Theorem 2.4.3

$$\lim_{x\to a} f(g(x)) = f\left(\lim_{x\to a} g(x)\right) = f(g(a))$$

If f is continuous at b,

and $\lim_{x\to a} g(x) = b$,

then $\lim_{x\to a} f(g(x)) = f(b)$

$$\lim_{x\to -1} \cos\left(\frac{x^2-1}{x+1}\right)$$

$$\text{Define}$$

$$f(g(x)) = \cos\left(\frac{x^2-1}{x+1}\right)$$

$$f\left(\lim_{x\to a} g(x)\right) = \cos\left(\lim_{x\to -1} \frac{x^2-1}{x+1}\right)$$

$$f = \cos, \quad g(x) = \frac{x^2-1}{x+1}$$

$$=$$

$$\lim_{x\to -1} \cos\left(\frac{x^2-1}{x+1}\right) = \cos\left(\lim_{x\to -1} \frac{x^2-1}{x+1}\right)$$

$$=$$

$$\cos\left(\lim_{x\to -1} \frac{x^2-1}{x+1}\right)$$

$$=$$

$$\cos\left(\lim_{x\to -1} \frac{\cancel{(x+1)}(x-1)}{\cancel{(x+1)}}\right)$$

$$=$$

$$\cos\left(\lim_{x\to -1} (x-1)\right)$$

$$=$$

$$\cos\left(\lim_{x\to -1} x - \lim_{x\to -1} 1\right)$$

$$=$$

$$\cos(-1 - 1)$$

$$=$$

$$\cos(-1 + -1)$$

$$=$$

$$\cos(-2)$$

$$\boxed{\cos(-2) = f(g(a))}$$

Theorem 2.4.3

If f is continuous at b,

and $\lim_{x \to a} g(x) = b$, then $\lim_{x \to a} f(g(x)) = f(b)$

$$\boxed{\lim_{x \to a} f(g(x)) = f\left(\lim_{x \to a} g(x)\right)}$$

Theorem 2.4.4

If g is continuous at a,

and f is continuous at $g(a)$,

then $f \circ g$ is continuous at $x = a$

Theorem 2.4.4

If g is continuous at a, and f is continuous at $g(a)$,

the composition function $(f \circ g)(x) = f(g(x))$ is continuous at a.

"a continuous function of continuous function is a continuous function"

PROOF: Since g is continuous at a,

$$\lim_{x \to a} g(x) = g(a)$$

Since f is continuous at $g(a)$, we can apply Theorem 2.4.3 (Limit of a Composite Function)

$$\lim_{x \to a} f(g(x)) = f\left(\lim_{x \to a} g(x)\right) = f(g(a))$$

$$\boxed{f(g(x)) \text{ is continuous at } a}$$

$$\lim_{x \to 1} \sin\left(\frac{\pi x}{2}\right) = 1$$

$$= \sin\left(\frac{\pi(1)}{2}\right) = \sin\left(\frac{\pi}{2}\right) = 1$$

$$\lim_{x \to -1} \cos\left(\frac{x^2-1}{x+1}\right) = \lim_{x \to -1} 1$$

$$\frac{\cancel{(x+1)}(x-1)}{\cancel{(x+1)}} = \lim_{x \to -1} (x-1) = \lim_{x \to -1} x - \lim_{x \to -1} 1 = -1 + -1 = -2$$

Find the value of : $\lim_{x \to -1} \cos\left(\frac{x^2-1}{x+1}\right)$

$$= \cos(-2)$$

$$\lim_{x \to -1} \frac{x^2-1}{x+1} = \frac{\cancel{(x+1)}(x-1)}{\cancel{(x+1)}} = \lim_{x \to -1} (x-1) = \lim_{x \to -1} x - \lim_{x \to -1} 1 = -1 + -1 = -2$$

Theorem 2.4.3

$$\lim_{x \to a} f(g(x)) = f\left(\lim_{x \to a} g(x)\right)$$