SOLUTIONS

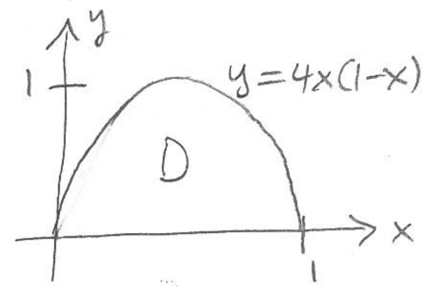
Math 253 Calculus III (Bueler) 6 April 2018

Worksheet: Setting up double integrals

Set up but do not (yet) evaluate the following integrals.

A. Choose the order of integration you prefer.

$$\iint_D x\,dA \quad \text{where} \quad D = \{(x,y) \,|\, 0 \le y \le 4x(1-x)\}$$



$$= \int_0^1 \int_{y=0}^{y=4x(1-x)} x\,dy\,dx$$

(type I)

B. Same integral but in other order.

curve $y=4x(1-x)$ can be solved for x to get

$$x = \frac{1 \pm \sqrt{1-y}}{2}$$

$\to$ $-4x^2+4x-y=0$

$$x = \frac{-4 \pm \sqrt{16-16y}}{-8}$$

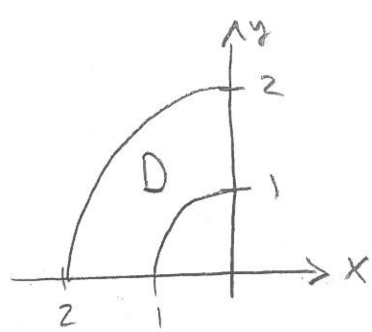
$$x = \frac{1 \pm \sqrt{1-y}}{2}$$

So (as type II):

$$\iint_D x\,dA = \int_0^1 \int_{x=\frac{1-\sqrt{1-y}}{2}}^{x=\frac{1+\sqrt{1-y}}{2}} x\,dx\,dy$$

C. Set up in polar. D is sketched at right.

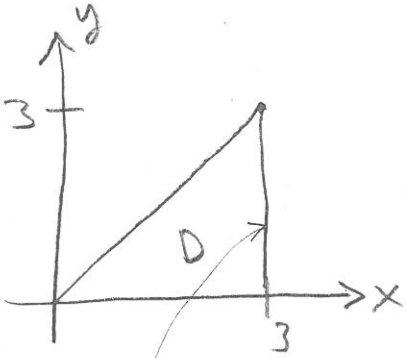
$$\iint_D x e^{-x^2-y^2}\,dA$$



$$= \int_{\theta=\frac{\pi}{2}}^{\theta=\pi} \int_{r=1}^{r=2} r\cos\theta\, e^{-r^2}\, r\,dr\,d\theta$$

D. Set up in polar. D is sketched at right.

$$\iint_D xy \, dA$$

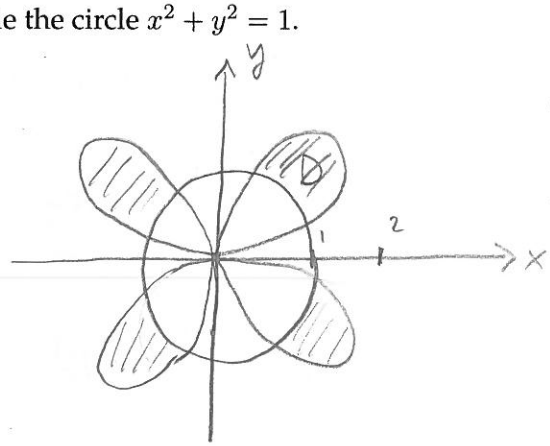


$$= \int_{\theta=0}^{\theta=\frac{\pi}{4}} \int_{r=0}^{r=3\sec\theta} r\cos\theta \, r\sin\theta \; r \, dr \, d\theta$$

line $x=3$
is $r\cos\theta = 3$
or $r = \frac{3}{\cos\theta} = 3\sec\theta$

E. Find the area inside the rose $r = 2\sin(2\theta)$ and outside the circle $x^2 + y^2 = 1$.

$$A(D) = 4 \int_{\theta=\frac{\pi}{12}}^{\theta=\frac{5\pi}{12}} \int_{r=1}^{r=2\sin(2\theta)} 1 \, r \, dr \, d\theta$$



note: $A(D) = \iint_D 1 \, dA$

limits on θ from solving:

$2\sin(2\theta) = 1$

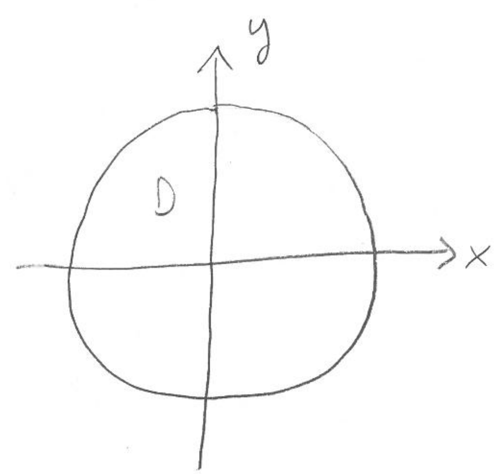
$\sin(2\theta) = \frac{1}{2}$

$2\theta = \frac{\pi}{6}, \frac{5\pi}{6}$

F. Find the volume of the solid bounded by the cylinders $x^2 + y^2 = r^2$ and $y^2 + z^2 = r^2$.

using R for cylinders radii

$D = \{(x,y) \mid x^2+y^2 \le R^2\}$



upper surface:

$y^2 + z^2 = R^2$

$z = +\sqrt{R^2 - y^2}$

so:

$$V = 2\iint_D z \, dA = 2 \int_0^{2\pi} \int_{r=0}^{r=1} \sqrt{R^2 - r^2\sin^2\theta} \; r \, dr \, d\theta$$