SOLUTIONS

1.

$$F(x) = x\sqrt{6-x}$$

(a) What is the domain of $F(x)$? $(-\infty, 6]$

(b) Find the intervals of increase or decrease and critical numbers.

(c) Find the intervals of concavity and the inflection points.

(d) Sketch the graph.

$$F'(x) = 1\cdot\sqrt{6-x} + x\cdot\tfrac{1}{2}(6-x)^{-1/2}(-1) = \sqrt{6-x} - \frac{x}{2\sqrt{6-x}}$$

$$= \frac{1}{\sqrt{6-x}}\left(6-x-\frac{x}{2}\right) = \frac{6-\frac{3}{2}x}{\sqrt{6-x}}, \quad F'(x)=0 \Leftrightarrow x=4$$

crit. #

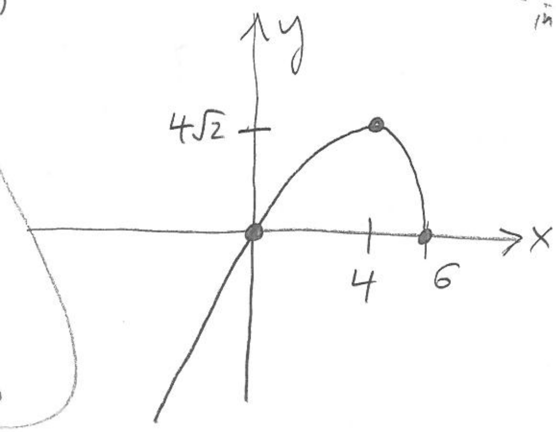
$$F''(x) = \frac{\left(-\frac{3}{2}\right)\sqrt{6-x} - \left(6-\frac{3}{2}x\right)\frac{1}{2}(6-x)^{-1/2}(-1)}{6-x} = \frac{(6-x)^{-1/2}\left(-\frac{3}{2}(6-x)+\frac{1}{2}\left(6-\frac{3}{2}x\right)\right)}{6-x}$$

$$= \frac{-3(6-x)+6-\frac{3}{2}x}{2(6-x)^{3/2}} = \frac{-12+\frac{3}{2}x}{2(6-x)^{3/2}}$$

$F''(x)=0 \Leftrightarrow x=8$? (not in domain)

x	F	F'	F''
-1			
0	0	$+$	
4	$4\sqrt{2}$	0	$-$
5		$-$	
6	0		

increasing $(-\infty, 4)$
decreasing $(4, 6)$
concave down $(-\infty, 6)$
never concave up
no pts of inflection



2. Compute the following limits; you may use L'Hopital's rule:

$$\lim_{x\to-\infty} \frac{e^x}{1-e^x} = \frac{0}{1-0} = 0$$

$$\lim_{x\to+\infty} \frac{e^x}{1-e^x} \overset{L'H}{=} \lim_{x\to\infty} \frac{e^x}{-e^x} = -1$$

(Can you compute the second limit without L'Hopital's rule? How?) $= \lim_{x\to\infty} \frac{e^x}{1-e^x}\cdot\frac{e^{-x}}{e^{-x}}$

$$= \lim_{x\to\infty} \frac{1}{e^{-x}-1} = \frac{1}{0-1} = -1$$

3.

$$g(x) = \frac{e^x}{1 - e^x}$$

(a) What is the domain of $g(x)$? $\leftarrow$ $\{x \neq 0\}$ or $(-\infty, 0) \cup (0, \infty)$

(b) Find the horizontal and vertical asymptotes. $\leftarrow$ horizontal: $y = 0$, $y = -1$

vertical: $x = 0$

(c) Find the intervals of increase or decrease and critical numbers.

(d) Find the intervals of concavity and the inflection points.

(e) Sketch the graph.

$$g'(x) = \frac{e^x(1-e^x) - e^x(-e^x)}{(1-e^x)^2} = \frac{e^x}{(1-e^x)^2} \left(\underset{\uparrow \text{never}}{= 0} \right)$$

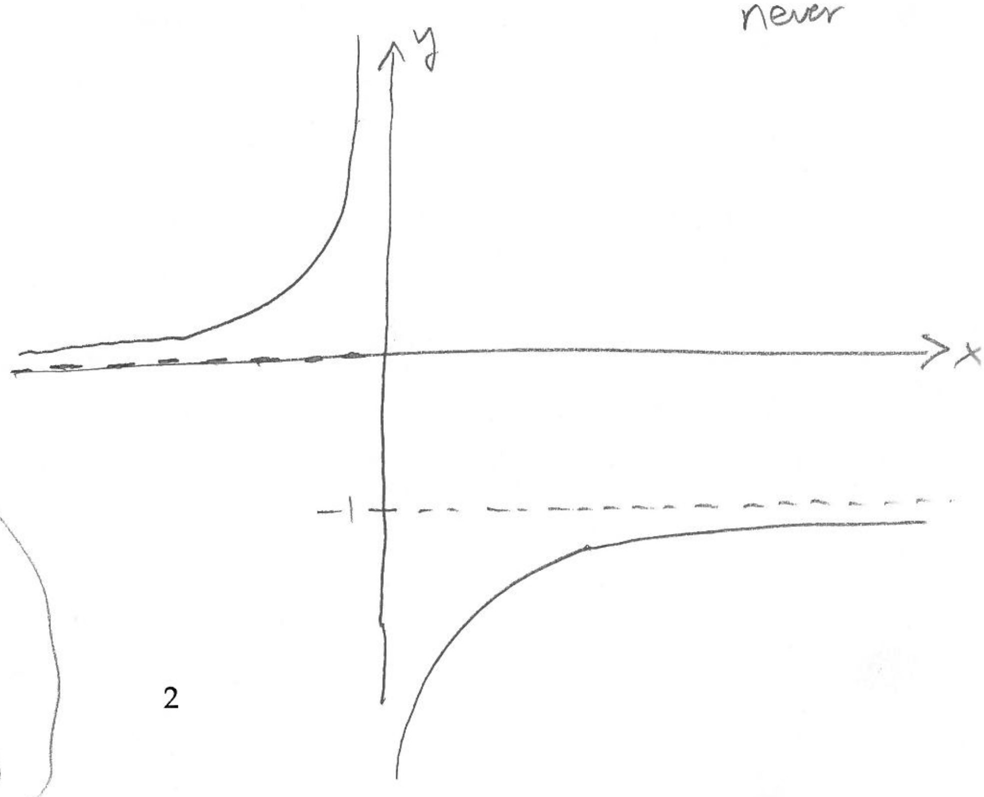
$$g''(x) = \frac{e^x(1-e^x)^2 - e^x \cdot 2(1-e^x)(-e^x)}{(1-e^x)^4}$$

$$= \frac{e^x(1-e^x) + 2e^{2x}}{(1-e^x)^3} = \frac{e^x(1+e^x)}{(1-e^x)^3}$$

$g''(x) = 0 \Leftrightarrow e^x(1+e^x) = 0$

never

x	g	g'	g''
$-\infty$	0		
		+	+
0	X	X	X
		+	−
$+\infty$	−1		



no crit. #s

no inflection pts

increasing $(-\infty, 0) \cup (0, \infty)$

never decreasing

concave up $(-\infty, 0)$

concave down $(0, \infty)$