

$$\frac{1}{x} = \frac{1}{1} \div \frac{x}{1} = \left[\frac{1}{1} \cdot \frac{1}{x} \right] = 1 \cdot \frac{1}{x}$$

I. $3x \leq f(x) \leq x^3 + 2$

for $0 \leq x \leq 2$, evaluate:

$$\lim_{x \rightarrow 1} f(x)$$

①

$$\lim_{x \rightarrow 1} 3x$$

$$\begin{aligned} &= 3 \cdot \lim_{x \rightarrow 1} x \\ &= 3 \cdot 1 \\ &= 3 \end{aligned}$$

③

$$\lim_{x \rightarrow 1} x^3 + 2$$

$$\begin{aligned} &= \lim_{x \rightarrow 1} x^3 + \lim_{x \rightarrow 1} 2 \\ &= (1)^3 + 2 \\ &= 1 + 2 \\ &= 3 \end{aligned}$$

③

②

$$\underbrace{\lim_{x \rightarrow 1} 3x}_3 \leq \lim_{x \rightarrow 1} f(x) \leq \underbrace{\lim_{x \rightarrow 1} x^3 + 2}_3$$

↓

Using Squeeze

$$\lim_{x \rightarrow 1} f(x) = 3$$

Cannot Substitute

Find $\lim_{x \rightarrow 0} x^2 \cdot \cos\left(\frac{1}{x^2}\right)$

$(0)^2 \cdot \cos\left(\frac{1}{0^2}\right) = \text{undef}$

① Domain of $\cos [-1, 1]$

$$-1 \leq \cos\left(\frac{1}{x^2}\right) \leq 1$$

↗ This is the same
multiplying by 1.

$$-1 \leq 1 \cdot \cos\left(\frac{1}{x^2}\right) \leq 1$$

↓
Using the same logic

$$-x^2 \leq x^2 \cdot \cos\left(\frac{1}{x^2}\right) \leq x^2$$

②

$$\lim_{x \rightarrow 0} -x^2 \leq \lim_{x \rightarrow 0} x^2 \cdot \cos\left(\frac{1}{x^2}\right) \leq \lim_{x \rightarrow 0} x^2$$

" " "

$$\lim_{x \rightarrow 0} -(0)^2 \leq \lim_{x \rightarrow 0} x^2 \cdot \cos\left(\frac{1}{x^2}\right) \leq \lim_{x \rightarrow 0} (0)^2$$

" "

$$0 \leq \lim_{x \rightarrow 0} x^2 \cdot \cos\left(\frac{1}{x^2}\right) \leq 0$$



$$\lim_{x \rightarrow 0} x^2 \cdot \cos\left(\frac{1}{x^2}\right) = 0$$

$$\lim_{x \rightarrow 0} x^4 \sin \frac{3}{x} \rightarrow \boxed{0^4 \sin \frac{3}{0} = \text{under}}$$

①

$$-1 \leq \sin \frac{3}{x} \leq 1, \text{ for all } x, x \neq 0$$

"

②

$$-1 \cdot x^4 \leq x^4 \sin \frac{3}{x} \leq x^4 \cdot 1$$

"

$$-x^4 \leq x^4 \sin \frac{3}{x} \leq x^4$$

"

$$\lim_{x \rightarrow 0} -x^4 \leq \lim_{x \rightarrow 0} x^4 \sin \frac{3}{x} \leq \lim_{x \rightarrow 0} x^4$$

"

0

$$\leq \lim_{x \rightarrow 0} x^4 \sin \frac{3}{x}$$

"

"

0

$$\boxed{\lim_{x \rightarrow 0} x^4 \sin \frac{3}{x} = 0}$$