SOLUTIONS

1. Find $f'(a)$ using the definition of the derivative:

$f(t) = 2t^2 + t$

$$f'(a) = \lim_{h\to 0} \frac{f(a+h) - f(a)}{h} = \lim_{h\to 0} \frac{2(a+h)^2 + (a+h) - (2a^2+a)}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{2a^2} + 4ah + 2h^2 + \cancel{a} + h - \cancel{2a^2} - \cancel{a}}{h}$$

$$= \lim_{h\to 0} \frac{4a\cancel{h} + 2h^2 + \cancel{h}}{\cancel{h}} = \lim_{h\to 0} 4a + 1 + 2h = 4a + 1$$

2. Find $f'(3)$ using the definition of the derivative:

$f(x) = x^{-2}$

$$f'(3) = \lim_{h\to 0} \frac{(3+h)^{-2} - 3^{-2}}{h} = \lim_{h\to 0} \frac{\frac{1}{(3+h)^2} - \frac{1}{9}}{h}$$

$$= \lim_{h\to 0} \frac{9 - (3+h)^2}{(3+h)^2 9h} = \lim_{h\to 0} \frac{\cancel{9} - \cancel{9} - 6h - h^2}{(3+h)^2 9h}$$

$$= \lim_{h\to 0} \frac{-6\cancel{h} - h^{\cancel{2}}}{(3+h)^2 9 \cancel{h}} = \frac{-6}{3^2 \cdot 9} = \frac{-2}{27}$$

3. Find $f'(a)$ using the definition of the derivative:

$f(x) = \sqrt{1+5x}$

$$f'(a) = \lim_{x\to a} \frac{\sqrt{1+5x} - \sqrt{1+5a}}{x-a}$$

$$\left(\underline{\text{or}}: = \lim_{h\to 0} \frac{\sqrt{1+5(a+h)} - \sqrt{1+5a}}{h} = \text{etc.} \right)$$

$$= \lim_{x\to a} \frac{\sqrt{1+5x} - \sqrt{1+5a}}{x-a} \cdot \frac{\sqrt{1+5x} + \sqrt{1+5a}}{\sqrt{1+5x} + \sqrt{1+5a}}$$

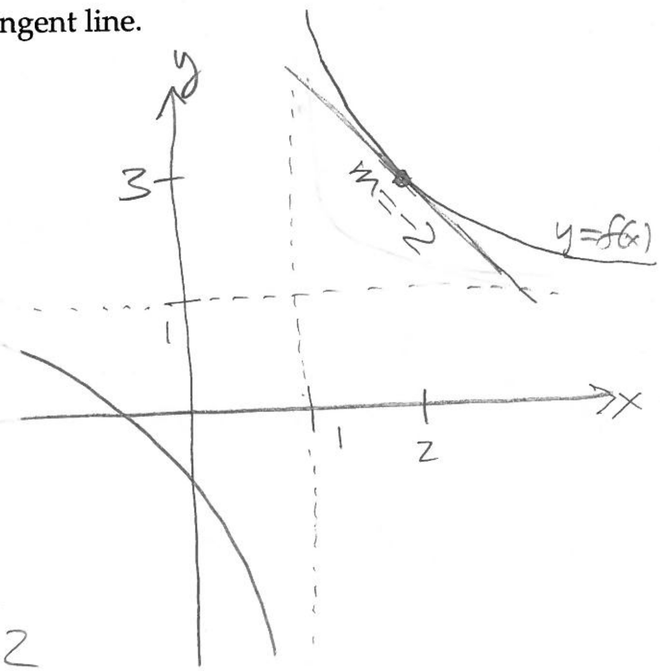
$$= \lim_{x\to a} \frac{\cancel{1}+5x - (\cancel{1}+5a)}{(x-a)(\sqrt{1+5x} + \sqrt{1+5a})} = \lim_{x\to a} \frac{5\cancel{(x-a)}}{\cancel{(x-a)}(\sqrt{1+5x} + \sqrt{1+5a})}$$

$$= \frac{5}{\sqrt{1+5a} + \sqrt{1+5a}} = \frac{5}{2\sqrt{1+5a}}$$

4. Find an equation of the tangent line to the curve at the given point:

$$f(x) = \frac{x+1}{x-1}, \qquad (2,3)$$

Also sketch both the curve $y = f(x)$ and the tangent line.



$$m = \lim_{x\to 2} \frac{f(x)-f(2)}{x-2}$$

$$= \lim_{x\to 2} \frac{\frac{x+1}{x-1} - 3}{x-2}$$

$$= \lim_{x\to 2} \frac{(x+1) - 3(x-1)}{(x-1)(x-2)}$$

$$= \lim_{x\to 2} \frac{-2x+4}{(x-1)(x-2}$$

$$= \lim_{x\to 2} \frac{-2\cancel{(x-2)}}{(x-1)\cancel{(x-2)}} = \frac{-2}{1} = -2$$

$\therefore$ tangent line is $y-3 = (-2)(x-2)$

5. A particle moves a distance $s = f(t)$ along a straight line, where s is measured in meters and t is in seconds:

$$f(t) = 40t - 5t^2$$

$f(4) = 80$

Find the velocity and speed when $t = 4$.

$$v(4) = f'(4) = \lim_{h\to 0} \frac{40(4+h) - 5(4+h)^2 - 80}{h}$$

$$= \lim_{h\to 0} \frac{160 + 40h - 5(16+8h+h^2) - 80}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{160} + \cancel{40h} - \cancel{80} - \cancel{40h} - 5h^2 - \cancel{80}}{h}$$

$$= \lim_{h\to 0} \frac{-5h^2}{h} = \lim_{h\to 0} -5h = 0$$

$\therefore$ velocity $= 0$ m/s, speed $= 0$ m/s