Find Domain and Range

$y = 2\cot(4x - \pi) + 3$

③ Solve for x

$4x - \pi = \pi k$

$+\pi \quad +\pi$

$4x = \pi + \pi k$

$\pi + \pi k = \pi k$

$\frac{\cancel{4}x}{\cancel{4}} = \frac{\pi k}{4}$

$\boxed{x = \frac{\pi}{4} k}$

Domain is all real numbers except $\frac{\pi}{4} k$

① Find Domain of $y = \cot(x)$

$\cot = \frac{adj}{opp} = \frac{x}{y}$

$y > 0$

y is 0 at π and 0. I'll use 0 since its the smallest reference angle.

$0 \pm \pi k$

Set 0 into this formula to get all coterminal angles.

Get Range

$\cot(x) = \frac{x}{y}$

$y = \frac{x}{y}$ ③ Solve for y

$y(y) = \left(\frac{x}{\cancel{y}}\right)\cancel{y}$

$y^2 = x$

I can input any y values here so my range is unrestricted.

Range: $(-\infty, \infty)$

$y = 2\cot(4x - \pi) + 3$

D: All real numbers except $\frac{\pi}{4} k$

R: $(-\infty, \infty)$

Find Domain and Range of

$y = 2\tan\left(2x - \frac{\pi}{2}\right) + 3$

① Find Domain of $y = \tan(x)$

$\tan = \frac{opp}{adj} = \frac{y}{x}$

$x \neq 0$

$\downarrow$

x is 0 at $\pi/2$ an

$3\pi/2$. I'll use

$\pi/2$ since its th

smallest reference

angle

$\frac{\pi}{2} \pm \pi k$

$\uparrow$

Set $\pi/2$ into this formula t

get all coterminal angles

② Set $2x - \frac{\pi}{2} = \frac{\pi}{2} \pm \pi k$

$$2x - \frac{\pi}{2} = \frac{\pi}{2} + \pi k$$

③ Solve for x

$$+\frac{\pi}{2} \quad = +\frac{\pi}{2}$$

$$2x = \frac{\pi}{2} + \frac{\pi}{2} + \pi k$$

$$\downarrow$$

$$2x = \frac{\cancel{2}\pi}{\cancel{2}} + \pi k$$

$$\downarrow$$

$$2x = \pi + \pi k \longleftrightarrow \pi + \pi k = \pi k \text{ for all integers } k$$

$\pi(1+k) \longleftrightarrow \pi k$ $\quad$ k will be my revolutions on unit circle.

1 is my constant

$(1+k) = k$

same with

$2\pi + 2\pi k = 2\pi k$

$$\downarrow$$

$$\frac{2x}{2} = \frac{\pi k}{2}$$

$$\downarrow$$

$$x = \frac{\pi}{2}k$$

Domain is all real numbers except $\frac{\pi}{2}k$

D: $\left(x \neq \frac{\pi}{2}k\right)$

R: $(-\infty, \infty)$

④ Get range

$\tan = \frac{y}{x}$ $\longleftarrow$ I can input any y values for tan so Range $(-\infty, \infty)$

Find Domain of

$$f(x) = \frac{1}{\sin(x)-1} + \sqrt{1-\cos^2 x}$$

$$f(x) = \frac{1}{\sin(x)-1} + \sqrt{1-\cos^2 x}$$

$\downarrow$

① Find Domain Restrictions $\frac{1}{\sin(x)-1}$

$$\begin{aligned} \sin(x) - 1 &> 0 \\ +1 \quad &+1 \\ \hline \sin(x) &> 1 \end{aligned}$$

$\sin(x) > 1$

② Solve for x $\sin(x) > 1$

I need to Know the value of sine where $\sin(x) = 1$.

$\sin(\pi/2) = 1$ or $\sin(90°) = 1$

$x > \pi/2$

x have to be greater than $\pi/2$

Sine is periodic with a period of 2π.
So I also need to restrict the coterminal angles

$\downarrow$

$x > \frac{\pi}{2}, \frac{\pi}{2} \pm 2\pi, \frac{\pi}{2} \pm 4\pi, \ldots$

1 rev... revolution

2 revs

$$\left(x > \frac{\pi}{2} \pm 2n\pi, \quad n = 0, 1, 2 \ldots \right)$$

② Find Domain Restrictions for

$\sqrt{1-\cos^2 x}$

$$1 - \cos^2 x > 0$$
$$+\cos^2 x \quad\quad +\cos^2 x$$

$$1 \geq \cos^2 x$$
$$\downarrow$$
$$\cos^2 x \leq 1$$

$\downarrow$ solve for x

$$\sqrt{y} = \sqrt{\cos^2 x}$$
$$\downarrow$$
$$\sqrt{x} = \cos x$$

I can input any x values there for there are no restrictions.

The are no domain restrictions for $\cos^2 x$

$\cos^2 x$ - $D: (-\infty, \infty)$

③ Domain for $f(x) = \dfrac{1}{\sin(x)-1} + \sqrt{1-\cos^2 x}$

$D: \{x \mid x > \frac{\pi}{2} \pm 2n\pi, n = 0, 1, 2 \ldots\}$