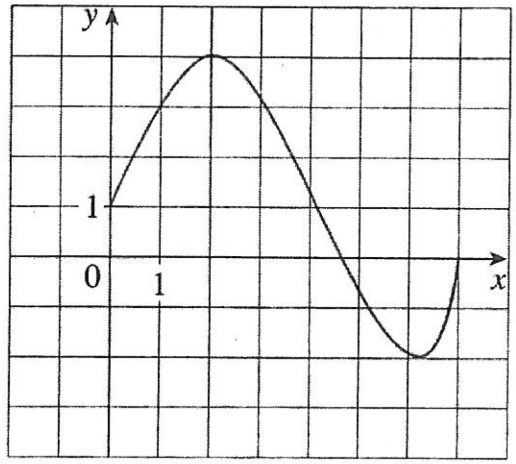
Some approximation is necessary

1. The graph of a function f is shown below. Find the following:



a) $f(1)$ and $f(5)$ $\quad f(1)=3$, $\quad f(5)=-0.7$

b) the domain of f $\quad [0,7]$

c) the range of f $\quad [-2,4]$

d) For which value of x is $f(x)=4$? $\quad x=2$

e) Where is f increasing? $\quad [0,2]\cup[6.2,7]$

2. Let $f(x)=3x^2-x+2$. Find and simplify the following expressions.

(a) $f(2)$

$$f(2)=3\cdot 2^2-2+2=12$$

(b) $f(a^2)$

$$f(a^2)=3a^4-a^2+2$$

(c) $[f(a)]^2$

$$[f(a)]^2=(3a^2-a+2)^2=9a^4-6a^3+13a^2-4a+4$$

(d) $\dfrac{f(2+h)-f(2)}{h}$

$$\frac{f(2+h)-f(2)}{h}=\frac{3(2+h)^2-(2+h)+2-12}{h}$$

$$=\frac{\cancel{12}+12h+3h^2\cancel{-2}-h\cancel{+2}\cancel{-12}}{h}$$

$$=\frac{3h^2+13h}{h}=3h+13$$

(e) $\dfrac{f(a+h)-f(a)}{h}$

$$\frac{f(a+h)-f(a)}{h}=\frac{3(a+h)^2-(a+h)+2-(3a^2-a+2)}{h}$$

$$=\frac{\cancel{3a^2}+6ah+3h^2\cancel{-a}-h\cancel{+2}\cancel{-3a^2}\cancel{+a}\cancel{-2}}{h}$$

$$=\frac{(6a+1)h+3h^2}{h}=3h+6a+1$$

3. Find the domain of each of the following functions. Use interval notation.

1. $f(x) = \dfrac{1}{x^4 - 16}$

[Work: $x^4 - 16 = 0 \Leftrightarrow x^2 = 4$ or $x^2 = -4$ ← impossible; $\Updownarrow$ $x = \pm 2$]

$(-\infty, -2) \cup (-2, 2) \cup (2, \infty)$

2. $f(x) = \sqrt{x} + \sqrt{11 - x}$

[Work: $x \geq 0$ and $11 - x \geq 0 \Leftrightarrow x \leq 11$]

$[0, 11]$

3. $g(x) = \ln(x - 4)$

[Work: $x - 4 > 0 \Leftrightarrow x > 4$]

$(4, \infty)$

4. $h(x) = \frac{1}{\sqrt{x^2-5x-6}}$

[Work: $x^2 - 5x - 6 > 0$
$(x-6)(x+1) > 0$]

$(-\infty, -1) \cup (6, \infty)$

4. Graph each of the following piecewise defined functions.

a) $f(x) = \begin{cases} -1 & \text{if } x \geq 2 \\ 7 - 2x & \text{if } x < 2 \end{cases}$

b) $f(x) = \begin{cases} x + 1 & \text{if } x \leq -1 \\ x^2 & \text{if } x > -1 \end{cases}$