If the average rate of change of the function $f(x) = x^2 - 6$ over the interval $[2, b]$ is 10, what is the value of b?

a, b

$[2, b]$

$$\frac{f(b) - f(a)}{b - a} = 10$$

$$\frac{f(b) - f(2)}{b - 2} = 10$$

$$\frac{(b^2 - 6) - (2^2 - 6)}{b - 2} = 10$$

$$\frac{(b^2 - 6) - (4 - 6)}{b - 2} = 10$$

$$\frac{(b^2 - 6) - (-2)}{b - 2} = 10$$

$$\frac{b^2 - 6 + 2}{b - 2} = 10$$

$$\frac{b^2 - 4}{b - 2} = 10$$

$$\frac{(b + 2)(b - 2)}{b - 2} = 10$$

$$b + 2 = 10$$

$$-2 \quad -2$$

$$b = 8$$

If the average rate of change of the function $f(x) = 4x^2 - 10$ over the interval $[2, b]$ is 48, what is the value of b?

$$[\overset{a}{2}, \overset{b}{b}]$$

$$\frac{f(b) - f(a)}{b - a} = 48$$

$$\Downarrow$$

$$\frac{f(b) - f(2)}{b - 2} = 48$$

$$\Downarrow$$

$$\frac{(4b^2 - 10) - 4(2)^2 - 10}{b - 2} = 48$$

$$\Downarrow$$

$$\frac{(4b^2 - 10) - (4(4) - 10)}{b - 2} = 48$$

$$\Downarrow$$

$$\frac{(4b^2 - 10) - (16 - 10)}{b - 2} = 48$$

$$\Downarrow$$

$$\frac{(4b^2 - 10) - 6}{b - 2} = 48$$

$$\Downarrow$$

$$\frac{4b^2 - 10 - 6}{b - 2} = 48$$

$$\Downarrow$$

$$\frac{4b^2 - 16}{b - 2} = 48$$

$$\Downarrow$$

$$\frac{4(b^2 - 4)}{b - 2} = 48$$

$$\Downarrow$$

$$\frac{4(b+2)(b-2)}{b-2} = 48$$

$$\Downarrow$$

$$\frac{4(b+2)}{4} = \frac{48}{4}$$

$$\Downarrow$$

$$b + 2 = 12$$

$$-2 \quad -2$$

$$b = 10$$

Check

$$(b+2)(b-2)$$

$$b^2 - 2b + 2b - 4$$

$$\Downarrow$$

$$b^2 - 4$$