Validate:

$$\lim_{x \to 0} \frac{|x|}{x}$$

$x = -0.1$

$$\lim_{x \to 0^-} \frac{|x|}{x}$$

$$\shortparallel$$

$$\lim_{x \to 0^-} \frac{|-0.1|}{-0.1}$$

$$\shortparallel$$

$$\lim_{x \to 0^-} \frac{0.1}{-0.1} = -1$$

$x = 0.1$

$$\lim_{x \to 0^+} \frac{|x|}{x} -$$

$$\shortparallel$$

$$\lim_{x \to 0^+} \frac{|0.1|}{0.1} = 1$$

Proof: If the $\lim_{x \to 0} \frac{|x|}{x}$ does not exist is true

$x < 0$

$$\lim_{x \to 0^-} \frac{|x|}{-x}$$

$$\shortparallel$$

$$\lim_{x \to 0^-} \frac{x}{-x}$$

$$\shortparallel$$

$\frac{x}{-x} = -1$ factor x

$$\shortparallel$$

$$\lim_{x \to 0^-} -1 = -1$$

$x > 0$

$$\lim_{x \to 0^+} \frac{|x|}{x}$$

$$\shortparallel$$

$$\lim_{x \to 0^+} \frac{x}{x}$$

$$\shortparallel$$

$\frac{x}{x} = 1$ factor x

$$\shortparallel$$

$$\lim_{x \to 0^+} 1 = 1$$

$$\lim_{x \to 0^-} \frac{|x|}{x} = -1 \neq \lim_{x \to 0^+} \frac{|x|}{x} = 1$$

$\therefore$ By T.T, the $\lim_{x \to 0} \frac{|x|}{x}$ does not exist is true