

Consistent and Independent  
"One Solution"

$y = mx + b$  (different slopes)

$$y = \boxed{\frac{1}{4}}x - 7$$

$$y = \boxed{\frac{1}{3}}x - 4$$

Consistent and Dependent  
"Many Solutions"

One Solution on top of the other

$y = mx + b$

Slopes and  
y-intercept  
are the same

$$y = \boxed{-4x + 9}$$
$$y = \boxed{-4x + 9}$$

Inconsistent  
"No Solutions"

$y = mx + b$

Slopes are the same

$$y = \boxed{\frac{1}{5}}x - 30$$

$$y = \boxed{\frac{1}{5}}x - 2$$



$$\begin{cases} 7x - 4y - 2z = 7 \\ 8x - 2y - 8z = 1 \\ x + 2y - 6z = -6 \end{cases}$$

↓

$$\boxed{9x - 14z = -5}$$

$$\begin{aligned} 9x - 14z &= -5 \\ -1(9x - 14z) &= -5(-1) \end{aligned}$$

$$\begin{aligned} 9x - 14z &= -5 \\ -9x + 14z &= 5 \end{aligned}$$

$$\boxed{0 = 0}$$

Many Solutions

let  $\boxed{z = a}$

z is called the "free variable"  
z can take many values

$$\begin{aligned} 9x - 14a &= -5 \\ +14a \quad +14a \\ \hline 9x &= \frac{14a - 5}{9} \end{aligned}$$

$$x = \frac{14a - 5}{9} \quad \text{or} \quad \boxed{x = \frac{14a - 5}{9}}$$

$$\left( \frac{14a - 5}{9}, \frac{40a - 49}{18}, a \right)$$

$$\begin{aligned} 7x - 4y - 2z &= 7 \\ 2(x + 2y - 6z) &= (-6)2 \end{aligned}$$

↓

$$\begin{aligned} 7x - 4y - 2z &= 7 \\ 2x + 4y - 12z &= -12 \end{aligned}$$

$$\boxed{9x - 14z = -5}$$

$$x + 2y - 6z = -6 \quad z = a$$

Solve for y

$$\frac{9}{1} \left( \frac{14a - 5}{9} + 2y - 6a \right) = -6 \left( \frac{9}{1} \right)$$

↓

$$\begin{aligned} 14a - 5 + 18y - 54a &= -54 \\ -5 - 40a + 18y &= -54 \\ +5 \qquad \qquad \qquad +5 \end{aligned}$$

$$\begin{aligned} -40a + 18y &= -49 \\ +40a \qquad \qquad +40a \end{aligned}$$

$$\frac{18y}{18} = \frac{40a - 49}{18}$$

$$\boxed{y = \frac{40a - 49}{18}}$$