Limits Involving Trigonometric Functions

$$\lim_{\theta \to 0} \frac{\sin\theta}{\theta} = \lim_{\theta \to 0} \frac{\theta}{\sin\theta} = 1$$

$$\lim_{\theta \to 0} \frac{\cos\theta - 1}{\theta} = 0$$

$$\lim_{x \to \pi/4} \frac{\sin x + \cos x}{\tan x}$$

$$\frac{\sin\left(\frac{\pi}{4}\right) + \cos\left(\frac{\pi}{4}\right)}{\tan\left(\frac{\pi}{4}\right)}$$

$$\frac{\sin\left(\frac{\pi}{4}\right)}{\cos\left(\frac{\pi}{4}\right)} = \frac{\frac{\sqrt{2}}{2}}{\frac{\sqrt{2}}{2}}$$

$$\downarrow$$

$$\frac{\sqrt{2}}{2} \div \frac{\sqrt{2}}{2}$$

$$\downarrow$$

$$\frac{\sqrt{2}}{2} \cdot \frac{2}{\sqrt{2}} = 1$$

$$\frac{\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}}{1}$$

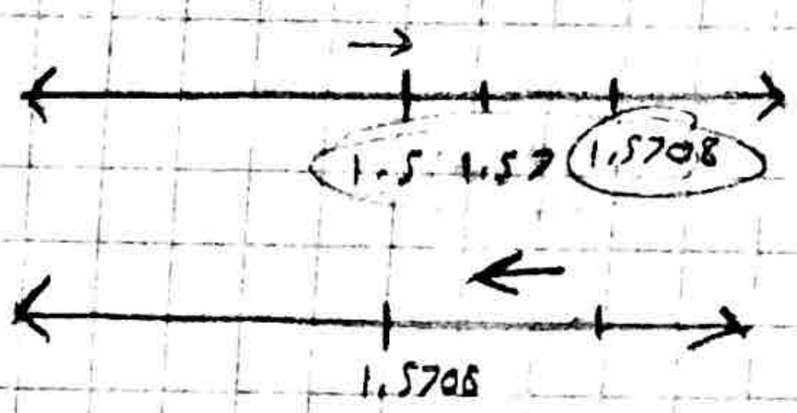
$$=$$

$$\frac{\sqrt{2} + \sqrt{2}}{2} = \frac{2\sqrt{2}}{2} = \sqrt{2}$$

$$\lim_{x \to \pi/4} \frac{\sin x + \cos x}{\tan x} = \sqrt{2}$$

$\lim_{x \to \pi/2^-} \tan(x)$

$\pi/2 = 1.5708$



x	tan(π/2)
1.5	14.1014
1.57	1255.77

$\lim_{x \to \pi/2^-} \tan(x) = \infty$ Vertical Asymptote at $x = \pi/2$

$\lim_{x \to} \tan(x)$

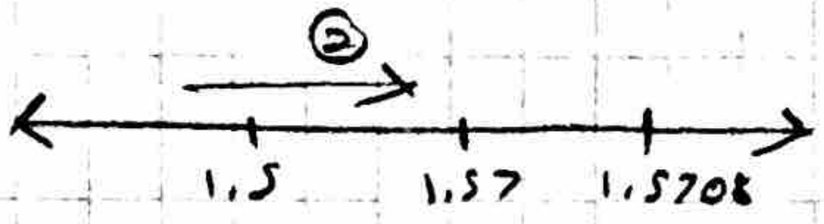
x	tan(π/2)
1.5710	-4909.83
1.5709	-9645.69

$\pi/2 = 1.5708$

$\lim_{x \to \pi/2^-} \tan(x)$

① $\pi/2 = 1.5708$

For now Im going to solve this in a tabular fashion.



③

x	tan(x)
1.5	14.1014
1.57	1255.77

← This have a high increment so this is going to $+\infty$

$\lim_{x \to \pi/2^-} \tan x = \infty$

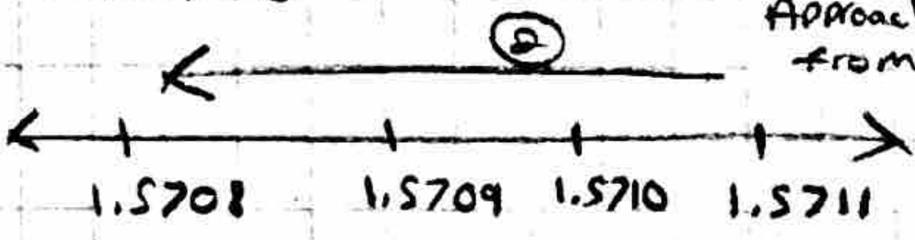
Vertical Asymptote
$x = \pi/2$

Refer to the graph of $\tan(x)$ to find the asymptote at $x = \pi/2$

$\lim_{x \to \pi/2^+} \tan(x)$

① $\pi/2 = 1.5708$

$\pi/2$ Approaching 1.5708 from the right



③

x	tan(x)
1.5711	-3293.01
1.5710	-4909.83
1.5709	-9645.69

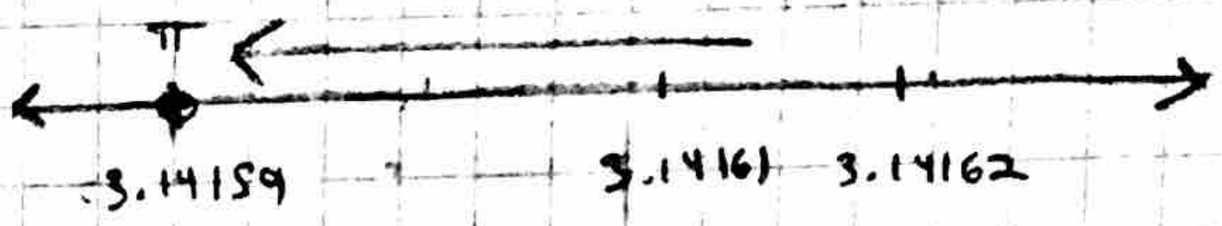
← High increments going down to $-\infty$

$\lim_{x \to \pi/2^+} \tan(x) = -\infty$

Vertical Asymptote
$x = \pi/2$

$\lim_{x \to \pi^+} \csc x$

① $\pi = 3.14159$



④

x	$\csc x = \frac{1}{\sin x}$
3.14162	-36567.9
3.14161	-57648.8

③ $\csc x = \frac{1}{\sin x}$

← High increments going toward the negative. $-\infty$

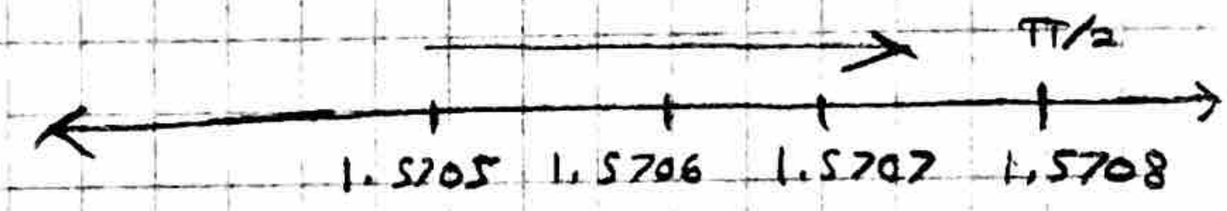
sin → csc
cos → sec
S's match
My way of remembering.

$\lim_{x \to \pi^+} \csc x = -\infty$

Vertical Asymptote At $x = \pi$
Refer to $\csc x$ graph

$\lim_{x \to \pi/2^-} \sec x$

① $\pi/2 = 1.5708$



x	$\sec x = \frac{1}{\cos x}$
1.5705	3374.65
1.5706	5093.55
1.5707	10381.3

③ $\sec x = \frac{1}{\cos x}$

← High increments in the positive direction. ∞

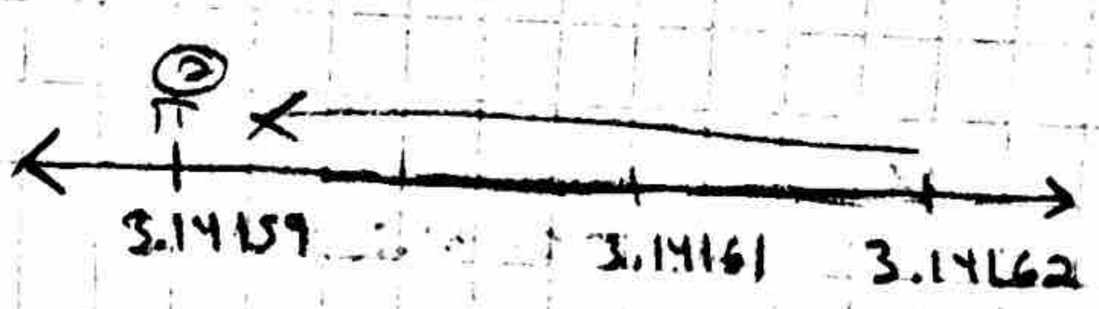
cos → sec
S's match

$\lim_{x \to \pi/2^-} \sec x = \infty$

Vertical Asymptote at $x = \pi/2$
Refer to $\sec x$ graph

$$\lim_{x \to \pi^+} \cot(x)$$

① $\pi = 3.14159$



③

x	$\cot(x) = \frac{\cos(x)}{\sin(x)}$
3.14162	-36567.9
3.14161	-57648.8

③ $\tan x = \frac{\sin x}{\cos x}$, $\cot = \frac{\cos x}{\sin x}$

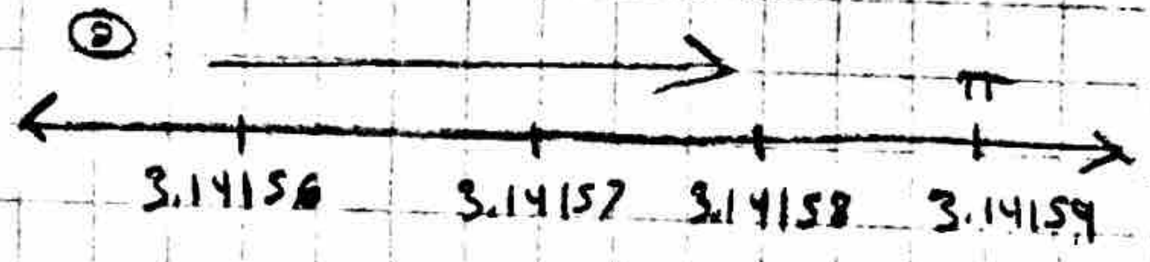
④ High increments in the positive direction. This tells me this is going to $+\infty$

$$\lim_{x \to \pi^+} \cot(x) = \infty$$

Vertical Asymptote at $x = \pi$

$$\lim_{x \to \pi^-} \cot(x)$$

① $\pi = 3.14159$



③ $\cot x = \frac{\cos x}{\sin x}$

④

x	$\cot(x) = \frac{\cos x}{\sin x}$
3.14156	-30624.5
3.14157	-44143.1
3.14158	-79029

High increments going in the negative direction. This tells me this is going to $-\infty$.

$$\lim_{x \to \pi^-} \cot(x) = -\infty$$

Vertical Asymptote at $x = \pi$