Exercises –Boolean functions

Exercise 7.1.

For the following Boolean functions of 3 variables, given by their tables of values, write the corresponding *disjunctive canonical form* (*DCF*) and ***conjunctive canonical form*** (*CCF*). Using Veitch diagrams simplify the functions.

x	y	z	f_1	f_2	f_3	f_4	f_5	f_6	f_7	f_8
0	0	0	0	1	1	1	0	1	0	1
0	0	1	1	1	1	0	1	0	0	0
0	1	0	0	0	1	0	1	1	1	1
0	1	1	1	0	0	1	0	1	0	0
1	0	0	1	0	1	0	0	0	1	0
1	0	1	0	1	0	0	1	0	1	1
1	1	0	0	0	0	1	1	1	0	1
1	1	1	1	1	0	1	0	0	1	0

Exercise 7.2.

Simplify the following Boolean functions of 4 variables using Veitch diagrams.

1. $f_1(x_1,x_2,x_3,x_4) = x_1\bar{x}_2\bar{x}_3\bar{x}_4 \vee x_1\bar{x}_2x_3\bar{x}_4 \vee x_1x_2\bar{x}_3\bar{x}_4 \vee \bar{x}_1\bar{x}_2\bar{x}_3\bar{x}_4 \vee \bar{x}_1x_2\bar{x}_3\bar{x}_4 \vee \bar{x}_1\bar{x}_2x_3\bar{x}_4$
$\vee\, x_1\bar{x}_2x_3x_4 \vee \bar{x}_1\bar{x}_2\bar{x}_3x_4 \vee \bar{x}_1x_2\bar{x}_3x_4;$
2. $f_2(x_1,x_2,x_3,x_4) = x_1\bar{x}_2x_3x_4 \vee x_1x_2\bar{x}_3x_4 \vee \bar{x}_1x_2x_3\bar{x}_4 \vee \bar{x}_1\bar{x}_2\bar{x}_3\bar{x}_4 \vee x_1\bar{x}_2\bar{x}_3\bar{x}_4 \vee \bar{x}_1x_2\bar{x}_3\bar{x}_4$
$\vee\, \bar{x}_1\bar{x}_2x_3x_4 \vee \bar{x}_1\bar{x}_2\bar{x}_3x_4 \vee \bar{x}_1x_2\bar{x}_3x_4;$
3. $f_3(x_1,x_2,x_3,x_4) = \bar{x}_1x_2x_3x_4 \vee \bar{x}_1x_2\bar{x}_3x_4 \vee \bar{x}_1x_2x_3\bar{x}_4 \vee x_1\bar{x}_2\bar{x}_3\bar{x}_4 \vee x_1\bar{x}_2x_3\bar{x}_4 \vee x_1x_2\bar{x}_3\bar{x}_4$
$\vee\, x_1\bar{x}_2x_3x_4 \vee x_1\bar{x}_2\bar{x}_3x_4 \vee \bar{x}_1\bar{x}_2\bar{x}_3x_4;$
4. $f_4(x_1,x_2,x_3,x_4) = \bar{x}_1\bar{x}_2x_3x_4 \vee x_1\bar{x}_2x_3x_4 \vee \bar{x}_1x_2\bar{x}_3x_4 \vee \bar{x}_1x_2x_3\bar{x}_4 \vee x_1\bar{x}_2\bar{x}_3\bar{x}_4 \vee x_1x_2\bar{x}_3\bar{x}_4$
$\vee\, x_1\bar{x}_2x_3x_4 \vee x_1\bar{x}_2\bar{x}_3x_4 \vee \bar{x}_1\bar{x}_2\bar{x}_3x_4;$
5. $f_5(x_1,x_2,x_3,x_4) = x_1\bar{x}_2x_3\bar{x}_4 \vee x_1\bar{x}_2\bar{x}_3x_4 \vee x_1x_2\bar{x}_3\bar{x}_4 \vee \bar{x}_1\bar{x}_2x_3\bar{x}_4 \vee \bar{x}_1x_2\bar{x}_3\bar{x}_4 \vee \bar{x}_1\bar{x}_2\bar{x}_3x_4$
$\vee\, \bar{x}_1\bar{x}_2x_3x_4 \vee x_1\bar{x}_2\bar{x}_3\bar{x}_4 \vee \bar{x}_1x_2\bar{x}_3x_4;$
6. $f_6(x_1,x_2,x_3,x_4) = x_1\bar{x}_2x_3x_4 \vee \bar{x}_1\bar{x}_2x_3x_4 \vee \bar{x}_1x_2\bar{x}_3x_4 \vee x_1x_2x_3\bar{x}_4 \vee \bar{x}_1x_2\bar{x}_3\bar{x}_4 \vee \bar{x}_1x_2x_3\bar{x}_4$
$\vee\, x_1\bar{x}_2\bar{x}_3\bar{x}_4 \vee x_1\bar{x}_2x_3x_4 \vee \bar{x}_1x_2\bar{x}_3x_4;$
7. $f_7(x_1,x_2,x_3,x_4) = x_1\bar{x}_2x_3x_4 \vee \bar{x}_1\bar{x}_2x_3x_4 \vee \bar{x}_1x_2\bar{x}_3x_4 \vee x_1x_2x_3\bar{x}_4 \vee x_1\bar{x}_2\bar{x}_3\bar{x}_4 \vee \bar{x}_1x_2\bar{x}_3\bar{x}_4$
$\vee\, \bar{x}_1\bar{x}_2x_3\bar{x}_4 \vee \bar{x}_1\bar{x}_2\bar{x}_3x_4 \vee \bar{x}_1x_2\bar{x}_3x_4;$
8. $f_8(x_1,x_2,x_3,x_4) = x_1\bar{x}_2x_3x_4 \vee x_1\bar{x}_2\bar{x}_3x_4 \vee \bar{x}_1x_2\bar{x}_3x_4 \vee \bar{x}_1\bar{x}_2x_3\bar{x}_4 \vee x_1\bar{x}_2\bar{x}_3\bar{x}_4 \vee \bar{x}_1x_2\bar{x}_3\bar{x}_4$
$\vee\, \bar{x}_1\bar{x}_2x_3x_4 \vee \bar{x}_1x_2\bar{x}_3x_4 \vee \bar{x}_1\bar{x}_2\bar{x}_3x_4.$

Exercise 7.3.

Using Karnaugh diagrams, simplify the following Boolean functions:

1. $f_1(x_1,x_2,x_3) = \bar{x}_1(x_2 \downarrow x_3) \vee \bar{x}_1\bar{x}_2x_3 \vee \overline{(x_1 \vee (x_2 \uparrow x_3))} \vee x_1x_2\bar{x}_3$
2. $f_2(x_1,x_2,x_3) = \bar{x}_1\bar{x}_2\bar{x}_3 \vee x_1(\bar{x}_2 \downarrow x_3) \vee \overline{(x_1 \vee (x_2 \uparrow \bar{x}_3))} \vee x_1\bar{x}_2x_3$
3. $f_3(x_1,x_2,x_3) = \bar{x}_1(x_2 \downarrow x_3) \vee x_1\bar{x}_2x_3 \vee \overline{(x_1 \vee (x_2 \uparrow x_3))} \vee x_1x_2x_3$
4. $f_4(x_1,x_2,x_3) = \bar{x}_1\bar{x}_2\bar{x}_3 \vee x_1(\bar{x}_2 \downarrow x_3) \vee \overline{(x_1 \vee (\bar{x}_2 \uparrow x_3))} \vee x_1\bar{x}_2x_3$
5. $f_5(x_1,x_2,x_3) = x_1(x_2 \downarrow x_3) \vee \bar{x}_1\bar{x}_2x_3 \vee \overline{(\bar{x}_1 \vee (\bar{x}_2 \uparrow x_3))} \vee \bar{x}_1x_2\bar{x}_3$
6. $f_6(x_1,x_2,x_3) = x_1\bar{x}_2x_3 \vee \bar{x}_1(\bar{x}_2 \downarrow x_3) \vee \overline{(\bar{x}_1 \vee (x_2 \uparrow x_3))} \vee x_1\bar{x}_2\bar{x}_3$
7. $f_7(x_1,x_2,x_3) = \bar{x}_1(x_2 \downarrow x_3) \vee x_1\bar{x}_2x_3 \vee \overline{(x_1 \vee (x_2 \uparrow x_3))} \vee x_1\bar{x}_2\bar{x}_3$
8. $f_8(x_1,x_2,x_3) = \bar{x}_1\bar{x}_2x_3 \vee x_1(x_2 \downarrow x_3) \vee \overline{(x_1 \vee (x_2 \uparrow x_3))} \vee \bar{x}_1x_2\bar{x}_3$

Exercise 7.4.

Simplify the following Boolean functions of 4 variables using Karnaugh diagrams.

1. $f_1(x_1,x_2,x_3,x_4) = x_1\bar{x}_4 \vee x_1x_2\bar{x}_3x_4 \vee \bar{x}_1x_2x_4 \vee \bar{x}_1x_3 \vee x_3\bar{x}_4$;
2. $f_2(x_1,x_2,x_3,x_4) = x_1x_2 \vee x_1\bar{x}_2\bar{x}_3x_4 \vee \bar{x}_1\bar{x}_2x_4 \vee \bar{x}_1x_3 \vee x_2x_3$;
3. $f_3(x_1,x_2,x_3,x_4) = x_1x_4 \vee x_1\bar{x}_2\bar{x}_3\bar{x}_4 \vee \bar{x}_1\bar{x}_2\bar{x}_4 \vee \bar{x}_1\bar{x}_3 \vee x_3x_4$;
4. $f_4(x_1,x_2,x_3,x_4) = x_1\bar{x}_2 \vee x_1x_2\bar{x}_3\bar{x}_4 \vee \bar{x}_1x_2\bar{x}_4 \vee \bar{x}_1x_3 \vee \bar{x}_2x_3$;
5. $f_5(x_1,x_2,x_3,x_4) = x_3\bar{x}_4 \vee x_1x_2x_3x_4 \vee \bar{x}_3x_2x_4 \vee \bar{x}_1\bar{x}_3 \vee \bar{x}_1\bar{x}_4$;
6. $f_6(x_1,x_2,x_3,x_4) = \bar{x}_1\bar{x}_4 \vee \bar{x}_1x_2x_3x_4 \vee x_1x_2x_4 \vee x_1\bar{x}_3 \vee \bar{x}_3\bar{x}_4$;
7. $f_7(x_1,x_2,x_3,x_4) = \bar{x}_3\bar{x}_4 \vee \bar{x}_1x_2\bar{x}_3x_4 \vee x_2x_3x_4 \vee x_1x_3 \vee x_1\bar{x}_4$;
8. $f_8(x_1,x_2,x_3,x_4) = x_3x_4 \vee x_1\bar{x}_2x_3\bar{x}_4 \vee \bar{x}_2\bar{x}_3\bar{x}_4 \vee \bar{x}_1\bar{x}_3 \vee \bar{x}_1x_4$.

Exercise 7.5.

Using Quine's method, simplify the following Boolean functions given by their values 0.

1. $f_1(0,1,0) = f_1(0,1,1) = f_1(1,0,1) = 0$;
2. $f_2(0,0,0) = f_2(0,0,1) = f_2(1,1,1) = 0$;
3. $f_3(0,0,1) = f_3(0,1,0) = f_3(1,1,0) = 0$;
4. $f_4(0,0,0) = f_4(0,1,1) = f_4(1,0,0) = 0$;
5. $f_5(0,0,0) = f_5(1,1,0) = f_5(1,1,1) = 0$;
6. $f_6(0,1,0) = f_6(1,0,0) = f_6(1,0,1) = 0$;
7. $f_7(0,1,1) = f_7(1,0,0) = f_7(1,1,1) = 0$;
8. $f_8(0,0,1) = f_8(1,0,1) = f_8(1,1,0) = 0$.

Exercise 7.6.

Using Quine's method, simplify the following Boolean functions given in *DCF* (disjunction of minterms):

1. $f_1(x_1,x_2,x_3) = m_0 \vee m_3 \vee m_4 \vee m_5 \vee m_6 \vee m_7$;
2. $f_2(x_1,x_2,x_3) = m_1 \vee m_2 \vee m_4 \vee m_5 \vee m_6 \vee m_7$;
3. $f_3(x_1,x_2,x_3) = m_1 \vee m_2 \vee m_3 \vee m_4 \vee m_5 \vee m_7$;
4. $f_4(x_1,x_2,x_3) = m_0 \vee m_1 \vee m_2 \vee m_3 \vee m_5 \vee m_6$;
5. $f_5(x_1,x_2,x_3) = m_0 \vee m_1 \vee m_2 \vee m_4 \vee m_6 \vee m_7$;
6. $f_6(x_1,x_2,x_3) = m_0 \vee m_1 \vee m_3 \vee m_5 \vee m_6 \vee m_7$;
7. $f_7(x_1,x_2,x_3) = m_0 \vee m_1 \vee m_2 \vee m_3 \vee m_4 \vee m_7$;
8. $f_8(x_1,x_2,x_3) = m_0 \vee m_2 \vee m_3 \vee m_4 \vee m_5 \vee m_6$.

Exercise 7.7.

Simplify the following Boolean functions of 4 variables given by their values 1, using Quine's method.

1. $f_1(1,1,1,1) = f_1(1,1,0,1) = f_1(0,1,1,1) = f_1(1,1,0,0) = f_1(0,1,0,0) = f_1(0,0,0,0) = f_1(0,0,0,1) = f_1(0,0,1,1) = 1$;
2. $f_2(1,1,0,1) = f_2(0,1,0,1) = f_2(0,1,0,0) = f_2(0,0,0,0) = f_2(0,0,1,0) = f_2(1,0,1,1) = f_2(1,0,0,1) = f_2(0,0,1,1) = 1$;
3. $f_3(0,1,0,1) = f_3(0,1,0,0) = f_3(0,1,1,0) = f_3(1,0,1,0) = f_3(1,0,0,0) = f_3(0,0,1,0) = f_3(1,0,0,1) = f_3(0,0,0,1) = 1$;
4. $f_4(0,1,0,1) = f_4(0,1,1,1) = f_4(1,1,1,0) = f_4(1,1,0,0) = f_4(0,1,1,0) = f_4(1,0,0,0) = f_4(0,0,0,0) = f_4(0,0,0,1) = 1$;
5. $f_5(1,1,1,1) = f_5(0,1,0,1) = f_5(0,1,1,1) = f_5(1,1,1,0) = f_5(1,1,0,0) = f_5(1,0,0,0) = f_5(1,0,0,1) = f_5(0,0,0,1) = 1$;
6. $f_6(1,1,0,1) = f_6(0,1,0,1) = f_6(0,1,1,1) = f_6(1,1,1,0) = f_6(0,1,1,0) = f_6(1,0,1,0) = f_6(1,0,1,1) = f_6(1,0,0,1) = 1$;
7. $f_7(1,1,1,1) = f_7(1,1,0,1) = f_7(0,1,0,1) = f_7(0,1,0,0) = f_7(0,1,1,0) = f_7(0,0,1,0) = f_7(1,0,1,1) = f_7(0,0,1,1) = 1$;
8. $f_8(1,1,1,1) = f_8(1,1,1,0) = f_8(1,1,0,0) = f_8(1,0,0,0) = f_8(0,0,0,0) = f_8(0,0,1,0) = f_8(1,0,1,1) = f_8(0,0,1,1) = 1$.

Exercise 7.8.

Using Moisil's method simplify the following Boolean functions of 3 variables:

1. $f_1(x_1,x_2,x_3) = m_0 \vee m_1 \vee m_2 \vee m_5 \vee m_6$;
2. $f_2(x_1,x_2,x_3) = m_0 \vee m_1 \vee m_3 \vee m_4 \vee m_7$;
3. $f_3(x_1,x_2,x_3) = m_1 \vee m_2 \vee m_3 \vee m_5 \vee m_6$;
4. $f_4(x_1,x_2,x_3) = m_0 \vee m_3 \vee m_4 \vee m_5 \vee m_7$;
5. $f_5(x_1,x_2,x_3) = m_1 \vee m_2 \vee m_5 \vee m_6 \vee m_7$;
6. $f_6(x_1,x_2,x_3) = m_0 \vee m_2 \vee m_3 \vee m_4 \vee m_7$;
7. $f_7(x_1,x_2,x_3) = m_1 \vee m_2 \vee m_4 \vee m_5 \vee m_6$;
8. $f_8(x_1,x_2,x_3) = m_0 \vee m_3 \vee m_4 \vee m_6 \vee m_7$.