Assignment #6

Due Monday 18 October, 2021 at the start of class.

Submit on paper or by email: `elbueler@alaska.edu`

Exercise 3.2.1. (*Do this by hand. Hints: The first step is to write down the normal equations* $A^\top Ax = A^\top b$*. You can* check *your work by* `x = A \ b` *in Matlab.*)

Exercise 3.2.3. (*Your argument needs to work for any size matrices. Hint: What is the* (i, j) *entry of* $A^\top A$*?*)

Exercise 3.2.5. (*Your argument in part* (a) *needs to work for any matrix* A *which satisfies the stated hypothesis. Hint: Start with formula* (3.2.2) *and compute* A^+A*.*)

Exercise 4.1.1.

Exercise 4.1.2.

Exercise 4.1.6. (*Hint: Given* x*, finding* $y = W(x)$ *is, indeed, a rootfinding problem.*)

P6. *Polynomial interpolation is badly-conditioned if some of the independent variables in the data, the* t*-values, are* close together*. This problem compares polynomial interpolation (section 2.1) versus polynomial fitting (regression; section 3.1) in this situation. We see that polynomial interpolation is badly impacted while polynomial regression is minimally affected. Note that each part below needs only a few lines of Matlab, and/or a short written answer.*

(a) Recall that if `t` is a Matlab vector of t-values then the Vandermonde matrix in equation (2.1.2) on page 32 can be computed by

```
>> V = fliplr(vander(t))
```

(*Why* `fliplr()`? *Matlab always starts each row with the highest power, while math books, and my lectures, start with the lowest power.*) For values $t_1 = 0$, $t_2 = 1$, $t_3 = 2$, $t_4 = 2.9$, $t_5 = 3$, $t_6 = 3.01$, and $t_7 = 4$, compute the Vandermonde matrix V and then find its condition number. About how many digits of accuracy do we expect to lose when solving a system with V?

(b) The matrix V in part (a) is for putting a degree-6 polynomial exactly through $m = 7$ points. (*This would require* y*-values, but they do not affect the matrix* V *itself.*) Suppose instead that we want to set up and solve the overdetermined system (3.1.2) on page 97 with $n = 3$, that is, suppose we want to fit a quadratic polynomial to the 7 points as closely as possible in the least-squares sense. In that case we merely extract the first 3 columns of V computed in part (a):

```
>> W = V(:,1:3)
```

What is the condition number of this matrix W? What is the condition number of the matrix $W^\top W$ if we set up the normal equations? About how many digits of accuracy do we expect to lose when solving these normal equations?

(c) We can show the sensitivity to noise in the y-values of polynomial interpolation and regression by graphing in a particular case. Using the same t-values as in parts (a) and (b), suppose the y-values are now given by

```
>> y = sin(t) + 0.01 * randn(size(t))
```

(That is, suppose the y-values are a noisy version of the $\sin$ function.) In a well-labeled single figure, plot the (t, y) data as markers, the interpolating polynomial from part (a), the regression polynomial from part (b), and finally the function $\sin(t)$. Use `legend()` to distinguish the four overlaid plots.

The figure in part (c) *will show that polynomial interpolation generates a bad interpolant that varies far outside of the range of the data, while, by contrast, polynomial regression does a reasonable job of modeling the y-values despite the added noise.*