Solutions - 1.4

1. $\angle a \cong \angle b$

2. $\angle c \cong \angle d$

3. $\overline{AB} \cong \overline{BC}$ or $\overline{AB} + \overline{BC} \cong \overline{AC}$

4. $\angle ABD \cong \angle DBC$ or $\angle ABD + \angle DBC \cong \angle ABC$

5. Answer: $\angle A = 40^{\circ}$
   Detailed Solution:
   $\angle B \cong \angle A + \angle C, \ \angle C = 50^{\circ}$ and $\angle B = 90^{\circ}$

   $B = A + C$
   $90 = A + 50$
   $90 - 50 = A$
   $A = 40$
   $\angle A = 40^{\circ}$

6. $\angle C = 80^{\circ}$

7. Answer: $\angle C = 110^{\circ}$
   Detailed Solution;
   $\angle B \cong \angle A + \angle C, \ \angle A = 50^{\circ}$ and $\angle B = 160^{\circ}$

   $B = A + C$
   $160 = 50 + C$
   $160 - 50 = C$
   $C = 110$
   $\angle C = 110^{\circ}$

8. $\angle A = 47^{\circ}$

9. Answer: $\angle B = 115^{o}$
   Detailed Solution:
   $\angle B \cong \angle A + \angle C, \quad \angle C = 84^{o} \quad$ and $\quad \angle A = 31^{o}$

   B = A + C
   B = 31 + 84
   B = 115
   $\angle B = 115^{o}$

10. $\angle B = 86^{o}$

11. Answer: $\angle B = 81^{o}$
    Detailed Solution;
    $\angle B \cong \angle A + \angle C, \quad \angle C = 38^{o} \quad$ and $\quad \angle A = 43^{o}$

    B = A + C
    B = 43 + 38
    B = 81
    $\angle B = 81^{o}$

12. $\angle B = 75^{o}$

13. Answer: $3(\angle A) = 72^{o}$
    Detailed Solution;
    $\angle A = 24^{o}$, find $3(\angle A)$

    3(24) = 72

    $3(\angle A) = 72^{o}$

14. $2(\angle A) = 122^{o}$

15. Answer: $4(\angle A) = 136^{\circ}$

Detailed Solution:

$\angle A = 34^{\circ}$, find $4(\angle A)$

$4(34) = 136$

$4(\angle A) = 136^{\circ}$

16. $5(\angle A) = 120^{\circ}$

17. Answer: $\dfrac{\angle B}{4} = 65^{\circ}$

Detailed Solution:

$\angle B = 260^{\circ}$, find $\dfrac{\angle B}{4}$

$\dfrac{260}{4} = 65$

$\dfrac{\angle B}{4} = 65^{\circ}$

18. $\dfrac{\angle B}{3} = 61^{\circ}$

19. Answer: $\dfrac{\angle B}{2} = 78°$

Detailed Solution:

$\angle B = 156°$, find $\dfrac{\angle B}{2}$

$\dfrac{156}{2} = 78$

$\dfrac{\angle B}{2} = 78°$

20. $\dfrac{\angle B}{5} = 49°$

21. Reflexive Property

22. Transitive Property

23. Additive Property

24. Symmetric Property

25. Transitive Property

26. Reflexive Property

27. Symmetric Property

28. Additive Property

29. Answer: $\angle a = 60^\circ$, $\angle b = 60^\circ$ and $\angle c = 60^\circ$

Detailed Solution:

Recall from lesson 1.1 that the angles in a triangle sum to 180°.

Therefore: $\angle a + \angle b + \angle c = 180^\circ$

The red arcs indicate that all three angles are congruent.
Since angles a and b are congruent, b can be replaced with a in the above equation. Also since angles a and c are congruent, c can be replaced with a in the above equation.

Therefore: $\angle a + \angle a + \angle a = 180^\circ$
$$a + a + a = 180$$
$$3a = 180$$
$$a = 60$$
$$\angle a = 60^\circ$$

Since angles a, b and c are all congruent, then $\angle b = 60^\circ$ and $\angle c = 60^\circ$.

Therefore: $\angle a = 60^\circ$, $\angle b = 60^\circ$ and $\angle c = 60^\circ$

Check your answer:
$\angle a + \angle b + \angle c = 180^\circ$
$60^\circ + 60^\circ + 60^\circ = 180^\circ$

30. $\angle a = 45^\circ$, $\angle b = 45^\circ$

31. Answer: $\angle a = 70^\circ$ and $\angle b = 70^\circ$

Detailed Solution:

The angles in a triangle sum to 180°.

Therefore: $\angle a + \angle b + 40^\circ = 180^\circ$

The red arcs indicate that angles a and b are congruent.
Since angles a and b are congruent, b can be replaced with a in the above equation.

Therefore:

$\angle a + \angle a + 40^\circ = 180^\circ$

$a + a + 40 = 180$

$2a + 40 = 180$

$2a = 140$

$a = 70$

$\angle a = 70^\circ$

Since angles a and b are congruent, then $\angle a = 70^\circ$ and $\angle b = 70^\circ$.

Check your answer:

$\angle a + \angle b + 40^\circ = 180^\circ$

$70^\circ + 70^\circ + 40^\circ = 180^\circ$

32. $\angle a = 76^\circ$ and $\angle b = 76^\circ$

33. Answer: $x = 56$, $\angle a = 60^\circ$, $\angle b = 58^\circ$

Detailed Solution:

The angles in a triangle sum to 180°.

Therefore: $\angle a + \angle b + 62^\circ = 180^\circ$

Angle $a = x + 4$ and angle $b = x + 2$.

Therefore:

$(x + 4) + (x + 2) + 62^\circ = 180^\circ$

$x + 4 + x + 2 + 62 = 180$

$2x + 68 = 180$

$2x = 112$

$x = 56$

Find: $\angle a$

$\angle a = x + 4$

$ = 56 + 4$

$ = 60$

$\angle a = 60^\circ$

Find: $\angle b$

$\angle b = x + 2$

$ = 56 + 2$

$ = 58$

$\angle b = 58^\circ$

Therefore: $x = 56$, $\angle a = 60^\circ$ and $\angle b = 58^\circ$

Check your answer:

$\angle a + \angle b + 62^\circ = 180^\circ$

$60^\circ + 58^\circ + 62^\circ = 180^\circ$

34. $x = 73$, $\angle a = 78^\circ$, $\angle b = 76^\circ$

35. Answer: x = 7, $\angle 1 = 13^\circ$ and $\angle 2 = 13^\circ$

Detailed Solution:

$\angle 1 = x + 6$ and $\angle 2 = 2x - 1$

The red arcs indicate that angles 1 and 2 are congruent.

Therefore:

x + 6 = 2x − 1

6 = x − 1

7 = x

Find: $\angle 1$

$\angle 1 = x + 6$

$\quad = 7 + 6$

$\quad = 13$

$\angle 1 = 13^\circ$

Find: $\angle 2$

$\angle 2 = 2x - 1$

$\quad = 2(7) - 1$

$\quad = 14 - 1$

$\quad = 13$

$\angle 2 = 13^\circ$

Therefore: x = 7, $\angle 1 = 13^\circ$ and $\angle 2 = 13^\circ$

36. x = 8, $\angle 1 = 30^\circ$ and $\angle 2 = 30^\circ$

37. Answer: x = 2, $\angle 1 = 20^\circ$ and $\angle 2 = 20^\circ$

Detailed Solution;

$\angle 1 = 8x + 4$ and $\angle 2 = 3x + 14$

The red arcs indicate that angles 1 and 2 are congruent.

Therefore:

8x + 4 = 3x + 14

5x + 4 = 14

5x = 10

x = 2

Find: $\angle 1$

$\angle 1 = 8x + 4$

$\quad = 8(2) + 4$

$\quad = 16 + 4$

$\quad = 20$

$\angle 1 = 20^\circ$

Find: $\angle 2$

$\angle 2 = 3x + 14$

$\quad = 3(2) + 14$

$\quad = 6 + 14$

$\quad = 20$

$\angle 2 = 20^\circ$

Therefore: x = 2, $\angle 1 = 20^\circ$ and $\angle 2 = 20^\circ$

38. x = 6, $\angle 1 = 22^\circ$ and $\angle 2 = 22^\circ$

39. Answer: $x = 3$, $\angle 1 = 18^{\circ}$ and $\angle 2 = 18^{\circ}$

Detailed Solution:

$\angle 1 = 4x + 6$ and $\angle 2 = x + 15$

The red arcs indicate that angles 1 and 2 are congruent.

Therefore:

$4x + 6 = x + 15$

$3x + 6 = 15$

$3x = 9$

$x = 3$

Find: $\angle 1$

$\begin{aligned} \angle 1 &= 4x + 6 \\ &= 4(3) + 6 \\ &= 12 + 6 \\ &= 18 \end{aligned}$

$\angle 1 = 18^{\circ}$

Find: $\angle 2$

$\begin{aligned} \angle 2 &= x + 15 \\ &= 3 + 15 \\ &= 18 \end{aligned}$

$\angle 2 = 18^{\circ}$

Therefore: $x = 3$, $\angle 1 = 18^{\circ}$ and $\angle 2 = 18^{\circ}$

40. $x = 6$, $\angle 1 = 40^{\circ}$ and $\angle 2 = 40^{\circ}$

41. Answer: $\angle 1 = 69^{\circ}$ and $\angle 2 = 23^{\circ}$

   Detailed Solution:

   If $\angle 1$ is three times $\angle 2$ and $\angle 1 + \angle 2 = 92^{\circ}$, find $\angle 1$ and $\angle 2$.

   $\angle 1 = 3(\angle 2)$

   $\angle 1 + \angle 2 = 92^{\circ}$

   $3(\angle 2) + \angle 2 = 92^{\circ}$

   $4(\angle 2) = 92^{\circ}$

   $\angle 2 = 23^{\circ}$

   $\angle 1 = 3(\angle 2)$

   $\quad = 3(23^{\circ})$

   $\quad = 69^{\circ}$

   $\angle 1 = 69^{\circ}$

   Therefore: $\angle 1 = 69^{\circ}$ and $\angle 2 = 23^{\circ}$

42. $\angle 1 = 48^{\circ}$ and $\angle 2 = 96^{\circ}$

43. Answer: $\angle 1 = 134^\circ$ and $\angle 2 = 33^\circ$

    Detailed Solution:

    If $\angle 1$ is two more then four times $\angle 2$ and $\angle 1 + \angle 2 = 167^\circ$, find $\angle 1$ and $\angle 2$.

    $\angle 1 = 2 + 4(\angle 2)$

    $\angle 1 + \angle 2 = 167^\circ$

    $2 + 4(\angle 2) + \angle 2 = 167^\circ$

    $2 + 5(\angle 2) = 167^\circ$

    $5(\angle 2) = 165^\circ$

    $\angle 2 = 33^\circ$

    $\angle 1 = 2 + 4(\angle 2)$

    $\quad = 2 + 4(33^\circ)$

    $\quad = 2 + 132^\circ$

    $\angle 1 = 134^\circ$

    Therefore: $\angle 1 = 134^\circ$ and $\angle 2 = 33^\circ$

44. $\angle 1 = 17^\circ$ and $\angle 2 = 37^\circ$

45. Answer: $\angle 1 = 127^\circ$ and $\angle 2 = 44^\circ$

    Detailed Solution:

    If $\angle 1$ is five less then three times $\angle 2$ and $\angle 1 + \angle 2 = 171^\circ$, find $\angle 1$ and $\angle 2$.

    $\angle 1 = 3(\angle 2) - 5$

    $\angle 1 + \angle 2 = 171^\circ$

    $3(\angle 2) - 5 + \angle 2 = 171^\circ$

    $4(\angle 2) - 5 = 171^\circ$

    $4(\angle 2) = 176^\circ$

    $\angle 2 = 44^\circ$

    $\angle 1 = 3(\angle 2) - 5$

    $\quad = 3(44^\circ) - 5$

    $\quad = 132^\circ - 5$

    $\angle 1 = 127^\circ$

    Therefore: $\angle 1 = 127^\circ$ and $\angle 2 = 44^\circ$

46. $\angle 1 = 27^\circ$ and $\angle 2 = 131^\circ$

47. Answer: $\angle 1 = 88^{\circ}$ and $\angle 2 = 164^{\circ}$

Detailed Solution:

If $\angle 1$ is six more then half $\angle 2$ and $\angle 1 + \angle 2 = 252^{\circ}$, find $\angle 1$ and $\angle 2$.

$$\angle 1 = 6 + \frac{\angle 2}{2}$$

$$\angle 1 + \angle 2 = 252^{\circ}$$

$$6 + \frac{\angle 2}{2} + \angle 2 = 252^{\circ}$$

$$6 + \frac{\angle 2}{2} + \frac{2(\angle 2)}{2} = 252^{\circ}$$

$$6 + \frac{3(\angle 2)}{2} = 252^{\circ}$$

$$\frac{3(\angle 2)}{2} = 246^{\circ}$$

$$\angle 2 = 246^{\circ}\left(\frac{2}{3}\right)$$

$$\angle 2 = 164^{\circ}$$

$$\angle 1 = 6 + \frac{\angle 2}{2}$$

$$= 6 + \frac{164^{\circ}}{2}$$

$$= 6 + 82^{\circ}$$

$$= 88^{\circ}$$

$$\angle 1 = 88^{\circ}$$

Therefore: $\angle 1 = 88^{\circ}$ and $\angle 2 = 164^{\circ}$

48. $\angle 1 = 184^{\circ}$ and $\angle 2 = 102^{\circ}$