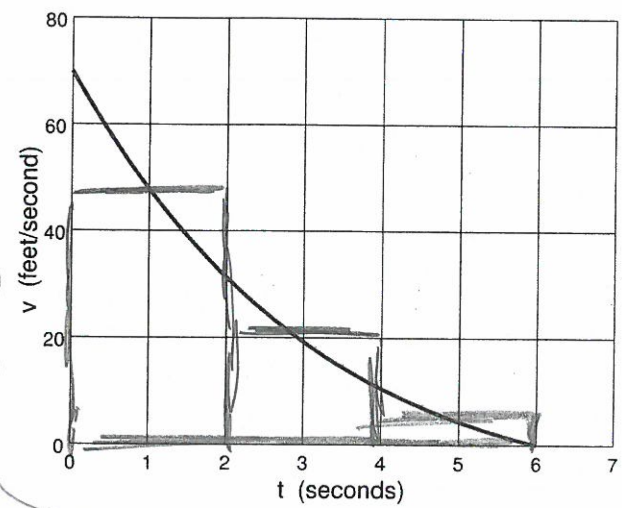
SOLUTIONS

1. The velocity graph $v(t)$ of a braking car is shown.



(a) Use the graph to estimate the distance traveled by the car when the brakes are applied. (*Suggestion: Use 3 or 6 rectangles.*)

$n=3$: (as shown, using midpts)

$d \approx 48\cdot 2 + 21\cdot 2 + 5\cdot 2$

$= 148$ ft

$n=6$:

$d \approx 58\cdot 1 + 38\cdot 1 + 25\cdot 1 + 14\cdot 1 + 6\cdot 1 + 3\cdot 1$

$= 144$ ft

your answer may be a bit different

(b) Write the exact distance as a definite integral.

$$d = \int_0^6 v(t)\,dt$$

2. Evaluate the upper and lower sums for $f(x) = 2 + \sin x$ on $0 \le x \le \pi$ with $n = 4$. Illustrate with a diagram.

(upper sum)

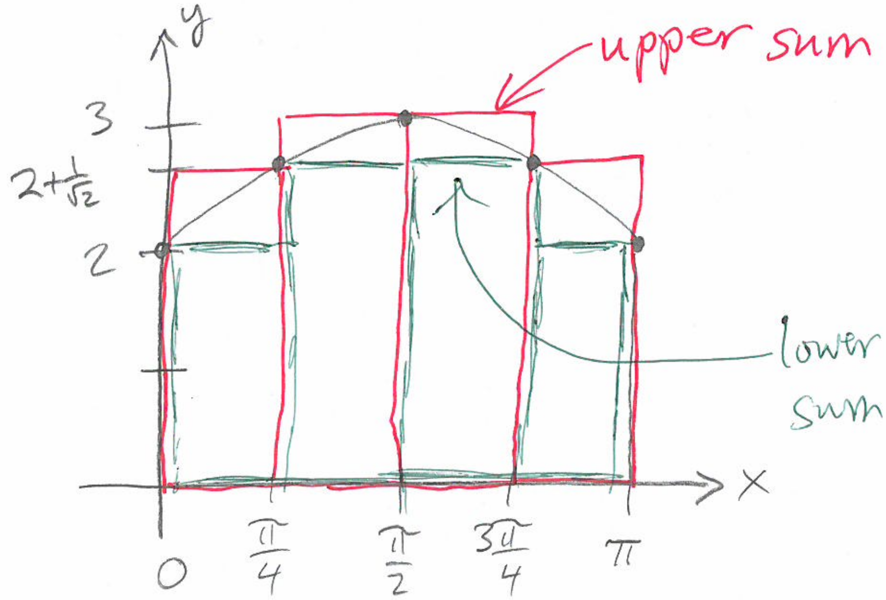
$= \frac{\pi}{4}\left(2+\frac{1}{\sqrt{2}} + 3 + 3 + 2 + \frac{1}{\sqrt{2}}\right)$

$= \frac{(10+\sqrt{2})\pi}{4}$

(lower sum)

$= \frac{\pi}{4}\left(2 + 2 + \frac{1}{\sqrt{2}} + 2 + \frac{1}{\sqrt{2}} + 2\right)$

$= \frac{(8+\sqrt{2})\pi}{4}$

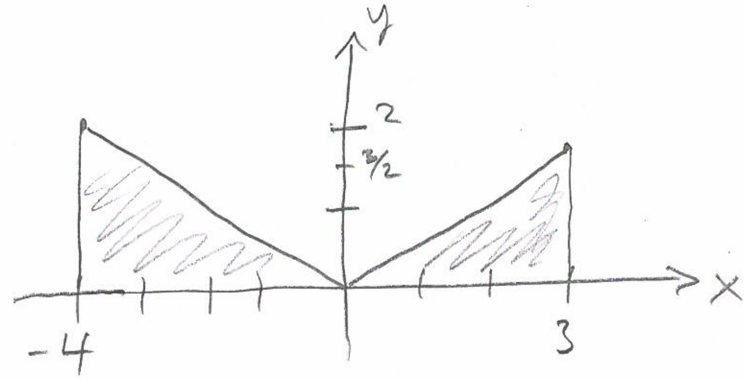


3. Evaluate the integral by interpreting it in terms of areas. (*Hint: Start by sketching the integrand.*)

$$\int_{-4}^{3} \left|\tfrac{1}{2}x\right| \, dx$$

$$= \frac{1}{2}\cdot 4 \cdot 2 + \frac{1}{2}\cdot 3 \cdot \frac{3}{2}$$

$$= 4 + \frac{9}{4} = \frac{25}{4}$$



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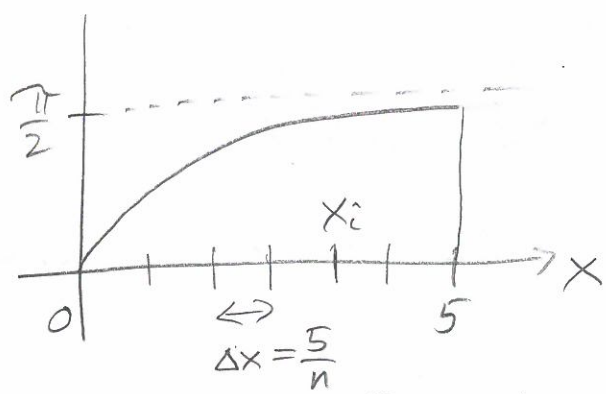
4. (a) Set up an expression for the following integral as a limit of sums; you will not be able to compute the limit:

$$\int_{0}^{5} \arctan x \, dx$$

$$= \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\,\Delta x$$

$$= \lim_{n\to\infty} \sum_{i=1}^{n} \arctan\left(\frac{5i}{n}\right)\frac{5}{n}$$



$$x_i = 0 + i\,\Delta x = i\,\frac{5}{n} = \frac{5i}{n}$$

my error: should be this

(b) Using a graph of $y = \arctan x$, sketch a diagram which shows that

$$\frac{5\arctan(5)}{2}\ \cancel{\frac{5\pi}{4}} \leq \int_{0}^{5} \arctan x \, dx \leq \frac{5\pi}{2}$$

$$\begin{pmatrix}\text{area of}\\ \text{rectangle}\end{pmatrix} = \frac{\pi}{2}\cdot 5 = \frac{5\pi}{2}$$

$$\begin{pmatrix}\text{area of}\\ \text{triangle}\end{pmatrix} = \frac{1}{2}\cdot \arctan 5 \cdot 5 = \frac{5\arctan(5)}{2}$$