Interpret the function $g(x) = \int_0^x f(t)\,dt$ for $x > 0$, where $f(t) = t$.
Use the interpretation to find $g(1)$, $g(2)$ and $g(0)$. Find $g'(x)$.

① Define $g(x)$

$g(x)$ represents the area under the curve $f(t) = t$ from $t = 0$ to $t = x$.

② Define area under curve.

Area under the curve $y = t$ from $t=0$ to $t=x$ is the area of a triangle with base length x and height x, then

$$\boxed{g(x) = \int_0^x f(t)\,dt \rightarrow \text{area of triangle} \\ \parallel \\ g(x) = \frac{1}{2}x^2}$$

③ Get $g(1)$, $g(2)$ and $g(0)$

$$\boxed{\text{Area of triangle} \quad \frac{1}{2} \cdot \text{base} \cdot \text{height}}$$

$g(x) = \frac{1}{2}x^2$

$g(1) = \frac{1}{2}(1)^2$

$\parallel$

$\frac{1}{2}(1)^2$

$\parallel$

$\boxed{g(1) = \frac{1}{2}}$

$g(2) = \frac{1}{2}(2)^2$

$\parallel$

$\frac{1}{2}(2)^2$

$\parallel$

$\frac{1}{\cancel{2}_1}(\cancel{4}^2)$

$\parallel$

$\boxed{g(2) = 2}$

$g(0) = \frac{1}{2}(0)^2$

$\parallel$

$\frac{1}{2}(0)^2$

$\parallel$

0

$\parallel$

$\boxed{g(0) = 0}$

④ Get $g'(x)$

$$g(x) = \frac{1}{2}x^2$$

$$\Downarrow$$

$$g'(x) = \frac{d}{dx}\left[\frac{1}{2}x^2\right]$$

$$= \frac{1}{2} \cdot \frac{d}{dx}\left[x^2\right]$$

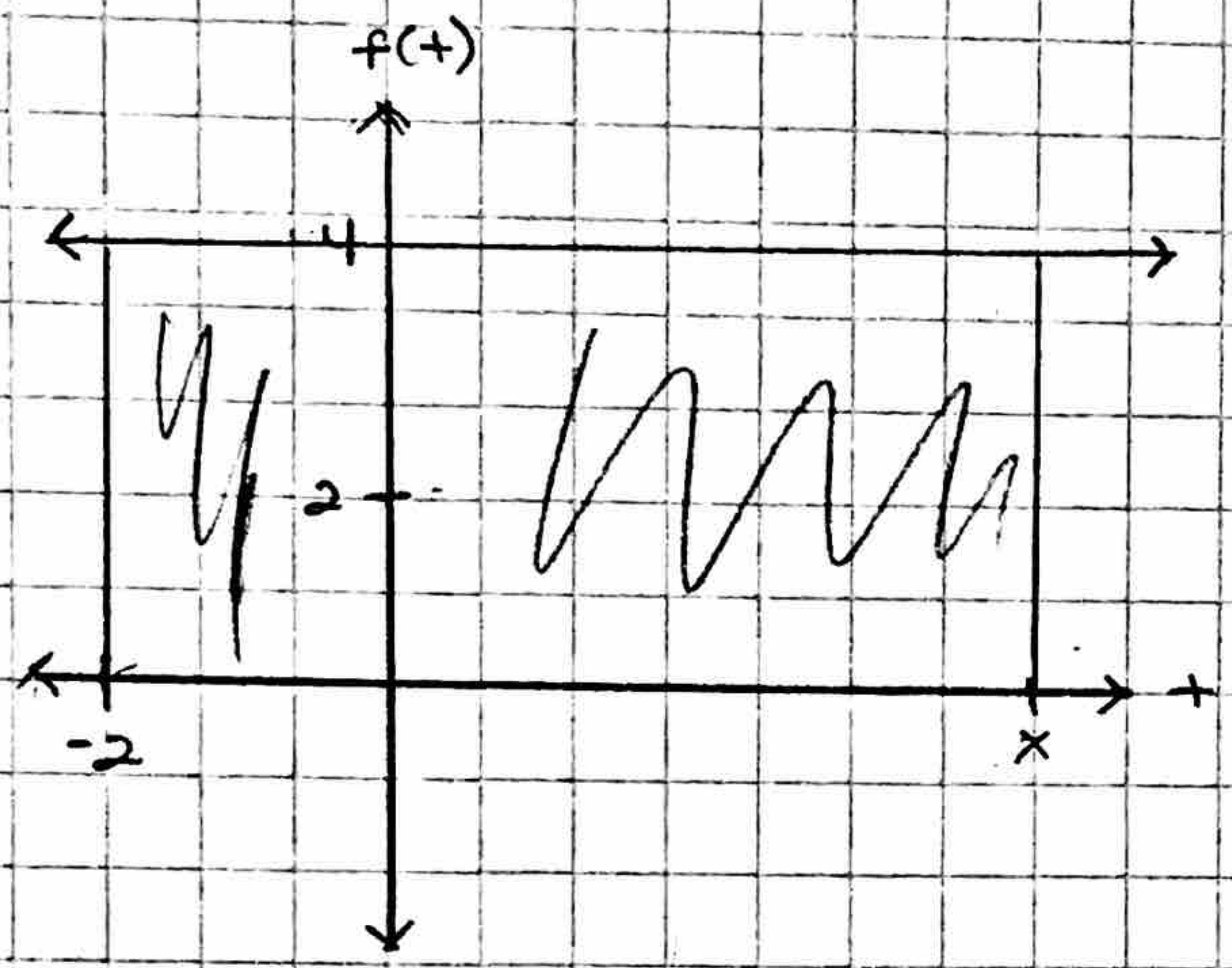
$$= \frac{1}{2} \cdot 2x^{2-1}$$

$$= \frac{1}{\cancel{2}_1} \cdot \cancel{2}^1 x^1$$

$$= x^1$$

$$\boxed{g'(x) = x}$$

Interpret the function $g(x) = \int_{-2}^{x} 4 \, dt$ for $x > -2$ and $g(t) = t$.
Use the interpretation to find $g(1)$, $g(2)$ and $g(0)$.
Get $g'(x)$



① Define $g(x)$

$g(x)$ represents the area under the curve $f(t) = 4$ from $t = -2$ to $t = x$.

② Define area under curve

Area under curve $y = 4$ from $t = -2$ to $t = x$ is area of rectangle with base length $x + 2$ and height 4, then

$$g(x) = \int_{-2}^{x} 4 \, dt = 4(x+2) \rightarrow \text{area of rectangle}$$

$$g(x) = 4(x+2)$$

③ Get $g(1)$, $g(2)$ and $g(0)$

$g(x) = 4(x+2)$ Area of rectangle
base · height

$g(1) = 4(1+2) = 4(3)$
$\boxed{g(1) = 12}$

$g(2) = 4(2+2) = 4(4)$
$\boxed{g(2) = 16}$

$g(0) = 4(0+2) = 4(2)$
$\boxed{g(0) = 8}$

④ Get $g'(x)$

$$g(x) = 4(x+2)$$

$$\downarrow$$

$$g'(x) = \frac{d}{dx}\left[4(x+2)\right]$$

$f(x) = 4, \quad g(x) = (x+2)$

$$4 \cdot \frac{d}{dx}[(x+2)] + (x+2) \cdot \frac{d}{dx}[4]$$

$f(x) = x$
$g(x) = 2$

$$=$$

$$4 \cdot \left(\frac{d}{dx}[x] + \frac{d}{dx}[2]\right) + (x+2) \cdot 0$$

$$=$$

$$4(1x^{1-1} + 0) + 0$$

$$=$$

$$4(x^0 + 0)$$

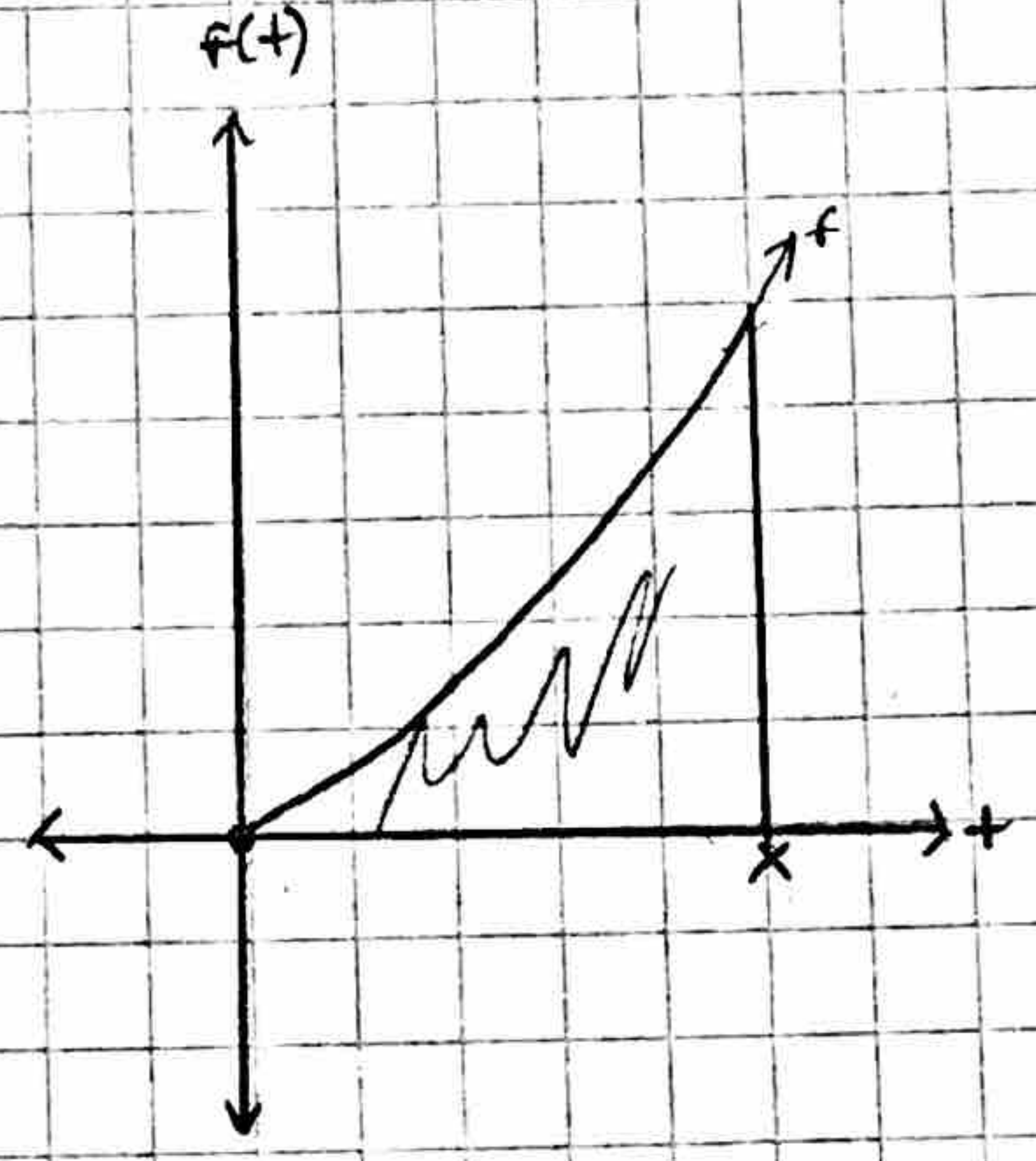
$$=$$

$$4(1)$$

$$=$$

$$\boxed{g'(x) = 4}$$

Interpret the function $g(x) = \int_0^x t^2\,dt$ for $x > 0$.
Use the interpretation to find $g(1)$, $g(2)$ and $g(0)$
Get $g'(x)$



① Before drawing this out understand what $g(x) = \int_0^x t^2\,dt$ for $x > 0$ mean

$\int_0^x \rightarrow$ interval from 0 to x

$t^2 = f(t)$ from $t = 0$ to $t = x$

Function $g(x)$ represents the area under the curve $f(t) = t^2$ from $t = 0$ to $\boxed{t = x.}$

② Express $g(x) = \int_0^x t^2\,dt$ as a limit

The shaded area will not be easily computed since it is not a defined shape (triangle, rectangle, etc).

$[0, x]$

$\downarrow$

$[0, x/n], [x/n, 2x/n], \ldots [(n-1)(x/n), x]$

$\uparrow$

n subintervals of equal width

$\Delta t = (x - 0)/n = x/n$

$\downarrow$

$t_i^* = ix/n$

$f(t) = t^2$, for any value of t

$f(t_i^*) =$

$\downarrow$

$$\downarrow$$
$$f(ix/n)$$
$$\downarrow$$
$$f(t_i^*) = (ix/n)^2$$

$$\boxed{g(x) = \int_0^x t^2\,dt = \lim_{n\to\infty} \sum_{i=1}^{n} f(t_i^*)\,\Delta t = \lim \sum_{i=1}^{n} \left(\frac{ix}{n}\right)^2 \frac{x}{n}}$$

$$= \left(\frac{x}{n}\right)^2 \cdot \frac{x}{n} \cdot i$$

③ Get Limit

$$= \lim_{n\to\infty} \sum_{i=1}^{n} \frac{x^3}{n^3}\, i \cdot i$$

$$= \boxed{\lim_{n\to\infty} \sum_{i=1}^{n} \frac{x^3}{n^3}\, i^2}$$

$$\lim_{n\to\infty} \sum_{i=1}^{n} \frac{x^3}{n^3}\, i^2 \longrightarrow x^3/n^3 \text{ do not depend on } \Sigma$$

$$= \lim_{n\to\infty} \frac{x^3}{n^3} \cdot \sum_{i=1}^{n} i^2 \rightarrow \sum_{i=1}^{n} = \frac{n(n+1)(2n+1)}{6}$$

Consecutive Integer Proof

$$= \lim_{n\to\infty} \frac{x^3}{n^3} \left[\frac{n(n+1)(2n+1)}{6}\right]$$

$$= \lim_{n\to\infty} \frac{x^3}{1} \cdot \frac{1}{\cancel{n}^3} \left[\frac{\cancel{n}(n+1)(2n+1)}{6}\right]$$

$$= \lim_{n\to\infty} x^3 \cdot \frac{1}{n^2} \left[\frac{(n+1)(2n+1)}{6}\right]$$

$$x^3 \cdot \lim_{n\to\infty} \left[\frac{1}{n^2} \cdot \frac{(n+1)(2n+1)}{6}\right]$$

$$= x^3 \cdot \lim_{n\to\infty} \left[\frac{(n+1)(2n+1)}{6n^2}\right]$$

$$= x^3 \lim_{n\to\infty} \left[\frac{2n^2 + n + 2n + 1}{6n^2}\right]$$

$$x^3 \cdot \lim_{n\to\infty}\left[\frac{2n^2+3n+1}{6n^2}\right]$$

$$=$$

$$x^3 \cdot \lim_{n\to\infty}\left[\frac{2\cancel{n^2}}{6\cancel{n^2}}+\frac{3n}{6n^2}+\frac{1}{6n^2}\right]$$

$$=$$

$$x^3 \cdot \lim_{n\to\infty}\left[\frac{1}{3}+\frac{1}{2n}+\frac{1}{6n^2}\right]$$

$$=$$

$$x^3 \cdot \lim_{n\to\infty}\frac{1}{3}+\lim_{n\to\infty}\frac{1}{2n}+\lim_{n\to\infty}\frac{1}{6n^2}$$

$$=$$

$$x^3 \cdot \frac{1}{3}+\frac{\lim_{n\to\infty} 1}{\lim_{n\to\infty} 2n}+\frac{\lim_{n\to\infty} 1}{\lim_{n\to\infty} 6n^2}$$

$$=$$

$$x^3 \cdot \frac{1}{3}+\frac{1}{2\cdot\lim_{n\to\infty} n}+\frac{1}{6\cdot\lim_{n\to\infty} n^2}$$

$$=$$

$$x^3 \cdot \frac{1}{3}+\frac{1}{2\cdot\infty}+\frac{1}{6\cdot\lim_{n\to\infty} n\cdot\lim_{n\to\infty} n}$$

$$=$$

$$x^3 \cdot \frac{1}{3}+\frac{1}{\text{Indef}}+\frac{1}{6\cdot\infty\cdot\infty}$$

$$=$$

$$x^3 \cdot \frac{1}{3}+\frac{1}{\text{Indef}}+\frac{1}{6\cdot\infty}$$

$$=$$

$$x^3 \cdot \frac{1}{3}+\frac{1}{\text{Indef}}+\frac{1}{\text{Indef}}$$

$$=$$

$$\frac{x^3}{3}$$

or

$$\boxed{\frac{1}{3}\cdot x^3}$$

Area Under Curve

$$\boxed{g(x)=\int_0^x t^2\,dt=\frac{x^3}{3}}$$

④ Get $g(1)$, $g(2)$ and $g(0)$

$$g(x) = \frac{x^3}{3}$$

$g(1) = \frac{1^3}{3}$ $\quad$ $g(2) = \frac{2^3}{3}$ $\quad$ $g(0) = \frac{0^3}{3}$

$\quad = \quad$ $\quad = \quad$ $\quad = \quad$

$g(1) = 1/3$ $\quad$ $g(2) = 8/3$ $\quad$ $g(0) = 0$

⑤ Get $g'(x)$

$g(x) = \frac{x^3}{3} \longrightarrow$ Extract Constant $x^3 \div 3 = \boxed{x^3 \cdot \frac{1}{3}}$

$$=$$

$$g(x) = x^3 \cdot \frac{1}{3}$$

$$\downarrow$$

$$g'(x) = \frac{d}{dx}\left[x^3 \cdot \frac{1}{3}\right]$$

$$=$$

$$\frac{1}{3} \cdot \frac{d}{dx}[x^3]$$

$$=$$

$$\frac{1}{3} \cdot 3x^{3-1}$$

$$=$$

$$\frac{1}{3} \cdot 3x^2$$

$$=$$

$$\frac{3x^2}{3}$$

$$=$$

$$x^2$$

$$\boxed{g'(x) = x^2}$$