**Solutions - 1.2**

1.  ∠CDE  or  ∠EDC
    ∠1
    ∠D


2.  ∠AOB  or  ∠BOA
    ∠1
    ∠O


3.  Answer:  Both angles are 20°
    Detailed Solution:
    Since these are vertical angles, refer to theorem 1.2.1:
    If two angles are vertical angles, then they are congruent.

    Therefore:
    $x + 5 = 3x - 25$
    $5 = 2x - 25$
    $30 = 2x$
    $15 = x$

    To find the congruent angles, use either expression:
    $x + 5$          or          $3x - 25$
    $(15) + 5$      or          $3(15) - 25$
    20                or          $45 - 25$
    20                or          20
    Both expressions produce 20 degrees.

    Therefore, the two angles are 20°.


4.  Both angles are 24°.

5.  Answer:  Both angles are 11$^{\circ}$
    Detailed Solution:
    Since these are vertical angles, refer to theorem 1.2.1:
    If two angles are vertical angles, then they are congruent.

    Therefore:
    3x + 2 = 2x + 5
    x + 2 = 5
    x = 3

    To find the congruent angles, use either expression:
    3x + 2          or          2x + 5
    3(3) + 2        or          2(3) + 5
    11              or          6 + 5
    11              or           11
    Both expressions produce 11 degrees.

    Therefore, the two angles are 11$^{\circ}$.


6.  Both angles are 38$^{\circ}$.


7. Supplementary Angles


8. Vertical Angles


9. Supplementary Angles


10. Supplementary Angles


11. Complementary Angles


12. Complementary Angles

13. Vertical Angles

14. Vertical Angles

15.  Answer:  a = 50$^o$
    Detailed Solution:
    Since these are vertical angles, refer to theorem 1.2.1:
    If two angles are vertical angles, then they are congruent.

    Angle a and 50$^o$ are vertical angles, and thus congruent.
    Therefore:  a = 50$^o$

16.  a = 60$^o$

17.  Answer:  b = 130$^o$
    Detailed Solution:
    Since angle a and 50$^o$ are supplementary angles, the sum of their measure equals 180$^o$.

    Therefore:
    b + 50 = 180
    b = 130$^o$

18.  b = 150$^o$

19. Answer:  x = 31
    Detailed Solution:
    Since both the unknown angles are supplementary to the same angle, refer to
     theorem 1.2.2:
    If two angles are supplementary to the same angle, then they are congruent.

    Therefore:
    $5x - 5 = 3x + 57$
    $2x - 5 = 57$
    $2x = 62$
    $x = 31$

    Check to make sure the two angles sum to $180^\circ$:
    $(5x - 5)^\circ + 30^\circ = 180^\circ$
    $5(31)^\circ - 5^\circ + 30^\circ = 180^\circ$
    $155^\circ - 5^\circ + 30^\circ = 180^\circ$
    $180^\circ = 180^\circ$
    Therefore, the answer of  x = 31 is indeed correct.

20. x = 16

21. Answer:  x = 16
     Detailed Solution:
     Since both the unknown angles are supplementary to the same angle, refer to
     theorem 1.2.2:
     If two angles are supplementary to the same angle, then they are congruent.

    Therefore:
    $8x + 2 = 4x + 66$
    $4x + 2 = 66$
    $4x = 64$
    $x = 16$

    Check to make sure the two angles sum to $180^\circ$.
    $(8x + 2)^\circ + 50^\circ = 180^\circ$
    $8(16)^\circ + 2^\circ + 50^\circ = 180^\circ$
    $128^\circ + 2^\circ + 50^\circ = 180^\circ$
    $180^\circ = 180^\circ$
    Therefore, the answer of  x = 16 is indeed correct.

22. $x = 26$

23. Answer: $x = 28$
    Detailed Solution:
    Since the angles are in a linear pair and one angle is $90^o$, refer to theorem 1.2.3:
    If one angle in a linear pair is a right angle, then the other angle is also a right angle.

    Therefore:
    $3x + 6 = 90$
    $3x = 84$
    $x = 28$

24. $x = 22$

25. Answer: $x = 16$
    Detailed Solution:
    Since the $90^o$ angle and $(5x+10)$ are vertical angles, refer to theorem 1.2.1:
    If two angles are vertical angles, then they are congruent.

    Therefore:
    $5x + 10 = 90$
    $5x = 80$
    $x = 16$

26. $x = 12$

27. Answer: $\angle 1 = 44^o$, $\angle 2 = 46^o$

Detailed Solution:
Since angles one and two are complementary angles, refer to the definition:
Complementary angles are angles whose sum equals $90^o$.

Therefore:
x + x + 2 = 90
2x + 2 = 90
2x = 88
x = 44
$\angle 1 = x$, $\angle 2 = x + 2$

Since angle 1 equals x, angle 1 equals $44^o$.
Since angle 2 equals (x + 2), angle 2 equals (44 + 2) = $46^o$.

Therefore:
$\angle 1 = 44^o$

$\angle 2 = 46^o$

Check to make sure the two angles sum to $90^o$.
$44^o + 46^o = 90^o$
Therefore, the answer of x = 44 is indeed correct.

28. $\angle 1 = 20^o$, $\angle 2 = 70^o$

29.  Answer:  $\angle a = 15°$, $\angle b = 30°$, $\angle c = 45°$

Detailed Solution:
Since all three angles are complementary, refer to the definition:
Complementary angles are angles whose sum equals 90°.

Therefore:
x + 2x + 3x = 90
6x = 90
x = 15
$\angle a = x$,  $\angle b = 2x$,  $\angle c = 3x$

Since angle a equals x, angle a equals 15°.
Since angle b equals 2x, angle b equals 2(15) = 30°.
Since angle c equals 3x, angle b equals 3(15) = 45°.
Therefore:

$\angle a = 15°$

$\angle b = 30°$

$\angle c = 45°$

Check to make sure the three angles sum to 90°.
15° + 30° + 45° = 90°
Therefore, 15°, 30° and 45° are indeed the correct angles.


30.  $\angle a = 20°$, $\angle b = 30°$, $\angle c = 40°$

31.  Answer:  $\angle 1 = 141^\circ, \ \angle 2 = 39^\circ$

Detailed Solution:
Since these are supplementary angles, refer to the definition:
Supplementary angles are angles whose sum equals $180^\circ$.

Therefore:
$3x + 9 + x - 5 = 180$
$4x + 4 = 180$
$4x = 176$
$x = 44$
$\angle 1 = 3x + 9, \quad \angle 2 = x - 5$

Since angle 1 equals (3x +9), angle 1 equals $[3(44)+ 9] = 141^\circ$.
Since angle 2 equals (x – 5), angle 2 equals $(44 – 5) = 39^\circ$.
Therefore:
$\angle 1 = 141^\circ$
$\angle 2 = 39^\circ$

Check to make sure the two angles sum to $180^\circ$.
$141^\circ + 39^\circ = 180^\circ$
Therefore, $141^\circ$ and $39^\circ$ are indeed the correct angles.


32.  $\angle 1 = 142^\circ, \ \angle 2 = 38^\circ$

33. Answer: $\angle a = 90^\circ$, $\angle b = 135^\circ$, $\angle c = 45^\circ$

Detailed Solution:

Since angle a and the $90^\circ$ angle are supplementary angles, refer to the definition:
Supplementary angles are angles whose sum equals $180^\circ$.
Therefore:
a + 90 = 180
a = $90^\circ$

Angle b and the $45^\circ$ are supplementary angles, refer to the definition:
Supplementary angles are angles whose sum equals $180^\circ$.

Therefore:
b + 45 = 180
b = $135^\circ$

Angle c and the $45^\circ$ are complementary angles, refer to the definition:
Complementary angles are angles whose sum equals $90^\circ$.

Therefore:
c + 45 = 90
c = $45^\circ$

Therefore $\angle a = 90^\circ$, $\angle b = 135^\circ$, $\angle c = 45^\circ$.


34. $\angle a = 90^\circ$, $\angle b = 120^\circ$, $\angle c = 60^\circ$

35. Answer: $37^o$ and $143^o$
    Detailed Solution;
    The measure of an angle is 5 less than four times the measure of the other angle.
    The other angle is unknown, so it is represented by x.  This gives us two angles
     $4x - 5$ and x.  It is given that these angles are supplementary, thus by definition
     these angles sum to $180^o$.

    Let angle 2 = x.

    (angle 1) + (angle 2) = $180^o$
    $(4x - 5) + (x) = 180$
    $4x - 5 + x = 180$
    $5x - 5 = 180$
    $5x = 185$
    $x = 37^o$  (angle 2)

    The other angle is $4x - 5$.
    Insert x.
    Angle 1 = $4x - 5$
           $= 4(37) - 5$
           $= 143^o$

    Therefore, the two angles are $143^o$ and $37^o$.

    Check to make sure the two angles sum to $180^o$.
    $37^o + 143^o = 180^o$

36. $57^o$ and $123^o$

37. Answer: $90^o$ and $90^o$
    Detailed Solution:
    The measure of an angle is the square root of the measure of the other angle squared. The other angle is unknown, so it is represented by x. This gives us two angles $\sqrt{x^2}$ and x. It is given that these angles are supplementary, thus by definition these angles sum to $180^o$.

    Let angle 2 = x.

    (angle 1) + (angle 2) = $180^o$
    $\sqrt{x^2} + x = 180$
    x + x = 180
    2x = 180
    x = $90^o$ (angle 2)

    The other angle is $\sqrt{x^2}$.
    Insert x.
    Angle 1 $= \sqrt{x^2}$
    $\qquad$ = x
    $\qquad$ = $90^o$

    Therefore, the two angles are $90^o$ and $90^o$.

    Check to make sure the two angles sum to $180^o$.
    $90^o + 90^o = 180^o$

38. $88^o$ and $92^o$

39. Answer: $90^o$ and $90^o$
    Detailed Solution:
    The measure of an angle is 0 more than the measure of the other angle. The other angle is unknown, so it is represented by x. This gives us two angles (0 + x) and x. It is given that these angles are supplementary, thus by definition these angles sum to $180^o$.

    Let angle 2 = x.

    (angle 1) + (angle 2) = $180^o$
    (0 + x) + x = 180
    2x = 180
    x = $90^o$ (angle 2)

    The other angle is 0 + x.
    Insert x.
    Angle 1 = 0 + x
    $\quad\quad$ = x
    $\quad\quad$ = $90^o$

    Therefore, the two angles are $90^o$ and $90^o$.

    Check to make sure the two angles sum to $180^o$.
    $90^o$ + $90^o$ = $180^o$


40. $87.5^o$ and $92.5^o$

41. Answer: $30^o$ and $150^o$

Detailed Solution:

The measure of an angle is 45 less than half the measure of the other angle. The other angle is unknown, so it is represented by x. This gives us two angles $\left(\dfrac{1}{2}x - 45\right)$ and x. It is given that these angles are supplementary, thus by definition these angles sum to $180^o$. Let angle 2 = x.

(angle 1) + (angle 2) = $180^o$

$$\dfrac{1}{2}x - 45 + x = 180$$

$$\dfrac{3}{2}x - 45 = 180$$

$$\dfrac{3}{2}x = 225$$

$$x = 225\left(\dfrac{2}{3}\right)$$

$$x = 150^o \quad \text{(angle 2)}$$

The other angle is $\dfrac{1}{2}x - 45$.

Insert x.

$$\text{Angle 1} = \dfrac{1}{2}x - 45$$

$$= \dfrac{1}{2}(150) - 45$$

$$= 75 - 45$$
$$= 30^o$$

Therefore, the two angles are $30^o$ and $150^o$.

Check to make sure the two angles sum to $180^o$.
$30^o + 150^o = 180^o$

42. $28^o$ and $152^o$

43.  The measure of an angle is the difference between 5 times the measure of the other angle and the measure of the other angle.  The other angle is unknown, so it is represented by x.  This gives us two angles (5x – x) and x.  It is given that these angles are supplementary, thus by definition these angles sum to $180^o$.

Let:  angle 2 = x

(angle 1) + (angle 2) = $180^o$
(5x – x) + x = 180
5x = 180
x = $36^o$ (angle 2)

The other angle is 5x - x.
Insert x.
Angle 1 = 5x - x
          = 4x
          = 4(36)
          = $144^o$

Therefore the two angles are $144^o$ and $36^o$.

Check to make sure the two angles sum to $180^o$.
$144^o$ + $36^o$ = $180^o$

44.  $18^o$  and  $162^o$

45. Answer: $19^o$ and $71^o$
    Detailed Solution:
    The measure of an angle is 5 less than four times the measure of the other angle.
    The other angle is unknown, so it is represented by x. This gives us two angles
    $(4x - 5)$ and x. It is given that these angles are complementary, thus by definition
    these angles sum to $90^o$.

    (angle 1) + (angle 2) = $90^o$
    $(4x - 5) + x = 90$
    $5x - 5 = 90$
    $x = 19^o$ (angle 2)

    The other angle is $4x - 5$.
    Insert x.
    Angle 1 = $4x - 5$
    $\qquad = 4(19) - 5$
    $\qquad = 76 - 5$
    $\qquad = 71^o$

    Therefore the two angles are $71^o$ and $19^o$.

    Check your answer:

    Check to make sure the two angles sum to $90^o$.
    $71^o + 19^o = 90^o$

46. $27^o$ and $63^o$

47. Answer: $45^o$ and $45^o$
    Detailed Solution:
    The measure of an angle is the square root of the measure of the other angle
    squared.  The other angle is unknown, so it is represented by x.  This gives us
    two angles $\sqrt{x^2}$ and x.  It is given that these angles are complementary, thus by
    definition these angles sum to $90^o$.

    Let:  angle 2 = x

    (angle 1) + (angle 2) = $90^o$
    $\sqrt{x^2} + x = 90$
    x + x = 90
    2x = 90
    x = $45^o$ (angle 2)

    The other angle is $\sqrt{x^2}$ .
    Insert x.
    Angle 1 = $\sqrt{x^2}$
    $\qquad$ = x
    $\qquad$ = $45^o$

    Therefore, the two angles are $45^o$ and $45^o$.

    Check to make sure the two angles sum to $90^o$.
    $45^o + 45^o = 90^o$

48. $43^o$ and $47^o$

49. Answer: $45^o$ and $45^o$
    Detailed Solution:
    The measure of an angle is 0 more than the measure of the other angle. The other angle is unknown, so it is represented by x. This gives us two angles (0 + x) and x. It is given that these angles are complementary, thus by definition these angles sum to $90^o$.

    Let: angle 2 = x

    (angle 1) + (angle 2) = $90^o$
    (0 + x) + x = 90
    2x = 90
    x = $45^o$ (angle 2)

    The other angle is 0 + x.
    Insert x.
    Angle 1 = 0 + x
            = x
            = $45^o$

    Therefore the two angles are $45^o$ and $45^o$.

    Check to make sure the two angles sum to $90^o$.
    $45^o + 45^o = 90^o$

50. $15^o$ and $75^o$

51. Answer: $12.71^\circ$ and $77.26^\circ$
    Detailed Solution:
    The measure of an angle is 6 times greater than the measure of the other angle plus 1. The other angle is unknown, so it is represented by x. This gives us two angles (6x + 1) and x. It is given that these angles are complementary, thus by definition these angles sum to $90^\circ$.

    Let: angle 2 = x

    (angle 1) + (angle 2) = $90^\circ$
    (6x + 1) + x = 90
    7x + 1 = 90
    7x = 89
    x = $12.71^\circ$ (angle 2)

    The other angle is 6x + 1.
    Insert x.
    Angle 1 = 6x + 1
        = 6(12.71) + 1
        = 76.26 + 1
        = $77.26^\circ$

    Therefore, the two angles are $77.26^\circ$ and $12.71^\circ$.

    Check to make sure the two angles sum to $90^\circ$
    $77.26^\circ$ + $12.71^\circ$ = $89.97^\circ$. This doesn't exactly equal $90^\circ$ due to rounding.

    Note: If we round to more decimal places we get closer to exactly 90 degrees.

    If we round to three decimal place, then we would have:
    Angle 2 = $12.714^\circ$ and Angle 1 = $77.284^\circ$

    Then $12.714^\circ$ + $77.284^\circ$ = $89.998^\circ$

    If we round to four decimal places, then we would have:
    Angle 2 = $12.7143^\circ$ and Angle 1 = $77.2858^\circ$

    Then $12.7143^\circ$ + $77.2858^\circ$ = $90.0001^\circ$.

52. $18^\circ$ and $72^\circ$

53. Answer: $18^o$  and  $72^o$
    Detailed Solution;
    The measure of an angle is the difference between 5 times the measure of the
    other angle and the measure of the other angle.  The other angle is unknown, so it
    is represented by x.  This gives us two angles (5x – x) and x.  It is given that these
    angles are complementary, thus by definition these angles sum to $90^o$.

    Let:  angle 2 = x

    (angle 1) + (angle 2) = $90^o$
    $5x - x + x = 90$
    $5x = 90$
    $x = 18^o$ (angle 2)

    The other angle is 5x – x.
    Insert x.
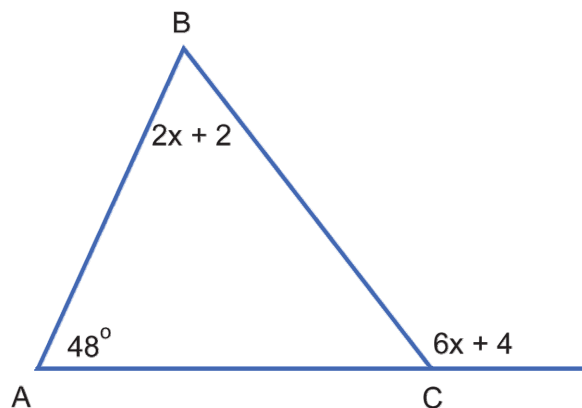    Angle 1 = 5x – x
    $\qquad$ = 4x
    $\qquad$ = 4(18)
    $\qquad$ = $72^o$

    Therefore, the two angles are $72^o$ and $18^o$.

    Check to make sure the two angles sum to $90^o$.
    $72^o + 18^o = 90^o$

54. $30^o$  and  $60^o$

55.  Answer:  x = 11.5
     Detailed Solution:
     Find x.

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                    B

               2x + 2



      48°                     6x + 4
   A                      C
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Use Theorem:  1.2.6

48 + 2x + 2 = 6x + 4
2x + 50 = 6x + 4
50 = 4x + 4
46 = 4x
x = 11.5

56.  x = 17

57.  Answer:  x = 5
     Detailed Solution:
     Find x.

B

4x - 3

2x + 1          2x + 18

A                  C
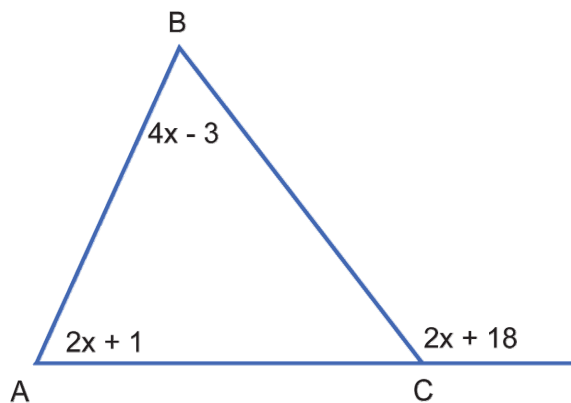
Use Theorem: 1.2.6

$2x + 1 + 4x - 3 = 2x + 18$
$6x - 2 = 2x + 18$
$4x - 2 = 18$
$4x = 20$
$x = 5$

58.  x = 8