Intermediate Value Theorem

Show that f has a zero in the interval $(0,\pi)$

$f(x) = \cos(x) + \sin(x) \qquad (0,\pi)$

① Trig functions and their sums are continuous everywhere.

$\cos x + \sin x = 0$

②

$\cos(0) + \sin(0) = 1 + 0 = 1$

$\cos(\pi) + \sin(\pi) = -1 + 0 = -1$

③

We can pick any number n in the interval $(-1,1)$ and find a value c in the interval $(0,\pi)$ such that $f(c) = n$.

Picking $n = 0$, we have shown f has a zero in the interval $(0,\pi)$.

$\therefore$ Therefore, f has a zero in the interval $(0,\pi)$

Intermediate Value Theorem

Show that there is a root of the given equation on the given interval.

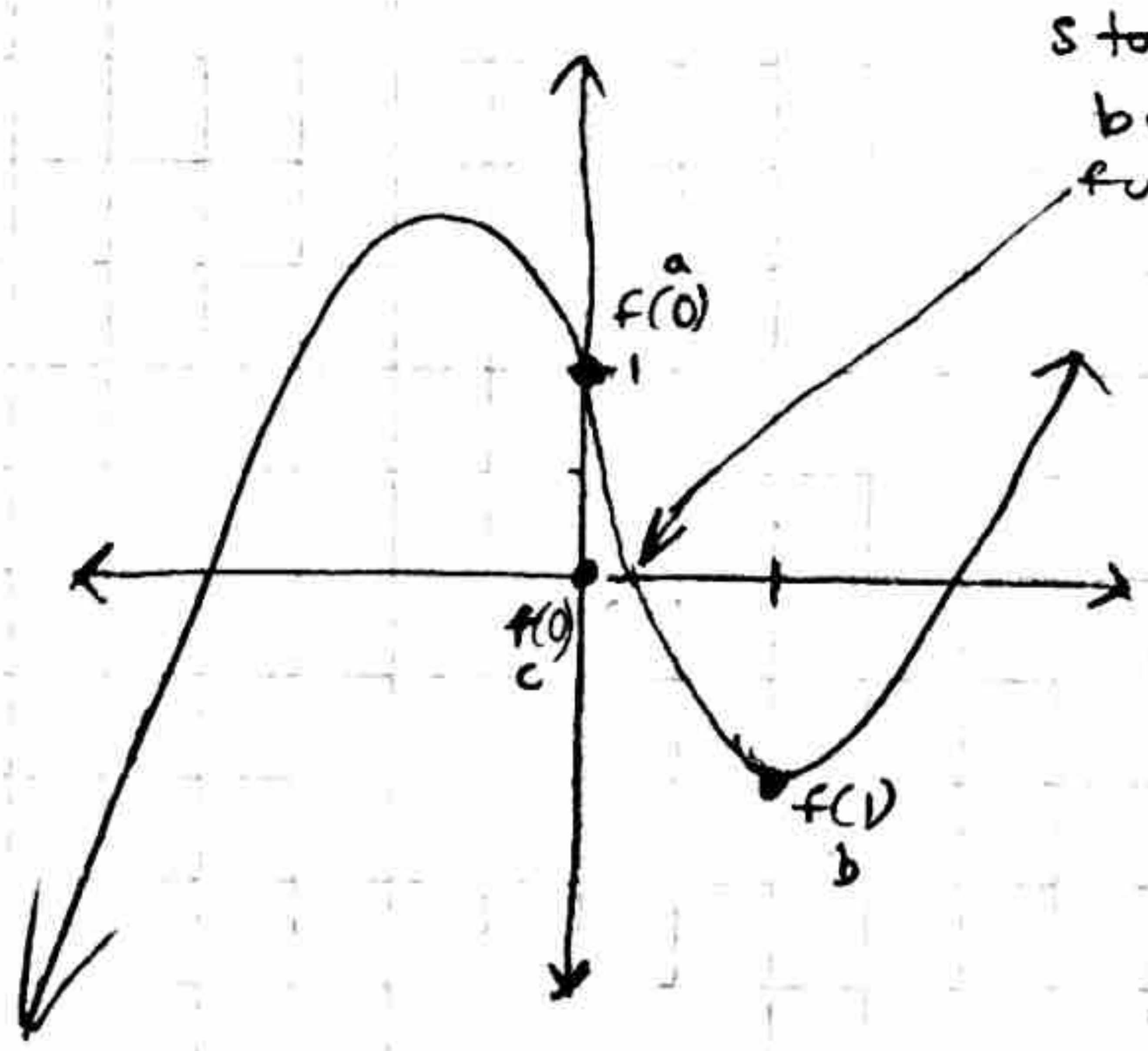
This polynomial is continuous everywhere

1) $\underbrace{x^3 - 3x + 1}_{f(x)} = 0$ $\quad (0, 1)$ Trying to find something between the interval (0, 1).

$f(0) = (0)^3 - 3(0) + 1 = 1$

$f(1) = (1)^3 - 3(1) + 1 = 1 - 3 + 1 = -2 + 1 = -1$

$x^3 - 3x + 1 = 0$ ← Intermediate Value Theorem states that there have to be a place where this function equals 0.



Polynomial and root functions are continuous everywhere. I can use the Intermediate Value Theorem

$x^2 = \sqrt{x+1}$ $(1,2)$ Try to find something between the interval $(1,2)$

①

$x^2 = \sqrt{x+1}$

$-\sqrt{x+1} \quad -\sqrt{x+1}$

$\underbrace{x^2 - \sqrt{x+1}}_{f(x)} = 0$

$f(1) = (1)^2 - \sqrt{(1)+1}$

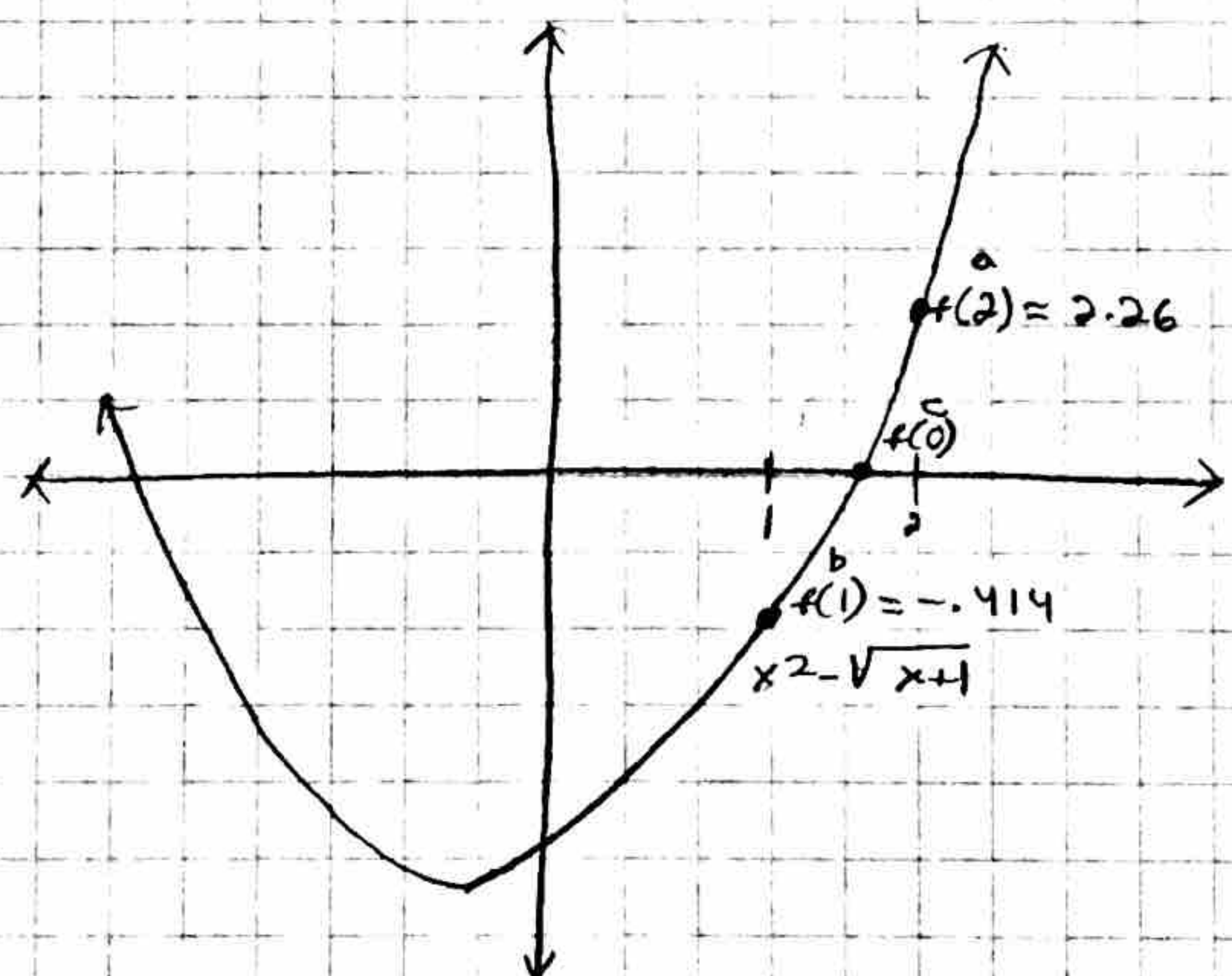
$= 1 - \sqrt{2}$

$\approx -.414214$

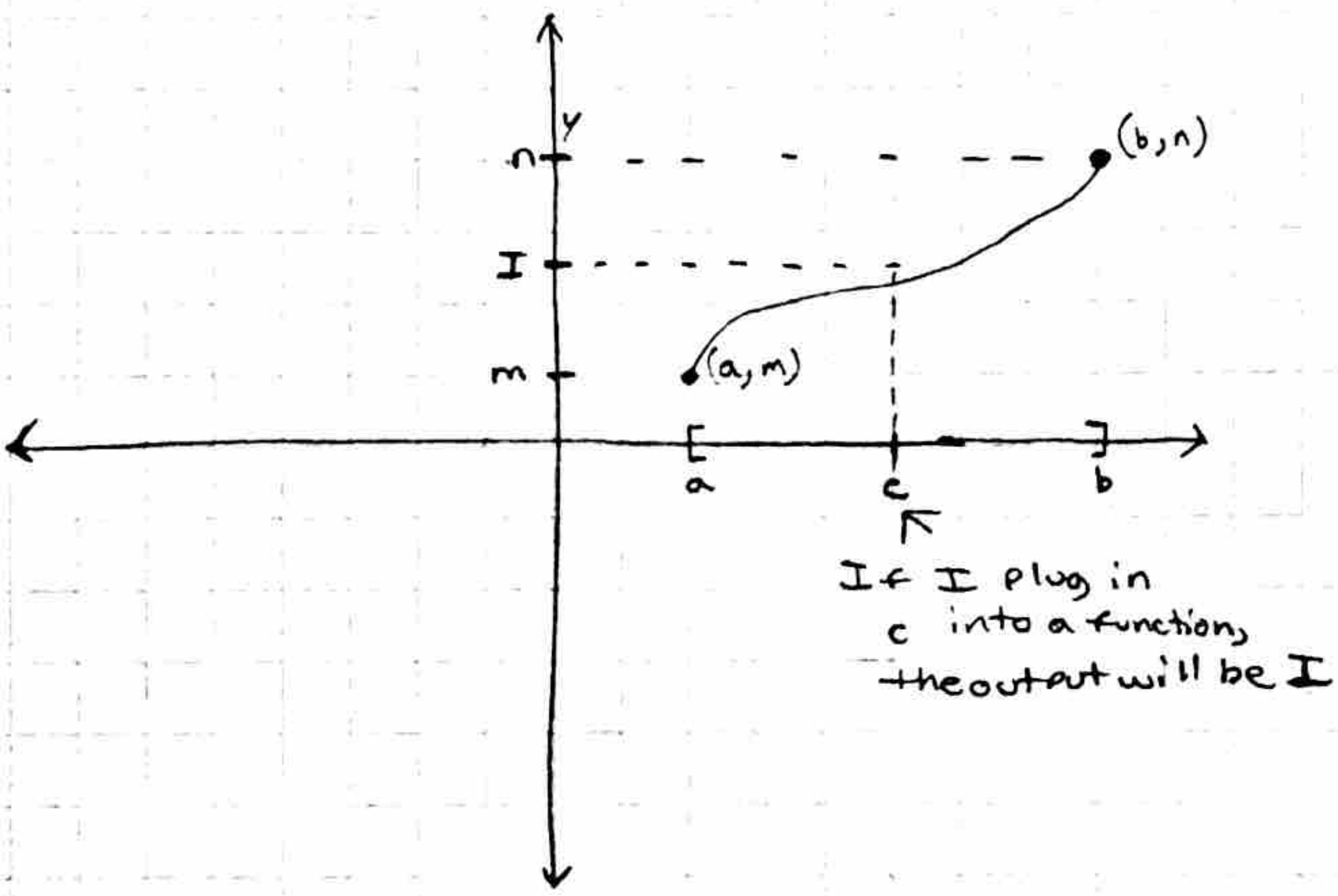
$f(2) = (2)^2 - \sqrt{(2)+1}$

$= 4 - \sqrt{3}$

≈ 2.26795



Intermediate Value Theorem



Suppose $f(x)$ is continuous on $[a, b]$;

let $f(a) = m$, $f(b) = n$;

let $m \leq I \leq n$

there exist c in $[a,b]$, where $f(c) = I$