1. $\forall\ x \in \mathbb{R}\ \ \forall\ y \in \mathbb{R},\ x^2 > y$

Rewrite as:

$\forall\ x,y \in \mathbb{R},\ x^2 > y$

False. Let x = 0, y = 1; then, $0^2 > 1$ is a false statement.

Note: x = 0, y = 1 is a counterexample.

Negation:

$\exists\ x,y \in \mathbb{R} \ni x^2 \le y$

True. Let x = 0, y = 1; then, $0^2 \le 1$ is a true statement.

Note: x = 0, y = 1 is a witness.

2. $\forall\ x \in \mathbb{R}\ \ \exists\ y \in \mathbb{R} \ni\ x + y = 0$

True. Let y = -x. Then, for every x in R, x + (-x) = 0 is a true statement.

Note: y = -x is an equation that shows this statement is true for every x in R.

Common error answer: True. Let x = 1, y = -1. This is not correct because you have NOT shown for every x there is a y. You have just shown for one x there is a y.

Negation: $\exists\ x \in \mathbb{R} \ni\ \forall\ y \in \mathbb{R},\ x + y \ne 0$

False. Let y = -x. Then for every x in R, $x + (-x) \ne 0$ is a false statement. Which means there is no x in R that is true for every y in R.

Common error answer: False. Let x = 1, y = -1. This is not correct because you have just shown it fails for one x, when you are suppose to show it fails for EVERY x!

3. $\exists\, x \in \mathbb{R} \;\; \exists\, y \in \mathbb{R} \ni x < y^2$

Rewrite as:

$\exists\; x,y \in \mathbb{R} \;\ni\; x < y^2$

True. Let x = 0, y = 1; then, $0 < 1^2$ is a true statement.

Note: x = 0, y = 1 is a witness.

Negation:

$\forall\; x,y \in \mathbb{R},\;\; x \geq y^2$

False. Let x = 0, y = 1; then, $0 \geq 1^2$ is a false statement.

Note: x = 0, y = 1 is a counterexample.

4. $\forall\; y \in \mathbb{R} \;\; \exists\, x \in \mathbb{R} \;\ni\; x < y^2 + 1$ (Note: y is the dependant variable.)

True. Let x = -1, then for every y in R, $-1 < y^2 + 1$ is a true statement.

Note: x = -1 is an equation that shows this statement is true for every y.

Negation:

$\exists\, y \in \mathbb{R} \;\ni\; \forall\; x \in \mathbb{R},\;\; x \geq y^2 + 1$

False: Let x = -1, then $-1 \geq y^2 + 1$ is false for all y in R. Which means there is no y in R, that is true for every x in R.

5. $\forall\; x \in \mathbb{R} \;\; \exists\, y \in \mathbb{R} \;\ni\; x^2 + y^2 = 1$

At first glance this proposition seems like it is true, and $y = \sqrt{1 - x^2}$, is an equation that can find a y for every x. However, if the domain for both variables were the complex numbers, then this proposition would be true. If you notice a value, such as x = 2 inserted into our equation produces the complex number $y = i\sqrt{3}$. Therefore, our conclusion is:

False: Let x = 2. Then, there is no y in the real numbers, such that $x^2 + y^2 = 1$.

Note: x = 2 is a counterexample.

Negation: $\exists\, x \in \mathbb{R} \;\ni\; \forall\; y \in \mathbb{R},\;\; x^2 + y^2 \neq 1$

True. Let x = 2, then for every y in R, $(2)^2 + y^2 \neq 1$ or $[y^2 \neq -3]$ is a true statement.

Note: x = 2 is a witness.

6. $\forall\, x \in \mathbb{C} \quad \exists\, y \in \mathbb{C} \ni \; x^2 + y^2 = 1$

True. Let $y = \sqrt{1-x^2}$. Then, for every x in C, let $y = \sqrt{1-x^2}$, and $x^2 + y^2 = 1$ will be a true statement.

Common error answer: Let x = 0 + 0i, y = 1 + 0i. This is not correct because you have not shown for every x there is a y. You have just shown for one x there is a y.

Negation:

$\exists\, x \in \mathbb{C} \ni \forall\, y \in \mathbb{C}, \quad x^2 + y^2 \neq 1$

False. Let $y = \sqrt{1-x^2}$. Then, for every x in C, $y = \sqrt{1-x^2}$ will fail in $x^2 + y^2 \neq 1$, which means there is no x in C that is true for every y in C.

Common error answer: False. Let x = 0+0i, y = 1+0i. This is not correct because you have just shown it fails for one x, when you are suppose to show it fails for EVERY x!

7. $\exists\, x \in \mathbb{R} \quad \exists\, y \in \mathbb{R} \ni \; x^2 + y^2 = 25$

Rewrite as:

$\exists\, x,y \in \mathbb{R} \ni \; x^2 + y^2 = 25$

True. Let x = 0, y = 5. Then, $0^2 + 5^2 = 25$ is a true statement.

Note: x = 0, y = 5 is a witness.

Negation: $\forall\, x,y \in \mathbb{R}, \quad x^2 + y^2 \neq 25$

False. Let x = 0, y = 5. Then, $0^2 + 5^2 \neq 25$ is a false statement.

Note: x = 0, y = 5 is a counterexample.

8. $\forall\ x \in \mathbb{R}\ \ \exists\ y \in \mathbb{R}\ \ni\ x^2 + y^2 = 25$

False: Let x = 6. Then, there is no y in the real numbers, such that $(6)^2 + y^2 = 25$. i.e., $[y^2 = -11]$ is false for all y in R.
Note: x = 6 is a counterexample.

Negation: $\exists\ x \in \mathbb{R}\ \ni\ \forall\ y \in \mathbb{R},\ x^2 + y^2 \neq 25$

True. Let x = 6, then for every y in R, $x^2 + y^2 \neq 25$ is a true statement.
i.e., $[y^2 \neq -11]$ is true for all y in R.
Note: x = 6 is a witness.

9. $\forall\ x \in \mathbb{Z}\ \ \forall\ y \in \mathbb{Z},\ x^2 + y^2 \geq 0$

Rewrite as:

$\forall\ x, y \in \mathbb{Z},\ x^2 + y^2 \geq 0$

True. $\forall\ x \in \mathbb{Z},\ x^2 \geq 0$ is true and $\forall\ y \in \mathbb{Z},\ y^2 \geq 0$ is true. Thus, the sum of the two inequalities is $x^2 + y^2 \geq 0$. Thus, this is true for all x and y in Z.

Common error answer: True. Let x = 1, y = 1. This is not correct because you have NOT shown for every x and y produce a true statement. You have just shown for one x and y produces a true statement.

Negation:

$\exists\ x, y \in \mathbb{Z}\ \ni\ x^2 + y^2 < 0$

False. $\forall\ x \in \mathbb{Z},\ x^2 \geq 0$ is true and $\forall\ y \in \mathbb{Z},\ y^2 \geq 0$ is true. Thus, there is no x, y in Z that can be true for $x^2 + y^2 < 0$. Therefore, there is no x, y in Z where $x^2 + y^2 < 0$.
Common error answer: False. Let x = 1, y = 1. This is not correct because you have just shown it fails for one x and one y. You are suppose to show it fails for EVERY x and y combination!

10. $\forall\, x \in \mathbb{N}\ \ \exists\, y \in \mathbb{N} \ni\ y < x$

False. Let x = 1. Then, there is no y in N such that y < 1.

Note: x = 1 is a counterexample.

Negation: $\exists\, x \in \mathbb{N}\ \ni\ \forall\, y \in \mathbb{N},\ \ y \geq x$

True. Let x = 1, then for all y in N, $y \geq 1$ is true.

Note: x = 1 is a witness.

11. $\exists\, x \in W\ \ni\ \forall\, y \in W,\ \ x < y + 1$

True. Let x = 0, then for all y in W, 0 < y + 1 is a true statement.

Note: x = 0 is a witness.

Negation: $\forall\, x \in W\ \ \exists\, y \in W \ni\ \ x \geq y + 1$

False. Let x = 0, then there is no y in W, such that $0 \geq y + 1$.

Note: x = 0 is a counterexample.

12. $\exists\, x \in \mathbb{R}\ \ni\ \forall\, y \in \mathbb{R},\ \ x^2 \leq y - 1$

This is a false 'there exist' statement. Thus, we need to justify why out of all the real numbers, there is no x that produces a true statement. The x that we are suppose to come up with, must work with EVERY y. Thus, it must work with y = -1. However, if we input y = -1 into $x^2 \leq (-1) - 1$. We get $x^2 \leq -2$. There is no x in R that satisfies this inequality. Therefore, the 'there exist' statement is false. i.e., all x in R produce a false statement.

Common error answer: False. Let x = 1, y = 1. This is not correct because you have just shown it fails for one x, when you are suppose to show it fails for EVERY x!

Negation: $\forall\, x \in \mathbb{R}\ \ \exists\, y \in \mathbb{R} \ni\ \ x^2 > y - 1$

This is a true 'for all' statement. For every x, let y = -1 then $x^2 > (-1) - 1$ is a true statement. Therefore, the 'for all' statement is true.

Common error answer: True. Let x = 1, y = 1. This is not correct because you have NOT shown for every x there is a y. You have just shown for one x there is a y.

13. $\exists\ x \in \mathbb{N} \ni$ if $x > 1$, then $x^2 - x > 0$

True. Let x = 2, then 2 > 1 and $(2)^2 - 2 > 0$ is a true statement.

Note: x = 2 is a witness.

Negation: $\forall\ x \in \mathbb{N},\ \ x > 1$ and $x^2 - x \leq 0$

False. Let x = 2 then 2 > 1, but $(2)^2 - 2 \leq 0$ is a false statement.

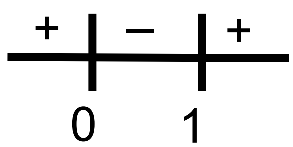
Note: x = 2 is a counterexample.

14. $\forall\ x \in \mathbb{N}$ if $x > 1$, then $x^2 - x > 0$

This is a true "for all" conditional statement. Solve the inequality, and show that it has a solution set for all naturals on (1, ∞).

Consider: $x^2 - x > 0$ (This is a non-linear inequality and must be solved as such.)

$x(x - 1) > 0$



So $x(x - 1) > 0$ is true on x < 0 or x > 1. (We only need the x > 1 solutions.)

Therefore, $x^2 - x > 0$ is true for all x in N on (1, ∞).

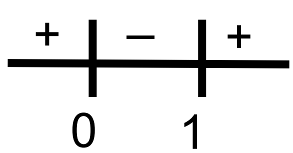
Common error answer: True. Let x = 2. This is not correct because you have NOT shown for every x > 1, $x^2 - x > 0$ is true. You have just shown for one x > 1 $x^2 - x > 0$ is true.

Negation: $\exists\ x \in \mathbb{N} \ni\ x > 1$ and $x^2 - x \leq 0$

This is a false "there exists" conditional statement. Write the proposition. Work on $x^2 - x \leq 0$ until it contradicts $\exists\ x \in \mathbb{N} \ni\ x > 1$.

Consider: $x^2 - x \leq 0$ (This is a non-linear inequality and must be solved as such.)

$x(x - 1) \leq 0$



So $x(x - 1) \leq 0$ is true on [0, 1]. We now have a contradiction to $\exists\ x \in \mathbb{N} \ni\ x > 1$. There is no x in N that is greater than one and on [0, 1] at the same time. Therefore, there is no x in N x > 1 and satisfies $x^2 - x \leq 0$.

15. $\forall\ x \in \mathbb{R}$ if $x > 0$, then $x^2 > x$

False. Let x = 1/2, then $(1/2)^2 > (1/2)$ is a false statement.

Note: x = 1/2 is a counterexample.

Negation: $\exists\ x \in \mathbb{R} \ni x > 0$ and $x^2 \le x$

True. Let x = 1/2, then $(1/2)^2 \le (1/2)$ is a true statement.

Note: x = 1/2 is a witness.

16. $\forall\ x \in \mathbb{R}$ if $x > 1$, then $\dfrac{x}{x^2 - 1} > 0$

This statement says that for all x in the real numbers, which are greater than 1, the non-linear inequality is true. Solve the non-linear inequality and verify the solution set does indeed have solutions on $(1, \infty)$.

The critical points are: -1, 0, 1

```
____-__|__+___|__-___|__+___
      -1      0      1
```

Thus, the solution set is: $(-1, 0] \cup (1, \infty)$ (We only need the solutions for $(1, \infty)$.

Therefore, the inequality is true for all x in R on $(1, \infty)$.

Therefore, the statement is true.

Common error answer: True. Let x = 2. This is not correct because you have NOT shown for every x > 1, $x^2/(x^2 - 1) > 0$ is true. You have just shown for one x > 1 $x^2/(x^2 - 1) > 0$ is true.

Negation: $\exists\ x \in \mathbb{R} \ni x > 1$ AND $\dfrac{x}{x^2 - 1} \le 0$

This statement says there exists at least one x in the real numbers, which is great than one AND satisfies the non-linear inequality.

```
____-__|__+___|__-___|__+___
      -1      0      1
```

The solution set for the non-linear inequality is: $(-\infty, -1) \cup [0, 1)$

Therefore, there are no real numbers greater than one which satisfy the non-linear inequality. Therefore, the negation is false.

17. $\forall\ x \in \mathbb{R} \quad \exists\ y \in \mathbb{R},\ $ if $x < y$, then $x^2 \le y^2$

This statement says that for all x in R, I can find at least one y in R that satisfies x < y and $x^2 \le y^2$ will also be true.
This is true. Let y = |x| + 1, then x < |x| + 1 and $x^2 \le (|x| + 1)^2$ is true.
Common error answer: True. Let x = 1, y = 2. This is not correct because you have NOT shown for every x there is a y. You have just shown for one x there is a y.

Negation: $\exists\ x \in \mathbb{R} \ni \forall\ y \in \mathbb{R}\quad x < y$ and $x^2 > y^2$

This statement says that I can find at least one x in R, such that both x < y AND $x^2 > y^2$ for every y in R. This is false. For any x in R, x < y for all y is false. Thus, we are never going to get a true "x < y AND $x^2 > y^2$" statement. Therefore, there is no x in R.
Common error answer: False. Let x = 1, y = 2. This is not correct because you have just shown it fails for one x, when you are suppose to show it fails for EVERY x!

18. $\forall\ x \in \mathbb{R} \quad \forall\ y \in \mathbb{R},\ $ if $x < y$, then $x^2 \le y^2$

Rewrite as: $\forall\ x, y \in \mathbb{R},\ $ if $x < y$, then $x^2 \le y^2$

False. Let x = -1, y = 0. Then, -1 < 0, but $(-1)^2 \le (0)^2$ is a false statement.
Note: x = -1, y = 0 is a counterexample.

Negation: $\exists\ x, y \in \mathbb{R} \ni \quad x < y$ and $x^2 > y^2$

True. Let x = -1, y = 0 then -1 < 0 and $(-1)^2 > (0)^2$ is a true statement.
Note: x = -1, y = 0 is a witness.

19. $\exists\ x \in \mathbb{N} \quad \exists\ y \in \mathbb{N} \ni$ if $x < y$, then $x^2 \le y^2$

True. Let x = 1, y = 2, then 1 < 2 and $(1)^2 \le (2)^2$ is a true statement.
Note: x = 1, y = 2 is a witness.

Negation: $\forall x \in \mathbb{N} \quad \forall\ y \in \mathbb{N}\quad x < y$ and $x^2 > y^2$

False. Let x = 1, = 2, then 1 < 2, but $(1)^2 > (2)^2$ is a false statement.
Note: x = 1, y = 2 is a counterexample.