Calculus I Solutions - 2.3

1. Answer: $\lim_{x \to 1} (4x - 2) = 2$

   Detailed Solution:

$$\lim_{x \to 1} (4x - 2) = \lim_{x \to 1} 4x - \lim_{x \to 1} 2 \qquad \text{(Law 4)}$$

$$= 4 \cdot \lim_{x \to 1} x - 2 \qquad \text{(Laws 5 \& 1)}$$

$$= 4 \cdot 1 - 2 \qquad \text{(Law 2)}$$

$$= 4 - 2$$

$$= 2$$

2. $\lim_{x \to 2} (3x + 1) = 7$

3. Answer: $\lim_{x \to 1} 2x^4 = 2$

   Detailed Solution:

$$\lim_{x \to 1} 2x^4 = 2 \cdot \lim_{x \to 1} x^4 \qquad \text{(Law 5)}$$

$$= 2 \cdot \left(\lim_{x \to 1} x\right)^4 \qquad \text{(Law 8)}$$

$$= 2 \cdot (1)^4 \qquad \text{(Law 2)}$$

$$= 2 \cdot 1$$

$$= 2$$

4. $\lim_{x \to 1.5} 3x^3 = 10.125$

5. Answer: $\lim\limits_{x \to 10} \sqrt{2x-4} = 4$

Detailed Solution:

$$\lim_{x \to 10} \sqrt{2x-4} = \sqrt{\lim_{x \to 10} 2x-4} \qquad (\text{Law } 9)$$

$$= \sqrt{\lim_{x \to 10} 2x - \lim_{x \to 10} 4} \qquad (\text{Law } 4)$$

$$= \sqrt{2\lim_{x \to 10} x - \lim_{x \to 10} 4} \qquad (\text{Law } 5)$$

$$= \sqrt{2(10)-4} \qquad (\text{Laws } 2 \ \& \ 1)$$

$$= \sqrt{16}$$

$$= 4$$

6. Answer: $\lim\limits_{x \to -11} \sqrt{3-2x} = 5$

7. Answer: $\lim\limits_{x \to 2} \dfrac{4}{x+1} = \dfrac{4}{3}$

Detailed Solution:

$$\lim_{x \to 2} \frac{4}{x+1} = \frac{\lim\limits_{x \to 2} 4}{\lim\limits_{x \to 2} (x+1)} \qquad (\text{Law } 7)$$

$$= \frac{4}{\lim\limits_{x \to 2} x + \lim\limits_{x \to 2} 1} \qquad (\text{Laws } 1 \ \& \ 3)$$

$$= \frac{4}{2+1} \qquad (\text{Laws } 2 \ \& \ 1)$$

$$= \frac{4}{3}$$

8. $\lim\limits_{x\to 0} \dfrac{-1}{x-2} = \dfrac{1}{2}$

9. Answer: $\lim\limits_{x\to 0} \left(\dfrac{1}{x+1}\right)^2 = 1$

Detailed Solution:

$$\lim_{x\to 0} \left(\frac{1}{x+1}\right)^2 = \left(\lim_{x\to 0} \frac{1}{x+1}\right)^2 \qquad (\text{Law } 8)$$

$$= \left(\frac{\lim\limits_{x\to 0} 1}{\lim\limits_{x\to 0}(x+1)}\right)^2 \qquad (\text{Law } 7)$$

$$= \left(\frac{1}{\lim\limits_{x\to 0} x + \lim\limits_{x\to 0} 1}\right)^2 \qquad (\text{Laws } 1 \,\&\, 3)$$

$$= \left(\frac{1}{0+1}\right)^2 \qquad (\text{Laws } 2 \,\&\, 1)$$

$$= 1$$

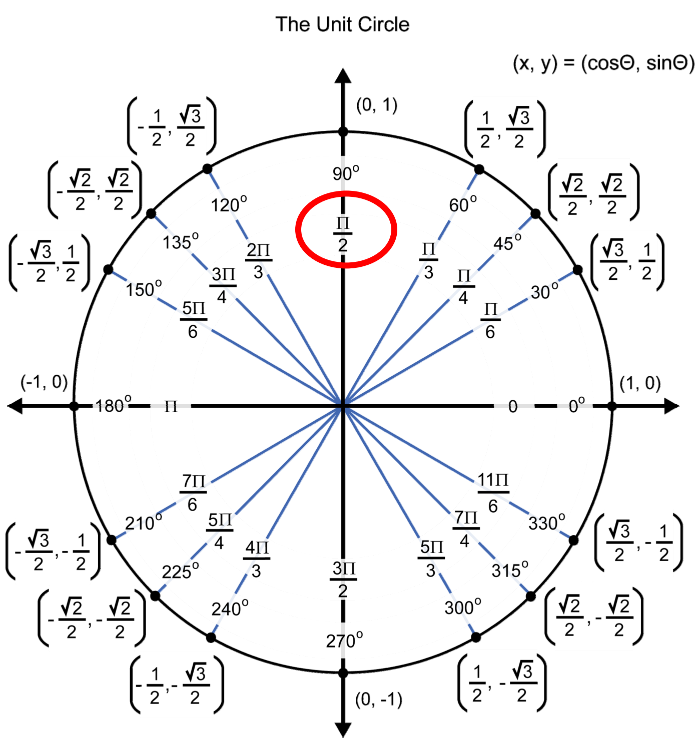
10. $\lim\limits_{x\to 1} (2x+1)^3 = 27$

11. Answer: $\lim_{x \to 1} \sin\left(\frac{\pi x}{2}\right) = 1$

Detailed Solution:

$$\lim_{x \to 1} \sin\left(\frac{\pi x}{2}\right) = \sin\left(\frac{\pi}{2}\right) \qquad (\text{Law } 2)$$

$$= 1$$



12. $\lim_{x \to 3} \tan\left(\frac{\pi x}{4}\right) = -1$

13. Answer: $\lim_{x\to\pi} \tan(x) = 0$

Detailed Solution:

$$\lim_{x\to\pi} \tan(x) = \tan(\pi) \qquad (\text{Law } 2)$$

$$= \frac{\sin(\pi)}{\cos(\pi)}$$

$$= \frac{0}{-1}$$

$$= 0$$

14. $\lim_{x\to 7} \sec\left(\frac{\pi x}{6}\right) = \frac{-2\sqrt{3}}{3}$

15. $\lim_{x\to\pi/2} \frac{1-\cos(2x)}{\sin(x)} = 2$

Detailed Solution:

$$\lim_{x\to\pi/2} \frac{1-\cos(2x)}{\sin(x)} = \frac{1-\cos(\pi)}{\sin(\pi/2)} \qquad (\text{Law } 2)$$

$$= \frac{1-(-1)}{1}$$

$$= \frac{1+1}{1}$$

$$= 2$$

16. $\lim_{x\to 0} \frac{1-\sin(2x)}{\cos(x)} = 1$

17. Answer: $\lim\limits_{x \to 0} \dfrac{\sqrt{x+25}-5}{x} = \dfrac{1}{10}$

Detailed Solution:

$$\lim_{x \to 0} \frac{\sqrt{x+25}-5}{x}$$

$$\lim_{x \to 0} \frac{\left(\sqrt{x+25}-5\right)\left(\sqrt{x+25}+5\right)}{x\left(\sqrt{x+25}+5\right)}$$

$$\lim_{x \to 0} \frac{x+25-25}{x\left(\sqrt{x+25}+5\right)}$$

$$\lim_{x \to 0} \frac{x}{x\left(\sqrt{x+25}+5\right)}$$

$$\lim_{x \to 0} \frac{1}{\sqrt{x+25}+5}$$

$$\frac{\lim\limits_{x \to 0} 1}{\lim\limits_{x \to 0} \sqrt{x+25}+5} \quad \text{(Law 7)}$$

$$\frac{1}{\lim\limits_{x \to 0} \sqrt{x+25} + \lim\limits_{x \to 0} 5} \quad \text{(Laws 1 \& 3)}$$

$$\frac{1}{\sqrt{0+25}+5} \quad \text{(Laws 2 \& 1)}$$

$$\frac{1}{\sqrt{25}+5}$$

$$\frac{1}{5+5}$$

$$\frac{1}{10}$$

18. $\lim\limits_{x \to 0} \dfrac{\sqrt{x+4}-2}{x} = \dfrac{1}{4}$

19. Answer: $\lim\limits_{x\to 1}\dfrac{\sqrt{x+8}-3}{x-1}=\dfrac{1}{6}$

Detailed Solution:

$$\lim_{x\to 1}\frac{\sqrt{x+8}-3}{x-1}$$

$$\lim_{x\to 1}\frac{\left(\sqrt{x+8}-3\right)\left(\sqrt{x+8}+3\right)}{(x-1)\left(\sqrt{x+8}+3\right)}$$

$$\lim_{x\to 1}\frac{x+8-9}{(x-1)\left(\sqrt{x+8}+3\right)}$$

$$\lim_{x\to 1}\frac{x-1}{(x-1)\left(\sqrt{x+8}+3\right)}$$

$$\lim_{x\to 1}\frac{1}{\sqrt{x+8}+3}$$

$$\frac{\lim\limits_{x\to 1}1}{\lim\limits_{x\to 1}\sqrt{x+8}+3} \quad \text{(Law 7)}$$

$$\frac{1}{\lim\limits_{x\to 1}\sqrt{x+8}+\lim\limits_{x\to 1}3} \quad \text{(Laws 1 \& 3)}$$

$$\frac{1}{\sqrt{1+8}+3} \quad \text{(Laws 1 \& 2)}$$

$$\frac{1}{\sqrt{9}+3}$$

$$\frac{1}{3+3}$$

$$\frac{1}{6}$$

20. $\lim\limits_{x\to 1}\dfrac{\sqrt{x+15}-4}{x-1}=\dfrac{1}{8}$

21. Answer: $\lim\limits_{x\to 0}\left(\dfrac{1}{x+1}\right)^2 = 1$

Detailed Solution:

Note: $\left(\dfrac{1}{x+1}\right)^2 = \dfrac{1^2}{(x+1)^2} = \dfrac{1}{x^2+2x+1}$

$$\lim_{x\to 0}\left(\frac{1}{x+1}\right)^2 = \lim_{x\to 0}\frac{1}{x^2+2x+1}$$

$$= \frac{\lim\limits_{x\to 0} 1}{\lim\limits_{x\to 0}\left(x^2+2x+1\right)} \qquad (\text{Law } 7)$$

$$= \frac{1}{\lim\limits_{x\to 0} x^2 + \lim\limits_{x\to 0} 2x + \lim\limits_{x\to 0} 1} \qquad (\text{Laws } 1 \,\&\, 3)$$

$$= \frac{1}{\left(\lim\limits_{x\to 0} x\right)^2 + 2\cdot\lim\limits_{x\to 0} x + 1} \qquad (\text{Laws } 8 \,\&\, 5 \,\&\, 1)$$

$$= \frac{1}{(0)^2 + 2\cdot 0 + 1} \qquad (\text{Law } 2)$$

$$= 1$$

Note: This is the same final answer we obtained in problem #9.

22. $\lim\limits_{x\to 1}(2x+1)^3 = 27$

Note: This is the same final answer we obtained in problem #10.

23. Answer: $\lim\limits_{x\to 1}(f\circ g)(x)=3$

Detailed Solution:

Note: $(f\circ g)(x)=f(g(x))=f\left(\dfrac{1}{x}\right)=2\left(\dfrac{1}{x}\right)+1=\dfrac{2}{x}+1$

$$\lim_{x\to 1}(f\circ g)(x)=\lim_{x\to 1}\left(\frac{2}{x}+1\right)$$

$$=\frac{\lim\limits_{x\to 1}2}{\lim\limits_{x\to 1}x}+\lim_{x\to 1}1 \qquad \text{(Laws 3 \& 7)}$$

$$=\frac{2}{1}+1$$

$$=2+1$$

$$=3$$

24: $\lim\limits_{x\to 1}(f\circ g)(x)=0$

25. Answer: $\lim\limits_{x\to 2}\dfrac{4}{x+1}=\dfrac{4}{3}$

Detailed Solution:

$$\lim_{x\to 2}\frac{4}{x+1}=\frac{4}{2+1}=\frac{4}{3}$$

Yes, our final answer agrees with the final answer we obtained in problem #7.

26. $\lim_{x \to 1} (2x + 1)^3 = 27$

Yes, our final answer agrees with the final answer we obtained in problem #10.

27. Answer: $\lim_{x \to -2} \frac{x^2 + 5x + 6}{x + 2} = 1$

Detailed Solution:

$$\lim_{x \to -2} \frac{x^2 + 5x + 6}{x + 2} = \lim_{x \to -2} \frac{(x+2)(x+3)}{x+2}$$

$$= \lim_{x \to -2} (x+3)$$

$$= -2 + 3$$

$$= 1$$

28. $\lim_{x \to -3} \frac{x^2 - 9}{x + 3} = -6$

29. Answer: $\lim_{x\to 0} x\cos\left(\frac{1}{x}\right) = 0$

Detailed Solution:

$$-1 \le \cos\left(\frac{1}{x}\right) \le 1$$

$$-x \le x\cos\left(\frac{1}{x}\right) \le x$$

$$\lim_{x\to 0} -x \le \lim_{x\to 0} x\cos\left(\frac{1}{x}\right) \le \lim_{x\to 0} x$$

$$0 \le \lim_{x\to 0} x\cos\left(\frac{1}{x}\right) \le 0$$

Therefore, by the squeeze theorem $\lim_{x\to 0} x\cos\left(\frac{1}{x}\right) = 0.$

30. $\lim_{x\to 0} |x|\sin\left(\frac{1}{x}\right) = 0$

31. $\lim_{x\to a} f(x) = b$

Detailed Solution:

$$b - |x - a| \le f(x) \le b + |x - a|$$

$$\lim_{x\to a} b - |x - a| \le \lim_{x\to a} f(x) \le \lim_{x\to a} b + |x - a|$$

$$b - |a - a| \le \lim_{x\to a} f(x) \le b + |a - a|$$

$$b - |0| \le \lim_{x\to a} f(x) \le b + |0|$$

$$b \le \lim_{x\to a} f(x) \le b$$

Therefore: $\lim_{x\to a} f(x) = b$

32. $\lim\limits_{x \to 0} f(x) = 5$

33. Given: $\lim\limits_{x \to c} f(x) = 0$

$|g(x)| \leq M$ for a fixed number M and all $x \neq c$

Proof: Since $|g(x)| \leq M$ for a fixed number M and all $x \neq c$

$-M \leq g(x) \leq M$

$-Mf(x) \leq f(x)g(x) \leq Mf(x)$

$$\lim_{x \to c} -Mf(x) \leq \lim_{x \to c} f(x)g(x) \leq \lim_{x \to c} Mf(x)$$

$$-M \lim_{x \to c} f(x) \leq \lim_{x \to c} f(x)g(x) \leq M \lim_{x \to c} f(x)$$

$$-M(0) \leq \lim_{x \to c} f(x)g(x) \leq M(0)$$

$$0 \leq \lim_{x \to c} f(x)g(x) \leq 0$$

Therefore, by the squeeze theorem $\lim\limits_{x \to c} f(x)g(x) = 0$.

34. $\lim\limits_{r \to 0.06} 1000\left(1 + \frac{r}{4}\right)^{40} = 1814.02$

35. Answer: $\lim\limits_{r \to 0.09} 5000\left(1+\frac{r}{12}\right)^{96} = 10244.61$

Detailed Solution:

$$\lim_{r \to 0.09} 5000\left(1+\frac{r}{12}\right)^{96} = 5000 \lim_{r \to 0.09}\left(1+\frac{r}{12}\right)^{96} \qquad \text{(Law 5)}$$

$$= 5000\left(\lim_{r \to 0.09}\left(1+\frac{r}{12}\right)\right)^{96} \qquad \text{(Law 8)}$$

$$= 5000\left(\lim_{r \to 0.09} 1 + \lim_{r \to 0.09}\frac{r}{12}\right)^{96} \qquad \text{(Law 3)}$$

$$= 5000\left(1+\frac{0.09}{12}\right)^{96} \qquad \text{(Law 1 \& 2)}$$

$$= 5000(1.0075)^{96}$$

$$= 10244.61$$

Therefore: $\lim\limits_{r \to 0.09} 5000\left(1+\frac{r}{12}\right)^{96} = 10244.61$