$f(x) = x^2 - x + 1$, Find the average rate of change of the function f with respect to x over the interval $[0, 4]$.

① Average Speed

$$\frac{\text{Distance}}{\text{Time}}$$

Average Rate of Change

$$\frac{\Delta y}{\Delta x} = \boxed{\frac{f(b)-f(a)}{b-a}} = \frac{f(a+h)-f(a)}{h}$$

↓

where $b=a+h$ and $h>0$

② $\dfrac{f(b)-f(a)}{b-a}$ $\quad [0, 4]$

$\quad a, b$

$$= \frac{f(4)-f(0)}{4-0}$$

$f(4) = (4)^2 - (4) + 1$

$f(0) = (0)^2 - (0) + 1$

$$= \frac{(4^2-4+1)-(0^2-0+1)}{4-0}$$

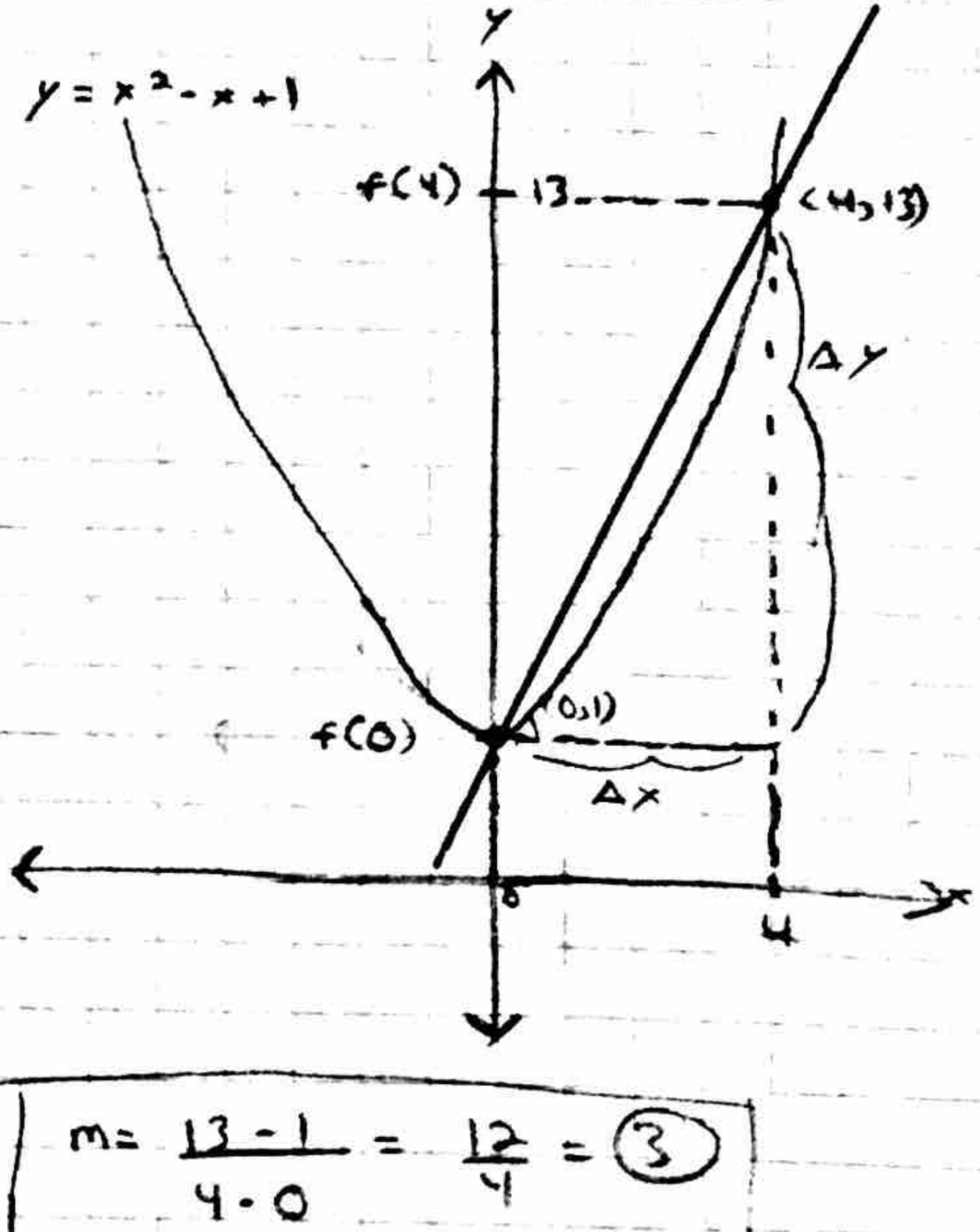
$$= \frac{(16-4+1)-(1)}{4}$$

$$= \frac{13-1}{4}$$

$$= \frac{12}{4}$$

$$= ③$$

Average Rate of Change of the function f with respect to x over the interval [1,2] is 12.



$$\boxed{m = \frac{13-1}{4-0} = \frac{12}{4} = ③}$$

$m=13$

Slope of secant line passing through (0,1) and (4,13) is 3.

$f(x) = 4x^2$, Find Average rate of change of function f over

$$\underset{a,\ b}{[1, 2]}$$

$$\frac{\Delta y}{\Delta x} = \frac{f(b)-f(a)}{b-a}$$

$$= \frac{f(2)-f(1)}{2-1}$$

$$= \frac{4(2)^2 - 4(1)^2}{2-1}$$

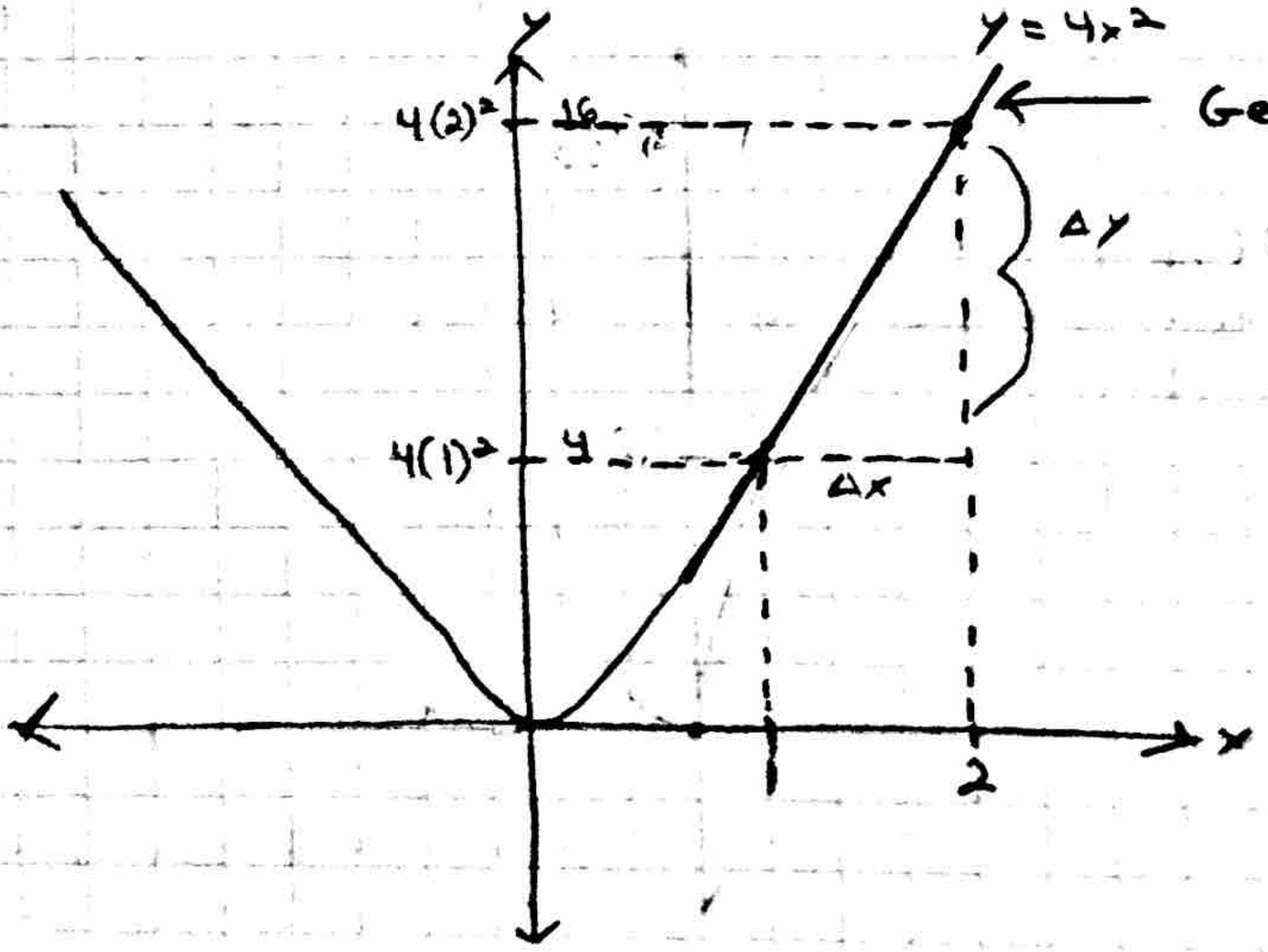
$$= \frac{4 \cdot 4 - 4 \cdot 1}{2-1}$$

$$= \frac{16-4}{1}$$

$$= \frac{12}{1}$$

$$= \textcircled{12}$$

Average rate of change of the function f with respect to x over the interval [1, 2] is 12.



Geometrically, the slope of the secant line passing through the points (1, 4) and (2, 16) is 12.

$$m = \frac{16-4}{2-1} = \frac{12}{1} = \textcircled{12}$$

Find the average rate of change of the function $f(x) = \sqrt{x} + 1$ over the interval $[1, 4]$.

Average Rate of Change

$$\frac{\Delta y}{\Delta x} = \frac{f(b) - f(a)}{b - a}$$

$$\frac{f(b) - f(a)}{b - a} \qquad \underset{a,\ b}{[1, 4]}$$

$f(x) = \sqrt{x} + 1$

$f(1) = \sqrt{1} + 1$

$f(4) = \sqrt{4} + 1$

$f(a) = f(1)$

$f(b) = f(4)$

$$= \frac{(\sqrt{4} + 1) - (\sqrt{1} + 1)}{4 - 1}$$

$$= \frac{(2 + 1) - (1 + 1)}{3}$$

$$= \frac{3 - 1 - 1}{3}$$

$$= \frac{3 - 1 + (-1)}{3}$$

$$= \frac{2 + (-1)}{3}$$

$$= \frac{1}{3} \approx 0.33$$

The average rate of change of the function f over the interval $[1, 4]$ is approximately 0.33

Consider the function $y = x^2$. Calculate the average rate of change of the function f over the interval [1, 1+h] as h gets closer to 0. Specifically, consider the values h = 1, 0.5, 0.25, 0.1, 0.01, and 0.001. What do you notice about the rate of change of f over the interval [1, 1+h] as h gets closer to 0?

①

Average Rate of Change (Slope of Secant)

$$\frac{\text{Distance}}{\text{Time}} = \frac{f(1+h) - f(1)}{(1+h) - 1} = \frac{f(1+h) - f(1)}{1+h-1} = \frac{f(1+h) - f(1)}{\underset{\uparrow}{1+-1}+h} = \boxed{\frac{f(1+h) - f(1)}{h}}$$

1's cancel

② Input Values $[\overset{a}{1}, \overset{b}{1+h}]$

h = 1

$$\frac{f(1+h) - f(1)}{1+h-1} = \frac{(1+1)^2 - 1^2}{1} = \boxed{3}$$

h = 0.5

$$\frac{f(1+h) - f(1)}{1+h-1} = \frac{(1+0.5)^2 - 1^2}{0.5} = \boxed{2.5}$$

As h get smaller and closer to 0, the rate of change of f over the interval [1, 1+h] becomes closer to 2.

h = 0.25

$$\frac{f(1+h) - f(1)}{1+h-1} = \frac{(1+0.25)^2 - 1^2}{1+0.25-1} = \boxed{2.25}$$

h = 0.1

$$\frac{f(1+h) - f(1)}{1+h-1} = \frac{(1+0.1)^2 - 1^2}{0.1} = \boxed{2.1}$$

h = 0.01

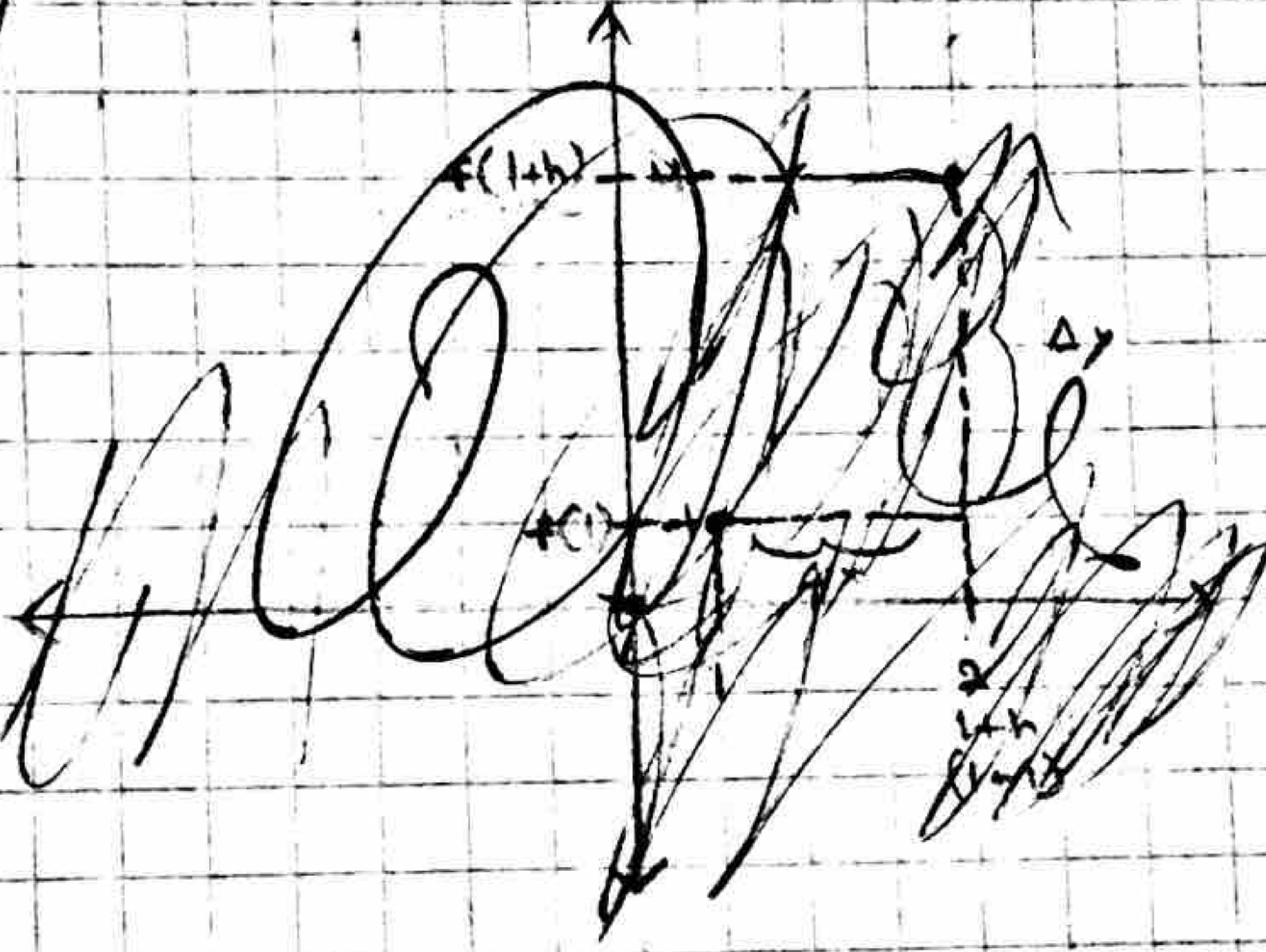
$$\frac{f(1+h) - f(1)}{1+h-1} = \frac{(1+0.01)^2 - 1^2}{0.01} = \boxed{2.01}$$

h = 0.001

$$\frac{f(1+h) - f(1)}{1+h-1} = \frac{(1+0.001)^2 - 1^2}{0.001} = \boxed{2.001}$$

The rate of change of f over the interval [1, 1+h] is the slope of the secant line passing through the points (1, f(1)) and (1+h, f(1+h)).

~~At h =~~

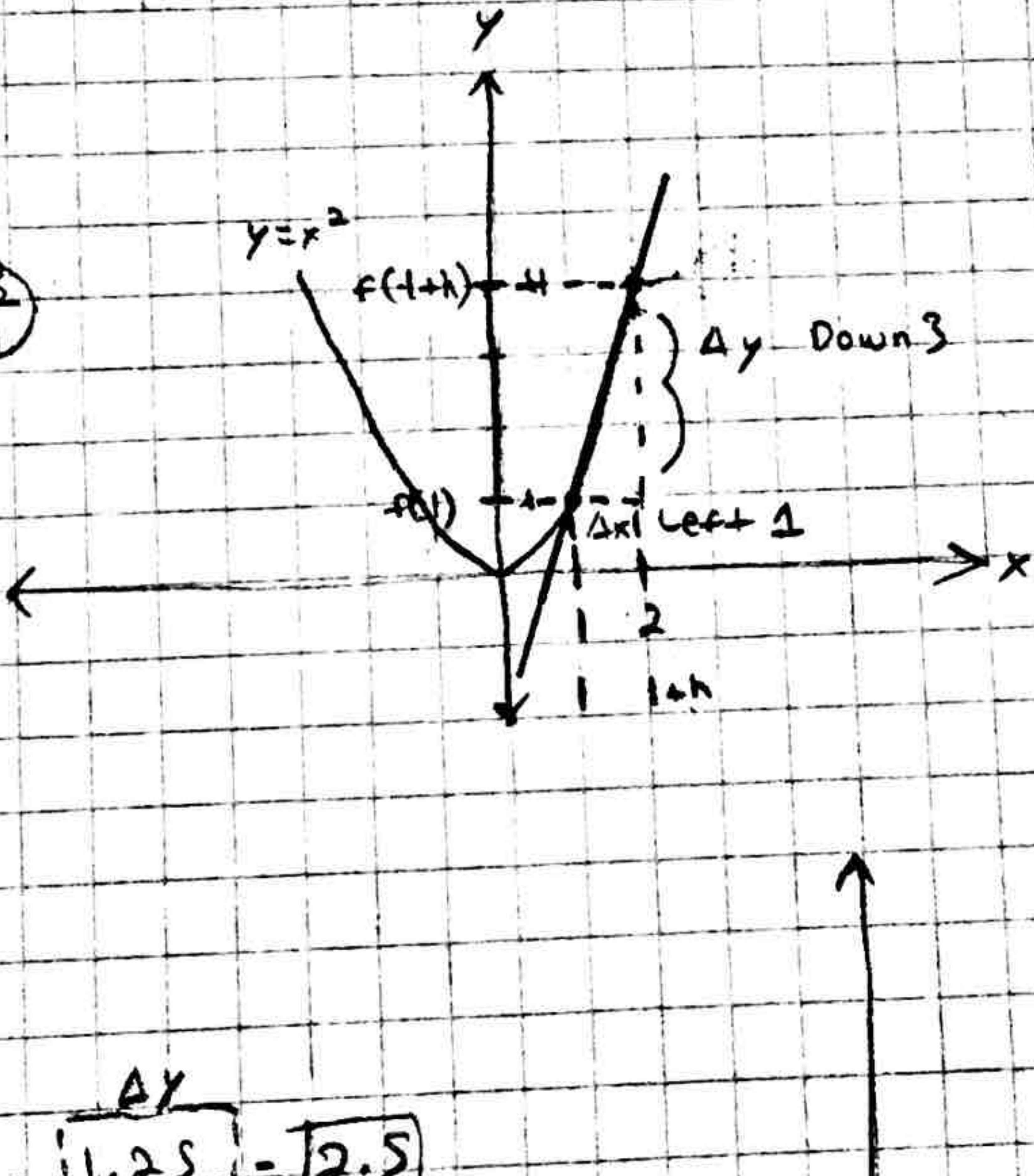


~~$f(1) = (1)^2 = 1$~~
~~$f(1+h) = (1+1)^2 = (2)^2 = 4$~~

At h = 1

Secant Slope

$$\frac{f(1+h)-f(1)}{1+h-1} = \frac{3}{1}$$



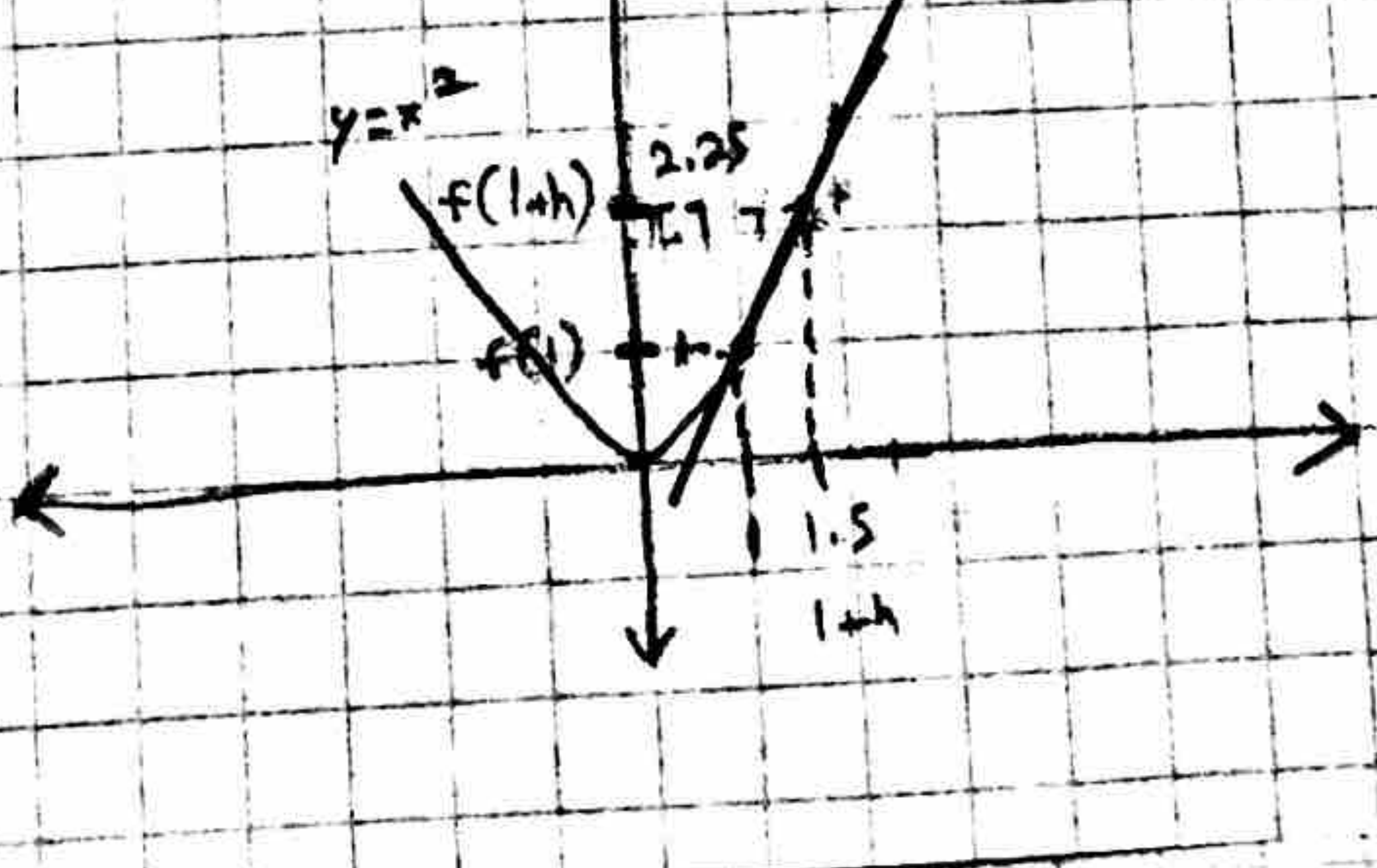
$f(1) = (1)^2 = 1$
$f(1+h) = (1+1)^2 = (2)^2 = 4$
$1+h = 1+1 = 2$
1

$f(1) = (1)^2 = 1$
$f(1+h) = (1+0.5)^2 = 2.25$
$1+h = 1+0.5 = 1.5$
1

At h = 0.5

Secant Slope

$$\frac{f(1+h)-f(1)}{1+h-1} = \underset{\Delta x}{\overset{\Delta y}{\frac{1.25}{.5}}} = \boxed{2.5}$$

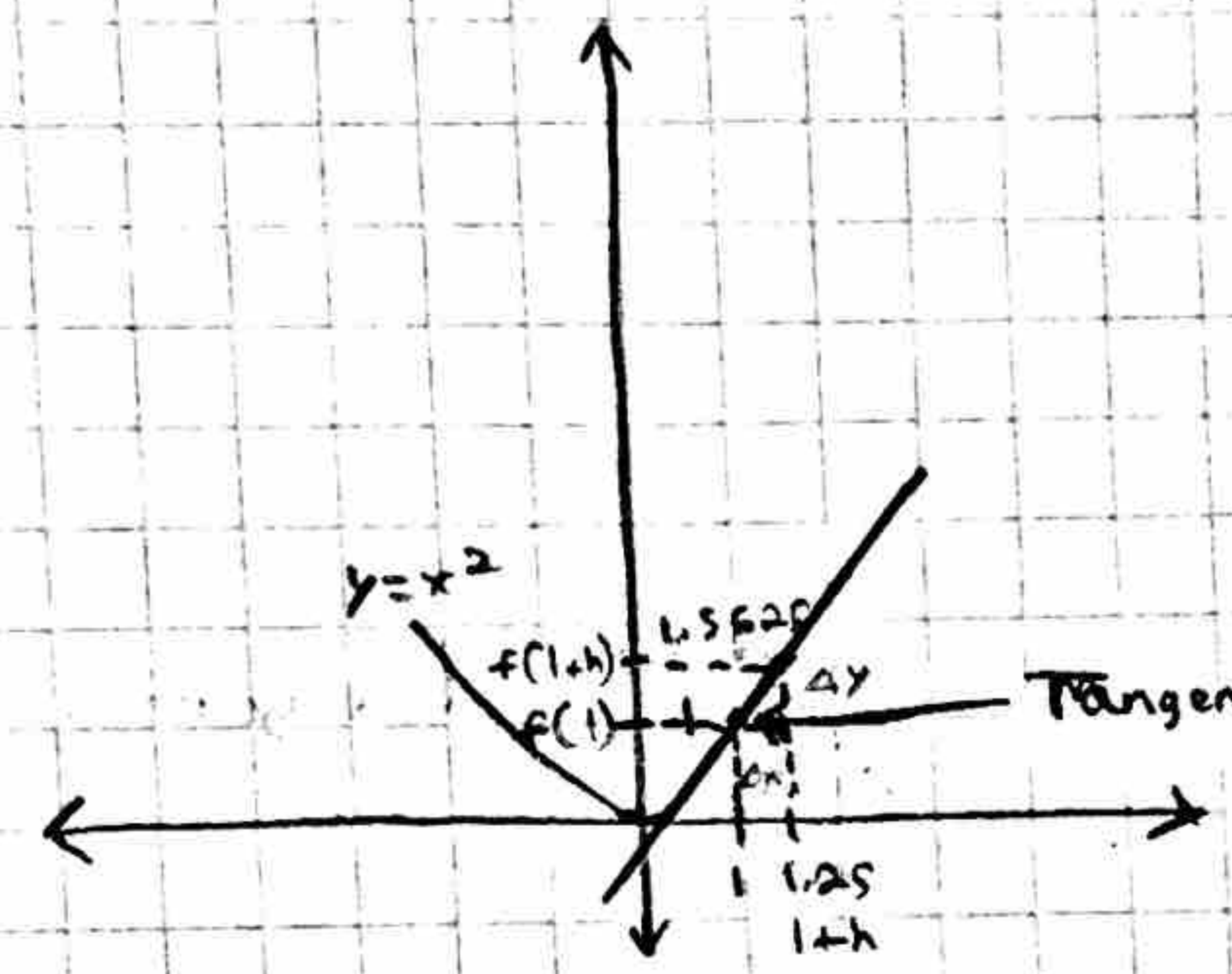


At $h = 0.25$

Secant Slope

$$\frac{f(1+h)-f(1)}{1+h-1} = \underset{\Delta x}{\overset{\Delta y}{\boxed{\frac{.5625}{.25}}}} = \boxed{2.25}$$

$f(1) = (1)^2 = 1$
$f(1+h) = (1+0.25)^2 = 1.5625$
$1+h = 1+0.25 = 1.25$



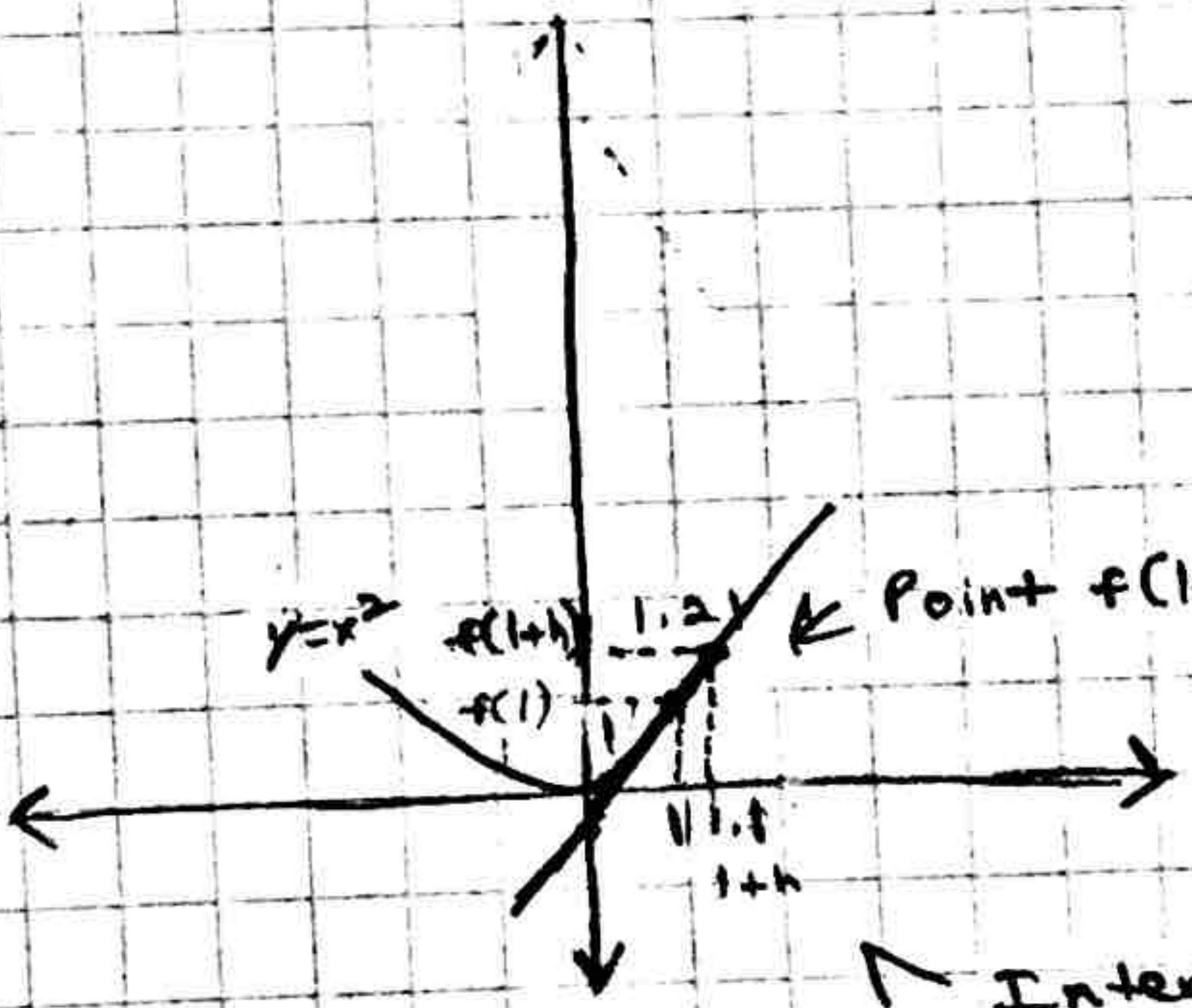
Tangent line will touch point (1,1) once. Graph of x² is funky. I'm not an artist.

At $h = 0.1$

Secant Slope

$$\frac{f(1+h)-f(1)}{1+h-1} = \underset{\Delta x}{\overset{\Delta y}{\boxed{\frac{.21}{0.1}}}} = \boxed{2.1}$$

$f(1) = (1)^2 = 1$
$f(1+h) = (1+0.1)^2 = 1.21$
$1+h = 1+0.1 = 1.1$



Point f(1+h) is getting closer to 1

Interval is getting smaller.

h= 0.01

Secant Slope

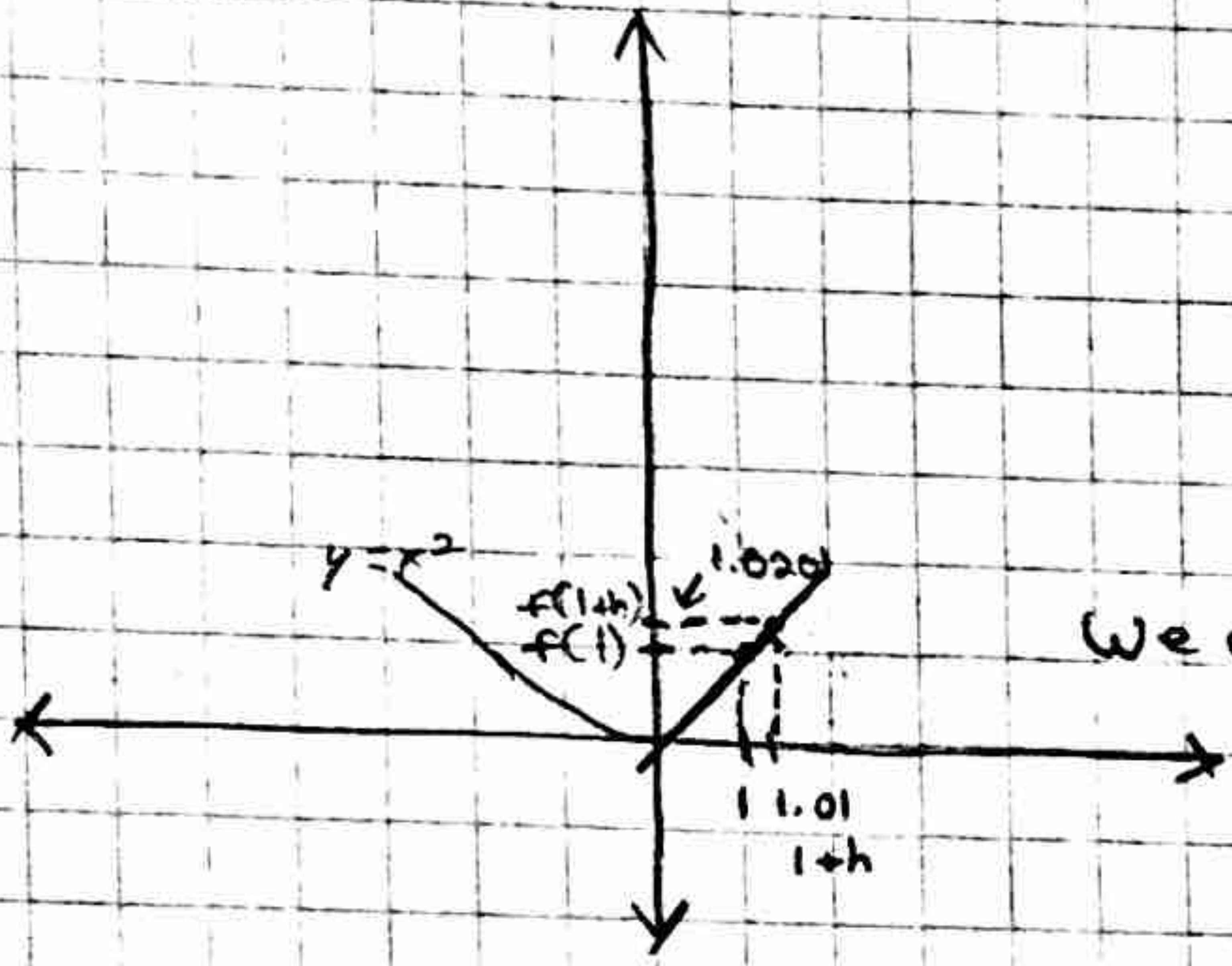
$$\frac{f(1+h)-f(1)}{1+h-1} = \boxed{\frac{\overset{\Delta y}{.0201}}{\underset{\Delta x}{.01}}} = \boxed{2.01}$$

$f(1) = (1)^2 = 1$

$f(1+h) = (1+0.01)^2 = 1.0201$

$1+h = 1+0.01 = 1.01$

1



We getting mad close to 1

h= 0.001

Secant Slope

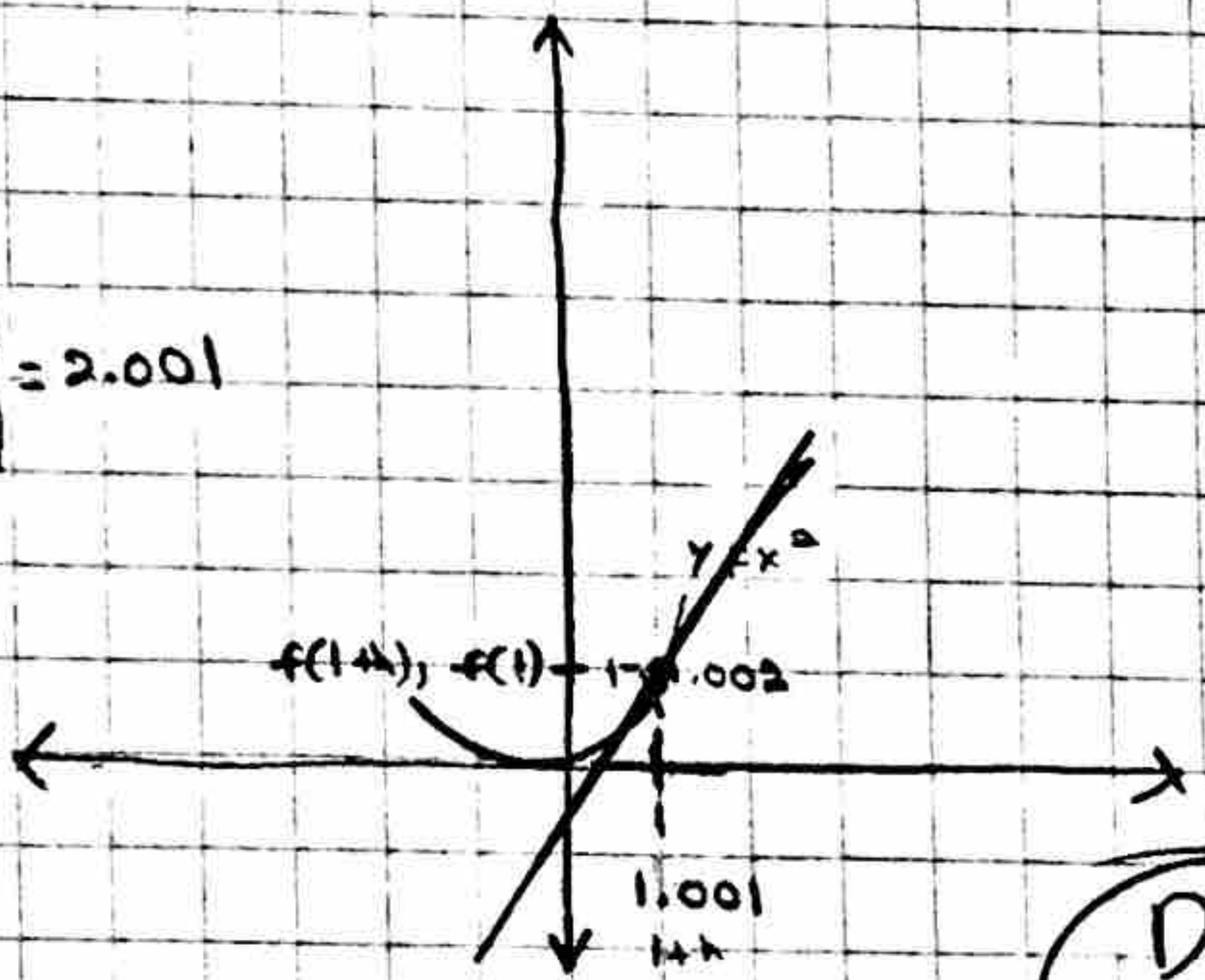
$$\frac{f(1+h)-f(1)}{1+h-1} = \boxed{\frac{\overset{\Delta y}{.002}}{\underset{\Delta x}{.001}}} = 2.001$$

$f(1) = (1)^2 = 1$

$f(1+h) = (1+0.001)^2 = 1.002$

$1+h = 1+0.001 = 1.001$

1



Done