Worksheet: Easy ODE problems by calculus

For ordinary differential equation initial value problems (ODE IVPs) like

$$(*) \qquad y' = f(t,y), \quad y(t_0) = y_0$$

our *major themes* are that:

i) problem $(*)$ describes a simulation using a law for how things behave (the ODE) and a starting point of the object (the IVP), and

ii) most simulations require approximations by computers (numerical methods).

However, in easy cases we may not need approximations. We can be precise!

In these easy cases one can either solve problem $(*)$ for $y(t)$ or check (verify) that a stated function $y(t)$ is a solution to $(*)$. One only needs calculus, as on this worksheet.

On this worksheet, and on the final exam, it is important to read the question; are you asked to *verify* or *solve*?

A. Verify that $y(t) = 3e^{-t^2}$ solves the ODE IVP

$$y' = -2ty, \quad y(0) = 3$$

B. Solve the ODE IVP:

$$y' = t^2 + \cos t, \quad y(1) = -2$$

Then find $y(3)$.

C. Solve the ODE IVP:

$$u' = u^{1/2}, \quad u(0) = 1$$

Sketch the solution in the t, u plane, clearly indicating the initial value.

D. Using a different color, add the direction field for the ODE $u' = u^{1/2}$ to the above sketch.

E. Verify that both $y(t) = 0$ and $y(t) = t^{4/3}$ solve the ODE IVP

$$y' = \frac{4}{3} y^{1/4}, \quad y(0) = 0$$

Sketch both solutions in the t, y plane, clearly indicating the initial value. Again add the direction field in a different color.