Assignment #7

Due *Monday 7 April, 2014.*

Read sections 2.13, 2.14, 2.15, 2.17, 3.1 of Morton & Mayers, 2nd ed.

1. Exercise 3.1 in Morton & Mayers, 2nd ed (page 83).

2. For the constant-coefficient diffusion-advection equation, namely

$$u_t = b_0 u_{xx} - a_0 u_x,$$

I claimed in lecture that the explicit method which is centered for the u_{xx} term and which uses upwinding for the u_x term has truncation error $T(x,t) = O(\Delta t^1) + O(\Delta x^1)$. Show this. (*Note that $b_0 > 0$ and $a_0 \in \mathbb{R}$. You are allowed to only compute the truncation error in the $a_0 \geq 0$ case, as long as you write a line or two which describes the changes in the $a_0 < 0$ case.*)

3. Reproduce two of the three curves in Figure 2.10. You get to choose which two methods to implement. Please identify the ones you implement by their equation numbers in the book: (2.103), (2.110), (2.114). (*Note that you need to read the text to find out what initial condition, boundary conditions, value for μ, and value for Δt were used in producing Figure 2.10. Yes, the book does give this info!*)

4. Reproduce Figures 3.2, 3.3, 3.4, and 3.5 from Morton & Mayers, 2nd ed; they are on pages 68–69. Even though they appear in section 3.2, they were produced by the explicit scheme in 3.1. Thus I am asking you to implement the explicit scheme, exactly the same one analysed in problem **1** above. Of course you need to figure out a stable timestep! Use only a $J_x = 100$ by $J_y = 100$ grid; don't worry about finer grids.

Your solution will be greatly assisted by using this code which I have already written. It computes the initial condition:

`http://bueler.github.io/M615S14/formM.m`

(*Extra Credit. You can get one extra credit point by redoing this with a letter other than "M". You can get three extra credit points by* advecting *the letter, whether "M" or not, as it also diffuses; you will need to choose the velocity in some sensible way.*)