Example 11

Investigate the curve $y = 2x^4 - 8x^3$ by discussing increasing, decreasing, concavity, points of inflection and relative extrema.

① Get $f'(x)$

$$f'(x) = \frac{d}{dx}\left[2x^4 - 8x^3\right]$$

$$\frac{d}{dx}[2x^4] - \frac{d}{dx}[8x^3]$$

$$2 \cdot \frac{d}{dx}[x^4] - 8 \cdot \frac{d}{dx}[x^3]$$

$$2 \cdot 4x^{4-1} - 8 \cdot 3x^{3-1}$$

$$2 \cdot 4x^3 - 8 \cdot 3x^2$$

$$\boxed{f'(x) = 8x^3 - 24x^2}$$

② Get $f''(x)$

$$f''(x) = \frac{d}{dx}\left[8x^3 - 24x^2\right]$$

$$\frac{d}{dx}[8x^3] - \frac{d}{dx}[24x^2]$$

$$8 \cdot \frac{d}{dx}[x^3] - 24 \cdot \frac{d}{dx}[x^2]$$

$$8 \cdot 3x^{3-1} - 24 \cdot 2x^{2-1}$$

$$8 \cdot 3x^2 - 24 \cdot 2x^1$$

$$\boxed{f''(x) = 24x^2 - 48x}$$

③ Get increasing/decreasing stuff

④ Set $f'(x) = 0$

$$8x^3 - 24x^2 = 0$$

$$8x^2(x - 3) = 0$$

$$\frac{8x^2}{8} = \frac{0}{8}$$

$$\sqrt{x^2} = \sqrt{0}$$

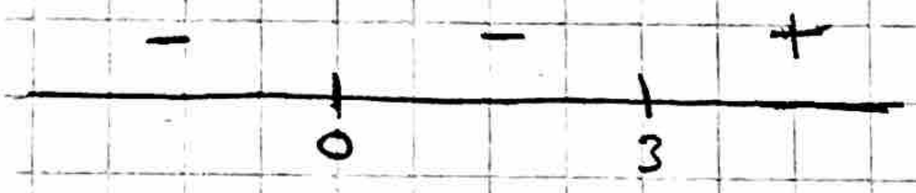
$$\boxed{x = 0}$$

$$x - 3 = 0$$

$$+3 \quad +3$$

$$\boxed{x = 3}$$

⑤ Set intervals



$(-1): 8(-1)^2(-1-3)$

$\parallel$

$8(1)(-4)$

$\parallel$

$8(-4)$

$\parallel$

-32

$(1): 8(1)^2(1-3)$

$\parallel$

$8(1)(-2)$

$\parallel$

$8(-2)$

$\parallel$

-16

$f'(x) = -16$

$(4): 8(4)^2(4-3)$

$\parallel$

$8(16)(1)$

$\parallel$

$128(1)$

$\parallel$

$+128$

⑥ Increasing / Decreasing Summary

f or $y = 2x^4 - 8x^3$ is decreasing on $(-\infty, 3)$ and increasing on $(3, \infty)$

⑦ Get Concavity

⑧ Determine Intervals

$f''(x) = 24x^2 - 48x$

$24x(x-2)$

$\parallel$

$\frac{24x}{24} = \frac{0}{24}$

$\parallel$

$\boxed{x = 0}$

$x - 2 = 0$

$+2 \quad +2$

$\boxed{x = 2}$

$(-1): 24(-1)(-1-2)$

$-24(-3)$

$+72$

$(1): 24(1)(1-2)$

$24(-1)$

-24

$(3): 24(3)(3-2)$

$72(1)$

$+72$

⑨ Concavity Summary

$y = 2x^4 - 8x^3$ concaves upward on $(-\infty, 0) \cup (2, \infty)$

concaves downward on $(0, 2)$

⑩ Get inflection Points

⑪ Get critical values of $f''(x)$

$x = 0$, $x = 2$

⑫ Check $y = 2x^4 - 8x^3$ is continuous at $x = 0$ and $x = 2$

$y = 2x^4 + 8x^3$ is a polynomial

Polynomial functions are continuous everywhere on their domains

$x = 0$ and $x = 2$ are in the domain of $y = 2x^4 - 8x^3$

$y = 2x^4 - 8x^3$ is continuous at $x = 0$ and $x = 2$

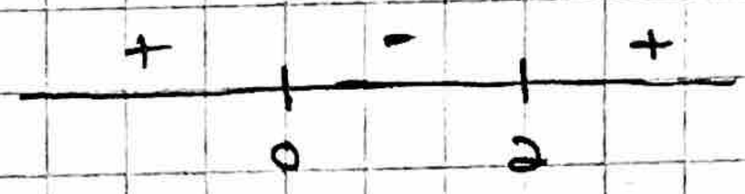
⑬ Inflection Point Summary

$y = 2x^4 - 8x^3$ have two inflection points at $x = 0$ and $x = 2$.

⑭ Get Relative Extrema Using Second Derivative Test

$f'(x) = 8x^3 - 24x^2$

$f''(x) = 24x^2 - 48x$



$f'(3) = 8(3)^3 - 24(3)^2$

$=$

$8(27) - 24(9)$

$216 - 216$

$=$

$f'(3) = 0$

$\downarrow$

$f'(x) = 0$

$f''(3) = 24(3)^2 - 48(3)$

$=$

$24(9) - 144$

$=$

$216 - 144$

$=$

$f''(3) = 72$

$f''(x) > 0$

If $f'(x) = 0$ and $f''(x) > 0$, the f has a relative minimum at $x = 3$.