

Assignment #1

Due Wednesday, 7 September 2016, at the start of class

Please read Chapters 1, 2, 3 in the textbook.¹ Do the following exercises.

Chapter 2, pages 7–8, Exercises:

- 2.1 (Both show a Venn diagram *and* write a proof that does not refer to the diagram.)
- 2.2 (Ditto.)
- 2.4

Chapter 3, pages 15, Exercises:

- 3.2
- 3.3
- 3.4
- 3.5

Problem P1. In this problem, “ (a, b) ” denotes an open interval of the real line, not an ordered pair. You may use the fact that an interval I of the real line is open if and only if for every $x \in I$ there is some $\epsilon > 0$ so that $(x - \epsilon, x + \epsilon) \subseteq I$.

(a) Give an example of an index set J , and a collection of open intervals $(a_j, b_j) \subseteq \mathbb{R}$ for $j \in J$, so that

$$\bigcap_{j \in J} (a_j, b_j)$$

is a closed interval $[c, d]$ which is not empty. Prove your example works.

(b) Prove that for any index set J and collection of open intervals $(a_j, b_j) \subseteq \mathbb{R}$ for $j \in J$, if

$$\bigcup_{j \in J} (a_j, b_j)$$

is an interval then it is an open interval.

¹W. Sutherland, *Introduction to Metric and Topological Spaces*, 2nd ed., Oxford Univ. Press 2009