Find and classify all points of discontinuity for

$$f(x) = \begin{cases} x^2 - 1, & x < 0 \\ x, & x \geq 0 \end{cases}$$

$f(x) = x^2 - 1$ $\qquad\qquad$ $f(x) = x$

$$\lim_{x \to 0} x^2 - 1 = \lim_{x \to 0} x^2 - \lim_{x \to 0} 1 = \lim_{x \to 0} x \cdot \lim_{x \to 0} x - \lim_{x \to 1} 1 = 0 - 1 = \boxed{-1}$$

$$\lim_{x \to 0} x = \boxed{0}$$

$$\lim_{x \to 0^-} x^2 - 1 = -1$$

$x < 0$

moving towards 0 from the left

$$\lim_{x \to 0^+} x = 0$$

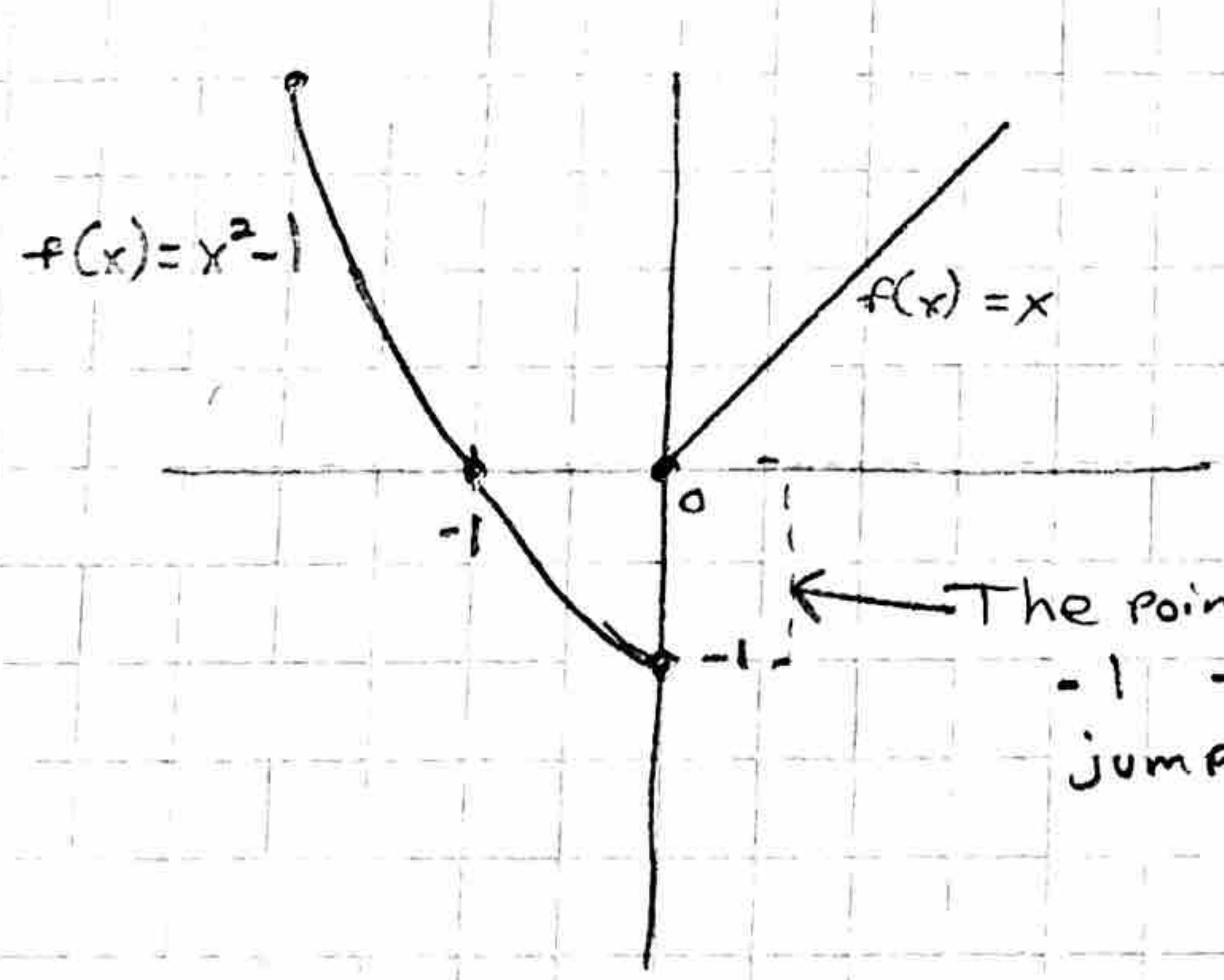
$x \geq 0$

moving towards 0 from the right

$-1 \neq 0$

Limit do not exist.

f is discontinuous at $x = 0$



The point at $a=0$ jumps from -1 to 0. This discontinuity is jumping.

Find the value c that makes the function defined by $f(x) = \begin{cases} x^2 + c, & x < 1 \\ x - 1, & x \geq 1 \end{cases}$
continuous everywhere.

$f(x) = x^2 + c$

Polynomial function is continuous everywhere except where $x < 1$, $(-\infty, 1)$

$\boxed{\text{Discontinuity is at } x = 1}$

$f(x) = x - 1$

Polynomial Function is continuous everywhere except where $x \geq 1$, $[1, \infty)$

this is going to change to $(1, \infty)$ since $x = 1$ or $a = 1$ is not continuous on the point itself.

$\boxed{\text{Discontinuity is at } x = 1}$

Force f to be continuous at $x = 1$.

$\lim_{x \to 1} f(x) = f(1)$

<u>Get Limits</u>

$\lim_{x \to 1^-} x^2 + c$

$=$

$\lim_{x \to 1^-} x^2 + \lim_{x \to 1^+} c$

$=$

$\lim_{x \to 1^-} x \cdot \lim_{x \to 1^-} x + c$

$=$

$1 \cdot 1 + c$

$=$

$\boxed{1 + c}$

$\lim_{x \to 1^+} x - 1$

$=$

$\lim_{x \to 1^+} x - \lim_{x \to 1^+} 1$

$=$

$1 - 1$

$=$

$\boxed{0}$

$\boxed{\lim_{x \to 1^-} x^2 + c = 1 + c}$

$\boxed{\lim_{x \to 1^+} x - 1 = 0}$

Set $\lim_{x \to 1^-} x^2 + c$ equal to $\lim_{x \to 1^+} x - 1 = 0$

$$\downarrow$$

$$1 + c = 0$$
$$-1 \qquad -1$$
$$c = -1$$

$$\lim_{x \to 1^-} x^2 + (-1)$$
$$=$$
$$\lim_{x \to 1^-} x^2 + \lim_{x \to 1^-} -1$$
$$=$$
$$\lim_{x \to 1^-} x \cdot \lim_{x \to 1^-} x + -1$$
$$=$$
$$1 \cdot 1 + -1$$
$$=$$
$$1 + (-1)$$
$$=$$
$$0$$

$\lim_{x \to 1^-} x^2 + c = 0$, where $c = -1$ $\qquad \lim_{x \to 1^+} x - 1 = 0$

When $c = -1$, $f(x) = \begin{cases} x^2 + c, & x < 1 \\ x - 1, & x \geq 1 \end{cases}$ is continuous everywhere