Example 1

$x^2 + y^2 = 1$

Find $\frac{dy}{dx}$ and equation of tangent line to the circle at (x, y)

$$= \left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$$

$$\frac{d}{dx}\left[x^2 + y^2\right] = \frac{d}{dx}[1]$$

$$= \frac{d}{dx}[x^2] + \frac{d}{dx}[y^2] = \frac{d}{dx}[1]$$

This Rate of Change Applies to Any Point on Unit Circle

$$= 2x^{2-1} + \frac{d}{dx}[y^2] = 0$$

$$2x + 2y^{2-1} = 0$$

$$= 2x + 2y = 0$$

$$2x + 2y \cdot \frac{d}{dx}[y] = 0$$

$$= 2x + 2y\frac{dy}{dx} = 0$$

$$-2x \qquad -2x$$

$$\frac{2y\frac{dy}{dx}}{2y} = \frac{-2x}{2y}$$

$$= \frac{dy}{dx} = -2x \div 2y$$

$$= \frac{dy}{dx} = -2x \cdot \frac{1}{2y}$$

$$= \frac{dy}{dx} = -\frac{2x}{2y}$$

$$= \boxed{\frac{dy}{dx} = -\frac{x}{y}}$$

Slope of tangent line to $x^2+y^2=1$ at Point $\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$ is

$$\boxed{-x/y}$$

$$x = \frac{\sqrt{2}}{2}, \quad y = \frac{\sqrt{2}}{2}$$

$$\downarrow$$

$$\frac{-\frac{\sqrt{2}}{2}}{\frac{\sqrt{2}}{2}} \quad \frac{-x}{y}$$

$$=$$

$$-\frac{\sqrt{2}}{2} \div \frac{\sqrt{2}}{2}$$

$$=$$

$$-\frac{\cancel{\sqrt{2}}}{\cancel{2}} \cdot \frac{\cancel{2}}{\cancel{\sqrt{2}}}$$

$$=$$

$$-1$$

$$\boxed{\text{Slope of tangent line at point } \left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right) \text{ to unit circle } x^2+y^2=1 \text{ is } -1}$$

Get Equation of Tangent Line

$x = \frac{\sqrt{2}}{2}$, $y = \frac{\sqrt{2}}{2}$, $m = -1$

$$y = mx + b$$

$$\frac{\sqrt{2}}{2} = -1\left(\frac{\sqrt{2}}{2}\right) + b$$

$$\frac{\sqrt{2}}{2} = -\frac{\sqrt{2}}{2} + b$$

$$+\frac{\sqrt{2}}{2} \qquad +\frac{\sqrt{2}}{2}$$

$$\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} = b$$

$$\frac{1\sqrt{2} + 1\sqrt{2}}{2}$$

$$\frac{\cancel{2}\sqrt{2}}{\cancel{2}} = b$$

$$\boxed{\begin{array}{c}\sqrt{2} = b \\ b = \sqrt{2}\end{array}}$$

Equation of tangent line to unit circle $x^2 + y^2 = 1$ is

$$\boxed{y = -x + \sqrt{2}}$$

Example 2

$x^2+y^2=1 \quad (x,y)=\left(\frac{\sqrt{2}}{2},\frac{\sqrt{2}}{2}\right)$

Get Slope

$$x^2+y^2=1$$
$$-x^2 \qquad -x^2$$
$$\sqrt{y^2}=\sqrt{1-x^2}$$
$$\parallel$$
$$y=\pm\sqrt{1-x^2}$$

This Rate of Change Applies to ordered pairs that satisfy $\boxed{y=\sqrt{1-x^2}}$

Point $\left(\frac{\sqrt{2}}{2},\frac{\sqrt{2}}{2}\right)$ is positive in Quadrant 1 of Unit Circle so we use $\boxed{y=\sqrt{1-x^2}}$

Get $\frac{dy}{dx}$

$$\parallel$$

$$\frac{d}{dx}\left[\sqrt{1-x^2}\right] \qquad f(x)=\sqrt{1-x^2} \quad g(x)=-x^2$$

$$\parallel$$

Chain It

$$f'(g(x))\,g'(x)$$
$$\parallel$$
$$\frac{d}{dx}\left[\sqrt{1-x^2}\right]\cdot\frac{d}{dx}\left[-x^2\right]$$
$$\parallel \qquad \parallel$$
$$\frac{d}{dx}\left[(1-x^2)^{1/2}\right]\cdot -2x^{2-1}$$
$$\parallel \qquad \parallel$$
$$\frac{1}{2}(1-x^2)^{1/2-1}\cdot -2x^1$$
$$\parallel \qquad \parallel$$
$$\frac{1}{2}(1-x^2)^{-1/2}\cdot -2x$$
$$\parallel$$
$$\frac{1}{2}\cdot\frac{1}{(1-x^2)^{1/2}}\cdot -2x$$
$$\parallel$$
$$\frac{1}{2\sqrt{1-x^2}}\cdot -2x$$
$$\parallel$$
$$\frac{-2x}{2\sqrt{1-x^2}}$$
$$\parallel$$
$$\boxed{\frac{-x}{2\sqrt{1-x^2}}}$$

$$x = \frac{\sqrt{2}}{2}$$

$$\downarrow$$

$$\frac{-x}{2\sqrt{1-x^2}}$$

$$=$$

$$\frac{-\left(\frac{\sqrt{2}}{2}\right)}{2\sqrt{1-\left(\frac{\sqrt{2}}{2}\right)^2}} \longrightarrow \sqrt{1-\left(\frac{\sqrt{2}}{2}\right)^2}$$

$$= \sqrt{1-\frac{\sqrt{2}}{2}\cdot\frac{\sqrt{2}}{2}}$$

$$= \sqrt{1-\frac{\sqrt{4}}{4}}$$

$$= \sqrt{1-\frac{2}{4}}$$

$$= \sqrt{1-\frac{1}{2}}$$

$$\frac{1}{1} = \frac{2}{2}$$

$$-\frac{1}{2} = \frac{1}{2}$$

$$\frac{1}{2}$$

$$= \sqrt{\frac{1}{2}}$$

$$\sqrt{\frac{1}{2}}$$

$$=$$

$$-\frac{\frac{\sqrt{2}}{2}}{2\sqrt{\frac{1}{2}}}$$

$$=$$

$$-\frac{\frac{\sqrt{2}}{2}\cdot\frac{2\sqrt{\frac{1}{2}}}{1}}{2\sqrt{\frac{1}{2}}\cdot 2\sqrt{\frac{1}{2}}}$$

$$=$$

$$-\frac{\frac{2\sqrt{2\cdot\frac{1}{2}}}{2}}{4\sqrt{\frac{1}{2}\cdot\frac{1}{2}}}$$

$$=$$

$$-\frac{2\sqrt{\frac{2}{2}}}{4\sqrt{\frac{1}{4}}}$$

$$=$$

$$-\frac{2\sqrt{1}}{4\,\frac{1}{2}}$$

$$=$$

$$-\frac{2\cdot 1}{\frac{4}{2}}$$

$$=$$

$$-\frac{2}{2}$$

$$=$$

$$-1$$

$$\boxed{m = -1}$$