SOLUTIONS

1. Find the derivative of the function. You do not need to simplify your answer.

(a) $y = \left(x + \dfrac{1}{x}\right)^7$

$$\frac{dy}{dx} = 7\left(x+\frac{1}{x}\right)^6\left(1-\frac{1}{x^2}\right)$$

(b) $f(\theta) = \cos(\theta^2)$

$$f'(\theta) = -\sin(\theta^2)\cdot 2\theta$$

(c) $g(t) = e^{(t^3)}$

$$g'(t) = e^{(t^3)}(3t^2)$$

(d) $y = \sqrt{x + \sqrt{x + \sqrt{x}}}$

$$\frac{dy}{dx} = \frac{1}{2}\left(x+(x+x^{1/2})^{1/2}\right)^{-1/2}\cdot\left(1+\frac{1}{2}(x+x^{1/2})^{-1/2}\cdot\left(1+\frac{1}{2}x^{-1/2}\right)\right)$$

2. Find an equation of the tangent line to the curve at the given point.

$$y = \sqrt{1+x^3}, \quad (2,3)$$

$$\frac{dy}{dx} = \frac{1}{2}(1+x^3)^{-1/2}(3x^2)$$

$$m = \left.\frac{dy}{dx}\right|_{x=2} = \frac{1}{2}(1+8)^{-1/2}(3\cdot 2^2) = \frac{6}{\sqrt{9}} = 2$$

$$y-3 = 2(x-2)$$

3. If $F(x) = f(g(x))$, and if $f(-2) = 8$, $f'(-2) = 4$, $f'(5) = 3$, $g(5) = -2$, and $g'(5) = 6$, find $F'(5)$.

$$F'(5) = f'(g(5))\,g'(5)$$

$$= f'(-2)\cdot 6$$

$$= 4\cdot 6 = 24$$

4. Find the 49th derivative of $f(x) = x\,e^{-x}$.

$$f'(x) = 1\cdot e^{-x} + x\cdot e^{-x}(-1) = (1-x)e^{-x}$$

$$f''(x) = (-1)e^{-x} + (1-x)e^{-x}(-1) = (-2+x)e^{-x}$$

$$f'''(x) = (1)e^{-x} + (-2+x)e^{-x}(-1) = (3-x)e^{-x}$$

$$f^{(4)}(x) = (-1)e^{-x} + (3-x)e^{-x}(-1) = (-4+x)e^{-x}$$

$\vdots$ pattern!

$$f^{(49)}(x) = (49-x)e^{-x}$$