SOLUTIONS

1. Find the most general antiderivative of the function. (*Check your answer by differentiation.*)

(a) $f(x) = 3\sqrt{x} - 2\sqrt[3]{x}$ $= 3x^{\frac{1}{2}} - 2x^{\frac{1}{3}}$

$F(x) = 3 \cdot \frac{2}{3} x^{3/2} - 2 \cdot \frac{3}{4} x^{4/3} + C$

$= 2x^{3/2} - \frac{3}{2} x^{4/3} + C$

(b) $h(\theta) = 2\sin\theta - \sec^2\theta$

$H(\theta) = -2\cos\theta - \tan\theta + C$

(c)

$$f(x) = \frac{2x^4 + 4x^3 - x^2}{x^3}, \qquad x > 0$$

$f(x) = 2x + 4 - \frac{1}{x}$

$F(x) = x^2 + 4x - \ln|x| + C$

(or "$\ln(x)$" in <u>this</u> case, because $x > 0$)

2. Find f.

(a) $f'(t) = 4/(1+t^2),\ \ f(1) = 0$

$$f'(t) = 4 \cdot \frac{1}{1+t^2} \implies f(t) = 4\arctan(t) + c$$

$$0 = f(1) = 4\arctan(1) + c \quad \therefore \quad c = -4\arctan(1)$$

$$f(t) = 4(\arctan(t) - \arctan(1))$$

(b) $f''(x) = 8x^3 + 5,\ \ f(1) = 0,\ \ f'(1) = 8$

$$f'(x) = 2x^4 + 5x + c, \quad 8 = f'(1) = 2 + 5 + c \ \therefore c = 1$$

$$f'(x) = 2x^4 + 5x + 1$$

$$f(x) = \frac{2}{5}x^5 + \frac{5}{2}x^2 + x + \hat{c}, \quad 0 = f(1) = \frac{2}{5} + \frac{5}{2} + 1 + \hat{c}$$

$$\therefore \ \hat{c} = -3.9$$

$$f(x) = \frac{2}{5}x^5 + \frac{5}{2}x^2 + x - 3.9$$

3. The graph of f' is shown in the figure. Sketch the graph of f if f is continuous and $f(0) = -1$.