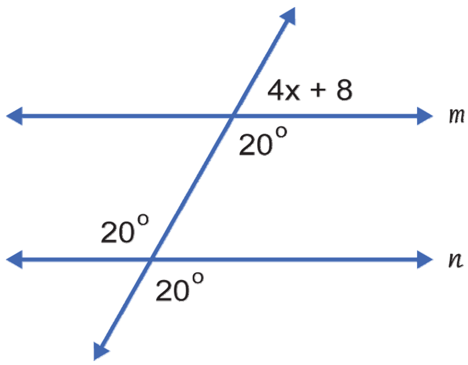
Solutions - 1.6

1. Alternate Exterior Angles
2. Alternate Exterior Angles
3. Alternate Interior Angles
4. Alternate Interior Angles
5. Alternate Interior Angles
6. Alternate Interior Angles
7. Corresponding Angles
8. Corresponding Angles
9. Corresponding Angles
10. Corresponding Angles
11. Corresponding Interior Angles
12. Corresponding Interior Angles
13. Alternate Exterior Angles
14. Alternate Exterior Angles
15. Corresponding Interior Angles
16. Corresponding Interior Angles
17. Alternate Interior Angles
18. Alternate Interior Angles
19. Corresponding Exterior Angles
20. Corresponding Exterior Angles
21. $\angle 1 = 80^o$
22. $\angle 2 = 100^o$
23. $\angle 3 = 80^o$
24. $\angle 4 = 100^o$
25. $\angle 5 = 80^o$
26. $\angle 6 = 100^o$
27. $\angle 7 = 80^o$
28. $\angle 8 = 100^o$

29. Answer: $x = 38$
Detailed Solution;
Vertical angles are congruent and alternate interior angles are congruent, therefore:

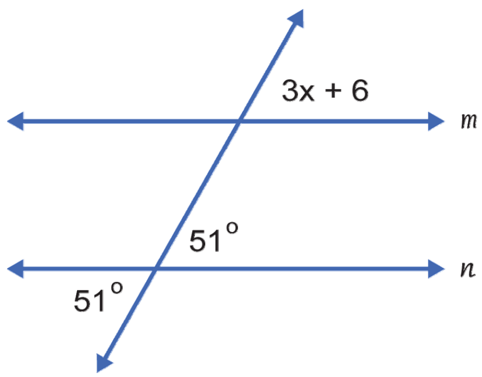


$4x + 8$ and 20^o are supplementary angles, therefore:

$4x + 8 + 20 = 180$
$4x + 28 = 180$
$4x = 152$
$x = 38$

30. $x = 30$

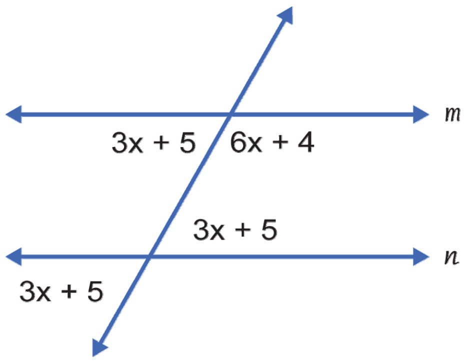
31. Answer: x = 15
Detailed Solution:
Vertical angles are congruent, therefore:



$3x + 6$ and 51° are corresponding angles, therefore:
$3x + 6 = 51$
$3x = 45$
$x = 15$

32. $x = 19$

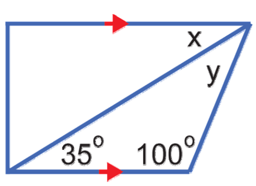
33. Answer: $x = 19$
Detailed Solution:
Vertical angles are congruent and alternate interior angles are congruent, therefore:



$3x + 5$ and $6x + 4$ are supplementary angles, therefore:
$3x + 5 + 6x + 4 = 180$
$9x + 9 = 180$
$9x = 171$
$x = 19$

34. $x = 19$

35. Answer: $x = 35^o$, $y = 45^o$
Detailed Solution:
Find x and y:

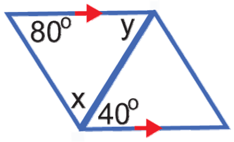


$x = 35^o$ by alternate interior angles

$35^o + 100^o + y = 180^o$
$135^o + y = 180^o$
$y = 45^o$

36. $x = 90^o$, $y = 40^o$

37. Answer: $y = 40^o$, $x = 60^o$
Detailed Solution:
Find x and y:



$y = 40^o$ by alternate interior angles

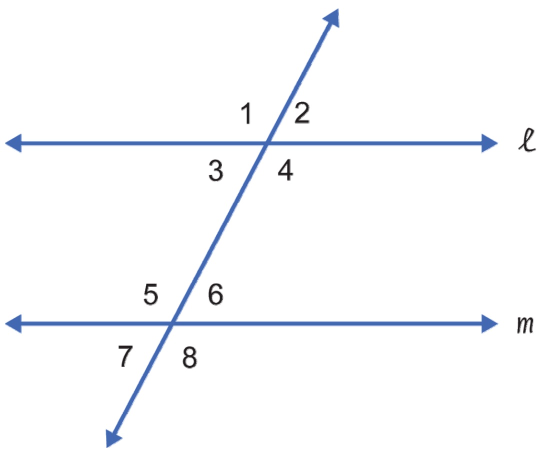
$x + y + 80^o = 180^o$
$x + 40^o + 80^o = 180^o$
$x + 120^o = 180^o$
$x = 60^o$

38. $x = 60^o$, $y = 45^o$

39. Prove Theorem 1.6.1:

If two parallel lines are cut by a transversal, then the alternate interior angles are congruent.

Given: $\ell \parallel m$
Prove: $\angle 3 \cong \angle 6$



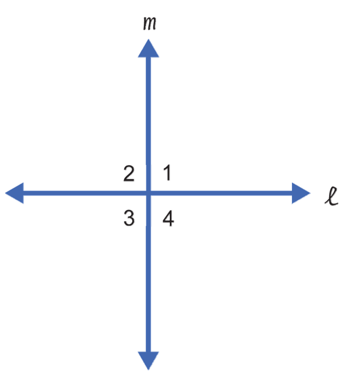
Statement:	Reason:
1. $\ell \parallel m$	1. Given.
2. $\angle 3 \cong \angle 7$	2. Postulate 1.6.1: If two parallel lines are cut by a transversal, then the corresponding angles are congruent.
3. $\angle 7$ and $\angle 6$ are vertical angles	3. Definition of vertical angles.
4. $\angle 7 \cong \angle 6$	4. Theorem 1.2.1: If two angles are vertical angles, then they are congruent.
5. $\angle 3 \cong \angle 6$	5. Transitive.

40. Prove Theorem 1.6.5: Two lines are perpendicular if and only if they form four right angles.

To show the if and only if is true, show two proofs:
If two intersecting lines form four right angles, then the lines are perpendicular.
AND
If two lines are perpendicular then they form four right angles.

Part 1: Show: If two intersecting lines form four right angles, then the lines are perpendicular.

Given: $\angle 1, \angle 2, \angle 3$, and $\angle 4$ are right angles.
Prove: $\ell \perp m$



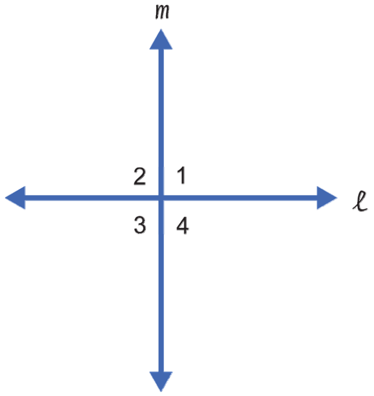
Statement	Reason
1. $\angle 1, \angle 2, \angle 3$, and $\angle 4$ are right angles.	1. Given
2. $\angle 1, \angle 2, \angle 3$, and $\angle 4$ are all 90^o	2. Definition of right angle.
3. $\ell \perp m$	3. Postulate 1.6.4: Two lines that intersect and form a 90^o angle are perpendicular.

Part 2: Show: If two lines are perpendicular then they form four right angles.

Given: $\ell \perp m$

Prove: $\angle 1, \angle 2, \angle 3$, and $\angle 4$ are right angles.

40. Continued:

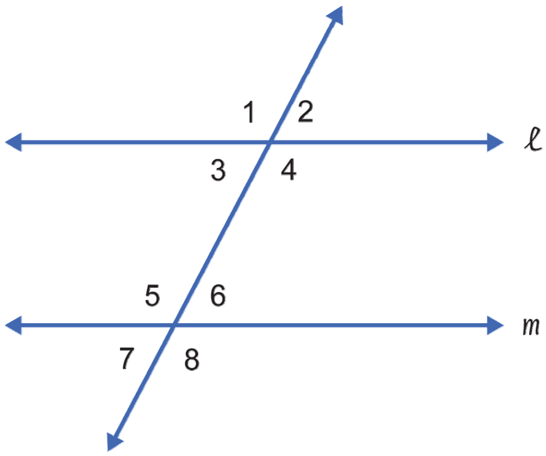


Statement	Reason
1. $\ell \perp m$	1. Given
2. $\angle 1$ is a right angle.	2. Postulate 1.6.4: Two lines that intersect are perpendicular if and only if they form a right angle.
3. $\angle 1$ and $\angle 3$ are vertical angles.	3. Definition of vertical angles.
4. $\angle 1 \cong \angle 3$	4. Theorem1.2.1: If two angles are vertical angles, then they are congruent.
5. $\angle 3$ is a right angle.	5. Definition of congruence.
6. $\angle 1 = 90^o$, $\angle 3 = 90^o$	6. Definition of right angle (lines 2, 5)
7. $\angle 1$ and $\angle 2$ form a linear pair. $\angle 3$ and $\angle 4$ form a linear pair.	7. Definition of linear pair.
8. $\angle 1$ and $\angle 2$ are supplementary angles. $\angle 3$ and $\angle 4$ are supplementary angles.	8. Postulate 1.2.1: If two angles form a linear pair, then they are supplementary angles.
9. $\angle 1 + \angle 2 = 180^o$ $\angle 3 + \angle 4 = 180^o$	9. Definition of supplementary angles.
10. $90^o + \angle 2 = 180^o$ $90^o + \angle 4 = 180^o$	10. Substitution from lines 6 and 9.
11. $\angle 2 = 90^o$ $\angle 4 = 90^o$	11. Subtraction of 90^o.
12. $\angle 2$ and $\angle 4$ are right angles.	12. Definition of right angles.
13. $\angle 1$, $\angle 2$, $\angle 3$, and $\angle 4$ are right angles	13. Lines: 2, 5, 12

41. Prove Theorem 1.6.2: If two lines are cut by a transversal and one pair of alternate interior angles are congruent, then the other pair of alternate interior angles also are congruent.

Given: $\angle 3 \cong \angle 6$

Prove: $\angle 4 \cong \angle 5$

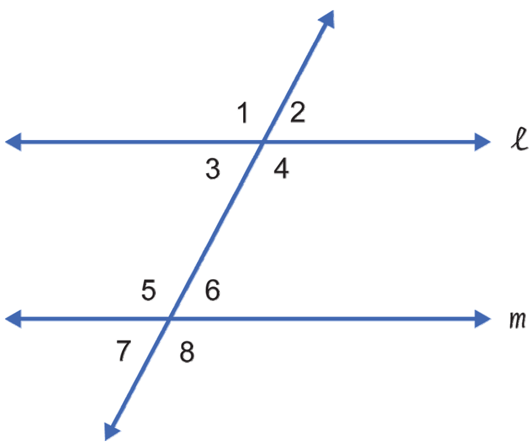


Statement:	Reason:
1. $\angle 3$ & $\angle 4$ form a linear pair $\angle 5$ & $\angle 6$ form a linear pair	1. Definition of linear pair.
2. $\angle 3$ & $\angle 4$ are supplementary angles $\angle 5$ & $\angle 6$ are supplementary angles	2. Postulate 1.2.1: If two angles form a linear pair, then they are supplementary angles.
3. $\angle 3 \cong \angle 6$	3. Given.
4. $\angle 6 \cong \angle 4$ are supplementary angles $\angle 5 \cong \angle 6$ are supplementary angles	4. Substitution from line 2 and 3.
5. $\angle 4 \cong \angle 5$	5. Theorem 1.2.2: If two angles are supplementary to the same angle, then they are congruent.

42. Prove Theorem 1.6.3: If two lines are cut by a transversal and one pair of corresponding angles are congruent, then all pairs of corresponding angles are congruent.

Given: $\angle 1 \cong \angle 5$

Prove: $\angle 3 \cong \angle 7$
$\angle 2 \cong \angle 6$
$\angle 4 \cong \angle 8$



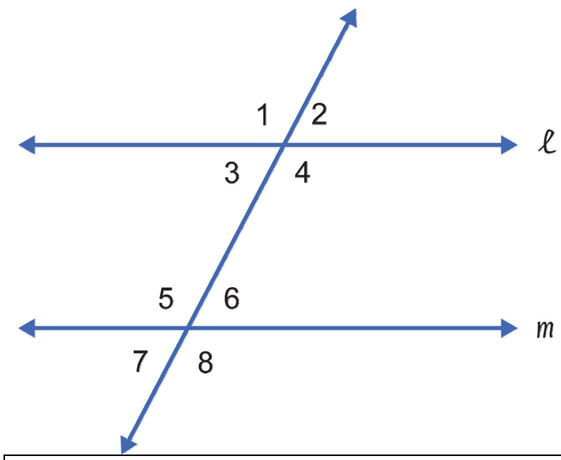
42. Continued:

Statement	Reason
1. $\angle 1$ and $\angle 4$ are vertical angles $\angle 5$ and $\angle 8$ are vertical angles	1. Definition of vertical angles.
2. $\angle 1 \cong \angle 4$ $\angle 5 \cong \angle 8$	2. Theorem 1.2.1: If two angles are vertical angles, then they are congruent.
3. $\angle 1 \cong \angle 5$	3. Given.
4. $\angle 1 \cong \angle 4$ $\angle 1 \cong \angle 8$	4. Substitution from line 3.
5. $\angle 4 \cong \angle 8$	5. Substitution from line 4.
6. $\angle 3$ and $\angle 4$ form a linear pair $\angle 7$ and $\angle 8$ form a linear pair	6. Definition of linear pair.
7. $\angle 3$ and $\angle 4$ are supplementary angles $\angle 7$ and $\angle 8$ are supplementary angles	7. Postulate 1.2.1: If two angles form a linear pair, then they are supplementary angles.
8. $\angle 4 \cong \angle 8$	8. Line 5.
9. $\angle 3$ and $\angle 8$ are supplementary angles. $\angle 7$ and $\angle 8$ are supplementary angles.	9. Substitution from line 8.
10. $\angle 3 \cong \angle 7$	10. Theorem 1.2.2: If two angles are supplementary to the same angle, then they are congruent.
11. $\angle 1$ and $\angle 2$ form a linear pair. $\angle 5$ and $\angle 6$ form a linear pair.	11. Definition of linear pair.
12. $\angle 1$ and $\angle 2$ are supplementary angles $\angle 5$ and $\angle 6$ are supplementary angles	12. Postulate 1.2.1: If two angles form a linear pair, then they are supplementary angles.
13. $\angle 1 \cong \angle 5$	13. Given.
14. $\angle 5$ and $\angle 2$ are supplementary angles $\angle 5$ and $\angle 6$ are supplementary angles	14. Substitution from line 13.
15. $\angle 2 \cong \angle 6$	15. Theorem 1.2.2: If two angles are supplementary to the same angle, then they are congruent.
16. $\angle 3 \cong \angle 7$, $\angle 2 \cong \angle 6$, $\angle 4 \cong \angle 8$	16. Lines 10, 15 and 5.

43. Prove Theorem 1.6.4: If two lines are cut by a transversal and a pair of alternate interior angles are congruent, then the two lines are parallel.

Given: $\angle 3 \cong \angle 6$

Prove: $\ell \parallel m$

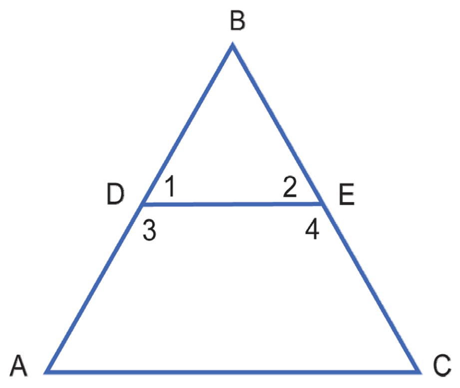


Statement:	Reason:
1. $\angle 1$ & $\angle 3$ form a linear pair $\angle 5$ & $\angle 6$ form a linear pair	1. Definition of linear pair.
2. $\angle 1$ & $\angle 3$ are supplementary angles $\angle 5$ & $\angle 6$ are supplementary angles	2. Postulate 1.2.1: If two angles form a linear pair, then they are supplementary angles.
3. $\angle 3 \cong \angle 6$	3. Given.
4. $\angle 1 \cong \angle 6$ are supplementary angles $\angle 5 \cong \angle 6$ are supplementary angles	4. Substitution from line 2 and 3.
5. $\angle 1 \cong \angle 5$	5. Theorem 1.2.2: If two angles are supplementary to the same angle, then they are congruent.
6. $\ell \parallel m$	6. Postulate 1.6.3: If two lines are cut by a transversal and a pair of corresponding angles are congruent, then the two lines are parallel.

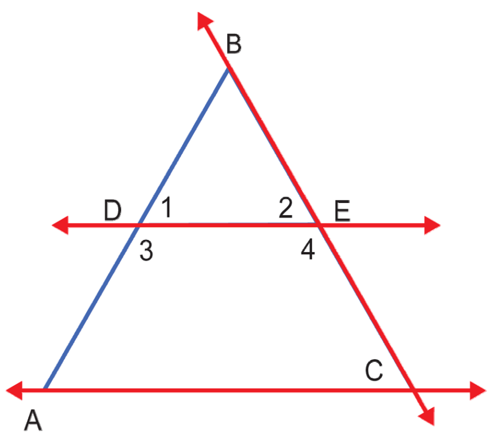
44.

Given: $\angle 3 \cong \angle 4$
$\angle 1 \cong \angle C$

Prove: $\overline{DE} \parallel \overline{AC}$



Redraw the above figure by extending lines $\overleftrightarrow{DE}$ and $\overleftrightarrow{AC}$ and the transversal line $\overleftrightarrow{CB}$.



Notice: $\angle 2 \cong \angle C$ are corresponding angles.

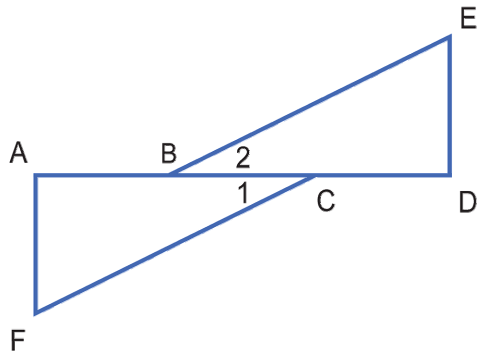
44. Continued:

Statement	Reason
1. $\angle 1$ and $\angle 3$ form a linear pair. $\angle 2$ and $\angle 4$ form a linear pair.	1. Definition of linear pair.
2. $\angle 1$ and $\angle 3$ are supplementary angles. $\angle 2$ and $\angle 4$ are supplementary angles.	2. Postulate 1.2.1: If two angles form a linear pair, then they are supplementary angles.
3. $\angle 3 \cong \angle 4$	3. Given.
4. $\angle 1$ and $\angle 3$ are supplementary angles. $\angle 2$ and $\angle 3$ are supplementary angles.	4. Substitution from line 3.
5. $\angle 1 \cong \angle 2$	5. Theorem 1.2.2: If two angles are supplementary to the same angle, then they are congruent.
6. $\angle 1 \cong \angle C$	6. Given.
7. $\angle 2 \cong \angle C$	7. Substitution from line 5.
8. $\overline{DE} \parallel \overline{AC}$	8. Postulate 1.6.3: If two lines are cut by a transversal and a pair of corresponding angles are congruent, then the two lines are parallel.

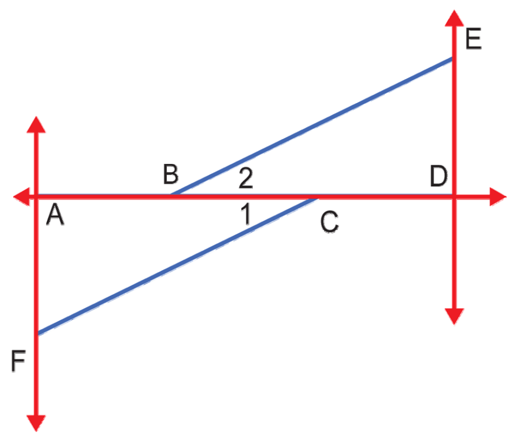
45.

Given: $\angle F$ and $\angle 1$ are complementary
$\angle E$ and $\angle 2$ are complementary

Prove: $\overline{AF} \parallel \overline{DE}$



Redraw the above figure by extending lines $\overleftrightarrow{AF}$, $\overleftrightarrow{DE}$ and the transversal line $\overleftrightarrow{AD}$.



Note: $\angle A$ and $\angle D$ are alternate interior angles.

45. Continued:

Statement:	Reason:
1. $\angle F$ and $\angle 1$ are complementary $\angle E$ and $\angle 2$ are complementary	1. Given.
2. $\angle F + \angle 1 = 90^o$ $\angle E + \angle 2 = 90^o$	2. Definition of complementary.
3. $\angle A + \angle F + \angle 1 = 180^o$ $\angle D + \angle E + \angle 2 = 180^o$	3. Definition of angle sum in a triangle.
4. $\angle A + 90^o = 180^o$ $\angle D + 90^o = 180^o$	4. Substitution from lines 2 & 3.
5. $\angle A = 90^o$ $\angle D = 90^o$	5. Subtraction of 90^o.
6. $\angle A \cong \angle D$	6. Definition of congruence.
7. $\overline{AF} \parallel \overline{DE}$	7. Theorem 1.6.4: If two lines are cut by a transversal and a pair of alternate interior angles are congruent, then the two lines are parallel.