Example 1:

$f(x) = \sin(x^2)$ Get $f'(x)$

$u = x^2$ $f(u) = \sin(x)$ $g(u) = x^2$

$f'(u)$ $g'(u)$

$\frac{d}{dx}[\sin(x)]$ $\frac{d}{dx}[x^2]$

$\cos(x)$ $2x^{2-1}$

$2x^1$

$\cos(x)$ $2x$

Chain Rule

$f'(g(x))\, g'(x)$

$\cos(x^2) \cdot 2x$

$2x\cos(x^2)$

Example 2

$f(x) = (2x+1)^4$ Get $f'(x)$

$u = 2x+1$, $f(u) = x^4$, $g(u) = 2x+1$

$f'(u)$

$= \frac{d}{dx}[x^4]$

$= 4x^{4-1}$

$= 4x^3$

$= f'(u) = 4x^3$

$g'(u)$

$= \frac{d}{dx}[2x+1]$

$= \frac{d}{dx}[2x] + \frac{d}{dx}[1]$

$= 2 \cdot \frac{d}{dx}[x^1] + 0$

$= 2 \cdot 1x^{1-1} + 0$

$= 2 \cdot 1x^0 + 0$

$= 2 \cdot 1 \cdot 1 + 0$

$= 2 + 0$

$= g'(u) = 2$

Chain Rule

$f'(g(u))\, g'(u)$, where $u = x$

$= 4(2x+1)^3 \cdot 2$

$= 2 \cdot 4(2x+1)^3$

$= f'(x) = 8(2x+1)^3$

Example 3

$f(x) = \sin^2(x)$ Get $f'(x)$

$\Downarrow$

$f(x) = (\sin x)^2$

$u = \sin(x)$

$f(u) = x^2$ $g(u) = \sin(x)$

$f'(u)$

$\Downarrow$

$\frac{d}{dx}[x^2]$

$\Downarrow$

$2x^{2-1}$

$\Downarrow$

$2x^1$

$\Downarrow$

$f'(u) = 2x$

$g'(u)$

$\Downarrow$

$\frac{d}{dx}[\sin(x)]$

$\Downarrow$

$\cos(x)$

$\Downarrow$

$g'(u) = \cos(x)$

Chain Rule

$f'(g(u))\ g'(u)$, where $u = x$

$\Downarrow$

$2(\sin(x)) \cdot \cos(x)$

$\Downarrow$

$f'(x) = 2\sin(x)\cos(x)$

Example 4

$$f(x) = \frac{1}{(x+1)^2}$$

Get $f'(x)$

$u = x+1$, $f(u) = \frac{1}{u^2}$, $g(x) = x+1$

$$f'(u) = \frac{d}{dx}\left[\frac{1}{u^2}\right]_{g(x)}^{f(x)}$$

$$= \frac{u^2 \cdot \frac{d}{dx}[1] - 1 \cdot \frac{d}{dx}[u^2]}{[u^2]^2}$$

$$= \frac{u^2 \cdot 0 - 1 \cdot 2u^{2-1}}{u^4}$$

$$= \frac{0 - 1 \cdot 2u^1}{u^4}$$

$$= \frac{-1 \cdot 2u^1}{u^4}$$

$$= \frac{-2u^1}{u^4}$$

$$= -2u^1 u^{-4}$$

$$= -2u^{-3}$$

$$= \frac{-2}{u^3}$$

$$\boxed{f'(u) = \frac{-2}{u^3}}$$

$$g'(u) = \frac{d}{dx}[x+1]$$

$$= \frac{d}{dx}[x^1] + \frac{d}{dx}[1]$$

$$= 1x^{1-1} + 0$$

$$= 1x^0 + 0$$

$$= 1 \cdot 1 + 0$$

$$= 1$$

$$\boxed{g'(u) = 1}$$

Chain Rule It

$$f'(g(x))\, g'(x)$$

$$= \frac{-2}{(x+1)^3} \cdot 1$$

$$\boxed{f'(x) = \frac{2}{(x+1)^3}}$$

Example 5

$f(x) = \sqrt{\sin(x)}$ Get $f'(x)$

$u = \sin(x)$ $f(u) = \sqrt{u}$ $g(u) = \sin(x)$

$f'(u)$

$=$

$\frac{d}{dx}[\sqrt{u}]$

$=$

$\frac{d}{dx}[u^{1/2}]$

$=$

$\frac{1}{2}u^{(1/2-1)}$

$=$

$\frac{1}{2}u^{-1/2}$

$=$

$\frac{u^{-1/2}}{2}$

$=$

$\frac{1}{2 \cdot u^{1/2}}$

$=$

$\frac{1}{2\sqrt{u}}$

$=$

$\frac{1}{2\sqrt{\sin(x)}}$, where $\sin(x) = u$

$g'(u)$

$=$

$\frac{d}{dx}[\sin(x)]$

$=$

$g'(u) = \cos(x)$

$f'(u) = \frac{1}{2\sqrt{\sin(x)}}$

Example 5 cont.

Chain Rule I+

$f'(g(u))\,g'(u)$, where $u = x$

$$\frac{1}{2\sqrt{u}} \cdot \cos(x)$$

"

$$\frac{\cos(x)}{2\sqrt{u}}$$

"

$$f'(x) = \frac{\cos(x)}{2\sqrt{\sin(x)}}, \text{ where } \sin(x) = u$$

Example 6

$f(x) = \sec(x^2+1)$ Get $f'(x)$

$v = x^2+1$

$f(v) = \sec(v)$ or $\frac{1}{\cos(v)}$

$g(v) = x^2+1$

$f'(v)$ = $\frac{d}{dx}\left[\frac{1}{\cos(v)}\right]$ ($f(x)$ over $g(x)$)

= $\frac{\cos(v)\cdot\frac{d}{dx}[1] - 1\cdot\frac{d}{dx}[\cos(v)]}{[\cos(v)]^2}$

= $\frac{\cos(v)\cdot 0 - 1\cdot -\sin(v)}{[\cos(v)]^2}$

= $\frac{0+\sin(v)}{[\cos(v)]^2}$

= $\frac{\sin(v)}{\cos(v)^2}$

= $\frac{\sin(v)}{\cos(v)}\cdot\frac{1}{\cos(v)}$

= $\tan(v)\cdot\sec(v)$

$f'(v) = \tan(v)\sec(v)$

$g'(v)$ = $\frac{d}{dx}[x^2] + \frac{d}{dx}[1]$

= $2x^{2-1} + 0$

= $2x^1 + 0$

$g'(v) = 2x$

Chain Rule It

$f'(g(v))\,g'(v)$, where $v = x$

$\tan(x^2+1)\sec(x^2+1)\cdot 2x$

$f'(x) =$ $2x\tan(x^2+1)\sec(x^2+1)$ or $2x\sec(x^2+1)\tan(x^2+1)$