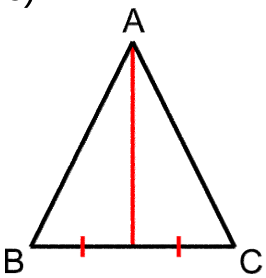


b)

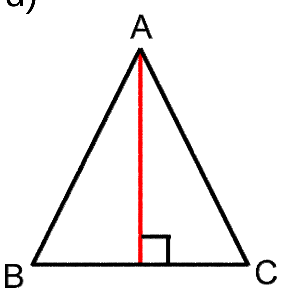


2. Continued:

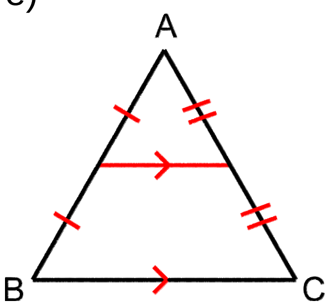
c)



d)



e)



3. Altitude

4. Median

5. Altitude, perpendicular bisector, median, angle bisector

6. Altitude

7. Median

8. Angle bisector

9. Angle bisector, median

10. Perpendicular bisector

11. Median

12. Altitude, median, perpendicular bisector, angle bisector

13. Altitude

14. Median

15. Perpendicular bisector

16. None

17. Answer: a) $K = \left(\frac{3}{2}, \frac{17}{2}\right)$

b) Slope of $\overline{CL} = \frac{13}{3}$

c) No, $\overline{NA}$ is not an altitude of ΔABC

d) Yes, $\overline{NA}$ is an altitude of ΔABC.

Detailed Solution:

a. A(-5, 10), B(8, 7) and C(-4, -8). What are the coordinates of K, if $\overline{CK}$ is a median of ΔABC.

Since $\overline{CK}$ is the median of ΔABC, then K is a midpoint of $\overline{AB}$. A median starts at the vertex of an angle, and is drawn to the midpoint of the opposite side.

Midpoint formula: $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

Midpoint of $\overline{AB}$ is: $\left(\frac{-5+8}{2}, \frac{10+7}{2}\right) = \left(\frac{3}{2}, \frac{17}{2}\right)$

Therefore: $K = \left(\frac{3}{2}, \frac{17}{2}\right)$

17. Continued:

b. A(-5, 10), B(8, 7) and C(-4, -8). What is the slope of $\overline{CL}$, if $\overline{CL}$ is the altitude from C.

Since $\overline{CL}$ is the altitude from C, then $\overline{CL}$ and $\overline{AB}$ are perpendicular by definition of altitude.

To find the slope of $\overline{CL}$, first find the slope of $\overline{AB}$ then take the negative reciprocal.

Slope formula: $m = \dfrac{y_2 - y_1}{x_2 - x_1}$

Slope of $\overline{AB} = \dfrac{7-10}{8-(-5)}$

$= \dfrac{7-10}{8+5}$

$= \dfrac{-3}{13}$

The slope of $\overline{CL}$ is the negative reciprocal:

Therefore, the slope of $\overline{CL} = \dfrac{13}{3}$.

17. Continued:

c. Given A(-5, 10), B(8, 7) and C(-4, -8). Point N on $\overline{BC}$ has coordinates $\left(\frac{8}{15}, \frac{20}{3}\right)$.

Is $\overline{NA}$ an altitude of ΔABC?

If $\overline{NA}$ is an altitude, then $\overline{NA}$ and $\overline{BC}$ are perpendicular lines and their slopes are negative reciprocals of each other.

Find the slope of $\overline{NA}$ and the slope of $\overline{BC}$. If the slopes are negative reciprocals, then $\overline{NA}$ is an altitude. If the slopes are not negative reciprocals then $\overline{NA}$ is not an altitude.

$$\text{Slope of } \overline{BC} = \frac{-8-7}{-4-8} = \frac{-15}{-12} = \frac{5}{4}$$

$$\text{Slope of } \overline{NA} = \frac{\frac{20}{3}-10}{\frac{8}{15}-(-5)} = \frac{\frac{20}{3}-\frac{30}{3}}{\frac{8}{15}+\frac{75}{15}} = \frac{-\frac{10}{3}}{\frac{83}{15}} = -\frac{10}{3}\bullet\frac{15}{83} = -\frac{50}{83}$$

Since $\frac{5}{4}$ and $\frac{-50}{83}$ are not negative reciprocals, $\overline{NA}$ is not an altitude of ΔABC.

17. Continued:

d. Given A(-5, 10), B(8, 7) and C(-4, -8). Point N on $\overline{BC}$ has coordinates $\left(\frac{180}{41}, \frac{102}{41}\right)$.

Is $\overline{NA}$ an altitude of ΔABC?

From part c, we have the slope of $\overline{BC} = \frac{5}{4}$.

$$\text{Slope of } \overline{NA} = \frac{\frac{102}{41} - 10}{\frac{180}{41} - (-5)}$$

$$= \frac{\frac{102}{41} - \frac{410}{41}}{\frac{180}{41} + \frac{205}{41}}$$

$$= \frac{-\frac{308}{41}}{\frac{385}{41}}$$

$$= -\frac{308}{41} \bullet \frac{41}{385}$$

$$= -\frac{4}{5}$$

Since $\frac{5}{4}$ and $\frac{-4}{5}$ are negative reciprocals, $\overline{NA}$ is an altitude of ΔABC.

18. a) k = (-1, 10)

b) Slope of $\overline{CL} = -\frac{7}{3}$

c) Yes, $\overline{NA}$ is an altitude of ΔABC.

d) No, $\overline{NA}$ is not an altitude of ΔABC

19. Answer: a) K = (-4, -5)

b) Slope of the perpendicular bisector of $\overline{AB} = -\frac{1}{12}$.

c) Yes, $\overline{NA}$ is an altitude of ΔABC.

d) No, $\overline{NA}$ is not an altitude of ΔABC.

Detailed Solution:

a. Given A(-3, 7), B(-5, -17) and C(4, 10). What are the coordinates of K, if $\overline{CK}$ is a median of ΔABC?

Since $\overline{CK}$ is the median of ΔABC, then K is a midpoint of $\overline{AB}$. A median starts at the vertex of an angle, and is drawn to the midpoint of the opposite side.

Midpoint formula: $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

Midpoint of $\overline{AB}$ is: $\left(\frac{-3+(-5)}{2}, \frac{7+(-17)}{2}\right) = (-4, -5)$

Therefore: K = (-4, -5)

b. Given A(-3, 7), B(-5, -17) and C(4, 10). What is the perpendicular bisector of $\overline{AB}$?

The perpendicular bisector of $\overline{AB}$, is perpendicular to $\overline{AB}$.

To find the slope of the perpendicular bisector of $\overline{AB}$, first find the slope of $\overline{AB}$, then take the negative reciprocal.

Slope formula: $m = \frac{y_2 - y_1}{x_2 - x_1}$

$$\text{Slope of } \overline{AB} = \frac{-17-7}{-5-(-3)}$$

$$= \frac{-24}{-2}$$

$$= 12$$

Take the negative reciprocal to find the slope of the perpendicular bisector of $\overline{AB}$.

Therefore, the slope of the perpendicular bisector of $\overline{AB} = -\frac{1}{12}$.

19. Continued:

c. Given A(-3, 7), B(-5, -17) and C(4, 10). Point N on $\overline{BC}$ has coordinates $\left(\frac{12}{5}, \frac{26}{5}\right)$. Is $\overline{NA}$ an altitude of ΔABC?

If $\overline{NA}$ is an altitude, then $\overline{NA}$ and $\overline{BC}$ are perpendicular lines and their slopes are negative reciprocals of each other.

Find the slope of $\overline{NA}$ and the slope of $\overline{BC}$. If the slopes are negative reciprocals then $\overline{NA}$ is an altitude. If the slopes are not negative reciprocals then $\overline{NA}$ is not an altitude.

$$\text{Slope of } \overline{BC} = \frac{-17-10}{-5-4}$$

$$= \frac{-27}{-9}$$

$$= 3$$

$$\text{Slope of } \overline{NA} = \frac{\frac{26}{5}-7}{\frac{12}{5}-(-3)}$$

$$= \frac{\frac{26}{3}-\frac{35}{5}}{\frac{12}{5}+\frac{15}{5}}$$

$$= \frac{-\frac{9}{5}}{\frac{27}{5}}$$

$$= -\frac{9}{5}\bullet\frac{5}{27}$$

$$= -\frac{1}{3}$$

Since 3 and $\frac{-1}{3}$ are negative reciprocals, $\overline{NA}$ is an altitude of ΔABC.

19. Continued:

d. Given A(-3, 7), B(-5, -17) and C(4, 10). Point N on $\overline{BC}$ has coordinates $\left(\frac{-2}{9}, \frac{-8}{3}\right)$. Is $\overline{NA}$ an altitude of ΔABC?

From part c, we have the slope of $\overline{BC} = 3$.

$$\text{Slope of } \overline{NA} = \frac{\frac{-8}{3} - 7}{\frac{-2}{9} - (-3)}$$

$$= \frac{\frac{-8}{3} - \frac{21}{3}}{\frac{-2}{9} + \frac{27}{9}}$$

$$= \frac{-\frac{29}{3}}{\frac{25}{9}}$$

$$= -\frac{29}{3} \bullet \frac{9}{25}$$

$$= -\frac{87}{25}$$

Since 3 and $\frac{-87}{25}$ are not negative reciprocals, $\overline{NA}$ is not an altitude of ΔABC.

20. a) $K = \left(-\frac{15}{2}, 2\right)$

b) Slope of the perpendicular bisector of $\overline{AB} = -\frac{1}{4}$

c) No, $\overline{NA}$ is not an altitude of ΔABC

d) Yes, $\overline{NA}$ is an altitude of ΔABC.

21. Answer: Not possible
Detailed solution:
Use the Triangle Inequality to determine if it is possible to draw a triangle with the given measures as sides:

215, 204, 7

$215 + 204 > 7$
$419 > 7$ ✔

$215 + 7 > 204$
$222 > 204$ ✔

$204 + 7 > 215$
$211 > 215$ NO

This triangle is not possible.

22. Not possible

23. Answer: Yes, this triangle is possible
Detailed Solution:
Use the Triangle Inequality to determine if it is possible to draw a triangle with the given measures as sides:

16, 12, 17

$16 + 12 > 17$
$28 > 17$ ✔

$16 + 17 > 12$
$33 > 12$ ✔

$12 + 17 > 16$
$29 > 16$ ✔

This triangle exists.

24. Yes, this triangle is possible

25. Answer: Not possible
Detailed Solution:
Use the Triangle Inequality to determine if it is possible to draw a triangle with the given measures as sides:

2.2, 12, 14.3

$2.2 + 12 > 14.3$
$14.2 > 14.3$ NO

This is not a triangle.

26. Not possible

27. Answer: $4 < x < 10$
    Detailed Solution:
    First side measure: 3
    Second side measure: 7

    Let the third side be x.
    The third side is greater than:
    $3 + x > 7$
    $x > 7 - 3$
    $x > 4$

    Third side is less than:

    $3 + 7 > x$
    $10 > x$

    Therefore: $4 < x < 10$

28. $4 < x < 16$

29. Answer: $3 < x < 27$
    Detailed Solution:
    First side measure: 15
    Second side measure: 12

    Let the third side be x.
    The third side is greater than:
    $12 + x > 15$
    $x > 15 - 12$
    $x > 3$

    Third side is less than:

    $12 + 15 > x$
    $27 > x$

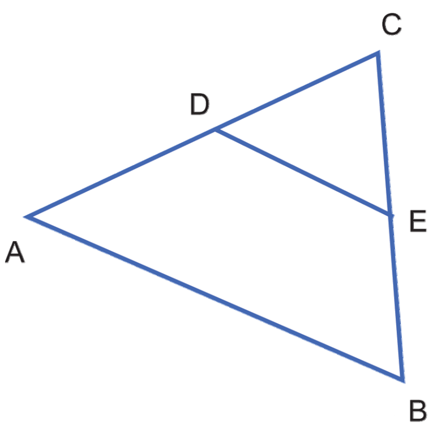
    Therefore: $3 < x < 27$

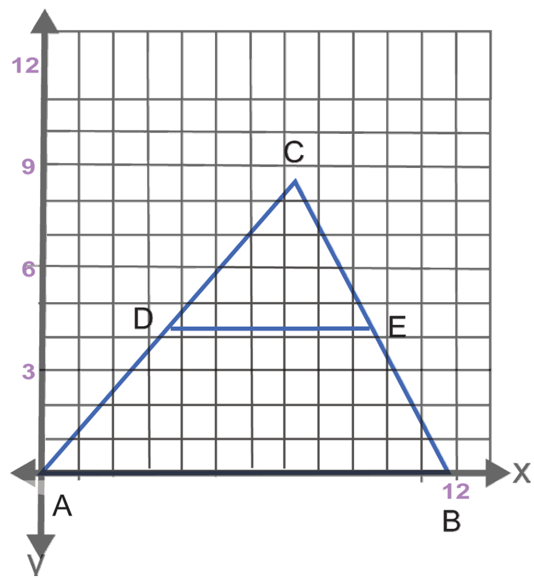
30. 19

31. Prove theorem 2.1.2:

If the mid-segment is drawn in a triangle, then it is parallel to the side that is not included in the mid-segment.

Note: Any triangle can be placed on the xy plane with coordinates: A(0,0), B(x,0), C(m,n)





31. Continued.

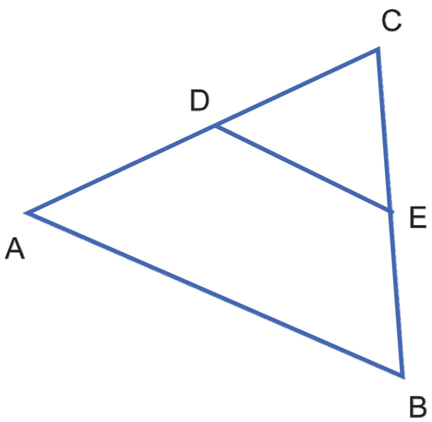
Given: A(0, 0), B(x, 0), C(m,n)
$\overline{DE}$ is a mid-segment.

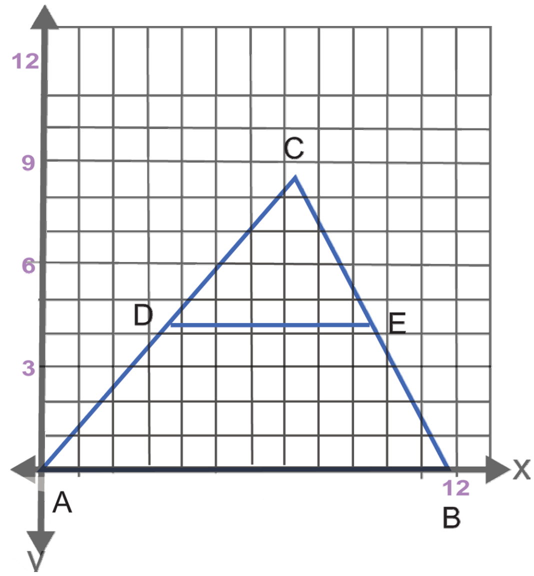
Prove: $\overline{DE} \parallel \overline{AB}$

Statement	Reason
1. $\overline{DE}$ is a mid-segment.	1. Given.
2. D is a midpoint of $\overline{AC}$.	2. Definition of mid-segment.
3. $D = \left(\frac{m+0}{2}, \frac{n+0}{2}\right) = \left(\frac{m}{2}, \frac{n}{2}\right)$	3. Midpoint formula.
4. E is a midpoint of $\overline{BC}$.	4. Definition of mid-segment.
5. $E = \left(\frac{m+x}{2}, \frac{n+0}{2}\right) = \left(\frac{m+x}{2}, \frac{n}{2}\right)$	5. Midpoint formula.
6. slope of $\overline{AB} = \frac{0-0}{x-0} = 0$	6. Slope formula.
7. slope of $\overline{DE} = \dfrac{\frac{n}{2} - \frac{n}{2}}{\frac{m+x}{2} - \frac{m}{2}} = 0$	7. Slope formula.
9. $\overline{DE} \parallel \overline{AB}$	9. Lines that have the same slope are parallel. Since $\overline{DE}$ and $\overline{AB}$ both have a slope of zero, they are parallel.

32. Prove theorem 2.1.3: If the mid-segment is drawn in a triangle, then it is half the length of the side not included in the mid-segment.

Note: Any triangle can be place on the xy plane with coordinates: A(0,0), B(x,0), C(m,n)





32. Continued:

Given: A(0, 0), B(x, 0), C(m,n).
$\overline{DE}$ is a mid-segment.

Prove: $\overline{DE} = \frac{1}{2}\overline{AB}$

Statement	Reason
1. $\overline{DE}$ is a mid-segment.	1. Given.
2. D is a midpoint of $\overline{AC}$.	2. Definition of mid-segment.
3. $D = \left(\frac{m+0}{2}, \frac{n+0}{2}\right)$ $= \left(\frac{m}{2}, \frac{n}{2}\right)$	3. Midpoint formula.
4. E is a midpoint of $\overline{BC}$.	4. Definition of mid-segment.
5. $E = \left(\frac{m+x}{2}, \frac{n+0}{2}\right)$ $= \left(\frac{m+x}{2}, \frac{n}{2}\right)$	5. Midpoint formula.
6. $\overline{AB} = \sqrt{(x-0)^2 + (0-0)^2}$ $= \sqrt{(x)^2}$ $= x$	6. Distance formula.
7. $\overline{DE} = \sqrt{\left(\frac{m+x}{2} - \frac{m}{2}\right)^2 + \left(\frac{n}{2} - \frac{n}{2}\right)^2}$ $= \sqrt{\left(\frac{x}{2}\right)^2}$ $= \sqrt{\frac{x^2}{4}}$ $= \frac{1}{2}x$	7. Distance formula.
9. $\overline{DE} = \frac{1}{2}\overline{AB}$	9. Substitution from line 6.