Example 4

Find the differential of $y = x^2$, and interpret its meaning.

① Get $y'(x)$

$$y'(x) = \frac{d}{dx}[x^2]$$

$$"$$

$$2x^{2-1}$$

$$"$$

$$\boxed{y'(x) = 2x}$$

↓   ② Set $\frac{dy}{dx}$

$$\cdot \quad \frac{dy}{dx} = 2x$$

③ Solve for dy

$$"$$

$$dx \cdot \frac{dy}{dx} = 2x \cdot dx$$

$$"$$

$$\cancel{dx} \cdot \frac{dy}{\cancel{dx}} = 2x \cdot dx$$

$$"$$

$$\boxed{dy = 2x\, dx}$$

④ Interpret

The dy differential mean the tangent line rises or falls by the amount $2x\, dx$. when x changes by the amount dx.

when $x = 1$ and $dx = 0.1$, then

$$dy = 2(1)(0.1)$$

$$2(0.1)$$

$$\boxed{dy = 0.2}$$

Consider the function $y = \sin(x)$

Evaluate the differential dy when $x = 0$ and $dx = 0.02$

① Get $y'(x)$

$$y'(x) = \frac{d}{dx}[\sin(x)]$$

$$\boxed{y'(x) = \cos(x)}$$

② Set $\frac{dy}{dx}$

$$\frac{dy}{dx} = \cos(x) \qquad ③ \text{ Solve for } dy$$

$$dx \cdot \frac{dy}{dx} = \cos(x) \cdot dx$$

$$\boxed{dy = \cos(x)\, dx}$$

④ Evaluate and Interpret

$$x = 0, \quad dx = 0.02$$

$$dy = \cos(x)\, dx$$

$$\cos(0)\,(0.02)$$

$$1 \cdot 0.02$$

$$\boxed{dy = 0.02}$$

Tangent line rises by the amount of $dy = 0.02$ when $x = 0$ and changes by the amount of $dx = 0.02$ in the positive x-direction.

$$\frac{dy}{dx} = \frac{0.02}{0.02} = 1 \quad \text{at } x = 0$$

$\cos(0) = 1$ at $x = 0$ agreeing with the derivative