$f(x) = \cos(x)$

$$\cos(A + B)$$
$$"$$
$$\cos(A)\cos(B) - \sin(A)\sin(B)$$

$f'(x) = \lim\limits_{h \to 0} \dfrac{f(x+h) - f(x)}{h}$

$$\cos(x+h)$$
$$"$$
$$\cos(x)\cos(h) - \sin(x)\sin(h)$$

$$"$$

$$\lim\limits_{h \to 0}\left[\dfrac{\cos(x+h) - \cos x}{h}\right]$$

Cosine Sum Formula

$$\cos(x+h)$$
$$"$$
$$\cos(x)\cos(h) + \sin(x)\sin(h)$$

$$\lim\limits_{h \to 0}\left[\dfrac{\cos(x)\cos(h) - \sin(x)\cdot\sin(h) - \cos(x)}{h}\right]$$

$$"$$

$$\lim\limits_{h \to 0}\left[\dfrac{\cos(x)\cos(h) - \cos(x) - \sin(x)\cdot\sin(h)}{h}\right]$$

$\lim\limits_{h \to 0} \dfrac{\sin(h)}{h} = 1$

$\lim\limits_{h \to 0} \dfrac{\cos(h) - 1}{h} = 0$

$$\lim\limits_{h \to 0}\left[\cos(x)\cdot\dfrac{\cos(h) - 1}{h} - \sin(x)\cdot\dfrac{\sin(h)}{h}\right]$$

$$"$$

$$\cos(x)\cdot\lim\limits_{h \to 0}\left[\dfrac{\cos(h) - 1}{h}\right] - \sin(x)\cdot\lim\limits_{h \to 0}\left[\dfrac{\sin(h)}{h}\right]$$

$$"$$

$$\cos(x)\cdot 0 - \sin(x)\cdot 1$$

$$"$$

$$0 - \sin(x)$$

$$"$$

$$-\sin(x)$$

$$"$$

$$f'(x) = -\sin(x)$$