Three Requirements Are Needed For Function F to be Continuous

function f is continuous at a number a if $\lim_{x \to a} f(x) = f(a)$

1. $f(a)$ is defined $\to$ a is in the domain of f

2. $\lim_{x \to a} f(x)$ exists

3. $\lim_{x \to a} f(x) = f(a)$

IT LOOKS LIKE IM DEFINING BOTH $\boxed{f(a)}$ and $\boxed{\lim_{x \to a} f(x)}$

Once They Are Defined AND output the same Value. Then $\lim_{x \to a} f(x) = f(a)$

$f(x) = x + 1$ Show that f is continuous at $x = 1$

1. $f(1)$ is defined, 1 is in the domain of f. $f(1) = 1 + 1 = 2$

$f(a)$ is defined, a is in domain of f

2. $\lim_{x \to 1} f(x) \to \lim_{x \to 1} (x+1)$

$$= \lim_{x \to 1} x + \lim_{x \to 1} 1$$

$$= 1 + 1$$

$$= 2$$

limit of $f(x)$ exists as x approaches 1

$\lim_{x \to a} f(x)$ exists

3. $f(1) = 2$, so $\lim_{x \to 1} f(x) = f(1)$

$\boxed{\text{i } f \text{ is continuous at } x = 1 \text{ by definition}}$

$f(1) = 2$ And $\lim_{x \to 1} f(x) = 2$

Both of those are defined and output the same value 2

this makes f continuous at $x = 1$

Using Definition of Continuity and Limit Laws

Show that the function $f(x) = \frac{1}{x} + \sqrt{x+1}$ is continuous at $x = 1$

① $a = x$, $a = 1$

$f(a) = \frac{1}{a} + \sqrt{a+1}$, $a = 1$, $g(a) = 1/a$, $h(a) = \sqrt{a+1}$

Domain of $g(a)$: $(-\infty, 0) \cup (0, \infty)$

Domain of $h(a)$: $[-1, \infty)$ ← $a = 1$ falls within these intervals

Both $g(a)$ and $h(a)$ are continuous at $a = 1$, then $f(a)$ is also continuous at $a = 1$.

② $$\lim_{x \to 1} \underbrace{\frac{1}{x}}_{g(a)} + \underbrace{\sqrt{x+1}}_{h(a)}$$

$$= \lim_{x \to 1} \frac{1}{x} + \lim_{x \to 1} \sqrt{x+1}$$

$$= \frac{\lim_{x \to 1} 1}{\lim_{x \to 1} x} + \sqrt{\lim_{x \to 1} x+1}$$

Domain of $g(a)$: $(-\infty, 0) \cup (0, \infty)$
Domain of $h(a)$: $[-1, \infty)$
→ Same intervals as $f(a)$

③ $f(a)$ and $\lim_{x \to 1} \frac{1}{x} + \sqrt{x+1}$ have same domains,

f is continuous at $x = 1$

Investigate the continuity of the function $f(x) = 1/x$

① $f(a)$ is defined, a is in the domain of f.

Domain of $f(x) = 1/x$

$(-\infty, 0) \cup (0, \infty)$

This is where a is defined.

② $\lim_{x \to a} f(x)$ exists

$\lim_{x \to a} \frac{1}{x}$ exists for values except 0

$a \neq 0$

③ $\lim_{x \to a} f(x) = f(a)$

$\parallel$

$\lim_{x \to a} \frac{1}{x} = f(a)$, where $a \neq 0$

f is continuous at
$(-\infty, 0) \cup (0, \infty)$

f is not continuous at
$(-\infty, 0] \cup [0, \infty)$

$f(x) = 1/x$, Show that f is continuous at $x=2$

(1) $f(2)$ is defined, $f(2) = \frac{1}{2}$ not undef

(2) $\lim_{x \to 2} f(x) = \lim_{x \to 2} \frac{1}{x}$

$$= \frac{\lim_{x \to 2} 1}{\lim_{x \to 2} x}$$

$$= \frac{1}{2}$$

$$\boxed{\lim_{x \to 2} f(x) = 1/2}$$

(3) Since $f(2) = 1/2$, $\therefore \lim_{x \to 2} f(x) = f(2)$

$\boxed{f \text{ is continous a } x=2}$

$f(x) = 1/x$, show that f is not continuous at $x=0$

$f(0) = \frac{1}{0}$ ← undef

0 is not in the domain of f

f is not continous at $x=0$

Investigate the continuity of function : $f(x) = \sqrt{x+1}$

① Domain for $f(x) = \sqrt{x+1}$, $x \geq -1$, $[-1, \infty)$
$f(a)$ is defined $[-1, \infty)$

② $\lim_{x \to a} \sqrt{x+1}$

$$= \sqrt{\lim_{x \to a} x+1}$$

$$= \sqrt{\lim_{x \to a} x + \lim_{x \to a} 1}$$

$$= \sqrt{a+1}$$

← $\lim_{x \to a} \sqrt{x+1}$ is defined at $[-1, \infty)$

$a \geq -1$

③ $\lim_{x \to a} \sqrt{x+1} = f(x)$

why?

This exists at $[-1, \infty)$ | This exists at $[-1, \infty)$

$\boxed{f \text{ is continuous on the interval } [-1, \infty)}$

For:

$f(x) = \sqrt{2x+1}$

What is the least value of a such that f is continuous on the interval $[a, \infty)$?

① $a = x$

$\downarrow$

$f(a) = \sqrt{2a+1}$

$\downarrow$

$$2a + 1 \geq 0$$
$$-1 \quad = -1$$
$$\frac{2a}{2} = \frac{-1}{2}$$
$$\parallel$$
$$a = -1/2$$
$$\text{SS}$$
$$-0.5$$

② $\lim_{x \to a} \sqrt{2x+1}$

$$= \sqrt{\lim_{x \to a} 2x + 1}$$
$$= \sqrt{\lim_{x \to a} 2x + \lim_{x \to a} 1}$$
$$= \sqrt{2 \cdot \lim_{x \to a} x + 1}$$
$$= \sqrt{2 \cdot a + 1}$$
$$= \sqrt{2a + 1} \rightarrow 2a + 1 \geq 0$$
$$-1 \quad = -1$$
$$\frac{\cancel{2}a}{\cancel{2}} = \frac{-1}{2} \rightarrow a = -1/2 \quad \text{SS} \quad -0.5$$

Same outputs

③ $\lim_{x \to a} \sqrt{2x+1}$ is defined at $[-0.5, \infty)$

AND

$f(a)$ is defined at $[-0.5, \infty)$

The least value of a is -0.5