Babeş-Bolyai University, Faculty of Mathematics and Computer Science
Analysis for Computer Science
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Exercise Set #9

1. Let $f : [0,2] \times [0,4] \to \mathbb{R}$, $f(x,y) = x^2 - 2xy + 2y$. Find the global minimum and global maximum points of f.

2. Let $f : \mathbb{R}^2 \to \mathbb{R}$, $f(x,y) = (x^2 - y)(x^2 - 3y)$.
a) Prove that 0_2 is a stationary point of f. Is 0_2 a local minimum point of f?
b) Prove that the restriction of f to any line through 0_2 attains a local minimum at 0_2.

3. Find the stationary points and the local extremum points (specifying their type) of the following functions:
a) $f : \mathbb{R}^2 \to \mathbb{R}$, $f(x,y) = x^3 - 3x + y^2$, b) $f : \mathbb{R}^2 \to \mathbb{R}$, $f(x,y) = x^3 + y^3 - 3xy$,
c) $f : (0,\infty) \times \mathbb{R} \to \mathbb{R}$, $f(x,y) = x(y^2 + \ln^2 x)$, d) $f : \mathbb{R}^2 \to \mathbb{R}$, $f(x,y) = x^4 + y^4 - 4(x-y)^2$,
e) $f : \mathbb{R}^3 \to \mathbb{R}$, $f(x,y,z) = z^2(1 + xy) + xy$, f) $f : \mathbb{R}^3 \to \mathbb{R}$, $f(x,y,z) = x^3 - 3x + y^2 + z^2$.