

SUBJECT 1

1. The semantics of propositional logic.
Using a semantic proof method, prove that the syllogism rule is a tautology.
2. Check the distributivity property of the existential quantifier over implication using a syntactic proof method. The theorem of soundness and completeness of the method.
3. Draw a logic circuit having 3 inputs and containing all basic and derived gates. Write the corresponding Boolean function and simplify it. Draw the simplified circuit.

SUBJECT 2

1. Using lock resolution check if: $p \rightarrow q, \neg(q \rightarrow r) \rightarrow \neg p \vdash p \rightarrow r$.
Resolution as a formal system.
2. Using a semantic proof method check if the formula: $(\forall x)A(x) \vee (\forall x)B(x)$ is a logic consequence of the formula: $(\forall x)(A(x) \wedge B(x))$. Theory.
3. Simplify the following Boolean function using Veitch diagram:
 $f(x_1, x_2, x_3) = x_1x_3 \vee x_1x_2x_3 \vee \bar{x}_1x_3 \vee \bar{x}_1\bar{x}_2x_3$. Implement the logic circuits corresponding to the initial form of f and to all the simplified forms of f .

SUBJECT 3

1. The formal (axiomatic) system of propositional logic. What is a theorem?
Using a refutation proof method prove that the separation of the premises law is a theorem.
2. Evaluate the formula $U = ((\forall x)p(x) \rightarrow (\forall x)q(x)) \rightarrow (\forall x)(p(x) \wedge q(x))$ under two interpretations: one with a finite domain and the other with an infinite domain. How many possible interpretations has U ? Is predicate logic decidable? Argue your answer.
3. Using Quine's method simplify the Boolean function:
 $f(x_1, x_2, x_3, x_4) = x_1\bar{x}_2x_3 \vee x_1x_2x_3\bar{x}_4 \vee x_1x_2\bar{x}_3x_4 \vee x_1\bar{x}_2\bar{x}_3\bar{x}_4 \vee \bar{x}_1x_2x_3\bar{x}_4$.
Implement the logic circuit associated to a simplified form of f .

SUBJECT 4

1. Write all the models and the anti-models of the formula: $V = ((p \wedge \neg r) \rightarrow q) \rightarrow \neg p \wedge \neg q \wedge r$.
Theory.
2. Using linear resolution check if the following set of formulas is inconsistent.
 $S = \{p(x) \wedge q(x) \vee r(x), \neg q(y) \vee r(y), r(a) \wedge \neg p(a)\}$. Theory.
3. Definitions for: minterm, maxterm, central monom, maximal monom, factorization.
Examples of 4 minterms and 4 maxterms of 4 variables: expressions, notations and tables of values. Draw the logic circuit associated to the Boolean function:
 $f(x_1, x_2, x_3, x_4) = m_1 \vee m_{13} \vee m_8 \vee m_5$.