Math E-3: ASSIGNMENT 5 - SOLUTIONS

TOTAL POSSIBLE POINTS: 50

Show work for full or partial credit.

For some of the following problems, it's quite helpful to draw diagrams. Use Pascal's Triangle when you are working with coins or with any event that can occur in only two ways. Use the dice table for dice.

Problems 1- 4

Hint: These are simple probability problems using the basic formula for probability.

An urn contains 4 blue balls, 6 red balls, and 3 green balls and 6 yellow balls. You stick your hand in and pull out a ball without looking. What is the probability of the following outcomes? **(Please give your answers as decimals rounded to TWO decimal places.)**

1) selecting a red ball?

$$\frac{\text{6 red balls}}{\text{19 total balls}} = \frac{6}{19} = .3157\ldots = .32 \qquad \text{(2 pts)}$$

2) selecting a blue ball?

$$\frac{\text{4 blue balls}}{\text{19 total balls}} = \frac{4}{19} = .21052\ldots = .21 \qquad \text{(2 pts)}$$

3) selecting a yellow ball or a green ball?

$$\frac{\text{6 yellow + 3 green}}{\text{19 total balls}} = \frac{9}{19} = .47368\ldots = .47 \qquad \text{(2 pts)}$$

4) Assume you chose a red ball. Do NOT put it back into the urn. You then pick another ball from the urn. Find the probability it will be another red ball.

$$\frac{\text{5 remaining red balls}}{\text{18 remaining total balls}} \quad \frac{5}{18} = .27777\ldots = .28 \qquad \text{(2 pts)}$$

Problems 5-6

Dice. These two questions involve one die.
(Please give your answers as decimals rounded to TWO decimal places.)

5) What is the probability of obtaining a 3 **or** a 5 on ONE roll of a single die? (2 pts)

$\frac{1+1}{6} = \frac{2}{6} = .333\ldots = .33$

6) What is the probability of obtaining an odd number on one roll of a die? (2 pts)

$\frac{3}{6} = .50$

Problems 7-13

Dice continued. For these problems, you must draw a dice table, and answer questions relating to two dice. (Please give your answers as decimals rounded to TWO decimal places.)

7) Draw Dice Table (4 pts)

Dice	**1**	**2**	**3**	**4**	**5**	**6**
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

Now that you have drawn the dice table, find the probability. These problems use some abbreviations: P(4, 2 dice) means what is the probability of getting a 4 when you roll two dice. (Please give your answers as decimals rounded to TWO decimal places.)

If you roll a **PAIR** of dice, what is the probability that the total will be a:

8) P(7, 2 dice) (2 pts)

$\frac{6}{36}$ = .16666… = .17

9) P(3, 2 dice) (2 pts)

$\frac{2}{36}$ = .05555… = .06

10) P(10, 2 dice) (2 pts)

$\frac{3}{36}$ = .083333…. =.08

11) P(5, 2 dice) (2 pts)

$\frac{4}{36}$ = .1111… =.11

12) P(<6, 2 dice) (2 pts)

$\frac{10}{36}$ = .27777… = .28

13) P(>5, 2 dice) (2 pts)

$\frac{26}{36}$ = .72222… = .72

Problem 14 –18

Coins. Draw Pascal's Triangle and calculate the probability of the following coin flip problems. (Please give your answers as decimals rounded to TWO decimal places.)

14) Draw Pascal's Triangle (4 pts)

			1 1			
			1 2 1			
			1 3 3 1			
			1 4 6 4 1			
5 coin row	1	5	10	10	5	1
	5h	**4h**	**3h**	**2h**	**1h**	**0h**
	0t	**1t**	**2t**	**3t**	**4t**	**5t**

If you flip five coins what is the probability that you will get: (Please give your answers as decimals rounded to TWO decimal places.)

15) Two heads? (That means exactly 2 heads.) (2 pts)

$\frac{10}{32} = .3125 = .31$

16) Three tails? (2 pts)

$\frac{10}{32} = .3125 = .31$

17) At least three heads? (2 pts)

$\frac{10+5+1}{32} = \frac{16}{32} = .50$

18) Fewer than two tails? (2 pts)

$\frac{5+1}{32} = \frac{6}{32} = .1875 = .19$

Problem 19

In class we looked at the probability of winning a lottery such as Megabucks. To play, one has to pick six numbers between **1 and 48**, with no repeated numbers. The chance of your number being the winning number is

$$\frac{\mathbf{1}}{\mathbf{12271512}} \text{ or } \mathbf{0.0000000815.}$$

When the lottery first came out in Massachusetts, you only had to pick six different numbers between **1 and 36**. Do the following:

19) If you bought one ticket to this type of lottery, calculate your chance of having the winning number under these older conditions i.e. pick six numbers between 1 and 36. Please give your answer both as a fraction and as a decimal – this time don't round. Just write down all the numbers you see in the screen of your calculator, as above. (If you are using a computer, please don't write down more than 15 digits.) (4 pts)

- Calculate total possible combinations of numbers 1 to 36:

  36 * 35 *34 *33* 32 *31 = 1,402,410,240 total number of combinations

  The 1,402, 410, 240 possible combinations include many of the same set of numbers just arranged in different orders.

- Now, calculate the number of ways each set of 6 digits could be arranged within these combinations:

  6 *5 *4* 3* 2*1 =720

- Divide total possible combinations by number of times each number can be represented in different orders.

  $$\frac{1{,}402{,}410{,}240}{720} = 1{,}947{,}792$$

- Now calculate probability:

  $$\frac{\text{1 ticket}}{1{,}947{,}792} = \mathbf{.000000513}$$

Problem 19 (cont)

Interesting thought questions but no need to answer:
Why do you think the rules were changed?
How do you think the gambling public perceived this change?

The rules were changed to decrease the probability of someone winning. The probability of winning with six digits from 1 to 36, while still low, are much better than the probability of winning with combinations of six digits from 1 to 48. Unless they actually calculated the new probability of winning, the public may have perceived the change as something good because there are more numbers from which to choose.

Problems 20-25

Calculate the following-you may use the factorial key on your calculator. Some of these are a bit tricky – you may need to do some simplifying *before* using your calculator.

20) 7! = 5040 (1 pt)

7*6*5*4*3*2*1 = 5040 (or, use your factorial key on the calculator)

21) 15! $=1.307674368 * 10^{12}$ or 1,307,674,368,000 (1 pt)

22) $\frac{15!}{6!}$ =1,816,214,400 (1 pt)

Note: You can't use canceling here as you would if ! was a variable, factoring out a common factor of 3 from 15 and 6. 15! represents 15*14*13*12*11*10*9*8*7*6*5*4*3*2*1, and 6! represents 6*5*4*3*2*1. What could be cancelled is the 6!(6*5*4*3*2*1) from each. You would be left with 15*14*13*12*11*10*9*8*7 =1,816,214,400.

23) $\frac{25!}{(25-10)!} = \frac{25!}{15!} = 1.186167629*10^{13}$ or 11,861,676,290,000 (1 pt)

- You can't use canceling here as you would if ! was a variable, factoring out a common factor of 5 from 25 and 15.

- You could recognize that 25!=25*24*23*22*21*20*19*18*17*16*15!, and the 15! of the numerator would cancel with the 15! in the denominator.
- You would be left with 25*24*23*22*21*20*19*18*17*16=1.186167629*10^{13}.

24) $\frac{48!}{6!\times 42!}$ = 12, 271,512 (1 pt)

- If you put 48! into your calculator it will give you a very large number in scientific notation: 1.241391559 * 10^{61}. If you put (6! *42!) into your calculator you will get another very large number in scientific notation: 1.011604405 *10^{54}. Dividing the first by the second will give you 12,271,512.

- You can also do this using the rules of exponents to solve this as follows: divide 1.1241391559 by 1.011604405 which gives you 1.227151199, and divide 10^{61} by 10^{54}= $10^{(61-54)}$ = 10^{7}. The answer would then be 1.227151199 * 10^{7} or 12,271, 511.99 which rounds up to 12,271,512.

- A third way to work on this problem is to recognize that you can rewrite the numerator as 48 * 47 * 46 * 45 * 44* 43 * 42!. The 42! in the denominator would cancel out the 42! in the numerator and you would be left with (48*47*46*45*44*43) ÷ 6! =8,835,488,640/720 =12,271,512.

25) 0! = 1 (1 pt)