Find the value c that mates the function defined by $f(x) = \begin{cases} x^2 + c, & x < 1 \\ x - 1, & x \geq 1 \end{cases}$

continuous everywhere

$$f(x) = \begin{cases} x^2 + c, & x < 1 \\ x - 1, & x \geq 1 \end{cases}$$

① $\lim_{x \to 1} f(x) = f(1)$

② $\lim_{x \to 1^-} = \lim_{x \to 1^+}$

③

$\lim_{x \to 1^-} x^2 + c = (1)^2 + c = 1 + c$

$\lim_{x \to 1^+} x - 1 = (1) - 1 = 0$

④

$1 + c = 0$

$$1 + c = 0$$
$$-1 \quad -1$$
$$c = -1$$

When $c = -1$, f is continuous everywhere

For what value of 'a' is the piecewise function $f(x)$ continuous on the entire real line?

$$f(x) = \begin{cases} x^2+3 \; ; & x<1 \\ ax+6 \; ; & x \geq 1 \end{cases}$$

① $\lim\limits_{x \to 1} f(x) = f(1)$ $\longrightarrow$ $\lim\limits_{x \to a} f(x) = f(a)$

② $\lim\limits_{x \to 1^-} = \lim\limits_{x \to 1^+}$

$\lim\limits_{x \to 1^-} (x^2+3) = (1)^2+3 = 1+3 = 4$

$\lim\limits_{x \to 1^+} (ax+6) = a(1)+6 = a+6$

$\underline{4} = \underline{a+6}$

③ Find the value for a

$$4 = a+6$$
$$\underline{-6 \quad -6}$$
$$-2 = a$$

$\boxed{\text{Limit exists at } a = -2}$

Find the value of c such that $f(x) = \begin{cases} cx+1, & x \le 2 \\ x, & x > 2 \end{cases}$ is

continuous everywhere

① $\lim_{x \to 2} f(x) = f(2)$

② $\lim_{x \to 2^-} = \lim_{x \to 2^+}$

③

$\lim_{x \to 2^-} c+1 = cx+1$

$= c(2)+1$

$= 2c+1$

$\lim_{x \to 2^+} x = x$

$= 2$

④ $2c+1 = 2$

$$2c+1 = 2$$
$$-1 \quad -1$$
$$\frac{2c}{2} = \frac{1}{2}$$
$$c = \frac{1}{2}$$
$$c = 0.5$$

When $c = 0.5$, f is continuous everywhere