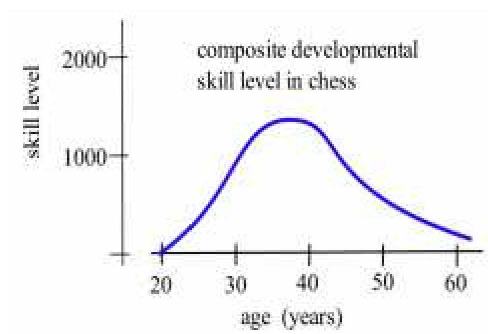
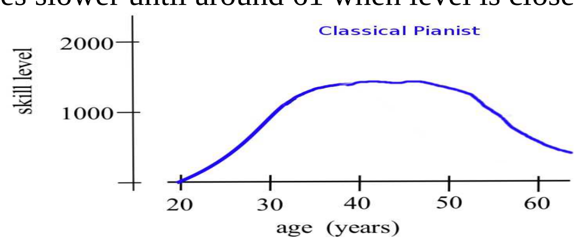
<u>QUIZ #2 Solutions</u>

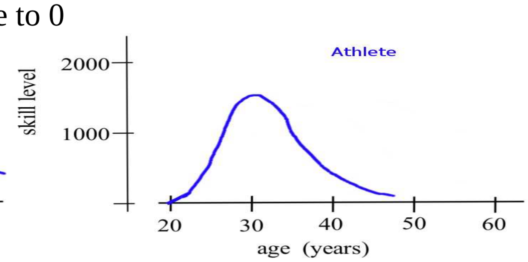
Section 1.0 #8: The figure below shows the composite developmental skill level of chessmasters at different ages as determined by their performance against other chessmasters. (From “Rating Systems for Human Abilities,” by W.H. Batchelder and R.S. Simpson, 1988. UMAP Module 698.)

(a) At what age is the “typical” chessmaster playing the best chess?
(b) At approximately what age is the chessmaster’s skill level increasing most rapidly?
(c) Describe the development of the “typical” chessmaster’s skill in words.
(d) Sketch graphs that you think would reasonably describe the performance levels versus age for an athlete, a classical pianist, a rock singer, a mathematician and a professional in your major field.

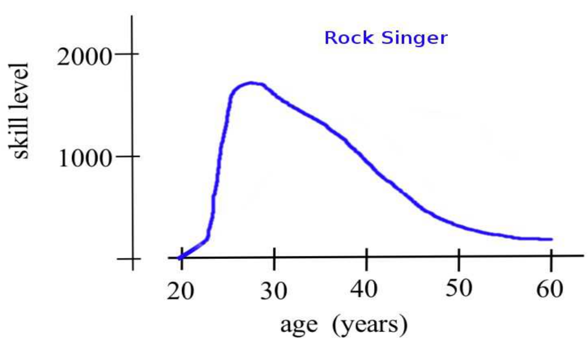


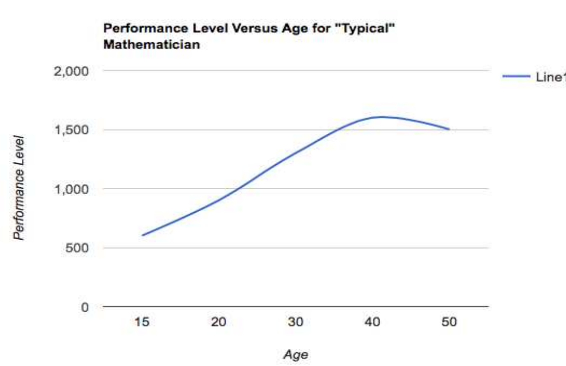
(a) around 38
(b) around 29
(c) begins playing at 20 with skill level of 0, skill level increases quicker and quicker until age 30 when skill level is 1000, skill level increases less and less until peaks at age 38 with skill level of 1400, skill decreases more and more until age 48 and skill level is 800, skill level decreases slower until around 61 when level is close to 0





(d)





Section 1.1 #12: Find the one- and two-sided limits of f (x) as x $\to$ 0, 1 and 2, if f (x) is defined as:

$$f(x)=\begin{cases} x & \text{if } x<0 \\ \sin(x) & \text{if } 0<x\le 2 \\ 1 & \text{if } 2<x \end{cases}$$

$\lim_{x\to 0+} f(x)=\lim_{x\to 0+}\sin(x)=0$, $\lim_{x\to 0-} f(x)=0$, so $\lim_{x\to 0} f(x)=0$

$\lim_{x\to 1+} f(x)=\sin(1)$, $\lim_{x\to 1-} f(x)=\sin(1)$, so $\lim_{x\to 1} f(x)=\sin(1)$

$\lim_{x\to 2+} f(x)=1$, $\lim_{x\to 2-} f(x)=\sin(2)$, so $\lim_{x\to 2} f(x)$ **d.n.e. (NOTE: error in problem)**

Section 1.1 #16: use a calculator or computer to get an approximate answer accurate to 2 decimal places.

(a) $\lim_{x\to 5}\frac{\sqrt{x-1}-2}{x-5}$ **(b)** $\lim_{x\to 0}\frac{\sin(7x)}{2x}$

(a) $$\lim_{x\to 5}\frac{\sqrt{x-1}-2}{x-5}=\lim_{x\to 5}\frac{\sqrt{x-1}-2}{x-5}\times\frac{\sqrt{x-1}+2}{\sqrt{x-1}+2}=\lim_{x\to 5}\frac{(x-1)-4}{x-5}\times\frac{1}{\sqrt{x-1}+2}$$
$$=\lim_{x\to 5}\frac{1}{\sqrt{x-1}+2}=\frac{1}{4}$$

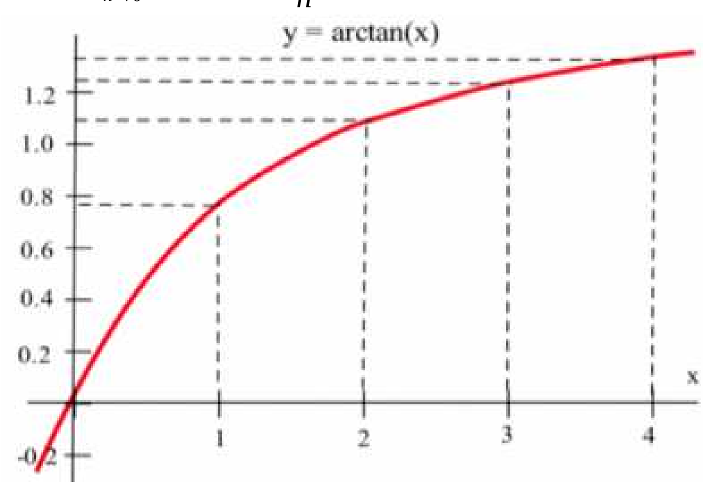
(b) $$\lim_{x\to 0}\frac{\sin(3x)}{5x}=\frac{3}{5}\lim_{x\to 0}\frac{\sin(3x)}{3x}=\frac{3}{5}\times 1=\frac{3}{5}$$

Section 1.2 #12: Give geometric interpretations for each limit and use a calculator to estimate its value.

(a) $\lim_{h\to 0}\frac{\arctan(0+h)-\arctan(0)}{h}$

(b) $\lim_{h\to 0}\frac{\arctan(1+h)-\arctan(1)}{h}$

(c) $\lim_{h\to 0}\frac{\arctan(2+h)-\arctan(2)}{h}$



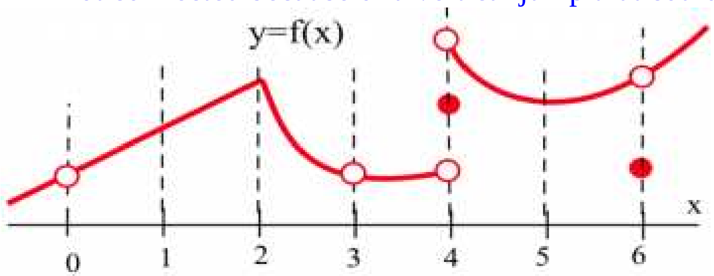
(a) slope of arctan at x = 0; $\lim\limits_{h\to 0}\dfrac{\arctan(0+h)-\arctan(0)}{h}=1$

(b) slope of arctan at x = 1; $\lim\limits_{h\to 0}\dfrac{\arctan(1+h)-\arctan(1)}{h}=.5$

(c) slope of arctan at x = 2; $\lim\limits_{h\to 0}\dfrac{\arctan(2+h)-\arctan(2)}{h}=.2$

Section 1.2 #18: describe the behavior at each integer of the function $y = f(x)$ in the figure provided, using one of these phrases:

- "connected and smooth"
- "connected with a corner"
- "not connected because of a simple hole that could be plugged by adding or moving one point"
- "not connected because of a vertical jump that could not be plugged by moving one point"



@ x = 0 not connected because of a simple hole that could be plugged by adding or moving one point
@ x = 1 connected and smooth
@ x = 2 connected with a corner
@ x = 3 not connected because of a simple hole that could be plugged by adding or moving one point
@ x = 4 not connected because of a vertical jump that could not be plugged by moving one point
@ x = 5 connected and smooth
@ x = 6 not connected because of a simple hole that could be plugged by adding or moving one point

Section 1.2 #20: Show that $\lim\limits_{x\to 0}\sin\left(\dfrac{1}{x}\right)$ **does not exist. (Suggestion: Let $f(x) = \sin(1/x)$ and let $a_n = 1/n\pi$ so that $f(a_n) = \sin(1/a_n) = \sin(n\pi) = 0$ for every n. Then pick $b_n = 1/(2n\pi + \pi/2)$ so that $f(b_n) = \sin(2n\pi + \pi/2) = \sin(\pi/2) = 1$ for every n.)**

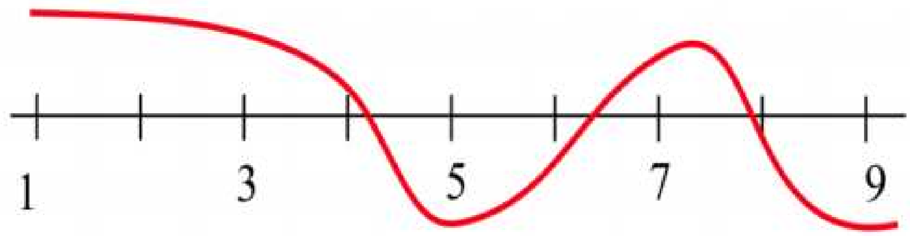
Let $f(x) = \sin(1/x)$, let $a_n = 1/n\pi$, and let $b_n = 1/(2n\pi + \pi/2)$.
We have $f(a_n) = \sin(1/a_n) = \sin(n\pi) = 0$ for every n.
We have $f(b_n) = \sin(1/b_n) = \sin(2n\pi + \pi/2) = \sin(\pi/2) = 1$ for every n.
Thus a_n and b_n both go to 0 so

$$0=\lim_{n\to\infty}\sin(n\pi)=\lim_{n\to\infty}\sin\left(\frac{1}{a_n}\right)=\lim_{a_n\to 0}\sin(a_n)$$

$$1=\lim_{n\to\infty}\sin\left(2n\pi+\frac{\pi}{2}\right)=\lim_{n\to\infty}\sin\left(\frac{1}{b_n}\right)=\lim_{b_n\to 0}\sin(b_n)$$

We have $\lim_{x\to 0}\sin\left(\frac{1}{x}\right)$ has two different values so the limit does not exist.

Section 1.3 #8: Two students claim that they both started with the points x = 1 and x = 9 and applied the Bisection Algorithm to the function graphed below. The first student says that the algorithm converged to the root near x = 8, but the second claims that the algorithm will converge to the root near x = 4. Who is correct?



The second is correct. The first student did not use [1, 9]. Instead used [7, 9] to get root near x = 8. The second student used [1, 9], [1, 5], then [3, 5] to get root near x = 4.

Section 1.3 #22:

a) A square sheet of paper has a straight line drawn on it from the lower left corner to the upper right corner. Is it possible for you to start on the left edge of the sheet and draw a "connected" line to the right edge that does not cross the diagonal line?

(b) Prove: If f is continuous on the interval [0, 1] and $0 \le f(x) \le 1$ for all x, then there is a number c with $0 \le c \le 1$ such that $f(c) = c$. (The number c is called a "fixed point" of f because the image of c is the same as c: f does not "move" c.) Hint: Define a new function $g(x) = f(x) - x$ and start by considering the values $g(0)$ and $g(1)$.

(c) What does part (b) have to do with part (a)?

(d) Is the theorem in part (b) true if we replace the closed interval [0, 1] with the open interval (0, 1)?

(a) No.

(b) Let $g(x) = f(x) - x$. Then $g(0) = f(0) - 0 = f(0) \ge 0$ and $g(1) = f(1) - 1 \le 0$.
Thus $g(1) \le 0 \le g(0)$. $f(x)$ and x are continuous on [0,1] so $g(x)$ is continuous on [0,1]. We just showed that 0 is a value between $g(0)$ and $g(1)$. Then by the IVT there is a number c between 0 and 1 so that $0 = g(c) = f(c) - c$ or $f(c) = c$.

(c) A connected line from the left edge to the right edge is $f(x)$ and the diagonal line is x so part (b) is equivalent to part (a). Part (a) is the graphical explanation of part (b).

(d) No. We are unable to calculate $g(0)$ in the proof.

Section 1.4 #12: You need to cut four pieces of wire (exactly the same length after the cut) and form them into a square. If the area of the square must be within 0.06 inches of 100 inches, then each piece of wire must be within how many inches of 10?

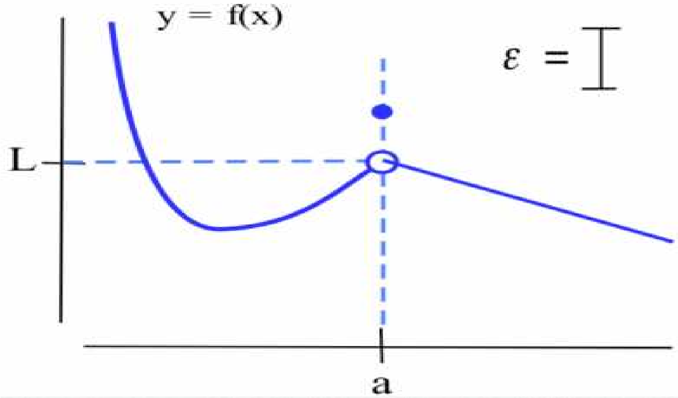
$99.94 \le$ area of square ≤ 100.06

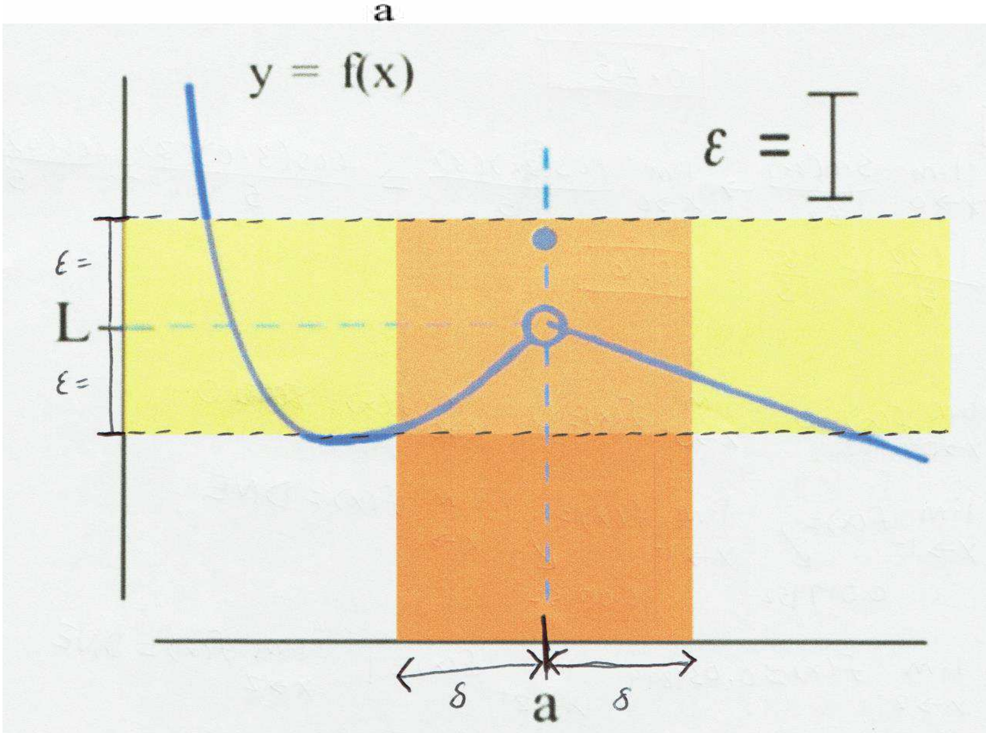
$\sqrt{99.94} = 9.9969995499$ $\quad$ $10 - 9.9969995499 = .003000450135$

$\sqrt{100.06} = 10.0029995501$ $\quad$ $10.0029995501 - 10 = .002999550135$

Each piece must be within .002999550135 inches of 10

Section 1.4 #16: $\lim_{x \to a} f(x) = L$ and the function f and a value for ε are given graphically. Find a length for δ that satisfies the limit definition for the function and value of ε.





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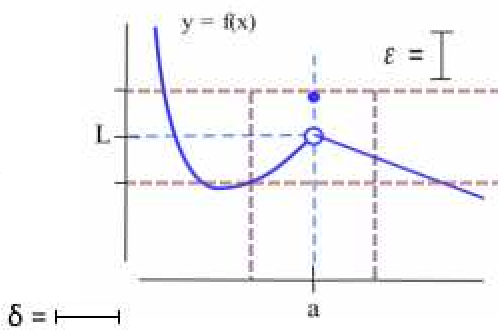

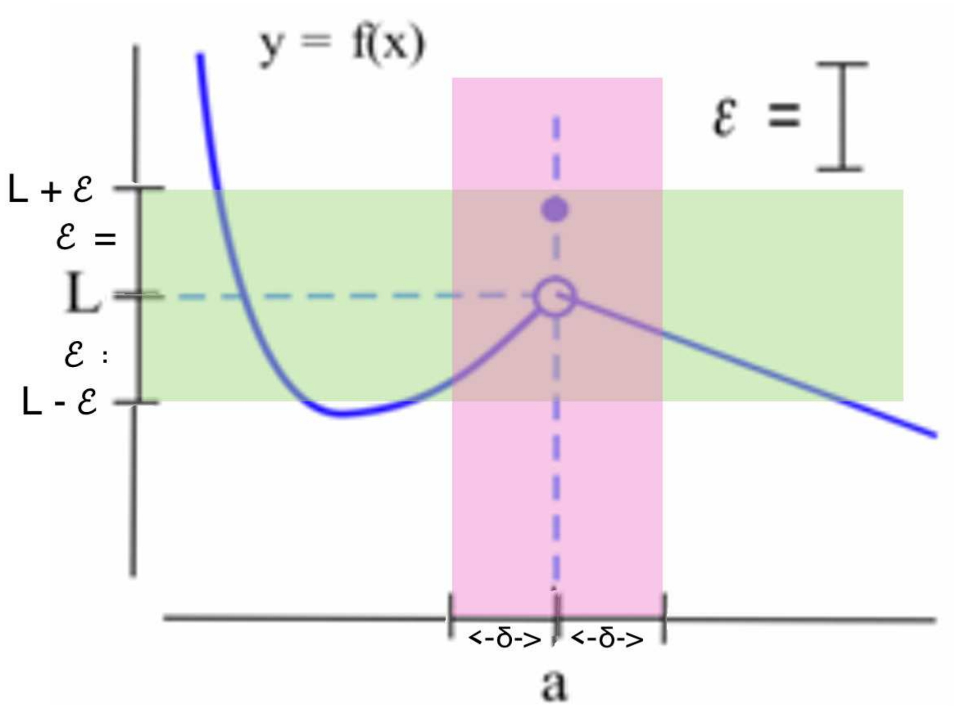

δ= <--->

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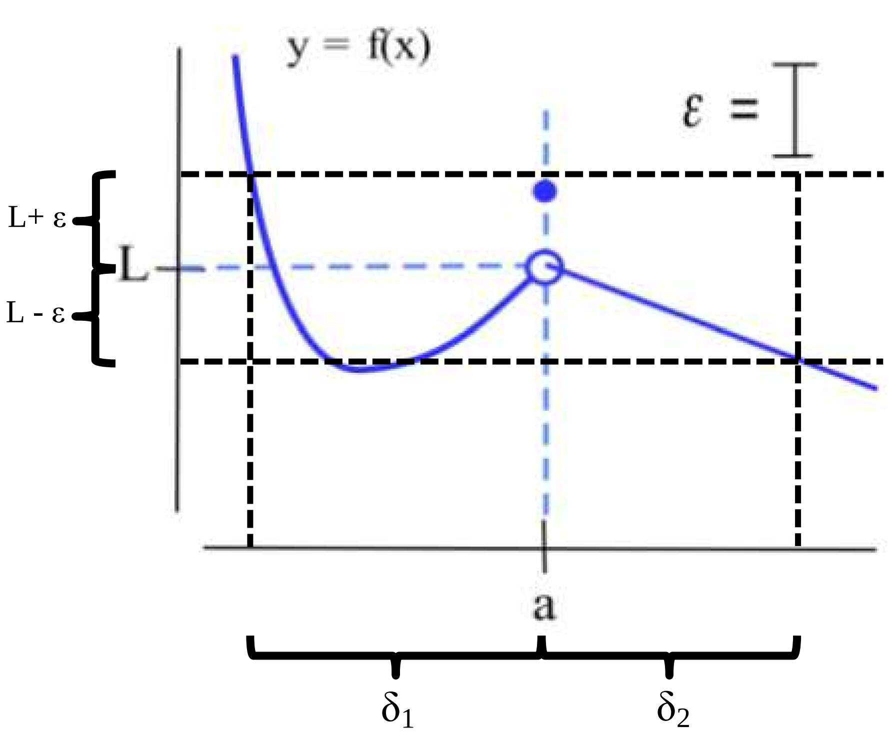

$\delta_1 =$ $\longleftrightarrow$

$\delta_2 =$ $\longleftrightarrow$

so $\delta =$ $\longleftrightarrow$