Calculating a Limit by Multiplying by a Conjugate

Conjugate : $a \pm b \leftrightarrow a \mp b$ Multiply with sign's being reversed

$$\lim_{x \to 7} \frac{\sqrt{x+2} - 3}{x-7} = (6)$$

Multiply the expression by numerator's conjugate

$$\frac{(\sqrt{x+2} - 3)}{(x-7)} \cdot \frac{(\sqrt{x+2} + 3)}{(\sqrt{x+2} + 3)}$$

$$\frac{\sqrt{x^2+4} + 3\sqrt{x+2} \cdot 3\sqrt{x+2} - 9}{(x-7)(\sqrt{x+2} + 3)}$$

$$\frac{x+2-9}{(x-7)(\sqrt{x+2}+3)}$$

$$\frac{(x-7)}{(x-7)(\sqrt{x+2}+3)}$$

$$\lim_{x \to 7} \frac{1}{(\sqrt{x+2}+3)}$$

$$\frac{1}{\sqrt{7+2}+3}$$

$$\frac{1}{\sqrt{9}+3}$$

$$\frac{1}{3+3}$$

$$\lim_{x \to 7} \frac{1}{6}$$

$$1/6$$

$$\lim_{x\to 1}\frac{\sqrt{x+15}-4}{x-1} = \frac{1}{8}$$

$$=\frac{\left(\sqrt{x+15}-4\right)}{(x-1)}\cdot\frac{\left(\sqrt{x+15}+4\right)}{\left(\sqrt{x+15}+4\right)}$$

$$=\frac{\sqrt{x^2+225}+4\sqrt{x+15}-4\sqrt{x+15}-16}{(x-1)\left(\sqrt{x+15}+4\right)}$$

$$=\frac{x+15-16}{(x-1)\left(\sqrt{x+15}+4\right)}$$

$$=\frac{\cancel{x-1}}{\cancel{(x-1)}\left(\sqrt{x+15}+4\right)}$$

$$=\lim_{x\to 1}\frac{1}{\sqrt{x+15}+4}$$

$$=\frac{\lim_{x\to 1} 1}{\lim_{x\to 1}\sqrt{x+15}+\lim_{x\to 1} 4}$$

$$=\frac{1}{\sqrt{\lim_{x\to 1} x+15}+4}$$

$$=\frac{1}{\sqrt{\lim_{x\to 1} x+15}+4}$$

$$=\frac{1}{\sqrt{1+15}+4}$$

$$=\frac{1}{\sqrt{16}+4}$$

$$=\frac{1}{4+4}$$

$$=\frac{1}{8}$$

$$\lim_{x \to 1} \frac{\sqrt{x+8} - 3}{x - 1}$$

$$=$$

$$\frac{\left(\sqrt{x+8} - 3\right)}{(x-1)} \cdot \frac{\left(\sqrt{x+8} + 3\right)}{\left(\sqrt{x+8} + 3\right)}$$

$$=$$

$$\frac{\sqrt{x^2+64} + 3\sqrt{x+8} - 3\sqrt{x+8} - 9}{x\sqrt{x+8} + 3x - 1\sqrt{x+8} - 3}$$

$$=$$

$$\frac{x+8-9}{x\sqrt{x+8} + 3x - 1\sqrt{x+8} - 3}$$

$$=$$

$$\lim_{x \to 1} \frac{x-1}{x\sqrt{x+8} + 3x - 1\sqrt{x+8} - 3}$$

$$=$$

$$\frac{1-1}{1\sqrt{1+8} + 3(1) - 1\sqrt{1+8} - 3}$$

$$=$$

$$\frac{0}{0}$$

$$\frac{x-1}{(x-1)\left(\sqrt{x+8}\right) + 3}$$

$$=$$

$$\lim_{x \to 1} \frac{1}{\sqrt{x+8} + 3}$$

$$=$$

$$\frac{1}{\sqrt{1+8} + 3}$$

$$=$$

$$\frac{1}{\sqrt{9} + 3}$$

$$=$$

$$\frac{1}{3+3}$$

$$=$$

$$\boxed{1/6}$$

$$\lim_{x \to 1} \frac{\sqrt{x+8} - 3}{x - 1}$$

$$\frac{(\sqrt{x+8} - 3)}{(x-1)} \cdot \frac{(\sqrt{x+8} + 3)}{(\sqrt{x+8} + 3)}$$

$$\frac{\sqrt{x^2+64} + 3\sqrt{x+8} - 3\sqrt{x+8} - 9}{x\sqrt{x+8} + 3x - 1\sqrt{x+8} - 3}$$

$$\frac{x + 8 - 9}{x\sqrt{x+8} + 3x - 1\sqrt{x+8} - 3}$$

$$\lim_{x \to 0} \frac{\sqrt{x+25} - 5}{x} = 1/10$$

$$=$$

$$\frac{(\sqrt{x+25} - 5)}{x} \cdot \frac{(\sqrt{x+25} + 5)}{(\sqrt{x+25} + 5)}$$

$$=$$

$$\frac{\sqrt{x^2 + 625} + 5\sqrt{x+25} - 5\sqrt{x+25} - 25}{x\sqrt{x+25} + 5x}$$

$$=$$

$$\frac{x + 25 - 25}{x\sqrt{x+25} + 5x}$$

$$=$$

$$\frac{x}{x\sqrt{x+25} + 5x}$$

$$=$$

$$\lim_{x \to 0} \frac{1}{\sqrt{x+25} + 5x}$$

$$=$$

$$\frac{1}{\sqrt{0+25} + 5(0)}$$

$$=$$

$$\frac{1}{\sqrt{25} + 5}$$

$$=$$

$$\frac{1}{5+5}$$

$$=$$

$$\lim_{x \to 0} \frac{1}{10}$$

$$=$$

$$\boxed{\frac{1}{10}}$$

$$\lim_{x \to 0} \frac{\sqrt{x+4} - 2}{x}$$

$$=$$

$$\frac{(\sqrt{x+4} - 2)}{x} \cdot \frac{(\sqrt{x+4} + 2)}{(\sqrt{x+4} + 2)}$$

$$=$$

$$\frac{\sqrt{x^2+16} + 2\sqrt{x+4} - 2\sqrt{x+4} - 4}{x\sqrt{x+4} + 2x}$$

$$=$$

$$\frac{x + 4 - 4}{x\sqrt{x+4} + 2x}$$

$$=$$

$$\frac{x}{x\sqrt{x+4} + 2x}$$

$$=$$

$$\lim_{x \to 0} \frac{1}{\sqrt{x+4} + 2x}$$

$$=$$

$$\frac{1}{\sqrt{0+4} + 2(0)}$$

$$=$$

$$\frac{1}{4+0}$$

$$=$$

$$\lim_{x \to 0} \frac{1}{4}$$

$$=$$

$$\boxed{\frac{1}{4}}$$