1. $R = \{(0,8), (3,4), (5,7), (6,2), (9,1)\}$ Func

$R^{-1} = \{(8,0), (4,3), (7,5), (2,6), (1,9)\}$ Func

3. $R = \{(1,6), (2,5), (3,4), (-2,0), (-1,5)\}$ Func

$R^{-1} = \{(6,1), (5,2), (4,3), (0,-2), (5,-1)\}$ 5 repeat, not func

5. $R = \{(2,5), (5,6), (6,8), (2,-2), (7,6)\}$ 2 rep, not func

$R^{-1} = \{(5,2), (6,5), (8,6), (-2,2), (6,7)\}$ 6 rep, not func

7. $f = \{(0,1), (1,2), (2,3), (3,4)\}$

$f^{-1} = \{(1,0), (2,1), (3,2), (4,3)\}$

Both are funcs so they are one-to-one

Each relation have distinctive x any y coordinates

9. $f = \{(-1,0), (0,2), (1,3), (2,3)\}$

$f^{-1} \{(0,-1), (2,0), (3,1), (3,2)\}$

No

f is not a one-to-one because its inverse is not a function

f do not have distinctive y-coordinates. The y value of 3 repeats.

For a one-to-one function, both a function and its inverse need to have distinctive x and y coordinates or vales

$f = \{(x, x-1) \mid x \text{ is any real number}\}$

① Define f^{-1}

$f(x) = x - 1$

$\downarrow$

$y = x - 1$

$\downarrow$

$x = y - 1$

$+1 \qquad +1$

$x + 1 = y$

$\downarrow$

$y = x + 1$

$$\boxed{f^{-1}(x) = x + 1}$$

$f = \{(x, x-1) \mid x \text{ is any real number}\}$

Domain of $f(x)$: $(-\infty, \infty)$

$f(x) = x - 1$

$$\boxed{f(a) = f(b) = \quad a \neq b}$$

A have to Equal B to be one-to-one

$a - 1 = b - 1$

$+1 \qquad +1$

$\downarrow$

$a = b$

On a graph if $f(x)$ passes the horizontal line test, then it is one-to-one.

Each x coordinate have a unique y coordinate

$f(x) = \sqrt{x+6}$

$f(a) = f(b) \rightarrow a=b$

$\sqrt{a+6} = \sqrt{b+6}$ solve for a

$\downarrow$

$(\sqrt{a+6})^2 = (\sqrt{b+6})^2$

$\downarrow$

$$\begin{array}{rcl} a+6 & = & b+6 \\ -6 & & -6 \\ \hline \end{array}$$

$\boxed{a=b}$ This is one-to-one

$f(x) = \sqrt{x-4}$

$\downarrow$

$f(a) = f(b) \rightarrow a=b$

$\downarrow$

$\sqrt{a-4} = \sqrt{b-4}$

$\downarrow$

$(\sqrt{a-4})^2 = (\sqrt{b-4})^2$

$\downarrow$

$$\begin{array}{rcl} a-4 & = & b-4 \\ +4 & & +4 \\ \hline \end{array}$$

$\boxed{a=b}$ This is one-to-one

$f(x) = 4x - 2$ Find $f^{-1}(10)$

$$4x - 2 = 10$$
$$+2 \quad +2$$
$$\frac{4x}{4} = \frac{12}{4}$$
$$x = 3$$
$$\boxed{f^{-1}(10) = 3}$$

$f = (x+2)^2 + 3 \mid x \geq -2$ Find $f^{-1}(19)$

$$f(x) = (x+2)^2 + 3$$
$$y = (x+2)^2 + 3$$
$$x = (y+2)^2 + 3$$
$$-3 \qquad -3$$
$$\sqrt{x-3} = \sqrt{(y+2)^2}$$
$$\sqrt{x-3} = y + 2$$
$$-2 \qquad -2$$
$$\sqrt{x-3} - 2 = y$$
$$y = \sqrt{x-3} - 2$$
$$f^{-1}(x) = \sqrt{x-3} - 2$$

$$f^{-1}(19) = \sqrt{19-3} - 2$$
$$\sqrt{16} - 2$$
$$4 - 2$$
$$2$$

2nd Approach

$$f(x) = (x+2)^2 + 3$$
$$(x+2)^2 + 3 = 19$$
$$-3 \quad -3$$
$$\sqrt{(x+2)^2} = \sqrt{16}$$
$$x + 2 = 4$$
$$-2 = -2$$
$$x = 2$$
$$f^{-1}(19) = 2$$

$(6, -2)$

$f(6) = -2$

$(-2, 6)$

$f^{-1}(-2) = 6$

c ✓

d ✓

f ✓

23. $f = \{(-2,6), (-1,9), (0,12), (1,15)\}$

$f^{-1} = \{(6,-2), (9,-1), (12,0), (15,1)\}$ $f^{-1}(12) = 0$

5. $f = \{(x, 4x-2) \mid x$ is any real number$\}$

$f^{-1}(-6)$

$f(x) = 4x - 2$

$\downarrow$

$y = 4x - 2$

$\downarrow$

$x = 4y - 2$

$+2 \quad\quad +2$

$\frac{x+2}{4} = \frac{\cancel{4}y}{\cancel{4}}$

$\downarrow$

$\frac{x+2}{4} = y$

$y = \frac{x+2}{4}$

$\downarrow$

$f^{-1}(x) = \frac{x+2}{4}$

$f^{-1}(x) = \frac{x+2}{4}$

$\downarrow$

$f^{-1}(-6) = \frac{(-6)+2}{4}$

$\downarrow$

$\frac{-4}{4}$

$\parallel$

$f^{-1}(-6) = -1$

$4x - 2 = -6$

$+2 \quad +2$

$\frac{4x}{4} = \frac{-4}{4}$

$x = -1$

$f(x) = \sqrt{x-3}$ Find $f^{-1}(2)$

$$(\sqrt{x-3})^2 = (2)^2$$

$$x - 3 = 4$$

$$\underline{\quad +3 \quad +3}$$

$$x = 7$$

$$\boxed{f^{-1}(2) = 7}$$

$f(x) = \sqrt{x-3}$ Find $f^{-1}(3)$

$$(\sqrt{x-3})^2 = (3)^2$$

$$x - 3 = 9$$

$$\underline{\quad +3 \quad +3}$$

$$x = 12$$

$$f^{-1}(3) = 12$$

46. $f(x) = \sqrt{3x+18} - 2$

$\downarrow$

$y = \sqrt{3x+18} - 2$

$x = \sqrt{3y+18} - 2$

$+2 \qquad +2$

$(x+2)^2 = (\sqrt{3y+18})^2$

$(x+2)^2 = 3y+18$

$-18 \qquad -18$

$\frac{(x+2)^2 - 18}{3} = \frac{\cancel{3}y}{\cancel{3}}$

$\frac{(x+2)^2}{3} - \frac{18}{3} = y$

$\frac{1(x+2)^2}{3} - 6 = y$

$\frac{1}{3}(x+2)^2 - 6 = y$

$y = \frac{1}{3}(x+2)^2 - 6$

$f^{-1}(x) = \frac{1}{3}(x+2)^2 - 6, \quad x \geq -2$

Get Range for Inverse since we have a restriction

$3x + 18 \geq 0$

$-18 \quad \geq -18$

$\frac{3x}{3} \geq \frac{-18}{3} \quad x \geq -6$

$D: x \geq -6$

$\sqrt{3x+18-2}$

$3x+18-2 \geq 0$

$3x + 16 \geq 0$

$-16 \quad -16$

$\frac{3x}{3} = \frac{-16}{3}$

$x =$

$2x - 6 \geq 0$

$+6 \quad +6$

$\frac{2x}{2} \geq \frac{6}{2}$

$x \geq 3$

$(3)\ y = \cancel{\tfrac{1}{3}}(x+2)^2 - 6$

$3y = (x+2)^2 - 6$

$+6 \qquad +6$

$\sqrt{3y+6} = \sqrt{(x+2)^2}$

$\sqrt{3y+6} = x+2$

$-2 \qquad -2$

$\sqrt{3y+6} = x$

$\hookrightarrow 3y+6 \geq 0$

$-6 \quad -6$

$\frac{3y}{3} \geq \frac{-6}{3} \quad y \geq -2$