Math E-3
Assignment 7 Solutions **TOTAL POSSIBLE POINTS = 100**

SHOW YOUR WORK TO RECEIVE FULL OR PARTIAL CREDIT.

Z scores

Please show your normal distribution curve. Refer to Z score tables at the end of the homework assignment to answer problems 2, 3, 6 and 11.

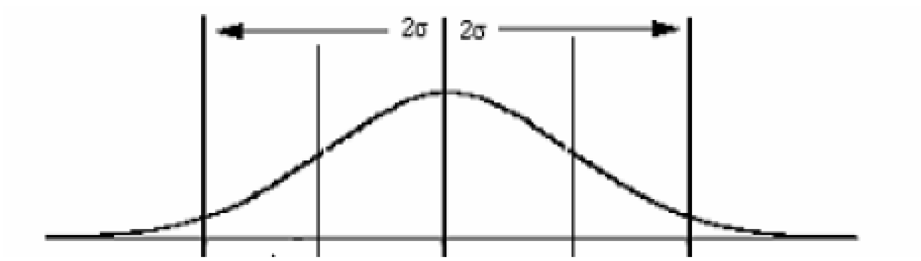
Round Z scores, standard deviations and percentages to 1 DP.
Remember to round at the end.

Problems 1-3

In a given population of males it is found that the mean weight is 170 lbs. Furthermore, the calculated standard deviation is 22.4 pounds. Assume the distribution of weights is normal.

1) Draw curve of the normal distribution of male weights. **3 points**

Diagram



125.2 lbs 147.6 lbs 170 lbs 192.4 lbs 214.8 lbs

2) Use the Z score tables at the end of the assignment to calculate the percentage of males who have weights between 140 lbs and 170 lbs? **4 points**

Calculate z:

$$Z = \frac{140-170}{22.4} = \frac{-30}{22.4} = -1.3392$$

Ignore negative and round = **1.3**
From our z table of z-scores, the percentage associated with 1.3 s.d. is .4032 or **40.32%.**
Rounded to 1 dp = 40.3%.

3) Use the Z score tables at the end of the assignment to calculate the percentage of males who have weights between 155 lbs and 195 lbs? **5 points**

You need to break this down into two pieces:

$Z = \frac{155-170}{22.4} = \frac{-15}{22.4} = -.6696$ Ignore negative and round to **.7**

From our z table of z-scores, the percentage associated with .7 s.d. is .2580 or **25.80%.**

$Z = \frac{195-170}{22.4} = \frac{25}{22.4} = 1.1160$ round to **1.1**

From our z table of z-scores, the percentage associated with 1.1 s.d. is .3643 or **36.43%.**

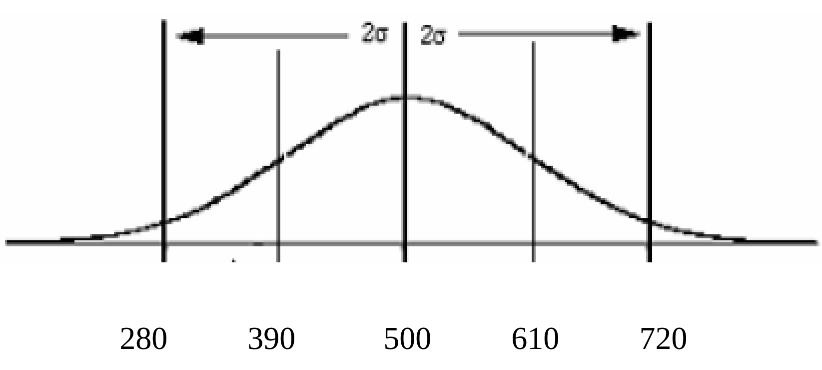
Add these two percentages together: 25.80 %+36.43% = **62.23%**
Rounded to 1 dp = 62.2%.

Problems 4-6

Suppose the scores on the Graduate Record Exam (GRE) are normally distributed with a mean of 500 and a standard deviation of 110.[1]

4) Draw curve of the distribution of GRE scores. **3 points**

Diagram



[1] Problem adapted from *Using and Understanding Mathematics*. Jeffrey Bennett, William Briggs. Addison, Wesley, Longman, 1998.

5) If the graduate school you are interested in attending requires a GRE score of 630 for admission, how many standard deviations above the mean do you need to score? **2 points**

$\frac{630-500}{110} = \frac{130}{110} = 1.1818\ldots = \mathbf{1.2}$ standard deviations

6) If the admission's officer chooses and applicant at random, what is the probability of finding a student who scores a grade over 700 on the GRE? Use the Z score table at the end of the assignment to help you answer this. **4 points**

$\frac{700-500}{110} = \frac{200}{110} = 1.8181\ldots = \mathbf{1.8\%}$

Use Table B at 1.8 standard deviations and that is .0359 or 3.59%. The probability of finding a student who scores a grade over 700 is **3.59%.**

Rounded to 1 dp = 3.6%.

Hypothesis Testing

For problems 7 through 12, assume all samples were randomly chosen even if not stated in the problem.

Make sure you follow the steps as outlined in class and the reading; be sure to include a diagram for each question, and remember to clearly state both the Null Hypothesis and the conclusion in each case.

Round standard deviations and percentages to 1 DP. <u>Remember to round at the end.</u>

Follow this format for full credit for problems 7, 8, 9, 10, and 12:

Step 1) State your Null Hypothesis – use words not just a percentage.
Step 2) Calculate the Standard Deviation.
Step 3) Draw your diagram with the mean and 1 and 2 standard deviations identified.
Step 4) Calculate (if necessary), state, and compare the observed percentage.
Step 5) Construct the proper sentence either rejecting or not rejecting the Null Hypothesis. Use the proper statistical language.
Step 6) Give an informal conclusion.

Problem 11 does not require a new hypothesis test.

Problem 7 15 points

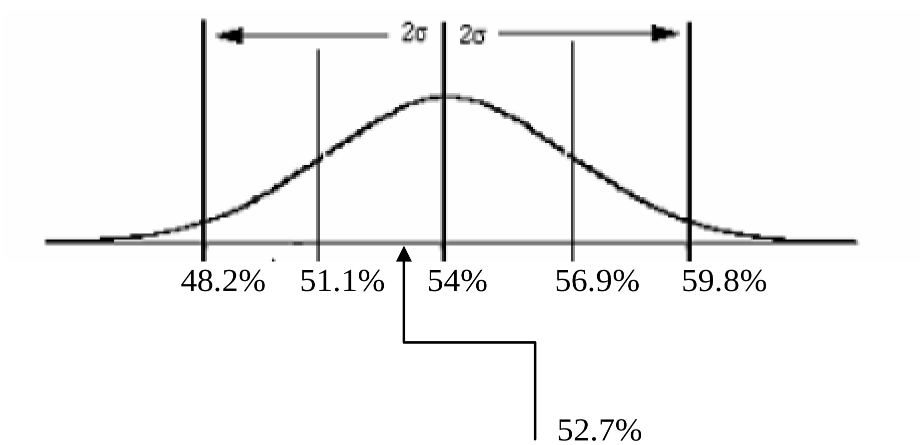
The Beautiful Body Cosmetics Company claims that its new wart cream dissolves 54% of all warts with one application. A scientist from a competing company is given the job of disproving this claim. She purchases a few jars of the product and does her own tests. If this scientist tries the cream on several randomly selected people (and randomly selected warts!) and finds that after applying the cream to 300 warts, 158 of the unattractive warts disappeared with one application. Perform a hypothesis test and determine what the scientist would conclude about Beautiful Body's product.

1. **NH:** The Beautiful Body Company's new wart cream dissolves 54% of all warts with one application.

2. Standard deviation using claim percentage as p and the survey size as n.

$$\sigma = \sqrt{\frac{.54(1-.54)}{300}} = \sqrt{\frac{.54(.46)}{300}} = \sqrt{\frac{.2484}{300}} = \sqrt{.000828} = .028774989 = \mathbf{2.9\%}$$

3. Draw the diagram:



4. Calculate the observed percentage and compare to the interval of $p \pm 2\sigma$.

$$\frac{158}{300} = .5266\ldots = \mathbf{52.7\%}$$

5. Construct the proper sentence either rejecting or not rejecting the null hypothesis.

Since our observed percentage falls <u>inside</u> the likely region, we <u>cannot reject</u> Beautiful Body Company's claim that their new wart cream dissolves 54% of warts, at a 5% level of significance.

6. Write your conclusion in your words.

   Beautiful Body Company's claim may be fairly accurate.

Problem 8 (This was an issue a few years ago, when Trent Lott was in the Senate)

15 points

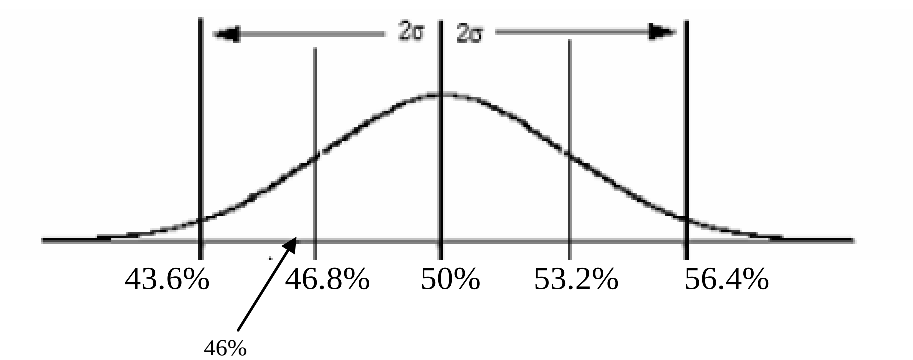
Senator Lott claims that 50% of Americans want federal funding for PBS **cut.** You, a devout fan of "Barney" and "Lamb Chop" (sadly, Shari Lewis passed away in 1998), are deeply suspicious of this claim. So, you decide to take your own *unbiased* poll. You sample 250 people and find that 46% want to see the cuts in funding. Perform a hypothesis test, and give your conclusions.

1. **NH:** 50% of Americans want federal funding for PBS cut.

2. Standard deviation using claim percentage as p and the survey size as n.

$$\sigma = \sqrt{\frac{.50(1-.50)}{250}} = \sqrt{\frac{.50(.50)}{250}} = \sqrt{\frac{.25}{250}} = \sqrt{.001} = .031622777 = \mathbf{3.2\%}$$

3. Draw the diagram:



4. Calculate the observed percentage and compare to the interval of $p \pm 2\sigma$.

   The percentage was given as **46%**.

5. Construct the proper sentence either rejecting or not rejecting the null hypothesis.

   Since our observed percentage falls <u>inside</u> the likely region, we <u>cannot reject</u> Senator Trent Lott's claim that 50% of Americans want federal funding for PBS cut, at a 5% level of significance.

6. Write your conclusion in your words.

   It's possible Trent Lott is fairly accurate about Americans' desire to cut funding for PBS.

Problem 9   15 points

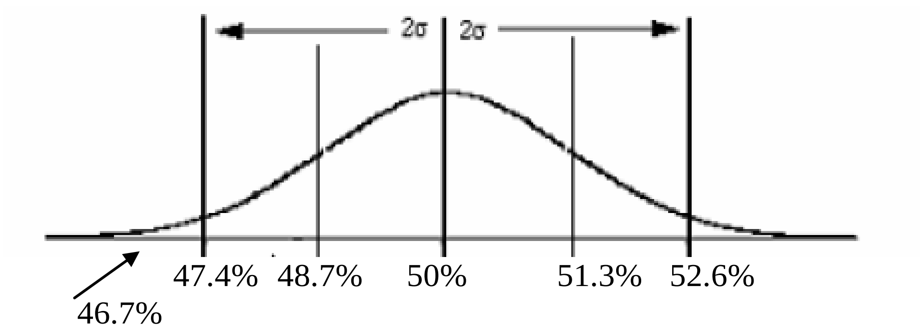
You decide to take another sample of people to find out for sure about the public's opinion on cuts to PBS, this time of 1500 people. From this sample, 700 folks favor the cuts. Perform another hypothesis test. What is your conclusion this time? If your conclusion is different than in problem 8 above, explain *why* there is a difference.

1. **NH:** 50% of Americans want federal funding for PBS cut.

2. Standard deviation changes because the sample size increases:

$$\sigma = \sqrt{\frac{.50(1-.50)}{1500}} = \sqrt{\frac{.50(.50)}{1500}} = \sqrt{\frac{.25}{1500}} = \sqrt{.000166667} = .012909944 = \mathbf{1.3\%}$$

3. Draw the diagram with new standard deviation:



4. Calculate the observed percentage and compare to the interval of p$\pm$ 2σ.

$\frac{700}{1500}$ =.46666…=**46.7%**

5. Construct the proper sentence either rejecting or not rejecting the null hypothesis.

Since our observed percentage falls <u>outside</u> the likely region, we <u>reject</u> Senator Trent Lott's claim that 50% of Americans want federal funding for PBS cut, at a 5% level of significance.

6. Write a short concluding statement in your own words:
It looks like Trent Lott could be wrong.

The conclusion changes because with the increased sample size, our margin of error decreases, and the new survey percentage falls outside the likely region.

Problem 10 15 points

The Mars Company, maker of M&M's, recently claimed that M&M's were so much fun because they were a perfect rainbow, that is, each bag they made contained equal numbers of each of the five colors. An enterprising student decided to test this and bought one small bag, chosen at random. The number of M&M's found for each color was:

Brown:	19
Blue:	5
Green:	5
Orange:	9
Yellow:	10

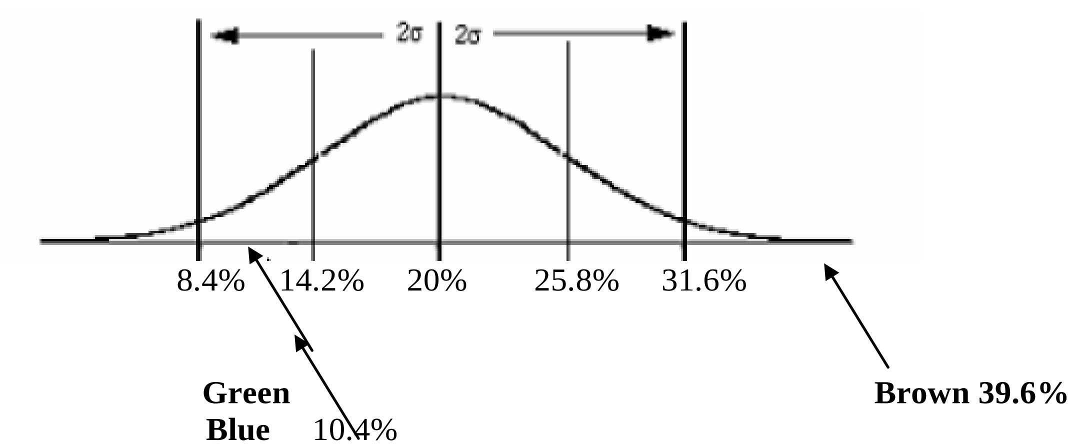
Perform a hypothesis test and state your conclusions about Mars' claim.

1. **NH:** Mars makes it bags of M&M's with equal numbers of each of the five colors with 20% apiece of the browns, blues, greens, oranges, and yellows.

2. Standard deviation using claim percentage as p and the survey size as n.

$$\sigma = \sqrt{\frac{.20(1-.20)}{48}} = \sqrt{\frac{.20(.80)}{48}} = \sqrt{\frac{.16}{48}} = \sqrt{.00033} = .0577350 = \mathbf{5.8\%}$$

3. Draw the diagram:



4. Calculate the observed percentage and compare to the interval of $p \pm 2\sigma$.

Brown %= $\frac{\text{19 brown M\&M's}}{\text{48 Total M\&M's}}$ =.3958 = **39.6 %**

Blue % = $\frac{5}{48}$ = **10.4%** Green % = $\frac{5}{48}$ = **10.4%** Orange = $\frac{9}{48}$ = **18.8%** Yellow = $\frac{10}{48}$ = **20.8%**

5. Construct the proper sentence either rejecting or not rejecting the null hypothesis.

Since our observed percentage falls outside the likely region, we reject Mars' claim that bags of their M&M's contain equal numbers of each of their 5 colors, at a 5% level of significance.

6. Write your conclusion in your words.

It seems that there are not equal numbers of each color of M&M's.

Problem 11 4 points

Refer to problem 10 above**.** If the colors were in equal numbers, what would be the probability that a random sample of 48 M&M's will have the above proportion of brown ones? (Hint: Go back to the Normal Distribution Curve and look at the probabilities under the curve.) No need to perform another hypothesis test here.

- Any answer that is 2.5 % or less is acceptable. Common answers will be: less than 2.5%, less than .15%, or less than .1%.

- Looking at where the brown sample percent fell in the diagram above tells you that probability is **less than 2.5%.** If you drew in the third standard deviation, you would see there is less than a **.15%** chance of getting this result.

- For the most accurate result calculate the z score:

$$z = \frac{39.6 - 20}{5.8} = 3.379\ldots$$

Round to 3.4, look at table B for the area beyond and that has a value of .0003 = **.03%,** a very small probability.

Problem 12 15 points

A psychologist wants to test whether very young children are especially attracted to bright 'crayon box' colors. Thus, she places her young subjects in a playroom with three balls to choose from:

one is striped with black and white
one is made up of two assorted pastels shades
the last is striped with two bright primary colors.

The balls are identical in all other respects and their position is shuffled between trials so that it will not influence the results. From her observations she found that of the 80 toddlers she tested, 42 went for the brightly colored ball.

Formulate an appropriate null hypothesis as if you were the researcher, and perform a hypothesis test. (Remember that in hypothesis testing, you must have a proportion (percent) to work with. Think carefully about this percentage.) At the end, state your conclusion explicitly, i.e. do children tend to show a preference for bright colors?

Step 1: State the Null Hypothesis (There are several ways to express this hypothesis.)

NH: Young children show no preference in their choice of colors.

OR NH: Young children choose each colored ball an equal number

of times.

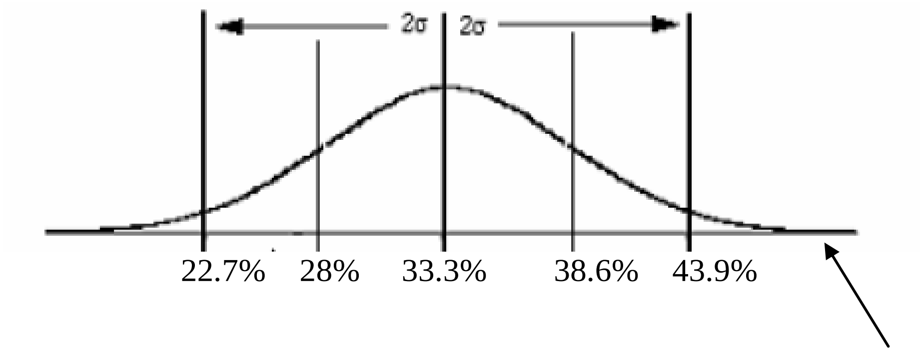
In this case, in order to test this hypothesis we need to have a percentage. The psychologist uses three balls of different colors. Therefore, we could have a null hypothesis that says:

NH: Children will be equally attracted to all three balls.

Step 2: Calculate the standard deviation using the claim for p and the number of toddlers in the sample for n. **p is rounded to one decimal place:**

$\sigma = \sqrt{\frac{.333(1-.333)}{80}} = \sqrt{\frac{.333(.667)}{80}} = \sqrt{\frac{.222111}{80}} = \sqrt{.002776388}$ =.052691437 =5.2691437% = **5.3%**

Step 3: Draw the picture (calculate the likely region, p + 2σ and p - 2σ)



52.5%

Step 4: Compare the observed percentage that you got in your sample to this interval. For this part, we need to calculate the test percent.

$\frac{42}{80}$ = .525 =52.5%. That is **52.5%** of the time, children chose the brightly colored ball.

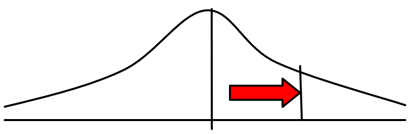
Step 5: Since our test percent falls outside the likely region, we reject the null hypothesis that children are equally attracted to all colored balls, at a 5% level of significance.

Step 6: We conclude, therefore, that the children do indeed show a preference toward bright colors!

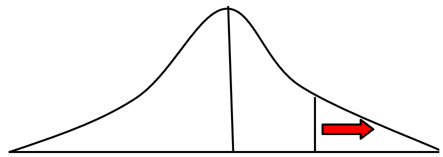
Z -Scores*

$Z = \frac{x - x\ bar}{\sigma}$ $x\ bar$=mean

A



B



Z	Area between the Mean and X (curve A)	Area beyond X (curve B)
.1	.0398	.4602
.2	.0793	.4207
.3	.1179	.3821
.4	.1554	.3446
.5	.1915	.3085
.6	.2257	.2743
.7	.2580	.2420
.8	.2881	.2119
.9	.3159	.1841
1	.3413	.1587
1.1	.3643	.1357
1.2	.3849	.1151
1.3	.4032	.0968
1.4	.4192	.0808
1.5	.4332	.0668
1.6	.4452	.0548
1.7	.4554	.0446
1.8	.4641	.0359
1.9	.4713	.0287
2	.4772	.0228
2.1	.4821	.0179
2.2	.4861	.0139
2.3	.4893	.0107
2.4	.4918	.0082
2.5	.4838	.0062
2.6	.4953	.0047
2.7	.4965	.0035
2.8	.4974	.0026
2.9	.4981	.0019
3	.4987	.0013
3.1	.4990	.0010
3.2	.4993	.0007
3.3	.4995	.0005
3.4	.4997	.0003
3.5	.4998	.0002
3.6	.4998	.0002

*Adapted from "*Understanding Social Statistics*" by Jane Fielding and Nigel Gilbert.