MATH E-3 Assignment 6

TOTAL POINTS: 83

NAME:

Please show your work in the space provided.

Problems 6.1 and 6.2
The Normal Distribution Curve

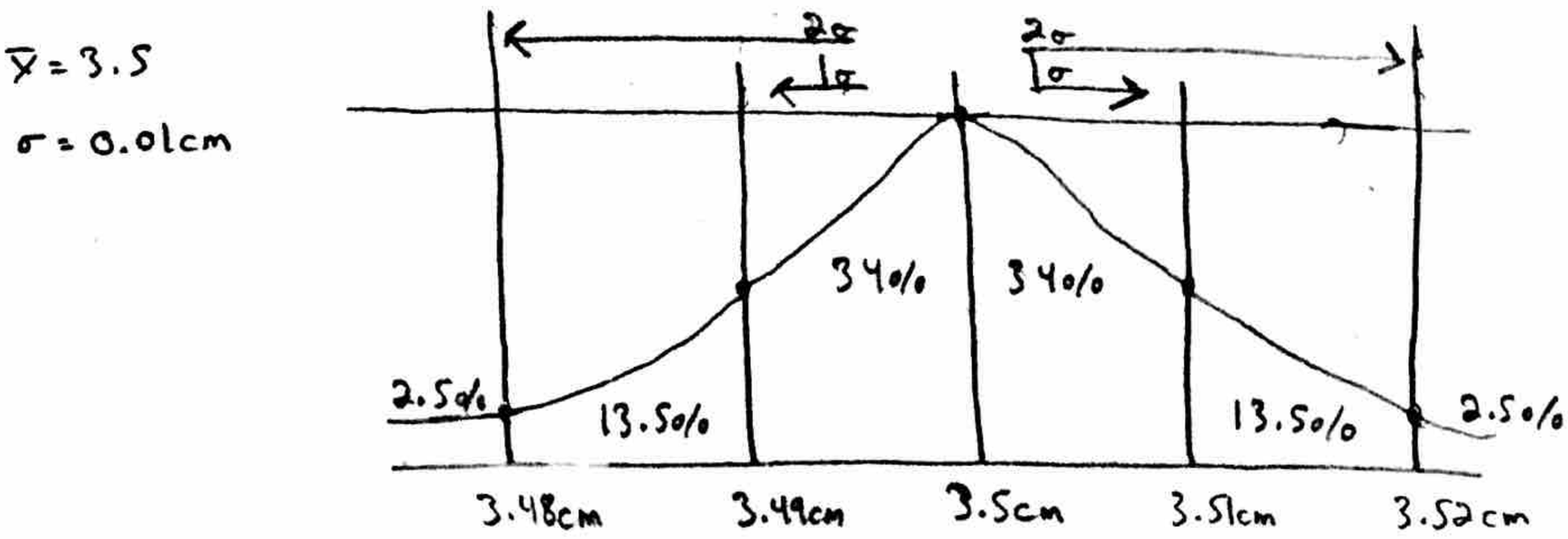
Draw your distribution curves by hand, and please be neat. You will need to show the mean, the values for plus or minus 1 standard deviation and 2 standard deviations, along with the rough and ready percentages that apply. Be sure to indicate, with label arrows, 1 and 2 standard deviations (see readings and lecture slides for example). There is no need to show 3 standard deviations.

Problem 6.1 (6 points)

A quality control engineer for a major toy company recently performed some statistical analysis on the length of plastic screws produced by its 'Plastics Division.' These particular screws are used in the construction of certain children's toys, so their length is important in the structural integrity of the toys. Thus, a check is periodically performed. He found that the mean length of the screws is **3.5** centimeters and the standard deviation is **0.01cm.**

Answer the following. Make sure you draw the appropriate diagram for the normal distribution of these screws.

Draw Diagram here:



Problem 6.1

Assume the quality control engineer chooses one screw at random. What percentage of the time will the length of the screw be:

a) More than 3.51 cm? **(2 points)**

13.5% + 2.5% = 16%

b) Less than 3.48 cm? **(2 points)**

2.5%

c) More than 3.49 cm? **(2 points)**

2(34%) + 13.5% + 2.5% = 84%

d) Less than 3.52 cm? **(2 points)**

2(34%) + 2(13.5%) + 2.5 = 97.5%

e) Less than 3.5 cm? **(2 points)**

34% + 13.5% + 2.5% = 50%

f) Between 3.48 and 3.51 cm? **(2 points)**

2(34%) + 13.5% = 81.5%

g) Between 3.51cm and 3.52 cm? 13.5% **(2 points)**

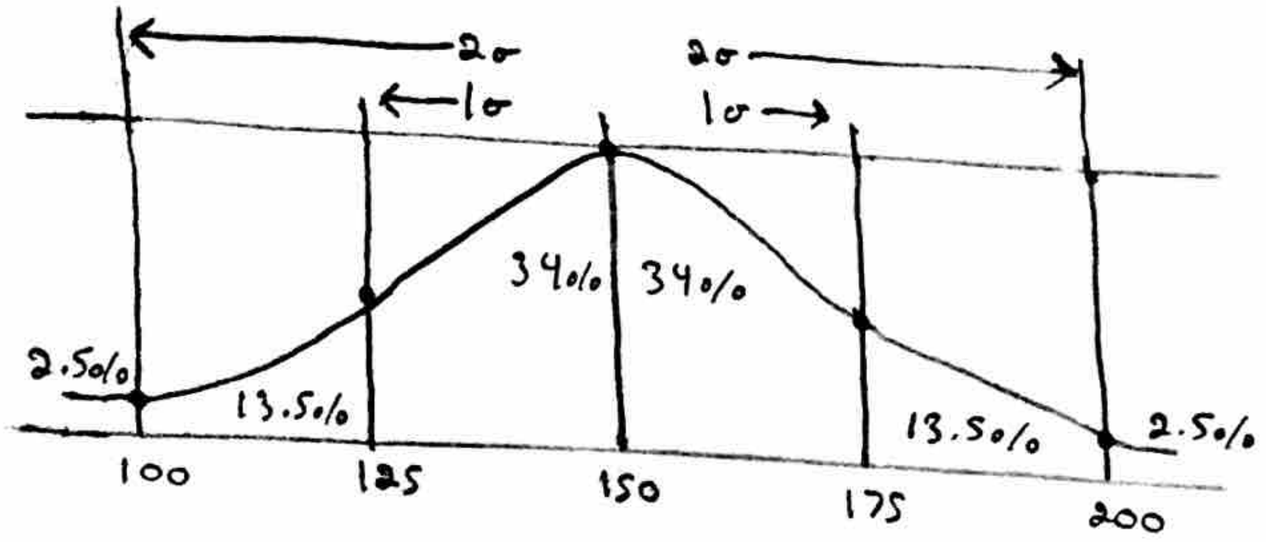
Problem 6.2 (6 points)

The 911 emergency center in a certain city receives various numbers of calls on different days. A record of the center shows that the number of daily calls is normally distributed with mean of 150 and standard deviation of 25. Remember to sketch a diagram.

Draw diagram here:

$\bar{x} = 150$

$\sigma = 25$



a) What percent of the days in this record have more than 100 calls? **(2 points)**

$2(34\%) + 2(13.5\%) + 2.5\% = 97.5\%$

b) What percent of the days in this record have fewer than 175 calls? **(2 points)**

$2(34\%) + 13.5\% + 2.5\% = 84\%$

c) What percent of the days in this record have between 125 and 200 calls? **(2 points)**

$2(34\%) + 13.5\% = 81.5\%$

d) What percent of the days in this record have more than 200 calls? **(2 points)**

2.5%

Problems 6.3-6.7
The new formula for standard deviation, and 95% Confidence Intervals.

Please give standard deviations as a <u>percent</u>, rounded to one decimal point. Round <u>after</u> turning into a percent.

Problem 6.3

For the following problems, calculate the **standard deviation** using our new formula. Show all your work as done in class. No diagram necessary for these questions.

a) Assume p = 54% and our sample size, n = 375. **(4 points)**

$p = .54,\ n = 375$

$\sigma = \sqrt{\frac{p(1-p)}{n}} = \sqrt{\frac{.54(1-.54)}{375}} = \sqrt{\frac{.54(.46)}{375}} = \sqrt{\frac{.2484}{375}} = \sqrt{.000662} = .025729$

$\downarrow$

02.5729

$\sigma = 2.6\%$

b) Assume p = 68% and our sample size, n = 1105. **(4 points)**

$p = .68,\ n = 1105$

$\sigma = \sqrt{\frac{p(1-p)}{n}} = \sqrt{\frac{.68(1-.68)}{1105}} = \sqrt{\frac{.68(.32)}{1105}} = \sqrt{\frac{.2176}{1105}} = \sqrt{.000197} = .014036$

$\downarrow$

01.4036

$\sigma = 1.4\%$

c) Assume p = 4% and our sample size, n = 250 **(4 points)**

$p = .04,\ n = 250$

$\sigma = \sqrt{\frac{p(1-p)}{n}} = \sqrt{\frac{.04(1-.04)}{250}} = \sqrt{\frac{.04(.96)}{250}} = \sqrt{\frac{.0384}{250}} = \sqrt{.000154} = .01241$

$\downarrow$

01.241

$\sigma = 1.2\%$

Problem 6.4

The University of Michigan Survey Research Center conducted a survey of 2000 people In 1976. 20% of those surveyed said they felt unhappy about their jobs.

Find a 95% Confidence Interval for the true proportion of people who felt unhappy about their jobs. **(6 points)**

$p = 20\%$

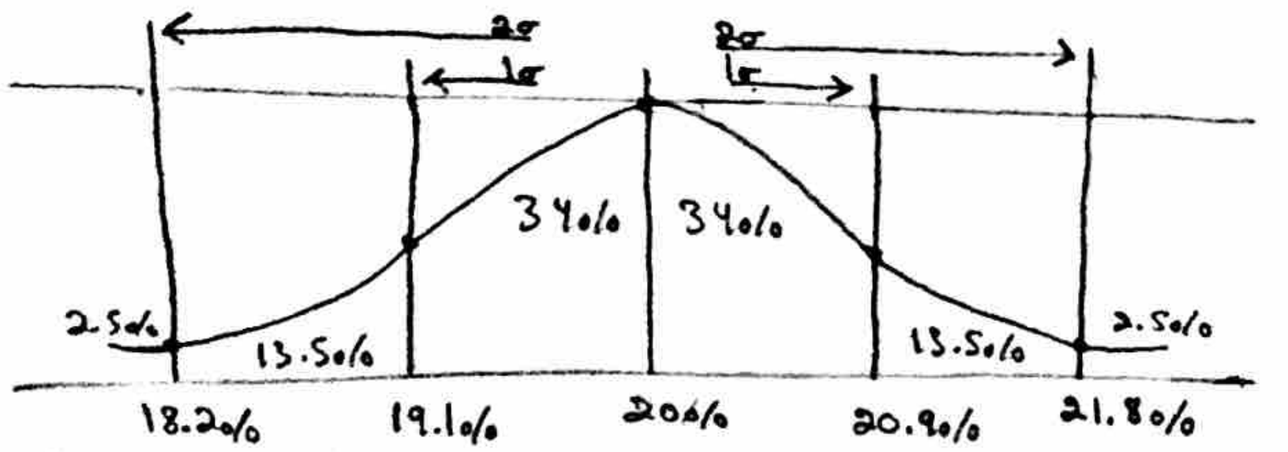
$n = 2000$

$$\sigma = \sqrt{\frac{p(1-p)}{n}} = \sqrt{\frac{.2(1-.2)}{2000}} = \sqrt{\frac{.2(.8)}{2000}} = \sqrt{\frac{.16}{2000}} = \sqrt{.00008} = .008944$$

0.8944

$\sigma = 0.9\%$

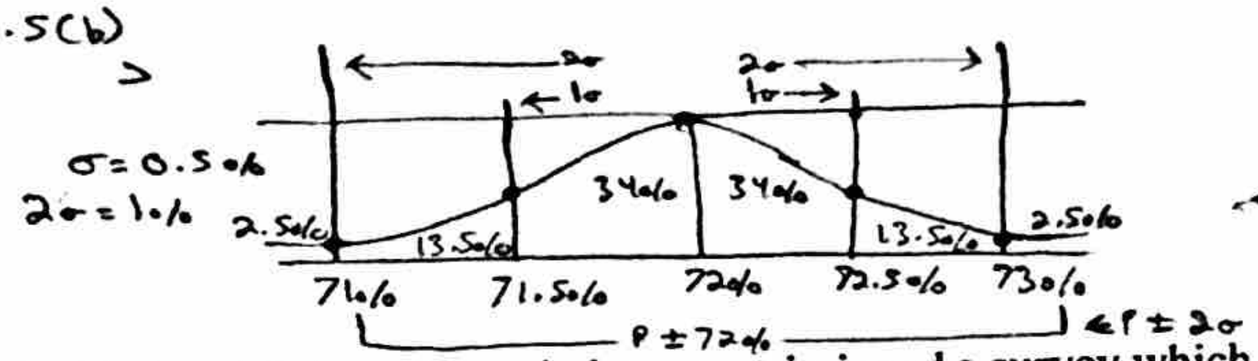
Margin of Error : $2\sigma \rightarrow 2(0.9) = 1.8\%$



The 95% Confidence Interval for the true proportion of people who felt unhappy about their jobs is between 18.2% and 21.8%.

$18.2\% \longleftarrow\cdots\cdots\cdots\longrightarrow 21.8\%$

Problem 6.5



In July, 1998, the National Science Foundation commissioned a survey which revealed that vast numbers of American adults are scientifically illiterate. A Boston Globe article describing the results of this study reported that of 2041 adults questioned, only 72% knew the earth revolved around the sun and not the other way around!!!

a) Assuming this was a random sample of all American adults, find a 95% confidence interval for the true proportion of scientifically knowledgeable American adults. **(6 points)**

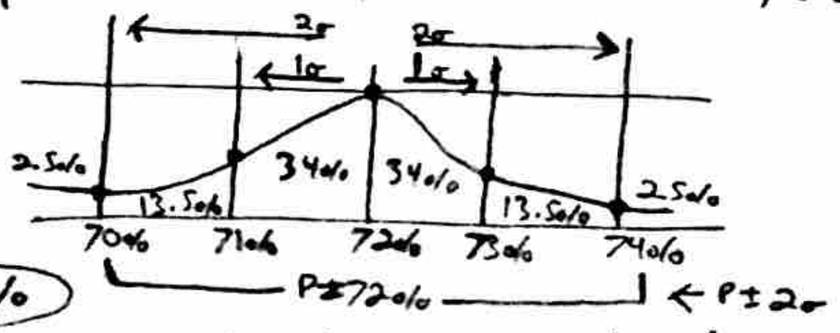
$P = 72\%$, $n = 2041$

$\sigma = \sqrt{\frac{P(1-P)}{n}} = \sqrt{\frac{.72(1-.72)}{2041}} = \sqrt{\frac{.72(.28)}{2041}} = \sqrt{\frac{.2016}{2041}} = \sqrt{.000099} = .00995 \rightarrow 0.995,\ \sigma = 1\%$

$2\sigma \rightarrow$ Margin of Error: $2(1\%) = 2\%$

The 95% Confidence Interval for the true proportion of scientifically knowledgeable American adults is between 70% and 74%.

70% <------> 74%



b) How would the size of the Confidence Interval change if four times as many American adults were surveyed? Explain why your answer is different (if it is) and <u>show</u> your calculations to prove your answer. **(6 points)** Normal distribution is above to allow space.

$P = 72\%$, $n = 4(2041) = 8164$

$\sigma = \sqrt{\frac{P(1-P)}{n}} = \sqrt{\frac{.72(1-.72)}{8164}} = \sqrt{\frac{.72(.28)}{8164}} = \sqrt{\frac{.2016}{8164}} = \sqrt{.000025} = .005 \rightarrow 0.5,\ \sigma = 0.5\%$

Margin of Error: $2\sigma \rightarrow 2(0.5\%) = 1\%$

The 95% Confidence Interval for the true proportion of scientifically knowledgeable American adults is between 71% and 73%, given $n = 4(204) = 8164$ 71% <---> 73%

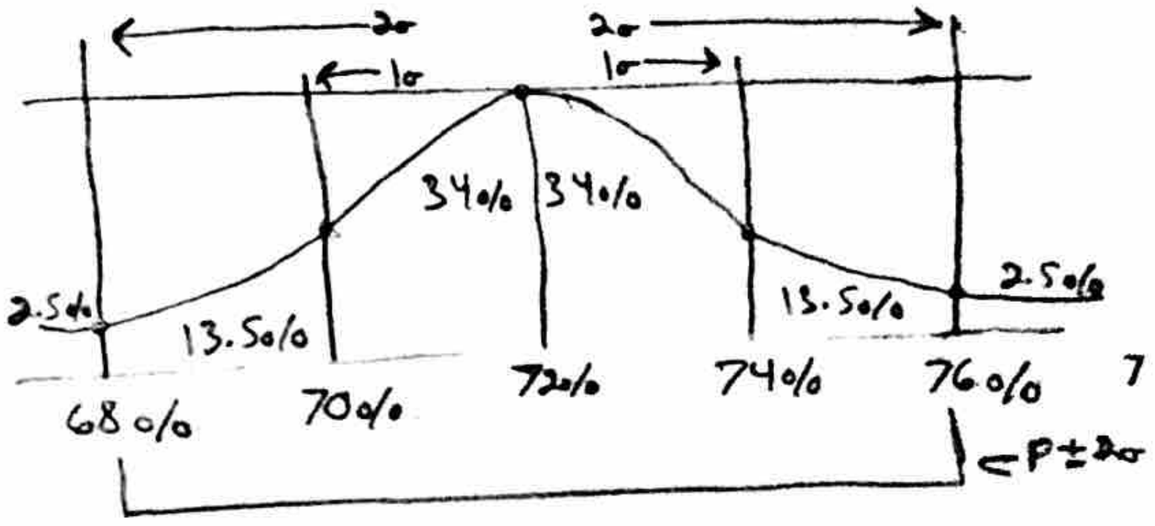
For a larger sample size of $n = 8164$, the standard deviation is smaller from 1% to 0.5%. The margin of error is also smaller from 2% to 1%.

c) What if the size of the survey was one-fourth as large? i.e. 1/4 of 2041 (with rounding)? Again, explain and <u>show</u> your calculations. **(6 points)**

$P = 72\%$, $n = .25(2041) = 510.25 \approx 510$

$\sigma = \sqrt{\frac{P(1-P)}{n}} = \sqrt{\frac{.72(1-.72)}{510}} = \sqrt{\frac{.72(.28)}{510}} = \sqrt{\frac{.2016}{510}} = \sqrt{.000395} = .019875 \rightarrow 1.9875;\ \sigma = 2\%$

Margin of Error $\rightarrow 2\sigma \rightarrow 2(2\%) = 4\%$



The 95% Confidence Interval for the true proportion of scientifically knowledgeable American adults is between 68% and 76% given a sample size of 510 American adults.

68% <-------> 76%

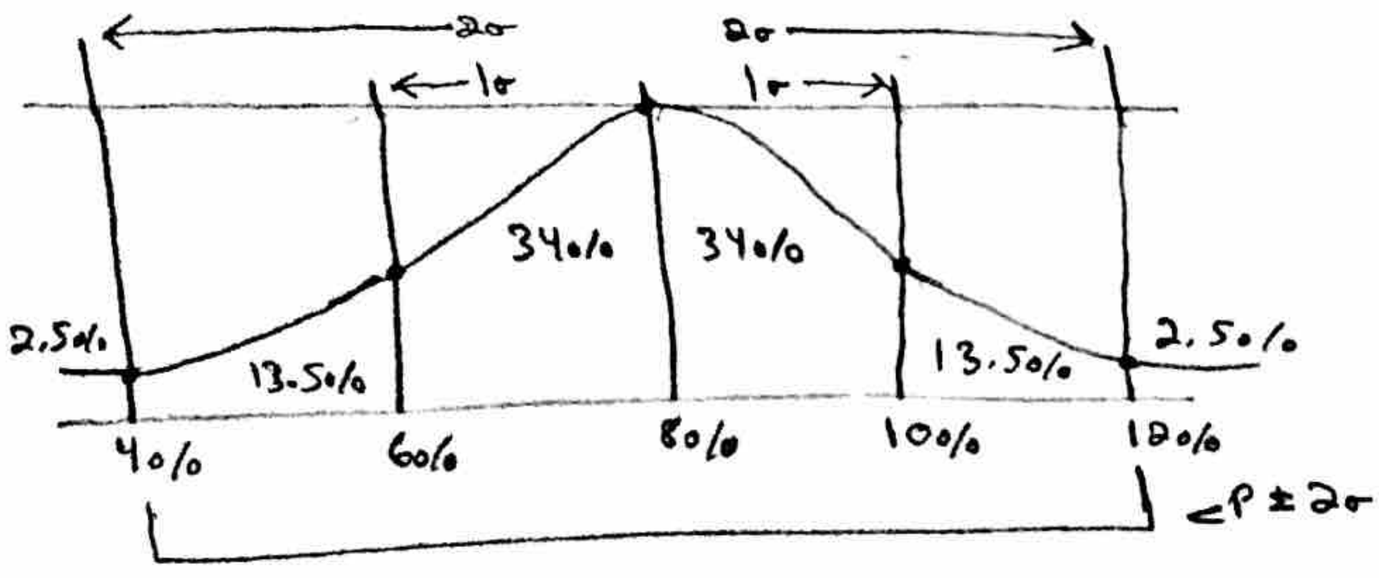
Problem 6.6

The *Encyclopaedia Britannica* reports that 8% (use this value for your mean=p) of all North American males have some form of red-green color blindness. Imagine that you take a random sample of 184 North American males. What is the probability that in this sample over 10% of these people will be color blind? **Hint:** You should draw a normal distribution curve to answer this question. **(6 points)**

$p = 8\%$, $n = 184$

$\sigma = \sqrt{\frac{p(1-p)}{n}} = \sqrt{\frac{.08(1-.08)}{184}} = \sqrt{\frac{.08(.92)}{184}} = \sqrt{\frac{.0736}{184}} = \sqrt{.0004} = .02 \rightarrow 2.\ \ \sigma = 2\%$

Margin of Error $2\sigma \rightarrow 2(2\%) = 4\%$



Over $10\% \rightarrow 13.5\% + 2.5\% = 16\%$

There is a 16% probability that 10% of 184 people will be color blind.

Problem 6.7

In yet another article in the Boston Globe (back in May of 1989) it was reported that in a study of 854 young women (ages 12-23) about 67% of them were dissatisfied with their weight.

a) Assuming once again that this was a random sample, find a 95% Confidence Interval for the true proportion of young women (ages 12-23) who were dissatisfied with their weight. **(6 points)**

$p = 67\%$

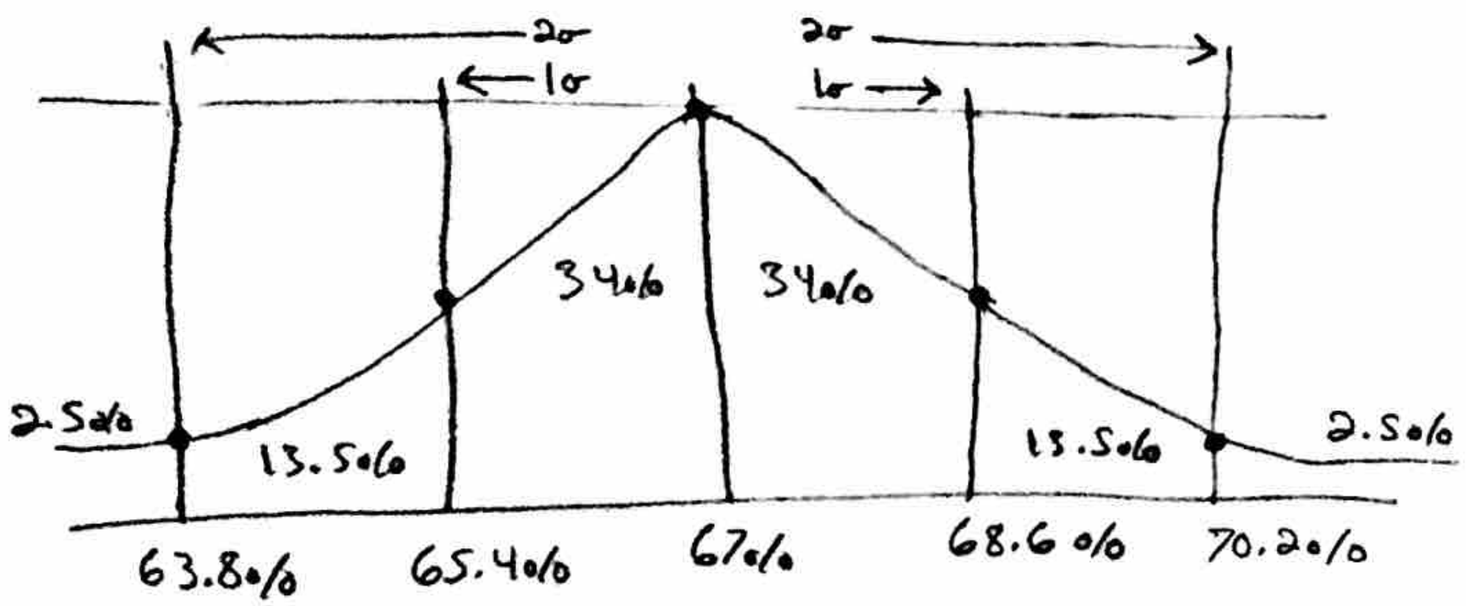
$n = 854$

$$\sigma = \sqrt{\frac{p(1-p)}{n}} = \sqrt{\frac{.67(1-.67)}{854}} = \sqrt{\frac{.67(.33)}{854}} = \sqrt{\frac{.2211}{854}} = \sqrt{.000259} = .016093$$

1.6093

$\sigma = 1.6\%$

Margin of Error $2\sigma \rightarrow 2(1.6\%) = 3.2\%$



The 95% Confidence Interval for the true proportion of young women (ages 12-23) who were dissatisfied with their weight is between 63.8% and 70.2%.

$63.8\% \longleftrightarrow 70.2\%$

Problem 6.7 (continued)

b) How much smaller a group could you have surveyed if you were willing to accept a 95% Confidence Interval of **67% ± 5%? (1 pt extra credit)**

Hint: You can use either trial and error or algebra. There is nothing wrong with trial and error! In either case, we will give you 1 point extra credit if you get within 5 of the true answer. Round your answer to a whole number since we can't survey parts of people! To solve the problem algebraically, you will have to use the new formula "in reverse." I.e. instead of knowing p and n, and then solving for the standard deviation σ, you will start by knowing the standard deviation σ and p, and then solving for n. That's one of the great things about formulas – you can generally use them in more than one way.

$p = 67\%$

$\sigma = \frac{5\%}{2} = 2.5\%$

$\sigma = \sqrt{\frac{p(1-p)}{n}}$

$.025 = \sqrt{\frac{.67(1-.67)}{n}}$

$.025 = \sqrt{\frac{.67(.33)}{n}}$

$.025 = \sqrt{\frac{.2211}{n}}$

$(.025)^2 = \left(\sqrt{\frac{.2211}{n}}\right)^2$

$n(.000625) = \left(\frac{.2211}{n}\right)n$

$\frac{.000625n}{.000625} = \frac{.2211}{.000625}$ → $n = 353.76$

$\sigma = \sqrt{\frac{p(1-p)}{n}}$

$p = 67\%$, $n = 354$ ↓ rounded from $n = 353.76$

$\sigma = \sqrt{\frac{.67(1-.67)}{354}}$

$\sigma = \sqrt{\frac{.67(.33)}{354}}$

$\sigma = \sqrt{\frac{.2211}{354}}$

$\sigma = \sqrt{.000625}$

$\sigma = .025 \rightarrow 2.5$, $\sigma = 2.5\%$

$2\sigma = 2(2.5\%) = 5\%$

Gotta Love Algebra!

Answer: A group size of 354 should suffice.