SOLUTIONS

Math F251: Section 4.5 Worksheet — Friday 1 November 2018

Recall the guidelines:

A. domain
B. intercepts
C. symmetry
D. asymptotes
E. increase/decrease (and critical numbers)
F. local maxima/minima
G. concavity (and inflection points)
H. sketch the graph

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1. Sketch the graph by applying the guidelines:

$$y = \frac{\sin x}{2 + \cos x}, \qquad 0 \le x \le 2\pi$$

$$y' = \frac{\cos x(2+\cos x) - \sin x(-\sin x)}{(2+\cos x)^2}$$

$$= \frac{2\cos x + 1}{(2+\cos x)^2}$$

$$y'' = \frac{-2\sin x(2+\cos x)^2 - (2\cos x+1)2(2+\cos x)(-\sin x)}{(2+\cos x)^4}$$

$$= \frac{-2\sin x(2+\cos x) + 2(2\cos x+1)\sin x}{(2+\cos x)^3}$$

$$= \frac{2\sin x(\cos x - 1)}{(2+\cos x)^3}$$

domain: all real numbers

x-intercepts: $\sin x = 0 \Leftrightarrow x = 0, \pi, 2\pi$

y-intercept: $y = 0$

symmetry: periodic w. period 2π

asymptotes: none

crit #s: $2\cos x + 1 = 0 \Leftrightarrow \cos x = -\frac{1}{2}$

$x = \frac{2\pi}{3}, \frac{4\pi}{3}$

infl. pts: $\sin x = 0$ or $\cos x - 1 = 0$

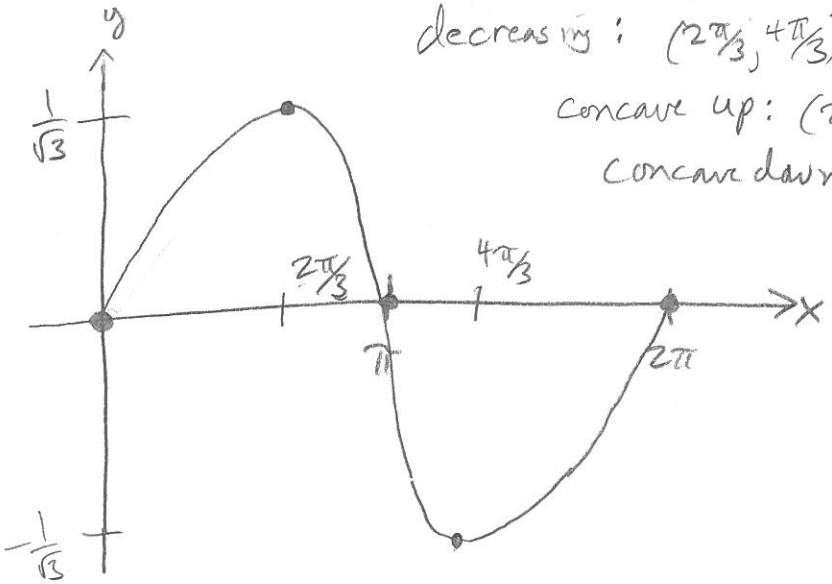
$x = 0, \pi, 2\pi$

increasing: $(0, 2\pi/3) \cup (4\pi/3, 2\pi)$

decreasing: $(2\pi/3, 4\pi/3)$

concave up: $(\pi, 2\pi)$

concave down: $(0, \pi)$



x	y	y′	y″
0	0	+	0
$2\pi/3$	$1/\sqrt{3}$	0	−
π	0	−	0
$4\pi/3$	$-1/\sqrt{3}$	0	+
2π	0	+	0

2. Sketch the graph by applying the guidelines:

$y = \dfrac{1}{x^2-4}$ → $y' = -(x^2-4)^{-2}(2x) = \dfrac{-2x}{(x^2-4)^2}$, $y'' = \dfrac{2(3x^2+4)}{(x^2-4)^3}$

domain: $(-\infty,-2)\cup(-2,2)\cup(2,\infty)$

intercepts: $(0,-1/4)$

symmetry: even

asymptotes: $x=-2$, $x=2$, $y=0$ ($\lim_{x\to\pm\infty} \frac{1}{x^2-4} = 0$)

crit #s: $x=0$

infl. pts: none

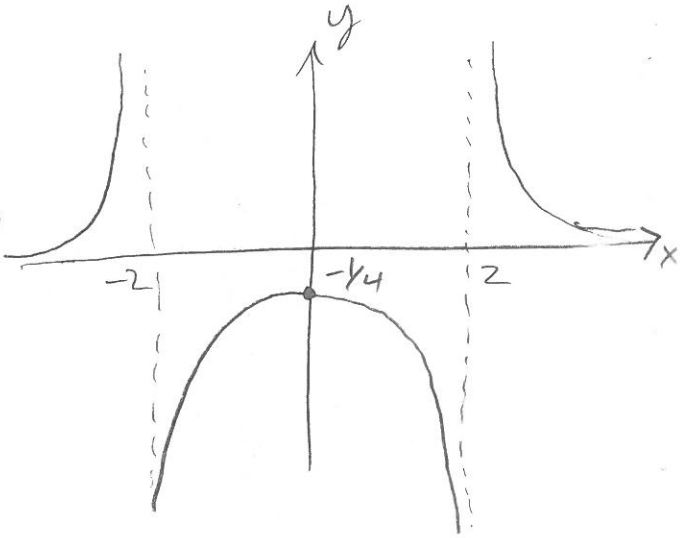
x	y	y'	y''
$-\infty$	0	+	+
-2	$\pm\infty$	+	
0	$-1/4$	0	−
$+2$	$\pm\infty$	−	+
$+\infty$	0		

increasing: $(-\infty,-2)\cup(-2,0)$

decreasing: $(0,2)\cup(2,\infty)$

concave up: $(-\infty,-2)\cup(2,\infty)$

concave down: $(-2,2)$



3. Sketch the graph by applying the guidelines:

$y = \dfrac{x}{\sqrt{x^2+1}}$ → $y' = \dfrac{1}{(x^2+1)^{3/2}}$, $y'' = \dfrac{-3x}{(x^2+1)^{5/2}}$

domain: $(-\infty,\infty)$

intercepts: $(0,0)$

symmetry: odd

asymptotes: $y=-1$, $y=+1$ ($\lim_{x\to-\infty} \frac{x}{\sqrt{x^2+1}} = -1$, $\lim_{x\to+\infty} \frac{x}{\sqrt{x^2+1}} = +1$)

crit #s: none

infl. pts: $x=0$

x	y	y'	y''
$-\infty$	-1		
		+	+
0	0	+	0
		+	−
$+\infty$	$+1$		

increasing: $(-\infty,\infty)$

decreasing nowhere

concave up: $(-\infty,0)$

concave down: $(0,\infty)$