0.3 Linear Equations and Inequalities

In the introduction to this chapter we said that we were going to review "the concepts, skills and vocabulary we believe are prerequisite to a rigorous, college-level Precalculus course." So far, we've presented a lot of vocabulary and concepts but we haven't done much to refresh the skills needed to survive in the Precalculus wilderness. Thus over the course of the next few sections we will focus our review on the Algebra skills needed to solve basic equations and inequalities. In general, equations and inequalities fall into one of three categories: conditional, identity or contradiction, depending on the nature of their solutions. A **conditional** equation or inequality is true for only *certain* real numbers. For example, $2x+1=7$ is true precisely when $x=3$, and $w-3 \leq 4$ is true precisely when $w \leq 7$. An **identity** is an equation or inequality that is true for *all* real numbers. For example, $2x-3=1+x-4+x$ or $2t \leq 2t+3$. A **contradiction** is an equation or inequality that is *never* true. Examples here include $3x-4=3x+7$ and $a-1 > a+3$.

As you may recall, solving an equation or inequality means finding all of the values of the variable, if any exist, which make the given equation or inequality true. This often requires us to manipulate the given equation or inequality from its given form to an easier form. For example, if we're asked to solve $3-2(x-3)=7x+3(x+1)$, we get $x=\frac{1}{2}$, but not without a fair amount of algebraic manipulation. In order to obtain the correct answer(s), however, we need to make sure that whatever maneuvers we apply are reversible in order to guarantee that we maintain a chain of **equivalent** equations or inequalities. Two equations or inequalities are called **equivalent** if they have the same solutions. We list these 'legal moves' below.

Procedures which Generate Equivalent Equations

- Add (or subtract) the same real number to (from) both sides of the equation.
- Multiply (or divide) both sides of the equation by the same **nonzero** real number.[a]

Procedures which Generate Equivalent Inequalities

- Add (or subtract) the same real number to (from) both sides of the equation.
- Multiply (or divide) both sides of the equation by the same **positive** real number.[b]

[a]Multiplying both sides of an equation by 0 collapses the equation to $0=0$, which doesn't do anybody any good.

[b]Remember that if you multiply both sides of an inequality by a negative real number, the inequality sign is reversed: $3 \leq 4$, but $(-2)(3) \geq (-2)(4)$.

0.3.1 Linear Equations

The first type of equations we need to review are **linear** equations as defined below.

Definition 0.10. An equation is said to be **linear** in a variable X if it can be written in the form $AX=B$ where A and B are expressions which do not involve X and $A \neq 0$.

One key point about Definition 0.10 is that the exponent on the unknown 'X' in the equation is 1, that is $X = X^1$. Our main strategy for solving linear equations is summarized below.

Strategy for Solving Linear Equations

In order to solve an equation which is linear in a given variable, say X:

1. Isolate all of the terms containing X on one side of the equation, putting all of the terms not containing X on the other side of the equation.

2. Factor out the X and divide both sides of the equation by its coefficient.

We illustrate this process with a collection of examples below.

Example 0.3.1. Solve the following equations for the indicated variable. Check your answer.

1. Solve for x: $3x - 6 = 7x + 4$
2. Solve for t: $3 - 1.7t = \dfrac{t}{4}$
3. Solve for a: $\dfrac{1}{18}(7 - 4a) + 2 = \dfrac{a}{3} - \dfrac{4 - a}{12}$
4. Solve for y: $8y\sqrt{3} + 1 = 7 - \sqrt{12}(5 - y)$
5. Solve for x: $\dfrac{3x - 1}{2} = x\sqrt{50} + 4$
6. Solve for y: $x(4 - y) = 8y$

Solution.

1. The variable we are asked to solve for is x so our first move is to gather all of the terms involving x on one side and put the remaining terms on the other.[1]

$$\begin{array}{rcll} 3x - 6 & = & 7x + 4 & \\ (3x - 6) - 7x + 6 & = & (7x + 4) - 7x + 6 & \text{Subtract } 7x\text{, add } 6 \\ 3x - 7x - 6 + 6 & = & 7x - 7x + 4 + 6 & \text{Rearrange terms} \\ -4x & = & 10 & 3x - 7x = (3 - 7)x = -4x \\ \dfrac{-4x}{-4} & = & \dfrac{10}{-4} & \text{Divide by the coefficient of } x \\ x & = & -\dfrac{5}{2} & \text{Reduce to lowest terms} \end{array}$$

To check our answer, we substitute $x = -\frac{5}{2}$ into each side of the **orginial** equation to see the equation is satisfied. Sure enough, $3\left(-\frac{5}{2}\right) - 6 = -\frac{27}{2}$ and $7\left(-\frac{5}{2}\right) + 4 = -\frac{27}{2}$.

[1] In the margin notes, when we speak of operations, e.g.,'Subtract $7x$,' we mean to subtract $7x$ from **both** sides of the equation. The 'from both sides of the equation' is omitted in the interest of spacing.

2. In our next example, the unknown is t and we not only have a fraction but also a decimal to wrangle. Fortunately, with equations we can multiply both sides to rid us of these computational obstacles:

$$\begin{array}{rcll}
3-1.7t & = & \dfrac{t}{4} & \\
40(3-1.7t) & = & 40\left(\dfrac{t}{4}\right) & \text{Multiply by } 40 \\
40(3)-40(1.7t) & = & \dfrac{40t}{4} & \text{Distribute} \\
120-68t & = & 10t & \\
(120-68t)+68t & = & 10t+68t & \text{Add } 68t \text{ to both sides} \\
120 & = & 78t & 68t+10t=(68+10)t=78t \\
\dfrac{120}{78} & = & \dfrac{78t}{78} & \text{Divide by the coefficient of } t \\
\dfrac{120}{78} & = & t & \\
\dfrac{20}{13} & = & t & \text{Reduce to lowest terms}
\end{array}$$

To check, we again substitute $t=\frac{20}{13}$ into each side of the original equation. We find that $3-1.7\left(\frac{20}{13}\right)=3-\left(\frac{17}{10}\right)\left(\frac{20}{13}\right)=\frac{5}{13}$ and $\frac{(20/13)}{4}=\frac{20}{13}\cdot\frac{1}{4}=\frac{5}{13}$ as well.

3. To solve this next equation, we begin once again by clearing fractions. The least common denominator here is 36:

$$\begin{array}{rcll}
\dfrac{1}{18}(7-4a)+2 & = & \dfrac{a}{3}-\dfrac{4-a}{12} & \\
36\left(\dfrac{1}{18}(7-4a)+2\right) & = & 36\left(\dfrac{a}{3}-\dfrac{4-a}{12}\right) & \text{Multiply by } 36 \\
\dfrac{36}{18}(7-4a)+(36)(2) & = & \dfrac{36a}{3}-\dfrac{36(4-a)}{12} & \text{Distribute} \\
2(7-4a)+72 & = & 12a-3(4-a) & \text{Distribute} \\
14-8a+72 & = & 12a-12+3a & \\
86-8a & = & 15a-12 & 12a+3a=(12+3)a=15a \\
(86-8a)+8a+12 & = & (15a-12)+8a+12 & \text{Add } 8a \text{ and } 12 \\
86+12-8a+8a & = & 15a+8a-12+12 & \text{Rearrange terms} \\
98 & = & 23a & 15a+8a=(15+8)a=23a \\
\dfrac{98}{23} & = & \dfrac{23a}{23} & \text{Divide by the coefficient of } a \\
\dfrac{98}{23} & = & a &
\end{array}$$

The check, as usual, involves substituting $a = \frac{98}{23}$ into both sides of the original equation. The reader is encouraged to work through the (admittedly messy) arithmetic. Both sides work out to $\frac{199}{138}$.

4. The square roots may dishearten you but we treat them just like the real numbers they are. Our strategy is the same: get everything with the variable (in this case y) on one side, put everything else on the other and divide by the coefficient of the variable. We've added a few steps to the narrative that we would ordinarily omit just to help you see that this equation is indeed linear.

$$\begin{array}{rcll}
8y\sqrt{3}+1 & = & 7-\sqrt{12}(5-y) & \\
8y\sqrt{3}+1 & = & 7-\sqrt{12}(5)+\sqrt{12}y & \text{Distribute} \\
8y\sqrt{3}+1 & = & 7-(2\sqrt{3})5+(2\sqrt{3})y & \sqrt{12}=\sqrt{4\cdot 3}=2\sqrt{3} \\
8y\sqrt{3}+1 & = & 7-10\sqrt{3}+2y\sqrt{3} & \\
(8y\sqrt{3}+1)-1-2y\sqrt{3} & = & (7-10\sqrt{3}+2y\sqrt{3})-1-2y\sqrt{3} & \text{Subtract 1 and } 2y\sqrt{3} \\
8y\sqrt{3}-2y\sqrt{3}+1-1 & = & 7-1-10\sqrt{3}+2y\sqrt{3}-2y\sqrt{3} & \text{Rearrange terms} \\
(8\sqrt{3}-2\sqrt{3})y & = & 6-10\sqrt{3} & \\
6y\sqrt{3} & = & 6-10\sqrt{3} & \text{See note below} \\
\dfrac{6y\sqrt{3}}{6\sqrt{3}} & = & \dfrac{6-10\sqrt{3}}{6\sqrt{3}} & \text{Divide } 6\sqrt{3} \\
y & = & \dfrac{2\cdot\sqrt{3}\cdot\sqrt{3}-2\cdot 5\cdot\sqrt{3}}{2\cdot 3\cdot\sqrt{3}} & \\
y & = & \dfrac{\cancel{2}\cancel{\sqrt{3}}(\sqrt{3}-5)}{\cancel{2}\cdot 3\cdot\cancel{\sqrt{3}}} & \text{Factor and cancel} \\
y & = & \dfrac{\sqrt{3}-5}{3} &
\end{array}$$

In the list of computations above we marked the row $6y\sqrt{3} = 6 - 10\sqrt{3}$ with a note. That's because we wanted to draw your attention to this line without breaking the flow of the manipulations. The equation $6y\sqrt{3} = 6 - 10\sqrt{3}$ is in fact linear according to Definition 0.10: the variable is y, the value of A is $6\sqrt{3}$ and $B = 6 - 10\sqrt{3}$. Checking the solution, while not trivial, is good mental exercise. Each side works out to be $\frac{27-40\sqrt{3}}{3}$.

5. Proceeding as before, we simplify radicals and clear denominators. Once we gather all of the terms containing x on one side and move the other terms to the other, we factor out x to

identify its coefficient then divide to get our answer.

$$
\begin{array}{rcll}
\dfrac{3x-1}{2} & = & x\sqrt{50}+4 & \\
\dfrac{3x-1}{2} & = & 5x\sqrt{2}+4 & \sqrt{50}=\sqrt{25\cdot 2} \\
2\left(\dfrac{3x-1}{2}\right) & = & 2\left(5x\sqrt{2}+4\right) & \text{Multiply by 2} \\
\dfrac{\cancel{2}\cdot(3x-1)}{\cancel{2}} & = & 2(5x\sqrt{2})+2\cdot 4 & \text{Distribute} \\
3x-1 & = & 10x\sqrt{2}+8 & \\
(3x-1)-10x\sqrt{2}+1 & = & (10x\sqrt{2}+8)-10x\sqrt{2}+1 & \text{Subtract } 10x\sqrt{2}\text{, add 1} \\
3x-10x\sqrt{2}-1+1 & = & 10x\sqrt{2}-10x\sqrt{2}+8+1 & \text{Rearrange terms} \\
3x-10x\sqrt{2} & = & 9 & \\
(3-10\sqrt{2})x & = & 9 & \text{Factor} \\
\dfrac{(3-10\sqrt{2})x}{3-10\sqrt{2}} & = & \dfrac{9}{3-10\sqrt{2}} & \text{Divide by the coefficient of } x \\
x & = & \dfrac{9}{3-10\sqrt{2}} &
\end{array}
$$

The reader is encouraged to check this solution - it isn't as bad as it looks if you're careful! Each side works out to be $\dfrac{12+5\sqrt{2}}{3-10\sqrt{2}}$.

6. If we were instructed to solve our last equation for x, we'd be done in one step: divide both sides by $(4-y)$ - assuming $4-y\neq 0$, that is. Alas, we are instructed to solve for y, which means we have some more work to do.

$$
\begin{array}{rcll}
x(4-y) & = & 8y & \\
4x-xy & = & 8y & \text{Distribute} \\
(4x-xy)+xy & = & 8y+xy & \text{Add } xy \\
4x & = & (8+x)y & \text{Factor}
\end{array}
$$

In order to finish the problem, we need to divide both sides of the equation by the coefficient of y which in this case is $8+x$. Since this expression contains a variable, we need to stipulate that we may perform this division only if $8+x\neq 0$, or, in other words, $x\neq -8$. Hence, we write our solution as:

$$y=\frac{4x}{8+x},\quad \text{provided } x\neq -8$$

What happens if $x=-8$? Substituting $x=-8$ into the original equation gives $(-8)(4-y)=8y$ or $-32+8y=8y$. This reduces to $-32=0$, which is a contradiction. This means there is no solution when $x=-8$, so we've covered all the bases. Checking our answer requires some Algebra we haven't reviewed yet in this text, but the necessary skills *should* be lurking somewhere in the mathematical mists of your mind. The adventurous reader is invited to show that both sides work out to $\frac{32x}{x+8}$. $\square$

0.3.2 Linear Inequalities

We now turn our attention to linear inequalities. Unlike linear equations which admit at most one solution, the solutions to linear inequalities are generally intervals of real numbers. While the solution strategy for solving linear inequalities is the same as with solving linear equations, we need to remind ourselves that, should we decide to multiply or divide both sides of an inequality by a **negative** number, we need to reverse the direction of the inequality. (See page 38.) In the example below, we work not only some 'simple' linear inequalities in the sense there is only one inequality present, but also some 'compound' linear inequalities which require us to use the notions of intersection and union.

Example 0.3.2. Solve the following inequalities for the indicated variable.

1. Solve for x: $\dfrac{7-8x}{2} \geq 4x+1$
2. Solve for y: $\dfrac{3}{4} \leq \dfrac{7-y}{2} < 6$
3. Solve for t: $2t-1 \leq 4-t < 6t+1$
4. Solve for x: $5+\sqrt{7}x \leq 4x+1 \leq 8$
5. Solve for w: $2.1-0.01w \leq -3$ or $2.1-0.01w \geq 3$

Solution.

1. We begin by clearing denominators and gathering all of the terms containing x to one side of the inequality and putting the remaining terms on the other.

$$\begin{array}{rcll}
\dfrac{7-8x}{2} & \geq & 4x+1 & \\
2\left(\dfrac{7-8x}{2}\right) & \geq & 2(4x+1) & \text{Multiply by } 2 \\
\dfrac{\cancel{2}(7-8x)}{\cancel{2}} & \geq & 2(4x)+2(1) & \text{Distribute} \\
7-8x & \geq & 8x+2 & \\
(7-8x)+8x-2 & \geq & 8x+2+8x-2 & \text{Add } 8x \text{, subtract } 2 \\
7-2-8x+8x & \geq & 8x+8x+2-2 & \text{Rearrange terms} \\
5 & \geq & 16x & 8x+8x=(8+8)x=16x \\
\dfrac{5}{16} & \geq & \dfrac{16x}{16} & \text{Divide by the coefficient of } x \\
\dfrac{5}{16} & \geq & x &
\end{array}$$

We get $\frac{5}{16} \geq x$ or, said differently, $x \leq \frac{5}{16}$. We express this set[2] of real numbers as $\left(-\infty, \frac{5}{16}\right]$. Though not required to do so, we could partially check our answer by substituting $x = \frac{5}{16}$ and a few other values in our solution set ($x = 0$, for instance) to make sure the inequality holds. (It also isn't a bad idea to choose an $x > \frac{5}{16}$, say $x = 1$, to see that the inequality *doesn't* hold there.) The only real way to actually show that our answer works for *all* values in our solution set is to start with $x \leq \frac{5}{16}$ and reverse all of the steps in our solution procedure to prove it is equivalent to our original inequality.

2. We have our first example of a 'compound' inequality. The solutions to

$$\frac{3}{4} \leq \frac{7-y}{2} < 6$$

must satisfy

$$\frac{3}{4} \leq \frac{7-y}{2} \qquad \underline{\text{and}} \qquad \frac{7-y}{2} < 6$$

One approach is to solve each of these inequalities separately, then intersect their solution sets. While this method works (and will be used later for more complicated problems), since our variable y appears only in the middle expression, we can proceed by essentially working both inequalities at once:

$$\begin{array}{rcll}
\frac{3}{4} \leq & \frac{7-y}{2} & < 6 & \\
4\left(\frac{3}{4}\right) \leq & 4\left(\frac{7-y}{2}\right) & < 4(6) & \text{Multiply by 4} \\
\frac{\cancel{4} \cdot 3}{\cancel{4}} \leq & \frac{\cancelto{2}{4}(7-y)}{\cancel{2}} & < 24 & \\
3 \leq & 2(7-y) & < 24 & \\
3 \leq & 2(7) - 2y & < 24 & \text{Distrbute} \\
3 \leq & 14 - 2y & < 24 & \\
3 - 14 \leq & (14 - 2y) - 14 & < 24 - 14 & \text{Subtract 14} \\
-11 \leq & -2y & < 10 & \\
\frac{-11}{-2} \geq & \frac{-2y}{-2} & > \frac{10}{-2} & \text{Divide by the coefficient of } y \\
& & & \text{Reverse inequalities} \\
\frac{11}{2} \geq & y & > -5 &
\end{array}$$

Our final answer is $\frac{11}{2} \geq y > -5$, or, said differently, $-5 < y \leq \frac{11}{2}$. In interval notation, this is $\left(-5, \frac{11}{2}\right]$. We could check the reasonableness of our answer as before, and the reader is encouraged to do so.

[2] Using set-builder notation, our 'set' of solutions here is $\{x \,|\, x \leq \frac{5}{16}\}$.

3. We have another compound inequality and what distinguishes this one from our previous example is that 't' appears on both sides of both inequalities. In this case, we need to create two separate inequalities and find all of the real numbers t which satisfy both $2t - 1 \leq 4 - t$ *and* $4 - t < 6t + 1$. The first inequality, $2t - 1 \leq 4 - t$, reduces to $3t \leq 5$ or $t \leq \frac{5}{3}$. The second inequality, $4 - t < 6t + 1$, becomes $3 < 7t$ which reduces to $t > \frac{3}{7}$. Thus our solution is all real numbers t with $t \leq \frac{5}{3}$ *and* $t > \frac{3}{7}$, or, writing this as a compound inequality, $\frac{3}{7} < t \leq \frac{5}{3}$. Using interval notation,[3] we express our solution as $\left(\frac{3}{7}, \frac{5}{3}\right]$.

4. As before, with this inequality we have no choice but to solve each inequality individually and intersect the solution sets. Starting with the leftmost inequality, we first note that the in the term $\sqrt{7}x$, the vinculum of the square root extends over the 7 only, meaning the x is not part of the radicand. In order to avoid confusion, we will write $\sqrt{7}x$ as $x\sqrt{7}$.

$$\begin{array}{rcll} 5 + x\sqrt{7} & \leq & 4x + 1 & \\ (5 + x\sqrt{7}) - 4x - 5 & \leq & (4x + 1) - 4x - 5 & \text{Subtract } 4x \text{ and } 5 \\ x\sqrt{7} - 4x + 5 - 5 & \leq & 4x - 4x + 1 - 5 & \text{Rearrange terms} \\ x(\sqrt{7} - 4) & \leq & -4 & \text{Factor} \end{array}$$

At this point, we need to exercise a bit of caution because the number $\sqrt{7} - 4$ is negative.[4] When we divide by it the inequality reverses:

$$\begin{array}{rcll} x(\sqrt{7} - 4) & \leq & -4 & \\ \dfrac{x(\sqrt{7} - 4)}{\sqrt{7} - 4} & \geq & \dfrac{-4}{\sqrt{7} - 4} & \begin{array}{r}\text{Divide by the coefficient of } x \\ \text{Reverse inequalities}\end{array} \\ x & \geq & \dfrac{-4}{\sqrt{7} - 4} & \\ x & \geq & \dfrac{-4}{-(4 - \sqrt{7})} & \\ x & \geq & \dfrac{4}{4 - \sqrt{7}} & \end{array}$$

We're only half done because we still have the rightmost inequality to solve. Fortunately, that one seems rather mundane: $4x + 1 \leq 8$ reduces to $x \leq \frac{7}{4}$ without too much incident. Our solution is $x \geq \frac{4}{4 - \sqrt{7}}$ *and* $x \leq \frac{7}{4}$. We may be tempted to write $\frac{4}{4 - \sqrt{7}} \leq x \leq \frac{7}{4}$ and call it a day but that would be nonsense! To see why, notice that $\sqrt{7}$ is between 2 and 3 so $\frac{4}{4 - \sqrt{7}}$ is between $\frac{4}{4-2} = 2$ and $\frac{4}{4-3} = 4$. In particular, we get $\frac{4}{4 - \sqrt{7}} > 2$. On the other hand, $\frac{7}{4} < 2$. This means that our 'solutions' have to be simultaneously greater than 2 AND less than 2 which is impossible. Therefore, this compound inequality has no solution, which means we did all that work for nothing.[5]

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[3] If we intersect the solution sets of the two individual inequalities, we get the answer, too: $\left(-\infty, \frac{5}{3}\right] \cap \left(\frac{3}{7}, \infty\right) = \left(\frac{3}{7}, \frac{5}{3}\right]$.
[4] Since $4 < 7 < 9$, it stands to reason that $\sqrt{4} < \sqrt{7} < \sqrt{9}$ so $2 < \sqrt{7} < 3$.
[5] Much like how people walking on treadmills get nowhere. Math is the endurance cardio of the brain, folks!

5. Our last example is yet another compound inequality but here, instead of the two inequalities being connected with the conjunction '*and*', they are connected with '*or*', which indicates that we need to find the *union* of the results of each. Starting with $2.1 - 0.01w \leq -3$, we get $-0.01w \leq -5.1$, which gives[6] $w \geq 510$. The second inequality, $2.1 - 0.01w \geq 3$, becomes $-0.01w \geq 0.9$, which reduces to $w \leq -90$. Our solution set consists of all real numbers w with $w \geq 510$ *or* $w \leq -90$. In interval notation, this is $(-\infty, -90] \cup [510, \infty)$. □

[6] Don't forget to flip the inequality!

0.3.3 Exercises

In Exercises 1 - 9, solve the given linear equation and check your answer.

1. $3x - 4 = 2 - 4(x - 3)$
2. $\dfrac{3 - 2t}{4} = 7t + 1$
3. $\dfrac{2(w - 3)}{5} = \dfrac{4}{15} - \dfrac{3w + 1}{9}$
4. $-0.02y + 1000 = 0$
5. $\dfrac{49w - 14}{7} = 3w - (2 - 4w)$
6. $7 - (4 - x) = \dfrac{2x - 3}{2}$
7. $3t\sqrt{7} + 5 = 0$
8. $\sqrt{50}y = \dfrac{6 - \sqrt{8}y}{3}$
9. $4 - (2x + 1) = \dfrac{x\sqrt{7}}{9}$

In equations 10 - 27, solve each equation for the indicated variable.

10. Solve for y: $3x + 2y = 4$
11. Solve for x: $3x + 2y = 4$
12. Solve for C: $F = \dfrac{9}{5}C + 32$
13. Solve for x: $p = -2.5x + 15$
14. Solve for x: $C = 200x + 1000$
15. Solve for y: $x = 4(y + 1) + 3$
16. Solve for w: $vw - 1 = 3v$
17. Solve for v: $vw - 1 = 3v$
18. Solve for y: $x(y - 3) = 2y + 1$
19. Solve for π: $C = 2\pi r$
20. Solve for V: $PV = nRT$
21. Solve for R: $PV = nRT$
22. Solve for g: $E = mgh$
23. Solve for m: $E = \dfrac{1}{2}mv^2$

In Exercises 24 - 27, the subscripts on the variables have no intrinsic mathematical meaning; they're just used to distinguish one variable from another. In other words, treat 'P_1' and 'P_2' as two different variables as you would 'x' and 'y.' (The same goes for 'x' and 'x_0,' etc.)

24. Solve for V_2: $P_1V_1 = P_2V_2$
25. Solve for t: $x = x_0 + at$
26. Solve for x: $y - y_0 = m(x - x_0)$
27. Solve for T_1: $q = mc(T_2 - T_1)$
28. With the help of your classmates, find values for c so that the equation: $2x - 5c = 1 - c(x + 2)$
    (a) has $x = 42$ as a solution.
    (b) has no solution (that is, the equation is a contradiction.)

    Is it possible to find a value of c so the equation is an identity? Explain.

In Exercises 29 - 46, solve the given inequality. Write your answer using interval notation.

29. $3-4x \geq 0$

30. $2t-1 < 3-(4t-3)$

31. $\dfrac{7-y}{4} \geq 3y+1$

32. $0.05R+1.2 > 0.8-0.25R$

33. $7-(2-x) \leq x+3$

34. $\dfrac{10m+1}{5} \geq 2m-\dfrac{1}{2}$

35. $x\sqrt{12}-\sqrt{3} > \sqrt{3}x+\sqrt{27}$

36. $2t-7 \leq \sqrt[3]{18}t$

37. $117y \geq y\sqrt{2}-7y\sqrt[4]{8}$

38. $-\dfrac{1}{2} \leq 5x-3 \leq \dfrac{1}{2}$

39. $-\dfrac{3}{2} \leq \dfrac{4-2t}{10} < \dfrac{7}{6}$

40. $-0.1 \leq \dfrac{5-x}{3}-2 < 0.1$

41. $2y \leq 3-y < 7$

42. $3x \geq 4-x \geq 3$

43. $6-5t > \dfrac{4t}{3} \geq t-2$

44. $2x+1 \leq -1$ or $2x+1 \geq 1$

45. $4-x \leq 0$ or $2x+7 < x$

46. $\dfrac{5-2x}{3} > x$ or $2x+5 \geq 1$

0.3.4 Answers

1. $x = \frac{18}{7}$
2. $t = -\frac{1}{30}$
3. $w = \frac{61}{33}$
4. $y = 50000$
5. All real numbers.
6. No solution.
7. $t = -\frac{5}{3\sqrt{7}} = -\frac{5\sqrt{7}}{21}$
8. $y = \frac{6}{17\sqrt{2}} = \frac{3\sqrt{2}}{17}$
9. $x = \frac{27}{18+\sqrt{7}}$
10. $y = \frac{4-3x}{2}$ or $y = -\frac{3}{2}x + 2$
11. $x = \frac{4-2y}{3}$ or $x = -\frac{2}{3}y + \frac{4}{3}$
12. $C = \frac{5}{9}(F - 32)$ or $C = \frac{5}{9}F - \frac{160}{9}$
13. $x = \frac{p-15}{-2.5} = \frac{15-p}{2.5}$ or $x = -\frac{2}{5}p + 6$.
14. $x = \frac{C-1000}{200}$ or $x = \frac{1}{200}C - 5$
15. $y = \frac{x-7}{4}$ or $y = \frac{1}{4}x - \frac{7}{4}$
16. $w = \frac{3v+1}{v}$, provided $v \neq 0$.
17. $v = \frac{1}{w-3}$, provided $w \neq 3$.
18. $y = \frac{3x+1}{x-2}$, provided $x \neq 2$.
19. $\pi = \frac{C}{2r}$, provided $r \neq 0$.
20. $V = \frac{nRT}{P}$, provided $P \neq 0$.
21. $R = \frac{PV}{nT}$, provided $n \neq 0, T \neq 0$.
22. $g = \frac{E}{mh}$, provided $m \neq 0, h \neq 0$.
23. $m = \frac{2E}{v^2}$, provided $v^2 \neq 0$ (so $v \neq 0$).
24. $V_2 = \frac{P_1 V_1}{P_2}$, provided $P_2 \neq 0$.
25. $t = \frac{x - x_0}{a}$, provided $a \neq 0$.
26. $x = \frac{y - y_0 + mx_0}{m}$ or $x = x_0 + \frac{y - y_0}{m}$, provided $m \neq 0$.
27. $T_1 = \frac{mcT_2 - q}{mc}$ or $T_1 = T_2 - \frac{q}{mc}$, provided $m \neq 0, c \neq 0$.
29. $\left(-\infty, \frac{3}{4}\right]$
30. $\left(-\infty, \frac{7}{6}\right)$
31. $\left(-\infty, \frac{3}{13}\right]$
32. $\left(-\frac{4}{3}, \infty\right)$
33. No solution.
34. $(-\infty, \infty)$
35. $(4, \infty)$
36. $\left[\frac{7}{2 - \sqrt[3]{18}}, \infty\right)$
37. $[0, \infty)$

38. $\left[\frac{1}{2}, \frac{7}{10}\right]$

39. $\left(-\frac{23}{6}, \frac{19}{2}\right]$

40. $\left(-\frac{13}{10}, -\frac{7}{10}\right]$

41. $(-4, 1]$

42. $\{1\} = [1, 1]$

43. $\left[-6, \frac{18}{19}\right)$

44. $(-\infty, -1] \cup [0, \infty)$

45. $(-\infty, -7) \cup [4, \infty)$

46. $(-\infty, \infty)$