Section 1.3

Continuity Condition

1. $f(a)$ is defined
2. $\lim_{x \to a} f(x)$ exists
3. $\lim_{x \to a} f(x) = f(a)$

1. 1, 3, 4,

3a. $\frac{x+5}{x-3}$

$x - 3 = 0$
$+3 \quad +3$
$x = 3$

Domain
$(-\infty, 3) \cup (3, \infty)$

$a = 3$ $\quad \frac{3+5}{3-3} = \frac{8}{0}$

$\uparrow$ $\quad$ = under

Fails condition 1

3b. $\frac{x^2 + x - 6}{x - 2}$

$x - 2 = 0$
$+2 \quad +2$
$x = 2$

Domain
$(-\infty, 2) \cup (2, \infty)$

$a = 2$ $\quad \frac{(2)^2 + 2 - 6}{2 - 2} = \frac{4 + 2 - 6}{0}$

$=$ → under

Fails Condition 1

3c. $\sqrt{\cos(x)}$

Domain

$\downarrow$

$x = \frac{\pi}{2} + \pi k$, where $x > \frac{\pi}{2}$ and $x < \frac{3\pi}{2}$

$a = \pi$ $\quad \sqrt{\cos(\pi)}$

$\downarrow$

$\cos(\pi) = -1$

$=$

$\sqrt{-1}$

$\nearrow$

non-real

Fail Condition 1

7a 3d. $\text{Int}(x^2)$

$\downarrow$

$\text{Int}(1^2)$

$=$

1

$\lim_{x \to 1^-} \text{Int}(x^2) = 0$

$\lim_{x \to 1^+} \text{Int}(x^2) = 1$

x	$\text{Int}(x^2)$
0.0001	0
0.001	0
0.01	0

x	$\text{Int}(x^2)$
1.0001	1
1.001	1
1.01	1

$\lim_{x \to 1} \text{Int}(x^2) =$ DNE Fails condition 2

3e. $\frac{x}{\sin(x)} =$

Domain
$\sin(x) \neq 0$

$\sin(0) = 0$
$\sin(\pi) = 0$

$=$

$\frac{x}{\sin(\pi)} = \frac{x}{0}$ ← undef Fails condition 1

3f. $\frac{x}{x}$

Domain
$x \neq 0$

$\frac{x}{0} =$ undef Fails Condition 1

3g. $\ln(x^2)$

Domain
$x \neq 0$

$\ln((0)^2) =$ undef Fails Condition 1

7a. $f(x) = x^2$, $\overset{a,b}{[0,3]}$, $V=2$ $\quad a=0,\ b=3,\ f(a)=0,\ f(b)=9$

Test Continuity

$a=0$

$f(a) = f(0) = 0^2 = 0$

$\lim_{x \to 0} x^2 \to f(0) = 0^2 = 0$

$=$

0

$\lim_{x \to 0} x^2 = 0$

$f(x) = x^2$ is continuous

$f(0) = 0^2 = 0$

$f(3) = 3^2 = 9$

$0 \le 2 \le 3$

$\sqrt{2} = \sqrt{x^2}$

$=$

$\sqrt{2} = x$ or $x = \sqrt{2}$

$\sqrt{2} = x$ $\quad x \approx 1.41$

$\sqrt{2} = c$ or $c \approx 1.41$

7b. $f(x) = x^2$ on $[-1, 2]$, $V=3$

$f(-1) = (-1)^2 = 1$

$f(2) = (2)^2 = 4$

$1 \le 3 \le 4$

$\sqrt{3} = \sqrt{x^2}$

$=$

$\sqrt{3} = x$

$=$

$x = \sqrt{3}$ or $x \approx 1.73$

$c = \sqrt{3}$ or $c \approx 1.73$

7c. $f(x) = \sin(x)$ $[0, \pi/2]$, $V = 1/2$

$f(0) = \sin(0) = 0$
$f(\pi/2) = \sin(\pi/2) = 1$

$0 \leq 1/2 \leq 1$

$\frac{1}{2} = \sin x$

$\sin^{-1} x \approx 0.524$
$c \approx 0.524$

7d. $f(x) = x$ $[0, 1]$, $V = 1/3$

$f(0) = 0$
$f(1) = 1$

$0 \leq 1/3 \leq 1$

$1/3 = x$ or $x = 1/3$
$c = 1/3$

7e. $f(x) = x^2 - x$ on $[2, 5]$, $V = 4$

$f(2) = 2^2 - 2 = 4 - 2 = 2$
$f(5) = 5^2 - 5 = 25 - 5 = 20$

$2 \leq 4 \leq 20$

$4 = x^2 - x$
$-4 \quad\quad -4$

$0 = x^2 - x - 4$

Hard to Factor Out

$a = 1,\ b = -1,\ c = -4$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \frac{-1 \pm \sqrt{1^2 - 4(1)(-4)}}{2a}$

$\boxed{c = \frac{1 + \sqrt{17}}{2}} \leftarrow x = \frac{1 + \sqrt{17}}{2}$, $x = \frac{1 - \sqrt{17}}{2}$

$x = 2.56155$, $x = -1.56155$

7f. $f(x) = \ln(x)$ $[1, 10]$, $v = 2$

$\ln(1) = 0$
$\ln(10) = 2.30259$

$0 \leq 2 \leq 2.30259$

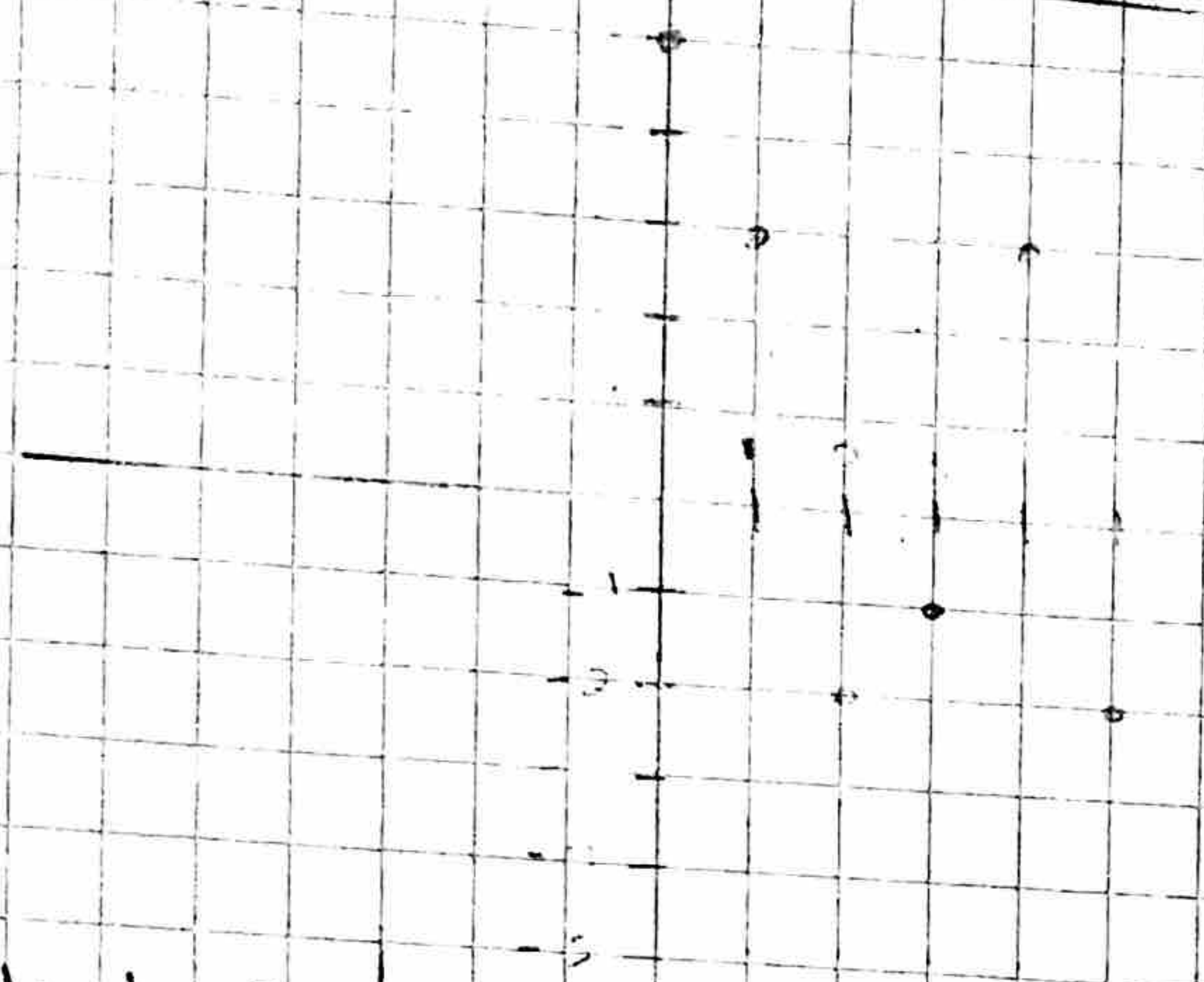
$2 = \ln x$ $\qquad \ln(x) = \log_e(x)$, $e \approx 2.71828$

$2 = \log_e(x)$
$\quad ||$
$e^2 = x$
$\quad ||$
$e^2 \approx c$
$\quad ||$
$c = 7.38905$

5.

See Desmos Graph For Points

Bisection Algorithm



5a. f has at least 3 roots between 0 and 5

a, b
[0, 5] $a = 0,\ b = 5,\ f(a) = 5,\ f(b) = -2$

1st Iteration

$m = (a+b)/2$

$= (0+5)/2$

$= 5/2$

$= m = 2.5$

$f(m) = f(2.5) = -1.5$

$f(a) = 5,\ f(m) = -1.5 \rightarrow [a, m],\ b = m$

$[0, 2.5],\ b = 2.5$ ← root

$a = 0,\ b = 2.5,\ f(a) = 5,\ f(b) = -1.5$

$[0, 2.5]$

2nd Iteration

$m = (0 + 2.5)/2$
$= 2.5/2$
$m = 1.25$

$f(m) = f(1.25) = 1.75$

$f(b) = -1.5,\ f(m) = 1.75 \to [m, b],\ a = m$
$[1.25, 2.5]\ a = 1.25$

$a = 1.25,\ b = 2.5\ f(a) = -1.75\ f(b) = -1.5$

3rd Iteration

$[1.25, 2.5]$

$m = (1.25 + 2.5)/2$
$m = 1.875$

$f(m) = f(1.875) = -1.375$

$f(a) = 1.25,\ f(m) = -1.375 \to [a, m],\ b = m$
$[1.25, 1.875]\ b = 1.875$ ← root

4th Iteration

$a = 1.25,\ b = 1.875,\ f(a) = 1.75\ f(b) = -1.375$

$[1.25, 1.875]$

$m = (1.25 + 1.875)/2$
$m = 1.5625$

$f(m) = f(1.5625) = 0.1875 \approx 0$ ← root

5b. $f(x) = 4$ at least 1 time between 0 and 5.
$f(x) = 2$ at least 3 times between 0 and 5.
$f(x) = 3$ at least 2 times between 0 and 5.

5c. Its possible applying another point.

9. $[0,5]$ $a=0$, $b=5$

$x = (a+b)/2$

$(0+5)/2$

$5/2$

$x = 2.5$,

f at 2.5 is negative

new interval $[2.5, 5]$

$x = (2.5+5)/2$

$7.5/2$

$x = 3.75$

f at 3.75 is negative

new interval $[3.75, 5]$

$x = (3.75+5)/2$

$8.75/2$

$x = 4.375$

f at 4.375 $= 0$

Bisection Algorithm converges at C.