Example 3

$$x^3 + xy^2 = \cos(y)$$

Get $\frac{dy}{dx}$

$$\frac{d}{dx}[x^3 + xy^2] = \boxed{\frac{d}{dx}[\cos(y)]}$$

$$\frac{d}{dx}[x^3] + \frac{d}{dx}[xy^2] = -\sin(y)\frac{dy}{dx}$$

$f(x) = x$, $g(x) = y^2$

$$3x^{3-1} + x \cdot \frac{d}{dx}[y^2] + y^2 \cdot \frac{d}{dx}[x] =$$

$$3x^2 + x \cdot 2y \cdot \frac{dy}{dx} + y^2 \cdot \frac{dx}{dx} = -\sin(y)\frac{dy}{dx}$$

$$3x^2 + 2xy \cdot \frac{dy}{dx} + y^2 \cdot 1 = -\sin(y)\frac{dy}{dx}$$

$$3x^2 + 2xy\frac{dy}{dx} + y^2 = -\sin(y)\frac{dy}{dx}$$

$f(u) = \cos(u)$ $\quad u = g(x) = y$

Outside Function: $\cos(u)$ $\quad \frac{d}{dx}[\cos(u)] = -\sin(u)$

Inside Function: y $\quad \frac{d}{dx}[y] = \frac{dy}{dx}$

$$= -\sin(u)\frac{dy}{dx}$$

$$= \boxed{-\sin(y)\frac{dy}{dx}, \text{ where } y = u}$$

$$\begin{array}{rcl} -2xy\frac{dy}{dx} & = & -2xy\frac{dy}{dx} \\ \hline 3x^2 + y^2 & = & -\sin(y)\frac{dy}{dx} - 2xy\frac{dy}{dx} \end{array}$$

$$3x^2 + y^2 = \frac{dy}{dx}[-\sin(y) - 2xy]$$

$$\frac{3x^2 + y^2}{-1[\sin(y) + 2xy]} = \frac{\frac{dy}{dx} \cdot -1[\sin(y) + 2xy]}{-1[\sin(y) + 2xy]}$$

$$-\frac{3x^2 + y^2}{\sin(y) + 2xy} = \frac{dy}{dx}$$

or

$$\boxed{\frac{dy}{dx} = -\frac{3x^2 + y^2}{\sin(y) + 2xy}}$$

Example 4

$\frac{\sin(y)}{\cos(x)} = y$ Get $\frac{dy}{dx}$

$\frac{d}{dx}\left[\frac{\sin(y)}{\cos(x)}\right] = \frac{d}{dx}[y]$

$f(x) = \sin(y)$
$g(x) = \cos(x)$

$\parallel$

$\frac{\cos(x)\frac{d}{dx}[\sin(y)] - \sin(y)\frac{d}{dx}[\cos(x)]}{\cos^2(x)} = \frac{dy}{dx}$

Chain It

$f(u) = \sin(u)$ $u = g(x) = y$

Outside: $\sin(u)$, $\frac{d}{dx}[\sin(u)] = \cos(u)$

Inside: y, $\frac{d}{dx}[y] = \frac{dy}{dx}$

$\parallel$

$\cos(y)\frac{dy}{dx}$, where $y = u$

$\cancel{\cos^2(x)} \cdot \frac{\cos(x)\cos(y)\frac{dy}{dx} - \sin(y)\cdot(-1)\cdot\sin(x)}{\cancel{\cos^2(x)}} = \frac{dy}{dx} \cdot \cos^2(x)$

$\parallel$

$\cos(x)\cos(y)\frac{dy}{dx} - \sin(y)\cdot(-1)\cdot\sin(x) = \frac{dy}{dx}\cos^2(x)$

$\parallel$

$\cos(x)\cos(y)\frac{dy}{dx} + \sin(y)\sin(x) = \frac{dy}{dx}\cos^2(x)$

$-\cos(x)\cos(y)\frac{dy}{dx}$ $\qquad -\cos(x)\cos(y)\frac{dy}{dx}$

$\sin(y)\sin(x) = \frac{dy}{dx}\cos^2(x) - \cos(x)\cos(y)\frac{dy}{dx} = 0$

$\parallel$

$\frac{\sin(y)\sin(x)}{\cos^2(x) - \cos(x)\cos(y)} = \frac{\frac{dy}{dx}[\cos^2(x) - \cos(x)\cos(y)]}{[\cos^2(x) - \cos(x)\cos(y)]}$

$\parallel$

$$\boxed{\frac{\sin(y)\sin(x)}{\cos^2(x) - \cos(x)\cos(y)} = \frac{dy}{dx} \quad \text{or} \quad \frac{dy}{dx} = \frac{\sin(y)\sin(x)}{\cos^2(x) - \cos(x)\cos(y)}}$$

Chain It

$f(u) = u^2$, $u = g(x) = y$

Outside u^2 $\qquad \frac{d}{dx}[u^2] = 2u$

Inside y $\qquad \frac{d}{dx}[y] = \frac{dy}{dx}$

$\qquad\qquad\qquad =$

$2y\frac{dy}{dx}$, where $y = u$

Solve $\frac{d}{dx}[x^2y^2]$

$f(x) = x^2$

$g(x) = y^2$

$x^2 \cdot \frac{d}{dx}[y^2] + y^2 \cdot \frac{d}{dx}[x^2]$

$=$

$x^2 \cdot 2y \cdot \frac{dy}{dx} + y^2 \cdot 2x^{2-1}$

$=$

$x^2 \cdot 2y\frac{dy}{dx} + 2xy^2$

$=$

$$\boxed{\begin{gathered} 2x^2y\frac{dy}{dx} + 2xy^2 \\ \frac{d}{dx}[x^2y^2] = 2x^2y\frac{dy}{dx} + 2xy^2 \end{gathered}}$$

Example 5

$xy = \cos(x)$ Get dx/dy

$\frac{d}{dy}[xy] = \frac{d}{dy}[\cos(x)]$

$f(x)=x$

$g(x)=y$

$\Downarrow$ Product Rule

$x \cdot \frac{d}{dy}[y] + y \cdot \frac{d}{dy}[x] = -\sin(x)\frac{dx}{dy}$

Chain It

$f(u) = \cos(u)$ $u = g(x) = x$

Outside: $\cos(u)$ $\frac{d}{dy}[\cos(u)] = -\sin(u)$

Inside: x $\frac{d}{dy}[x] = \frac{dx}{dy} =$

$-\sin(x)\frac{dx}{dy}$, where $x = u$

$$x \cdot \frac{dy}{dy} + y \cdot \frac{dx}{dy} = -\sin(x)\frac{dx}{dy}$$

$$x \cdot 1 + y\frac{dx}{dy} = -\sin(x)\frac{dx}{dy}$$

$$x + y\frac{dx}{dy} = -\sin(x)\frac{dx}{dy}$$

$$+\sin(x)\frac{dx}{dy} \quad +\sin(x)\frac{dx}{dy}$$

$$x + y\frac{dx}{dy} + \sin(x)\frac{dx}{dy} = 0$$

$$-x \quad -x$$

$$y\frac{dx}{dy} + \sin(x)\frac{dx}{dy} = -x$$

$$\frac{\frac{dx}{dy}[y + \sin(x)]}{[y+\sin(x)]} = \frac{-x}{[y+\sin(x)]}$$

$$\boxed{\frac{dx}{dy} = \frac{-x}{[y+\sin(x)]}}$$

Chain It, 3 chains

$$x^3 + xy^2 = \cos(y) \qquad \text{Get } dx/dy$$

$$\frac{d}{dy}\left[x^3 + xy^2\right] = \boxed{\frac{d}{dy}\left[\cos(y)\right]}$$

$$=$$

$$\frac{d}{dy}\left[x^3\right] + \frac{d}{dy}\left[xy^2\right] = \boxed{-\sin(y)}$$

$f(x) = x$, $g(x) = y^2$

$$=$$

$$\boxed{3x^2\frac{dx}{dy}} + x\cdot\boxed{\frac{d}{dy}\left[y^2\right]} + y^2\cdot\frac{d}{dy}\left[x\right] = -\sin(y)$$

$$\boxed{3x^2\frac{dx}{dy}} + x\cdot\boxed{2y} + y^2\frac{dx}{dy} = -\sin(y)$$

$$3x^2\frac{dx}{dy} + 2xy + y^2\frac{dx}{dy} = -\sin(y)$$

$$-2xy \qquad\qquad -2xy$$

$$3x^2\frac{dx}{dy} + y^2\frac{dx}{dy} = -\sin(y) - 2xy$$

$$=$$

$$\frac{\frac{dx}{dy}\cancel{\left[3x^2+y^2\right]}}{\cancel{\left[3x^2+y^2\right]}} = \frac{-\sin(y) - 2xy}{\left[3x^2+y^2\right]}$$

$$=$$

$$\boxed{\frac{dx}{dy} = -\frac{\sin(y) - 2xy}{\left[3x^2+y^2\right]}}$$

$f(u) = \cos(u)$, $u = g(x) = y$

$\cos(u)$ $\quad \frac{d}{dy}\cos(u) = -\sin(u)$

y $\quad \frac{d}{dy}[y] = \frac{dy}{dy} = 1$

$-\sin(y)$, where $y = u$

$f(u) = u^2$, $u = g(x) = y$

u^2 $\quad \frac{d}{dy}[u^2] = 2u$

y $\quad \frac{d}{dy}[y] = \frac{dy}{dy} = 1$

$=$

$2y$, where $y = u$

$f(u) = u^3$, $u = g(x) = x$

u^3, $\quad \frac{d}{dy}[u^3] = 3u^2$

x, $\quad \frac{d}{dy}[x] = \frac{dx}{dy}$

$=$

$3x^2\frac{dx}{dy}$, where $x = u$