Example 1

A. Find an equation of the tangent line to the curve $f(x) = x^2$ at $x = 1$.
B. Use tangent line to approximate $f(1.1)$
C. Calculate $f(1.1)$ directly.
D. Construct table comparing $f(x)$ with its linear approximation at $x=1$ using various values near 1.

A. Find an equation of the tangent line to the curve $f(x) = x^2$ at $x=1$

① Get $f'(x)$

$$f'(x) = \frac{d}{dx}[x^2]$$

$$= 2x^{2-1}$$

$$= \boxed{f'(x) = 2x}$$

② Get slope of tangent at $x=1$

$$f'(1) = 2(1) = 2$$

$$\boxed{f'(1) = 2}$$

③ Set equation of tangent line

$y = mx + b$, $f'(x) = 2$

$$= \boxed{y = 2x + b}$$

3.1 Get Domain and Range

Domain of x for $y = 2x + b$ $(-\infty, \infty)$ No Restrictions

3. Get Range

$$y = 2x + b$$
$$-b \quad -b$$
$$\frac{y-b}{2} = \frac{2x}{2}$$

$$= \frac{y}{2} - \frac{b}{2} = x$$

No Restrictions For y

Range: $(-\infty, \infty)$

$\boxed{\text{Domain: } (-\infty, \infty) \qquad \text{Range: } (-\infty, \infty)}$

(3.2) Get y-intercept

Use $(1,1)$ for $y = 2x+b$

$1 = 2(1) + b$

$1 = 2 + b$

$-2 \quad -2$

$-1 = b$

$b = -1$

(3.3) Summarize Equation

Equation of tangent line to the curve $y=f(x)$ at $x=1$ is

$\boxed{y = 2x - 1}$

B. Use tangent line to approximate $f(1.1)$

C. Calculate $f(1.1)$ directly

(4) Calculate $f(1.1)$ and $f'(1.1)$

$\boxed{f(1.1) = (1.1)^2 = 1.21}$

for

$f(x) = x^2$

$\boxed{f'(1.1) = 2(1.1) - 1 = 1.2}$

for

$f'(x) \doteq 2x - 1$

D. Construct Table comparing $f(x)$ with it's linear approximation at $x=1$ using various values near 1.

(5)

x	Linear Approximation $y=2x-1$	$y = x^2$
0.9	0.8	0.81
0.95	0.9	0.9025
1	1	1
1.05	1.1	1.1025
1.1	1.2	1.21

Linear Approximation of $f'(x)$ is close to exact values of $f(x)$ for various values of x

Example 2

A. Find equation of tangent line to the curve $f(x) = \sqrt{x+2}$ at $x=2$.
B. Use line to approximate $\sqrt{4.1}$ and calculate $\sqrt{4.1}$ directly using $f(x)$
C. Finally, construct table comparing $f(x)$ with its linear approximation at $x=2$ using various values near 2.

(A1) Get $f'(x)$

$$f'(x) = \frac{d}{dx}\left[\sqrt{x+2}\right] = \frac{d}{dx}\left[(x+2)^{1/2}\right] = \frac{1}{2}(x+2)^{1/2-1} = \frac{1}{2}\cdot(x+2)^{-1/2} = \frac{1}{2}\cdot\frac{1}{(x+2)^{1/2}}$$

$$\boxed{f'(x) = \frac{1}{2\sqrt{x+2}}}$$

(A2) Get $f'(2)$, $x=2$

$$f'(2) = \frac{1}{2\sqrt{2+2}} = \frac{1}{2\sqrt{4}} = \frac{1}{2\cdot 2}$$

$$\boxed{f'(2) = \frac{1}{4}}$$

(A3) Set Equation of Tangent Line

$y = mx + b \qquad f'(2) = \frac{1}{4}$

$$\boxed{y = \frac{1}{4}x + b}$$

(A4) Get Domain and Range : Domain and Range of Linear Equation $(-\infty, \infty)$

Domain for $y = \frac{1}{4}x + b$ $(-\infty, \infty)$, x have no restrictions

Get Range

$$y = \frac{1}{4}x + b$$
$$-b \qquad -b$$
$$y - b = \frac{1}{4}x$$
$$\|$$
$$\frac{4}{1}(y - b) = \frac{\cancel{4}}{\cancel{1}} \cdot \frac{\cancel{1}}{\cancel{4}} x$$
$$\|$$
$$\frac{4}{1}(y - b) = x$$

y have no restrictions

Domain	Range
$(-\infty, \infty)$	$(-\infty, \infty)$

(A4.1) Get y-intercept

Use (2,2)

$$y = \frac{1}{4}x + b$$
$$2 = \frac{1}{4}(2) + b$$
$$\|$$
$$2 = \frac{1}{2} + b$$
$$-\frac{1}{2} \quad -\frac{1}{2}$$
$$\frac{2}{1} = \frac{4}{2}$$
$$-\frac{1}{2} = \frac{1}{2} \qquad = b$$
$$\frac{3}{2} = b$$

or

$$\boxed{b = 3/2}$$

(A4.5) Summarize $f'(x)$ equation

$$\boxed{\begin{array}{l} y = \frac{1}{4}x + \frac{3}{2} \\ \text{at } x = 2 \end{array}}$$

B1

y

$f(x) = \sqrt{x+2}$

Get Rid of Radical

$\sqrt{4.1} = 2.1$, $x = 2.1$

$f(2.1) = \sqrt{2.1+2}$

SS

$\boxed{f(2.1) \approx 2.02485}$

y

$f'(x) = \frac{1}{4}x + \frac{3}{2}$

$f'(2.1) = \frac{1}{4}(2.1) + \frac{3}{2}$

SS

$\boxed{f'(2.1) = 2.025}$

x	Linear Approximation $y = \frac{1}{4}x + \frac{3}{2}$	$y = \sqrt{x+2}$
1.9	1.975	1.9748
1.95	1.9875	1.9875
2	2	2
2.05	2.0125	2.0125
2.10	2.025	2.0248

Example 3

Get linearization of $f(x) = \sin(x)$ at 0.

① Get $f'(x)$

$$f'(x) = \frac{d}{dx}[\sin(x)]$$

$$\parallel$$

$$\boxed{f'(x) = \cos(x)}$$

② Get $f'(0)$

$$\parallel$$

$$f'(0) = \cos(0)$$

$$\parallel$$

$$f'(0) = 1$$

③ Set equation of tangent line

$$y = mx + b \qquad f'(0) = 1$$

$$y = 1x + b \quad \text{or} \quad \boxed{y = x + b}$$

④ Domain and Range of $y = x + b$ $(-\infty, \infty)$

⑤ Get y-intercept

$$y = x + b \qquad \text{Use } (0,0)$$

$$\downarrow$$

$$0 = 0 + b$$

$$-0 \quad -0$$

$$0 = b \quad \text{or} \quad \boxed{b = 0}$$

⑥ Summarize $f'(x)$ equation

$$y = x + 0$$

$$\parallel$$

$$y = x$$

7. Approximate $f(x)$ and $f'(x)$. Test values close to $x = 0$

$f(x) = \sin(x)$

$f(0.1) = \sin(0.1)$

$\approx$

$f(0.1) = .099833$

$\approx$

$\boxed{f(0.1) = 0.1}$

$f'(x) = \cos(x)$

$f'(0.1) = \cos(0.1)$

$\approx$

$f'(0.1) = .995004$

$\approx$

$\boxed{f'(0.1) = 0.1}$