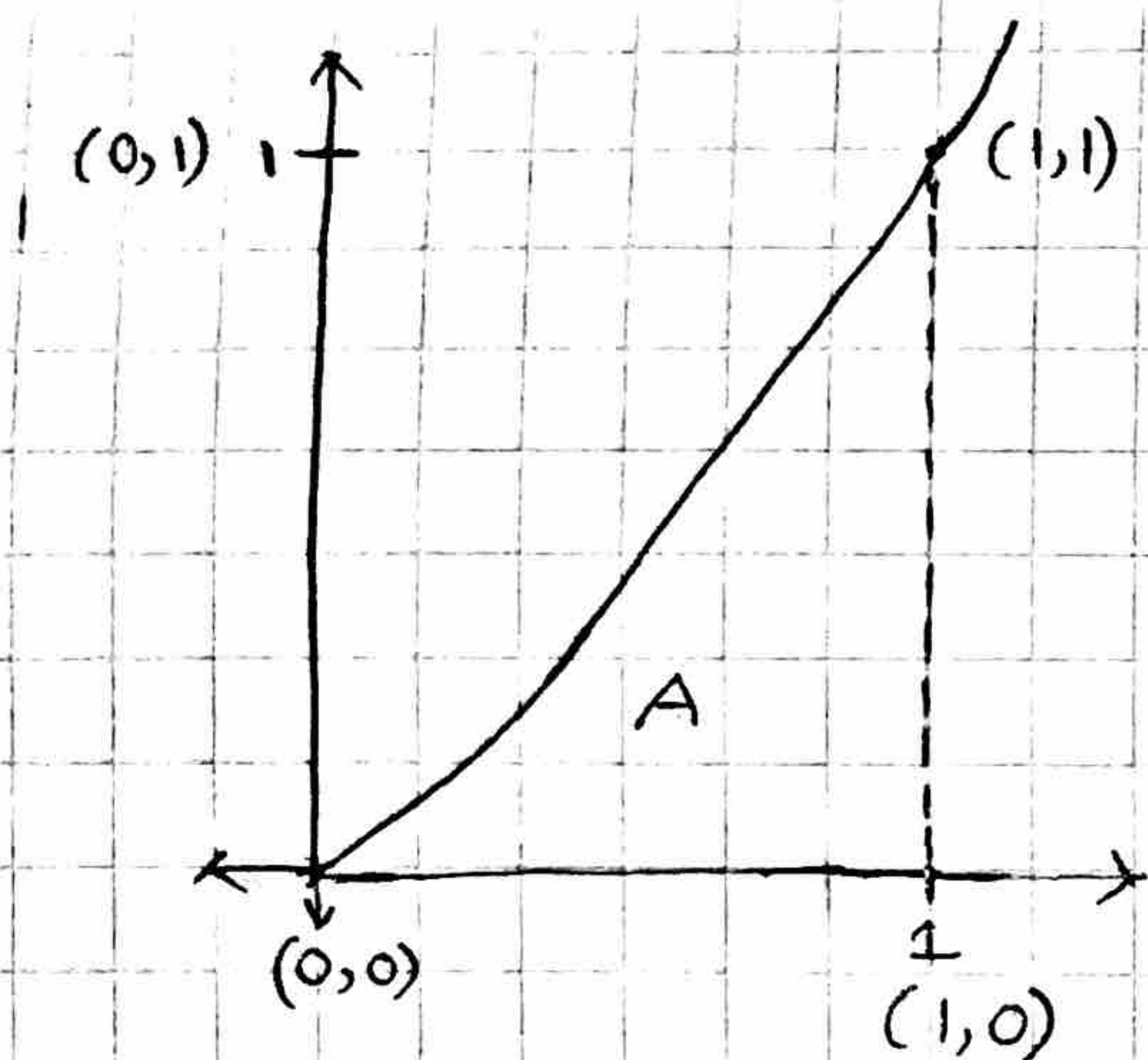
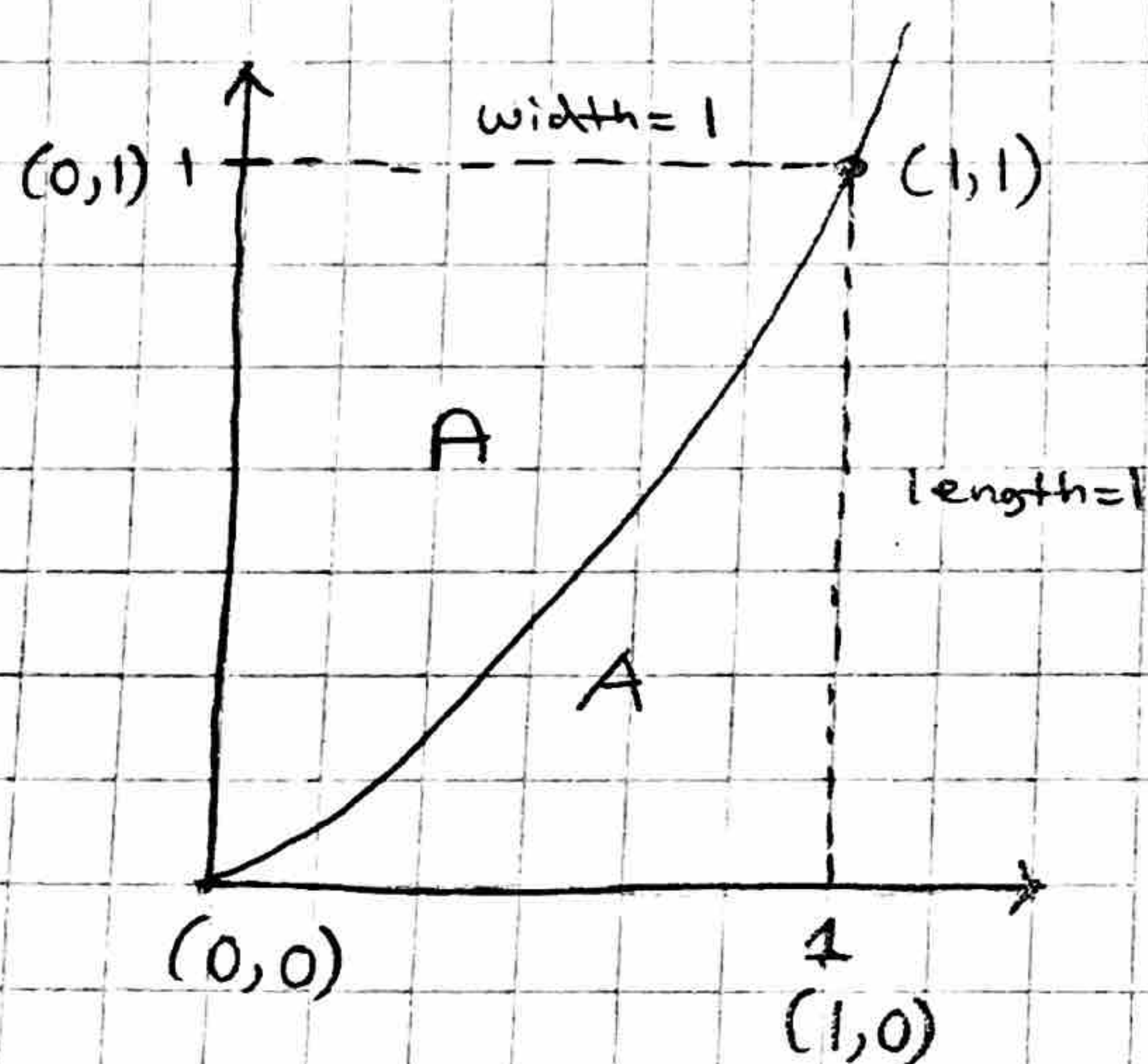


Use Rectangles to Estimate the area  $A$  between  $f(x) = x^2$  and the  $x$ -axis over interval  $[0, 1]$ .

① Set Vertices at  $(0,1), (0,0), (1,0), (1,1)$



② Set Area of Rectangle



Area of Rectangle  $A = wL$

$$A = 1 \cdot 1$$

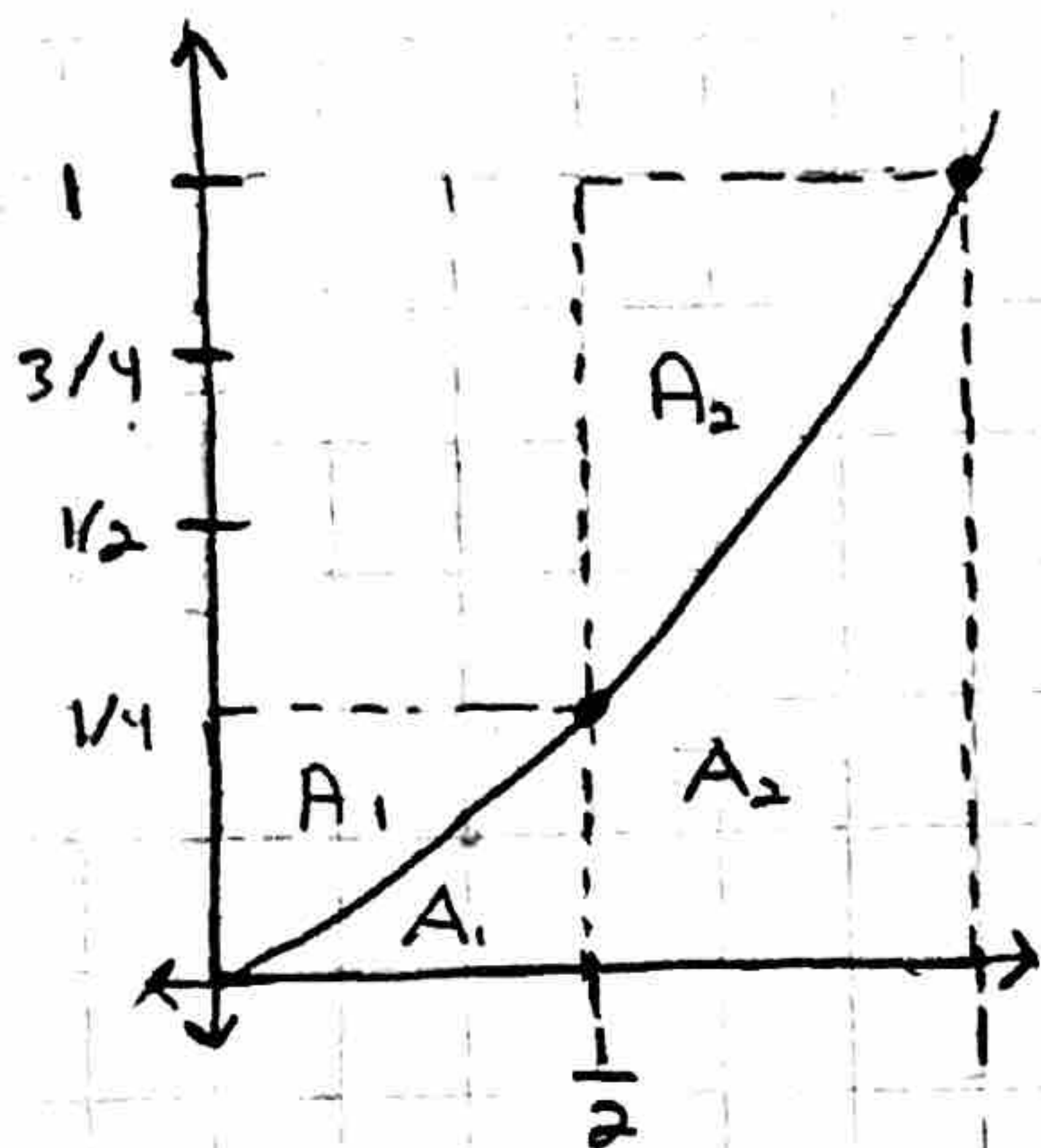
$$A = 1$$

Area  $A$  under Curve is less than 1

$$A < 1$$



③ Set Two Rectangles to Improve Approximation  
Area under Curve



Area of Left Rectangle

$$A_1 = (1/2)(1/4)$$

$$A_1 = 1/8$$

Area of Right Rectangle

$$A_2 = (1/2)(1)$$

$$A_2 = 1/2$$

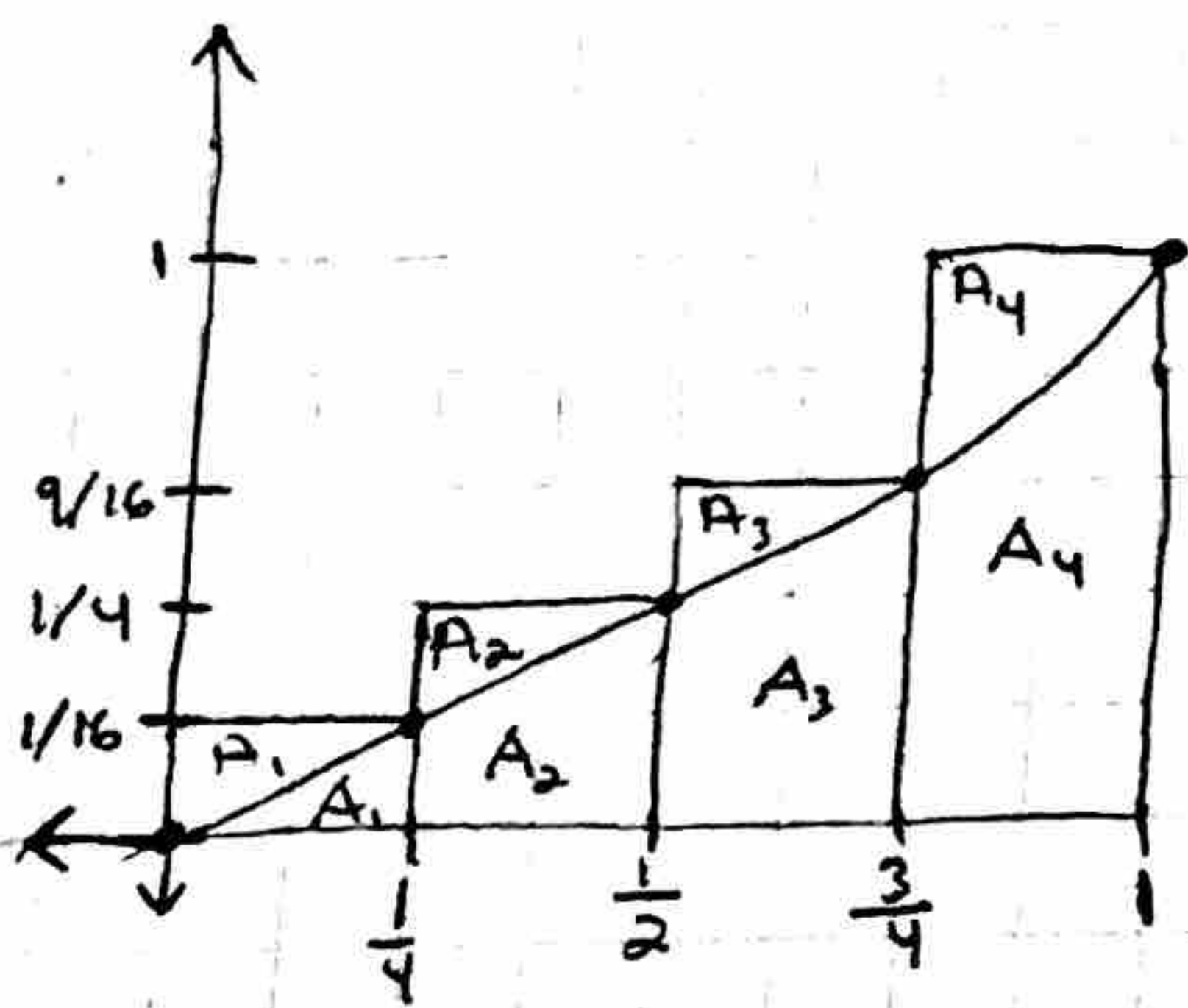
$$A_1 + A_2 = \frac{1}{8} + \frac{1}{2}$$

$$0.625$$

Area under curve  $< 0.625$



④ Set Four Rectangles to Improve Approximation of Area Under Curve



$$\underbrace{\frac{1}{4} \cdot f\left(\frac{1}{4}\right)}_{A_1} + \underbrace{\frac{1}{4} \cdot f\left(\frac{1}{2}\right)}_{A_2} + \underbrace{\frac{1}{4} \cdot f\left(\frac{3}{4}\right)}_{A_3} + \underbrace{\frac{1}{4} \cdot f(1)}_{A_4}$$

$$A_1 + A_2 + A_3 + A_4$$

$$\left(\frac{1}{4}\right)\left(\frac{1}{16}\right) + \left(\frac{1}{4}\right)\left(\frac{1}{4}\right) + \left(\frac{1}{4}\right)\left(\frac{9}{16}\right) + \left(\frac{1}{4}\right)(1)$$

$$\frac{15}{32} \approx 0.46875$$

Area of Curve < 0.46875

⑤ Set Ten Rectangles to Improve Approximation of Area Under Curve.

See Slide