Example 6

$h(x) = x\sec(x^2+1)$ Get $h'(x)$

$x = x^2+1$ $\quad f(x) = x \quad g(x) = \sec(x^2+1)$

$$h'(x) = \frac{d}{dx}\left[x\sec(x^2+1)\right]$$

$$=$$

$$x \cdot \frac{d}{dx}\left[\sec(x^2+1)\right] + \sec(x^2+1)\cdot\frac{d}{dx}[x^1] \quad \leftarrow \text{Product Rule}$$

$f(x) = \sec(x)$

$g(x) = x^2+1$

$$\frac{d}{dx}[\sec(x)] = \sec(x)\tan(x)$$

Chain It

$$f'(g(x))\,g'(x)$$

$$= f'(g(x)) \cdot g'(x)$$

$$= \frac{d}{dx}\left[\sec(x^2+1)\right] \cdot \frac{d}{dx}[x^2+1]$$

$$= \sec(x^2+1)\tan(x^2+1) \cdot \frac{d}{dx}[x^2] + \frac{d}{dx}[1]$$

$$= 2x^{2-1} + 0$$

$$= \boxed{\sec(x^2+1)\tan(x^2+1) \cdot 2x}$$

$$x\left[2x \cdot \sec(x^2+1)\tan(x^2+1)\right] + \sec(x^2+1)\cdot\frac{d}{dx}[x^1]$$

$$= x\left[2x \cdot \sec(x^2+1)\tan(x^2+1)\right] + \sec(x^2+1)\cdot 1$$

$$= 2x^2\sec(x^2+1)\tan(x^2+1) + \sec(x^2+1)$$

$$\boxed{\begin{aligned} h'(x) &= 2x^2\sec(x^2+1)\tan(x^2+1) + \sec(x^2+1) \\ &\text{or} \\ &\sec(x^2+1) + 2x^2\sec(x^2+1)\tan(x^2+1) \end{aligned}}$$

Example 7

$$h(x) = \frac{\sqrt{4x^2-x}}{x+1}$$

Find equation of the tangent line to the curve $y=h(x)$ at $x=1$

$$h'(x) = \frac{d}{dx}\left[\frac{\overset{f(x)}{\sqrt{4x^2-x}}}{\underset{g(x)}{x+1}}\right]$$

Quotient Rule It $f(x) = \sqrt{4x^2-x}$, $g(x) = x+1$

$$= \frac{(x+1)\cdot\frac{d}{dx}\left[\sqrt{4x^2-x}\right] - \sqrt{4x^2-x}\cdot\frac{d}{dx}[x+1]}{(x+1)^2}$$

$$= \frac{(x+1)\cdot\frac{d}{dx}\left[(4x^2-x)^{1/2}\right] - \sqrt{4x^2-x}\cdot\frac{d}{dx}[x] + \frac{d}{dx}[1]}{(x+1)^2}$$

$$\frac{d}{dx}[x] = \frac{dx}{dx} = 1$$

$$= \frac{(x+1)\cdot\frac{d}{dx}\left[(4x^2-x)^{1/2}\right] - \sqrt{4x^2-x}\cdot 1 + 0}{(x+1)^2}$$

$$= \frac{(x+1)\cdot\frac{d}{dx}\left[(4x^2-x)^{1/2}\right] - \sqrt{4x^2-x}}{(x+1)^2}$$

Chain Rule It

$f'(g(x))\, g'(x)$

$$f'(g(x)) = \frac{d}{dx}\left[(4x^2-x)^{1/2}\right] = \frac{1}{2}(4x^2-x)^{1/2-1} = \boxed{\frac{1}{2}(4x^2-x)^{-1/2}}$$

$\cdot$

$$g'(x) = \frac{d}{dx}[4x^2-x] = \frac{d}{dx}[4x^2] - \frac{d}{dx}[x] = 4\cdot\frac{d}{dx}[x^2] - \frac{dx}{dx} = 4\cdot 2x^{2-1} - 1 = 4\cdot 2x^1 - 1 = \boxed{8x-1}$$

$$\frac{(x+1)\left[\frac{1}{2}(4x^2-x)^{-1/2}(8x-1)\right]-\sqrt{4x^2-x}}{(x+1)^2}$$

$$\boxed{\frac{1}{2}(4x^2-x)^{-1/2} = \frac{1}{2\sqrt{4x^2-x}}}$$

Need To Rationalize The Denominator For The Complete Rational Expression

$$=$$

$$\left[\frac{(x+1)\left[\frac{1}{2}(4x^2-x)^{-1/2}(8x-1)\right]-\sqrt{4x^2-x}}{(x+1)^2}\right]\cdot\frac{2\sqrt{4x^2-x}}{2\sqrt{4x^2-x}}$$

$$=$$

$$\frac{\left[(x+1)\left[\frac{1}{2}(4x^2-x)^{-1/2}(8x-1)\right]-\sqrt{4x^2-x}\right]2\sqrt{4x^2-x}}{2(x+1)^2\sqrt{4x^2-x}}$$

$$=$$

Distribute

$$\frac{\left[\left[\frac{1}{2}(4x^2-x)^{-1/2}(8x-1)\right](x+1)-\sqrt{4x^2-x}\right]2\sqrt{4x^2-x}}{2(x+1)^2\sqrt{4x^2-x}}$$

$$\frac{1}{2}(4x^2-x)^{-1/2} = \frac{1}{2}\cdot\frac{1}{\sqrt{4x^2-x}} = \boxed{\frac{1}{2\sqrt{4x^2-x}}} = \frac{1}{2\sqrt{4x^2-x}} = \frac{1}{2\sqrt{4x^2-x}}\cdot 2\sqrt{4x^2-x} = \frac{2\sqrt{4x^2-x}}{2\sqrt{4x^2-x}} = 1$$

$$2(x+1)^2\sqrt{4x^2-x}$$

$$=$$

$$2\sqrt{4x^2-x}\cdot\sqrt{4x^2-x}$$

$$=$$

$$2\sqrt{16x^4-x^2}$$

$$=$$

$$\boxed{2(4x^2-x)}$$

$$\frac{1\cdot(8x-1)(x+1)-2(4x^2-x)}{2(x+1)^2\sqrt{4x^2-x}}$$

$$=$$

$$\frac{(8x-1)(x+1)-2(4x^2-x)}{2(x+1)^2\sqrt{4x^2-x}}$$

$$=$$

$$\frac{8x^2+8x-x-1-8x^2+2x}{2(x+1)^2\sqrt{4x^2-x}}$$

$$=$$

$$\frac{8x^2-8x^2+8x+2x-x-1}{2(x+1)^2\sqrt{4x^2-x}}$$

$$=$$

$$\boxed{h'(x)=\frac{9x-1}{2(x+1)^2\sqrt{4x^2-x}}}$$

$$=$$

$$h'(1)=\frac{9(1)-1}{2(1+1)^2\sqrt{4(1)^2-(1)}}\qquad \rightarrow \boxed{8}$$

$$\longrightarrow 2(1+1)^2\sqrt{4(1)^2-1} = 2(2)^2\sqrt{4-1} = 2(4)\sqrt{3} = 8\sqrt{3} = \boxed{8\sqrt{3}}$$

$$=\frac{8}{8\cdot\sqrt{3}} = \frac{1}{\sqrt{3}}\cdot\frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{3}}{\sqrt{9}}$$

$$=$$

$$\boxed{h'(1)=\frac{\sqrt{3}}{3}}$$

?

$$h(x) = \frac{\sqrt{4x^2 - x}}{x+1} \qquad h'(1) = \frac{\sqrt{3}}{3}, \quad x = 1$$

$\downarrow$

Get $h(1)$

$$h(1) = \frac{\sqrt{4(1)^2 - 1}}{1+1}$$

$$= \frac{\sqrt{4-1}}{2}$$

$$= \boxed{h(1) = \frac{\sqrt{3}}{2}}$$

$$h'(1) = \frac{\sqrt{3}}{3} \qquad \left(1, \frac{\sqrt{3}}{2}\right)$$

$\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad$ x_1, y_1

or

$$m = \frac{\sqrt{3}}{3}$$

$$y_1 = mx + b$$

$$\frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{3}(1) + b$$

$$= $$

$$\frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{3} + b$$

$$-\frac{\sqrt{3}}{3} \qquad -\frac{\sqrt{3}}{3}$$

$$\frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{3} = b$$

$$\frac{\sqrt{3}}{2} = \frac{3\sqrt{3}}{6}$$

$$-\frac{\sqrt{3}}{3} = \frac{2\sqrt{3}}{6}$$

$$\frac{3\sqrt{3}}{6} - \frac{2\sqrt{3}}{6}$$

$$= \frac{3\sqrt{3} - 2\sqrt{3}}{6}$$

$$= \frac{3 - 2\sqrt{3}}{6}$$

$$= \boxed{\frac{\sqrt{3}}{6}}$$

$$= \boxed{b = \frac{\sqrt{3}}{6}}$$

Equation of Tangent Line Is

$$\boxed{y = \frac{\sqrt{3}}{3}x + \frac{\sqrt{3}}{6}}$$