Get general indefinite integral for $\int \frac{1}{x^3}\,dx$

① Convert $\int \frac{1}{x^3}\,dx$ to $\boxed{\int x^{-3}\,dx}$

② Get antiderivative

$\int x^{-3}\,dx$ Apply rule #3

$$\int x^k\,dx = \frac{x^{k+1}}{k+1}$$

$$\int x^{-3}\,dx = \frac{x^{-3+1}}{-3+1}, \quad k=-3$$

$$\frac{x^{-2}}{-2}$$

$$\parallel$$

$$\frac{1}{x^2 \cdot -2}$$

$$\parallel$$

$$\frac{1}{-2x^2}$$

$$\parallel$$

$$-\frac{1}{2x^2}$$

③ Interpret Domain and General Antiderivative

$\int \frac{1}{x^3}\,dx$, where $x>0$ or $x<0$

$\int \frac{1}{x^3}\,dx$ is undefined for $x=0$

$$f(x) = \begin{cases} -\frac{1}{2x^2} + C_1, & \text{where } x<0 \text{ and } C_1 \text{ is an arbitrary constant} \\ -\frac{1}{2x^2} + C_2, & \text{where } x>0 \text{ and } C_2 \text{ is an arbitrary constant} \end{cases}$$

$$\int \frac{1}{x^3}\,dx = \int x^{-3}\,dx = \frac{x^{-2}}{-2} + C \quad \text{for intervals } (-\infty, 0) \cap (0, \infty)$$

Get general indefinite integral $\int 2x + \sin(x)\,dx$

① Decompose $\int 2x + \sin(x)\,dx \rightarrow$ $\boxed{\text{Rule \#4 } \int f(x) + g(x)\,dx = \int f(x)\,dx + \int g(x)\,dx}$

② Get antiderivatives

$\int 2x\,dx$ + $\int \sin(x)\,dx$

ⓐ Extract Constant

$2 \cdot \int x^1\,dx$

ⓑ Apply rule #3

$\int x^k\,dx = \frac{x^{k+1}}{k+1}$

$\int x^1\,dx = \frac{x^{1+1}}{1+1}$, $k=1$

$2 \cdot \frac{x^{1+1}}{1+1}$

$=$

$\cancel{2}^{1} \cdot \frac{x^2}{\cancel{2}_1}$

$=$

x^2

$\boxed{\int 2x\,dx = x^2}$

$\int \sin(x)\,dx$

$=$

$\frac{d}{dx}[\sin(x)]$

$=$

$\cos(x)$

Integral is opposite sign

$-\cos(x) + C_2$

$x^2 + C_1$ Apply arbitrary constant for indefinite integral + $-\cos(x) + C_2$

③ Summarize

$\int 2x + \sin(x)\,dx = x^2 \underbrace{+ C_1 - \cos(x) + C_2}$

Combine Constants into One

$\boxed{\int 2x + \sin(x)\,dx = x^2 - \cos(x) + C}$

Get general indefinite integral $\int \frac{1+t}{\sqrt{t}}\, dx$ $\quad \sqrt{t} = t^{1/2}$

① Decompose $\int \frac{1+t}{\sqrt{t}}\, dt$

$$= \int \frac{1}{\sqrt{t}} + \frac{t}{\sqrt{t}}\, dt$$

$$= \int \frac{1}{t^{1/2}} + \frac{t^1}{t^{1/2}}\, dt$$

Exponent Properties

$$\frac{t^1}{t^{1/2}} = t^{1-1/2} = t^{1/2}$$

$$= \int t^{-1/2} + t^{1/2}\, dt$$

$$= \int t^{-1/2}\, dt + \int t^{1/2}\, dt$$

② Get antiderivatives

$$\int t^{-1/2}\, dt \quad + \quad \int t^{1/2}\, dt$$

$$\int x^k\, dx = \frac{x^{k+1}}{k+1} \qquad \int x^k\, dx = \frac{x^{k+1}}{k+1}$$

$$\downarrow \qquad \downarrow$$

$$\int t^{-1/2}\, dt = \frac{t^{-1/2+1}}{-1/2+1} \quad + \quad \int t^{1/2}\, dt = \frac{t^{1/2+1}}{1/2+1}$$

$$= \frac{t^{1/2}}{1/2} \quad + \quad = \frac{t^{3/2}}{3/2} + C$$

Apply Constant

$$= \frac{\sqrt{t}}{2} \qquad = \frac{\sqrt{t^3}}{3/2}$$

$$= \sqrt{t} \div \frac{2}{1} \qquad = \sqrt{t^3} \div \frac{3}{2}$$

$$= \sqrt{t} \cdot \frac{2}{1} \qquad = \sqrt{t^3} \cdot \frac{2}{3}$$

$$= 2\sqrt{t} \qquad = \frac{2\sqrt{t^3}}{3}$$

$\sqrt{t} \cdot \sqrt{t} \cdot \sqrt{t}$

$\sqrt{t^2} \cdot \sqrt{t}$

$$\frac{2}{3} t \sqrt{t} \leftarrow t \cdot \sqrt{t}$$

③ Summary

$$\int \frac{1+t}{\sqrt{t}}\, dx = 2t^{1/2} + \frac{2}{3} t^{3/2} + C$$

Get the general indefinite integral $\int \frac{\cos(\theta)}{\sin^2(\theta)} d\theta$

① Decompose $\int \frac{\cos(\theta)}{\sin^2(\theta)} d\theta$

$$=$$

$$\int \frac{\cos(\theta)}{\sin(\theta)} \cdot \frac{1}{\sin(\theta)} d\theta$$

$$=$$

$$\int \cot(\theta)\csc(\theta)\, d\theta$$

$$=$$

$$-\csc(\theta) + C$$

$\frac{d}{dx}[\csc(\theta)] = \ominus \cot(\theta)\csc(\theta)$

Use opposite sign for integral

$$\boxed{\int \frac{\cos(\theta)}{\sin^2(\theta)} d\theta = -\csc(\theta) + C}$$

Get the general indefinite integral $\int 6x^2 + 2x + 1 \, dx$

① Decompose $\int 6x^2 + 2x + 1 \, dx$ → $\boxed{\text{Rule \#4 } \int f(x) + g(x) \, dx = \int f(x) \, dx + \int g(x) \, dx}$

$$= 6\int x^2 \, dx + 2\int x \, dx + \int 1 \, dx$$

② Get antiderivatives

$6\int x^2 \, dx$ + $2\int x \, dx$ + $\int 1 \, dx$

$\int x^k \, dx = \frac{x^{k+1}}{k+1}$ ↓ (for $6\int x^2 \, dx$)

$= 6 \cdot \frac{x^{2+1}}{2+1}, \; k = 2$

$= \cancel{6}^{2} \cdot \frac{x^3}{\cancel{3}_1}$

$= 2x^3$

$\int x^k \, dx = \frac{x^{k+1}}{k+1}$ ↓ (for $2\int x \, dx$)

$= 2 \cdot \frac{x^{1+1}}{1+1}, \; k = 1$

$= \cancel{2}^{1} \cdot \frac{x^2}{\cancel{2}_1}$

$= x^2$

Rule #2

$\int k \, dx = kx + C$

$\int 1 \, dx$

$= 1x + C$

$= x + C$

$2x^3 + x^2 + x + C$

③ Summary

$$\boxed{\int 6x^2 + 2x + 1 \, dx = 2x^3 + x^2 + x + C}$$