Determining Symmetry

Even functions are symmetric about the y-axis;

$$f(-x) = f(x)$$

Odd functions are symmetric about the origin;

$$f(-x) = -f(x)$$

* Determine if each of the following functions is either odd, even, both or neither.

$$f(x) = 3x^4 + x^2 + 5$$
$$\shortparallel$$
$$f(-x) = 3(-x)^4 + (-x)^2 + 5$$
$$\shortparallel$$
$$f(-x) = 3x^4 + x^2 + 5$$
$$\shortparallel$$
$$f(-x) = f(x)$$
$$\downarrow$$

Even Function
Symmetric about y-axis

$$f(x) = x^3 - x$$
$$\shortparallel$$
$$f(-x) = (-x)^3 - (-x)$$
$$\shortparallel$$
$$f(-x) = -x^3 + x$$

you can stop here

factor -1 out

$$\shortparallel$$
$$f(-x) = -1(x^3 - x)$$
$$\shortparallel$$
$$f(-x) = -(x^3 - x)$$
$$\shortparallel$$
$$f(-x) = -f(x)$$
$$\downarrow$$

Odd Function
Symmetric about the origin

Check Even, ODD, or Neither Functions

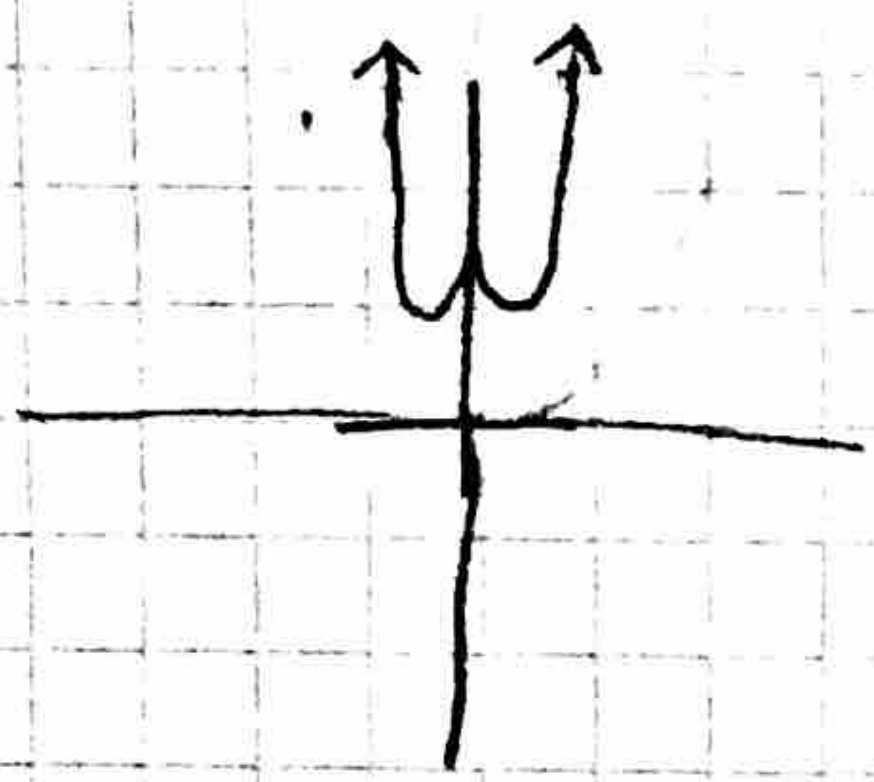
$f(x) = 3x^4 - 5x^2 + 12$

$f(-x) = 3(-x)^4 - 5(-x)^2 + 12$

$f(-x) = 3x^4 - 5x^2 + 12$

$f(-x) = f(x)$

Even function symmetric about the y-axis



$f(x) = 2x^5 + x^3 - 9x$

$f(-x) = 2(-x)^5 + (-x)^3 - 9(-x)$

$f(-x) = -2x^5 - x^3 + 9x$

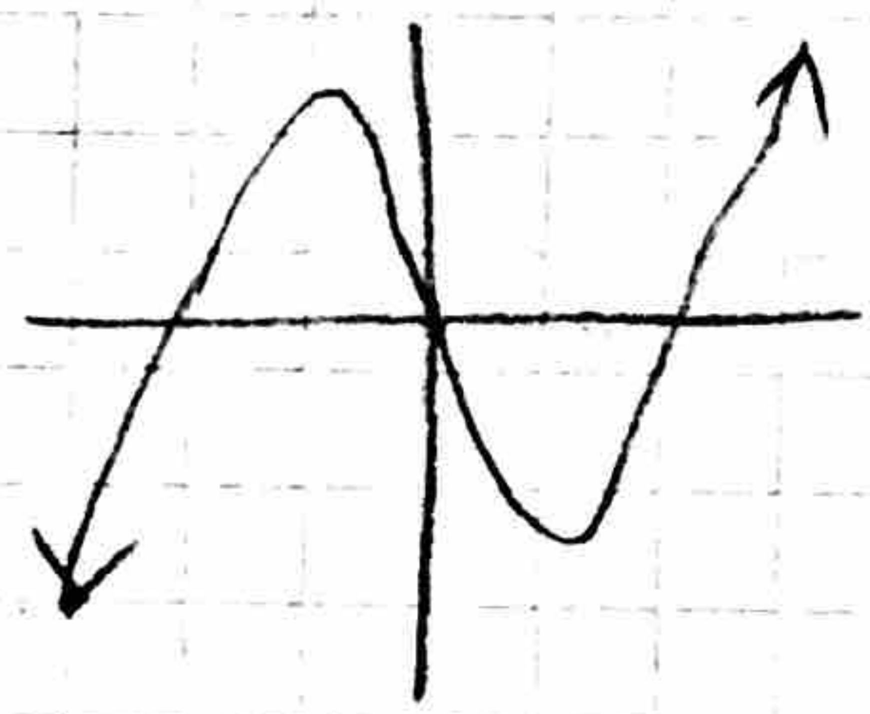
$f(-x) = -(2x + x^3 - 9x)$

Factor out -1

$f(-x) = -(2x + x^3 - 9x)$

$f(-x) = -f(x)$

Odd function symmetric about the origin



Check Even $f(-x) = f(x)$

① $f(x) = x^4 - 2x^3 + 5$

$f(-x) = (-x)^4 - 2(-x)^3 + 5$

$f(-x) = x^4 + 2x^3 + 5$

$f(-x) \neq f(x)$

$x^4 + 2x^3 + 5 \neq x^4 - 2x^3 + 5$

② Check Odd $f(-x) = -f(x)$

$f(-x) = x^4 + 2x^3 + 5$

$f(-x) = -(x^4 - 2x^3 + 5)$

$f(-x) = -x^4 + 2x^3 - 5$

$f(-x) \neq f(x)$

$x^4 + 2x^3 + 5 \neq -x^4 + 2x^3 - 5$

$f(x) = x^4 - 2x^3 + 5$

This isnt odd

This function is Neither