Using Squeeze Theorem, Prove $\lim_{x \to c} f(x)\,g(x) = 0$

given $\lim_{x \to c} f(x) = 0$
where $|g(x)| \leq m$
and
$x \neq c$

① $|g(x)| \leq m$ — Given

② $|g(x)| = g(x)$ — Absolute value

③ $g(x) \leq m$ — Given

④ $g(x) = m$

⑤ $m \leq g(x) \leq m$ — Squeeze

⑥ $-m \lim_{x \to c} f(x) \leq \lim_{x \to c} f(x)\,g(x) \leq m \lim_{x \to c} f(x)$

$\lim_{x \to c} f(x) = 0$ — Given

$-m(0) \leq \lim_{x \to c} f(x)\,g(x) \leq m(0)$

$0 \leq \lim_{x \to c} f(x)\,g(x) \leq 0$

$\lim_{x \to c} f(x)\,g(x) = 0$