Assignment #1

Due Friday, 27 January at the start of class

Please read the Preface and Chapter 1 of the textbook R. LeVeque, *Finite Difference Methods for Ordinary and Partial Differential Equations*. Please get started reading Chapter 2 as well.

0. (*There is nothing to turn in on this problem. Problems 1–6 below relate to these two review topics.*) Find a standard textbook on *calculus* and an introductory textbook on *ordinary differential equations* (ODEs). You will need these references throughout the semester. Review these two topics:[1]

i) Taylor's theorem with remainder formula, and
ii) the solution of linear homogeneous constant-coefficient ODEs.

1. Calculate $(626)^{1/4}$ to within $0.00001 = 10^{-5}$ of the exact value *without* any computing machinery except a pencil or pen. Prove that your answer has this accuracy. (*Hint: You may, of course, use a computer to* check *your by-hand value.*)

2. Assume f' is continuous. Derive the remainder formula

$$\int_0^a f(x)\,dx = a f(0) + \frac{1}{2}a^2 f'(\nu) \tag{1}$$

for some (unknown) ν between zero and a. (*Hint: Start by showing* $f(x) = f(0) + f'(\xi)x$ *where* $\xi = \xi(x)$ *is some number between* 0 *and* x.) Use two sentences to explain the meaning of (1), as an answer to the question "What properties of an integral would make the left-endpoint rule $\int_0^a f(x)\,dx \approx a f(0)$ inaccurate?"

3. Download/install/purchase/find MATLAB or OCTAVE (or PYTHON etc.). Now work at the command line to compute a finite sum approximation to

$$\sum_{n=1}^{\infty} \frac{\cos n}{n^3}.$$

Compute at least three partial (finite) sums, with increasing numbers of terms N. Do you think you are getting close to the infinite sum, and if so, why? Turn your command line work into a function `sumstuff(N)`, defined in a file `sumstuff.m`, and show that it works. Turn in both the command line session and the code. (*Hint: These can be very brief.*)

[1] Taylor's theorem may be best explained by an additional undergraduate *numerical analysis* textbook.

4. Solve, by hand,

$$y'' + 2y' - 3y = 0, \quad y(2) = 0, \quad y'(2) = -1, \tag{2}$$

for the solution $y(t)$. Then find[2] $y(5)$. On t, y axes, give a reasonable by-hand sketch which shows the initial values, the solution, and the value $y(5)$.

5. Using Euler's method[3] for approximately solving ODEs, write your own MATLAB/OCTAVE/PYTHON etc. program (either script or function) to solve initial value problem (2) to find $y(5)$. A first step is to convert the second-order ODE into a system of two first-order ODEs. Use a few different step sizes, decreasing as needed, so that you get apparent four digit accuracy. (*Hint: You can use a built-in ODE solver to* check *your work, but this is not required.*)

6. Solve, by hand, the ODE boundary value problem

$$y'' + 2y' - 3y = 0, \quad y(0) = \alpha, \quad y(\tau) = \beta, \tag{3}$$

for the solution $y(t)$. Note that α, β, τ are the data of the problem. Are there values of τ for which this problem does not have a unique solution?

[2]This is a *prediction* of the outcome at $t = 5$, given initial data at $t = 2$ and a precise "law" about how $y(t)$ evolves in time, namely the differential equation itself.

[3]Look it up if needed!