Assignment #2

Due 16 September at the start of class

Please read Chapter 2, pages 10–29 in the textbook.¹

Now is the time to make sure that the notation used in the textbook is really clear to you. *Use* that notation in your homework solutions; it will help you understand the textbook in the future. And please read the ideas that the authors emphasize! All of the ideas in this Chapter will be built into various sophisticated algorithms later. In summary, *don't* merely do the minimum to get these homework problems done!

Exercise 2.1

Exercise 2.2

Exercise 2.3

Exercise 2.6

Exercise 2.9

Problem P5. Back on page 8 the textbook states several facts about convexity as though they are obvious. Perhaps they are, to experts who are familiar with the definitions and who know that the proofs follow directly from the definitions. This problem asks you to write a couple of such proofs.

(a) Let $b \in \mathbb{R}^k$, $d \in \mathbb{R}^m$ be fixed vectors. Let $A \in \mathbb{R}^{k \times n}$, $C \in \mathbb{R}^{m \times n}$ be fixed matrices. Show that

$$S = \left\{ x \in \mathbb{R}^n \,\middle|\, Ax = b, \, Cx \le d \right\}$$

is convex.

(b) Show that if f in (1.1) is convex and if the feasible set S in (1.1), defined by the equality and inequality constraints, is also convex, then any local solution to (1.1) is a global solution.

¹J. Nocedal & S. Wright, Numerical Optimization, 2nd ed., Springer 2006