$$\text{Velocity} = \frac{\Delta \text{ meters}}{\Delta \text{ seconds}} = \frac{dv}{dt}$$

Use $\frac{d}{ds}$

$$\frac{d}{dt}\left[t^3 - 6t^2 + 9t\right]$$

$$=$$

$$\frac{d}{dt}\left[t^3\right] - \frac{d}{dt}\left[6t^2\right] + \frac{d}{dt}\left[9t\right]$$

$$=$$

$$3t^{3-1} - 6 \cdot \frac{d}{dt}\left[t^2\right] + 9 \cdot \frac{d}{dt}\left[t\right]$$

$$=$$

$$3t^2 - 6 \cdot 2t^{2-1} + 9 \cdot 1$$

$$=$$

$$3t^2 - 12t + 9$$

$$\boxed{v(t) = 3t^2 - 12t + 9}$$

Get $t = 1$

$$v(1) = 3(1)^2 - 12(1) + 9$$

$$=$$

$$3 - 12 + 9$$

$$-9 + 9$$

$$=$$

$$\boxed{v(1) = 0}$$

Velocity of particle at time $t = 1$ is 0 meters/seconds

Example 4

$e = 10t$

Use d/dt

$$e'(t) = \frac{d}{dt}[10t] = 10 \cdot \frac{d}{dt}[t] = 10 \cdot \frac{dt}{dt} = 10 \cdot 1 = \boxed{e'(t) = 10}$$

Setup

$$\frac{de}{dt} = \frac{e\ (\text{dollars})}{t\ (\text{hours})} = \frac{\text{dollars}}{\text{hours}}$$

Example 3:

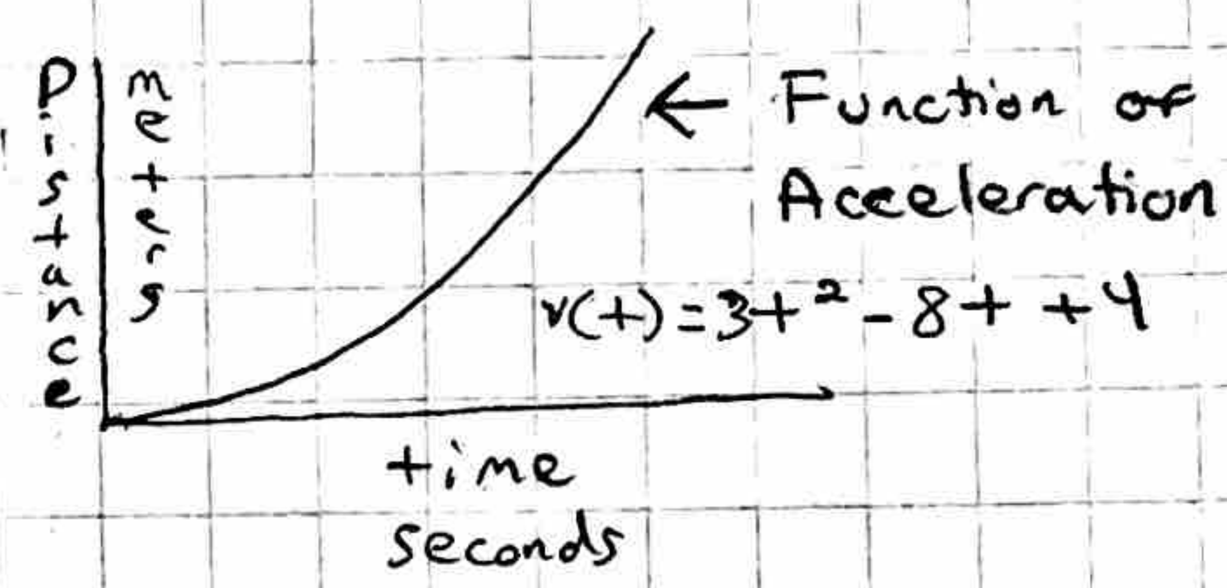
$s(t) = t^3 - 4t^2 + 4t$, where $t > 0$ for $\frac{\Delta \text{ meters}}{\Delta \text{ seconds}}$

Find acceleration of particle at time t.

Determine when the particle speeds up and slows down.

Velocity $= 3t^2 - 8t + 4$ for $t > 0$

Get Acceleration



Find Acceleration

$$\frac{\Delta \text{ meters}}{\Delta \text{ second}^2} = \frac{dv}{dt}$$

$=$

Use $\frac{d}{dt}$

Need to Get The Slope for $v(t) = 3t^2 - 8t + 4$ to get acceleration.

$$\frac{d}{dt}[3t^2 - 8t + 4]$$

$$=$$

$$\frac{d}{dt}[3t^2] - \frac{d}{dt}[8t] + \frac{d}{dt}[4]$$

$$=$$

$$3 \cdot \frac{d}{dt}[t^2] - 8 \cdot \frac{d}{dt}[t] + 0$$

$$=$$

$$3 \cdot 2t^{2-1} - 8 + 0$$

$$=$$

$$3 \cdot 2t - 8$$

$$=$$

$$\boxed{6t - 8}$$

Acceleration of the particle at time $t > 0$ is $a(t) = 6t - 8$

Determine when the particle speeds up and slows down.

Particle speeds up when both velocity and acceleration are positive, and when both velocity and acceleration are negative.

Acceleration

$6t - 8 > 0$

$2(3t - 4) > 0$

$\boxed{2 > 0}$ $\quad 3t - 4 > 0$

$\quad\quad +4 \quad +4$

$\frac{3t}{3} > \frac{4}{3}$

$\boxed{t > 4/3}$

$6t - 8 < 0$

$2(3t - 4) < 0$

$\boxed{2 > 0}$ $\quad 3t - 4 < 0$

$\quad\quad +4 \quad +4$

$\frac{3t}{3} < \frac{4}{3}$

$\boxed{t < 4/3}$

−   +   +

$4/3$ 2

$(0)\quad 2(3(0) - 4)$

$2(0 - 4)$

-8

$(1.5)\quad 2(3(1.5) - 4)$

$2(4.5 - 4)$

$2(.5) = 1$

$(3)\quad 2(3(3) - 4)$

$2(9 - 4)$

$2(5) = 10$

Velocity

$3t^2 - 8t + 4 > 0$

$(t-2)(3t-2) > 0$

$t - 2 > 0$ | $3t - 2 > 0$
$+2 \quad +2$ | $+2 \quad +2$
$t = 2$ | $\frac{3t}{3} > \frac{2}{3}$
 | $t > 2/3$

+ $2/3$ − 2 +

$v(0): (0-2)(3(0)-2)$
$(-2)(-2)$
(-4)

$v(1): (1-2)(3(1)-2)$
$(-1)(1)$
(-1)

$v(3): (3-2)(3(3)-2)$
$(1)(9-2)$
$(1)(7)$
7

Acceleration

− $4/3$ + 2 +

Postive: $(4/3, \infty)$
Negative: $(0, 4/3)$

Particle Speeds up when Both Acceleration and velocity are Positive.

Particle slows down when both acceloration and velocity are opposite signs.

Velocity

+ $2/3$ − 2 +

Positive: $(0, 2/3) \cup (2, \infty)$
Negative: $(2/3, 2)$

$\boxed{\text{Particle speeds up on } (2/3, 4/3) \cup (2, \infty) \text{ and slows down on } (0, 2/3) \cup (4/3, 2)}$

Example 5

Area of Circle $A = \pi r^2$

How fast does the area of the circle change with respect to the radius r?

$\boxed{\text{Use } d/dr}$

$$\frac{dA}{dr} = \frac{d}{dr}[\pi r^2] \qquad \frac{dA}{dr} = \frac{\text{meters}^2}{\text{meters}}$$

Differentiate Area with Respect to Radius R

$$= \pi \cdot \frac{d}{dr}[r^2]$$

$$= \pi \cdot 2r^{2-1}$$

$$= \pi \cdot 2r^1$$

$$= \boxed{2\pi r}$$

Circumference of Circle

The area rate of change with respect to the circle radius is

$$\frac{dA}{dr} = 2\pi r$$

How fast does the area of the circle change with respect to the radius r when r = 2 meters?

$r = 2$ $\quad 2\pi(2)$

$$= 2(2)\pi$$

$$= \boxed{4\pi}$$

Find rate of change of the volume of the cube with respect to side length s when $s=2$.

Differentiate Volume with respect to length s $\rightarrow$ $\frac{\Delta \text{ length}^3}{\Delta \text{ length}} = \frac{dV}{ds}$

Use $\frac{d}{ds}$

$\downarrow$

$$\frac{d}{ds}[s^3]$$

$$=$$

$$3s^{3-1}$$

$$=$$

$$\boxed{3s^2}$$

$$\boxed{\frac{dv}{ds}[s^3] = 3s^2}$$

$s=2$, $3(2)^2$

$$= 3 \cdot 4$$

$$= 12$$

Volume of cube with respect to side length s when $s=2$ is 12 inches^2

Example 6

$h = 100 - 4.9t^2$, where $t > 0$ seconds

Get velocity of object at time t

$\downarrow$

Differentiate height with respect to time t.

$\downarrow$

$$\text{velocity} = \frac{\Delta \text{height}}{\Delta \text{time}} = \frac{dh}{dt} = \frac{d}{dt}$$

$$\frac{d}{dt}\left[100 - 4.9t^2\right]$$

$f(x) = 100$

$g(x) = 4.9t^2$

$$\frac{d}{dt}[100] - \frac{d}{dt}[4.9t^2]$$

$$0 \quad - 4.9 \frac{d}{dt}[t^2]$$

$$= 4.9 \cdot 2t^{2-1}$$

$$= -4.9 \cdot 2t$$

$$= -9.8t$$

$$\boxed{\text{Velocity of object at time } t \text{ is } v(t) = -9.8t \text{ m/s}}$$

Get Acceleration of Object at time t

Velocity of object at time $t > 0$ is $v(t) = -9.8t$ m/s

$\uparrow$

Direction of Motion. In this case, the object is falling toward Earth.

Speed of Object $|-9.8t| = 9.8t$ m/s

Differentiate Meters with Respect to Seconds to Get Acceleration

$$\text{Acceleration} = \frac{\Delta \text{ meters}}{\Delta \text{ seconds}^2} = \frac{dv}{dt} = \frac{d}{dt}$$

$$v'(t) = \frac{d}{dt}[-9.8t]$$
$$= -9.8 \cdot \frac{d}{dt}[t]$$
$$= -9.8 \cdot 1$$
$$= -9.8$$

$$\boxed{\text{Acceleration of object at time } t \text{ is } a(t) = -9.8 \text{ m/s}^2}$$