$f(x) = \sqrt[3]{x} + 1$

$g(x) = (x-1)^3$

$x = 8 \rightarrow f(8) = \sqrt[3]{8} + 1 \rightarrow 2 + 1 \rightarrow \boxed{3}$

$x = 8 \leftarrow (2)^3 \leftarrow (3-1)^3 \leftarrow g(3) \leftarrow$

$f(x) = x + 4$

$\downarrow$

$y = x + 4$

$\downarrow$

$x = y + 4$

$-4 \qquad -4$

$\downarrow$

$x - 4 = y$

$\downarrow$

$x - 4 = f^{-1}(x) \longrightarrow f^{-1}(x) = x - 4$

$f^{-1}(1) = 1 - 4$

$\downarrow$

$f^{-1}(1) = -3$

$f(x) = x + 4$

$f^{-1}(x) = x - 4$

$f(f^{-1}(1))$

$\parallel$

$f(-3)$

$\parallel$

$f(f^{-1}(1)) = 1$

$f = \{x, x+4 \mid x \{(-\infty, \infty)\}$

Find $f^{-1}(1)$, verify $f(f^{-1}(1)) = 1$

verify $f^{-1}(f(1)) = 1$

①

$f(x) = x + 4$

$f(1) = (1) + 4$
$=$
$1 + 4$
$=$
5
$(1, 5)$

$f^{-1}(1) =$

$f(1) = (1) + 4 \rightarrow \boxed{5}$

$\leftarrow f^{-1}(5)$

$f = (1, 5)$

$f^{-1} = (5, 1)$

$f(x) = x + 4$

$f^{-1}(1) = -3$

$f(1) = 1 + 4 = \boxed{5}$

$f(x) = 2x - 1$ Find $f^{-1}(x)$

$\downarrow$

$f(x) = 2x - 1$

$\downarrow$

$y = 2x - 1$

$\downarrow$

$x = 2y - 1$

$+1 \qquad\quad +1$

$\frac{x+1}{2} = \frac{\cancel{2}y}{\cancel{2}}$

$\downarrow$

$\frac{x+1}{2} = y$

$\downarrow$

$y = \frac{x+1}{2}$

$\downarrow$

$\boxed{f^{-1}(x) = \frac{x+1}{2}}$ or $\boxed{f^{-1}(x) = \frac{1}{2}x + \frac{1}{2}}$

Let $f(x) = x^2 - 1,\ x \ge 1$. Find $f^{-1}(x)$

$\downarrow$

$f(x) = x^2 - 1$

$\downarrow$

$y = x^2 - 1$

$\downarrow$

$x = y^2 - 1$

$+1 \qquad +1$

$\downarrow$

$\sqrt{x+1} = \sqrt{y^2}$

$\downarrow$

$\sqrt{x+1} = y$

$\downarrow$

$y = \sqrt{x+1}$

$\downarrow$

$f^{-1}(x) = \sqrt{x+1},\ x \ge 1$

$$f(x) = 2x + 4$$

$$\downarrow$$

$$y = 2x + 4$$

$$\downarrow$$

$$x = 2y + 4$$

$$-4 \qquad -4$$

$$\frac{x-4}{2} = \frac{2y}{2}$$

$$\downarrow$$

$$\frac{x-4}{2} = y$$

$$\downarrow$$

$$y = \frac{x-4}{2}$$

$$\downarrow$$

$$y = \frac{x}{2} - \frac{4}{2}$$

$$\downarrow$$

$$y = \frac{1}{2}x - 2$$

$$f^{-1}(x) = 1/2x - 2$$

$$f(x) = x + 2 \qquad g(x) = x - 2$$

$$(f \circ g)(x) \rightarrow f(g(x))$$

$$f(g(x)) = (x-2) + 2$$

$$\downarrow$$

$$x - 2 + 2$$

$$\downarrow$$

$$x$$

$$g(f(x)) = (x+2) - 2$$

$$\downarrow$$

$$x + 2 - 2$$

$$\downarrow$$

$$x$$

Prove $f(x) = x^2$ and $g(x) = \sqrt{x}$ are Inverses of Each Other

① Domain of $f(x)$: $(-\infty, \infty)$
Domain of $g(x)$: $x \geq 0$

$f(g(1))$
$\downarrow$
$(\sqrt{1})^2$
$\downarrow$
$\sqrt{1}$
$\downarrow$
1

$g(f(1))$
$\sqrt{(1)^2}$
$\sqrt{1}$
1

$f(x) = 2x + 5$

$g(x) = \frac{x - 5}{2}$

$f(g(x))$
$\downarrow$
$\cancel{2}\left(\frac{x-5}{\cancel{2}}\right) + 5$
$\downarrow$
$x - 5 + 5$
$\downarrow$
x

$g(f(x))$
$\downarrow$
$\frac{(2x+5) - 5}{2}$
$\downarrow$
$\frac{2x + 5 - 5}{2}$
$\downarrow$
$\frac{2x}{2}$
$\downarrow$
x

$f(x) = x^2 + 3$

$g(x) = -\sqrt{x-3}$

$f(g(x))$

$\downarrow$

$(-\sqrt{x-3})^2 + 3$

$x - 3 + 3$

$\downarrow$

x

NOT

Inverses

$g(f(x))$

$\downarrow$

$-\sqrt{(x^2+3) - 3}$

$\downarrow$

$-\sqrt{x^2+3-3}$

$\downarrow$

$-\sqrt{x^2+3-3}$

$\downarrow$

$-\sqrt{x^2}$

$\downarrow$

$-x$

$$\begin{aligned} x - 3 &= 0 \\ +3 \quad &\;\; +3 \\ \hline x &= 3 \end{aligned}$$

$x > 3$

Prove Two Functions Are Inverses of Each Other

$f(x) = 5x + 3$ $\quad$ $g(x) = \frac{x-3}{5}$

⓪ Domain of x is all reall numbers so we can move to the next step.

① If $f(g(x)) = x$ and $g(f(x)) = x$ then $f(x)$ and $g(x)$ are inverses of each other.

②

$f(g(x)) = \cancel{5}\left(\frac{x-3}{\cancel{5}}\right) + 3$

↓

$x - 3 + 3$

↓

$\boxed{f(g(x)) = x}$

$g(f(x)) = \frac{(5x+3)-3}{5}$

↓

$\frac{5x+3-3}{5}$

↓

$\frac{\cancel{5}x}{\cancel{5}}$

↓

$\boxed{g(f(x)) = x}$

$f(g(x))$ and $g(f(x))$

ARE INVERSES OF EACH OTHER

Show that $f(x) = x^2$ and $g(x) = \sqrt{x}$ are not inverse functions of each other

$f(x) = x^2$
Domain $(-\infty, \infty)$

$g(x) = \sqrt{x}$
Domain $(0, \infty)$

$f(1) = (1)^2 = \boxed{1}$

$1 = \sqrt{1} = g(1)$

Inverse Ops

$f = \{(x, x+4) \mid x \in (-\infty, \infty)\}$

Find $f^{-1}(1)$, verify $f(f^{-1}(1)) = 1$ and $f^{-1}(f(1)) = 1$

① Define $f(x)$

$\downarrow$

$f(x) = x + 4$

② Define $f^{-1}(x)$

$\downarrow$

$f(x) = x + 4$

$\downarrow$

$y = x + 4$

$\downarrow$

$x = y + 4$

$\underline{-4 \qquad -4}$

$\downarrow$

$x - 4 = y$

$\downarrow$

$x - 4 = f^{-1}(x)$

$\downarrow$

$f^{-1}(x) = x - 4$

③

$f(x) = x + 4$

$\downarrow$

$f^{-1}(1) = 1 - 4 = -3$

$f^{-1}(x) = x - 4$

$f(f^{-1}(1))$

$=$

$f(-3) = (-3) + 4 = 1$

$f(f^{-1}(1)) = 1$

$f^{-1}(f(1)) = 1$

$=$

$f^{-1}(1 + 4)$

$=$

$f^{-1}(5)$

$=$

$f^{-1}(5 - 4) = 1 \rightarrow f^{-1}(f(1)) = 1$

$f = \{(-1,0), (0,1), (1,2), (2,4), (3,5), (4,6)\}$ Find $f^{-1}(2)$

A relation is one-to-one if both itself and inverse are functions

① Define f^{-1}

$f^{-1} = \{(0,-1), (1,0), (2,1), (4,2), (5,3), (6,4)\}$

$\downarrow$

$f^{-1}(2) = 1$

$f = \{(x, 2x+1) \mid x \in (-\infty, \infty)\}$ Find $f^{-1}(2)$

$\downarrow$

$f(x) = 2x + 1$ $\quad f^{-1}(x) = \dfrac{x-1}{2}$

$\downarrow$

$f^{-1}(2) = \dfrac{(2)-1}{2}$

$\downarrow$

$\dfrac{2-1}{2}$

$\downarrow$

$f^{-1}(2) = \dfrac{1}{2}$ or $.5$

Get $f^{-1}(x)$

$\downarrow$

$f(x) = 2x + 1$

$\downarrow$

$y = 2x + 1$

$\downarrow$

$x = 2y + 1$ Solve for y

$-1 \quad\quad -1$

$\dfrac{x-1}{2} = \dfrac{2y}{2}$

$\downarrow$

$\dfrac{x-1}{2} = y$

$\downarrow$

$y = \dfrac{x-1}{2}$

$\downarrow$

$\boxed{f^{-1}(x) = \dfrac{x-1}{2}}$

$R = \{(0,0), (1,2), (2,3), (3,3)\}$

Find the inverse relation R^{-1}

Domain = Range

Range = Domain

$R^{-1} = \{(0,0), (2,1), (3,2), (3,3)\}$

$$R^{-1} \neq \frac{1}{R}$$

$R = \{(-1,0), (0,1), (1,2), (2,4), (3,5), (4,6)\}$

$\downarrow$

$R^{-1} = \{(0,-1), (1,0), (2,1), (4,2), (5,3), (6,4)\}$

$R = \{(0,1), (1,2), (2,-1), (3,4)\}$

$R^{-1} = \{(1,0), (2,1), (-1,2), (4,3)\}$

Domain R $\{0, 1, 2, 3\}$

Range R $\{1, 2, -1, 4\}$

Domain R^{-1} $\{1, 2, -1, 4\}$

Range R^{-1} $\{0, 1, 2, 3\}$

$\boxed{\begin{array}{l} \text{Domain } R = \text{Range } R^{-1} \\ \text{Domain } R^{-1} = \text{Range } R \end{array}}$

$f = \{(0,1), (1,2), (2,-1), (3,4)\}$

Find $f(-1)$, $f(0)$, $f(2)$, $f^{-1}(-1)$ $f^{-1}(1)$

Verify $f(f^{-1}(-1)) = -1$ and $f^{-1}(f(1)) = 1$

$\downarrow$

① $f = \{(0,1), (1,2), (2,-1), (3,4)\}$

$f^{-1} = \{(1,0), (2,1), (-1,2), (4,3)\}$

$f(-1) = \text{undef}$

$f(0) = 1$

$f(2) = -1$

$f^{-1}(-1) = 2$

$f^{-1}(1) = 0$

$$\boxed{\begin{array}{l} f(\underset{\parallel}{f^{-1}(-1)}) = 1 \rightarrow f(f^{-1}(x)) = f \text{ Domain }(x) \text{ values} \\ f(2) = 1 \\ \\ f^{-1}(\underset{\parallel}{f(1)}) = 1 \rightarrow f^{-1}(f(x)) = f^{-1} \text{ Domain}(x) \text{ values} \\ f^{-1}(2) = 1 \end{array}}$$

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Let $f(x) = \frac{6x+5}{4}$. Find $f^{-1}(x)$

$\downarrow$

$f(x) = \frac{6x+5}{4}$

$\downarrow$

$y = \frac{6x+5}{4}$

$\downarrow$

$4(x) = \left(\frac{6y+5}{\cancel{4}}\right)\cancel{4}$

$\downarrow$

$$\begin{array}{rcl} 4x & = & 6y + 5 \\ -5 & & -5 \\ \hline \end{array}$$

$\downarrow$

$\frac{4x-5}{6} = \frac{\cancel{6}y}{\cancel{6}}$

$\downarrow$

$\frac{4x-5}{6} = y$

$\downarrow$

$y = \frac{4x-5}{6}$

$\downarrow$

$f^{-1}(x) = \frac{4x-5}{6}$