Can be done with either $\lim\limits_{h\to 0}$ or $\lim\limits_{x\to 3}$

1. Find $f'(1)$ using the definition of the derivative:

$f(t) = 2t^2 + t$

$$f'(1) = \lim_{h\to 0} \frac{f(1+h)-f(1)}{h} = \lim_{h\to 0} \frac{2(1+h)^2+(1+h)-(2\cdot 1^2+1)}{h}$$

$$= \lim_{h\to 0} \frac{2(1+2h+h^2)+1+h-3}{h} = \lim_{h\to 0} \frac{\cancel{2}+4h+2h^2+h-\cancel{2}}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{h}(5+2h)}{\cancel{h}} = \lim_{h\to 0} 5+2h = 5$$

2. Find $f'(3)$ using the definition of the derivative:

$f(x) = x^{-2}$

$$f'(3) = \lim_{x\to 3} \frac{f(x)-f(3)}{x-3} = \lim_{x\to 3} \frac{\frac{1}{x^2}-\frac{1}{3^2}}{x-3} = \lim_{x\to 3} \frac{9-x^2}{3^2x^2(x-3)}$$

$$= \lim_{x\to 3} \frac{{}^{-1}\cancel{(3-x)}(3+x)}{9x^2\cancel{(x-3)}} = \lim_{x\to 3} \frac{-(3+x)}{9x^2} = \frac{-6}{81} = \frac{-2}{27}$$

3. Find $f'(a)$ using the definition of the derivative:

$f(x) = \sqrt{1+5x}$

$$f'(a) = \lim_{h\to 0} \frac{\sqrt{1+5(a+h)}-\sqrt{1+5a}}{h} \cdot \frac{\sqrt{1+5(a+h)}+\sqrt{1+5a}}{\sqrt{1+5(a+h)}+\sqrt{1+5a}}$$

$$= \lim_{h\to 0} \frac{\cancel{1}+5(\cancel{a}+h)-(\cancel{1}+\cancel{5a})}{h\left(\sqrt{1+5(a+h)}+\sqrt{1+5a}\right)}$$

$$= \lim_{h\to 0} \frac{5\cancel{h}}{\cancel{h}\left(\sqrt{1+5(a+h)}+\sqrt{1+5a}\right)} = \frac{5}{\sqrt{1+5a}+\sqrt{1+5a}} = \frac{5}{2\sqrt{1+5a}}$$

4. Find an equation of the tangent line to the curve at the given point:

$$f(x) = \frac{x+1}{x-1}, \qquad (2,3)$$

$$m = \lim_{h\to 0} \frac{f(2+h)-f(2)}{h} = \lim_{h\to 0} \frac{\frac{2+h+1}{2+h-1} - 3}{h}$$

$$= \lim_{h\to 0} \frac{\frac{\cancel{3}+h-3(\cancel{1}+h)}{1+h}}{h} = \lim_{h\to 0} \frac{-2\cancel{k}}{\cancel{k}(1+h)}$$

$$= \lim_{h\to 0} \frac{-2}{1+h} = -2$$

$\therefore$ tangent line is $y-3 = (-2)(x-2)$

5. A particle moves a distance $s = f(t)$ along a straight line, where s is measured in meters and t is in seconds:

$$f(t) = 40t - 5t^2$$

Find the velocity and speed when $t = 4$.

$$V = \lim_{h\to 0} \frac{f(4+h)-f(4)}{h} = \lim_{h\to 0} \frac{40(\cancel{4}+h) - 5(4+h)^2 - (\cancel{40\cdot 4} - 5\cdot 4^2)}{h}$$

$$= \lim_{h\to 0} \frac{40h - 5(\cancel{4^2} + 2\cdot 4\cdot h + h^2) + \cancel{5\cdot 4^2}}{h}$$

$$= \lim_{h\to 0} \frac{h(\cancel{40} - \cancel{5\cdot 2\cdot 4} - 5h)}{h} = \lim_{h\to 0} -5h = 0$$

$\therefore$ $V = 0\ m/s$, speed $= 0\ m/s$