

# Lab 06

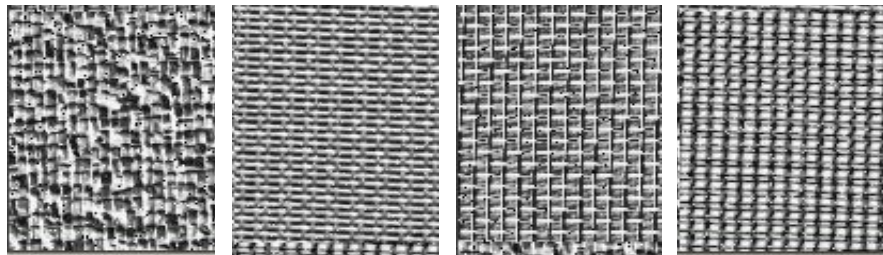
Basic Image Processing Algorithms  
Fall 2022

# Texture segmentation with Laws filters

Aim is to recognize trained textures on an image. **The algorithm has two phases:**

## Phase 1: Training

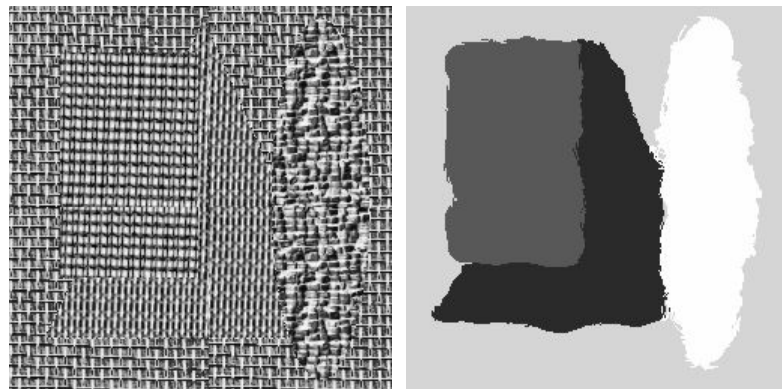
Calculate a set of scalars representing each texture, using different training images.



Training images

## Phase 2: Recognition

Find the known textures in the picture and classify its pixels into the appropriate bins.



Input test image

Output of the  
classifier

# Laws filter – mathematical background

Laws filter uses different types of **kernels** for convolution and then a **summation** on the convolved images is carried out resulting in a scalar **number** which represents a texture-kernel pair.

The convolution kernels are created from vector-vector products of the following 1D filters:

$$L = \frac{1}{6} \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$$

Level detection filter

$$E = \frac{1}{2} \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$$

Edge detection filter

$$S = \frac{1}{2} \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}$$

Spot detection filter

# Laws filter – kernels

$$L = \frac{1}{6} \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} \quad E = \frac{1}{2} \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} \quad S = \frac{1}{2} \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}$$

$$H_1 = LL^T = \frac{1}{36} \begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 1 \end{bmatrix} \quad H_2 = LE^T = \frac{1}{12} \begin{bmatrix} 1 & 0 & -1 \\ 2 & 0 & -2 \\ 1 & 0 & -1 \end{bmatrix} \quad H_3 = LS^T = \frac{1}{12} \begin{bmatrix} 1 & -2 & 1 \\ 2 & -4 & 2 \\ 1 & -2 & 1 \end{bmatrix}$$

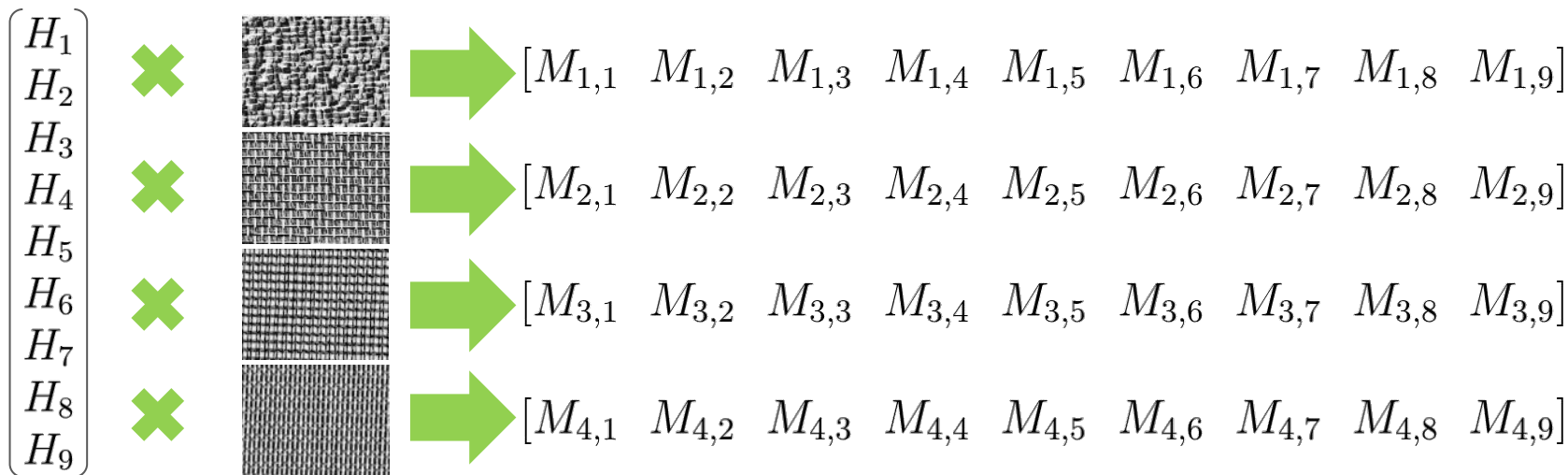
$$H_4 = EL^T = \frac{1}{12} \begin{bmatrix} 1 & 2 & 1 \\ 0 & 0 & 0 \\ -1 & -2 & -1 \end{bmatrix} \quad H_5 = EE^T = \frac{1}{4} \begin{bmatrix} 1 & 0 & -1 \\ 0 & 0 & 0 \\ -1 & 0 & 1 \end{bmatrix} \quad H_6 = ES^T = \frac{1}{4} \begin{bmatrix} 1 & -2 & 1 \\ 0 & 0 & 0 \\ -1 & 2 & -1 \end{bmatrix}$$

$$H_7 = SL^T = \frac{1}{12} \begin{bmatrix} 1 & 2 & 1 \\ -2 & -4 & -2 \\ 1 & 2 & 1 \end{bmatrix} \quad H_8 = SE^T = \frac{1}{4} \begin{bmatrix} 1 & 0 & -1 \\ -2 & 0 & 2 \\ 1 & 0 & -1 \end{bmatrix} \quad H_9 = SS^T = \frac{1}{4} \begin{bmatrix} 1 & -2 & 1 \\ -2 & 4 & -2 \\ 1 & -2 & 1 \end{bmatrix}$$

# Phase 1: Training

1/5

In the training phase, every texture sample gets convolved (and then summed) with the 9 different Laws filter kernels, resulting in 9 scalar numbers for a texture sample. The model of the texture is the 9-element long vector built from the  $M_{n,k}$  numbers, where  $n$  is the ID of the texture sample, and  $k$  is the kernel index ( $H_k$ ).



# Phase 1: Training

2/5

The training procedure is the following:

for each texture ( $n = 1, 2, \dots, N$ ):

for each filter ( $k = 1, 2, \dots, 9$ ):

Convolve the training texture  $T_n$  with the appropriate kernel:  $A = T_n \otimes H_k$

Compute the  $k$ -th element of the vector describing the  $n$ -th texture:

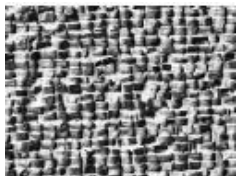
$$M_{n,k} = \frac{1}{hw} \sum_{x=1}^w \sum_{y=1}^h A^2(x, y)$$

Where  $h$  and  $w$  are the height and width of the training texture image  $T_n$

# Phase 1: Training

3/5

The  $i$ -th training texture image



$$T_i \in [0, 1]^{(h \times w)}$$

The different  
Laws filters

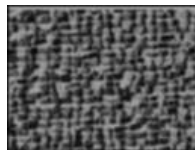
$$H_1 = \frac{1}{36} \begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 1 \end{bmatrix}$$

$$H_2 = \frac{1}{12} \begin{bmatrix} 1 & 0 & -1 \\ 2 & 0 & -2 \\ 1 & 0 & -1 \end{bmatrix}$$

$\vdots$

$$H_9 = \frac{1}{4} \begin{bmatrix} 1 & -2 & 1 \\ -2 & 4 & -2 \\ 1 & -2 & 1 \end{bmatrix}$$

Filtered training texture image



$$A_1 = T_i \otimes H_1$$



$$A_2 = T_i \otimes H_2$$

$\vdots$



$$A_9 = T_i \otimes H_9$$

Squared averaging

$$M_{i,1} = \frac{1}{hw} \sum_{x=1}^w \sum_{y=1}^h A_1^2(x, y)$$

$$M_{i,2} = \frac{1}{hw} \sum_{x=1}^w \sum_{y=1}^h A_2^2(x, y)$$

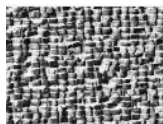
$\vdots$

$$M_{i,9} = \frac{1}{hw} \sum_{x=1}^w \sum_{y=1}^h A_9^2(x, y)$$

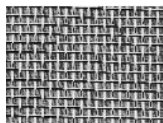
# Phase 1: Training

4/5

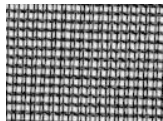
Finally, the result should be a vector of 9 scalar values for every texture.



$$[M_{1,1} \ M_{1,2} \ M_{1,3} \ M_{1,4} \ M_{1,5} \ M_{1,6} \ M_{1,7} \ M_{1,8} \ M_{1,9}]$$



$$[M_{2,1} \ M_{2,2} \ M_{2,3} \ M_{2,4} \ M_{2,5} \ M_{2,6} \ M_{2,7} \ M_{2,8} \ M_{2,9}]$$



$$[M_{3,1} \ M_{3,2} \ M_{3,3} \ M_{3,4} \ M_{3,5} \ M_{3,6} \ M_{3,7} \ M_{3,8} \ M_{3,9}]$$



$$[M_{4,1} \ M_{4,2} \ M_{4,3} \ M_{4,4} \ M_{4,5} \ M_{4,6} \ M_{4,7} \ M_{4,8} \ M_{4,9}]$$



# Phase 1: Training

5/5

In the program code we will store the “descriptors” of the textures in a so-called “model” matrix. We call the rows of this matrix “classes” and by “class index” we mean the row index (starting from 1).

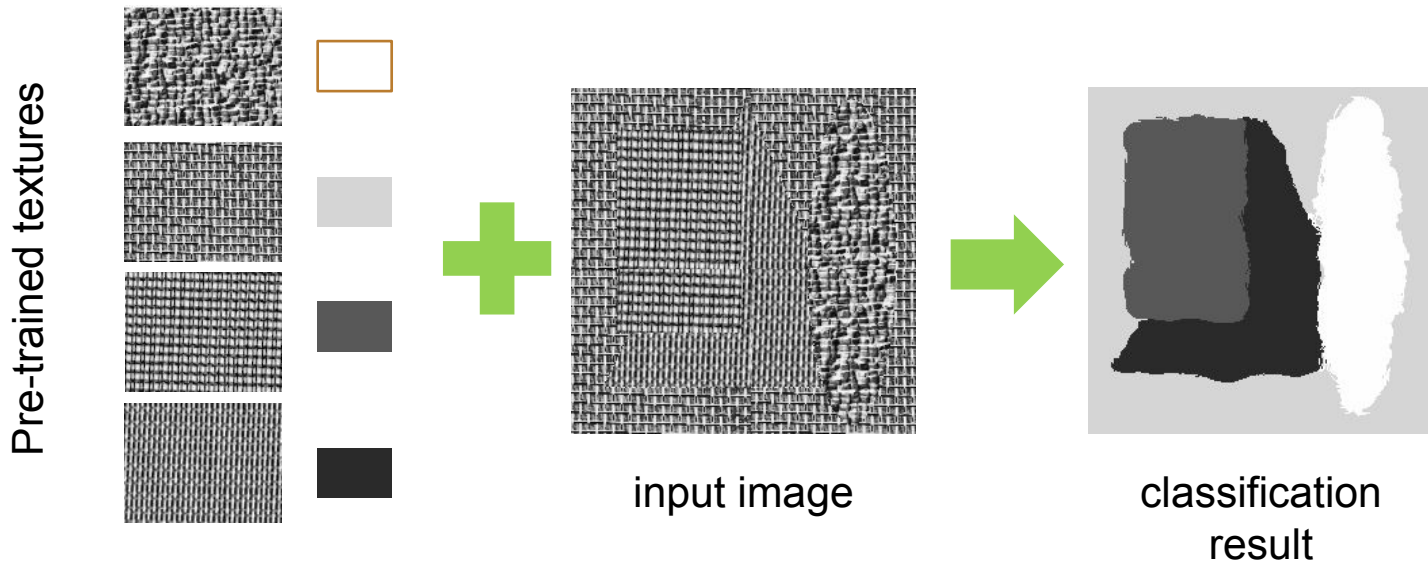
```
MODEL = [M11 M12 M13 M14 M15 M16 M17 M18 M19  
         M21 M22 M23 M24 M25 M26 M27 M28 M29  
         M31 M32 M33 M34 M35 M36 M37 M38 M39  
         M41 M42 M43 M44 M45 M46 M47 M48 M49];
```

With this matrix  $M_{i,j} = \text{MODEL}(i,j)$ , the parameters of *Class1* are in  $\text{MODEL}(1,:)$ .

# Phase 2: Recognition

1/6

Recognition takes the texture model (the  $M_{ij}$  scalars from the training phase) and an input image which consists of arbitrary regions of the pre-trained textures.



## Phase 2: Recognition

2/6

The recognition procedure is the following:

for each filter ( $k = 1, 2, \dots, 9$ ):

Convolve the input image with the appropriate kernel:  $B = I \otimes H_k$

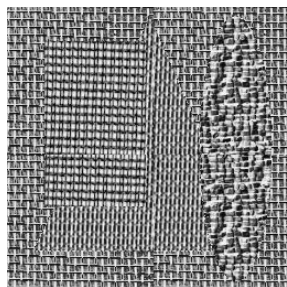
Square the result and apply a 15×15 averaging:  $B^* = B^2 \otimes N$

Where  $N = 1/(15*15) * \text{ones}(15)$

# Phase 2: Recognition

3/6

The input image  
to be recognized



The different  
Laws filters

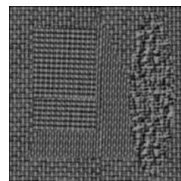
$$H_1 = \frac{1}{36} \begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 1 \end{bmatrix}$$

$$H_2 = \frac{1}{12} \begin{bmatrix} 1 & 0 & -1 \\ 2 & 0 & -2 \\ 1 & 0 & -1 \end{bmatrix}$$

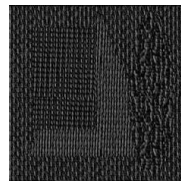
⋮

$$H_9 = \frac{1}{4} \begin{bmatrix} 1 & -2 & 1 \\ -2 & 4 & -2 \\ 1 & -2 & 1 \end{bmatrix}$$

Filtered input image



$$B_1 = I \otimes H_1$$



$$B_2 = I \otimes H_2$$

⋮



$$B_9 = I \otimes H_9$$

Squared, averaged result

$$B_1^* = B_1^2 \otimes N$$

$$B_2^* = B_2^2 \otimes N$$

⋮

$$B_9^* = B_9^2 \otimes N$$

## Phase 2: Recognition

4/6

Every  $B_i^*$  output of the previous step is a matrix (which has the same size as the image). If the image is  $h \times w$  and we concatenate our  $B_i^*$  outputs along the 3rd dimension then we get an  $(h \times w \times 9)$  size matrix (Let's call it **BB**.)

The task is to classify every pixel. (Classify := tell whether a pixel at location  $(x, y)$  belongs to *Class1* or *Class2* or ... or *ClassN* based on the texture in its  $15 \times 15$  neighborhood.)

For classification; at every pixel location we use the 9-element vector of the **BB** matrix at  $(x, y)$  and the pre-trained classes stored in **MODEL**.

## Phase 2: Recognition

5/6

After creating the  $(h \times w \times 9)$  size **BB** matrix we create an empty **ClassMap** and continue the recognition procedure with a pixel-by-pixel comparison; implementing the following formula:

$$\text{ClassMap}(x, y) = \arg \min_n \sum_k |B_k^*(x, y) - M_{n,k}|$$

$$n = 1, 2, \dots, N \quad k = 1, 2, \dots, 9$$

$$x = 1, 2, \dots, w \quad y = 1, 2, \dots, h$$

# Phase 2: Recognition

6/6

That is in a more pseudocode way:

for each pixel  $(x, y)$ :

for each texture class  $(n = 1, 2, \dots, N)$ :

Compute the summed absolute difference between the 9-element vectors  $BB(x, y, :)$  and  $MODEL(n, :)$  and store the result in a temporary variable:  
 $sum\_abs\_diff(n)$

After computing the sum of absolute differences for every class, find the  $n$  where the value of  $sum\_abs\_diff$  is the smallest (i.e. pick the class where the difference between the image texture and the training texture is the smallest).

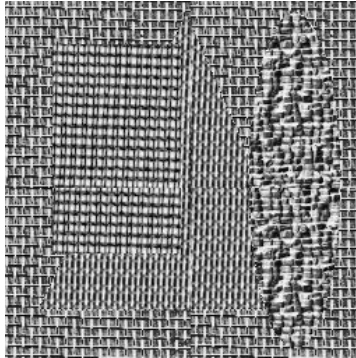
Put this  $n$  into  $ClassMap(x, y)$ .

Continue with the next pixel...

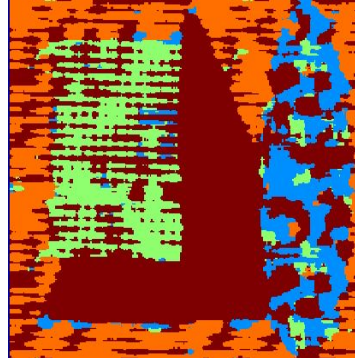
# Results

Output „winner class” maps for the four textures – enhanced with some morphology

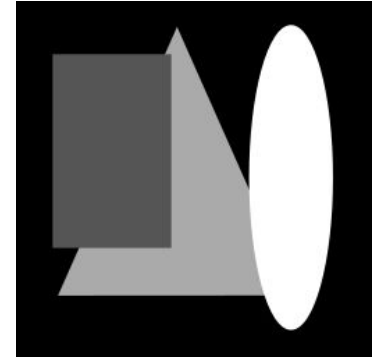
Input image



Laws segmentation results



Ground truth





# One more thing: Majority voting

1/4

Majority voting is a very common technique to smooth noisy images.

If we know that the image consist of large “constant” areas (i.e. the background, rectangle, triangle and ellipse in this case) then we can know “for sure” that a *Class1* label on its own surrounded by *Class2* labels is more likely to be an error than an actual good classification.



# One more thing: Majority voting

2/4

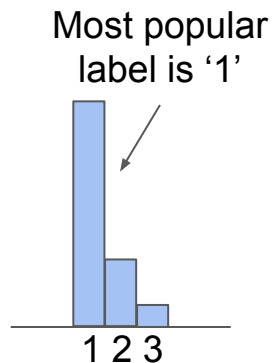
The idea of majority voting is that we define a window and count the “votes” (occurrences of the different class labels) inside the region. Then we get the “most popular vote” (the class label with the most frequent occurrence). Change all class labels in this window to the most popular one; this makes the “unpopular” (unlikely) class labels disappear.

1	2	1	1	2
1	3	1	2	1
1	1	1	1	1
1	1	2	1	1
3	3	1	1	1

Content of  
ClassMap

1	2	1	1	2
1	3	1	2	1
1	1	1	1	1
1	1	2	1	1
3	3	1	1	1

Selected  
window



Histogram of  
class labels

1	1	1	1	2
1	1	1	1	1
1	1	1	1	1
1	1	1	1	1
3	3	1	1	1

Updated map  
after voting

# One more thing: Majority voting

3/4

To implement majority voting you will need the diameter of the window (`w_dia`). You have to run this window through your input matrix (`IN`) without overlapping. Inside the window use `mode()` to get the most popular element and replace every element inside the window with the majority vote.

Use `x = 1:w_dia:width` and `x:x+w_dia-1` vectors to iterate through your image with step size `w_dia`. (same with `y`)

In this example the window diameter is 3.

Note that there are no overlapping windows.

Also note that if the number of remaining elements is less than a full window then we work with a smaller area.

1	2	1	1	2	1	2	1	1	2	1	2	1	1
1	3	1	2	1	1	3	1	2	1	1	3	1	2
1	1	1	1	1	1	1	1	1	1	1	1	1	1
1	1	2	1	1	1	1	1	2	1	1	1	1	1
3	3	1	1	1	3	3	3	1	1	3	3	3	3

# One more thing: Majority voting

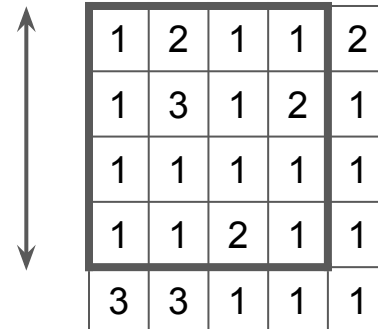
4/4

Pseudocode for majority voting:

```
for x=1:w_dia:size(IN,1)
  for y=1:w_dia:size(IN,2)
    startx = x; endx = min{x + w_dia - 1; size(IN,1)};
    starty = y; endy = similar
    SelectedWindow = IN(startx:endx, starty:endy)
    MajorityValue = mode{SelectedWindow}
    OUT(startx:endx, starty:endy) = MajorityValue
```

In this example

w\_dia = 4



1	2	1	1	2
1	3	1	2	1
1	1	1	1	1
1	1	2	1	1
3	3	1	1	1

Now please  
**download the 'Lab 06' code package**  
from the  
**[moodle system](#)**

# Exercise 1

Implement the **function** `laws_kernel` in which:

- Check if the input argument (`k`) is valid. It should be a number  $1 \leq k \leq 9$ . If this condition not holds, throw an error (use the `error()` function).
- If `k` is valid return the `k`-th Laws kernel. You can do this by computing the appropriate vector product or by hard-coding every kernel in the function; it's up to you. See [Slide 4](#) for the list of the Laws kernels.

You should return a 2D double matrix. Don't forget the normalization factors!

Run `test1.m` which will test your implementation.

## Exercise 2

Implement the **function** `training_phase` in which:

- The input is a cell of training textures: `T_cell = {T1, T2, ..., TN}`
- Get the number of training textures (N); create the empty (N×9) `MODEL` matrix.
- For each texture realize the training steps described in [Slide 6](#). Fill all the values of `MODEL`. Use `conv2` with the option `'same'` for convolution to get rid of the padding. Use your previously implemented function to get the kernels.

You can assume that all training textures are grayscale images, double, [0,1] range. **The training images might have different sizes!** (I.e. it can happen that T1 is 10×50, T2 is 33×33 etc...). **Pay attention to the order of the training textures!** (I.e. the M values describing T<sub>2</sub> must be in the second row of `MODEL`).

Run `test2.m` which will test your implementation.

## Exercise 3

Implement the **function** `recognition_phase` in which:

- The two inputs are the input image and the pre-trained `MODEL` matrix.
- Knowing the size of the input image create the same-sized empty `ClassMap`.
- Realize the recognition steps described on Slides [11](#) and [15](#). Fill all the values of `ClassMap`. Use `conv2` with the option `'same'` for convolution to get rid of the padding. Use your previously implemented function to get the kernels.

You can assume that the input is a grayscale image, double, from  $[0,1]$  range. **Pay attention to the order of the training textures!** (I.e. if the closest match is with the second row of `MODEL` then you should write 2 in the corresponding location of `ClassMap`.)

Run `test3.m` which will test your implementation.



# Exercise 4

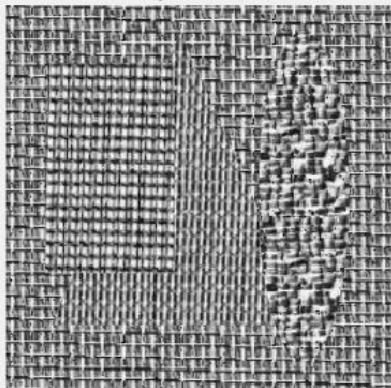
Implement the **function** `majority_voting` in which:

- The input is a matrix containing the class labels.
- Knowing the size of the input, create the same-sized empty `OUT`.
- Realize the majority voting described on [Slide 20](#). Fill all the values of `OUT`.  
Use `mode` and the smart, non-overlapping indexing.

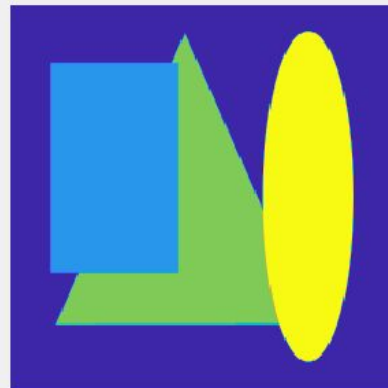
Run `test4.m` which will test your implementation.

If you are ready with all the function implementations, run `final_script.m` and check the results.

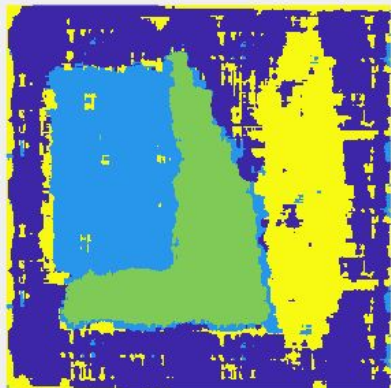
**Original input**



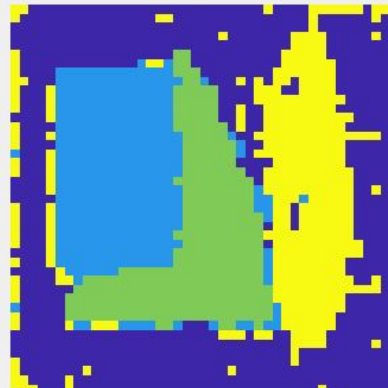
**Ground truth**



**Raw output**  
84.2148% accurate



**Output + majority voting**  
86.705% accurate



**THE END**