$R_i \leftrightarrow R_j$: Switch Row 'i' and Row 'j'

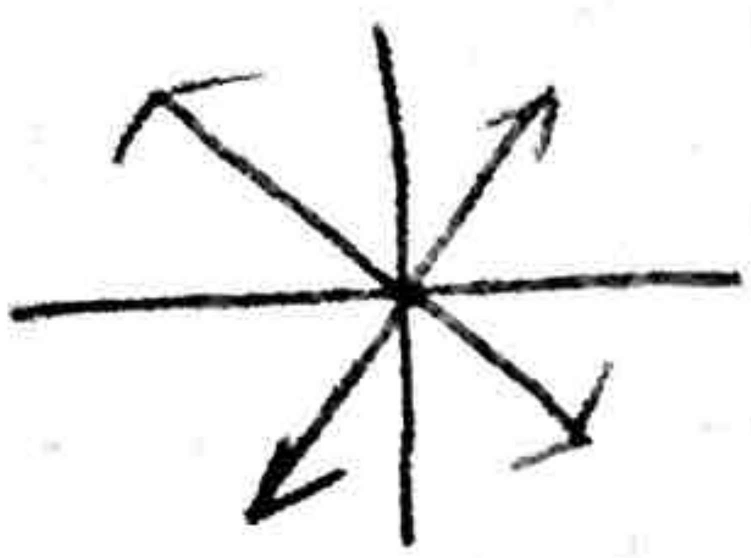
$_cR_i \rightarrow R_i$: Multiply Row 'i' by c, and make a new row 'i'.

$R_i + R_j \rightarrow$ Add Row 'i' and 'j' and replace row 'j'.

Consistent Independent

One Solution

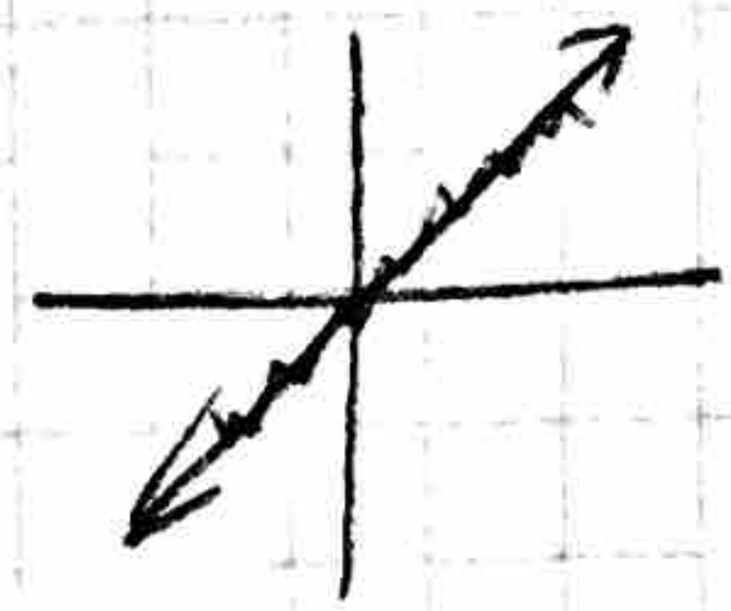
$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 1 & 5 \end{array}\right]$$



Consistent Dependent

Many Solutions

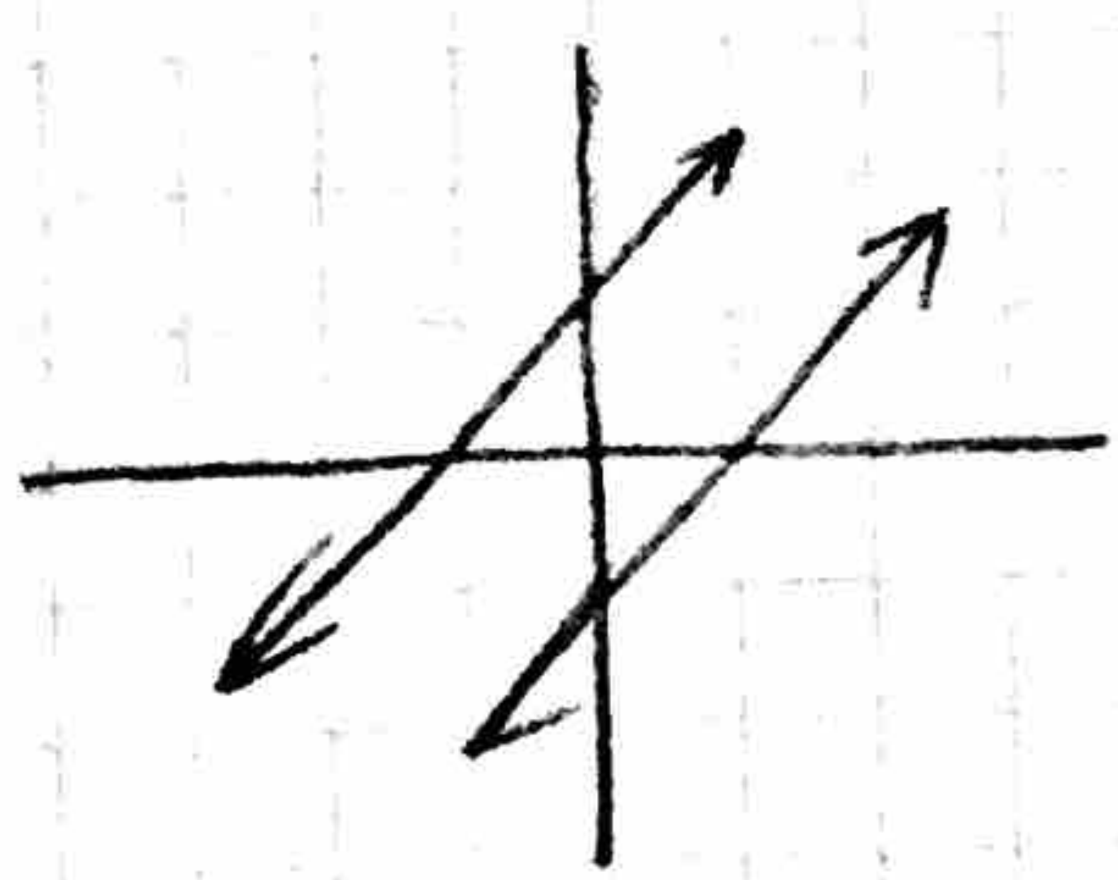
$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 0 & 0 \end{array}\right]$$



Inconsistent

No Solutions

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 0 & 14 \end{array}\right]$$



Gaussian Elimination

Gaussian and Gauss-Jordan are the same

$$\begin{aligned} x + y - z &= 9 \\ y + 3z &= 3 \\ -x - 2z &= 2 \end{aligned}$$

$$\left[\begin{array}{ccc|c} 1 & 1 & -1 & 9 \\ 0 & 1 & 3 & 3 \\ -1 & 0 & -2 & 2 \end{array}\right]$$

$1R_1 + R_3 \to R_3$

$$\left[\begin{array}{ccc|c} 1 & 1 & -1 & 9 \\ 0 & 1 & 3 & 3 \\ 0 & 1 & -3 & 11 \end{array}\right]$$

$-1R_2 + R_3 \to R_3$

$$\left[\begin{array}{ccc|c} 1 & 1 & -1 & 9 \\ 0 & 1 & 3 & 3 \\ 0 & 0 & -6 & 8 \end{array}\right]$$

$-\frac{1}{6} R_3 \to R_3$

$$\begin{array}{cccc} x & y & z & \text{constants} \end{array}$$
$$\left[\begin{array}{ccc|c} 1 & 1 & -1 & 9 \\ 0 & 1 & 3 & 3 \\ 0 & 0 & 1 & -4/3 \end{array}\right] \to \boxed{z = -4/3}$$

$$\left(\frac{2}{3}, 7, -\frac{4}{3}\right)$$

Row Echelon

1s along diagonal

$$\begin{array}{cccc} 1 & 1 & -1 & 9 \\ 0 & 1 & 3 & 3 \\ 0 & 0 & 1 & -4/3 \end{array}$$

0(s) under diagonal 1s

$$x + y - z = 9$$
$$x + (7) - \left(-\frac{4}{3}\right) = 9$$
$$x + \frac{25}{3} = 9$$
$$-\frac{25}{3} = -\frac{25}{3}$$
$$\boxed{x = 2/3}$$

$$\begin{aligned} x + y - z &= 9 \\ y + 3z &= 3 \\ \boxed{z = -4/3} \end{aligned}$$

$$y + 3z = 3$$
$$y + \frac{3}{1}\left(-\frac{4}{3}\right) = 3$$
$$y + (-4) = 3$$
$$+4 \quad +4$$
$$\boxed{y = 7}$$

Row Echelon Form

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 6 \\ 0 & 0 & 1 & 9 \end{array}\right]$$

Reduced - Row Echelon Form

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 3 \\ 0 & 1 & 1 & 6 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$$\begin{aligned} 3x - 2y + z &= -5 \\ x + 3y - z &= 12 \\ x + y + 2z &= 0 \end{aligned}$$

$$\left[\begin{array}{ccc|c} 3 & -2 & 1 & -5 \\ 1 & 3 & -1 & 12 \\ 1 & 1 & 2 & 0 \end{array}\right] \rightarrow \left[\begin{array}{ccc|c} 3 & -2 & 1 & -5 \\ 1 & 1 & 2 & 0 \\ 1 & 3 & -1 & 12 \end{array}\right] \rightarrow \left[\begin{array}{ccc|c} 3 & -2 & 1 & -5 \\ 1 & 1 & 2 & 0 \\ 0 & 2 & -3 & 12 \end{array}\right]$$

$R_3 \leftrightarrow R2$ $\qquad$ $-1R_2 + R_3 \rightarrow R_3$ $\qquad$ $\frac{1}{3}R_1 \rightarrow R_1$

$$\left[\begin{array}{ccc|c} 1 & -2/3 & 1/3 & -5/3 \\ 1 & 1 & 2 & 0 \\ 0 & 2 & -3 & 12 \end{array}\right] \quad \left[\begin{array}{ccc|c} 1 & -2/3 & 1/3 & -5/3 \\ 0 & 5/3 & 5/3 & 5/3 \\ 0 & 2 & -3 & 12 \end{array}\right] \rightarrow \left[\begin{array}{ccc|c} 1 & -2/3 & 1/3 & -5/3 \\ 0 & 1 & 1 & 1 \\ 0 & 1 & -3/2 & 6 \end{array}\right]$$

$-1R_1 + R_2 \rightarrow R_2$ $\qquad$ $\frac{3}{5}R_2 \rightarrow R_2$, $\frac{1}{2}R_3 \rightarrow R_3$ $\qquad$ $\frac{2}{3}R_2 + R_1 \rightarrow R_1$

$$\left[\begin{array}{ccc|c} 1 & 0 & 1 & -1 \\ 0 & 1 & 1 & 1 \\ 0 & 1 & -3/2 & 6 \end{array}\right] \rightarrow \left[\begin{array}{ccc|c} 1 & 0 & 1 & -1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & -5/2 & 5 \end{array}\right] \quad \left[\begin{array}{ccc|c} 1 & 0 & 1 & -1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & -2 \end{array}\right]$$

$-1R_2 + R_3 \rightarrow R_3$ $\qquad$ $-\frac{2}{5}R_3 \rightarrow R_3$ $\qquad$ $-1R_3 + R_1 \rightarrow R_1$, $-1R_3 + R_2 \rightarrow R_2$

$$\begin{array}{c} \begin{array}{cccc} x & y & z & c \end{array} \\ \left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & -2 \end{array}\right] \end{array} \quad \begin{aligned} x &= 1 \\ y &= 3 \\ z &= -2 \end{aligned} \qquad \boxed{\overset{x}{(1}, \overset{y}{3}, \overset{z}{-2})}$$