<u>MATH E-3   Assignment 11 Solutions</u>

For Problems 1-4, give the following. Use the example on page 10 in the chapter as a model.
To receive full credit, identify the following:

- growth rate (or decay rate)
- growth factor (or decay factor)
- a general equation using exponents
- answer the questions.

Setting up a table is sometimes helpful, but it is optional.   Round populations of people, animals, cells, to whole numbers. Round money to dollars and cents or just dollars. Round at the <u>end</u> of all your calculations.

Problem 1

Suppose the population of a small country is 950,000. If the population is growing at an annual rate of 4%,

a) What will the population be in 10 years?

b) What will it be in 25 years?

c) How long will it take for the population to reach 3 million? Remember "trial and error."

Growth rate = 4%
Growth factor = 1.04
General equation: Y= 950,000$(1.04)^n$

a) population in 10 years = 950,000$(1.04)^{10}$ =1,406,232
b) population in 25 years = 950,000$(1.04)^{25}$ = 2,532,544
c) Since 25 years gives you 2,532,544, try a number larger than 25.

- If you try 30, you get 950,000$(1.04)^{30}$ =3,081,227. This is a little too large.
- Try 29. You get 950,000$(1.04)^{29}$ = 2,962,718. This is a little too small but we are close.
- An answer between 29 and 30 would be fine.
- 29.32 years is the more exact answer.

Problem 2

Suppose that the population of an endangered species is 15,000. If the population is <u>decreasing</u> at an annual rate of 3.5%,

a) What will it be in 10 years?

b) What will it be in 25 years?

c) How long will it take for the population to reach 3,000?

Decay rate = 3.5%
Decay factor = .965
General equation: $Y = 15{,}000(.965)^n$

a) population in 10 years = $15{,}000(.965)^{10} = 10{,}504$
b) population in 25 years = $15{,}000(.965)^{25} = 6{,}156$
c) Since 25 years gives you 6156, try a number larger than 25.

- If you try 40, you get $15{,}000(.965)^{40} = 3{,}607$. This is a little too large.
- Try 45. You get $15{,}000(.965)^{45} = 3{,}018$. This is a little too large still but we are very close.
- An answer around 45 would be fine.
- 45.2 years is the more exact answer.

Problem 3

If you have one bacterium in a test tube and it doubles every minute, how many bacteria will you have in

a) one hour?

b) one day? Note: Your calculator may give you an error if the number is too large. Leave your answer as a number with an exponent.

EXTRA CREDIT-1 point: If you understand how to manipulate exponents, (given in the beginning of this handout) you can break down your answer into a number with an exponent that will work in your calculator. Write your final (correct) answer in correct Scientific Notation for extra credit.

$\boxed{\text{Growth Factor = } \mathbf{2 \text{ or } 200\%}}$ so $\boxed{\text{Growth Rate = } \mathbf{100\%}}$

Minutes	Number of Bacteria
0	1
1	1x2
2	$1 \times 2 \times 2 = 2^2 = 4$
3	$1 \times 2 \times 2 \times 2 = 2^3$
4	$1 \times 2 \times 2 \times 2 \times 2 = 2^4$
.	
n	$\mathbf{1 \times 2^n}$

Equation: $Y = 2^n$ Notice the '1' is not necessary. It's optional.

a) To find the number of bacteria after one hour, change the hour to 60 minutes.
Since n=60, the number of bacteria = $\boxed{\mathbf{2^{60} = 1.1529 \times 10^{18}}}$ This is a very large number

b) To find the number of bacteria after one day, find the number of minutes in 24 hours. Since there are 60 minutes in one hour and 24 hours in one day, there are 24x60 = 1440 minutes in a day. The number of bacteria after one day is $\boxed{\mathbf{2^{1440}}}$ **= Here you may get an error message on your calculator.**

This may be too large for your calculator to compute. **If so, just leave your answer as** number of bacteria after one day is 2^{1440}.

If you wish to calculate the answer for your own information. then note This may be broken down again by using the rule for exponents

$$\mathbf{a^b * a^c = a^{b+c}} \qquad \mathbf{2^{1440}} = 2^{720} * 2^{720}$$

My calculator will compute $2^{180} = 1.532495 \times 10^{54}$ and $2^{720} = 2^{180} * 2^{180} * 2^{180} * 2^{180}$
We need 8 of these 2^{180}, so $2^{1440} = (2^{180})^8 = (1.532495 \times 10^{54})^8$ using calculations for scientific notation we get $(1.532495^{\mathbf{8}} \mathrm{x} 10^{432})$ = $\mathbf{30.422333x10^{432}}$ **-** quite a staggering number! (we can rewrite in true scientific notation as $\mathbf{3.0422333x10^{433}}$

Problem 4

Suppose the time it takes for the earth to make one daily rotation increases 5% per billion years. If the length of an hour is assumed constant, how many hours long will a day be just before the sun novas (exploding just before its death), destroying Earth, five billion years in the future?
Hint: How many hours are there in a day right now? You will be increasing this amount.

Growth rate = 5% per billion years
Growth factor = 1.05
General equation: $Y= 24(1.05)^n$

Length of day = $24(1.05)^5$ =30.6 hours

Problem 5

Everything you need for compounding more than once per year is in the handout. Just be careful in substituting into the formula. Be careful not to round too soon or too much!

You deposit $7,500 into a bank account and leave it there for nine years at an interest rate of 2.4%. How much will you have at the end of nine years if the interest is

a) compounded annually

$$Y=7500\ (1 + .024)^9 = \$9284.55$$

b) compounded semi-annually?

$$Y=7500(1 + \frac{.024}{2})^{(2*9)} =9296.31$$

c) compounded quarterly?

$$Y=7500(1 + \frac{.024}{4})^{(4*9)} =9302.26$$

d) compounded monthly?

$$Y=7500(1+\frac{.024}{12})^{(12*9)}=9306.26$$

Problem 6 - Extra Credit – 1 point: No hints or extra help on this problem! It is not more difficult than any of the others. The difficulty is only in the conversions.

For the past few months the city of New Orleans has been plagued by a dangerously high level of unusual coliform bacteria in the drinking water. At the temperature of the water in their main reservoir, a coliform population is known to grow by 6.2% per day!!!

Assume the following:

a) Only 12 bacteria were initially introduced into the reservoir to start this outbreak.

b) The infected reservoir contains 2.3 million gallons of water. (Be careful of units!!)

c) Measurements show a bacteria count of 20 coliforms per quart.

QUESTION: How long ago were the original 12 coliforms introduced into the water?

First of all we must be careful of the units in this problem.

- Part (c) states that the bacteria count is 20 coliforms per quart.
- The reservoir is measured in gallons (2.3 million gallons.)
- Therefore, since there are 4 quarts in one gallon, there are 4*2.3 = 9.2 million quarts in the reservoir.
- Also there are 20 coliforms per quart so there are 20* 9.2 = 184 million coliforms in the reservoir.

We need to know how many days it took to grow that many bacteria. Set up the table:

Growth rate =6.2% Growth factor = 1.062

Number of Days Population of Coliforms

0	12
1	$12*(1.062)$
2	$12*(1.062)^2$
3	$12*(1.062)^3$
.	
n	$12*(1.062)^n$

We need $\mathbf{12*(1.062)^n}$ **=** 184,000,000

Once again, using trial and error we find that it took approximately 275 days to reach that amount, since $\boxed{\mathbf{12*(1.062)}}^{\mathbf{275}} \approx 183{,}410{,}144$.

THE NEXT FEW PROBLEMS ARE ON SCIENTIFIC NOTATION, PERCENTS, and EXPONENTS. Problem 7 - Write the following numbers in Scientific Notation.

a) The mean distance from the Sun to Mars is about Two Hundred Ten million miles.

$2.1 * 10^8$

b) The time it takes for light to travel 1 mile is .00000538 seconds.

$5.38 * 10^{-6}$

Problem 8 – Write the following numbers in expanded form. i.e. not in scientific notation.

a) 13.6×10^6 13,600,000

b) 3×10^{-5} .00003

Problem 9 – Do the following percent problems the **SHORT** WAY, i.e. in one step. Show your formula.

a) Increase $1500 by 7%. 1500 (1.07) = 1605

b) Decrease $600 by 3.6% 600(.964) =578.4

Problem 10 – Practicing with Exponents:

a) $x^2 \bullet x^5 =$ x^7

b) $a^{-3} \bullet a^6 =$ a^3

c) $\frac{b^{10}}{b^2}$ = b^8

d) $\frac{y^7 \bullet y^{-2}}{y^4}$ **Hint:** Do the numerator first. = y