SOLUTIONS

1. Use the quotient rule and the facts below to show that: $\frac{d}{dx}(\csc x) = -\csc x \cot x$

- $\csc x = 1/\sin x$
- $\cot x = \cos x/\sin x$
- $(\sin x)' = \cos x$

$$\frac{d}{dx}(\csc x) = \frac{d}{dx}\left(\frac{1}{\sin x}\right) = \frac{0 \cdot \sin x - 1 \cdot \cos x}{(\sin x)^2} = -\frac{\cos x}{\sin^2 x}$$

$$= -\frac{\cos x}{\sin x}\frac{1}{\sin x} = -\cot x \csc x$$

2. Differentiate the functions. $\boxed{\text{no request to simplify!}}$

$f(\theta) = \theta \cos\theta \sin\theta$

$$f'(\theta) = 1 \cdot (\cos\theta \sin\theta) + \theta(\cos\theta \sin\theta)'$$
$$= \cos\theta \sin\theta + \theta(-\sin\theta \sin\theta + \cos\theta\cos\theta)$$

$h(r) = \dfrac{ae^r}{b+e^r}$

$$h'(r) = \frac{ae^r(b+e^r) - ae^r(e^r)}{(b+e^r)^2}$$

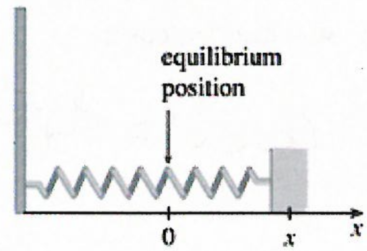
$y = \sec\theta \tan\theta$

$$\frac{dy}{dx} = (\sec\theta \tan\theta)\tan\theta + \sec\theta(\sec^2\theta)$$

$f(t) = \dfrac{\sin t}{3 - 5\cos t}$

$$f'(t) = \frac{\cos t(3-5\cos t) - \sin t(-5(-\sin t))}{(3-5\cos t)}$$

3. A mass on a spring vibrates horizontally on a surface with friction, as shown. Its equation of motion is $x(t) = \dfrac{2\sin t}{e^t}$, where t is in seconds and x is in inches.



(i) Find the velocity and acceleration at time t.

$$v = x' = \frac{2\cos t\, e^t - 2\sin t\, e^t}{(e^t)^2}$$

$$= \frac{2(\cos t - \sin t)}{e^t}$$

$$a = v' = \frac{2(-\sin t - \cos t)e^t - 2(\cos t - \sin t)e^t}{(e^t)^2} = \frac{2(-2\cos t)}{e^t}$$

$$= \frac{-4\cos t}{e^t}$$

(ii) Find the position and velocity at time $t = \dfrac{\pi}{2}$.

$$x\left(\frac{\pi}{2}\right) = \frac{2\cdot 1}{e^{\pi/2}} = \frac{2}{e^{\pi/2}} \text{ inches}$$

$$v\left(\frac{\pi}{2}\right) = \frac{2(0-1)}{e^{\pi/2}} = -\frac{2}{e^{\pi/2}} \ \frac{\text{inches}}{\text{sec.}}$$

(iii) Compute $\lim\limits_{t\to\infty} x(t)$.

$$\lim_{t\to\infty} x(t) = \lim_{t\to\infty} \frac{2\sin t}{e^t} = 0$$

(squeeze theorem: $|\sin t| \le 1$

so $\left|\dfrac{2\sin t}{e^t}\right| \le \dfrac{2}{e^t}$)

4. Find the derivative . . . by noticing the pattern:

$$\frac{d^{79}}{dx^{79}}(\cos x) = \frac{d^3}{dx^3}(\cos x) = +\sin x$$

pattern:

1st $(\cos x)' = -\sin x$

2nd $(-\sin x)' = -\cos x$

3rd $(-\cos x)' = +\sin x$

4th $(+\sin x)' = \cos x$

$\vdots$ repeats every 4 derivs