Domain and Range of a Composition of Functions

① Find Domain of the input/ inside function.

② Find Domain of the new function after performing the composition.

$f(x) = \frac{1}{x}$

$g(x) = \frac{1}{(x+2)(x-3)}$

Find Domain of $g(f(x))$ or $(g \circ f)(x)$

$g[f(x)]$

① Domain of $f(x) = \frac{1}{x}$

Domain have to be $x \neq 0$

② $g(\frac{1}{x}) = \frac{1}{((\frac{1}{x})+2)((\frac{1}{x})-3)}$

Domain have to be $x \neq 0$

③ Solve for x

$$\frac{1}{x} + 2 = 0$$
$$-2 \quad -2$$
$$\frac{1}{x} = -\frac{2}{1}$$
$$-\cancel{2}x = 1$$
$$\frac{\cancel{-2}x}{\cancel{-2}} = \frac{1}{-2}$$
$$x = -\frac{1}{2}$$

$$\frac{1}{x} - 3 = 0$$
$$+3 \quad +3$$
$$\frac{1}{x} = \frac{3}{1}$$
$$\frac{\cancel{3}x}{\cancel{3}} = \frac{1}{3}$$
$$x = \frac{1}{3}$$

$$\boxed{\text{Domain: } (-\infty, -1/2) \cup (-1/2, 0) \cup (0, 1/3) \cup (1/3, \infty)}$$

$\cup$: or

$\cap$: And

Get Range for $g(f(x))$ or $(g \circ f)(x)$

$f(x) = \frac{1}{x}$

$g(x) = \frac{1}{(x+2)(x-3)}$

① Range of $f(x) = \frac{1}{x}$

$$\frac{y}{1} = \frac{1}{x}$$

$$\frac{\cancel{y}x}{y} = \frac{1}{\cancel{y}}$$

$$x = \frac{1}{y}$$

$$y \neq 0$$

② Range of $g(f(x))$

$$g\left(\frac{1}{x}\right) = \frac{1}{\left(\left(\frac{1}{x}\right)+2\right)\left(\left(\frac{1}{x}\right)-3\right)}$$

$$y = \frac{1}{\left(\frac{1}{x}+2\right)\left(\frac{1}{x}-3\right)}$$

$$\frac{1}{x^2} - \frac{3}{x} + \frac{2}{x} - 6$$

$$\frac{1}{x^2} - \frac{1}{x} - 6$$

$$(y)^2 = \left(\sqrt{4-x^2}\right)^2 \qquad f(x) = \sqrt{4-x^2} \qquad g(x) = x^2$$

D: $(-\infty, \infty)$, R: $[0, \infty)$

$\sqrt{y} = \sqrt{x^2}$

$\sqrt{y} = x$

$y^2 = 4 - x$

$-4 \quad -4$

$\frac{y^2 - 4}{-1} = \frac{-x}{-1}$

$\sqrt{\frac{y^2-4}{-1}} = \sqrt{x}$

$\frac{\sqrt{y^2-4}}{\sqrt{-1}} = \sqrt{x}$

$y^2 - 4 \geq 0$

$+4 \quad +4$

$\sqrt{y^2} \geq \sqrt{4}$

$y \geq 2$

Getting The Range

$f(g(x))$

$f(x^2) = \sqrt{4-(x^2)^2}$

$y = \sqrt{4 - x^4}$

① Get Domain

$4 - x^4 = 0$

$-4 \qquad -4$

$\frac{-x^4}{-1} = \frac{-4}{-1}$

$\sqrt[4]{x^4} = \sqrt[4]{4}$

$x = \pm\sqrt{2} \approx 1.41421$

Domain: $(-\sqrt{2}, \sqrt{2})$

② Get Range

$y = \sqrt{4-(x^2)^2}$

$y = \sqrt{4-x^4}$

I'm going to input 0 into x which is the lowest valid x value.

$y = \sqrt{4-(0)^4}$

$\sqrt{4}$

2 — 2 is going to be my max range value

Range: $[0, 2]$

Overall it seems like my range is dependent on the restriction.