2. $\exists\ x \in N, \to (x/3) \in N$ (True) P $\quad P(x)$

witness

$x = 3$

$(x/3) \in N : \{3, 6, 9, \ldots\}$: True

$(x/3) \in N : \{1, 2, 4, \ldots\}$: False

$\forall\ x \in N, \quad (x/3) \notin N$ (False) $\overline{P}$ $\quad P(x)$

Counterexample

$3 \in N$,

$x = 3,\ 3/3 = 1 \in N$

$(x/3) \notin N : \{1, 2, 4, \ldots\}$: True

$(x/3) \notin N : \{3, 6, 9, \ldots\}$: False

$P(x)$

3. $\forall\, y \in N, (3/y) \notin N$ (False)

$(3/y) \notin N : \{2, 4, 6, \ldots\}$: True

$(3/y) \notin N : \{1, 3\}$: False $\quad y = 1 \in N$, $3/1 = 3 \in N$

$\exists\, y \in N, \exists\, (3/y) \in N$ (True)

$(3/y) \in N : \{1, 3\}$: True $\quad y = 1$

$(3/y) \in N : \{2, 4, 6, \ldots\}$: False

$P(x)$

4. $\forall\, y \in Z, y^2 \notin Z$ (False) P

$y^2 \notin Z : \{\ \}$ True

$y^2 \notin Z : \{1, 2, 3, \ldots\} \quad y = 1$ Any Integer That is Squared Is Always an Integer.

$\exists\, y \in Z, \exists\, y^2 \in Z$ (True) $\overline{P}$

$y^2 \in Z : \{1, 2, 3, \ldots\} \quad y = 1$

$y^2 \in Z : \{\ \}$ False

$P(x)$

5. $\exists\, x \in N, \exists\, x^2 \geq x$ (True) P

$x^2 \geq x \{1, 2, 3, 4 \ldots\}$ True $\quad x = 1$

$x^2 \geq x \{\ \}$ False

$\forall\, x \in N, x^2 < x$ (False) $\overline{P}$

$x^2 < x \{\ \}$ True $\quad x = 1 \in N$, $(1)^2 \leq x$

$x^2 < x \{1, 2, 3, 4, \ldots\}$ False

$P(x)$

6. $\exists \; x \in R, \to \; x^2 \geq x$ (True) P

$x^2 \geq x$: {1, 2} True $x = 1$

$x^2 \geq x$: {1/2} : False

$\forall \; x \in R, x^2 < x$ (False) $\overline{P}$

$x^2 < x$: {1/2} True

$x^2 < x$: {-1, 2} False $x = 0$

$(0)^2 \leq 0$

$P(x)$

7. $\forall \; x \in R, x^2 \geq x$ (False) P

$x^2 \geq x$: {1, 2} True

$x^2 \geq x$: {1/2} False

$\exists \; x \in R, \; x^2 < x$ (True) $\overline{P}$

$x^2 < x$: {1/2} True

$x^2 < x$: {1, 2} False

$P(x)$

8. $\forall \; x \in Q, \; x^2 \geq x$ (False) P

$x^2 \geq x$: { , } True

$x^2 \geq x$: {1/2} False

$\exists \; x \in Q, \to x^2 < x$ (True) $\overline{P}$

$x^2 < x$: {1/2} True $x = 1/2$

$x^2 < x$: { } False

9. $\forall\ x \in W,\ x^2 \geq 0$ (True) — $P(x)$

$x^2 \geq 0$: {0} True
$x^2 \geq 0$: { } False

$\exists\ x \in W, \not\rightarrow x^2 < 0$ (False)

$x^2 < 0$: { } True
$x^2 < 0$: {0} False

$x = 0$
$x^2 \not< 0$

10. $\forall\ x \in N,\ x^2 \geq 1$ (True) — $P(x)$

$x^2 \geq 1$: {1, 2, ...} True
$x^2 \geq 1$: { } False

$\exists\ x \in N, \not\rightarrow x^2 < 1$ (False) $x^2 < 1$: { } True
$x^2 < 1$: {1, 2, ...} False

11. $\forall\ x \in N,\ x+1 > x$ (True) P — $P(x)$

$x+1 > x$: {1, 2, 3, ...} True
$x+1 > x$: { } False

$\exists\ x \in N, \not\rightarrow x+1 \leq x$ (False) $\overline{P}$

$x+1 \leq x$: { } True
$x+1 \leq x$: {1, 2, 3, ...} False

12. $\forall\ x \in W,\ x < x^2 + 1$ (True) P — $P(x)$

$x < x^2 + 1$: {0} True
$x < x^2 + 1$: { } False

$\exists\ x \in W, \not\rightarrow x \geq x^2 + 1$ (False) $\overline{P}$

$x \geq x^2 + 1$: { } True
$x \geq x^2 + 1$: {0} False

$0 \not\geq (0)^2 + 1$
$0 \not\geq 1$

13. $\exists\ x \in R, \ni \underbrace{x+1>0}_{P(x)}$ (True) P

$x+1>0$ $[0,\infty)$ True — $x=1$, $0+1>0$, $1>0$

$x+1>0$ $(-\infty,-1]$ False

$\forall\ x \in R, \ni x+1 \leq 0$ (False) $\overline{P}$

$x+1 \leq 0$ $(-\infty,-1]$ True

$x+1 \leq 0$ $[0,\infty)$ False — $x=0$, $0+1 \not\leq 0$, $1 \not\leq 0$

14. $\exists\ x \in N, \ni \underbrace{x^2-4=0}_{P(x)}$ (True) P

$x^2-4=0$ $\{2\}$ True — $x=2$, $(2)^2-4=0$, $4-4=0$, $0=0$

$x^2-4=0$ $[-1,2) \cup (2,\infty)$ False

$\forall\ x \in N, \ni x^2-4 \neq 0$ (False) $\overline{P}$

$x^2-4 \neq 0$ $[1,2) \cup (2,\infty)$ True

$x^2-4 \neq 0$ $\{2\}$ False — $x=1$, $(1)^2-4=0$, $1-4=-3$

(15) $\forall\ x, y \in N, \underbrace{x<y}_{P(x)}$ (False) P

$x<y$ $\{x=1, y=2\}$ True

$x<y$ $\{x=3, y=2\}$ False — $3<2$

$\exists\ x, y \in N, \ni x \geq y$ (True) $\overline{P}$

$x \geq y$ $\{x=3, y=2\}$ True — $x=3, y=2$

$x \geq y$ $\{x=1, y=2\}$ False

P(x)

16. $\forall\ x, y \in N\ ,\ x^2 < y$ (False) P

$x^2 < y$ { $x=1$, $y=2$ } True

$x^2 < y$ { $x=2$, $y=1$ } False

$(1)^2 < 2$ $\quad x=2 \quad y=1$

$(2)^2 < 1$

"

$4 < 1$ CE

$\overline{P}$

$\exists\ x, y \in N\ ,\ni\ x^2 \geq y$ (True) $\overline{P}$

$x^2 \geq y$ { $x=2$, $y=1$ } True $\quad x=2,\ y=1$

$x^2 \geq y$ { $x=1$, $y=2$ } False