COURSE 10

Hash-Based Files

Hash files (Direct file organization)

- Hashing is a method of determining the address of a record based on the value of one of its fields (usually key value)
- Ideal case: function which calculates the address where each record is to be stored:

  $h: \{K_1, K_2, \ldots, K_n\} \rightarrow A$, $h(K_i)$ = memory add. of i^{th} record

  (K_i is the key value of the i^{th} record, A is the set of disk addresses)
- Hard to define such functions:
  - all possible key values must be known from the beginning
  - for big files is almost impossible to maintain bijectivity

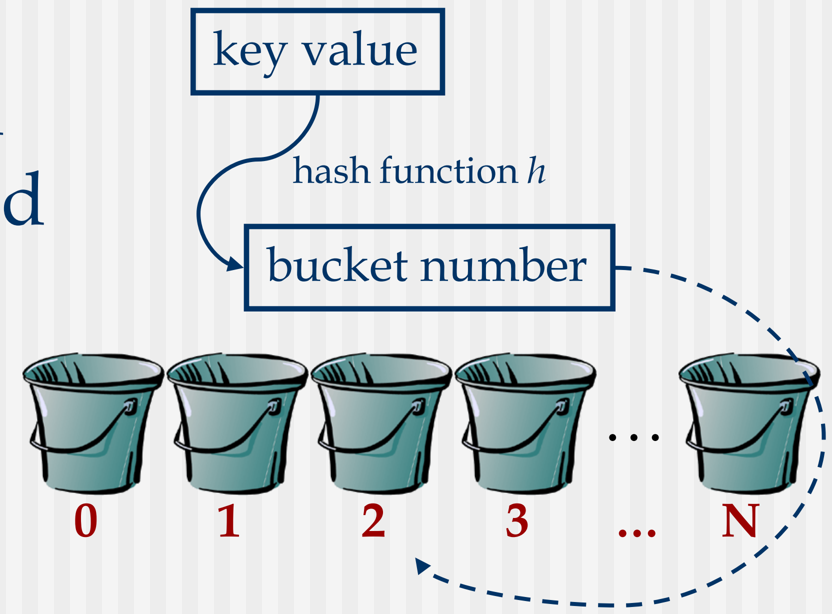
Buckets

- Solution - allow collisions:

$$h(K_i) = h(K_j), \text{ for } i \neq j \qquad (h \text{ – '}\textit{hashing function}\text{'})$$

- Records with key values K_i and K_j are synonyms

- All synonym records are stored into a "*bucket*" which starts at the address returned by *h*. A "*bucket*" is a unit of storage containing one or more records (a bucket is typically a disk block).



Main Concern When Using Hashed Files

- Distribution problem - once we have chosen the hash algorithm, we have no control over the distribution of records in mass storage.
- Clustering problem - majority of records are placed in the same bucket and the rest of buckets contain almost no records.
- Overflow problem - unless the buckets are extremely large, overflow may occur.

Defining Hashing Functions

- Requirements for a 'good' hashing function:

  - fast evaluation

  - minimizes number of collisions (evenly dispersion of records over addresses space)

    - Assume insert records into 41 buckets: the probability of placing the 1st record to an empty bucket is 41/41, the 2nd is 40/41, the 3rd is 39/41 and so on. The probability of placing 8 records into 8 empty buckets is

$$(41/41)(40/41)(39/41)\ldots(34/41) = 0.482$$

Less than 50%!!!

Defining Hashing Functions (cont)

- Approaches:
  - Division method
  - Mid-square method
  - Folding method
  - Multiplication method

- Typical hash functions perform computation on the internal ***binary representation*** of the search-key (for example, for a string search-key, the binary representations of all the characters in the string could be added and this sum could be the parameter of the hash function).

Defining Hashing Functions (cont)

- Division method:
  - Simply define h(k) = k *mod* N
  - This guarantees the range of h(k) to be [0 ...N-1]
  - Choosing N = 2^d for some *d*, only the least *d* bits of *k* will be effectively consider
  - Prime numbers were found to work best for N.
- Mid-square method:
  - Square the key and take some digits from the middle
- Folding method:
  - Divide the number in half and combine the two halves (e.g. add them together)

Defining Hashing Functions (cont)

- Multiplication method
  - Extract the fractional part of Z * k (for a specific Z) and multiply by hash table size N (N is arbitrary here):

$$h(k) = \lfloor N * (Z*k - \lfloor Z*k \rfloor) \rfloor = \lfloor N * \{Z*k\} \rfloor$$

  - Best results are obtained for

$Z = (\sqrt{5} - 1)/2 = 0.61803\ldots$ or $Z = (3 - \sqrt{5})/2 = 0.38196\ldots$

  - For $Z = Z'/2^w$ and $N = 2^d$ (w: number of bits in a CPU word)

$$h(k) = msb_d\ (Z'*k)$$

where $msb_d(x)$ denotes the d most significant bits of x!

(example: 42 has binary rep. 101010, $msb_3(42) = 5$ – bin. 101)

  - Theorem. Given an irrational number *x* and placing {x}, {2x},...,{nx} on [0,1] segment at most 3 different lengths of resulted n+1 segments are obtained. Also, the next value, {(n+1)x}, will be placed on one of the biggest segments

Example of Hashing Functions

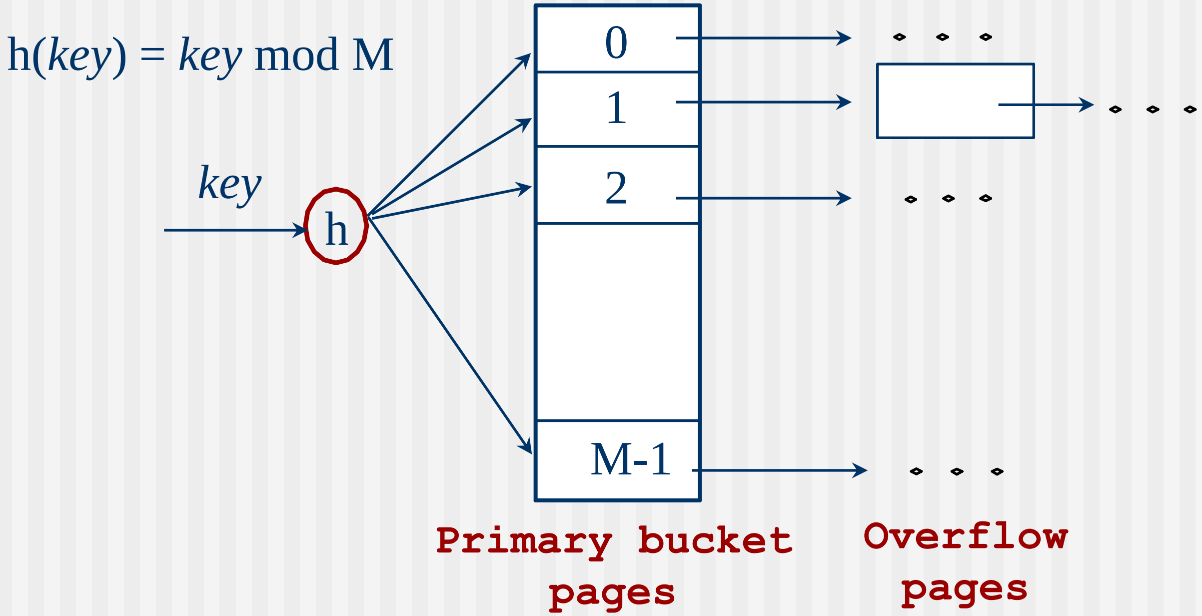
- Key value 'Toyota'
  - Concatenate together the alphabetic position of 1st and 2nd character: **To**yota ⇒ | **20** | **15** |
- Functions to hash-up the integer
  - Division method: mod by 97 → **2015** *mod* 97 = 75
  - Mid-square method: $\mathbf{2015}^2$ = 4060225 (take 2 middle dig)
  - Folding method: **2015** → 20+15 = 35
  - Multiplication method: $\lfloor 99 * \{\mathbf{2015} * 0.61803\} \rfloor = 32$
- Why not use **2015** as hashed value?
  - 4 digits → 10000 possible values! → table will be largely empty
  - our sample needs only 100 slots → we might overflow it

Collision Handling

- Open addressing.
- Store new record in an unsorted overflow area.
- Apply a second hash function to the hash key to get a “second choice” address.
- Store pointers instead of records. At the hash address will be stored:
    - All the pointers of synonym records - **bucket** of addresses.
    - The pointer to the first record. As part of the first record, store a pointer to the second synonym record, and so on - **linked list** addresses.

Static Hashing

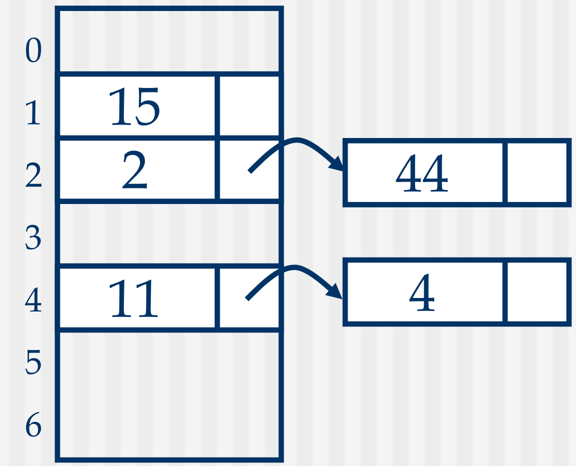
- Number of primary pages (or slots) is fixed, allocated sequentially, never de-allocated; some variants use overflow pages (slots) if needed.
- **h**(k) = k mod M = bucket to which data entry with key k belongs. (M = number of buckets)



Static Hashing: Independent Lists

- All synonyms are stored into a specific linked list
- The hashed file contains a list of M data entries, each is the head of a list of synonyms
- The order of synonyms in list can be:
  - the order of insertions
  - decreasing order of searching frequency
  - increasing order of key value (searching without success stops earlier)

k	h(k) = k mod 7
11	4
2	2
44	2
4	4
15	1

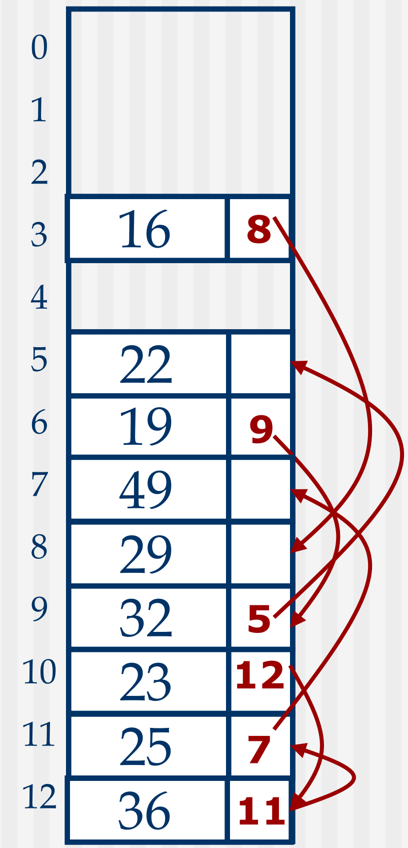


Static Hashing: Interleaved Lists

- Overflow pages (or slots) not used
- Insert a new record with key value K:
  - If the slot at h(K) address is empty: store the record
  - If the slot where the record must be stored is full:
    - occupy the first empty slot (searching from bottom to top of file)
    - insert the occupied slot at the end of the list which contains the slot referred by h(*K*)

Example:

k	h(k) = k mod 13
16	3
23	10
36	10
25	12
19	6
32	6
29	3
49	10
22	9



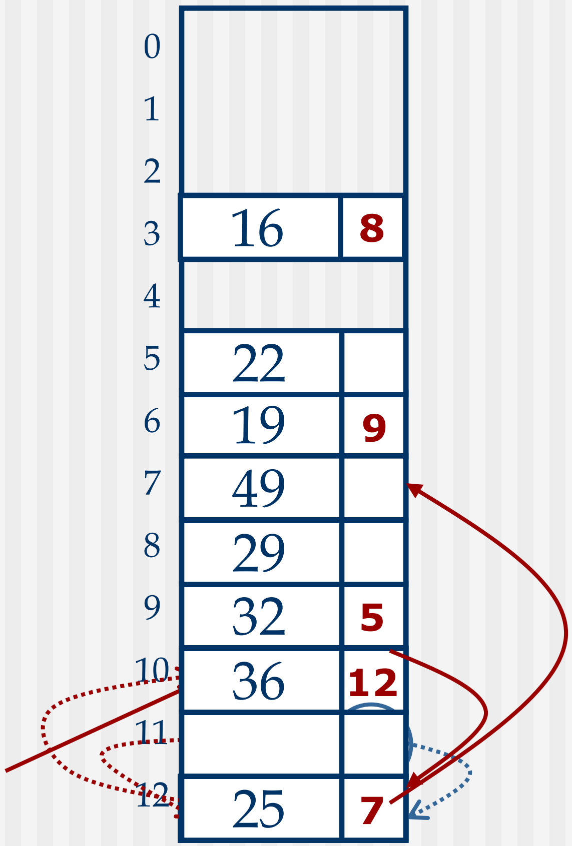
Static Hashing: Interleaved Lists (cont)

- Deleting a record with key value K:
  - If the slot from h(K) is empty: error message
  - If the slot from h(K) is full:

    1. remove the record
    2. search, following the link, a record *r* with $h(K_r) = h(K)$
        - if such record is found
            - move it to current slot.
            - repeat step 2 for the new empty slot
        - otherwise, continue with step 3
    3. copy the link of the empty slot in the previous element in list (if exist)

Static Hashing: Interleaved Lists (cont)

- Example for deleting records: delete record with key value 23.



k	h(k) = k mod 13
16	3
23	10
36	10
25	12
19	6
32	6
29	3
49	10
22	9

Static Hashing: Open Addressing

- The file contains only data entries (no link slot)
- Insert a new record with key value K:
  - If the slot at h(K) address is empty: store the record
  - If the slot where a record must be stored is full search for an empty slot at h(K)+1, h(K)+2,...,M-1,0,....,h(K)-1 addresses.
- Good for 75% occupancy

Example:

k	h(k) = k mod 13
5	5
21	8
24	11
22	9
23	10
34	8
35	9

0	35
1	
2	
3	
4	
5	5
6	
7	
8	21
9	22
10	23
11	24
12	34

Static Hashing: Open Addressing (cont)

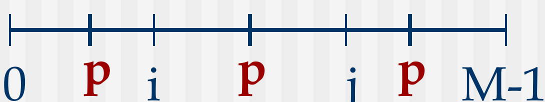
- Deleting a record with key value K:

A. Replace the key value K with a special code or character. (searching and inserting algorithms must be adjusted in order to interpret special values)

B. Delete the desired record and switch remaining records. Consider *i*, *j* and *p* memory addresses so that:

- *i* is the address of deleted record,
- no empty slots between *i* and *j*
- the record stored at *j* must be stored at *p*

$i<j$

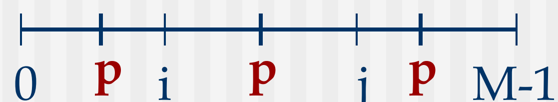


$i < p \leq j$: -

$j \leq p \leq m\text{-}1$: *record at j transfer at i*

$0 \leq p \leq i$: *record at j transfer at i*

$j < i$



$0 < p \leq j$: -

$j < p \leq i$ *record at j transfer at i*

$i < p \leq i$: -

Conclusions on Static Hashing

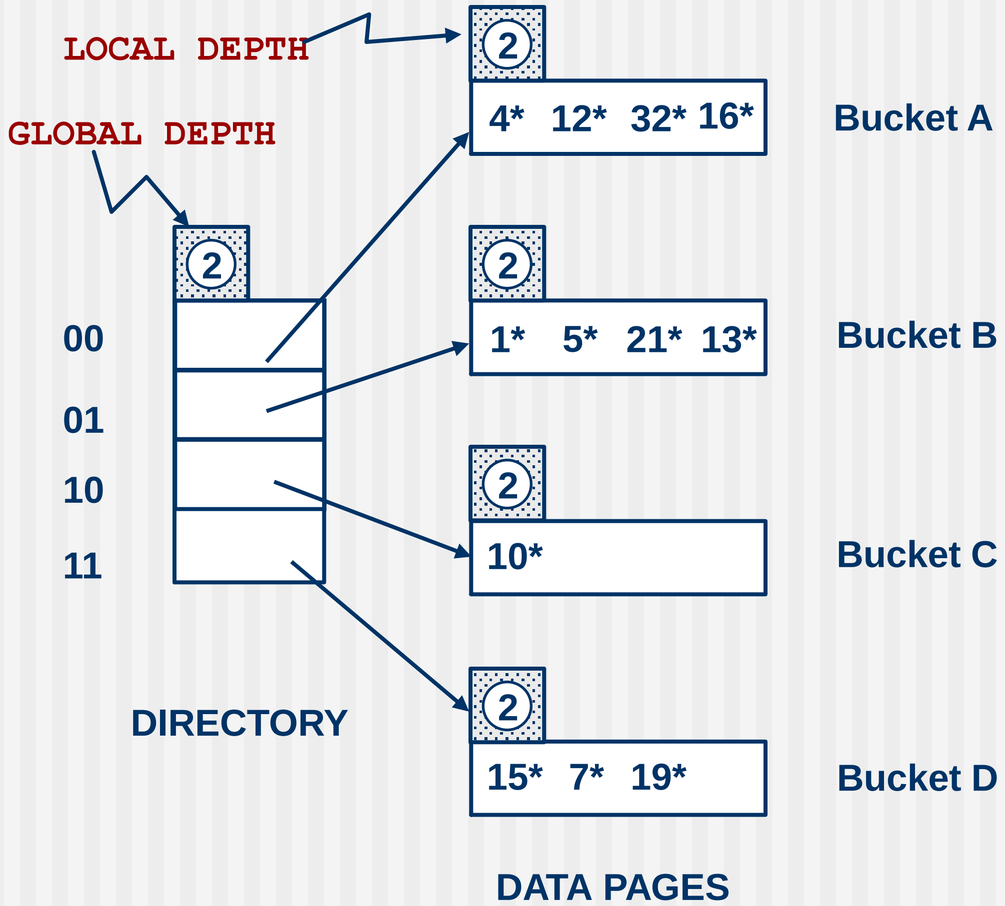
- Buckets contain *data entries*.
- Hash function works on *search key* field of record *r*. Must distribute values over range 0 ... M-1.
- If the underlying data file grows, the development of overflow chains spoils the otherwise predictable hash I/O behavior (1-2 I/Os).
- Similarly, if the file shrinks significantly, the static hash table may be a waste of space (data entry slots in the primary buckets remain unallocated).
- *Extendible* and *Linear Hashing*: Dynamic techniques to fix this problem.

Extendible Hashing

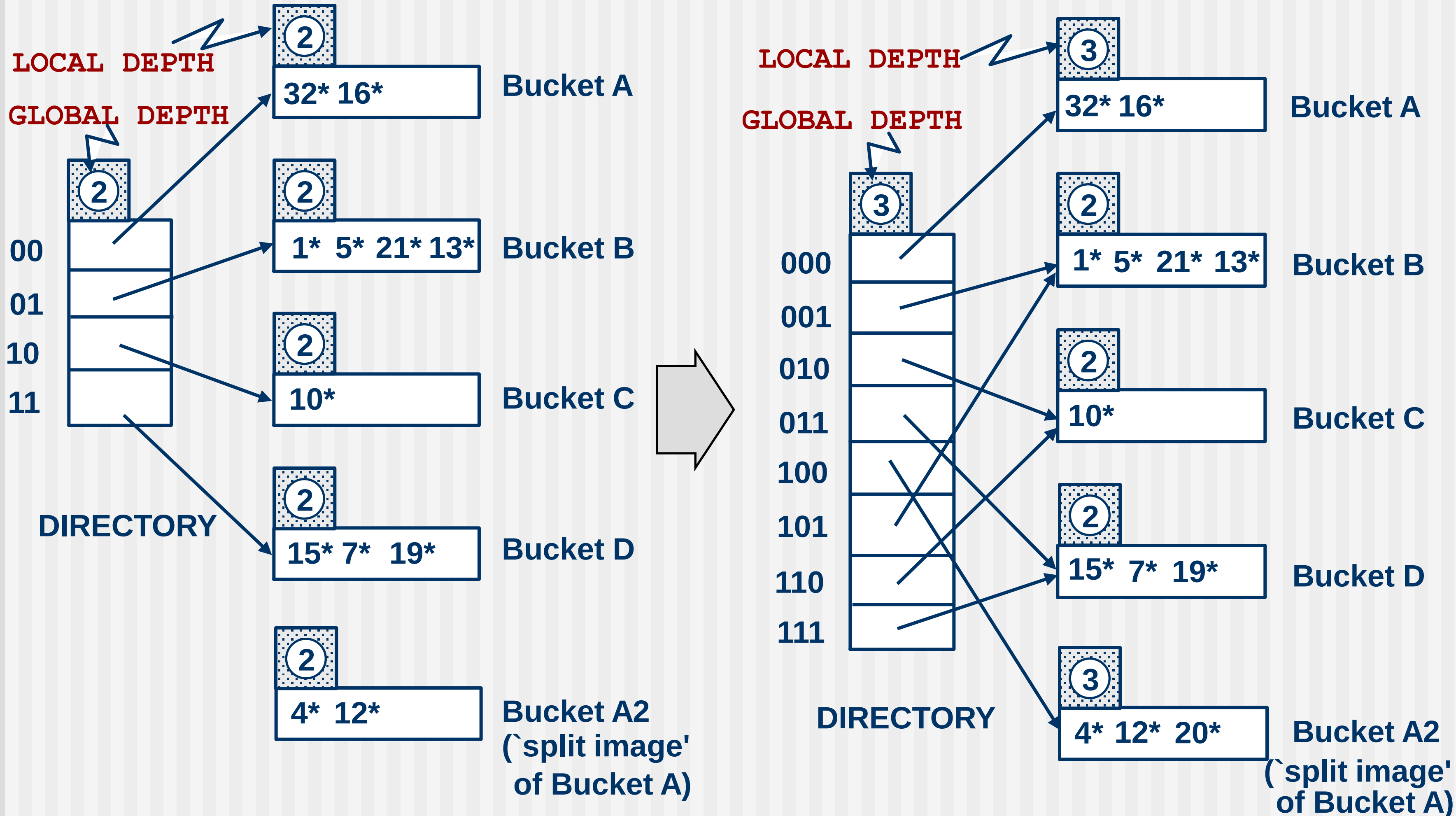
- Situation: Bucket (primary page) becomes full. Why not re-organize file by *doubling* number of buckets?
  - Reading and writing all pages is expensive!
  - *Idea*: Use *directory of pointers to buckets,* double number of buckets by *doubling the directory,* splitting just the bucket that overflowed!
  - Directory much smaller than file, so doubling it is much cheaper. Only one page of data entries is split. *No overflow page*!
  - Trick lies in how hash function is adjusted!

Example

- Directory is array of size 4.
- To find bucket for *r*, take last `*global depth*' number of bits of **h**(*r*); we denote *r* by **h**(*r*).
  - If **h**(*r*) = 5 = binary 101, it is in bucket pointed to by 01.
- **Insert**: If bucket is full, *split* it (*alloc. new page, re-distribute*).
- *If necessary*, double the directory. (As we will see, splitting a bucket does not always require doubling; we can tell by comparing *global depth* with *local depth* for the split bucket.)



Insert k: h(k) = 20 → Doubling

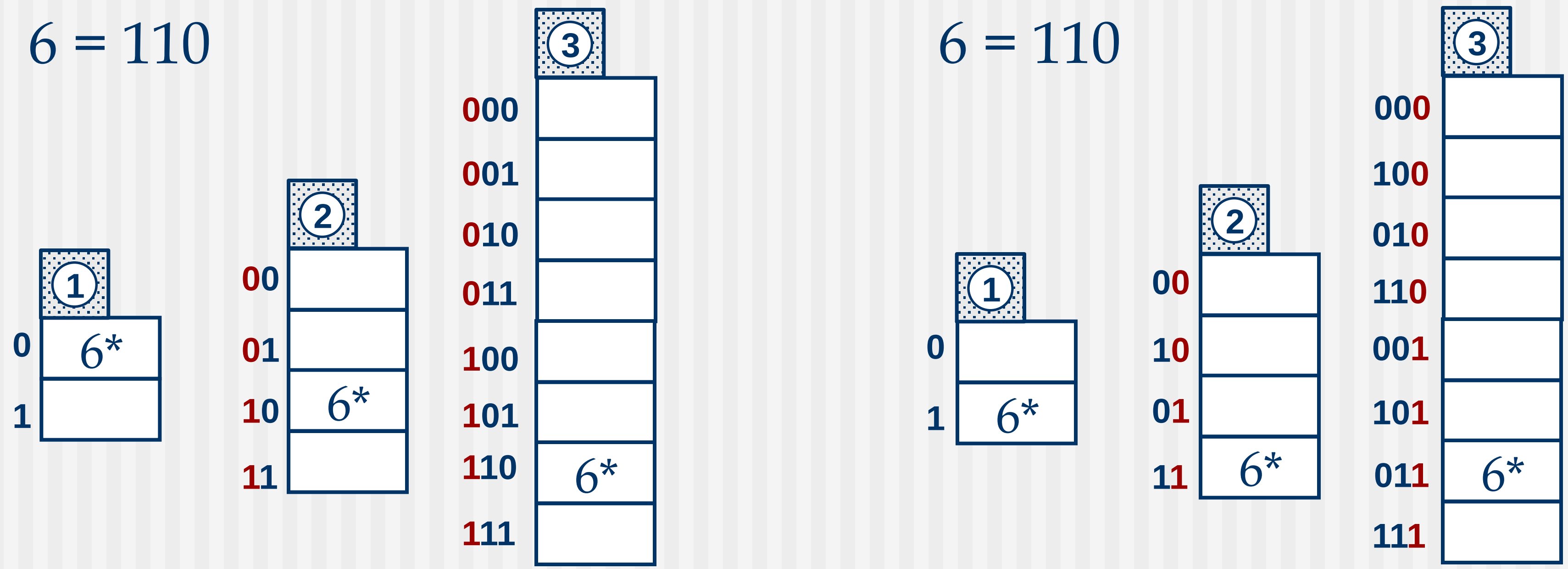


Extendible Hashing

- Inserting h(k) =20 (binary 10100). Last **2** bits (00) tell us *r* belongs in A or A2. Last **3** bits needed to tell which.
  - *Global depth of directory*: Max number of bits needed to tell which bucket an entry belongs to.
  - *Local depth of a bucket*: number of bits used to determine if an entry belongs to this bucket.
- When does bucket split cause directory doubling?
  - Before insert, *local depth* of bucket = *global depth*. Insert causes *local depth* to become > *global depth*; directory is doubled by *copying it over* and `fixing' pointer to split image page. (Use of least significant bits enables efficient doubling via copying of directory!)

Directory Doubling

- Using least significant bits in directory → doubling via copying!



Least Significant vs. Most Significant

Comments on Extendible Hashing

- If directory fits in memory, equality search answered with one disk access; else two.
  - 100MB file, 100 bytes/rec, 4K pages contains 1,000,000 records (as data entries) and 25,000 directory elements; chances are high that directory will fit in memory.
  - Directory grows in spurts, and, if the distribution *of hash values* is skewed, directory can grow large.
  - Multiple entries with same hash value cause problems!
- **Delete**: If removal of data entry makes bucket empty, can be merged with `split image'. If each directory element points to same bucket as its split image, can halve directory.

Evaluation of Hash-Based Files

- Advantages:
  - Very fast access from information you are likely to already have.
  - Best for *equality selections*, e.g.

```
SELECT * FROM R WHERE A = k
```

while the underlying data file - for relation R - grows or shrinks, we can answer such an equality query using a single I/O operation.

Evaluation of Hash-Based Files (cont)

- Disadvantages:
  - There can only be one hash arrangement based on one hash key. The other forms of access must be done by indexing.
  - The sequential order of the records is unlikely to be meaningful in any way, although there is work being done on order-preserving hash algorithms that keep records in the logical order of their hash key.
  - There are likely to be gaps of empty spaces of various sizes in the file – it won't be consistently loaded.
  - Cannot support range searches.
  - Does not support retrieval based on other fields than *exact* hash field
  - Not recommended when hash field is frequently updated

SQL Indexing

- Even the most recent version of the SQL standard, SQL3 or SQL-99, does not have standard commands for specifying indexes in your database during the design phase.
- However, most commercial databases implement their own variations of a create index command.

- The general syntax is:

```
CREATE [UNIQUE] INDEX indexName
ON tableName (colName [ASC|DESC] [,...])
DROP INDEX indexName;
```

- **UNIQUE** means that each value in the index is unique.
- **ASC/DESC** specifies the sorted order of index.

SQL Indexing - Examples

```
CREATE UNIQUE INDEX idxStudent ON Student(ID)
```

- Creates an index on the field **ID** in the table ***Student***
- **idxStudent** is the name of the index.

- The UNIQUE keyword ensures the uniqueness of **ID** values in the table (and index). This uniqueness is enforced even when adding an index to a table with existing data. If the **ID** field is non-unique then the index creation fails.

SQL Indexing - Examples

```
CREATE INDEX clMajor ON
     Student(Major) CLUSTER
```

- Creates a clustered index on the **Major** field of ***Student*** table.
- Remember a clustered index is another term for the search key that physically orders the file (i.e. primary index).
- Clustered index may or may not be on a key field.

```
CREATE INDEX idxMajorYear
     ON student(Major,Year)
```

- Creates an index with two fields.
- Duplicate search keys are possible.