Babeş-Bolyai University, Faculty of Mathematics and Computer Science
Analysis for Computer Science
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Exercise Set #4

1. Find the sum of the following series:

a) $\sum\limits_{n\geq 2}\left(-\frac{5}{9}\right)^n$, b) $\sum\limits_{n\geq 1}\left(\frac{1}{2}\right)^{2n}$, c) $\sum\limits_{n\geq 2}\ln\left(1-\frac{1}{n^2}\right)$, d) $\sum\limits_{n\geq 0}\frac{1}{(n+p)(n+1+p)}$, where $p>0$,

e) $\sum\limits_{n\geq 1}\frac{1}{n(n+1)(n+2)}$, f) $\sum\limits_{n\geq 1}\frac{1}{(3n-2)(3n+1)}$, g) $\sum\limits_{n\geq 1}\left(\sqrt{n+2}-2\sqrt{n+1}+\sqrt{n}\right)$, h) $\sum\limits_{n\geq 1}\frac{n+1}{2^n}$.

2. Let $\sum_{n\geq 1}x_n$ be a convergent series with nonnegative terms. Study which of the following series are convergent:

a) $\sum\limits_{n\geq 1}\frac{x_n}{1+x_n}$, b) $\sum\limits_{n\geq 1}x_n^2$, c) $\sum\limits_{n\geq 1}\sqrt{x_n}$, d) $\sum\limits_{n\geq 1}\frac{\sqrt{x_n}}{n}$.

3. Study if the following series are convergent or divergent:

a) $\sum\limits_{n\geq 1}\sin n$, b) $\sum\limits_{n\geq 1}\frac{5^{n/2}}{n2^n}$, c) $\sum\limits_{n\geq 1}\frac{e^n}{n+3^n}$, d) $\sum\limits_{n\geq 1}\frac{1}{\sqrt{n+1}}$, e) $\sum\limits_{n\geq 1}\frac{1}{n^2-\ln n+\sin n}$, f) $\sum\limits_{n\geq 1}\frac{2^n n!}{n^n}$,

g) $\sum\limits_{n\geq 1}\frac{n^2}{2^{n^2}}$, h) $\sum\limits_{n\geq 1}(\arctan n)^n$, i) $\sum\limits_{n\geq 1}\frac{n^2}{\left(2+\frac{1}{n}\right)^n}$, j) $\sum\limits_{n\geq 1}\left(1+\frac{1}{n}\right)^{-n^2}$, k) $\sum\limits_{n\geq 1}\frac{1\cdot 3\cdot\ldots\cdot(2n-1)}{2\cdot 4\cdot\ldots\cdot(2n)}$,

l) $\sum\limits_{n\geq 1}(2-\sqrt{e})\cdot(2-\sqrt[3]{e})\cdot\ldots\cdot(2-\sqrt[n]{e})$ Hint: $\forall n\in\mathbb{N}^*, e<\left(1+\frac{1}{n}\right)^{n+1}$,

m) $\sum\limits_{n\geq 1}\frac{n^n}{e^n n!}$ Hint: $\lim\limits_{n\to\infty}n\left(e-\left(1+\frac{1}{n}\right)^n\right)=\frac{e}{2}$, n) $\sum\limits_{n\geq 1}\frac{a(a+1)\cdot\ldots\cdot(a+n)}{n(n+1)\cdot\ldots\cdot(2n)}$, where $a\in\mathbb{R}$.

4. Study if the following series are convergent, absolutely convergent or divergent:

a) $\sum\limits_{n\geq 1}\frac{\sin n}{n^2}$, b) $\sum\limits_{n\geq 1}(-1)^{n+1}\frac{(n+1)^n}{n^{n+2}}$, c) $\sum\limits_{n\geq 1}\frac{(-1)^{n+1}}{\sqrt{n}}$, d) $\sum\limits_{n\geq 1}\frac{(-1)^{n+1}}{\sqrt{n(n+1)}}$,

e) $\sum\limits_{n\geq 1}\frac{a^n}{1+a^{2n}}$, where $a\in\mathbb{R}$.

5. Let (x_n) be a decreasing sequence in $[0,+\infty)$ such that $\lim\limits_{n\to\infty}x_n=0$. Show that the series $\sum\limits_{n\geq 1}(-1)^{n+1}\frac{x_1+x_2+\ldots+x_n}{n}$ is convergent.