Math 140 Midterm Exam
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INSTRUCTIONS

- The exam is worth 100 points. There are 20 problems (each worth 5 points).
- This exam is open book and open notes with unlimited time. This means you may refer to your text book, notes, and online classroom materials. You may take as much time as you wish provided you submit your exam no later than the due date posted in our course syllabus. **To be fair to others, late exams will not be accepted.**
- **You must show your work to receive full credit. If you do not show your work, you may earn only partial or no credit at the discretion of the instructor.**
- Emailed exams cannot be accepted as they crash my system (thank you for your cooperation and understanding on this one!)
- **You must put your answers on this page.**
- If you have any questions, please feel free to send me a message.
  Best wishes! ☺

(1)	$f(x)=0$ is both even and odd	(11)	b
(2)	d	(12)	a
(3)	b	(13)	a
(4)	b	(14)	a
(5)	e	(15)	a
(6)	d	(16)	a
(7)	e	(17)	b
(8)	d	(18)	b
(9)	d	(19)	c
(10)	a	(20)	b

Shaun C. Lewis

Midterm

1. Validate $f(x) = 0$ for odd

$f(-x) = 0$

$\boxed{f(x) = f(-x)}$

Validate $f(x) = 0$ for even

$-f(x) = -(0)$

$-f(x) = 0$

$\boxed{f(x) = -f(x)}$

Show if $f(x) = 0$ is both even and odd.

Proof: Since $f(x) = 0$ is both even and odd is true.

$f(-x) = 0$

$f(x) = f(-x)$, Definition of Odd Function

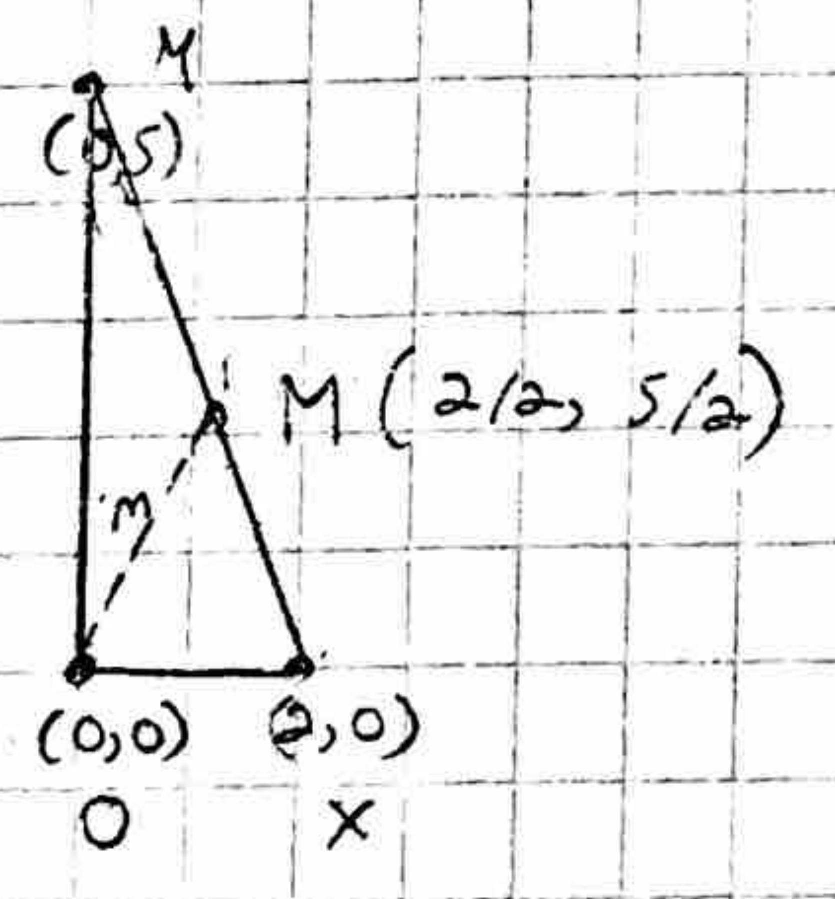
$-f(x) = -(0)$

$-f(x) = 0$

$f(x) = -f(x)$, Definition of Even Function

$\therefore$ By Truth Table if $f(x) = 0$, $f(x)$ is both even and odd.

2.



① Get midpoint

$$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

$$M = \left(\frac{0+2}{2}, \frac{5+0}{2}\right)$$

$$M = \left(\frac{2}{2}, \frac{5}{2}\right)$$

② Get Slope $\overline{OM}$

$$m = \frac{(5/2) - 0}{(2/2) - 0}$$

$$\|$$

$$m = \frac{\frac{5}{2}}{\frac{2}{2}} = \frac{5}{2} \cdot \frac{2}{2} = \frac{10}{4} = \frac{5}{2}$$

$m = 5/2$ D.

3. $f(x) = ax - b$ $g(x) = cx - d$

$(f \circ g)(x)$ $(g \circ f)(x)$

$a(\underbrace{cx-d) - b}_{\substack{-b \quad -d \\ f(b) = g(d)}}$ $c(\underbrace{ax - b) - d}_{f(b) = g(d)}$

(b)

4. $x + 8y = -9$

$-x \qquad\qquad -x$

$\frac{8y}{8} = \frac{-x}{8} - \frac{9}{8}$

$y = -\frac{1}{8}x - \frac{9}{8}$

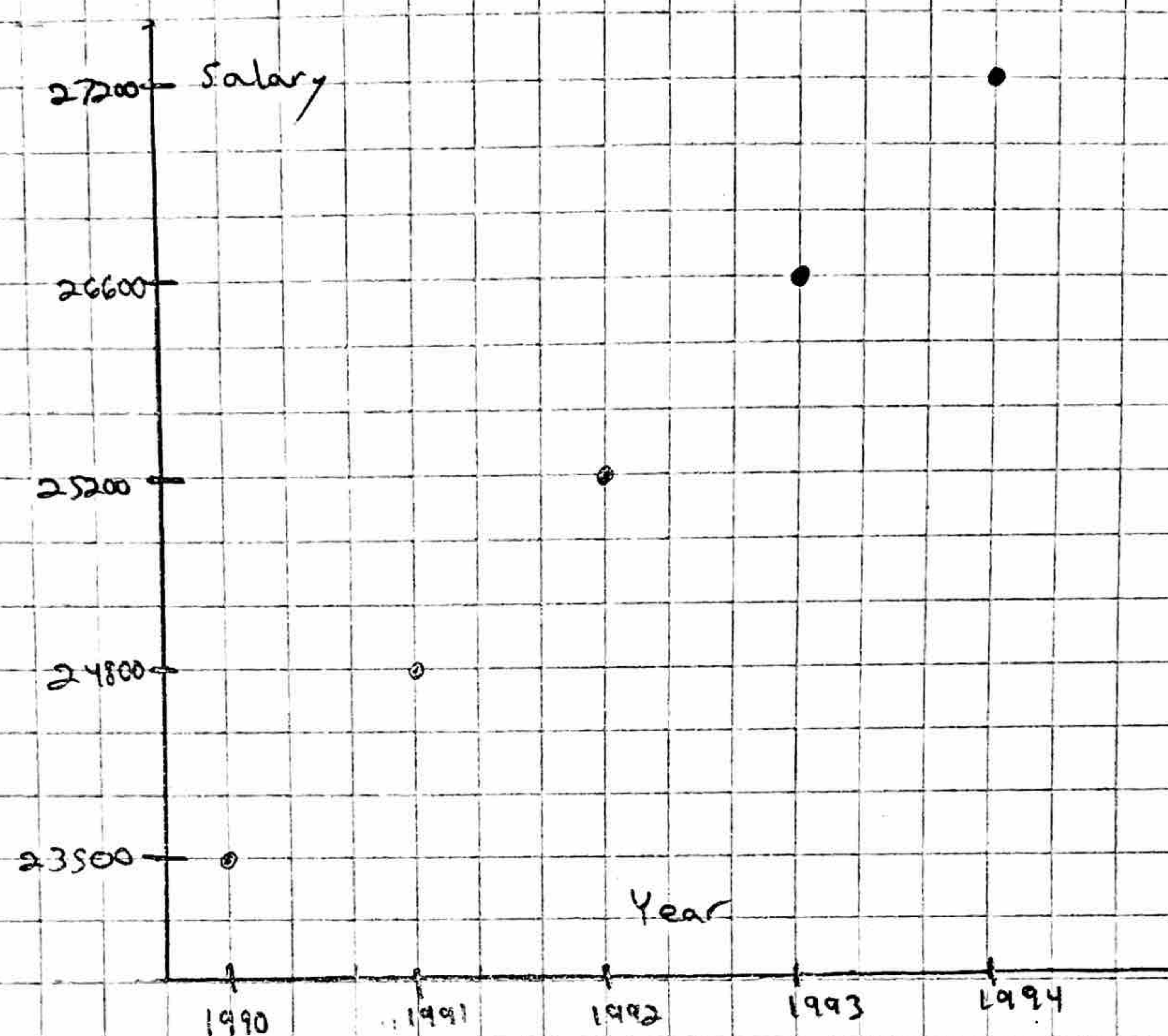
Find Equation

$b = -2, \quad m = 8$

$y = mx + b$

(b) $\boxed{y = 8x - 2}$

5. Point (124800, 326600) do not fall within linear model. (e)



$$m = \frac{27200 - 23500}{1994 - 1990} = \frac{3700}{4}$$

$$m = 925$$

$$y - y_1 = m(x - x_1)$$

$$y - 23500 = 925(x - 1990)$$

$$y - 23500 = 925x - 1.84075 \times 10^6$$

$$+23500 = +23500$$

$$y = 925x + 1.86425 \times 10^6$$

$$\boxed{y = 925x + 1.86425 \times 10^6}$$

6. $\lim_{x \to 8} f(x) = 9$ $\lim_{x \to 8} g(x) = 8$

$$\lim_{x \to 8} \left[\frac{3f(x) - 7g(x)}{9 + g(x)} \right] = -29/17$$ d

$$= \frac{\lim_{x \to 8} 3 \cdot \lim_{x \to 8} f(x) - \lim_{x \to 8} 7 \cdot \lim_{x \to 8} g(x)}{\lim_{x \to 8} 9 + \lim_{x \to 8} g(x)}$$

$$= \frac{3 \cdot 9 - 7 \cdot 8}{9 + 8}$$

$$= \frac{27 - 56}{17}$$

$$= -29/17$$

7. $\lim\limits_{x \to 16} \dfrac{\sqrt{x} - 4}{x - 16} \to \dfrac{\sqrt{x} - 4}{(16) - 16}$

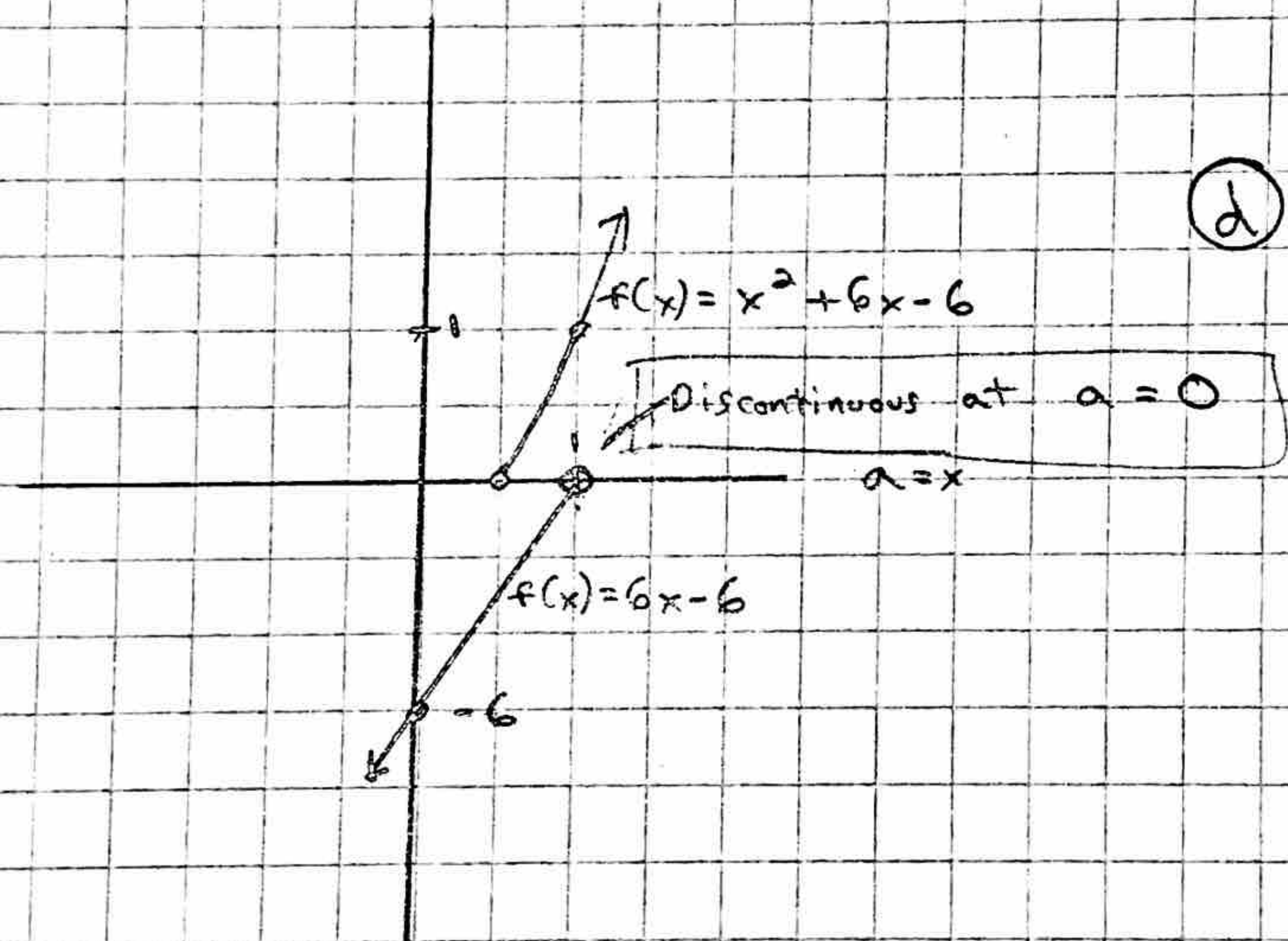
$$= \frac{\sqrt{x} - 4}{0}$$

$$= \text{undef}$$

limit do not exist

8. $f(x) = \begin{cases} 6x - 6, & x < 0 \\ x^2 + 6x - 6, & x \geq 0 \end{cases}$

(d)



9. $m = \dfrac{f(b) - f(a)}{b - a}$ $\quad [1, 9]$ (a, b) for $f(x) = x^2 + 7x$, $f(x) = y$

$f(a) = f(1) = (1)^2 + 7(1) = 1 + 7 = 8$

$f(b) = f(9) = (9)^2 + 7(9) = 144$

$m = \dfrac{f(b) - f(a)}{b - a}$

$\parallel$

$m = \dfrac{144 - 8}{9 - 1}$

$\parallel$

$m = \dfrac{136}{8}$

$\parallel$

$m = 17$ (d)

10. Condition $\quad$ Converse

$p \to q$ $\quad$ $q \to p$

Condition: If the sum of two angles is 180° (p), then the angles are supplementary (q).

Converse: If the angles are supplementary (q), then the sum of the angles is 180° (p).

(a)

11. $f(x) = \frac{1}{x}$ $\qquad g(x) = \sqrt{x+1}$

$(f \circ g)(x)$

$= \frac{1}{\sqrt{x+1}}$

$= \frac{1}{\sqrt{x+1}} \cdot \frac{\sqrt{x+1}}{\sqrt{x+1}}$

$= \frac{\sqrt{x+1}}{(\sqrt{x+1})^2}$

$= \frac{\sqrt{x+1}}{x+1}$

$$\boxed{(f \circ g)(x) = \frac{\sqrt{x+1}}{x+1}}$$

$(g \circ f)(x)$

$= \sqrt{\frac{1}{x} + 1}$

$= \sqrt{\frac{1+x}{x}}$

$$\boxed{(g \circ f)(x) = \sqrt{\frac{1+x}{x}}}$$

$(f \circ g)(3)$

$\frac{\sqrt{3+1}}{3+1}$

$= \frac{\sqrt{4}}{4}$

$= \frac{2}{4}$

$=$

$$\boxed{(f \circ g)(3) = \frac{1}{2}}$$

$(g \circ f)(3)$

$\sqrt{\frac{1+x}{x}}$

$= \sqrt{\frac{1+3}{3}}$

$= \sqrt{\frac{4}{3}}$

$= \frac{\sqrt{4}}{\sqrt{3}}$

$=$

$$\boxed{(g \circ f)(3) = \frac{2}{\sqrt{3}}}$$

b

12. $f(x) = \frac{1}{x+1}$, $\frac{f(1+h)-f(1)}{h}$

$f(1) = \frac{1}{1+1}$

$=$

$\boxed{f(1) = \frac{1}{2}}$

$f(1+h) = \frac{1}{(1+h)+1} = \frac{1}{2+h}$

$=$

$\frac{\frac{1}{2+h} - \frac{1}{2}}{h}$

$=$

$\frac{\frac{-h}{2(2+h)}}{h}$

$=$

$\frac{-h}{h \cdot 2(2+h)}$

$=$

$\frac{-h}{2h(2+h)}$

$\frac{-1}{2(h+2)}$

$=$

$\frac{-1}{2h+4}$

(a)

$\frac{\frac{b}{c}}{a} = \frac{b}{a \cdot c}$

$a=h, b=-h, c=2(2+h)$

$\rightarrow \frac{1}{2+h} - \frac{1}{2}$

$=$

$(2) \cdot \frac{1}{2+h} - \frac{1}{2} (h+2)$

$=$

$\frac{2}{2(2+h)} - \frac{h+2}{2(2+h)}$

$=$

$\frac{2-h+2}{2(2+h)}$

$=$

$\boxed{\frac{-h}{2(2+h)}}$

Test for Continuity at [-1, 0]

13. $f(x) = \frac{2x^3 + x^2 + 2x + 1}{2x - 1}$

Domain

$2x - 1 = 0$

$+1 \quad +1$

$\frac{2x}{2} = \frac{1}{2}$ $x = 1/2 = 0.5$

$\boxed{x \neq 1/2}$

$a = 0$ for $f(a)$

$f(0) = \frac{2(0)^3 + (0)^2 + 2(0) + 1}{2(0) - 1}$

$= \frac{2(0)^3 + (0)^2 + 2(0) + 1}{2(0) - 1}$

$= \frac{0 + 0 + 0 + 1}{0 - 1}$

$= \frac{1}{-1}$

$=$

$\boxed{f(0) = -1, \ f(a) = -1}$

$\lim_{x \to 0^-} \frac{2x^3 + x^2 + 2x + 1}{2x - 1}$

x	$f(x)$
-0.01	-0.96
-0.001	-0.99

$\lim_{x \to 0^+} \frac{2x^3 + x^2 + 2x + 1}{2x - 1}$

x	$f(x)$
0.01	-1.04
0.001	-1.00

$\lim_{x \to 0^-} \frac{2x^3 + x^2 + 2x + 1}{2x - 1} = -1$

$\lim_{x \to 0^+} \frac{2x^3 + x^2 + 2x + 1}{2x - 1} = -1$

$\boxed{\lim_{x \to 0} \frac{2x^3 + x^2 + 2x + 1}{2x - 1} = -1}$

$\boxed{\lim_{x \to 0} \frac{2x^3 + x^2 + 2x + 1}{2x - 1} = f(0)}$

$\boxed{f(x) = \frac{2x^3 + x^2 + 2x + 1}{2x - 1} \text{ is continuous at } [-1, 0]}$

13.

$f(x) = \dfrac{2x^3 + x^2 + 2x + 1}{2x - 1}$

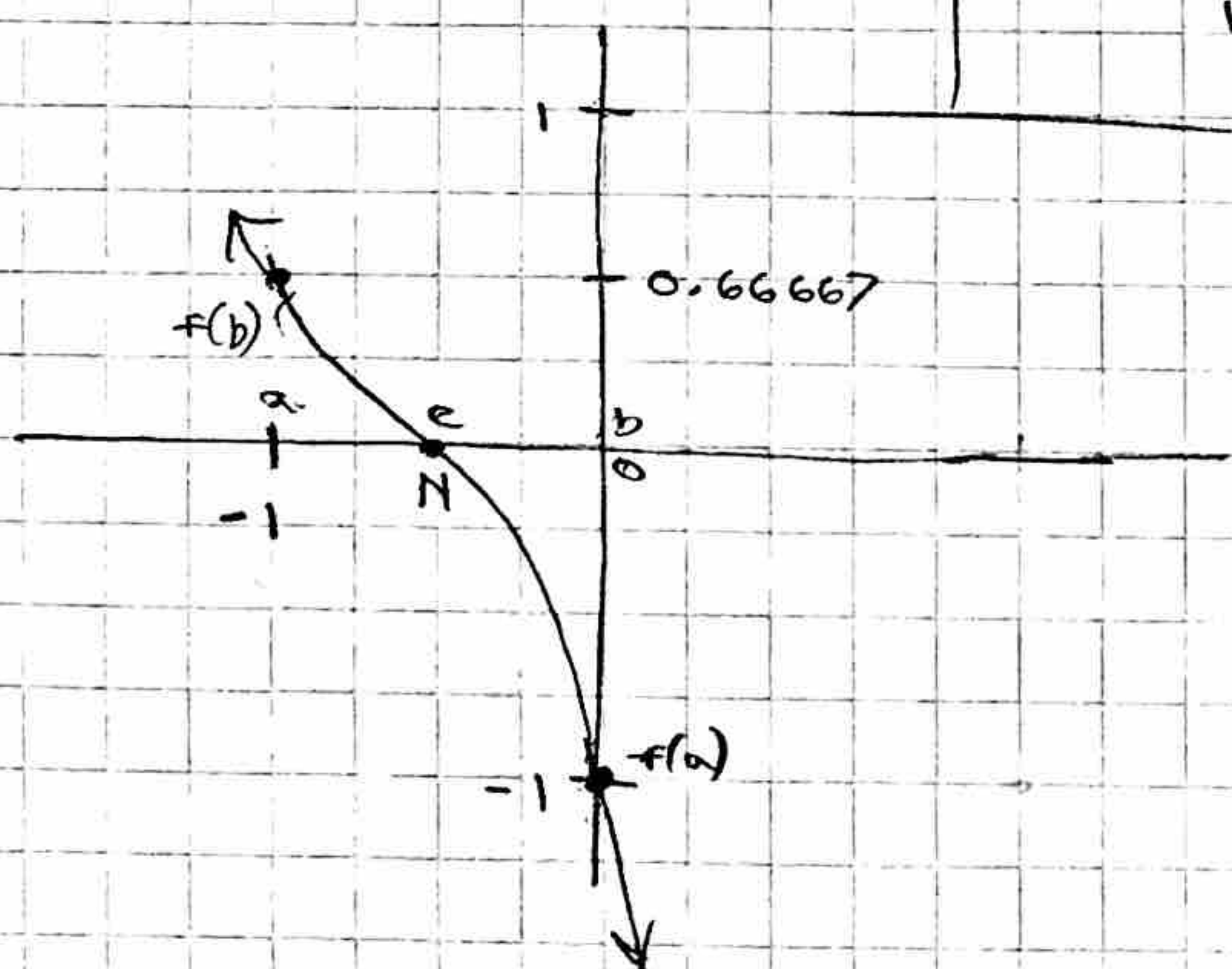
$[-1, 0]$ for $f(x) = \dfrac{2x^3 + x^2 + 2x + 1}{2x - 1} = 0$

$$f(-1) = \frac{2(-1)^3 + (-1)^2 + 2(-1) + 1}{2(-1) - 1}$$
$$= \frac{-2 + 1 - 2 + 1}{-2 - 1}$$
$$= \frac{-1 - 2 + 1}{-3}$$
$$= \frac{-3 + 1}{-3}$$
$$= \frac{-2}{-3}$$
$$=$$

$$\boxed{f(-1) = 2/3 \approx 0.66667}$$

$$f(0) = \frac{2(0)^3 + (0)^2 + 2(0) + 1}{2(0) - 1}$$
$$= \frac{0 + 0 + 0 + 1}{0 - 1}$$
$$= \frac{1}{-1}$$
$$=$$
$$\boxed{f(0) = -1}$$

a. f(x) has a solution between $[-1, 0]$



14. $\lim_{x \to a} f(x)$

$$\underbrace{\lim_{x \to a^-} f(x) = \lim_{x \to a^+} f(x)}$$

These need to be the same for the $\lim_{x \to a} f(x)$ to exist.

a

Definition

15. $\lim_{x \to t} f(x) = L$

For every given number $\varepsilon > 0$ there is a number $\sigma > 0$ where

If x is within σ units of t and $x \neq t$, then $f(x)$ is within ε units of L.

$|f(x) - L| < \varepsilon$ implies $|f(x) - L| < |x - t| < \sigma$

$0 < |x - t| < \varepsilon$ implies $|f(x) - L| < \sigma$

a

16. $f(x) = 2x - 6$

$L = -12$

$c = 3$

$\varepsilon = 0.01$

$$\lim_{x \to 3} 2x - 6 = -12$$

$$-12 - 0.01 < 2x - 6 < -12 + 0.01$$

$$\shortparallel$$

$$-12.01 < 2x - 6 < -11.99$$
$$+6 \qquad\qquad +6 \qquad\qquad +6$$

$$\frac{-6.01}{2} < \frac{2x}{2} < \frac{-5.99}{2}$$

$$\shortparallel$$

$$-3.005 < x < -2.995$$

$$-2.995 - (-3.005)$$

$$\shortparallel$$

$$-2.995 + 3.005$$

$$\shortparallel$$

$$0.01$$

$$\shortparallel$$

$$\boxed{0.01/2 = 0.005}$$

a

17. $\lim_{x \to a} [c \cdot f(a)]$ c = constant

a = arbitrary number

$=$

$\lim_{x \to a} c \cdot \lim_{x \to a} f(a)$ ← limit of product

$\boxed{c \cdot \lim_{x \to a} f(a)}$ ← product of limits

constant

b

18. $y = \ln(x^2)$, $x = e^2$ Get $f'(x)$

Apply Chain Rule

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

$y = \ln(u)$, $u = x^2$

$$\frac{d}{du}[\ln(u)] \cdot \frac{d}{dx}[x^2]$$

$$=$$

Ref

$$\boxed{\frac{d}{dx}[\ln(x)] = \frac{1}{x}}$$

$$\frac{1}{u} \cdot 2x$$

$$=$$

$$x^2 = u$$

$$\downarrow$$

$$\frac{1}{x^2} \cdot 2x$$

$$=$$

$$\frac{2x}{x^2}$$

$$=$$

$$f'(x) = \frac{2}{x}$$

$$\boxed{f'(x) = \frac{2}{e^2} \quad , x = e^2}$$

C

19. $F(x) = \overset{f(x)}{x}\,\overset{g(x)}{\sin(x)}$

$$F'(x) = \frac{d}{dx}[x\sin(x)] = x\frac{d}{dx}[\sin(x)] + \sin(x)\frac{d}{dx}[x]$$

$$=$$

$$x \cdot \cos(x) + \sin(x) \cdot 1x^{1-1}$$

$$=$$

$$x\cos(x) + \sin(x) \cdot 1x^{0}$$

$$=$$

$$x\cos(x) + \sin(x)$$

$$=$$

$$\boxed{F'(x) = \sin(x) + x\cos(x)}$$

$$=$$

$$F'(3\pi/2) = \sin\left(\frac{3\pi}{2}\right) + x\cos\left(\frac{3\pi}{2}\right)$$

$$=$$

$$-1 + x \cdot 0$$

$$=$$

$$\boxed{F'(3\pi/2) = -1}$$

D

20. $H(x) = x^3 - x^2 + \frac{1}{x}$ Get $H''(2)$

$$H'(x) = \frac{d}{dx}\left[x^3 - x^2 + \frac{1}{x}\right]$$

$$= \frac{d}{dx}[x^3] - \frac{d}{dx}[x^2] + \frac{d}{dx}\left[\frac{1}{x}\right]$$

$f(x)$ over $g(x)$

$$= 3x^{3-1} - 2x^{2-1} + \frac{x \cdot \frac{d}{dx}[1] - 1 \cdot \frac{d}{dx}[x^1]}{[x]^2}$$

$$= 3x^2 - 2x \qquad \frac{x \cdot 0 - 1 \cdot 1x^{1-1}}{x^2}$$

$$= \frac{0 - 1 \cdot 1}{x^2}$$

$$3x^2 - 2x \quad - \frac{1}{x^2}$$

$$\boxed{H'(x) = 3x^2 - 2x - \frac{1}{x^2}}$$

$$H''(x) = \frac{d}{dx}\left[3x^2 - 2x - \frac{1}{x^2}\right]$$

$$=$$

$$\frac{d}{dx}[3x^2] - \frac{d}{dx}[2x] - \frac{d}{dx}\left[\frac{\overset{f(x)}{1}}{\underset{g(x)}{x^2}}\right]$$

$$=$$

$$3 \cdot \frac{d}{dx}[x^2] - 2 \cdot \frac{d}{dx}[x^1] - \frac{x^2 \cdot \frac{d}{dx}[1] - 1 \cdot \frac{d}{dx}[x^2]}{[x^2]^2}$$

$$=$$

$$3 \cdot 2x^{2-1} - 2 \cdot 1x^{1-1} - \frac{x^2 \cdot 0 - 1 \cdot 2x^{2-1}}{x^4}$$

$$=$$

$$3 \cdot 2x - 2 \cdot 1x^0 - \frac{2x^1}{x^4}$$

$$=$$

$$6x - 2 - \frac{2}{x^3}$$

$$\boxed{H''(x) = 6x - 2 - \frac{2}{x^3}}$$

$$H''(2) = 6(2) - 2 - \frac{2}{(2)^3}$$

$$=$$

$$12 - 2 - \frac{2}{8}$$

$$=$$

$$12 - 2 - \frac{1}{4}$$

$$=$$

$$10 - \frac{1}{4}$$

$$=$$

$$\boxed{H''(x) = 39/4}$$

b