Newton's Method

Consider $x^2 - 1 = 0$

** Iteration 1 **

① Pick value starting at x_0, $x_0 = 3$. Try $x_0 = 3$, if this fail try another value.

② Get $f'(x)$

$$f'(x) = \frac{d}{dx}[x^2 - 1]$$

$$\frac{d}{dx}[x^2] - \frac{d}{dx}[1]$$

$$2x^{2-1} - 0$$

$$\boxed{f'(x) = 2x}$$

③ Get slope of tangent at $x = 3$

$$f'(3) = 2(3)$$

$$\boxed{f'(3) = 6}$$

④ Set equation of tangent line

$y = mx + b$ $\qquad$ $f'(3) = 6$, $m = 6$

$$y = 6x + b$$

⑤ Get Domain and Range

Domain of x, for $y = 6x + b$ $(-\infty, \infty)$ No Restrictions

Get Range

$$y = 6x + b$$
$$-b \qquad -b$$
$$y - b = 6x$$
$$\frac{y}{6} - \frac{b}{6} = \frac{6x}{6}$$
$$\boxed{\frac{y}{6} - \frac{b}{6} = x}$$

No Restrictions for y

⑥ Get y value and y-intercept

$$f(x) = x^2 - 1$$

$$f(3) = (3)^2 - 1$$

$$9 - 1$$

$$\boxed{f(3) = 8}$$

$$y = 8$$

Use $(3, 8)$

$$y = 6x + b$$

$$8 = 6(3) + b$$

$$8 = 18 + b$$

$$-18 \quad -18$$

$$\boxed{-10 = b}$$

⑦ Summarize Equation

Equation of tangent line to the curve $y = f(x)$ at $x = 3$ is

$$\boxed{y = 6x - 10}$$

⑧ Find where tangent line crosses x-axis.

Solve for x

$$6x - 10 = 0$$

$$+10 \quad +10$$

$$\frac{6x}{6} = \frac{10}{6}$$

$$\boxed{x = 5/3}$$

1st Iteration Complete

$$\boxed{x_1 = 5/3}$$

** Iteration 2 **

① Define $x_1 = 5/3$

② Get $f'(x)$

$$f'(x) = \frac{d}{dx}[x^2 - 1]$$

↓ refer to Iteration 1

$$\boxed{f'(x) = 2x}$$

③ Get slope of tangent at $x = 5/3$.

$$f'(5/3) = 2\left(\frac{5}{3}\right)$$

$$\boxed{f'(5/3) = \frac{10}{3}}$$

④ Set equation of tangent line

$y = mx + b$ $\quad f'(5/3) = 10/3$, $m = 10/3$

$$\boxed{y = \frac{10}{3}x + b}$$

⑤ Get Domain and Range →

Domain of x: for $y = \frac{10}{3}x + b$ $(-\infty, \infty)$ No Restrictions

Get Range

$$y = \frac{10}{3}x + b$$

$$-b \qquad -b$$

$$y - b = \frac{10}{3}x$$

$$\frac{3}{10}(y - b) = \frac{3}{10} \cdot \frac{10}{3}x$$

$$\boxed{\frac{3}{10}y - \frac{3}{10}b = x}$$

↖ No Restrictions for y

⑥ Get y value and y-int

$f(x) = x^2 - 1$

$f(5/3) = \left(\frac{5}{3}\right)^2 - 1$

$f(5/3) = \frac{25}{9} - 1 \rightarrow \frac{25}{9} - \frac{9}{9}$

$-\frac{1}{1} = \frac{9}{9}$

$\frac{16}{9}$

$\boxed{f(5/3) = \frac{16}{9}}$

$y = 16$

Use $(5/3, \frac{16}{9})$

$y = \frac{10}{3}x + b$

$\frac{16}{9} = \frac{10}{3}\left(\frac{5}{3}\right) + b$

$\frac{16}{9} = \frac{50}{9} + b$

$-\frac{50}{9} \quad -\frac{50}{9}$

$\boxed{-\frac{34}{9} = b}$

⑦ Summarize Equation

Equation of tangent line to the curve $y = f(x)$ at $x = 5/3$ is

$\boxed{y = \frac{10}{3}x - \frac{34}{9}}$

⑧ Find where tangent line crosses x-axis

Solve for x

$\frac{10}{3}x - \frac{34}{9} = 0$

$+\frac{34}{9} \quad +\frac{34}{9}$

$\left(\frac{3}{10}\right)\frac{10}{3}x = \frac{34}{9}\left(\frac{3}{10}\right) \rightarrow x = \frac{34}{30} = \frac{17}{15}$, $\boxed{x = 17/15}$ $\boxed{x_2 = 17/15}$

** 2nd Iteration Complete **

** Iteration 3 **

① Define $x_2 = 17/15$

② Get $f'(x)$

$$f'(x) = \frac{d}{dx}\left[x^2 - 1\right]$$

↓ refer to Iteration 1

$$\boxed{f'(x) = 2x}$$

③ Get slope of tangent at $x = 17/15$

$$f'(17/15) = 2\left(\frac{17}{15}\right)$$

$$\boxed{f'(17/15) = \frac{34}{15}}$$

④ Set equation of tangent line

$y = mx + b$ $\quad f'(17/15) = 34/15$, $m = 34/15$

$$\boxed{y = \frac{34}{15}x + b}$$

⑤ Get Domain and Range

Domain of x for $y = \frac{34}{15}x + b$ $\quad (-\infty, \infty)$ No Restrictions

Get Range

$$y = \frac{34}{3}x + b$$

$$-b \qquad\qquad -b$$

$$y - b = \frac{34}{3}x$$

$$\frac{3}{34}(y - b) = \frac{34}{3}x \cdot \frac{3}{34}$$

$$\boxed{\frac{3}{34}y - \frac{3}{34}b = x}$$

← no Restrictions for y

⑥ Get y value and y-int

$$f(x) = x^2 - 1$$

$$f(17/15) = (17/15)^2 - 1$$

$$\boxed{f(17/15) = \frac{64}{225}}$$

$$y = \frac{64}{225}$$

Use $(17/15, 64/225)$

$$y = \frac{34}{15}x + b$$

$$\frac{64}{225} = \frac{34}{15}\left(\frac{17}{15}\right) + b$$

$$\frac{64}{225} = \frac{578}{225} + b$$

$$-\frac{578}{225} \quad -\frac{578}{225}$$

$$\boxed{-\frac{514}{225} = b}$$

⑦ Summarize Equation

Equation of tangent line to the curve $y = f(x)$ at $x_2 = 17/15$ is

$$\boxed{y = \frac{34}{15}x - \frac{514}{225}}$$

⑧ Find where tangent line crosses x-axis. Solve for x

$$\frac{34}{15}x - \frac{514}{225} = 0$$

$$+\frac{514}{225} = \frac{514}{225}$$

$$\left(\frac{15}{34}\right) \quad \frac{34}{15}x = \frac{514}{225}\left(\frac{15}{34}\right)$$

$$\boxed{x = \frac{257}{255}}$$

$$\boxed{x_3 = \frac{257}{255}}$$ → Repeat for iteration 4. See slide.