Newton's Method (Different Root for $f(x) = x^2 - 1$)

Consider $x^2 - 1 = 0$

*** Iteration 1 ***

① Pick value starting at x_0, $x_0 = -2$

② Get $f'(x)$

$$f'(x) = \frac{d}{dx}[x^2 - 1]$$

$$\frac{d}{dx}[x^2] - \frac{d}{dx}[1]$$

$$2x^{2-1} - 0$$

$$\boxed{f'(x) = 2x^1 \quad f'(x) = 2x}$$

③ Get slope of tangent at $x = -2$

$$f'(-2) = 2(-2)$$

$$\boxed{f'(-2) = -4}$$

④ Set equation of tangent line

$y = mx + b$ $\quad f'(-2) = -4$, $m = -4$

$$y = -4x + b$$

⑤ Get Domain and Range

Domain of x for $y = -4x + b$ $(-\infty, \infty)$ No Restrictions

Get Range $\quad y = -4x + b$

$-b \qquad -b$

$$\frac{y - b}{-4} = \frac{-4x}{-4}$$

No Restrictions for y

Domain of y for $y = -4x + b$ $(-\infty, \infty)$

⑥ Get y value and y-intercept

$f(x) = x^2 - 1$

$f(-2) = (-2)^2 - 1$

$4 - 1$

$\boxed{f(-2) = 3}$

↓

$y = 3$

Use $(-2, 3)$

$y = -4x + b$

$3 = -4(-2) + b$

$3 = 8 + b$

$-8 \quad -8$

$\boxed{-5 = b}$

⑦ Summarize Equation

Equation of tangent line to the curve $y = f(x)$ at $x = -2$ is

$\boxed{y = -4x - 5}$

⑧ Find where tangent line crosses x-axis.

Solve for x

$-4x - 5 = 0$

$+5 \quad +5$

$\frac{-4x}{-4} = \frac{5}{-4}$

$\boxed{x = 5/-4 \text{ or } x = -5/4}$

Results

Iteration Number	Starting Value	Approximate Root
1	-2	$-5/4 = -1.25$
2	$-5/4$	$-41/40 = -1.025$
3	$-41/40$	$-3281/3280 = -1.0003$

⑨ Conclusion

Using a different starting value found a different root of the equation $x^2 - 1 = 0$