Example 1

Find where the function $f(x) = x^2$ is increasing and where it is decreasing.

① Get Derivative

$$f'(x) = \frac{d}{dx}[x^2] = 2x^{2-1} = 2x^1 = 2x$$

② Set $f'(x) = 0$

$$\frac{2x}{2} = \frac{0}{2} \quad = \quad x = 0$$

③ Set intervals

$$(-\infty, 0) \cup (0, \infty)$$

④ Set arbitrary values

-1 | 0 | 1

$$f'(x) = 2x$$
$$f'(-1) = 2(-1) = \boxed{f'(-1) = -2}$$

$$\downarrow$$

$$\frac{2x}{2} < \frac{0}{2}$$

$$x < 0$$

or

$$f'(x) = 2x$$
$$f'(1) = 2(1) = \boxed{f'(1) = 2}$$

$$\downarrow$$

$$\frac{2x}{2} > \frac{0}{2}$$

$$x > 0$$

⑤ Summary

$f'(x)$

$-$ | 0 | $+$

$\boxed{\text{The function } f \text{ is decreasing on } (-\infty, 0) \text{ and increasing on } (0, \infty).}$

① Get Derivative

Example 2

$$f'(x) = \frac{d}{dx}\left[\frac{\overset{f(x)}{1}}{\underset{g(x)}{x-1}}\right]$$

$$= \frac{(x-1)\frac{d}{dx}[1] - 1\frac{d}{dx}[x-1]}{(x-1)^2}$$

$$= \frac{(x-1)\cdot 0 - 1\cdot \frac{d}{dx}[x] - \frac{d}{dx}[1]}{(x-1)^2}$$

$$= \frac{0 - 1\cdot 1x^{1-1} - 0}{(x-1)^2}$$

$$= \frac{0 - 1\cdot 1 - 0}{(x-1)^2}$$

$$= \boxed{f'(x) = \frac{-1}{(x-1)^2}}$$

② Set $f'(x) = 0$

$$\frac{-1}{(x-1)^2} = 0$$

$$= (x-1)\cdot\frac{-1}{(x-1)^2} = 0\cdot(x-1)^2$$

$$= \boxed{-1 = 0}$$

False, no solutions

③ Set intervals $x = 1$ causes $-\frac{1}{(x-1)^2}$ to be under

Use $x = 1$ as an endpoint

$(-\infty, 1) \cup (1, \infty)$

$0 \qquad 2$

1

④ Set arbitrary values

$f'(x) = -1/(x-1)^2$

$f'(0) = -1/(0-1)^2$

$= -1/(-1)^2$

$= -1/1$

$=$

$\boxed{f'(0) = -1}$

$f'(x) = -1/(x-1)^2$

$f'(2) = -1/(2-1)^2$

$-1/(1)^2$

$= -1/1$

$=$

$\boxed{f'(2) = -1}$

④ Summary

$f'(x) \qquad - \qquad -$

$f'(x)$ is negative over entire interval $(1, \infty)$