

$$1. \exists y \in \mathbb{Z}, \ni \frac{y}{2} \in \mathbb{Z}$$

There exists a y element of the integers, such that $y/2$ is an integer.

Let $y = 2$, then $2/2 = 1$ is an element of the integers is a true statement.

Note: $y = 2$ is a witness.

Therefore, this statement is true.

$$\text{Negation: } \forall y \in \mathbb{Z}, \frac{y}{2} \notin \mathbb{Z}$$

For all y elements of the integers, $y/2$ is not an element of the integers.

Let $y = 2$, then $2/2$ is not an element of the integers is a false statement.

Note: $y = 2$ is a counterexample.

Therefore, this statement is false.

$$3. \forall y \in \mathbb{N}, \frac{3}{y} \notin \mathbb{N}$$

For all y elements of the natural numbers, $3/y$ is not a natural number.

Let $y = 3$, then $3/3$ is not an element of the natural numbers is a false statement.

Note: $y = 3$ is a counterexample.

Therefore, this statement is false.

$$\text{Negation: } \exists y \in \mathbb{N}, \ni \frac{3}{y} \in \mathbb{N}$$

There exists a y element of the natural numbers, such that $3/y$ is a natural number.

Let $y = 3$, then $3/3 = 1$ is an element of the natural numbers is a true statement.

Note: $y = 2$ is a witness.

Therefore, this statement is true.

5. $\exists x \in \mathbb{N}, x^2 \geq x$

There exists an element of the natural numbers, such that x^2 is greater than or equal to x .

Let $x = 2$, then $2^2 \geq 2$ is a true statement.

Note: $x = 2$ is a witness.

Therefore, this statement is true.

Negation: $\forall x \in \mathbb{N}, x^2 < x$

For all x elements of the natural numbers, $x^2 < x$.

Let $x = 0$, then $0^2 < 0$ is a false statement.

Note: $x = 0$ is a counterexample.

Therefore, this statement is false.

7. $\forall x \in \mathbb{R}, x^2 \geq x$

For all x elements of the real numbers, $x^2 < x$.

Let $x = 1/2$, then $(1/2)^2 \geq (1/2)$ is a false statement.

Note: $x = 1/2$ is a counterexample.

Therefore, this statement is false.

Negation: $\exists x \in \mathbb{R}, x^2 < x$

There exists an x element of the real numbers, such that $x^2 < x$.

Let $x = 1/2$, then $(1/2)^2 < (1/2)$ is a true statement.

Note: $x = 1/2$ is a witness.

Therefore, this statement is true.

9. $\forall x \in W, x^2 \geq 0$

For all x in the whole numbers, $x^2 \geq 0$.

This is a true for all statement, which is one of the two harder cases to justify.

$x = 0$ is the first element of the whole numbers, and $0^2 \geq 0$ is true.

$x = 1$ is the second element of the whole numbers, and $1^2 \geq 0$ is true.

$x = 2$ is the first element of the whole numbers, and $2^2 \geq 0$ is true.

... for all whole numbers

Therefore, this statement is true.

Negation: $\exists x \in W, x^2 < 0$

There exists an x element of the whole numbers, such that $x^2 < 0$.

This "there exists" statement is false, and that makes this one of the two harder cases to justify. You may think we just need to say something like let $x = 1$. But what have we really done? Absolutely nothing! The statement says "there exists and x in W , ..", and if this is false we have to verify NO X IN W WORKS! We have to make it clear to our reader that it is false "for all" x in whole numbers.

$x = 0$ is the first element of the whole numbers, and $(0)^2 < 0$ is false.

$x = 1$ is the second element of the whole numbers, and $(1)^2 < 0$ is false.

$x = 2$ is the third element of the whole numbers, and $(2)^2 < 0$ is false.

... for all elements of the whole numbers

Thus, no x in the whole numbers is true. Thus, the "there exists" statement is false.

Therefore, this statement is false.

11. $\forall x \in \mathbb{N}, x + 1 > x$

For all x in the natural numbers, $x + 1 > x$.

This is a true for all statement, which is one of the two harder cases to justify.

$x = 1$ is the first element of the natural numbers, and $1 + 1 > 1$ is true.

$x = 2$ is the second element of the natural numbers, and $2 + 1 > 2$ is true.

$x = 3$ is the first element of the natural numbers, and $3 + 1 > 3$ is true.

... for all natural numbers

Therefore, this statement is true.

Negation: $\exists x \in \mathbb{N}, x + 1 \leq x$

There exists an x element of the whole numbers, such that $x + 1 \leq x$.

This "there exists" statement is false, and that makes this one of the two harder cases to justify. You may think we just need to say something like let $x = 1$. But what have we really done? Absolutely nothing! The statement says "there exists an x in \mathbb{N} , ..", and if this is false we have to verify NO X IN \mathbb{N} WORKS! We have to make it clear to our reader that it is false "for all" x in natural numbers.

$x = 1$ is the first element of the natural numbers, and $1 + 1 \leq 1$ is false.

$x = 2$ is the second element of the natural numbers, and $2 + 1 \leq 2$ is false.

$x = 3$ is the third element of the natural numbers, and $3 + 1 \leq 3$ is false.

... for all elements of the natural numbers

Thus, no x in the natural numbers is true. Thus, the "there exists" statement is false.

Therefore, this statement is false.

13. $\exists x \in \mathbb{R}, x + 1 \geq 0$

There exists an a element of the real numbers, such that $x + 1$ is greater than or equal to 0.

Let $x = 2$, then $2 + 1 \geq 0$ is a true statement.

Note: $x = 2$ is a witness.

Therefore, this statement is true.

Negation: $\forall x \in \mathbb{R}, x + 1 < 0$

For all x elements of the real numbers, $x + 1 < 0$.

Let $x = 0$, then $0 + 1 < 0$ is a false statement.

Note: $x = 0$ is a counterexample.

Therefore, this statement is false.

15. $\forall x, y \in \mathbb{N}, x < y$

For all x and y elements of the natural numbers, x is less than y .

Let $x = 3$, and $y = 1$, then $3 < 1$ is a false statement.

Note: $x = 3, y = 1$ is a counterexample.

Therefore, this statement is false.

Negation: $\exists x, y \in \mathbb{N}, x \geq y$

There exists an x and y element of the natural numbers, such x is greater or equal to y .

Let $x = 3$, and $y = 1$, then $3 \geq 1$ is a true statement.

Note: $x = 3, y = 1$ is a witness.

Therefore, this statement is true.