Math 661 Optimization (Bueler)

20 October 2018

Review Guide for in-class Midterm Exam on Friday, 26 October 2018

The in-class Midterm Exam will cover the sections listed below from Griva, Nash, Sofer, Linear and Nonlinear Optimization, 2nd ed., 2009. It will only cover topics that have appeared on homework and in lecture. You are not responsible for any material in Chapters 7, 8, 9, 10, 13, 14, 15, or 16.

The Exam is *closed book* and *no calculator*. You may bring your own notes on *half* of a single sheet of letter-sized paper.

The problems will be of these types: state definitions, state or prove certain theorems (see below), prove or show propositions which follow directly from the definitions or from known facts, give examples with certain properties, describe or sketch concepts, or compute a step or two of certain algorithms.

I encourage you to get together with other students and work through this Review Guide. Be honest with yourself about what you do and don't know, and talk it through and learn!

The list of Definitions below has blanks for page numbers. Use it as a "worksheet;" you can study by finding the page where the term is defined. On Wednesday 10/24 I will post "solutions" for these page numbers at the website.

Sections. You are only responsible for material in these Sections:

- 1.1–1.5
- 2.1-2.7
- 3.1, 3.2
- 4.1-4.4
- 5.1, 5.2 (but not 5.2.3, 5.2.4)
- 6.1 (only page 177), 6.2 (but not 6.2.2)
- 11.1-11.3
- 12.1, 12.2 (but not the Lemmas in 12.2)
- Appendices A.1–A.5, A.7.1
- Appendices B.4, B.5.

Definitions. Be able to define the term (word or phrase). Understand and/or use the term correctly. Be able to prove things that follow immediately from the definition:

• transpose A^{\top} of an $m \times n$ matrix A	p. <u>661</u>
• symmetric matrix	p. <u>661</u>
• nonsingular matrix	p. <u>665</u>
• positive definite matrix	p. <u>677</u>
• gradient $\nabla f(x)$ of a function $f: \mathbb{R}^n \to \mathbb{R}$	p. <u>694</u>
• gradient $\nabla f(x)$ of a function $f: \mathbb{R}^n \to \mathbb{R}^m$	p. <u>695</u>
• Jacobian $\nabla f(x)^{\top}$ of a function $f: \mathbb{R}^n \to \mathbb{R}^m$	p. <u>695</u>
• Hessian $\nabla^2 f(x)$ of a function $f: \mathbb{R}^n \to \mathbb{R}$	p. <u>694</u>

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•	feasible set $S \subseteq \mathbb{R}^n$, e.g. as defined by constraints on page 43	p
•	feasible point	p. 44
•	active constraint at $x \in S$	p. 44
•	inactive inequality constraint at $x \in S$	p. <u>44</u>
•	interior point of S	p. 44
•	• global minimizer of f in S	p. <u>45</u>
•	strict global minimizer	p. <u>45</u>
•	• $local\ minimizer\ of\ f\ in\ S$	p. <u>46</u>
•	strict local minimizer	p. 46
(\bullet convex set S	p. <u>48</u>
(\bullet convex function f on a convex set S	p. <u>48</u>
(strictly convex function	p. <u>49</u>
(convex optimization problem	p. <u>49</u>
	convex combination of a finite set of points in \mathbb{R}^n	p. <u>50</u>
	search direction in an optimization algorithm	p. <u>55</u>
	step length	p. <u>55</u>
,	• descent direction	p. <u>56</u>
,	• feasible direction	p. <u>78</u>
,	• feasible descent direction	p. <u>58</u>
,	• line search	p. <u>57</u>
	• linear (rate of) convergence for a convergent sequence	p. <u>59</u>
	• superlinear (rate of) convergence	p. <u>59</u>
	• quadratic (rate of) convergence	p. <u>60</u>
	• $null\ space\ of\ an\ m imes n\ matrix\ A$	p. <u>82</u>
	• range space of A (or A^{\top})	p. <u>82</u>
	• orthogonal subspaces	p. <u>83</u>
)	• null space matrix of A	p. <u>83</u>
	• standard form of a linear programming problem (l.p.p.)	p. <u>101</u>
	• free variable	p. <u>102</u>
	• slack variable	p. <u>102</u>
	• excess variable	p. <u>102</u>
	• extreme point (or vertex) of a convex set S	p. <u> 07</u>
	• basic solution of a standard form l.p.p.	p. <u>107</u>
	• basic feasible solution of a standard form l.p.p.	p. <u>107</u>
	• optimal basic feasible solution of a standard form l.p.p.	p. <u>107</u>
	• degenerate vertex of a standard form l.p.p.	p. <u>// 2</u>
	• unbounded direction of a standard form l.p.p.	p. <u>112</u>
	• dual problem for a standard form l.p.p.	p. <u>177</u>
	• primal problem	p. <u>173</u>
	• stationary point	p. <u>358</u>
	• first-order necessary conditions	p. 359
	• second-order necessary and sufficient conditions	p. 359