Warm Up

b. $\overline{\forall x\, P(x)} = \exists x P(x)$

c. $\overline{\exists x\, P(x)} = \forall x P(x)$

d. The effect of the quantification of a proporitonal function turns the function into a proposition.

e.

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1. $\forall\ x \in R.\ \forall\ y \in R \quad \overset{P(x,y)}{x^2 > y}$ (False)

$\forall x \in R$	$\forall y \in R$	$x^2 > y$	Truth	Counter
$(x=1, x=2, x=3)$	$(y=1, y=2, y=3)$	$(1)^2 > 1$	F	$x=1, y=1$
			$\wedge$	
		$(2)^2 > 2$	T	
			$\wedge$	
		$(3)^2 > 3$	T	

$\exists\ x \in R.\ \exists\ y \in R \ni \overset{P(x,y)}{x^2 \leq y}$ (True)

$\exists x \in R$	$\exists y \in R$	$x^2 \leq y$	Truth	witness
$x=1$	$y=1$	$(1)^2 \leq 1$	T	$x=1, y=1$
			$\vee$	
$x=2$	$y=2$	$(2)^2 \leq 2$	F	
			$\vee$	
$x=3$	$y=3$	$(3)^2 \leq 3$	F	

2. $\forall x \in \mathbb{R} \quad \exists y \in \mathbb{R} \ni \overset{P(x,y)}{x + y = 0}$ (False) $y = -x \rightarrow$
$x + -x = 0$

$\forall x \in \mathbb{R}$	$\exists y \in \mathbb{R}$	$x + y = 0$	Truth	Witness
$x = 0$	$y = 0$	$0 + 0 = 0$	T	$x = 0, y = 0$
			$\wedge$	
$x = 1$	$y = 1$	$1 + 1 = 2$	F	

$\exists x \in \mathbb{R} \ni \forall y \in \mathbb{R}, \overset{P(x,y)}{x + y \neq 0}$ (True)

$\exists x \in \mathbb{R}$	$\forall y \in \mathbb{R}$	$x + y \neq 0$	Truth	Counter
$x = 0$	$(y = 0, y = 1, y = 2)$	$0 + 0 \neq 0$	F	$x = 0, y = 0$
			$\vee$	
		$0 + 1 \neq 0$	T	
			$\vee$	
		$0 + 2 \neq 0$	T	

3. $\exists x \in \mathbb{R} \quad \exists y \in \mathbb{R} \ni \overset{P(x,y)}{x < y^2}$ (True)

$\exists x \in \mathbb{R}$	$\exists y \in \mathbb{R}$	$x < y^2$	Truth	Witness
$x = 0$	$y = 1$	$0 < (1)^2$	T	$x = 0, y = 1$
			$\vee$	
$x = 1$	$y = 2$	$1 < (2)^2$	T	$x = 1, y = 2$
			$\vee$	
$x = 2$	$y = 3$	$2 < (3)^2$	T	$x = 2, y = 3$

$\forall x \in \mathbb{R} \quad \forall y \in \mathbb{R}, \overset{P(x,y)}{x \geq y^2}$ (False)

$\forall x \in \mathbb{R}$	$\forall y \in \mathbb{R}$	$x \geq y^2$	Truth	Counter
$(x = 0, x = 1, x = 2 \ldots)$	$(y = 1, y = 2, y = 3 \ldots)$	$0 \geq (1)^2$	F	$x = 0, y = 1$
			$\wedge$	
		$1 \geq (2)^2$	F	$x = 1; y = 2$
			$\wedge$	
		$2 \geq (3)^2$	F	$x = 2, y = 3$

5. $\forall x \in R \quad \exists y \in R \ni x^2 + y^2 = 1$ (False) $\rightarrow$

$$x^2 + y^2 = 1$$
$$-x^2 \quad -x^2$$
$$y = \sqrt{1 - x^2} = y = i\sqrt{3}$$

$\forall x \in R$	$\exists y \in R$	$x^2 + y^2 = 1$	Truth	Counter $\boxed{y \text{ is Complex}}$
$x = 1$	$y = 1$	$(1)^2 + (1)^2 = 1$	T	$x = 1, y = 1$
			$\wedge$	
$x = 2$	$y = 2$	$(2)^2 + (2)^2 = 8$	F	

$\exists x \in R \ni \forall y \in R \; x^2 + y^2 \neq 1$ (True)

$\exists x \in R$	$\forall y \in R$	$x^2 + y^2 \neq 1$	Truth	Witness
$x = 1$	$(y = 1, y = 2, \ldots)$	$(1)^2 + (1)^2 \neq 1$	F	
			$\vee$	
		$(1)^2 + (2)^2 \neq 1$	T	$x = 1, y = 2$

7. $\exists x \in R \quad \exists y \in R \ni x^2 + y^2 = 25$ (True)

$\exists x \in R$	$\exists y \in R$	$x^2 + y^2 = 25$	Truth	Witness
$x = 0$	$y = 5$	$(0)^2 + (5)^2 = 25$	T	$x = 0, y = 5$
			$\vee$	
$x = 4$	$y = 4$	$(4)^2 + (4)^2 = 25$	F	

(False)

$\forall x \in R$	$\forall y \in R$	$x^2 + y^2 \neq 25$	Truth	Counter
$x = 0$	$y = 5$	$(0)^2 + (5)^2 \neq 25$	F	$x = 0, y = 5$
			$\wedge$	
$y = 4$	$y = 4$	$(4)^2 + (4)^2 \neq 25$	T	

9. (True)

$\forall x \in \mathbb{Z}$	$\forall y \in \mathbb{Z}$	$x^2+y^2 \geq 0$	Truth	Counter
$(x=-1, x=-2, x=-3)$	$(y=-4, y=-5, y=-6, \ldots)$	$(-1)^2+(-4)^2 \geq 0$	T	
			$\wedge$	
		$(-2)^2+(-5)^2 \geq 0$	T	
			$\wedge$	
		$(-3)^2+(-6)^2 \geq 0$	T	

(False)

$\exists x \in \mathbb{Z}$	$\exists y \in \mathbb{Z}$	$x^2+y^2 < 0$	Truth	Witness
$x=-1$	$y=-4$	$(-1)^2+(-4)^2 < 0$	F	
			$\vee$	
$x=-2$	$y=-5$	$(-2)^2+(-5)^2 < 0$	F	
			$\vee$	
$x=-3$	$y=-6$	$(-3)^2+(-6)^2 < 0$	F	

11. (True)

$\exists x \in W \ni$	$\forall y \in W$	$x < y+1$	Truth	Witness
$x=0$	$(y=0, y=1, y=2, \ldots)$	$0 < 0+1$	T	$x=0, y=0$
			$\vee$	
		$0 < 1+1$	T	$x=0, y=1$
			$\vee$	
		$0 < 2+1$	T	$x=0, y=2$

$\forall x \in W$	$\exists y \in W$	$x \geq y+1$	Truth	Counter
$(x=0, x=1, x=2 \ldots)$	$y=0$	$0 \geq 0+1$	F	$x=0, y=0$
			$\wedge$	
		$1 \geq 0+1$	T	
			$\wedge$	
		$2 \geq 0+1$	T	

(P) $\forall x \in R$ if $\boxed{x > 1}$, then $x^2 > x$ (True)

Practice

$$x^2 > x$$
$$-x \quad -x$$
$$x^2 - x > 0$$
$$x(x-1) > 0$$

$x = 0$ $\qquad$ $x - 1 = 0$, $+1 \; +1$, $x = 1$

$+ \quad 0 \quad - \quad 1 \quad +$

$x < 0$ or $\boxed{x > 1}$

$p \to q \qquad p \wedge \bar{q}$

$\exists x \in R \ni x > 1$ and $x^2 \leq x$

$$x^2 - x \leq 0$$
$$x(x-1) \leq 0$$

$\exists x \in R \ni x > 1$ and $0 \leq x \leq 1$

(-1): $-1(-1-1)$
$-1(-2)$
$+$

$(.5)$: $.5(.5-1)$
$.5(-.5)$

(2): $2(2-1)$
$2(1)$

$$x^2 \leq x$$
$$-x \quad -x$$
$$x^2 - x \leq 0$$
$$x(x-1) \leq 0$$

$x = 0 \qquad x - 1 = 0$, $+1 \; +1$, $x = 1$

$+ \quad 0 \quad - \quad 1 \quad +$

$0 \leq x \leq 1$

$p \rightarrow q$

13. $\exists\, x \in \mathbb{N} \ni$ if $x > 1$, then $x^2 - x > 0$

$x(x-1) > 0$

$\exists\, x \in \mathbb{N} \ni$ if $x > 1$, then $x > 1$
(True)

$p \quad \wedge \quad \bar{q}$

$\forall x \in \mathbb{N} \quad x > 1$ and $x^2 - x \leq 0$

$x(x-1) \leq 0$

$\forall x \in \mathbb{N} \quad \underbrace{x > 1 \text{ and } 0 \leq x \leq 1}_{\text{Contradiction}}$ (False)

Solve Inequality

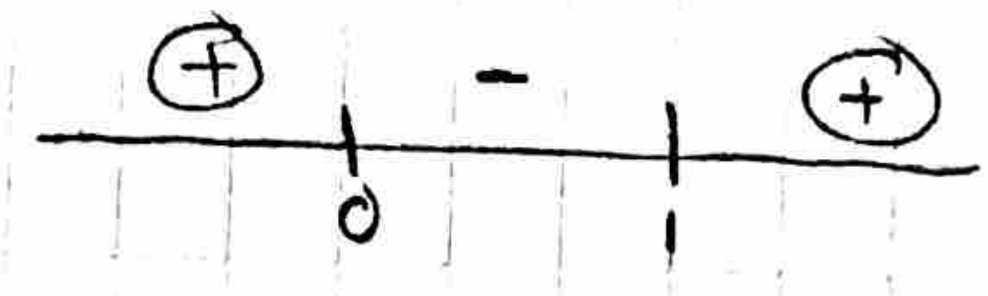
$x^2 - x > 0$

$=$

$x(x-1) > 0$

$x = 0$

$x - 1 = 0$
$+1 \quad +1$
$x = 1$



$(-1): -1(-1-1)$
$-1(-2)$
$+$

$(.5): .5(.5-1)$
$.5(-.5)$
$-$

$(2): 2(2-1)$
$2(1)$
$+$

$x < 0$ or $\boxed{x > 1}$

$0 \leq x \leq 1$

14. $\forall\ x \in \mathbb{N}$ if $x > 1$ then $x^2 - x > 0$

$$x(x-1)$$

$\forall\ x \in \mathbb{N}$ if $x > 1$, then $\boxed{x > 1}$

(True)

$\exists\ x \in \mathbb{N} \ni x > 1$ and $x^2 - x \leq 0$

$$x(x-1) \leq 0$$

$\exists\ x \in \mathbb{N} \ni \underbrace{x > 1 \quad \text{and} \quad 0 \leq x \leq 1}_{\text{contradiction}}$

(False)

$x(x-1) > 0$

$x = 0 \qquad x - 1 = 0$

$\qquad\qquad\quad +1\ \ +1$

$x = 0 \qquad x = 1$

(-1) : $-1(-1-1)$

$-1(-2)$

$+$

$(.5)$: $.5(.5-1)$

$.5(-.5)$

$-$

(2) : $2(2-1)$

$2(1)$

$+$

$x < 0$ or $\boxed{x > 1}$

$0 \leq x \leq 1$

15. $\forall x \in R$ if $x > 0$, then $x^2 > x$

(False)

$x^2 - x > 0$

$x(x-1) > 0$

$\forall x \in R$ if $x > 0$, then $x < 0$ or $x > 1$

$\forall x \in R$ if $x > 0$, then $x > 1$

(False)

This interval falls into the negative Region. Use .5 as a counter example.

Counter Example: $(.5)^2 > .5$

$.25 > .5$

$\exists x \in R \ni x > 0$ and $x^2 \le x$

$x^2 - x \le 0$

$x(x-1) \le 0$

$\exists x \in R \ni \underbrace{x > 0 \text{ and } 0 \le x \le 1}_{\text{(True)}}$

$x = .5$

$$
\begin{array}{r}
x^2 > x \\
-x \quad -x \\
\hline
x^2 - x > 0
\end{array}
$$

$x(x-1) > 0$

$x = 0$

$$
\begin{array}{r}
x - 1 = 0 \\
+1 \quad +1 \\
\hline
x = 1
\end{array}
$$

+ | 0 | − | 1 | +

(-1): $-1(-1-1)$

$-1(-2)$

+

$(.5)$: $.5(.5-1)$

$.5(-.5)$

−

(2): $2(2-1)$

$2(1)$

+

$x < 0$ or $x > 1$

+ | 0 | − | 1 | +

$0 \le x \le 1$

16. $\forall x \in R$ if $x > 1$, then $\frac{x}{x^2-1} > 0$

$$\frac{x}{(x+1)(x-1)} > 0$$

$\forall x \in R$, if $x > 1$, then $(1, \infty)$

True

$\exists x \in R \ni x > 1 \wedge \frac{x}{x^2-1} \leq 0$

$$\frac{x}{(x+1)(x-1)} \leq 0$$

$\exists x \in R \ni \underbrace{x > 1 \wedge (-\infty, 1) \cup [0, 1)}_{\text{Contradiction}}$

False

$x = 0$ $\quad \frac{x^2-1}{(x+1)(x-1)}$ $\quad \frac{x}{(x+1)(x-2)}$

$x = 0$	$x + 1 = 0$	$x - 1 = 0$
	$-1 \ -1$	$+1 \ +1$
$x = 0$	$x = -1$	$x = 1$

$-$ $\quad +$ $\quad -$ $\quad +$

$-1 \quad 0 \quad 1$

(-2): $\frac{-2}{(-2+1)(-2-1)} = \frac{-2}{(-1)(-3)}$ $-$ $+$

$(-.5)$: $\frac{-.5}{(-.5+1)(-.5-1)} = \frac{-.5}{(1)(-1.5)}$ $+$

$(.5)$: $\frac{.5}{(.5+1)(.5-1)} = \frac{.5}{(1.5)(-.5)}$ $-$

(2): $\frac{2}{(2+1)(2-1)} = \frac{2}{(3)(1)}$

$$x^2 - 1 = 0$$

$$(x+1)(x-1) = 0$$

$x + 1 = 0$	$x - 1 = 0$
$-1 \ -1$	$+1 \ +1$
$x = -1$	$x = 1$

$(-1, 0] \cup \boxed{(1, \infty)}$

$-$ $\quad +$ $\quad -$ $\quad +$

$-1 \quad 0 \quad 1$

$(-\infty, 1) \cup [0, 1)$

18.

$\forall x \in R \quad \forall y \in R$ if $x<y$, then $x^2 \le y^2$

$\forall x \in R$	$\forall y \in R$	$\overset{p}{x<y} \rightarrow$	$\overset{q}{x^2 \le y^2}$	Truth	Counter
$x=-1$	$y=0$	$-1<0$	$(-1)^2 \le (0)^2$	F	$x=-1, y=0$

$\exists x \in R \quad E y \in R \ni x<y$ and $x^2 > y^2$

$\exists x \in R$	$E y \in R$	$\overset{p}{x<y} \wedge$	$\overset{\bar{q}}{x^2>y^2}$	Truth	Witness
$x=-1$	$y=0$	$-1<0$	$(-1)^2>(0)$	T	$x=-1, y=0$

19. $\exists x \in N \quad \exists y \in N$ if $x<y$, then $x^2 \le y^2$

$\exists x \in N$	$\exists y \in N$	$\overset{p}{x<y} \rightarrow$	$\overset{q}{x^2 \le y^2}$	Truth	Witness
$x=1$	$y=2$	$1<2$	$(1)^2 \le (2)^2$	T	$x=1, y=2$

$\forall x \in N$	$\forall y \in N$	$\overset{p}{x<y} \wedge$	$\overset{\bar{q}}{x^2>y^2}$	Truth	Counter
$x=1$	$y=2$	$1<2$	$(1)^2>(2)^2$	F	$x=1, y=2$