Solving Linear Inequalities

$<, >, \leq, \geq$

$$\begin{array}{l} x + \cancel{5} \geq 8 \\ \quad -\cancel{5} \quad -5 \\ \hline \boxed{x \geq 3} \end{array}$$

Check

$\boxed{3} + 5 \geq 8$

$\boxed{8 \geq 8}$

True Statement

$$\begin{array}{l} -x + \cancel{5} \geq 8 \\ \quad -\cancel{5} \quad -5 \\ \hline \dfrac{-x}{-1} \geq \dfrac{3}{-1} \end{array} \longrightarrow x \leq -3$$

$$2(x+5) \leq 3(5-x)$$

$$\downarrow$$

$$\begin{array}{l} 2x + \cancel{10} \leq 15 - \cancel{3x} \\ +3x \; -\cancel{10} \qquad -10 + \cancel{3x} \\ \hline \dfrac{5x}{5} \leq \dfrac{5}{5} \\ \boxed{x \leq 1} \end{array}$$

$$2(3-x) + 4(x-8) < 3x + 5$$

$$\downarrow$$

$$6 \underbrace{- 2x + 4x}_{\downarrow} - 32 < 3x + 5$$

$$6 + 2x - 32 < 3x + 5$$

$$\downarrow$$

$$\begin{array}{l} -26 + \cancel{2x} < 3x + 5 \\ -\;\;6 \; - \cancel{2x} \qquad -2x - 6 \\ \hline -32 < x - 1 \\ +\;\;1 \qquad\quad +1 \\ \hline -31 < x \end{array}$$

$$\downarrow$$

$$\boxed{x > -31}$$

Solving Quadratic Inequalities

1. Turn Inequality into Equation
2. Find Solutions
3. Make a Number Line and Check Each Solution and Interval

Solutions always work with $\geq$ or ≤ 0

① $x^2 + 2x - 8 \geq 0$

$-8, +2$

$4; -2$

$\downarrow$

$x^2 + 2x - 8 = 0$

$\downarrow$

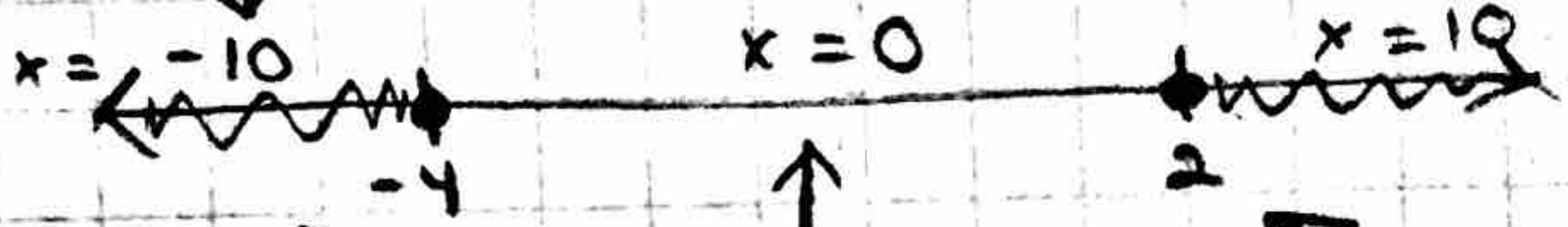
$(x+4)(x-2) = 0$

$\downarrow$

$x + 4 = 0$ $\quad$ $x - 2 = 0$

$-4 \;\; -4$ $\quad$ $+2 \;\; +2$

$\boxed{x = -4}$ $\quad$ $\boxed{x = 2}$



$((-4)+4)((-4)-2) \geq 0$

$\downarrow$

$(0)(-6) \geq 0$

$\downarrow$

$\boxed{0 \geq 0 \text{ TRUE}}$

$((2)+4)((2)-2) \geq 0$

$\downarrow$

$(6)(0) \geq 0$

$\downarrow$

$\boxed{0 \geq 0 \text{ TRUE}}$

$((0)+4)((0)-2) \geq 0$

$\downarrow$

$(4)(-2) \geq 0$

$\downarrow$

$\boxed{-8 \geq 0 \text{ FALSE}}$ ✗

$((-10)+4)((-10)-2) \geq 0$

$(-6)(-12) \geq 0$

$\boxed{72 \geq 0 \text{ TRUE}}$

Interval Notation Solution

$(-\infty, -4] \cup [2, \infty)$

$((10)+4)((10)-2) \geq 0$

$(14)(8) \geq 0$

$\boxed{112 \geq 0 \text{ TRUE}}$

Quadratic Equations

1) Factor
2) Quadratic Formula
3) Complete the Square

$$x^2 + 5x + 6 > 0$$

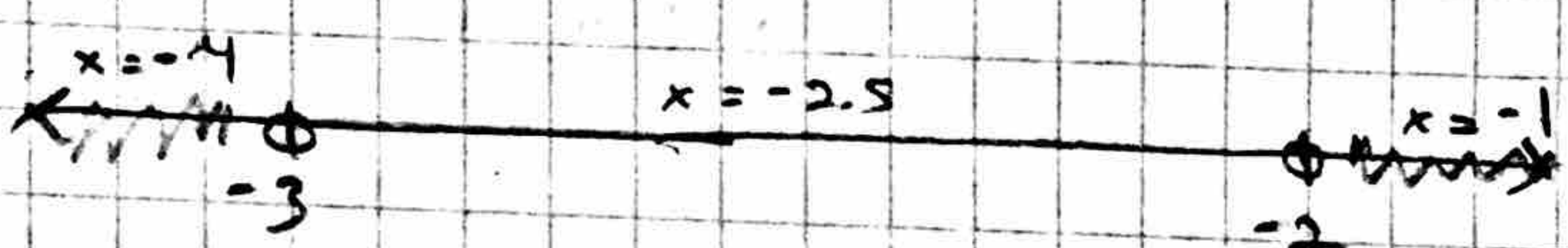
$\downarrow$

$$(x+2)(x+3) > 0$$

2, 3

$\downarrow$

$$\begin{array}{rl} x + 2 &= 0 \\ -2 & -2 \\ \hline x &= -2 \end{array} \qquad \begin{array}{rl} x + 3 &= 0 \\ -3 &= -3 \\ \hline x &= -3 \end{array}$$



-2

$$((-2)+2)((-2)+3) > 0$$

$\downarrow$

$$(0)(1) > 0$$

$$\boxed{\begin{array}{c} 0 > 0 \\ \text{FALSE} \end{array}}$$

-3

$$((-3)+2)((-3)+3) > 0$$

$\downarrow$

$$(-1)(0) > 0$$

$\downarrow$

$$\boxed{\begin{array}{c} 0 > 0 \\ \text{FALSE} \end{array}}$$

-4

$$((-4)+2)((-4)+3) > 0$$

$$(-2)(-1) > 0$$

$$\boxed{\begin{array}{c} 2 > 0 \\ \text{TRUE} \end{array}}$$

Solution
(Interval Notation)

$$(-\infty, -3) \cup (-2, \infty)$$

-2.5

$$((-2.5)+2)((-2.5)+3) > 0$$

$\downarrow$

$$(-.5)(.5) > 0$$

$\downarrow$

$$\boxed{\begin{array}{c} -.25 > 0 \\ \text{False} \end{array}}$$

$$((-1)+2)((-1)+3) > 0$$

$\downarrow$

$$(1)(2) > 0$$

$$\boxed{\begin{array}{c} 2 > 0 \\ \text{TRUE} \end{array}}$$

$x^2 + 4 \geq 0$

$x^2 + 4 = 0$

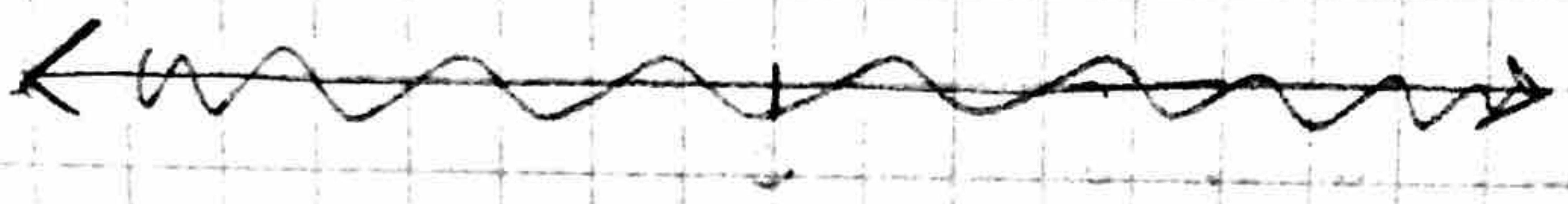
$\quad -4 \quad -4$

$\sqrt{x^2} = \sqrt{-4}$

$\boxed{x = \pm\sqrt{-4}}$

Non-Real Result

No Solutions in Real Numbers



$(0^2) + 4 \geq 0$

$4 \geq 0$

$\boxed{\text{Sol: } (-\infty, \infty)}$

Solving Rational Inequalities

Must be written as Form

$\frac{p(x)}{g(x)} < 0$

or

$>, \geq, \leq$

Form

$\left(\text{Single Fraction}\right) \begin{matrix} > \\ < \\ \leq \\ \geq \end{matrix} (0)$

1. Solve $p(x) = 0$
   $g(x) = 0$

2. Make # Line

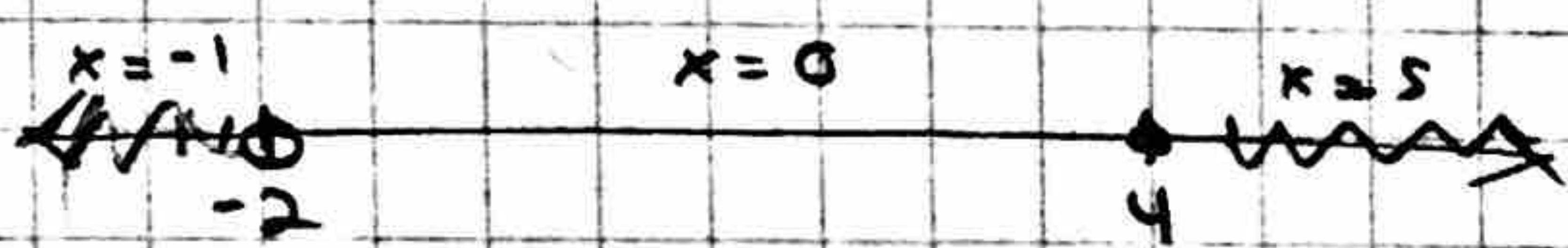
3. Test Points

$\frac{x-4}{x+2} > 0$

$\downarrow$

$x - 4 = 0$
$+4 \quad +4$
$x = 4$

$x + 2 = 0$
$-2 \quad -2$
$x = -2$



$\underline{-2}$

$\frac{(-2)-4}{(-2)+2} > 0$

$\downarrow$

$\frac{-6}{0} > 0$

undef

$\underline{-1}$

$\frac{(-1)-4}{(-1)+2} > 0$

$\downarrow$

$\frac{-5}{-1} > 0$

$\boxed{5 > 0}$
TRUE

$\underline{4}$

$\frac{((4)-4)}{((4)+2)} > 0$

$\downarrow$

$\frac{0}{6} > 0$

$\boxed{0 > 0}$
False

$\underline{0}$

$\frac{(0)-4}{(0)+2} > 0$

$\frac{-4}{2} > 0$

$\boxed{-2 > 0 \quad \text{False}}$

$\underline{5}$

$\frac{((5)-4)}{((5)+2)} > 0$

$\downarrow$

TRUE $\boxed{\frac{1}{7} > 0}$

Solution
$(-\infty, -2) \cup (4, \infty$

$$\frac{1}{x-3} \le \frac{5}{x-3}$$

$$-\frac{5}{x-3} \qquad -\frac{5}{x-3}$$

$$\downarrow$$

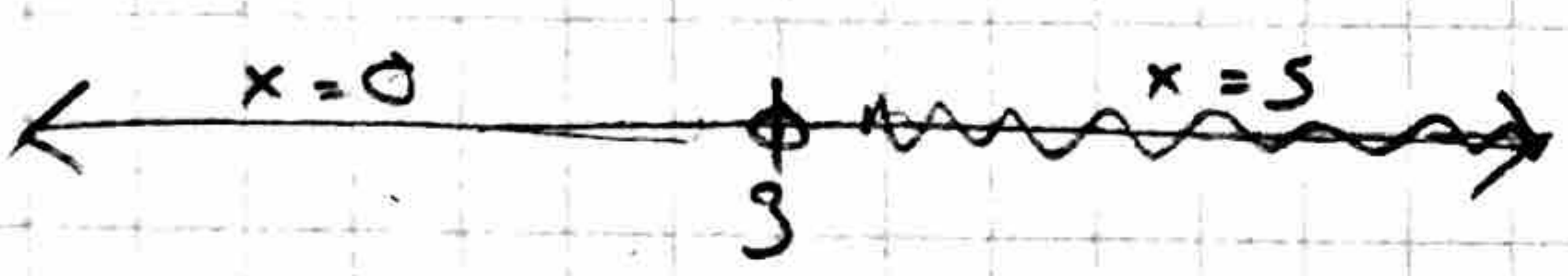
$$\frac{1}{x-3} - \frac{5}{x-3} \le 0$$

$$\downarrow$$

$$\boxed{\frac{-4}{x-3} \le 0}$$ Required Form

$$\downarrow$$

$$x - 3 = 0$$
$$+3 \quad +3$$
$$\boxed{x = 3}$$



3

$$\frac{-4}{(3)-3} \le 0$$
$$\downarrow$$
$$\frac{-4}{0} \le 0$$

under

$x = 5$

$$\frac{-4}{(5)-3} \le 0$$
$$\downarrow$$
$$\frac{-4}{2} \le 0$$
$$\downarrow$$
$$-2 \le 0$$

TRUE

$x = 0$

$$\frac{-4}{(0)-3} \le 0$$
$$\downarrow$$
$$\downarrow$$
$$\frac{-4}{-3} \le 0$$
$$\downarrow$$
$$\boxed{\frac{4}{3} \le 0}$$

False

$(3, \infty)$

$$\frac{1}{x} \le \frac{2}{x+1}$$

$$-\frac{2}{x+1} \qquad -\frac{2}{x+1}$$

↓

$$\frac{1}{x} - \frac{2}{x+1} \le 0$$

↓

$$\frac{1}{x}\left(\frac{x+1}{x+1}\right) - \frac{2}{x+1}\left(\frac{x}{x}\right) \le 0$$

↓

$$\frac{x+1-2x}{x(x+1)} \le 0$$

↓

$$\frac{1-x}{x(x+1)} \le 0$$

↓

$$\frac{1}{4} + \frac{3}{3}$$

↓

$$\frac{1}{4}\left(\frac{3}{3}\right) + \frac{3}{3}\left(\frac{4}{4}\right)$$

↓

$$\frac{3}{12} + \frac{12}{12} = \frac{15}{12}$$

$$\frac{1}{4} = \frac{3}{12}$$

$$+\frac{3}{3} = \frac{12}{12}$$

$$\frac{15}{12}$$

$$1 - x = 0$$

$$-1 \qquad = -1$$

$$\frac{-x}{-1} = \frac{-1}{-1}$$

↓

$$\boxed{x = 1}$$

$$x(x+1) = 0$$

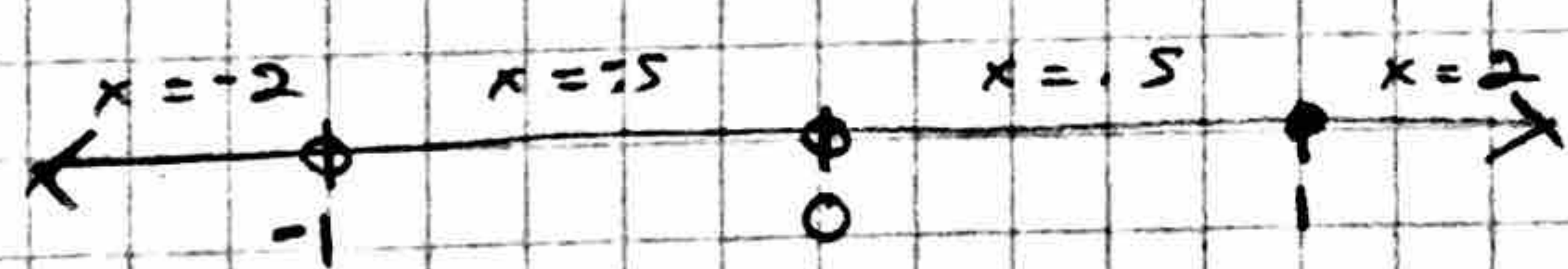
↓

$$\boxed{x=0}$$

$$x+1 = 0$$

$$-1 \quad -1$$

$$\boxed{x = -1}$$



$x = -1$

$$\frac{1-(-1)}{-1((-1)+1)} \le 0$$

↓

$$\frac{2}{-1(0)} \le 0$$

↓

$$\frac{2}{0} \le 0$$

under

$x = 0$

$$\frac{1-(0)}{0(0+1)} \le 0$$

↓

$$\frac{1}{0} \le 0$$

under

$x = 1$

$$\frac{1-(1)}{1(1+1)} \le 0$$

↓

$$\frac{0}{1(2)} \le 0$$

↓

$$\frac{0}{2} \le 0$$

$\boxed{0 \le 0}$ TRUE

$x=-2$

$$\frac{1-x}{x(x+1)} \le 0$$

$x=-2$, $x=-.5$, $x=.5$, $x=2$

$$\frac{1+(+2)}{-2(-2+1)} \le 0$$

$$\frac{3}{4-2} \le 0$$

$$\boxed{\frac{3}{2} \le 0}$$

false

$x=-.5$

$$\frac{1+(+.5)}{-.5(-.5+1)} \le 0$$

$$\frac{1.5}{-.25} \le 0$$

$$\boxed{-6 \le 0}$$

TRUE

$x=2$

$$\frac{1-(2)}{2(2+1)} \le 0$$

$$\frac{-1}{4+2} \le 0$$

$$\boxed{\frac{-1}{6} \le 0}$$

TRUE

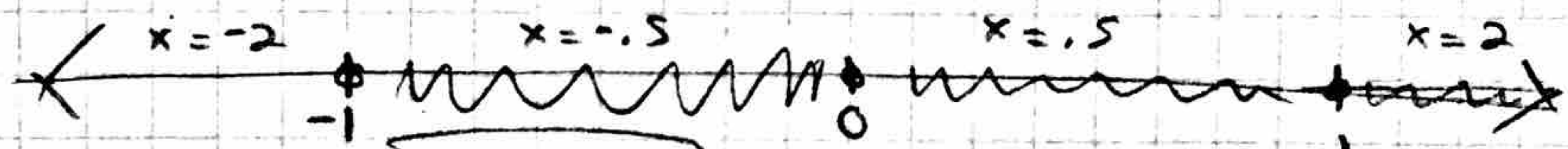
$x=.5$

$$\frac{1-(.5)}{.5(.5+1)} \le 0$$

$$\frac{-1.5}{.75} \le 0$$

$$\boxed{-2 \le 0}$$

TRUE



Solution
$(-1,\infty)$

Solving Absolute Value Inequalities

$|A| \boxed{\geq} B$, where $B \boxed{\geq} 0$, solve ① $A \geq B$
② $A \leq -B$

$|A| \boxed{\leq} B$, where $B \geq 0$, solve $-B \leq A \leq B$

$|2-4x| \geq 8$

$\downarrow$

$$\begin{array}{rcl} 2-4x & \geq & 8 \\ -2 & & -2 \\ \hline -4x & \geq & 6 \end{array}$$

$$\frac{-4x}{-4} \geq \frac{6}{-4}$$

$\downarrow$

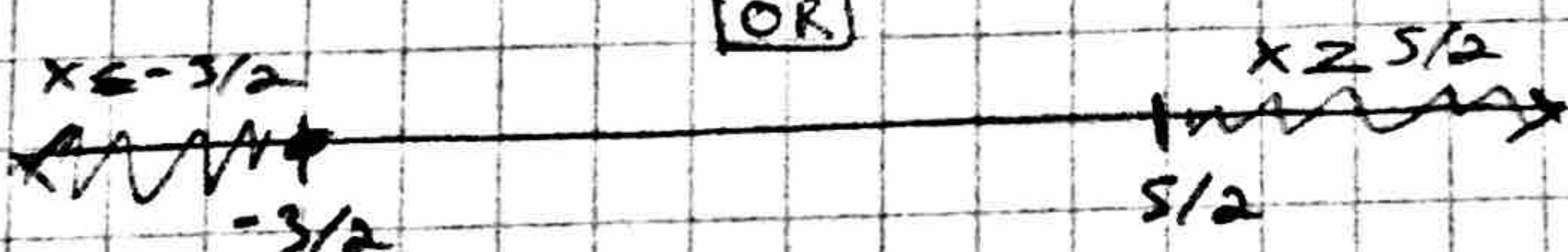
$$\boxed{x \leq -\frac{3}{2}}$$

$$\begin{array}{rcl} 2-4x & \leq & -8 \\ -2 & & -2 \\ \hline -4x & \leq & -10 \end{array}$$

$$\frac{-4x}{-4} \leq \frac{-10}{-4}$$

$$\boxed{x \geq \frac{5}{2}}$$

OR



$(-\infty, -3/2)$ OR $(5/2, \infty)$

$$\left| \frac{8-2d}{9} \right| \geq 5$$

$$\downarrow$$

$$\frac{\cancel{9}}{1}\left(\frac{8-2d}{\cancel{9}}\right) \geq 5\left(\frac{9}{1}\right)$$

$$\downarrow$$

$$8-2d \geq 45$$

$$\underline{-8 \qquad\qquad -8}$$

$$\frac{-2d}{-2} \geq \frac{37}{-2}$$

$$\downarrow$$

$$d \leq -\frac{37}{2}$$

$\boxed{\text{OR}}$

$$\frac{\cancel{9}}{1}\left(\frac{8-2d}{\cancel{9}}\right) \leq -5\left(\frac{9}{1}\right)$$

$$\downarrow$$

$$8-2d \leq -45$$

$$\underline{-8 \qquad\qquad -8}$$

$$\frac{-2d}{-2} \leq \frac{-53}{-2}$$

$$\downarrow$$

$$d \geq 53/2$$

$$\boxed{(-\infty, -37/2] \text{ OR } [53/2, \infty)}$$