$f = \{(0,-1), (1,0), (2,1), (3,2)\}$

$g = \{(0,1), (1,2), (2,3), (3,6), (4,8)\}$

Focus on getting the domain of the input function.

$(f \circ g)(x)$

① Domain for $g(x)$ is $\{0, 1, 2, 3, 4\}$

②

$f(g(1))$ → $f(2)$ → ①

$(f \circ g)(1) = 1$

$f(g(2))$ → $f(2)$ → ①

$(f \circ g)(2)$

$f(g(3))$ → $f(6)$ → undef

$(f \circ g)(3) =$ undef

$f(g(4))$ → $f(8)$ → undef

$(f \circ g)(4) =$ undef

$(f \circ g)(x)$ Domain: $\{0, 1, 2\}$

$f(g(0))$ → $f(1)$ → ◎

$(f \circ g)(0) = 0$

③ Input the domain values of $(f \circ g)(x)$ again for $(f \circ g)(x)$ to get the range.

$(f \circ g)(0)$

$f(g(0))$ → $f(1)$ → ◎

$(f \circ g)(1)$

$f(g(1))$ → $f(2)$ → ①

$(f \circ g)(2)$

$f(g(2))$ → $f(2)$ → ①

④ Range of $(f \circ g)(x)$

$\{0, 1\}$

$f = \{(0,1), (1,0), (2,1), (3,2)\}$

$h(x) = 2x - 2$

$(h \circ f)(x)$

$=$

② $h(f(x))$

① Domain for $f(x)$

$\{0, 1, 2, 3\}$

↑

This is also the domain for $(h \circ f)(x)$ or $h(f(x))$

$h(f(0)) = h(f(1)) = h(f(2)) = h(f(3))$

$h(1) = h(0) = h(1) = h(2)$

$2(1) - 2 = 2(0) - 2 = 2(1) - 2 = 2(2) - 2$

$2 - 2 = 0 - 2 = 2 - 2 = 4 - 2$

$0 = -2 = 0 = 2$

Range: $\{-2, 0, 2\}$

Get Domain and Range

$f = \{(0,1),(1,0),(2,1),(3,2)\}$

$h(x) = 2x - 2$

$(f \circ h)(x) = f(h(x))$

$h(x) = 2x - 2$

↑ Domain: $(-\infty, \infty)$

① My Domain for $h(x)$ is all real numbers.

② I know the domain for f is $\{0,1,2,3\}$ Im going to use these values to solve for x for $h(x)$

$2x - 2 = 0$
$+2 \quad +2$
$\frac{2x}{2} = \frac{2}{2}$
$x = 1$

$2x - 2 = 1$
$+2 \quad +2$
$\frac{2x}{2} = \frac{3}{2}$
$x = 3/2$

$2x - 2 = 2$
$+2 \quad +2$
$\frac{2x}{2} = \frac{4}{2}$
$x = 2$

$2x - 2 = 3$
$+2 \quad +2$
$\frac{2x}{2} = \frac{5}{2}$
$x = 5/2$

↓

I have the domain for $(f \circ h)(x)$

D: $\{1, 3/2, 2, 5/2\}$

③ Im going to get the range for $(f \circ h)(x)$

↓

$f(h(x))$

$f(2(1)-2)$
↓
$f(0) = -1$

$f(2(3/2)-2)$
↓
$f(3-2)$
↓
$f(1) = 0$

$f(2(2)-2)$
↓
$f(4-2)$
↓
$f(2) = 1$

$f(2(5/2)-2)$
↓
$f(5-2)$
↓
$f(3) = 2$

Range of $(f \circ h)(x)$
$\{-1, 0, 1, 2\}$

15. $f(x) = 4x + 4$

$g(x) = x + 1$

$f + g$

$y = (4x + 4) + (x + 1)$

$4x + 4 + x + 1$

$4x + x + 4 + 1$

$y = 5x + 5$

D; $(-\infty, \infty)$

R; $(-\infty, \infty)$

$f - g$

$(4x + 4) - 1(x + 1)$

$4x + 4 - x - 1$

$4x - x + 4 - 1$

$y = 3x + 3$

D; $(-\infty, \infty)$

R; $(-\infty, \infty)$

fg

$(4x + 4)(x + 1)$

$4x^2 + 4x + 4x + 4$

$y = 4x^2 + 8x + 4$

$a \quad b \quad c$

Domain $(-\infty, \infty)$

$x = -\frac{b}{2a}$

$x = -\left(\frac{8}{2(4)}\right) = -\left(\frac{8}{8}\right) = -(1) = -1$

$y = 4(-1)^2 + 8(-1) + 4$

$4(1) - 8 + 4$

$4 - 8 + 4$

$-4 + 4$

0

x, y

$(-1, 0)$ R; $[0, \infty)$

$\frac{f}{g}$

$\frac{f(x)}{g(x)} = \frac{4x + 4}{x + 1}$

$y = \frac{4x + 4}{x + 1}$

$x + 1 = 0$

$-1 \quad -1$

$x = -1$

D: $(-\infty, -1)$

R: $(-1, \infty)$

$f = \{(0,-1), (1,0), (2,1), (3,2)\}$ $g = \{(0,1), (1,2), (2,2), (3,6), (4,8)\}$

$h(x) = 2x - 2$ $s(x) = 2x + 1 \mid x > 0$

17.

$(f+g)(0)$

$\downarrow$

$f(0) + g(0)$

$\downarrow$

$-1 + 0$

$\downarrow$

-1

$(f+g)(0) = -1$ ✓

$(fg)(0)$

$f(0) \cdot g(0)$

$-1 \cdot 0$

0

$(fg)(0) = 0$ ✓

$(f \circ g)(0)$

$f(g(0))$

$f(1)$

$\downarrow$

0

$(f \circ g)(0) = 0$

$(f-g)(0)$

$\downarrow$

$f(0) - g(0)$

$\downarrow$

$-1 - 0$

$\downarrow$

-1

$(f-g)(0) = -1$ —

$\left(\frac{f}{g}\right)(0)$

$\downarrow$

$\frac{f(0)}{g(0)} = \frac{-1}{0} =$ undef

$f = \{(-1,0), (0,1), (1,2), (2,3), (3,4)\}$

$g = \{(-1,-4), (0,-2), (1,0), (2,2)\}$

$g = \{-1, 0, 1, 2\}$

$\{-4, -2, 0, 2\}$

$\wedge$

① Find Common Difference or Common Ratio for y or g(x)

$-4, -2, 0, 2$

common Diff

$-4 - (-2) = -2$ $\quad d = -2$

$-2 - (0) = -2$

$0 - 2 = -2$

$-4 = (-1) - 2$

↑ This is not true so I will try...

$-4 = -1z - 2$

solve for z

$-4 = -1z - 2$

$+2 \qquad +2$

$\frac{-2}{-1} = \frac{z}{-1}$

$2 = z$

② Construct equation

That's better

$-4 = 2(-1) - 2$

$-2 = 2(0) - 2$

$0 = 2(1) - 2$

$2 = 2(2) - 2$

$g(x) = 2x - 2$

$f = \{-1, 0, 1, 2, 3\}$

$\{0, 1, 2, 3, 4\}$

Get common Dif for y

$0 - 1 = -1$

$1 - 2 = -1$

$2 - 3 = -1$

$3 - 4 = -1$

$f = \{(0,-1), (1,0), (2,1), (3,2)\}$

$g = \{(0,0), (1,2), (2,4), (3,6), (4,8)\}$

Domain

$f \cap g = \{0, 1, 2, 3\}$

$f + g$

$f(0) + g(0)$

$-1 \quad 0$

(-1)

$f(1) + g(1)$

$0 + 2$

(2)

$f(2) + g(2)$

$1 + 4$

(5)

$f(3) + g(3)$

$2 + 6$

(8)

Domain of $f+g$

$\{0, 1, 2, 3\}$

Range of $f+g$

$\{-1, 2, 5, 8\}$

$f - g$

$f(0) - g(0)$

$-1 - 0$

(-1)

$f(1) - g(1)$

$0 - 2$

(-2)

$f(2) - g(2)$

$1 - 4$

(-3)

$f(3) - g(3)$

$2 - 6$

(-4)

Domain of $f-g$

$\{0, 1, 2, 3\}$

Range of $f-g$

$\{-1, -2, -3, -4\}$

$$y = \frac{3x-3}{2x}$$

$$\downarrow$$

$$\frac{3x}{2x} - \frac{3}{1}$$

$$y = \frac{3x-3}{2x}$$

$$\downarrow$$

$$y = \frac{3x}{2x} - \frac{3}{1}$$

$$\downarrow$$

$$y = \frac{3}{2} - \frac{3}{1}$$

$$\frac{3}{2} \cdot \frac{1}{1} = \frac{3}{2}$$

$$-\frac{3}{1} \cdot \frac{2}{2} = -\frac{6}{2}$$

$$y = -\frac{3}{2}$$

-1.5

$$y = \frac{3x-3}{2x}$$

$$\downarrow$$

$$\frac{3x}{2x} - \frac{3}{2x}$$

$$\frac{3}{2} - \frac{3}{2x}$$

$$\frac{3x}{2x}$$

$$-\frac{3}{2x}$$

$$\frac{3}{2} \cdot \frac{2x}{2x} = \frac{6x}{4x}$$

$$-\frac{3}{2x} \cdot \frac{2}{2} = -\frac{6}{4x}$$

$$y = \frac{4x+4}{x+1}$$

$$\downarrow$$

$$4x+4$$

$$\frac{4x+4}{1} \div \frac{x+1}{1}$$

$$\downarrow$$

$$\frac{4x+4}{1} \cdot \frac{1}{x+1}$$

$$\frac{4+4}{1} = 8$$

$$y = \frac{4x+4}{x+1}$$

$$\downarrow$$

$$\frac{4(x+1)}{x+1}$$

$$y = mx + b$$

$$y = 4$$

R: [4]

17

fg

$f(0) \cdot g(0)$

$-1 \cdot 0$

0

$f(1) \cdot g(1)$

$0 \cdot 2$

0

$f(2) \cdot g(2)$

$1 \cdot 4$

4

$f(3) \cdot g(3)$

$2 \cdot 6$

12

Domain of fg

$\{0, 1, 2, 3\}$

Range of fg

$\{0, 4, 12\}$

$\frac{f}{g}$

$\frac{f(0)}{g(0)} = \frac{-1}{0} =$ undef

$\frac{f(1)}{g(1)} = \frac{0}{2} = 0$

$\frac{f(2)}{g(2)} = \frac{1}{4}$

$\frac{f(3)}{g(3)} = \frac{2}{6} = \frac{1}{3}$

Domain of f/g

$\{1, 2, 3\}$

Range of f/g

$\{0, 1/4, 1/3\}$

13. $f = \{(x, 3x-3) \mid x \text{ is any real number}\}$

$g = \{(x, 2x) \mid x \text{ is any real number}\}$

Domain of f : $(-\infty, \infty)$
$f(x) = 3x-3$

Domain of g : $(-\infty, \infty)$
$g(x) = 2x$

$f+g$

$f(x) + g(x)$ Domain $(-\infty, \infty)$

$(3x-3) + (2x)$

$3x - 3 + 2x$

$3x + 2x - 3$

$y = 5x - 3$ Range $(-\infty, \infty)$

$f-g$

$f(x) - g(x)$ Domain $(-\infty, \infty)$

$(3x-3) - (2x)$

$3x - 3 - 2x$

$3x - 2x - 3$

$y = x - 3$ Range $(-\infty, \infty)$

fg

$f(x) \cdot g(x)$

$(3x-3)(2x)$

$2x(3x-3)$

$y = 6x^2 - 6x$ Domain : $(-\infty, \infty)$
Range : $(-\infty, \infty)$

$\frac{f}{g}$

$\frac{f(x)}{g(x)}$

$y = \frac{3x-3}{2x} \rightarrow \frac{2x}{2} = \frac{0}{2}$

$x = 0$

Domain : $(-\infty, 0) \cup (0, \infty)$

Range : $(-\infty, \infty)$

$\left(\frac{2x}{1}\right) y = \frac{3x-3}{2x}\left(\frac{2x}{1}\right)$

$\frac{2xy}{x} = \frac{3x-3}{x}$

$\left(\frac{1}{3x-3}\right) 2xy = \left(\frac{3x-3}{x}\right) \frac{1}{3x-3}$

$\frac{2xy}{3x-3} = \frac{1}{x}$

Express the Relations Using An Equation

$f = \{(0,1), (1,0), (2,1), (3,2)\}$ (x, y)

$g = \{(0,0), (1,2), (2,4), (3,6)\}$

$y = f(x)$ $y = g(x)$

$f(x) = y$

$f(0) = 0 - 1 \rightarrow 0 + (-1)$
$f(1) = 1 - 1 \quad 1 + (-1)$
$f(2) = 2 - 1 \quad 2 + (-1)$
$f(3) = 3 - 1 \quad 3 + (-1)$

$f(x) = x - 1$ or $x + (-1)$

$f = \{0, 1, 2, 3\}$ (x)

$\{-1, 0, 1, 2\}$ (y)

$-1 - 0 = -1$
$0 - 1 = -1$
$1 - 2 = -1$
$2 - 3 = -1$

$d = -1$ (-1 is my common difference (d))

$g(x) = y$

$0 = 0(2)$
$2 = 1(2)$
$4 = 2(2)$
$6 = 3(2)$

$g(x) = 2x$

Using Arithmetic and Geometric sequence perspective to set equation

$g = \{0, 1, 2, 3\}$ (x)

$\{0, 2, 4, 6\}$ (y)

$\frac{0}{0} =$ undef

$\frac{2}{1} = 2$

$\frac{4}{2} = 2$

$\frac{6}{3} = (2)$

2 is my common ratio (r)

Express the Relations f and g Using an Equation

$f = \{(x, 3x-3) \mid x \text{ is any real number}\}$

$g = \{(x, 2x) \mid x \text{ is any real number}\}$

$f(x) = 3x - 3$

$g(x) = 2x$

$f = \{(-1,0), (0,1), (1,2), (2,3), (3,4)\}$

$g = \{(-1,-4), (0,-2), (1,0), (2,2)\}$

$f^{x}\{-1, 0, 1, 2, 3\}$

$^{x}f\{0, 1, 2, 3, 4\}$

$g^{x}\{-1, 0, 1, 2\}$

$^{x}g\{-4, -2, 0, 2\}$

Check Common Difference (d)

$0 - (-1) = 1$

$1 - 0 = 1$

$2 - 1 = 1$

$3 - 2 = 1$

$4 - 3 = 1$

My Common Difference is ① here

$d = 1$

which makes this sequence arithmatic

$f(-1) = -1 + 1 = 0$

$f(0) = 0 + 1 = 1$ ←

$f(1) = 1 + 1 = 2$

$f(2) = 2 + 1 = 3$

$f(3) = 3 + 1 = 4$

↓

Equation of relation

$f(x) = x + 1$

$^{y}_{-4} = (^{x}_{-1})$

Use Trial and Error Here until I find a better way of getting the equation

↓

$-4 = 2(-1) - 2$

$-2 = 2(0) - 2$

$0 = 2(1) - 2$

$2 \quad 2(2) - 2$

$g(x) = 2x - 2$

21. $(f \circ g)(x)$

$f = \{0, 1, 2, 3\}$

$g = \{0, 1, 2, 3, 4\}$

$D: \{0, 1, 2, 3\}$

$(f \circ g)(x)$

$D: \{0, 1, 2, 3\}$

$R: \{0, 1, 1\}$

$(0,0), (1,1), (2,1)$

$(f \circ g)(x)$

$f(g(0))$	$f(g(1))$	$f(g(2))$	$f(g(3))$	$f(g(4))$
$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$f(2)$
$f(1)$	$f(2)$	$f(2)$	$f(6)$	$\downarrow$
$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	undef
(0)	(1)	(1)	undef	
$(0,0)$	$(1,1)$	$(2,1)$	undef	undef

$\{0, 1\}$

23. $(f \circ h)(x)$

$h(x) = 2x - 2$

$D: (-\infty, \infty)$

$f(h(x))$

$\downarrow$

$f(2x-2)$ where $x \geq 0$

$(f \circ h)x$:

$f(h(x))$

$\downarrow$

$f(2x-2)$

$\downarrow$

$f(2(1)-2)$	$f(2(2)-2)$	$f(2(3)-2)$	$f(2(0)-2)$
	$\downarrow$		$f(0-2)$
$f(2-2)$	$f(4-2)$	$f(6-2)$	$f(-2)$
$f(0)$	$f(2)$	$f(4)$	
(-1)	(1)	undef	

$2x - 2 = 0$

$+2 \quad +2$

$\frac{2x}{2} = \frac{2}{2}$

$x = 1$

19. $(s+h)(1)$

$s(1) = 2(1) + 1$

$2 + 1$

$s(1) = 3$

$h(1) = 2(1) - 2$

$2 - 2$

$h(1) = 0$

$3 + 0$

3

$(s+h)(1) = 3$

$(sh)(1)$

$s(1) \cdot h(1)$

$3 \cdot 0$

$(sh)(1) = 0$

$(s-h)(1)$

$s(1) = 2(1) + 1$

$2 + 1$

3

$h(1) = 2(1) - 2$

$2 - 2$

0

$3 - 0$

3

$(s-h)(1) = 3$

$\left(\frac{s}{h}\right)(1) = \frac{s(1)}{h(1)} = \frac{3}{0}$ = under

$(s \circ h)(1)$

$s(h(1))$

$s(0) =$ under where $x > 0$

$h(x) = 2x - 2$ $\quad\quad$ $a(x) = 3x - 5$

$$(h \circ a)(x)$$
$$\| $$
$$h(a(x))$$
$$\| $$
$$2(3x-5) - 2$$
$$\| $$
$$6x - 10 - 2$$
$$\| $$
$$6x - 10 + (-2)$$
$$\| $$
$$\boxed{6x - 12}$$

What Did The Student Do Wrong?

$f(x) = (x+1)(x+2)$ $\quad\quad$ $g(x) = x + 1$ $\quad\quad$ Find $\left(\frac{f}{g}\right)(x)$

Student Answer: $\left(\frac{f}{g}\right)(x) = x + 2$

Check

$$\frac{f(x)}{g(x)} = \frac{(x+1)(x+2)}{x+1}$$

Find Domain:

$x + 1 = 0$

$-1 = -1$

$x = -1$

The Student Forgot To Restrict The Domain

Answer: $x + 2 \mid x \neq -1$

Domain: $(-\infty, -1) \cup (-1, \infty)$

Express The Given Function Using an Equation

$h(x) = 2x - 2$

$s(x) = 2x + 1$

$(h \circ s)(x)$

$=$

$h(s(x))$

$=$

$2(2x+1) - 2$

$=$

$4x + 2 - 2$

$4x$

$a(x) = 3x - 5$

$\downarrow$

$3x - 5 = 0$

$+5 \quad +5$

$\frac{3x}{3} = \frac{5}{3}$

$x = 5/3$

$x > 5/3$ for $b(x)$

$b(x) = 4x + 1 \mid x > 0$

$(b \circ a)(x)$

$=$

$b(a(x))$

$4(3x - 5) + 1$

$=$

$12x - 20 + 1$

$=$

$y = 12x - 19$

$b(x) = 4x + 1$

$\downarrow$

$4x + 1 > 0$

$-1 \quad -1$

$\frac{4x}{4} > \frac{-1}{4}$

$x > -1/4$

Restriction

$(-1/4, \infty)$

$s(x) = 2x + 1 \mid x > 0$

$(s \circ b)(x)$

$=$

$s(b(x))$

$=$

$2(4x + 1) + 1$

$=$

$8x + 2 + 1$

$8x + 3$

(33) $f = \{(0,-1), (1,0), (2,1), (3,2)\}$ $g = \{(0,1), (1,2), (2,2), (3,6), (4,8)\}$

$(g \circ f)(x)$

↓

① $g(f(x))$

① Domain of $f(x)$

$\{0,1,2,3\}$

②

$g(f(0))$ = $g(-1)$ = undef

$g(f(1))$ = $g(0)$ = ①

$g(f(2))$ = $g(1)$ = ②

$g(f(3))$ = $g(2)$ = ②

③ Domain of $(g \circ f)(x)$

$\{1,2\}$

④ Input $\{1,2\}$ for $(g \circ f)(x)$

$g(f(1))$ = $g(0)$ = ①

$g(f(2))$ = $g(1)$ = ②

⑤ Range of $(g \circ f)(x)$

$\{1,2\}$

$h(x) = 2x - 2$

$s(x) = 2x + 1 \quad | \; x > 0$

$(h \circ s)(x)$ or $h(s(x))$

Domain of $s(x)$ and $h(s(x))$
$(0, \infty)$

Get Range

$2(0) - 2$
$=$
$0 - 2$
$=$
-2

0 is not in the domain so -2 is exclusive

Range
$(-2, \infty)$

$h(x) = 2x - 2$

$s(x) = 2x + 1 \quad | \; x > 0$

$(s \circ h)(x)$ or $s(h(x))$

$x > 0$

Domain of $h(x)$
$(-\infty, \infty)$

Domain $s(x)$
$(0, \infty)$

$2x - 2 = 0$
$+2 \quad +2$

$\frac{2x}{2} = \frac{2}{2}$

$x = 1$

Domain of $s(h(x))$
$(1, \infty)$

Get Range

$y = 2(2x - 2) + 1$
$4x - 4 + 1$
$y = 4x - 3$
$4(1) - 3$
$4 - 3$
$y = 1$

Range for $s(h(x))$
$=$
$(1, \infty)$