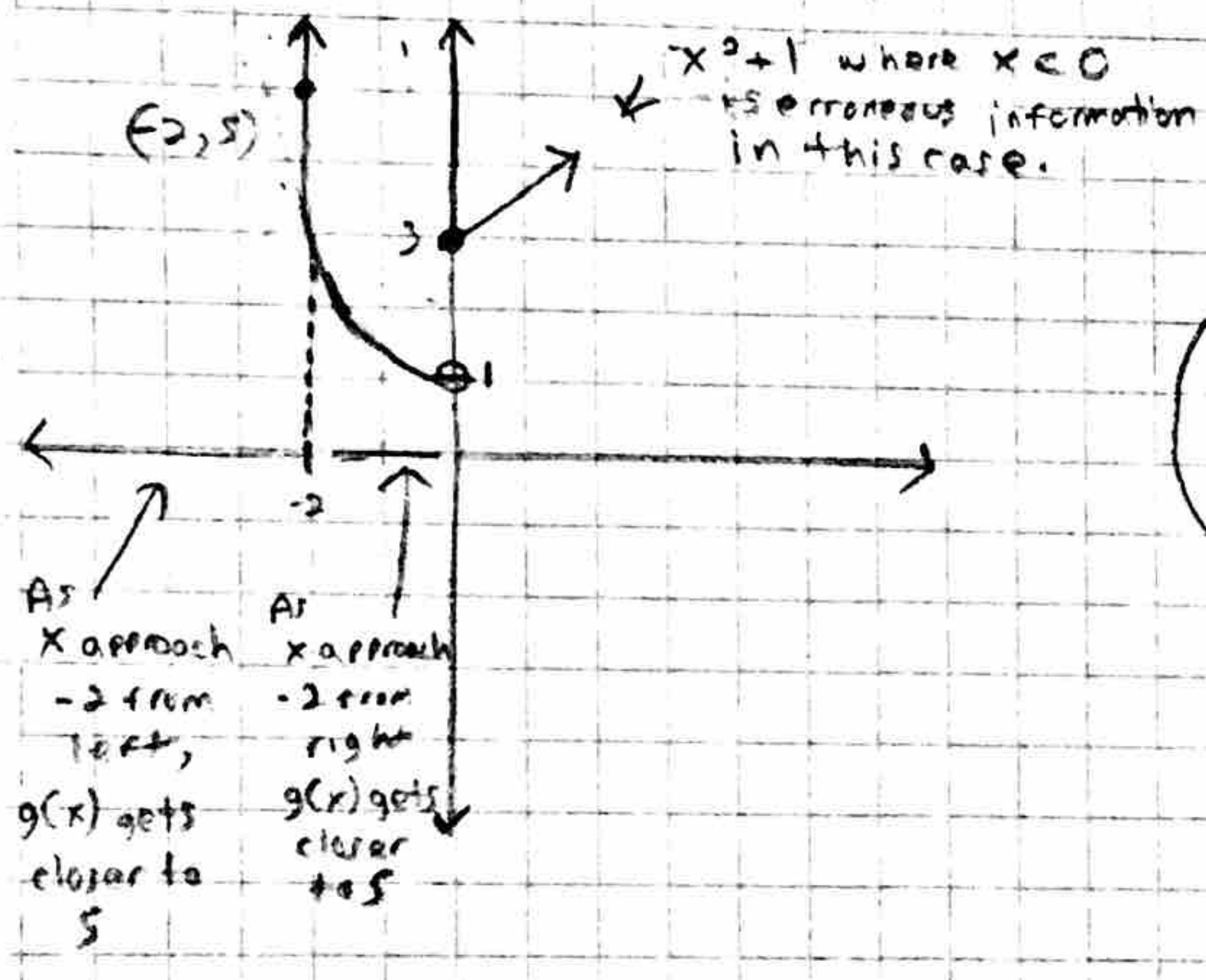
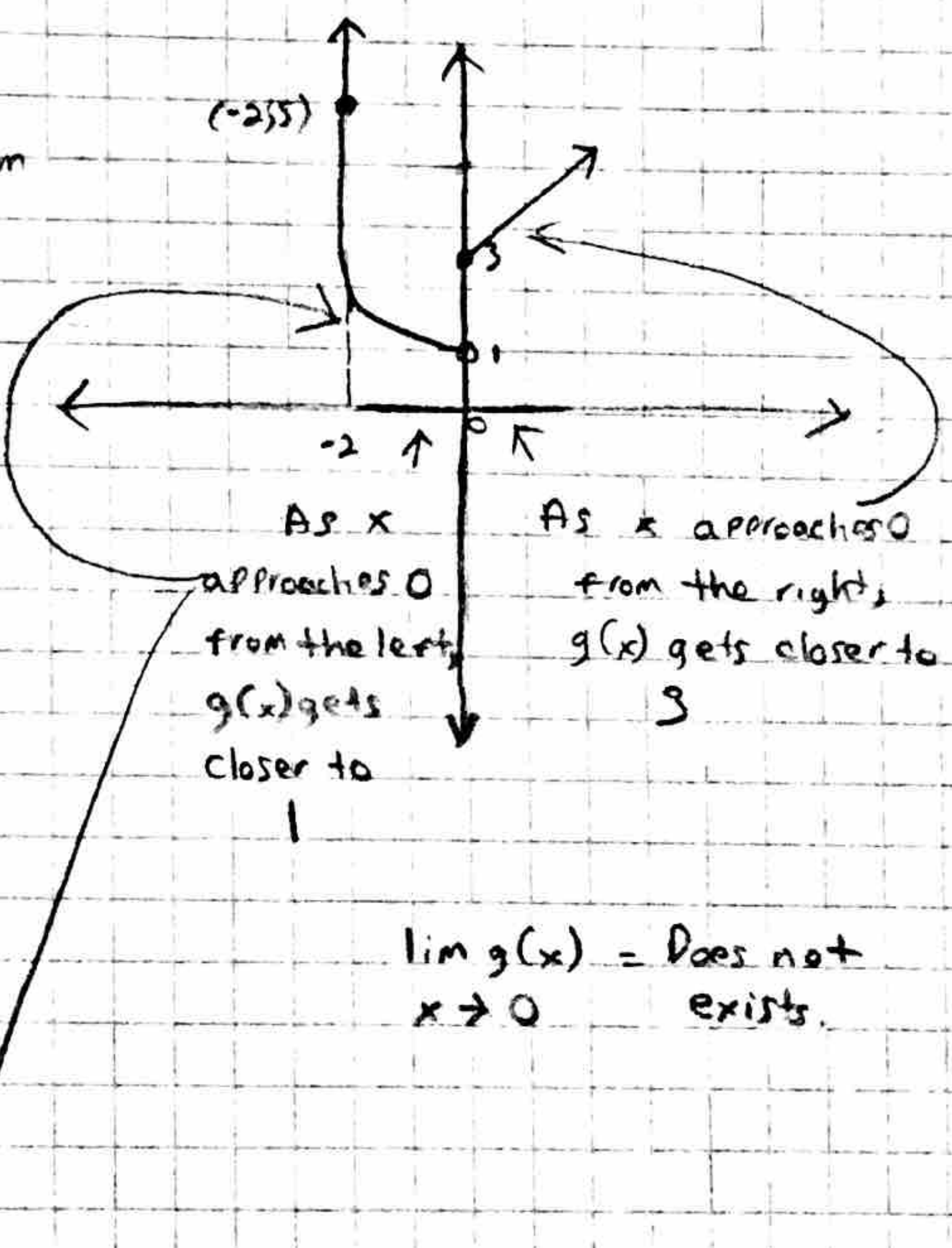
Finding a Limit of a Piecewise Function by Graphing

Let $g(x) = \begin{cases} x^2+1; & x<0 \\ 2x+3; & x \geq 0 \end{cases}$

What is the limit of $g(x)$ as x approaches -2?
What is the limit of $g(x)$ as x approaches 0?





$(-2)^2+1 = 5$ $\qquad \lim_{x \to -2} g(x) = 5$

$\lim_{x \to 0} g(x) =$ Does not exists.

This plays out the Theorem:

$\lim_{x \to a} f(x) = L$ if and only if $\lim_{x \to a^+} f(x) = L$ and $\lim_{x \to a^-} f(x) = L$.

If the limit from the left and right are not equal, than we say $\lim_{x \to a} f(x)$ does not exist.

Consider the function defined by: $f(x) \begin{cases} -x+2 & \text{if } x<-1 \\ 2x+1 & \text{if } -1 \le x \le 1 \\ 2x^2 & \text{if } x>1 \end{cases}$

Construct a table to guess the values of $\lim_{x\to 1^-} f(x)$, $\lim_{x\to 1^+} f(x)$, $\lim_{x\to 1} f(x)$

Using $2x+1$; $-1 \le x \le 1$

$x \to 1^-$

x	$2x+1$
0.9	2.8
0.99	2.98
0.999	2.998
0.9999	2.9998

Im going to be too far out to the left if I use $-x+2$, $x<1$. This seem to be to much extra information at this case.

Using $2x^2$; $x>1$

$x \to 1^+$

x	$2x^2$
1.1	2.42
1.01	2.0402
1.001	2.004
1.0001	2.0004

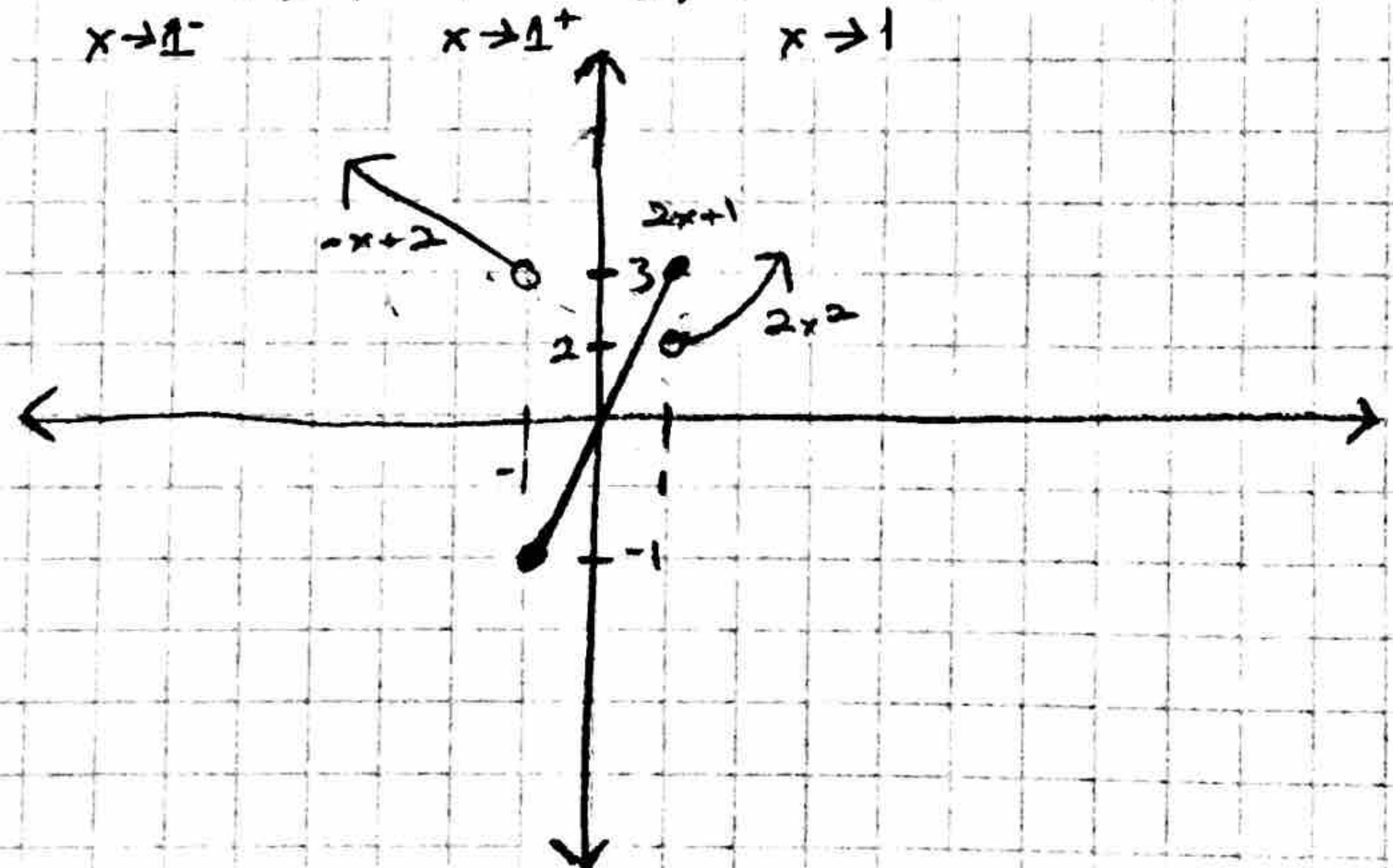
As x approaches 1 from the left, f(x) gets closer to 3

$\lim_{x\to 1^-} f(x) = 3$

As x approaches 1 from the right, f(x) gets closer to 2

$\lim_{x\to 1^+} f(x) = 2$

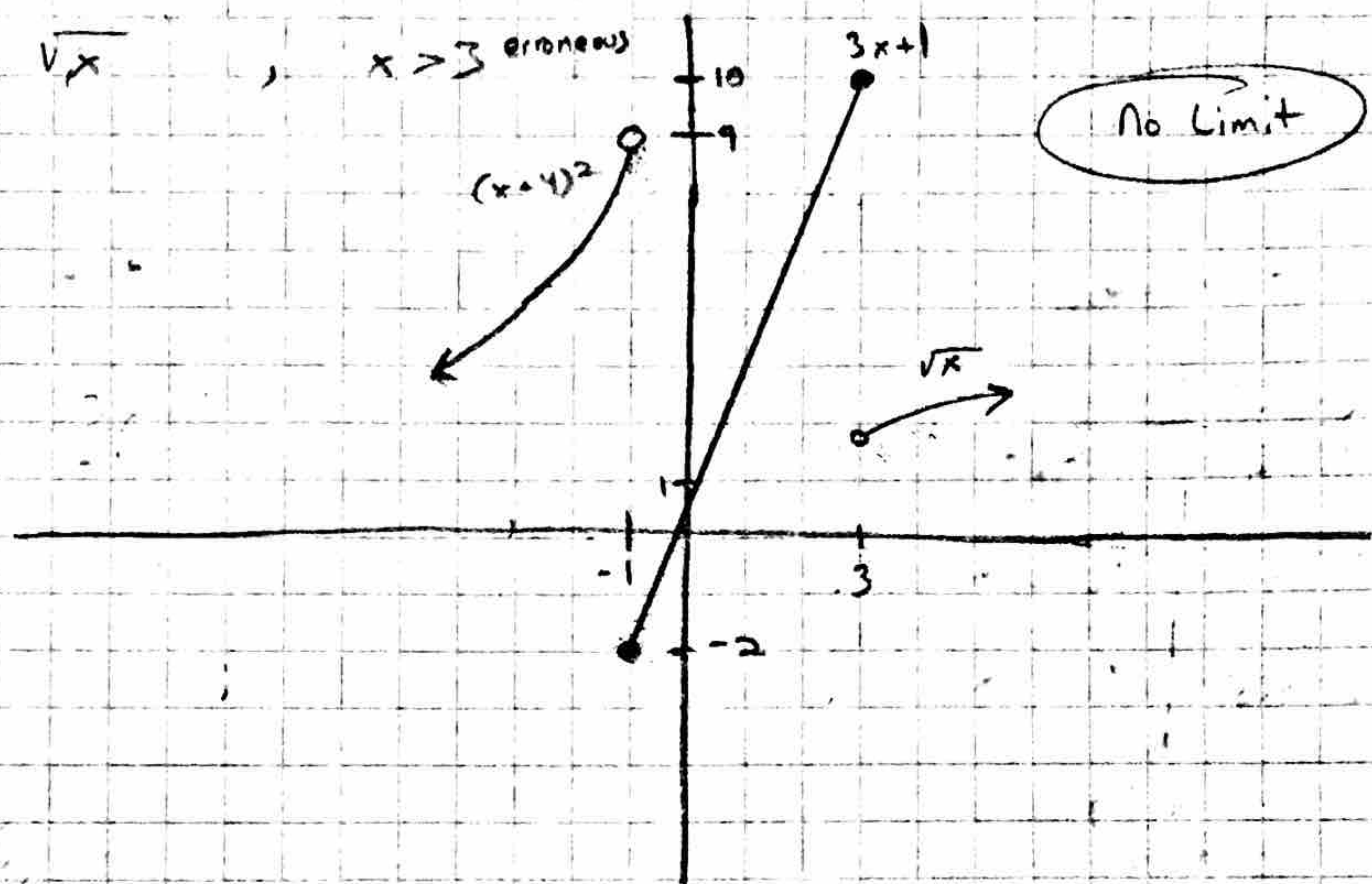
Since $\lim_{x\to 1^-} f(x) \ne \lim_{x\to 1^+} f(x)$, $\lim_{x\to 1} f(x)$ does not exist.



②

$$f(x) = \begin{cases} (x+4)^2 & , x < -1 \qquad x \to -1^- \\ 3x+1 & , -1 \le x \le 3 \qquad x \to +1^+ \\ \sqrt{x} & , x > 3 \text{ erroneous} \end{cases}$$

No Limit



③

$$f(x) = \begin{cases} x^2+4 & , x \le -2 \\ 2x+3 & , -2 \le x \le 3 \\ \sqrt{x+1}+7 & , x > 3 \end{cases}$$

erron

$$\lim_{x \to 3} f(x) = 9$$