1. Differentiate the functions.

$y = 3e^x + \dfrac{4}{\sqrt[3]{x}} = 3e^x + 4x^{-1/3}$

$$\frac{dy}{dx} = 3e^x - \frac{4}{3}x^{-4/3}$$

$G(q) = \left(1 + q^{-1}\right)^2 = (1+q^{-1})(1+q^{-1}) = 1 + 2q^{-1} + q^{-2}$

$$G'(q) = 0 - 2q^{-2} - 2q^{-3} = -2q^{-3}(q+1)$$

$y = \dfrac{\sqrt{x}}{2+x} = \dfrac{x^{1/2}}{2+x}$

$$\frac{dy}{dx} = \frac{\frac{1}{2}x^{-1/2}(2+x) - x^{1/2}(1)}{(2+x)^2} = \frac{\frac{1}{2}x^{-1/2}(2+x) - x^{1/2}}{(2+x)^2}$$

$g(x) = \left(\pi^{1/2} + 5\sqrt{x}\right)e^x$

$$g'(x) = \left(0 + \frac{5}{2}x^{-1/2}\right)e^x + \left(\pi^{1/2} + 5\sqrt{x}\right)e^x$$

$$= e^x\left(\frac{5}{2}x^{-1/2} + \pi^{1/2} + 5\sqrt{x}\right)$$

$f(x) = \dfrac{ax+b}{cx+d}$

$$f'(x) = \frac{a(cx+d) - (ax+b)(c)}{(cx+d)^2}$$

$$= \frac{ad - bc}{(cx+d)^2}$$

2. Find the derivative of $f(x) = (x + x^2)(x^{-1} + 3)$ in two ways:

(i) by the product rule:

$$f'(x) = (1+2x)(x^{-1}+3) + (x+x^2)(-x^{-2})$$
$$= x^{-1} + 3 + 2 + 6x - x^{-1} - 1 = 6x + 4$$

(ii) by first expanding the product:

$$f(x) = 1 + 3x + x + 3x^2 = 1 + 4x + 3x^2$$

$$f'(x) = 4 + 6x$$

3. Find an equation of a tangent line to the curve $y = x^4 + 1$ which is parallel to the line $32x - y = 15$.

slope of line: $y = 32x - 15$

$m = 32$

want: $32 = \frac{dy}{dx} = 4x^3$

$8 = x^3$

$x_0 = 2$

$y_0 = 2^4 + 1 = 17$

$$y - 17 = 32(x-2)$$

4. If $h(2) = 4$ and $h'(2) = -3$, find

$$\frac{d}{dx}\left(\frac{h(x)}{x}\right)\Big|_{x=2} = \frac{h'(x)\cdot x - h(x)\cdot 1}{x^2}\Bigg|_{x=2} = \frac{(-3)(2) - 4}{2^2}$$

quotient rule

$$= \frac{-10}{4} = \frac{-5}{2}$$