Simply typed Lambda-Calculus for developers

An introduction to the topic

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Abstract

Short abstract about why a developer should understand the lambda calculus. What is is how it works and why it is relevant in software engineering and other fields.

Keywords: Lambda-Calculus, type systems

1 Introduction

All modern programming languages have type systems. They are either of a more dynamic nature - like JavaScript - or statically typed like C-Sharp. Even if we talk about functional languages like Haskell, they have a type system that helps the developer to create programs.

But why are type systems helpful and how do they work is not a trivial question to be answered. Consider the following code statement:

```
1       foo = "Hello World"
2       bar = 42
3       foo + bar //?
```

Listing 1. Untyped language

As humans, we immediately understand that this statement is not going to terminate well - assuming we have an untyped language. Strings and numbers are not of the same type and cannot be added together. To determine that this is not going to work, the computer needs to execute the statements one by one and will encounter a wrong state.

A type system can prevent such errors and create a human readable message when compiling such a program. To understand a type system, it is necessary to use maths to explain how we can check and derive types. Such a mathematical system is the λ-Calculus.

The reader should have an understanding in programming languages and a brief understanding of the untyped λ-Calculus which is described in the first chapters of [Pie02].

The following chapters will define some knowledge about the untyped λ-Calculus and then introduce the reader to the simply typed λ-Calculus.

2 Lambda Calculus

A computer program can be described in multiple ways. A. M. Turing defined the turing machine in [Tur37] in the year 1937. This machine was an abstract model of computation. In contrast to a "machine", the mathematical model of a computer program was defined by Alonzo Church [Chu40]. It uses mathematical logic to run calculations. All elements can be described in mathematical terms.

The untyped λ-Calculus is turing complete, which means it can compute *any* program and therefore can run infinitely. This is a contrast to the simply typed calculus, which limits the executable terms in the way that they never reach an erroneous state. In an untyped λ system, it is possible to search for the successor of "true", which requires the argument to be a number and therefore results in a stuck state.

Since it is not desirable for computer programs to run to infinity, the mathematical system of types [Chu40] was defined.

3 Simply Typed Lambda-Calculus

The mathematical abstraction of the λ-Calculus is a topic that is not easy to understand if encountered for the first time. The purpose of the present paper is to describe the nature of the typed calculus with examples from programming languages so that a developer can understand the concepts.

3.1 Key Difference

The simply typed calculus relies upon static type analysis. This means, to determine if a program is *well-typed*, no machine needs to run the program. Also, the simply typed calculus is a system with the normalization property [Pie02] [BN98].

We extend the untyped λ grammar

$$\begin{aligned} \langle t \rangle \quad \models \quad & x \ \textit{variable} \\ & \lambda x\,.\,\langle t \rangle \ \textit{abstraction} \\ & \langle t \rangle \langle t \rangle \ \textit{application} \end{aligned}$$

And get the typed version that limits values and inputs to terms to certain types.

$$\begin{aligned} \langle t \rangle \quad \models \quad & x\,\texttt{:T} \ \textit{variable} \\ & \lambda x\,\texttt{:T}\,.\,\langle t \rangle \ \textit{abstraction} \\ & \langle t \rangle \langle t \rangle \ \textit{application} \end{aligned}$$

3.2 Types

Describe what types are and what types mean. In terms of mathematics and programming languages.

Why they are needed for certain tasks.

- example types
- describe in math language
- translate to "developer language"
- why do we need them?

3.3 Types in the Calculus

Describe the different types and their context. What are function types and how are they used. What are the properties of typing.

Ref to Curry/Church

typescript example of church notation with typings.

3.4 Extending types

- What hides behind "base types"
- the unit type (void)
- other typing constructs for the simple calculus

3.5 References

- What is a reference type

3.6 Error states

- What exactly is an exception
- How to throw / handle them

4 Usage and Relevancy

- describe the usefulness of the calculus
- describe where it is used (ref to dev ops / kubernetes maybe?)
- why it is relevant (for system F / system T among others)

5 Conclusion

Give the conclusion over the paper. What was shown, why it was shown.

What should the reader know about the calculus now and how the reader can translate that into typed languages

References

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