



Quiz 1 Review

(Lectures 1-5)

March 4, 2016

Quiz #1

(LECTURES 1-5)

Date: Tuesday, March 8

Time: 7:40 pm through March 9, 7:40 pm
(Eastern time)

Location: Online, MATH E-3 Canvas course site
(no proctor needed)

No class meeting

Quiz #1 is open book. You will have 75 minutes to complete the quiz.

Agenda

Jessica

- Operations and Numbers
- Percents
- Q&A

Sue

- Summary Statistics
- Probability
- Q&A

Know how to use your calculator

- Find + - * / : add, subtract, multiply, divide
- Find square root key
- Find the factorial key !
- Try PEMDAS problems with calculator



MATH E-3 Roadmap (so far)...

Working w/Numbers 1

- Types of Numbers
- Manipulation of decimals, percents, and fractions
- Order of Operations aka PEMDAS

Working w/Numbers 2

- Three Types of Percent Problems
- Problems involving Percent Increase
- Problems involving Percent Decrease

Introduction to Statistical Concepts

- Mean
- Median
- Mode
- Range
- Standard Deviation
- Histograms

Working with Numbers 1

- Types of Numbers
- Manipulation of decimals, percents, and fractions
- Order of Operations aka PEMDAS

The Complex Number System

Real Numbers

Rational numbers Irrational numbers
Integers
Whole numbers
Natural numbers

Imaginary Numbers

Although not used in MATH E-3, an example of an imaginary number is “i” which is the equivalent to $\sqrt{-1}$.

Real Numbers

Rational Numbers

Integers and Fractions

(...-3.5, -1/2, 0, 8/5, 3...)

Integers

Whole Numbers and Negatives

(...-3, -2, -1, 0, 1, 2, 3...)

Whole Numbers

Natural Numbers and Zero

(0, 1, 2, 3, 4, 5, ...)

Natural Numbers

Counting Numbers

(1, 2, 3, 4, 5, 6...)

Irrational Numbers

Numbers that cannot be expressed as a ratio or a fraction. The decimal equivalent of a fraction which either does not terminate or does not repeat a pattern.

(... $\sqrt{2}$, e , π ...)

Examples: Types of numbers

What type of number is $\frac{2}{3}$?

Is it a natural number?

Is it a whole number?

Is it an integer?

Is it rational?

Is it irrational?

Is it real?

Examples: Types of numbers

What type of number is $\frac{2}{3}$?

Is it a natural number?

No, it is a fraction and therefore does not fit in the counting numbers: 1,2,3...

Is it a whole number?

No, it is a fraction and therefore does not fit the in whole numbers which are the counting numbers plus 0.

Examples: Types of numbers

What type of number is $\frac{2}{3}$?

Is it an integer?

No, because it is a fraction, and not part of the integers which include all the whole numbers and their negatives:

... -3, -2, -1, 0, 1, 2, 3....

Examples Types of Numbers

What type of number is $\frac{2}{3}$?

Is it rational?

Yes, because it can be expressed as the ratio of 2 integers.

Is it irrational?

It can't be rational and irrational so:

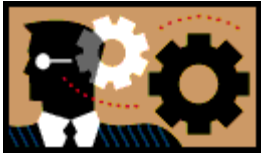
No, it is not irrational.

Examples: Types of Numbers

What type of number is $\frac{2}{3}$?

Is it real?

Yes, because it belongs to one of the sets of numbers that comprise the real number system. It is a rational number, and therefore is real.



Rational vs. Irrational



- Rational numbers can be expressed as the ratio of two integers
- They include natural numbers, whole numbers, integers
- So 1 is a rational number
- -6 is a rational number
- 0 is a rational number

Rational vs. Irrational

- Rational numbers also include other numbers like fractions and their decimal equivalent
- $\frac{1}{2}$ is rational
 - Its decimal equivalent is .50 which terminates
- $\frac{44}{45}$ is rational
 - Its decimal equivalent is .977777... which is a repeating decimal

Rational vs. Irrational

- So, what is left for the irrational set?
- It is a number that can't be expressed as ratio of two integers:
- And therefore its decimal equivalent does not terminate and does not repeat

$$\pi = 3.14159265359....$$

$$\sqrt{2} = 1.414213562...$$

Are all square roots irrational?

NO!

$\sqrt{4} = 2$ OR -2 both of these are rational,
and therefore $\sqrt{4}$ is rational

Square roots

The symbol $\sqrt{\quad}$ is called a radical sign

Means take the root of ...

Or find a number that when multiplied by itself equals what is under the radical sign

$$\sqrt{4} = 2 \text{ or } -2 \text{ since } 2 \times 2 = 4 \text{ and } -2 \times -2 = 4$$

Square Roots

$$\sqrt{49} = 7 \text{ or } -7 \text{ because } 7 * 7 = 49$$

$$\text{and } -7 * -7 = 49$$

- When there is no number outside the radical sign, it is the **square** root

Other Roots

- Cube roots:

- $\sqrt[3]{125} = 5$ because $5 * 5 * 5 = 125$

- Fourth root:

- $\sqrt[4]{81} = 3$ because $3 * 3 * 3 * 3 = 81$

Exponents

The opposite of roots is raising a number to a power

Means multiple the base by the number of times indicated by the exponent

Example:

5^2 : the 5 is called the base, and the 2 is the exponent

5^2 is 5 squared which means multiply 5 by itself or
 $5 * 5 = 25$

Exponents

- 3^3 is 3 cubed which means multiply $3*3*3=27$
- 2^6 is 2 to the 6th power which means
- $2*2*2*2*2*2=64$
- Special case: anything to zero power =1 so $(2,950,000)^0 = 1$

Factorials!



! is not just a punctuation mark that conveys enthusiasm

It is a mathematical function which means take the number indicated and multiply it by each preceding number until you get to 1

Therefore $8! = 8 * 7 * 6 * 5 * 4 * 3 * 2 * 1 = 40,320$

Percents, Decimals, Fractions

- A fraction is the ratio of two integers:
 $\frac{3}{4}, \frac{1}{5}, \frac{15}{4}, \frac{21}{8}$
- They can have a smaller number in the numerator ($\frac{3}{4}, \frac{1}{5}$) and are called proper fractions, or
- They can have a larger number in the numerator ($\frac{15}{4}, \frac{21}{8}$) and are called improper fractions

Fraction to a decimal

- Fractions can be converted to decimals simply by dividing the denominator (the bottom number) into the numerator (the top number)

Example:

$$\frac{4}{7} = .571428\dots \text{repeats pattern}$$

You get this by dividing 7 into 4

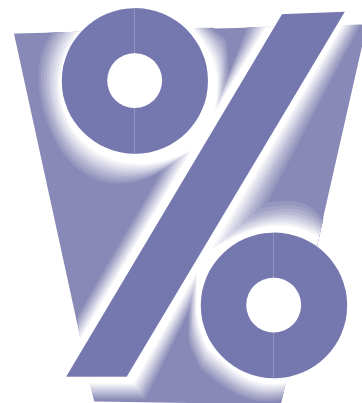
Fraction to decimal

- Another example:

- $\frac{19}{5} = 3.80$

We did this by dividing 19 by 5

Percents



Per Cent means “out of 100.”

Therefore:

65% means “65 out of 100” or $\frac{65}{100}$

5 ½ percent means 5 ½ out of 100 or $\frac{5.5}{100}$

So far, so good, yes?

Changing a decimal into a percent

To change from a decimal to a percent, multiply by 100 or move the decimal point two places to the right. Remember to attach the % sign.

Therefore:

$$.75 = 75\%$$

$$1 = 100\%$$

$$.025 = 2.5\%$$

$$1.5 = 150\%$$

Changing a percent into a decimal

To change from a **percent to a decimal**, divide by 100 or move the decimal point two places to the **left**. Remember to **remove** the % sign.

Therefore:

$$75\% = = 0.75$$

$$2.5\% = = .025$$

$$100\% = = 1$$

$$150\% = = 1.5$$



The Order of Operations



AKA

PEMDAS

You remember PEMDAS, right?

When working with expressions with parentheses, we need to be careful of the order in which the operations are performed. By convention, we follow the rules in the order given below.

P **Parentheses (innermost first)**

E **Exponents**

M} **Multiplication/Division (left to right whichever comes first)**

D} **Division/Multiplication (left to right whichever comes first)**

A} **Addition/Subtraction (left to right whichever comes first)**

S} **Subtraction/Addition (left to right whichever comes first)**

PEMDAS Revisited

$$6 + 7 + 8 \div 2(3+2) * 3^2 - 6 + 3$$

Step One: Do what's in the **Parentheses first** rewriting the entire problem to avoid confusion

$$= 6 + 7 + 8 \div 2(5) * 3^2 - 6 + 3$$

Step Two: Exponents

$$= 6 + 7 + 8 \div 2(5) * 9 - 6 + 3$$

PEMDAS Revisited

$$6 + 7 + 8 \div 2(3+2) * 3^2 - 6 + 3$$

Step Two: Exponents

$$= 6 + 7 + 8 \div 2(5) * 9 - 6 + 3$$

Step Three: Multiplication/Division (left to right)

$$= 6 + 7 + 4(5) * 9 - 6 + 3$$

$$= 6 + 7 + 20 * 9 - 6 + 3$$

$$= 6 + 7 + 180 - 6 + 3$$

PEMDAS revisited

$$18 - 5 * 2(6-9) - 3^2 \div 2 * 4$$

Step Three: Multiplication/Division (left to right)

$$= 6 + 7 + 180 - 6 + 3$$

Step Four: Addition/Subtraction

(left to right whichever comes first)

$$= 6 + 7 + 180 - 6 + 3$$

$$= 13 + 180 - 6 + 3$$

$$= 193 - 6 + 3$$

$$= 187 + 3$$

$$= 190$$

Working with Numbers 1

- Three Types of Percent Problems
- Problems involving Percent Increase
- Problems involving Percent Decrease

Translating English to Math...

Math is a language. You can translate from English to math:

“of” means multiply *

“is” means equal =

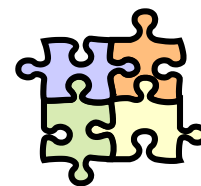
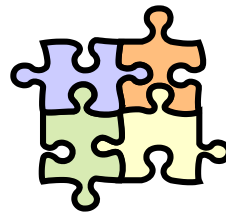
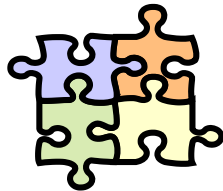
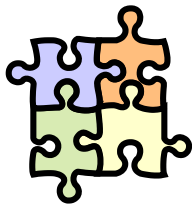
“what” is the unknown x

Three Types of Percent Problems

Type 1 asks: **What is?**

Type 2 asks: **What %?**

Type 3 asks: **of What?**



1st Type: What is?

What is 17.5% of \$814?

For this problem, we can “translate” the question into a math sentence – an equation.

What is $.175 * 814$?

$X = .175 * 814$

$X = 142.45$

So, our answer is : \$142.45

2nd Type: What %?

325 is what % of \$500?

Again, we can “translate” the question into a math sentence – an equation.

325 is what % * \$500?

$$325 = \frac{x}{100} * \$500$$

Because the x is not isolated, we'll need to use some algebra (but never fear!).

Let's take it step by step



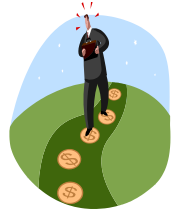
325 is what % * \$500? (what we want to know)

$$325 = \frac{x}{100} * \$500 \text{ (equation)}$$

Question: So how do we isolate x?

Answer: We must move the 100 and 500 to the other side of the equal sign (left hand side).

Let's take it step by step

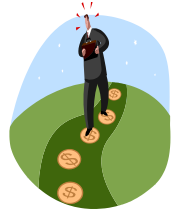


$$325 = \frac{x}{100} * \$500 \text{ (equation)}$$

Question: So how do we move the 100 and 500 to the left hand side of the equal sign?

Answer: We must “undo” their operations, by doing the opposite. In other words, we must do the same thing to both sides – to keep it as an equation.

Let's take it step by step



$$325 = \frac{x}{100} * \$500 \text{ (equation)}$$

Question: Exactly how do we “undo” their operations, by doing the opposite?

Answer: The 100 is being divided, so we multiply both sides by 100; the 500 is being multiplied, so we divide both sides by 500.



Let's take it step by step



$$325 = \frac{x}{100} * \$500 \text{ (equation)}$$

So let's start by multiplying both sides by 100.

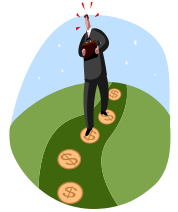
$$(100*) 325 = \frac{x}{100} (100*) * \$500$$

By doing this, we now have:

$$32,500 = x * 500$$



Let's take it step by step



$$32,500 = x * 500$$

Now let's divide both sides by 500.

$$\frac{32,500}{500} = \frac{x * 500}{500}$$

Now we have:

$$\frac{32,500}{500} = x \text{ (we're almost there)}$$



Let's take it step by step



Now we must divide 32,500 by 500.

$$\frac{32,500}{500} = \text{X}$$

$$65 = \text{X}$$

When we do this, we get 65. Now just add the percent sign.

So for our question: 325 is what percent of 500,

our answer is **65%**.

3rd Type: of what?

\$142.45 is 17.5% of what?

Again, we can “translate” the question into a math sentence – an equation.

142.45 is 17.5% of x

$142.45 = .175 * x$

Because the x is not isolated, we will again need to use some algebra.

3rd Type: of what?

$$142.45 = .175 * x$$

To isolate “x,” divide both sides by .175

$$\frac{142.45}{.175} = \frac{.175}{.175} * x$$

$$814 = x$$

So for our question: 142.45 is 17.5% of what, our answer is 814.

(Where have you seen these numbers before?)

Problems Involving Percent Change

Don't forget the formula!!



$$\% \text{ change} = \frac{(new - old)}{old} * 100$$

Percent Increase

Example: Let's say that the average gas price, in the United States has increased from \$3.51 to \$3.69. Yikes! What is the percent increase of this difference?

$$\% \text{ change} = \frac{(\text{new} - \text{old})}{\text{old}} * 100$$

Percent Increase

$$\% \text{ change} = \frac{(\text{new} - \text{old})}{\text{old}} * 100 \quad (\text{our formula})$$

$$\% \text{ change} = \frac{(3.69 - 3.51)}{3.51} * 100 =$$

$$\% \text{ change} = \frac{.18}{3.51} * 100 =$$

$$\% \text{ change} = .0512 * 100 = 5.12 \text{ or } 5.1\% (1 \text{ dp})$$

Percent Decrease

Example:

A car valued at \$84,000 in 2012 has decreased in value to \$72,000 in 2013. What is the percent decrease of this difference?

$$\% \text{ change} = \frac{(\text{New} - \text{Old})}{\text{Old}} * 100$$

Percent Decrease

$$\% \text{ change} = \frac{(\text{new} - \text{old})}{\text{old}} * 100 \quad (\text{our formula})$$

$$\% \text{ change} = \frac{(72,000 - 84,000)}{84,000} * 100 =$$

$$\% \text{ change} = \frac{-12,000}{84,000} * 100 =$$

$$\% \text{ change} = .142857143 * 100 = 14.28 \text{ or } 14.3\% (1 \text{ dp})$$

Other Examples of Percent Problems

Now we will look at problems in which we have the original amount and we want to see what the new amount will be depending upon a particular percentage increase or decrease.

These types of problems can be solved in two ways – a longer way and a shorter way. We ultimately would like you to solve these problems using the shorter way.

Increasing an original amount by a percent

What is 3018 increased by 37% What is 3018 increased by 37%

The Longer way:

1) Find 37% of 3018

$$.37 * 3018 = 1116.66$$

2) Add the 37% to the original
amount.

$$3018 + 1116.66 \\ = 4134.66$$

The Shorter way:

$$3018 * 1.37 \\ = 4,134.66$$

Decreasing an original amount by a percent

What is 553 decreased by
13%?

What is 553 decreased by
13%?

The Longer way:

1) Find 13% of 553.

$$.13 * 553 = 71.89$$

2) Subtract the 13% from the
original amount.

$$553 - 71.89 = 481.11$$

The Shorter way:

$$553 * .87 = 481.11$$

Introduction to Statistical Concepts

- Mean
- Median
- Mode
- Range
- Standard Deviation
- Histograms

Descriptive Statistics

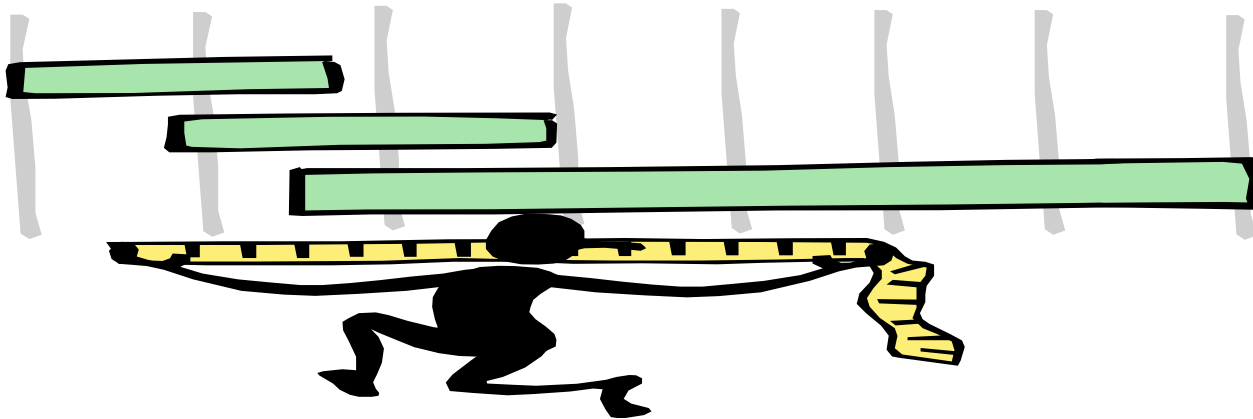
In Lecture 4, we introduced the topic of **descriptive statistics**.

Descriptive statistics provides a summary – either numeric or graphic – of collected quantitative data.

We will use **two measures** for this summary.

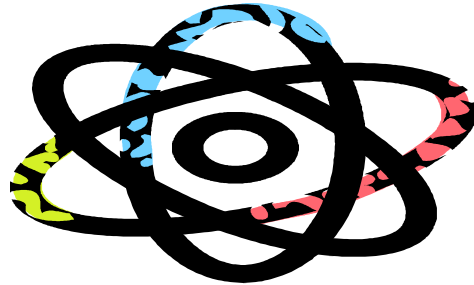
The Two Measures of Quantitative Data are:

1. measures of centrality
2. measures of spread or dispersion



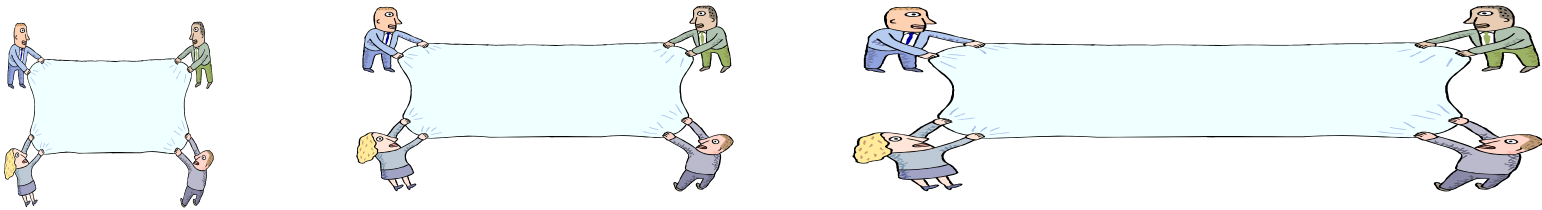
Measures of Centrality

- Mean
- Median
- Mode



Measures of Spread or Dispersion

- **Range**
- **Standard Deviation**

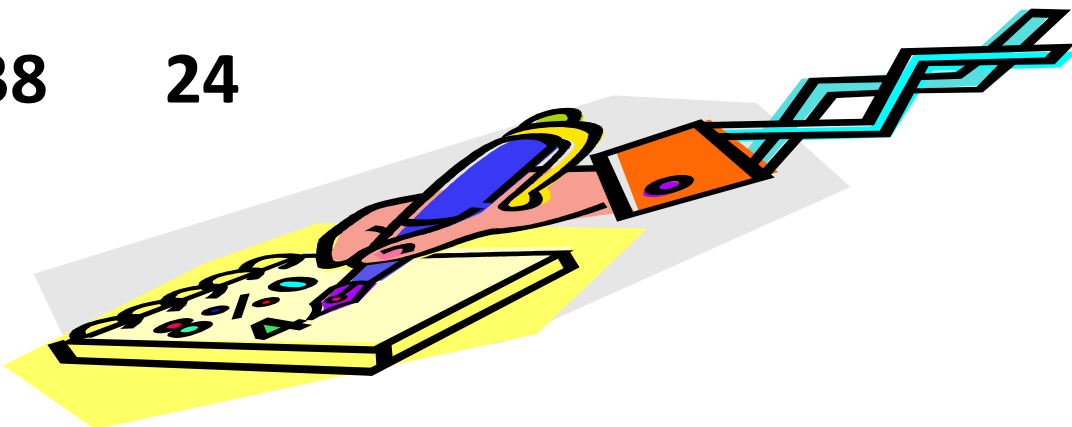


Example A

A Bit of Number Crunching (just to make sure it all makes sense)

Here are the ages of a group of Harvard Extension School Students:

19	57	28	34
28	36	38	24



Measures of Centrality

Calculate the following:

a. **mean** age of these students

Add the ages:

$$19 + 57 + 28 + 34 + 28 + 36 + 38 + 24 = 264$$

Divide by the number of ages:

$$264 \div 8 = 33$$

Answer: the **mean** age is **33** years old

Measures of Centrality

Calculate the following:

b. **median** age of these students

Place ages in order from smallest to largest

19 24 28 28 34 36 38 57

Find the location using the formula: $n \div 2$

$$\mathbf{8 \div 2 = 4}$$

Find the average of the 4th and 5th numbers:

$$\mathbf{28 + 34 \div 2 = 31}$$

Answer: the **median** age is **31**

Measures of Centrality

Ages	Frequency	Position
19	1	1 st
24	1	2 nd
28	2	3 rd and 4 th
34	1	5 th
36	1	6 th
38	1	7 th
57	1	8 th

Measures of Centrality

Calculate the following:

c. **mode** of these ages

Place ages in order from smallest to largest

19 24 **28 28** 34 36 38 57

Identify the most frequently occurring number

28

Answer: the **mode** is **28** years old

Measures of Dispersion (or spread/variability)

Calculate the following:

a. **range** of these ages

Subtract the lowest value from the highest value:

$$57 - 19 = 38$$

Answer: **38** years old

Measures of Dispersion (or spread/variability)

Calculate the following:

b. standard deviation (part 1)

x	X-xbar	(X-xbar) ²	frequency	f*(x-xbar) ²
19	19-33=-14	196	1	196
24	24-33=-9	81	1	81
28	28-33=-5	25	2	50
34	34-33=1	1	1	1
36	36-33=3	9	1	9
38	38-33=5	25	1	25
57	57-33=24	576	1	576
			8	938

Measures of Dispersion (or spread/variability)

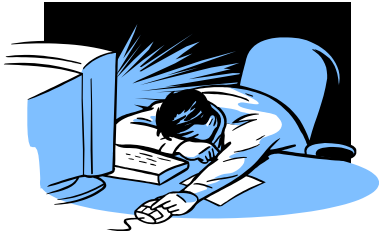
b. standard deviation (part 2)

$$\sigma = \sqrt{\frac{\sum (x - \bar{x})^2}{n}}$$

- Divide 938 by 8 = 117.25
- Take the square root of 117.25
- The answer is 10.82820391, **10.8**(1 dp)

Let's try one more problem in which we also will look at grouped data that can be displayed by a histogram. In this example, we will look at sleep time the previous night for a group of college students who were preparing for a final exam in statistics.





Example B

Are You Getting Enough Sleep?

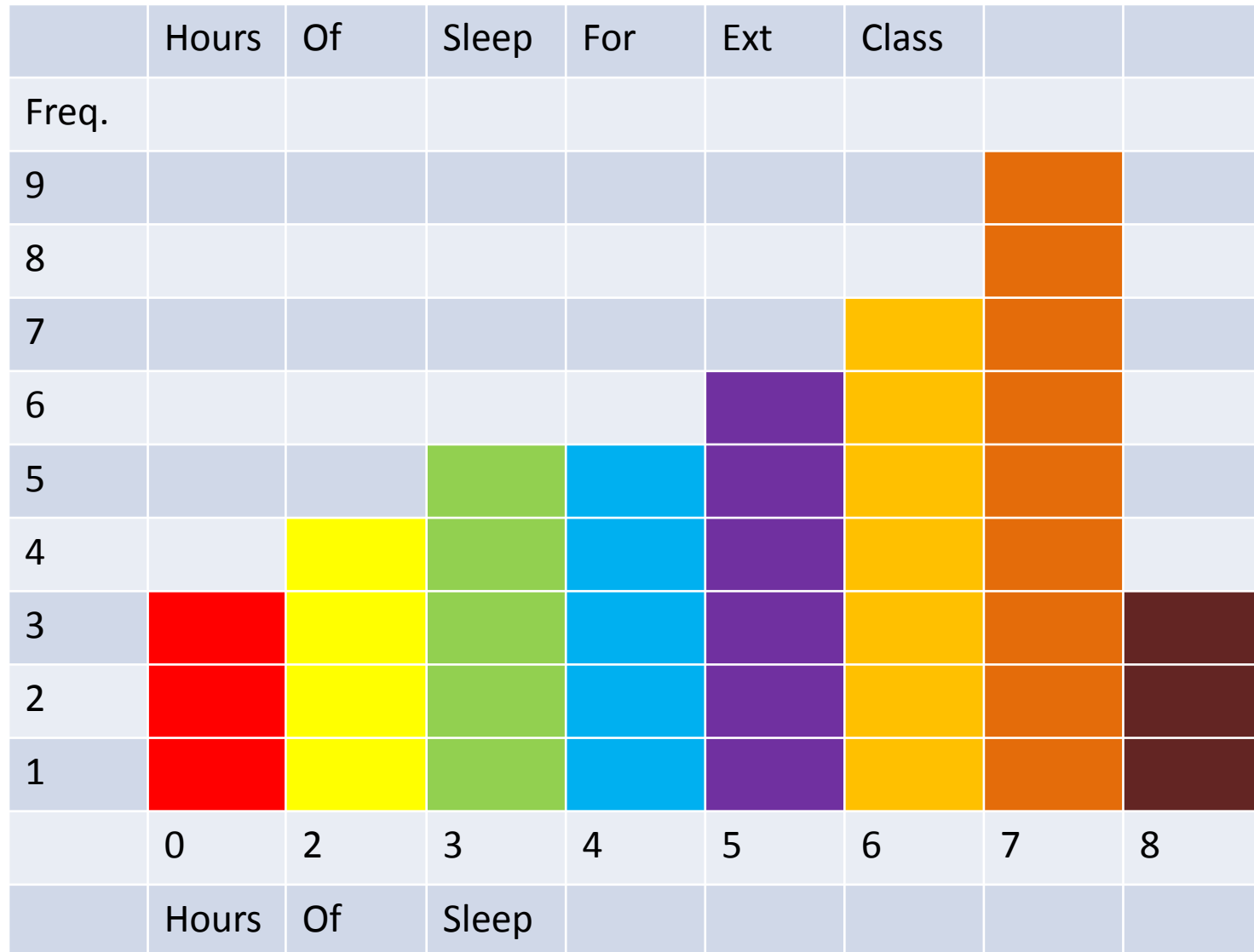


The data below shows group sleep time in hours. The least amount of sleep was 0 hours and the most amount of sleep was 8 hours.

Hours of Sleep	# of Students who slept the corresponding hours
no sleep	3
2 hours	4
3 hours	5
4 hours	5
5 hours	6
6 hours	7
7 hours	9
8 hours	3



Histogram



How would you describe the shape of the histogram?

Answer

The histogram is skewed left. In other words, the tail is on the left side of the graph.

How would you calculate the **mean** amount of sleep for these students?

Solution:

$$\text{Mean} = \frac{0(3)+2(4)+3(5)+4(5)+5(6)+6(7)+7(9)+8(3)}{42}$$

$$= \frac{0+8+15+20+30+42+63+24}{42}$$

$$= \frac{202}{42}$$

$$= 4.8 \text{ (rounded to 1 dp)}$$

How would you calculate the **median** amount of sleep for these students?

Solution:

- To find where the median will be, use the formula $n \div 2$.
- Because we have an even number of students, the location of the median will be a whole number.
- Thus $42 \div 2 = 21$
- Now look at the frequency table at the 21st and 22nd position and see the hours with which each position is associated.

How would you calculate the median amount of sleep for these students?

Here's our data:

Hours of Sleep # of Students who slept the corresponding hours

	no sleep	3	
	2 hours	4	
	3 hours	5	
	4 hours	5	
	5 hours	6	
	6 hours	7	
	7 hours	9	
	8 hours	3	

How would you calculate the median amount of sleep for these students?

- Here we see that the number of hours associated with the position 21st position is 5 and the number associated with the 22nd position also is 5.
- Next, we will take the average of these two numbers which is calculated by taking $(5+5) \div 2$. Thus, the average of the two numbers is simply 5.

Answer: the median number of hours is 5.

Which is greater, the mean or the median?

Why?

Answer:

- The mean, of 4.8 is less than the median of 5.
- Since the distribution is skewed left, the mean is pulled in that direction, making it slightly less than the median in this case.

What is the mode of these sleep hours?

Solution:

The most common number of sleep hours (the highest bar on the histogram) is 7.

no sleep	3
2 hours	4
3 hours	5
4 hours	5
5 hours	6
6 hours	7
7 hours	9
8 hours	3

Answer: the mode of these grades is 7.

Find the range of these sleep hours.

Solution:

8 (the highest number) – 0 (the lowest number)
= 8.

Answer: Thus, the range of these sleep hours is
8.

Find the standard deviation of these sleep hours.

x	$X - \bar{x}$	$(x \text{ minus } \bar{x})^2$	f	$(x \text{ minus } \bar{x})^2 * f$
0	$0 - 4.8 = -4.8$	23.04	3	69.12
2	$2 - 4.8 = -2.8$	7.84	4	31.36
3	$3 - 4.8 = -1.8$	3.24	5	16.2
4	$4 - 4.8 = -.8$.64	5	3.2
5	$5 - 4.8 = .2$.04	6	.24
6	$6 - 4.8 = 1.2$	1.44	7	10.08
7	$7 - 4.8 = 2.2$	4.84	9	43.56
8	$8 - 3.8 = 3.2$	10.24	3	30.72
			42	$\Sigma = 204.48$ (we're almost there!)



Solution:

Now that we have 204.48, we must divide it by 42 which equals 4.868571429

The final step is to take it's square root – don't forget the final step!

$$= \sqrt{4.868571429}$$

$$= 2.206483952$$

$$= \mathbf{2.2} \text{ rounded to one decimal place}$$



YEAH!!

Probability

- Making predictions and estimations of the chances of certain things happening
- Type I
 - Know in advance the chances of getting something

Probability Type I

Dichotomous Variable:

An event that can only occur two ways

➤ Coins

✓ Head/tail

➤ Births

✓ Girl/boy

➤ Switches

✓ On/Off

Probability Type I

Probability =

$$\frac{\text{number of outcomes satisfying the condition}}{\text{total number of possible outcomes}}$$

Provided each outcome is **equally**
likely to occur

Coins



- When you flip one coin, you have 2 possible outcomes Head or Tail
- When you flip 2 coins, you have 4 possible outcomes

HH HT
TH TT

- When you flip 3 coins, you have 8 possible outcomes

HHH HHT HTH THH
THT TTH TTT TTT

Pattern Emerging

Number of Coins	Number of Outcomes	$2^{\text{(number of coins)}}$
1	2	$(2)^1$
2	4	$(2)^2$
3	8	$(2)^3$
4	16	$(2)^4$

Total possible outcomes

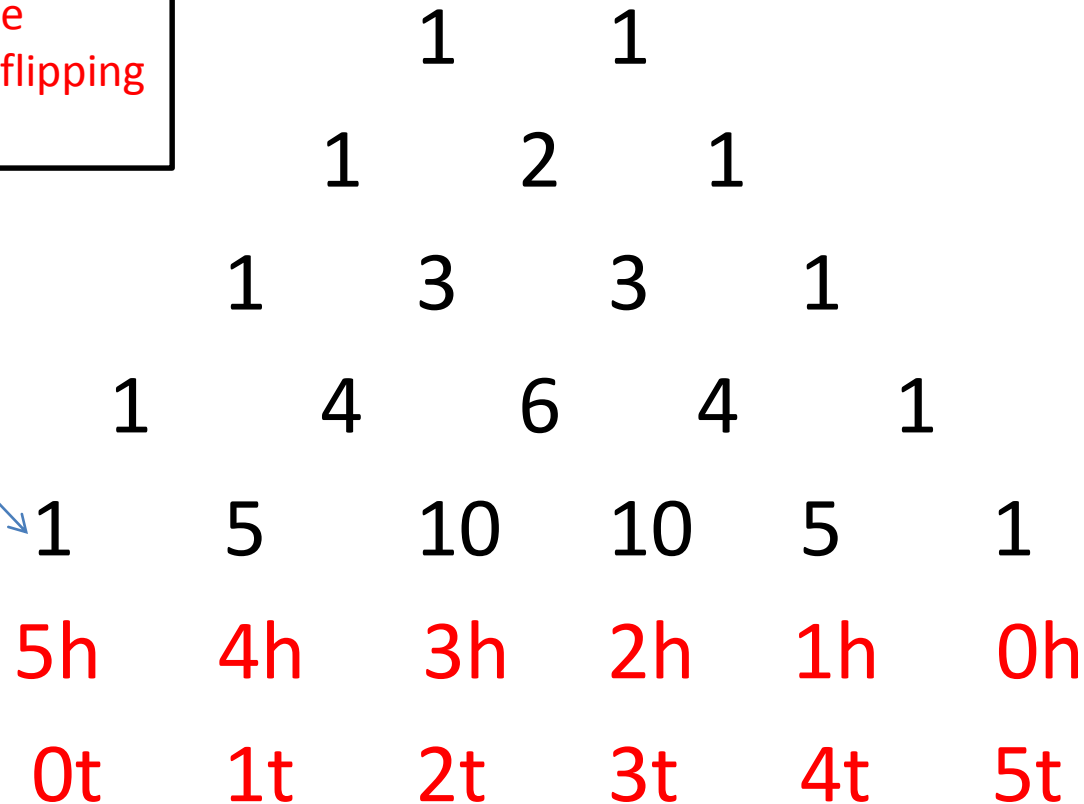
Number of Outcomes =

$$2^n$$

Where n = number of coins

Pascal's Triangle

This line gives the probabilities for flipping 5 coins.



Calculating Probability with Pascal's Triangle

Probability that you will get exactly 2 Heads when you flip 5 coins:

➤ Go to 5 Coin Row

1	5	10	10	5	1
5h	4h	3h	2h	1h	0h

➤ Find 2 Heads

➤ Look above: the number is 10

➤ This is your numerator

Probability of getting 2 Heads

- You have the numerator of your equation is 10
- Now get your denominator
 - You can add across the 5 coin row

$$\begin{array}{cccccc} 1 & 5 & 10 & 10 & 5 & 1 = 32 \\ 5h & 4h & 3h & 2h & 1h & 0h \end{array}$$

- Or, remember number of outcomes = 2^n

$$2^5 = 32$$

Exciting finale

- We have a numerator, 10, and denominator, 32, and we are ready to calculate the probability of getting two heads when you flip 5 coins!!!!
- Probability = $\frac{10}{32} = .3125 = .31$ (2 dp)

Some definitions

At least:

The smallest amount you can get from a group

- For example, if asked “What is the probability of getting at least 4 heads when you toss 5 coins?”
 - The condition is satisfied by getting 4 heads **or** 5 heads.

At most:

The largest amount you can get from group

For example, if asked “What is the probability of getting, at most, 4 heads when you toss 5 coins?”

The condition is satisfied by getting 4 heads, 3 heads, 2 heads, 1 head or 0 heads.

Example of “At least”

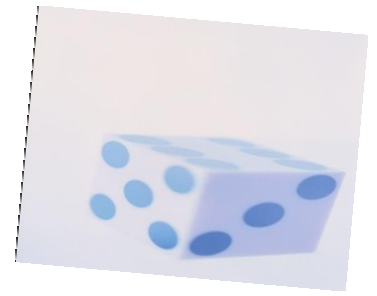
$$\begin{array}{cccccc} 1 & 5 & 10 & 10 & 5 & 1 = 32 \\ 5h & 4h & 3h & 2h & 1h & 0h \end{array}$$

- The probability of getting 4 heads = $5/32$
- The probability of getting 5 heads = $1/32$
- The probability of getting at least 4 heads =
 $5/32 + 1/32 = 6/32 = .1875 = .19$

“And/Or”

- When you see “**and**” in the probability problems you must **multiply**
- What is the probability that you will get a head when you toss a coin 3 times:
 - **$1/2 * 1/2 * 1/2 = 1/8 = .125 = .13$**
- When you see “**or**” in the probability problems you must **add**.

Dice



- When using 1 die, you have 6 possible outcomes: 1, 2, 3, 4, 5, 6
- To calculate the probability of getting a 2 on one roll of one die would simply be $\frac{1}{6} = .1666... = .17$ (2dp)
- To calculate the probability of getting an odd number with one roll: you could get a 1, a 3, or a 5 so you have $\frac{3}{6} = .50$

Two Dice



- The total outcomes grow exponentially!!!
- $6 \text{ sides}^2 = 36$ possible outcomes with 2 dice

	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

Example Dice

- What is the probability of getting a 6 when you throw two dice?

	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

Probability of getting a 6

- There are 5 sixes in the dice table
- They represent all the possible combinations of two dice that would give you a total of 6
- So the probability of getting a six=

5 ways to get six

36 total possibilities

- $5/36 = .13888... = .14(2 \text{ dp})$

More Dice

- What is the probability of getting a number less than 9?
 - This means your total would need to be 8, 7, 6, 5, 4, 3, 2, or 1. You would not include 9.

	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

Probability of number less than 9

- There are 26 combinations possible to get a number less than 9
- Probability = $26/36 = .72222... = .72$ (2 dp)

Megabucks



The Megabucks game in Massachusetts has you pick 6 numbers from 1 to 48

❖ Order doesn't matter so

16 45 23 18 4 37 is the same to the Mass State Lottery
as 18 37 16 4 45 23

❖ You can't have any duplicates

Megabucks

To calculate the total number of possible outcomes when you pick 6 numbers from 1 to 48:

First calculate how many different combinations there are :

$$48 * 47 * 46 * 45 * 44 * 43 =$$
$$8,835,488,640$$

Megabucks

- The 8,835,488,640 total combinations include multiples of the same 6 digit combination just in different orders
- So, you must reduce the above number by figuring out how many ways a six digit number can be represented:
- $6! = 6 * 5 * 4 * 3 * 2 * 1 = 720$

Megabucks

- This means the same group of six numbers can be generated 720 times just in differing orders
- So, to get the true number of possible outcomes, divide 8,835,488,640 by 720:
- The result =12,271,512
- We now have the denominator to our probability problem

Megabucks, the sad truth

If you buy 1 ticket, the probability of winning is...

$$1/12,271,512 = .000000081$$

Very

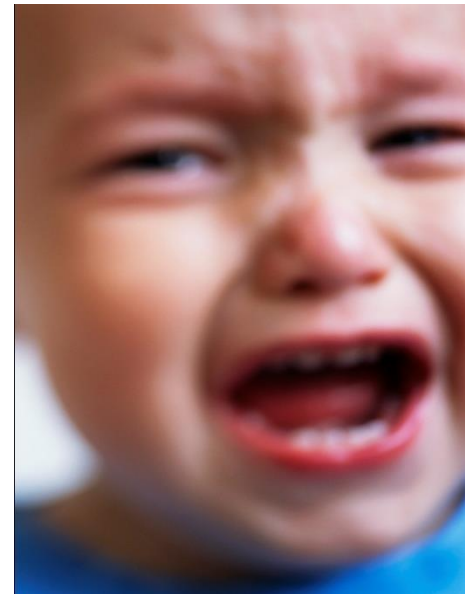
Very

Very

Very

Very

Very



Very

Low

Quiz #1 TIPS



- **Review** readings, homework assignments, lecture slides
- **Watch** the review section video and review section slides
- **Attend** online or on campus help section if needed
- **Complete** the Math E-3 Practice Test

GOOD LUCK!

