Theorem 2.4.3

$$\lim_{x \to a} f(g(x)) = f(\lim_{x \to a} g(x)) = f(g(a))$$

and
$$\lim_{x\to a} (x) = b$$
,

Then $\lim_{x\to a} (g(x)) = f(b)$

$$\lim_{x \to -1} \cos\left(\frac{x^2-1}{x+1}\right)$$

$$f(g(x)) = cos(x^2-1)$$

$$\lim_{x \to -1} \cos \left(\frac{x^2 - 1}{x + 1} \right) = \cos \left(\lim_{x \to -1} \frac{x^2 - 1}{x + 1} \right)$$

$$f\left(\lim_{x \to a} g(x)\right) = \cos\left(\lim_{x \to -1} \frac{x^2-1}{x+1}\right)$$

$$\frac{1}{x+1}$$

$$f = cos$$
, $g(x) = x^2 - 1$

(os/lim (x-1))

× 3-1

co (limx - lim)

cos (-1 -1)

cos (-1 +-1)

cos (-2)

Theorem 2 4.3

If f is continuous at b,

and $\lim_{x \to a} (x) = b$, then $\lim_{x \to a} f(g(x)) = f(b)$

 $\lim_{x \to a} f(g(x)) = f(\lim_{x \to a} f(x))$

Theorem 2.4.4

It g is continuous at a,

and f is continuous at g(a),

then fog is continuous at x = a

Theorem 2.4.4

It g is continuous at a, and + is continuous at g(a),

the composition function $(f \circ g)(x) = f(g(x))$ is continuous at a.

"a continuous function of continuous function is a continuous function"

PROOF: Since g is continuous at a,

 $\lim_{x \to \infty} g(x) = g(x)$

Since + is continuous at g(a), we can apply
Theorem 2.4.3 ((imit of a Composite Function)

$$\lim_{x\to a} f(g(x)) = f(\lim_{x\to a} g(x)) = f(g(a))$$

+(g(x)) is continuous at a

$$\lim_{x \to 1} \sin \left(\frac{\pi x}{2} \right) = 1$$

$$\sin \left(\frac{\pi (1)}{2} \right)$$

$$\sin \left(\frac{\pi}{2} \right)$$

$$\sin \left(\frac{\pi}{2} \right)$$

$$\begin{array}{ccc} 1 & i & m & cos \left(\frac{x^2 - 1}{x + 1} \right) \\ x & y - 1 & \left(\frac{x^2 - 1}{x + 1} \right) \end{array}$$

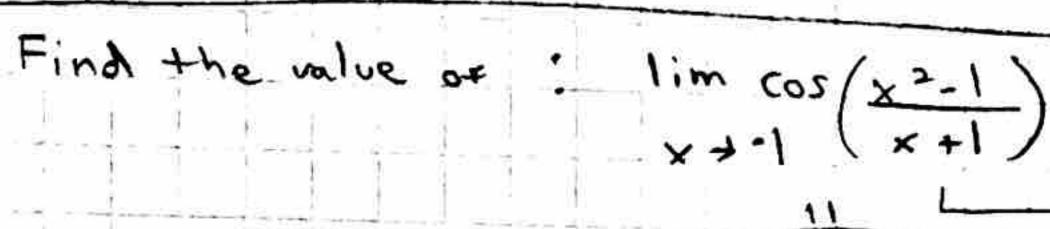
lim 1

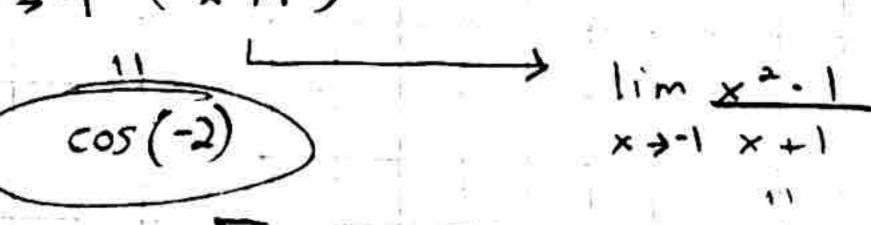
x +-1

lim (x-1)

x +-1

1/mx - 1/m1 x 3·1 - 1 - 1 + -1





Theorem 2.4.3 lim f(g(x)) = f(!limg(x)) x +a

(x+a)

1im x - 1im1 x - 1 x - 1