

### Example 4

### First Derivative Test

Find the relative minimum and maximum values of  $f(x) = x + \sin(x)$  for  $0 \leq x \leq 2\pi$

① Get Derivative

$$f'(x) = \frac{d}{dx} [x + \sin(x)]$$

"

$$\frac{d}{dx} [x] + \frac{d}{dx} [\sin(x)]$$

"

$$|x|^{-1} + \cos(x)$$

"

$$|x|^0 + \cos(x)$$

"

$$1 + \cos(x)$$

$$\boxed{f'(x) = 1 + \cos(x)}$$

② Set  $f'(x) = 0$

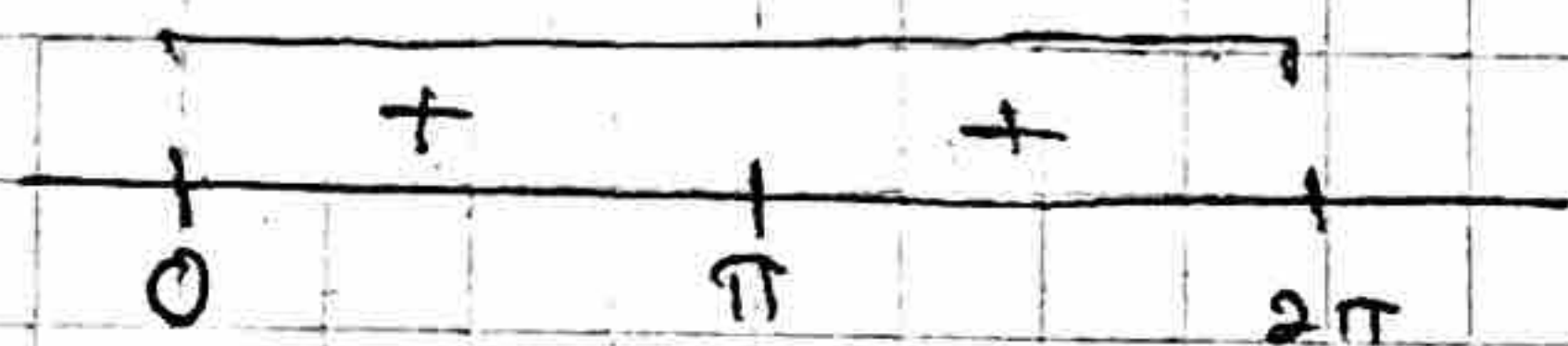
$$1 + \cos(x) = 0$$

$$\begin{array}{cc} -1 & -1 \\ \hline \cos(x) = -1 \end{array}$$

$\rightarrow x = \pi$  where  $\cos(x) = -1$  on unit circle

③ Set intervals

$$0 \leq x \leq 2\pi$$



$$\left(\frac{\pi}{3}\right) : 1 + \cos\left(\frac{\pi}{3}\right) \\ \quad \quad \quad + \frac{3}{2}$$

$$\left(\frac{4\pi}{3}\right) : 1 + \cos\left(\frac{4\pi}{3}\right) \\ \quad \quad \quad + \frac{1}{2}$$

④ Summary

Between interval  $[0, 2\pi]$  the signs do not change.

Based on the first derivative test,  $f(x) = x + \sin(x)$  do not have a relative minimum or maximum.

$\pi = c$



### Example 5

Find the relative minimum and maximum values of the function  $f(x) = x + 2\sin(x)$  for  $0 \leq x \leq 2\pi$

① Get Derivative

$$f'(x) = \frac{d}{dx} [x + 2\sin(x)]$$

$$\frac{d}{dx} [x] + 2 \cdot \frac{d}{dx} [\sin(x)]$$

$$1x^{1-1} + 2 \cdot \cos(x)$$

$$1x^0 + 2\cos(x)$$

$$f'(x) = 1 + 2\cos(x)$$

$$\boxed{f'(x) = 1 + 2\cos(x)}$$

② Set  $f'(x) = 0$

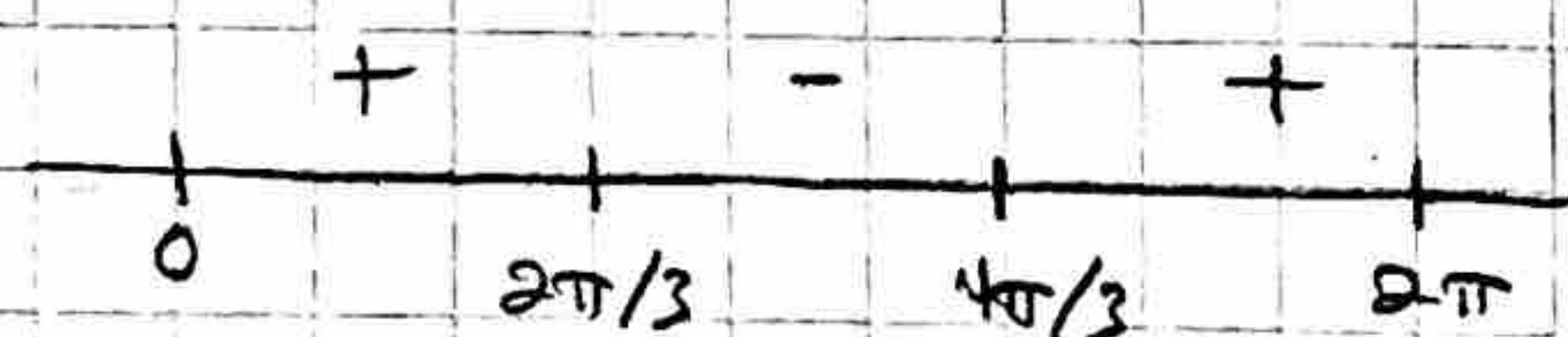
$$1 + 2\cos(x) = 0$$

$$\frac{-1}{2} = \frac{-2\cos(x)}{2}$$

$$\cos(x) = -\frac{1}{2}, \text{ where } x = \frac{2\pi}{3} \text{ and } x = \frac{4\pi}{3} \text{ on unit circle}$$

③ Set intervals

$$0 \leq x \leq 2\pi$$



$$\left(\frac{\pi}{2}\right) : 1 + 2\cos\left(\frac{\pi}{2}\right)$$
$$1 + 2(0)$$
$$1 + 0$$
$$\boxed{+1}$$



Example 5 cont

$$\begin{aligned} (\pi) &: 1 + 2 \cos(\pi) \\ &= 1 + 2(-1) \\ &= 1 - 2 \\ &= -1 \end{aligned}$$

$$\begin{aligned} \left(\frac{3\pi}{2}\right) : & 1 + 2\cos\left(\frac{3\pi}{2}\right) \\ & " \\ & 1 + 2(0) \\ & " \\ & 1 + 0 \\ & " \\ & (+1) \end{aligned}$$

### ④ Summary

Based on the first derivative test.

$f(x) = x + 2 \sin(x)$  have a relative maximum at

$x = 2\pi/3$  and a relative minimum at  $x = 4\pi/3$ .

$$c = 2\pi/3 \quad \text{and} \quad c = 4\pi/3.$$

⑤ Get relative maximum and minimum value for  $f(x) = x + 2 \sin(x)$

### Relative Maximum Value

$$f\left(\frac{2\pi}{3}\right) = \frac{2\pi}{3} + 2 \sin\left(\frac{2\pi}{3}\right)$$
$$= \frac{2\pi}{3} + 2 \left(\frac{\sqrt{3}}{2}\right)$$
$$= \frac{2\pi}{3} + \sqrt{3}$$
$$= 3.83$$

### Relative Minimum Value

$$f\left(\frac{4\pi}{3}\right) = \frac{4\pi}{3} + 2 \sin\left(\frac{4\pi}{3}\right)$$
$$\frac{4\pi}{3} + 2\left(-\frac{\sqrt{3}}{2}\right)$$
$$\frac{4\pi}{3} - \sqrt{3}$$
$$2.46$$