Chapter Zero Solutions to Odd Numbered Problems

Important Note about Precision of Answers:

In many of the problems in this book you are required to read information from a graph and to calculate with that information. You should take reasonable care to read the graphs as accurately as you can (a small straightedge is helpful), but even skilled and careful people make slightly different readings of the same graph. That is simply one of the drawbacks of graphical information. When answers are given to graphical problems, the answers should be viewed as the best approximations we could make, and they usually include the word "approximately" or the symbol "≈" meaning "approximately equal to." Your answers should be close to the given answers, but you should not be concerned if they differ a little. (Yes those are vague terms, but it is all we can say when dealing with graphical information.)

Section 0.1

1. approx. 1, 0, –1

3. (a) Approx. $\dfrac{70 - 150 \text{ deg.}}{10 - 0 \text{ min}} = -8$ deg/min. Avg. rate of cooling ≈ 8 deg/min. (b) Approx. 6 deg/min cooling, and 5 deg/min cooling. (c) Approx. 5.5 deg/min cooling, and 10 deg/min cooling. (d) When $t = 6$ min.

5. We estimate that the area is approximately (very approximate) 9 cm^2.

7. Method 1: Measure the diameter of the coffee can, then fill it about half full of wine and measure the height of the wine and calculate the volume. Submerge the bulb, measure the height of the wine again, and calculate the new volume. The volume of the bulb is the difference of the two calculated volumes.
   Method 2: Fill the can completely full of wine and weigh the full can. Submerge the bulb (displacing a volume of wine equal to the volume of the bulb), remove the bulb, and weigh the can again. By subtracting, find the weight of the displaced wine and then use the fact that the density of wine is approximately 1 gram per 1 cubic centimeter to determine the volume of the bulb.

Section 0.2

1. (a) –3/4 (b) 1/2 (c) 0 (d) 2 (e) undefined

3. (a) $\frac{4}{3}$ (b) $\frac{-9}{5}$ (c) $x + 2$ (if $x \neq 2$) (d) $4 + h$ (if $h \neq 0$) (e) $a + x$ (if $a \neq x$)

5. (a) $t = 5$: $\dfrac{5000}{1500} = \dfrac{10}{3}$, $t = 10$: $\dfrac{5000}{3000} = \dfrac{5}{3}$, $t = 20$: $\dfrac{5000}{6000} = \dfrac{5}{6}$ (b) any $t > 0$: $\dfrac{5000}{300t} = \dfrac{50}{3t}$
   (c) decreasing, since the numerator remains constant at 5000 while the denominator increases.

7. The restaurant is 4 blocks south and 2 blocks east. The distance is $\sqrt{4^2 + 2^2} = \sqrt{20} \approx 4.47$ blocks.

9. $y = \sqrt{20^2 - 4^2} = \sqrt{384} \approx 19.6$ feet, $m = \dfrac{\sqrt{384}}{4} \approx 4.9$. $\tan(q) = \dfrac{\sqrt{384}}{4} \approx 4.9$ so $q \approx 1.37$ ($\approx 78.5^{\text{o}}$).

11. The equation of the line through $P = (2,3)$ and $Q = (8,11)$ is $y - 3 = \frac{8}{6}(x - 2)$ or $6y - 8x = 2$. Substituting $x = 2a + 8(1-a) = 8 - 6a$ and $y = 3a + 11(1-a) = 11 - 8a$ into the equation for the line, we get $6(11 - 8a) - 8(8 - 6a) = 66 - 48a - 64 + 48a$ which equals 2 for every value of a, so the point with $x = 2a + 8(1-a)$ and $y = 3a + 11(1-a)$ is on the line through P and Q for every value of a.
    The $\text{Dist}(P,Q) = \sqrt{6^2 + 8^2} = 10$. $\text{Dist}(P,R) = \sqrt{(8-6a-2)^2 + (11-8a-3)^2}$
    $= \sqrt{(6 - 6a)^2 + (8 - 8a)^2} = \sqrt{6^2(1-a)^2 + 8^2(1-a)^2} = \sqrt{100(1-a)^2} = 10\cdot|1-a| = |1-a|\cdot\text{Dist}(P,Q)$.

13. (a) $m_1 \cdot m_2 = (1)(-1) = -1$ so the lines are perpendicular. (b) Because 20 units of x-values are physically wider on the screen than 20 units of y-values. (c) Set the window so (xmax − xmin) ≈1.7 (ymax − ymin).

15. (a) $y - 5 = 3(x - 2)$ or $y = 3x - 1$. (b) $y - 2 = -2(x - 3)$ or $y = 8 - 2x$ (c) $y - 4 = -\frac{1}{2}(x - 1)$ or $y = -\frac{1}{2}x + \frac{9}{2}$

17. (a) $y - 5 = \frac{3}{2}(x-2)$ or $y = \frac{3}{2}x + 2$ (b) $y - 2 = \frac{3}{2}(x+1)$ or $y = \frac{3}{2}x + \frac{7}{2}$ (c) $x = 3$.

19. The distance between the centers is $\sqrt{6^2 + 8^2} = 10$. (a) 10–2–4 = 4 (b) 10–2–7 = 1 (c) 0 (they intersect) (d) 15–10–3 = 2 (e) 12–10–1 = 1.

21. Find $\text{Dist}(P,C) = \sqrt{(x-h)^2 + (y-k)^2}$, and compare the value to r:

$$P \text{ is } \begin{cases} \text{inside the circle} & \text{if Dist}(P,C) < r \\ \text{on the circle} & \text{if Dist}(P,C) = r \\ \text{outside the circle} & \text{if Dist}(P,C) > r \end{cases}$$

23. A point P =(x,y) lies on the circle if and only if its distance from C = (h,k) is r : Dist(P,C) = r. So P is on the circle if and only if $\sqrt{(x-h)^2 + (y-k)^2} = r$ or $(x-h)^2 + (y-k)^2 = r^2$.

25. (a) slope is $-\frac{5}{12}$ (b) undefined (vertical line) (c) $\frac{12}{5}$ (d) 0 (horizontal line)

27. (a) distance ≈ 2.22 . (b) Distance ≈ 2.24 .
(c) (by inspection) 3 units which occurs at the point (5, 3).

29. (a) If $B \neq 0$, we may solve for y: $y = -\frac{A}{B}x + \frac{C}{B}$. The slope is the coefficient of x: $m = -\frac{A}{B}$.

(b) The required slope is B/A (the negative reciprocal of –A/B) so the equation is $y = \frac{B}{A}x$ or $Bx - Ay = 0$.

(c) Solve $\{ Ax + By = C, Bx - Ay = 0 \}$ to get $x = \frac{AC}{A^2 + B^2}$ and $y = \frac{BC}{A^2 + B^2}$.

(d) $$\text{Distance} = \sqrt{\left(\frac{AC}{(A^2 + B^2)}\right)^2 + \left(\frac{BC}{A^2 + B^2}\right)^2} = \sqrt{\frac{A^2C^2}{(A^2 + B^2)^2} + \frac{B^2C^2}{(A^2 + B^2)^2}}$$
$$= \sqrt{\frac{(A^2 + B^2)C^2}{(A^2 + B^2)^2}} = \sqrt{\frac{C^2}{A^2 + B^2}} = \frac{|C|}{\sqrt{A^2 + B^2}}$$

Section 0.3

1. A–a, B–c, C–d, D–b 3. A–b, B–c, C–d, D–a
5. (a)–C, (b)–A, (c)–B

6. The bottles are sketched in Fig. 0.3P6.

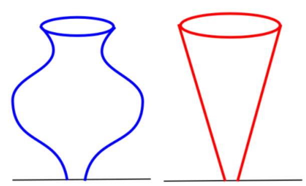

Fig. 0.3P6

7. $f(x) = x^2 + 3$, $g(x) = \sqrt{x - 5}$, $h(x) = \frac{x}{x - 2}$

(a) f(1) = 4, g(1) is undefined , h(1) = –1.
(b) Graphs of f, g and s are shown in Fig. 0.3P7.
(c) $f(3x) = (3x)^2 + 3 = 9x^2 + 3$, $g(3x) = \sqrt{3x - 5}$ (for $x \geq 5/3$) $h(3x) = \frac{3x}{3x - 2}$
(d) $f(x+h) = (x+h)^2 + 3 = x^2 + 2xh + h^2 + 3$, $g(x+h) = \sqrt{x + h - 5}$, $h(x + h) = \frac{x+h}{x+h - 2}$

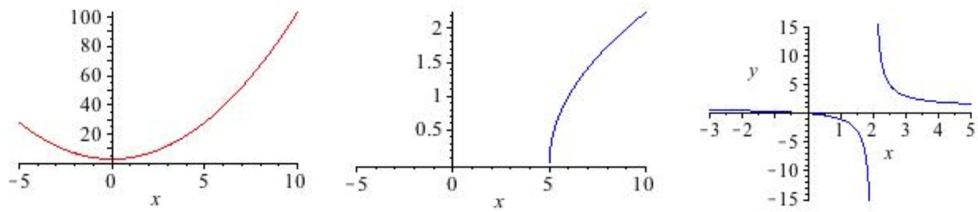

Fig. 0.3P7

9. (a) m = 2 (b) m = 2x + 3 + h . (c) m = x + 4 (if x≠1)
If x = 1.3, then m = 5.3 . If x = 1.1, then m = 5.1 . If x = 1.002, then m = 5.002 .

11. $f(x) = x^2 - 2x$, $g(x) = \sqrt{x}$.
$m = \dfrac{f(a+h) - f(a)}{h} = 2a + h - 2$ ($h \neq 0$). If a = 1, then m = h. If a = 2, then m = 2+h.
If a = 3, then m = 4 + h. If a = x, then m = 2x + h – 2.

$m = \dfrac{g(a+h) - g(a)}{h} = \dfrac{1}{\sqrt{a+h} + \sqrt{a}} = \dfrac{\sqrt{a+h} - \sqrt{a}}{h}$. If a = 1, then $m = \dfrac{\sqrt{1+h} - 1}{h}$.
If a = 2, then $m = \dfrac{\sqrt{2+h} - \sqrt{2}}{h}$. If a = 3, then $m = \dfrac{\sqrt{3+h} - \sqrt{3}}{h}$.
If a = x, then $m = \dfrac{\sqrt{x+h} - \sqrt{x}}{h}$.

13. (a) Approx. 250 miles, 375 miles. (b) Approx. 200 miles/hour.
(c) By flying along a circular arc about 375 miles from the airport.

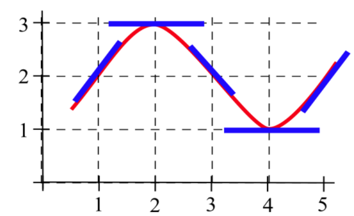

Fig. 0.3P15

15. (a) See Fig. 0.3P15. (b) Max at x = 2. Min at x = 4.
(c) Largest at x = 5. Smallest at x = 3.

17. The path of the slide is a straight line tangent to the graph of the path at the point of fall. See Fig. .

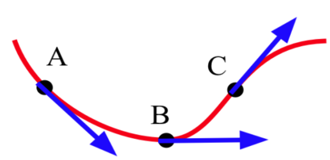

Fig. 0.3P17

19. (a) s(1) = 2, s(3) = 4/3. s(4) = 5/4. (b) $s(x) = \dfrac{x+1}{x}$.

21.

x	f(x)	**slope** of the line **tangent** to the graph of f at (x, f(x))
0	1	1
1	2	1
2	2	–1
3	1	0
4	1.5	0.5

23. On your own.

Section 0.4 Answers

1. (a) ≈ -18 . (b) -2.2

   (c) If $T = 11^{\circ}C$, WCI $= \begin{cases} 11 & \text{if } 0 \le v \le 6.5 \\ 33 - \dfrac{10.45 + 5.29\sqrt{v} - 0.279\,v}{22}(22) & \text{if } 6.5 < v \le 72 \\ -2.2 & \text{if } 72 < v \end{cases}$.

3. g(0)=3, g(1)=1, g(2)=2, g(3)=3, g(4)=1, g(5)=1. $g(x) = \begin{cases} 3 - x & \text{if } x < 1 \\ x & \text{if } 1 \le x \le 3 \\ 1 & \text{if } x > 3 \end{cases}$

5. (a) f(f(1)) = 1, f(g(2)) = 2, f(g(0)) = 2, f(g(1)) = 3
   (b) g(f(2)) = 0, g(f(3)) = 1, g(g(0)) = 0, g(f(0)) = 0
   (c) f (h(3)) = 3, f(h(4)) = 2, h(g(0)) = 0, h(g(1)) = –1

7. (a)

x	–1	0	1	2	3	4
f(x)	3	3	–1	0	1	1
g(x)	–2	0	1	2	3	4
h(x)	–3	–2	–1	0	1	2

   (b) f(g(1)) = –1, f(h(1)) = 3, h(f(1)) = –3, f(f(2)) = 3, g(g(3.5)) = 3

   (c) See Fig. 0.4P7 for the graphs of f, g, and h.

9. L(d) = $\begin{cases} \text{England} & \text{if d = Mon. or Tue.} \\ \text{France} & \text{if d = Wed.} \\ \text{Germany} & \text{if d = Thur. or Fri.} \\ \text{Italy} & \text{if d = Sat. or Sun.} \end{cases}$

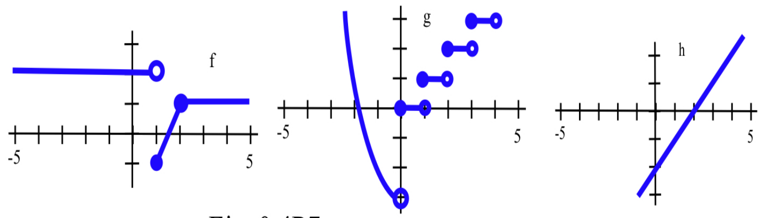

Fig. 0.4P7

11. $f(x) = \begin{cases} x^2 & \text{if } x < 2 \\ x - 1 & \text{if } x > 2 \end{cases}$

13. (a) $B(1) = 1 \cdot f(1) = 1 \cdot \frac{1}{1} = 1$,

   $B(2) = 2 \cdot f(2) = 2 \cdot \frac{1}{2} = 1$,

   $B(3) = 3 \cdot f(3) = 3 \cdot \frac{1}{3} = 1$.

   (b) For $x > 0$, $B(x) = x \cdot f(x) = x \cdot \frac{1}{x} = 1$.

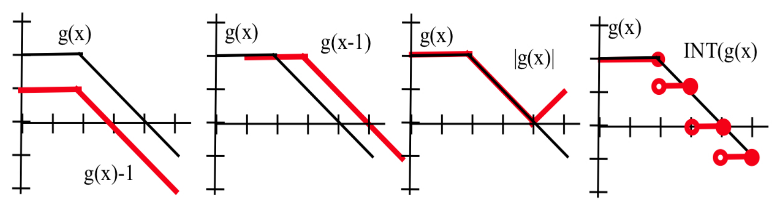

Fig. 0.4P15

15. See Fig. 0.4P15.

17. (a) f(g(x)) = 6x + 2 + 3A, g(f(x)) = g(3x+2) = 6x + 4 + A. If f(g(x)) = g(f(x)), then A = 1.
    (b) f(g(x)) = 3Bx – 1, g(f(x)) = 3Bx + 2B – 1. If f(g(x)) = g(f(x)), then B = 0.

19. See Fig. 0.4P19 for the graph of f(x) = x – [x] = x – INT(x).

21. f(x) = **[** 1.3 + 0.5•sin(x) **]** works. The value of 0.5 < A < 1.5 in
    f(x) = **[** A + 0.5•sin(x) **]** determines the relative lengths of the long and short parts of the pattern..

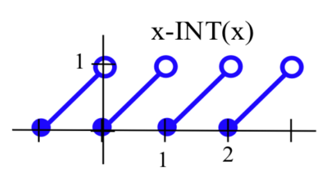

Fig. 0.4P19

23. (a) g(1) = 1, g(2) = 1, g(3) = 0, g(4) = –1. Now graph g

25. ≈ 0.739 starting with x = 1, 2, 10, or any value.

27. $f(x) = (x^2 + 1)/(2x)$. (note that this is the corrected version of the function f)
    $f(1) = 2/2 = 1$.
    $f(0.5) = 1.25$, $f(1.25) = 1.025$, $f(1.025) \approx 1.0003049$, $f(1.0003049) \approx 1.000000046$, ...
    $f(4) = 2.125$, $f(2.125) \approx 1.297794$, $f(1.297794) \approx 1.034166$, $f(1.034166) \approx 1.000564$, ...

29. (a) $f(2) = 14/3 \approx 4.7$, $f(14/3) = 50/9 \approx 5.6$, $f(50/9) = 158/27 \approx 5.85$, $f(158/27) = 482/81 \approx 5.95$
    $f(4) = 16/3 \approx 5.3$, $f(16/3) = 52/9 \approx 5.8$, $f(52/9) = 160/27 \approx 5.93$, $f(160/27) = 484/81 \approx 5.975$
    $f(6) = 6$.
    (b) $c = 6$.
    (c) Solve $c = g(c) = c/3 + A$ to get $3c = c + 3A$ and $2c = 3A$ so $c = \frac{3A}{2}$ is a fixed point of g.

31. On your own.

Section 0.5 Answers

1. (a) $x = 2, 4$ (b) $x = -2, -1, 0, 1, 2, 3, 4, 5$ (c) $x = -2, -1, 1, 3$

3. (a) all x (all real numbers) (b) $x > \sqrt[3]{-2}$ (c) all x

5. (a) $x = -2, -3, 3$ (b) no values of x (c) $x \geq 0$

7. (a) If $x \neq 2$ and $x \neq -3$, then $x^2 + x - 6 \neq 0$. True.
   (b) If an object does not have 3 sides, then it is not a triangle. True.

9. (a) If your car does not get at least 24 miles per gallon, then it is not tuned properly.
   (b) If you can not have dessert, then you did not eat your vegetables.

11. (a) If you will not vote for me, then you do not love your country.
    (b) If not only outlaws have guns, then guns are not outlawed. (poor English)
    If someone legally has a gun, then guns are not illegal.

13. (a) Both f(x) and g(x) are not positive. (b) x is not positive. ($x \leq 0$)
    (c) 8 is not a prime number.

15. (a) For some numbers a and b, $|a+b| \neq |a| + |b|$. (b) Some snake is not poisonous.
    (c) Some dog can climb trees.

17. If x is an integer, then 2x is an even integer. True.
    Converse: If 2x is an even integer, then x is an integer. True.
    (It is not likely that these were the statements you thought of. There are lots of other examples.)

19. (a) False. Put $a = 3$ and $b = 4$. Then $(a + b)^2 = (7)^2 = 49$, but $a^2 + b^2 = 3^2 + 4^2 = 9 + 16 = 25$.
    (b) False. Put $a = -2$ and $b = -3$. Then $a > b$, but $a^2 = 4 < 9 = b^2$.
    (c) True.

21. (a) True. (b) False. Put $f(x) = x + 1$ and $g(x) = x + 2$. Then $f(x)g(x) = x^2 + 3x + 2$ is not a linear function.
    (c) True.

23. (a) If a and b are prime numbers, then a + b is prime. False: take $a = 3$ and $b = 5$.
    (b) If a and b are prime numbers, then a + b is not prime. False: take $a = 2$ and $b = 3$.
    (c) If x is a prime number, then x is odd. False: take $x = 2$. (this is the only counterexample)
    (d) If x is a prime number, then x is even. False: take $x = 3$ (or 5 or 7 or ...)

25. (a) If x is a solution of $x + 5 = 9$, then x is odd. False: take $x = 4$.
    (b) If a 3–sided polygon has equal sides, then it is a triangle. True. (We also have nonequilateral triangles .)
    (c) If a person is a calculus student, then that person studies hard. False (unfortunately), but we won't mention names.
    (d) If x is a (real number) solution of $x^2 - 5x + 6 = 0$, then x is even. False: take $x = 3$.