

Assignment #1

Due Friday, 27 January at the start of class

Read the Preface and Chapter 1 of the textbook R. LeVeque, *FDMOPDEs*.
Get started reading Chapter 2 as well.

0. (There is nothing to turn in on this problem. Problems 1–6 on this assignment relate to these two review topics, along with a bit of actual computing.) Find a standard textbook on *calculus* and an introductory textbook on *ordinary differential equations* (ODEs). You will need these references throughout the semester. Review these two topics:¹

- i) Taylor's theorem with remainder formula, and
- ii) the solution of linear homogeneous constant-coefficient ODEs.

1. Calculate $(626)^{1/4}$ to within $0.00001 = 10^{-5}$ of the exact value *without* any computing machinery except a pencil or pen. Prove that your answer has this accuracy. (Hint: You may, of course, use a computer to check your by-hand value.)

2. Assume f' is continuous. Derive the remainder formula

$$(1) \quad \int_0^a f(x) dx = af(0) + \frac{1}{2}a^2 f'(\nu)$$

for some (unknown) ν between zero and a . (Hint: Start by showing $f(x) = f(0) + f'(\xi)x$ where $\xi = \xi(x)$ is some number between 0 and x .) Use two sentences to explain the meaning of (1), as an answer to the question "What properties of an integral would make the left-endpoint rule $\int_0^a f(x) dx \approx af(0)$ inaccurate?"

3. Download/install/purchase/find MATLAB or OCTAVE (or PYTHON etc.). Now work at the command line to compute a finite sum approximation to

$$\sum_{n=1}^{\infty} \frac{\cos n}{n^3}.$$

Compute at least three partial (finite) sums, with increasing numbers of terms N . Do you think you are getting close to the infinite sum, and if so, why? Turn your command line work into a function `sumstuff(N)`, defined in a file `sumstuff.m`, and show that it works. Turn in both the command line session (be brief) and the code.

¹Taylor's theorem may be best explained by an additional undergraduate *numerical analysis* textbook.

4. Solve, by hand,

$$(2) \quad y'' + 2y' - 3y = 0, \quad y(2) = 0, \quad y'(2) = -1,$$

for the solution $y(t)$. Then find² $y(5)$. On t, y axes, give a reasonable by-hand sketch which shows the initial values, the solution, and the value $y(5)$.

5. Using Euler's method³ for approximately solving ODEs, write your own MATLAB program (either script or function) to solve initial value problem (2) to find $y(5)$. A first step is to convert the second-order ODE into a system of two first-order ODEs. Use a few step sizes, decreasing as needed, so that you get apparent four digit accuracy. (*Hint: You can use a built-in ODE solver to check your work, but this is not required.*)

6. Solve, by hand, the ODE boundary value problem

$$(3) \quad y'' + 2y' - 3y = 0, \quad y(0) = \alpha, \quad y(\tau) = \beta,$$

for the solution $y(t)$. Note that α, β, τ are the data of the problem. Are there values of τ for which this problem does not have a unique solution?

²This is a *prediction* of the outcome at $t = 5$, given initial data at $t = 2$ and a precise "law" about how $y(t)$ evolves in time, namely the differential equation itself.

³Look it up if needed!