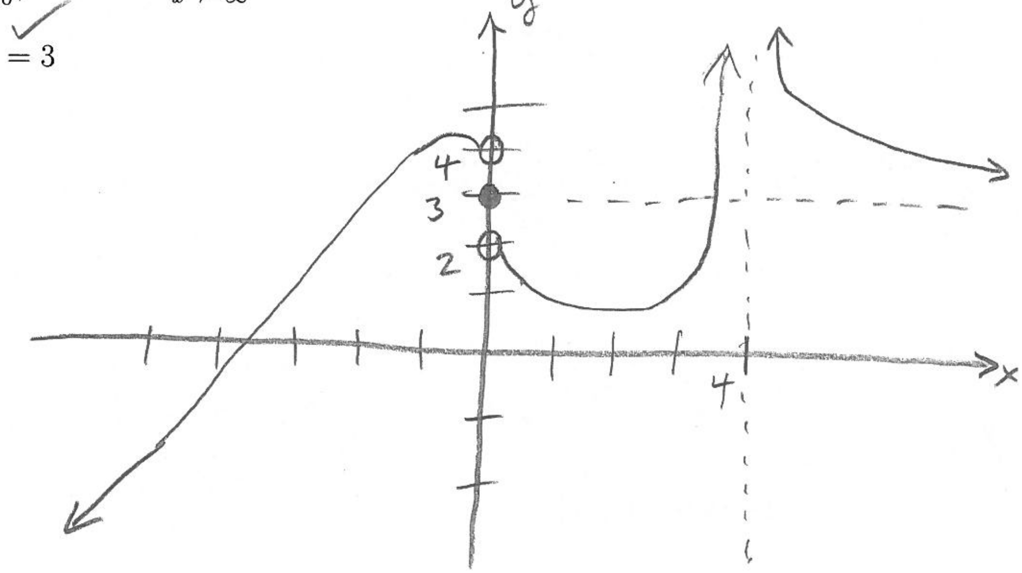
1. (§2.6 #9) Sketch the graph of a function that satisfies all these conditions:

$f(0) = 3$, $\lim_{x\to 0^-} f(x) = 4$, $\lim_{x\to 0^+} f(x) = 2$, $\lim_{x\to -\infty} f(x) = -\infty$,

$\lim_{x\to 4^+} f(x) = \infty$, $\lim_{x\to\infty} f(x) = 3$



2. Find $f'(x)$ using the definition if $f(x) = \sqrt{x}$.

$$f'(x) = \lim_{h\to 0} \frac{\sqrt{x+h}-\sqrt{x}}{h} = \lim_{h\to 0} \frac{\cancel{(x}+\cancel{h})-\cancel{x}}{\cancel{h}(\sqrt{x+h}+\sqrt{x})}$$

$$= \lim_{h\to 0} \frac{1}{\sqrt{x+h}+\sqrt{x}} = \frac{1}{2\sqrt{x}}$$

3. (§2.7 #7) Using the result of the last problem, find an equation of the tangent line to $y = \sqrt{x}$ at the point $(1, 1)$.

$$m = f'(1) = \frac{1}{2\sqrt{1}} = \frac{1}{2}$$

$$\therefore \quad y - 1 = \frac{1}{2}(x-1)$$

4. (§2.6 #50) Find the horizontal and vertical asymptotes of the curve, and state the limits which justify these asymptotes:

$$y = \frac{1+x^4}{x^2-x^4} = \frac{1+x^4}{x^2(1-x^2)} = \frac{1+x^4}{x^2(1-x)(1+x)} = f(x)$$

vertical asymptotes: $x=0$ $\left(\lim_{x\to 0} f(x) = +\infty\right)$

$x=-1$ $\left(\lim_{x\to -1^+} f(x) = +\infty\right)$

$x=+1$ $\left(\lim_{x\to 1^-} f(x) = +\infty\right)$

hor. asymptote: $y=-1$ $\left(\lim_{x\to\infty} f(x) = \lim_{x\to\infty} \frac{\frac{1}{x^4}+1}{\frac{1}{x^2}-1} = -1\right)$

5. (§2.3 #49) Let $g(x) = \frac{x^2+x-6}{|x-2|}$.

(a) Find $\lim_{x\to 2^-} g(x)$ and $\lim_{x\to 2^+} g(x)$.

(b) Does $\lim_{x\to 2} g(x)$ exist?

(a) $\lim_{x\to 2^-} g(x) = \lim_{x\to 2^-} \frac{x^2+x-6}{-(x-2)} = \lim_{x\to 2^-} \frac{\cancel{(x-2)}(x+3)}{-\cancel{(x-2)}} = -5$

$\lim_{x\to 2^+} g(x) = \lim_{x\to 2^+} \frac{x^2+x-6}{x-2} = \lim_{x\to 2^+} \frac{\cancel{(x-2)}(x+3)}{\cancel{x-2}} = 5$

(b) no, because one-sided limits are not equal

6. (like §2.7 #53) The cost of producing x ounces of gold from a new mine is $C = f(x)$ dollars.

(a) What is the meaning of the derivative $f'(x)$? What are its units?

(b) What does the statement $f'(80,000) = 17$ mean?

(a) it is the rate of change of the production cost as the ounces produced is increased, with units of dollars/ounce

(b) it means that at 80,000 ounces produced, each additional ounce costs \$17