Long Division of Polynomials

$$\frac{x^2+4x-8}{x-2}$$

$$\begin{array}{r} x \\ x-2\,\overline{)\,x^2+4x-8} \end{array}$$

Powers Should Be in Descending Order $x^2\ x^1\ x^0$

↓

$x \cdot x = x^2$

$$\begin{array}{rl} & x \\ (x-2) & \overline{)\,x^2+4x-8} \\ & x^2 \end{array}$$

x times what will give me: x^2 ; x

↓

$$\begin{array}{rl} & x \\ (x-2) & \overline{)\,x^2+4x-8} \\ & x^2-2x \end{array}$$

↓

$$\begin{array}{rl} & x \\ (x-2) & \overline{)\,x^2+4x-8} \\ \text{change signs} \rightarrow & \underline{\ominus(x^2-2x)} \\ & \quad 6x-8 \end{array}$$

$$\begin{array}{rl} & x+6 \\ (x-2) & \overline{)\,x^2+4x-8} \\ & \underline{-(x^2-2x)} \\ & \quad 6x-8 \end{array}$$

x times what will give me $6x$: 6

↓

$x \cdot \underline{6} = 6x$

$$\begin{array}{rl} & x+6 \\ (x-2) & \overline{)\,x^2+4x-8} \\ & \underline{-(x^2-2x)} \\ & \quad 6x-8 \\ & \quad 6x-12 \end{array}$$

↓

$$\begin{array}{rl} & x+6 \\ (x-2) & \overline{)\,x^2+4x-8} \\ & \underline{-(x^2-2x)} \\ & \quad 6x-8 \\ & \quad\underline{-(-6x\pm12)} \\ & \qquad -4 \end{array}$$

$$\frac{x^2+4x-8}{x-2}$$

$$\shortparallel$$

$$\boxed{x+6+\frac{4}{x-2}}$$

Solution

$$\frac{x^{3} + 4x + 6}{x^{2} + 1}$$

$x^{2} \cdot x = x^{3}$

Set Powers in Decerding Order

$$\begin{array}{r|l} & x \\ \hline x^{\boxed{2}} + 0x + 1 & x^{3} + 0x^{2} + 4x + 6 \\ & -(-x^{3} \overset{-}{\oplus} 0x^{2} \overset{-}{\oplus} 1x) \\ \hline & \boxed{3x^{\boxed{1}} + 6} \end{array}$$

3x + 6 has lower degree than $x^{2} + 0x + 1$

$$\boxed{\frac{x^{3} + 4x + 6}{x^{2} + 1} = x + \frac{3x + 6}{x^{2} + 1}}$$

$$\frac{x^{3} + 2x^{2} + x + 1}{x - 3}$$

$$\begin{array}{r|l} & \downarrow\ x^{2} + 5x + 16 \\ \hline x - 3 & x^{3} + 2x^{2} + x + 1 \\ & -(-x^{3} \overset{+}{\ominus} 3x^{2}) \\ \hline & 5x^{2} + x \\ & -(-5x^{2} \overset{+}{\ominus} 15x) \\ \hline & 16x + 1 \\ & -(-16x + 48) \\ \hline & 49^{\circ} \end{array}$$

x times what gives me a $5x^{2}$: 5x

x times what gives me a 16x : 16

49° → 49 has lesser degree than x - 3

$49^{\circ} : x^{1}$

$49^{\circ} : (x^{1} - 3)$

$$\frac{x^{3} + 2x^{2} + x + 1}{x - 3} = \boxed{x^{2} + 5x + 16 + \frac{49}{x - 3}}$$

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$$\frac{3x^6 - 2x^5 - x^4 + x^3 + 2}{x^3 + 2x^2 + 4}$$

← dividend

← Divisor

↓

$(x^3 + 2x^2 + 0x + 4)$

x^3 times what gives me

$$
\begin{array}{l}
\qquad 3x^3 - 8x^2 + 15x - 41 \\
\overline{\big)\, 3x^6 - 2x^5 - x^4 + x^3 + 0x^2 + 0x + 2} \\
-(\overset{-}{+}3x^6 \overset{-}{+} 6x^5 \overset{-}{+} 0x^4 \overset{-}{+} 12x^3) \\
\hline
\qquad -8x^5 - 1x^4 - 11x^3 + 0x^2 \\
\quad -(\overset{+}{-}8x^5 \overset{+}{-} 16x^4 \overset{-}{+} 0x^3 \overset{+}{-} 32x^2) \\
\hline
\qquad\quad 15x^4 - 11x^3 + 32x^2 + 0x \\
\qquad -(\overset{-}{+}15x^4 \overset{-}{+} 30x^3 \overset{-}{+} 0x^2 \overset{-}{+} 60x) \\
\hline
\qquad\qquad -41x^3 + 32x^2 - 60x + 2 \\
\qquad\qquad (+41x^3 + 82x^2 + 0x + 164) \\
\hline
\qquad\qquad\quad 114x^2 - 60x + 166
\end{array}
$$

Solution

$$\boxed{\frac{3x^6 - 2x^5 - x^4 + x^3 + 2}{x^3 + 2x^2 + 4} = 3x^3 - 8x^2 + 15x - 41 + \frac{114x^2 - 60x + 166}{x^3 + 2x^2 + 4}}$$

Multiplying Polynomials

3420 (3509)

93. $5x^2(x-3)$

$\downarrow$

$\boxed{5x^3 - 15x^2}$

94. $(x+4)(3x-5)$

$\downarrow$

$3x^2 - 5x + 12x - 20$

$\downarrow$

$\boxed{3x^2 + 7x - 20}$

95. $(x-y+6)(xy)$

$\downarrow$

$(xy)(x-y+6)$

$\downarrow$

$\boxed{x^2y - xy^2 + 6xy}$

96. $(2x-1)(x^2-x+4)$

$\downarrow$

$2x^3 - 2x^2 + 8x - 1x^2 + 1x - 4$

$\downarrow$

$2x^3 - 2x^2 - 1x^2 + 8x + 1x - 4$

$\downarrow$

$\boxed{2x^3 - 3x^2 + 9x - 4}$

97. $-x(x^4+3x^2+2)(x+3)$

$\downarrow$

$(-x^5 - 3x^3 - 2x)(x+3)$

$\downarrow$

$(x+3)(-x^5 - 3x^3 - 2x)$

$\downarrow$

$-x^6 - 3x^4 - 2x^2 - 3x^5 - 9x^3 - 6x$

$\downarrow$

$\boxed{-x^6 - 3x^5 - 3x^4 - 9x^3 - 2x^2 - 6x}$

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Subtracting Polynomials

88. $(5x - 3) - (-2x \overset{-}{\oplus} 4)$

$\downarrow$

$5x - 3 + 2x - 4$

$\downarrow$

$5x - 2x - 3 - 4$

$\downarrow$

$\boxed{3x - 7}$

89. $(x^2 - 3x + 1) - (-5x^2 + 2x$

$\downarrow$

$x^2 - 3x + 1 + 5x^2 - 2x + 4$

$\downarrow$

$x^2 + 5x^2 - 3x - 2x + 1 + 4$

$\downarrow$

$\boxed{6x^2 - 5x + 5}$

90. $(8x^3 + 5x^2 - 3x + 2) - 1(4x^3 + 5x - 12)$

$\downarrow$

$8x^3 + 5x^2 - 3x + 2 - 4x^3 - 5x + 12$

$\downarrow$

$8x^3 - 4x^3 + 5x^2 - 3x - 5x + 2 + 12$

$\downarrow$

$\boxed{4x^3 + 5x^2 - 8x + 14}$

Adding Polynomials

87. $(x^4 - 6x^2 + 3) + (5x^3 + 3x^2 - 3)$

$\downarrow$

$x^4 - 6x^2 + 3 + 5x^3 + 3x^2 - 3$

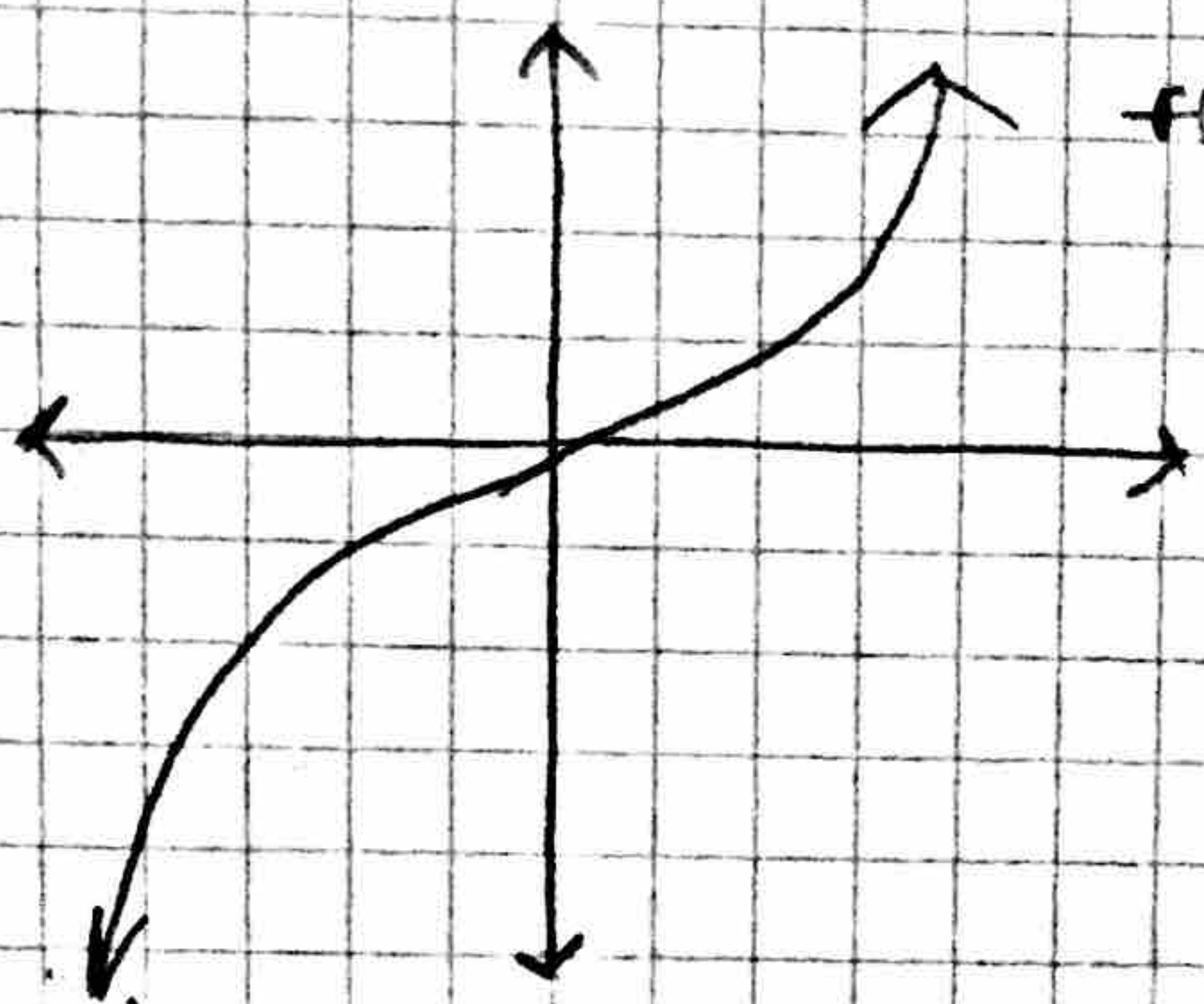
$\downarrow$

$x^4 + 5x^3 - 6x^2 + 3x^2 + 3 - 3$

$\downarrow$

$\boxed{x^4 + 5x^3 - 3x^2}$

End Behavior of Functions

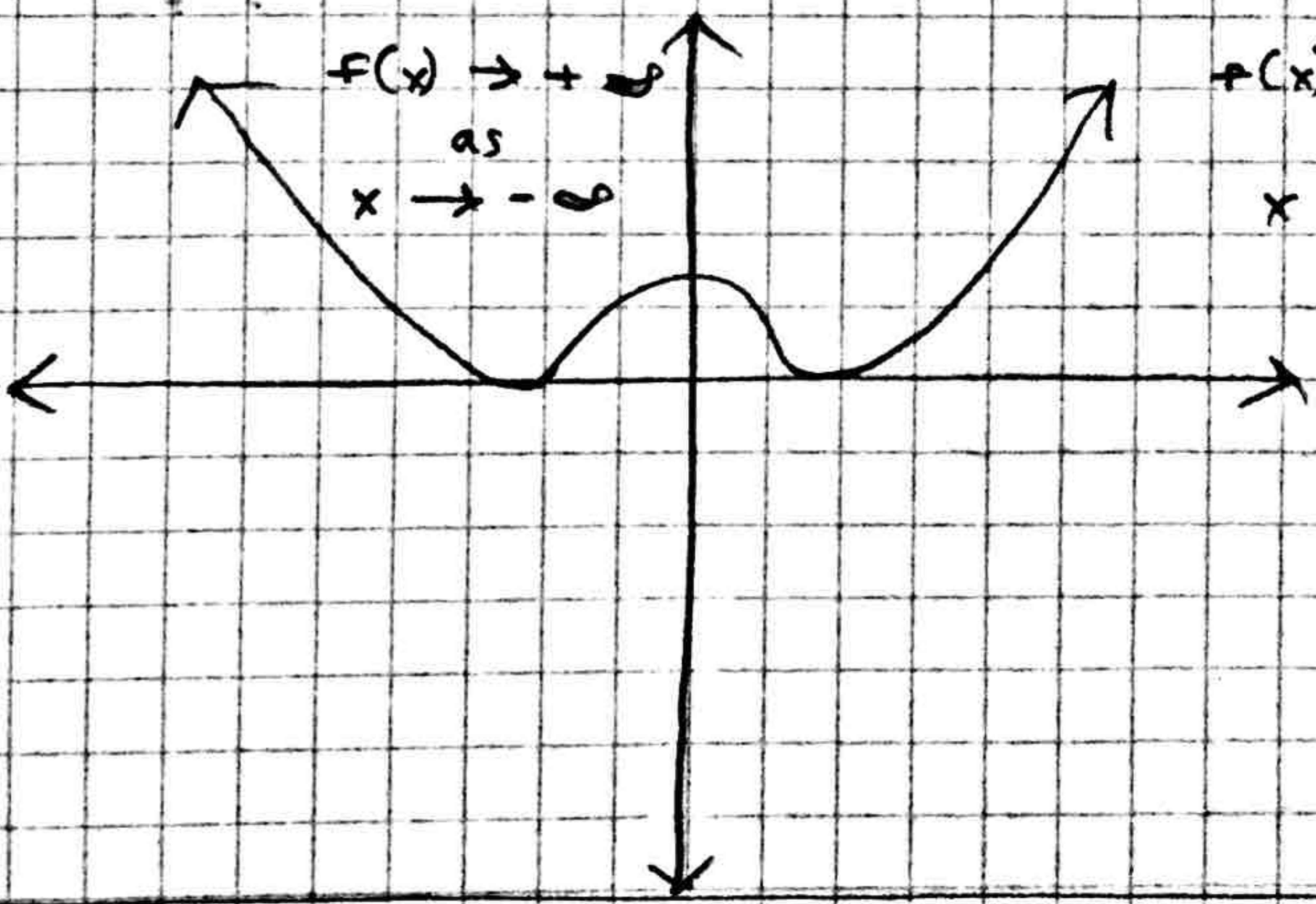


$f(x) \to +\infty$
as
$x \to +\infty$

f(x) approaches positive infinity as x approaches positive infinity.

$f(x) \to -\infty$
as
$x \to -\infty$

f(x) approaches negative infinity as x approaches negative infinity.



$f(x) \to +\infty$
as
$x \to -\infty$

$f(x) \to +\infty$
as
$x \to +\infty$

End Behavior of Polynomials

Find $\lim_{x \to -\infty} f(x)$ and $\lim_{x \to \infty} f(x)$

81. $f(x) = 3x^6 - 40x^5 + 33$

↓ Determine End Behavior of highest powered term

$$\lim_{x \to -\infty}(3x^6 - 40x^5 + 33) = \lim_{x \to \infty}(3x^6) = \infty$$

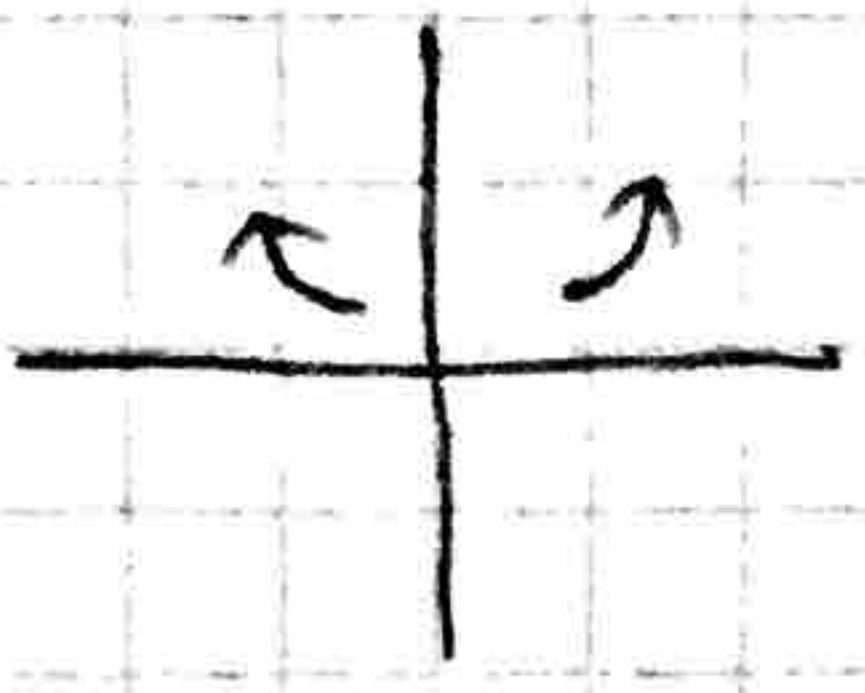
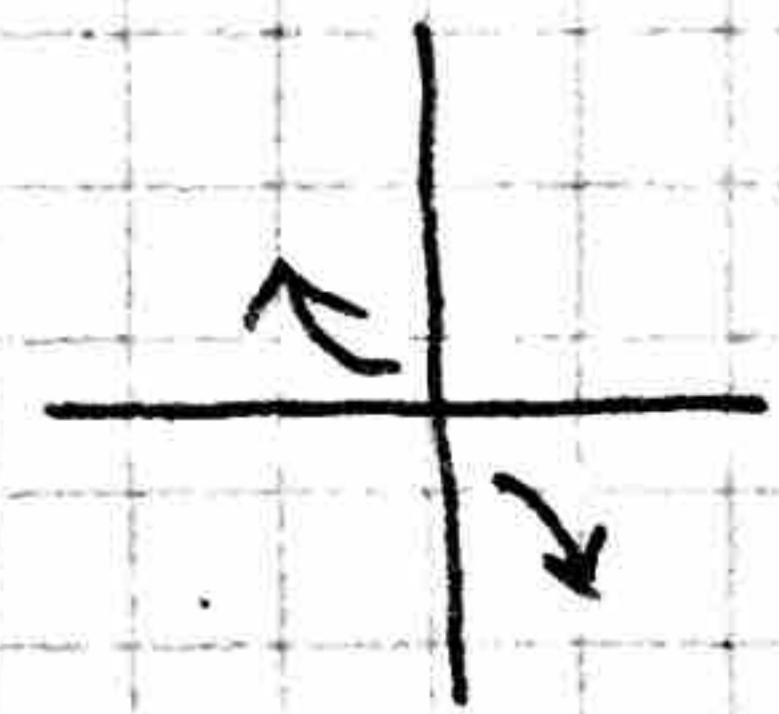
As the limit approaches $-\infty$ the limit is still positive due to the even exponent.

As x approaches ∞, you have

$$\lim_{x \to \infty}(3x^6 - 40x^5 + 33) = \lim_{x \to \infty}(3x^6) = \infty$$

Determine End Behavior of Polynomials

If the $\boxed{\text{Degree}}$ is ______ and the $\boxed{\text{leading term}}$ is ______

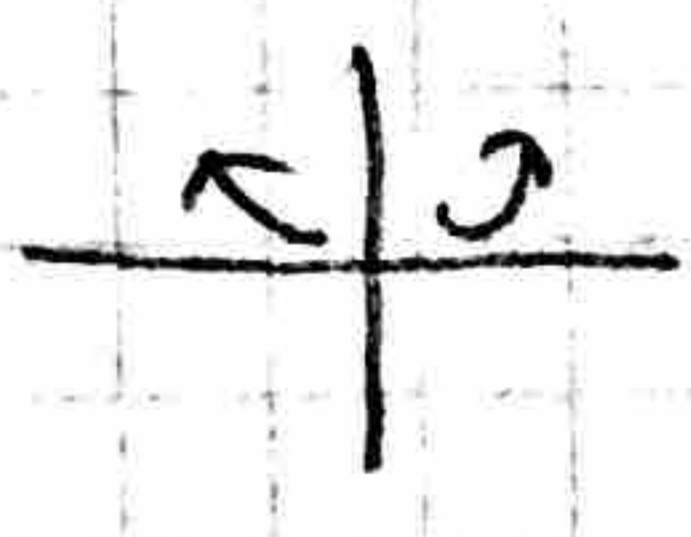
even	Positive	
even	negative	
odd	Positive	
odd	negative	

$f(x) = \boxed{4}x^{\textcircled{4}} + 2x^3 - 6x + 1$ → 4 degree is even → 4 is a even number

leading term is positive

Degree is even
Leading Term is Posititive

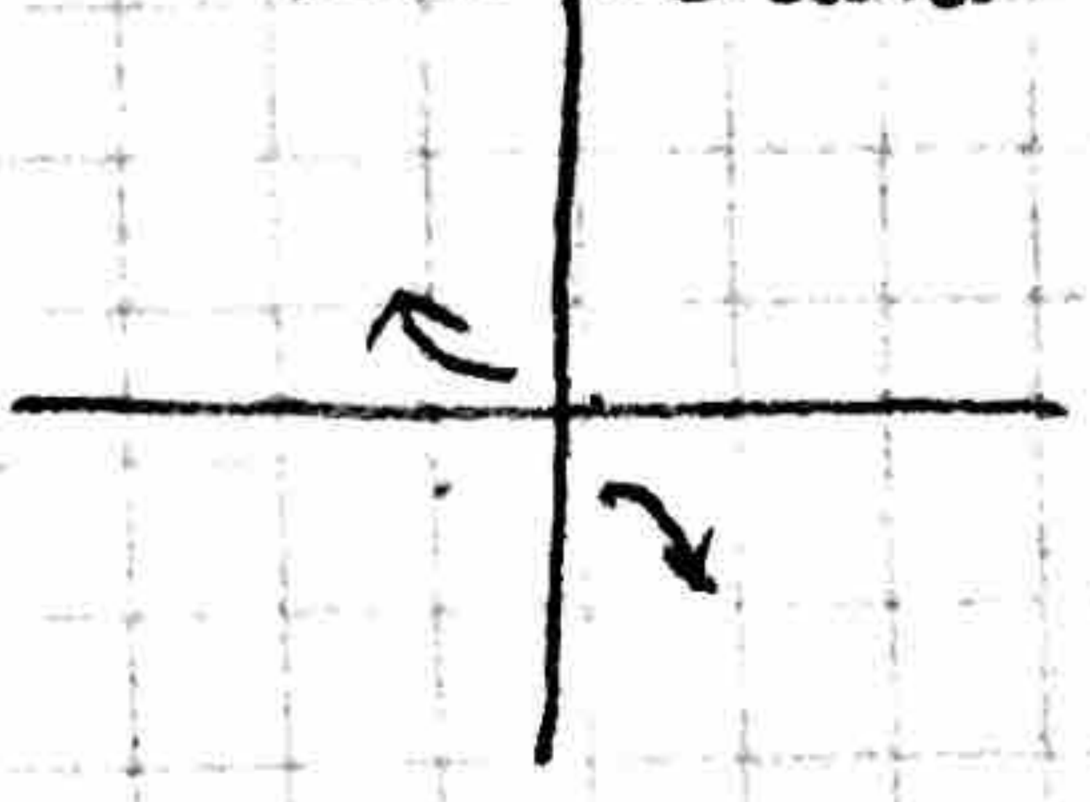
End Behavior



$g(x) = \underbrace{-x}^{\boxed{7}} + 2x^5 - 6$ ← odd degree

leading term is negative
leading term is -1

End Behavior



End Behavior of Polynomials

Find $\lim_{x \to -\infty} f(x)$ and $\lim_{x \to \infty} f(x)$

81. $f(x) = 3x^6 - 40x^5 + 33$

↓ Determine End Behavior of highest powered term

$$\lim_{x \to -\infty}(3x^6 - 40x^5 + 33) = \lim_{x \to \infty}(3x^6)$$

$$= \infty$$

As ~~the~~ limit approaches $-\infty$ the limit is still positive due to the even exponent

As x approaches ∞, you have

$$\lim_{x \to \infty}(3x^6 - 40x^5 + 33) = \lim_{x \to \infty}(3x^6)$$

$$= \infty$$

Find $\lim_{x \to -\infty} f(x)$ and $\lim_{x \to \infty} f(x)$

82. $f(x) = -7x^9 + 33x^8 - 51x^7 + 19x^4 - 1$

$$\downarrow$$

$$\lim_{x \to -\infty} \left(-7x^9 + 33x^8 - 51x^7 + 19x^4 - 1\right) = \lim_{x \to -\infty} \left(-7x^9\right)$$

$$= \infty$$