

# SOLUTIONS

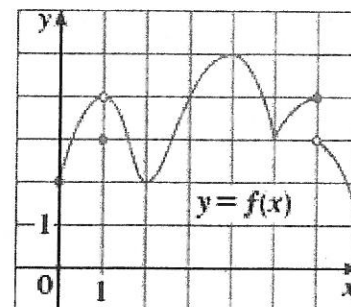
1. Use the graph to state the absolute and local maximum and minimum values of the function.

abs. max.:  $f(4) = 5$

no abs. min

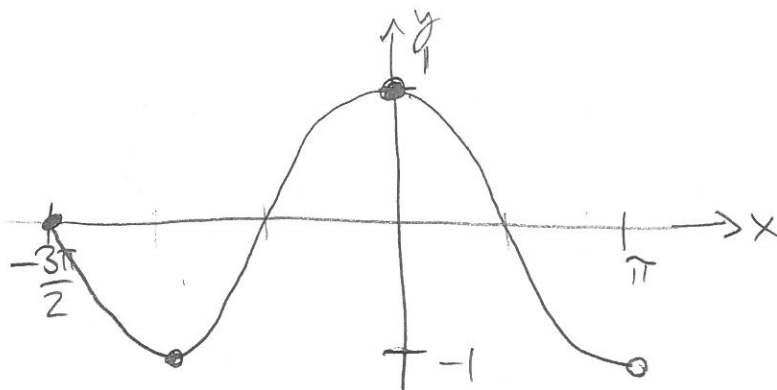
loc. min.:  $f(1) = 3, f(2) = 2, f(5) = 3$

loc. max.:  $f(4) = 5, f(6) = 4$



2. Sketch the graph  $f$  by hand and use your sketch to find the absolute and local maximum and minimum values of  $f$ .

$$f(t) = \cos(t), \quad -\frac{3\pi}{2} \leq t < \pi$$



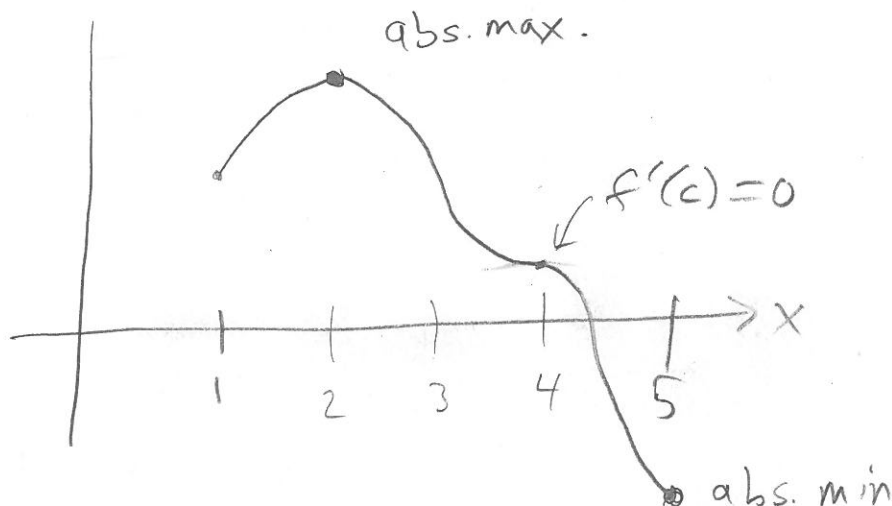
abs. max.  $f(0) = 1$

abs. min.  $f(-\pi) = -1$

loc. max.  $f(0) = 1$

loc. min.  $f(-\pi) = -1$

3. Sketch a graph of a function  $f$  which is continuous on  $[1, 5]$ , which has an absolute maximum at 2, has an absolute minimum at 5, and for which 4 is a critical number but neither a local maximum nor local minimum.



4. Find the absolute maximum and minimum values of  $f$  on the given interval:

$$f(x) = 2x^3 - 3x^2 - 12x + 1, \quad [-2, 3]$$

$$\begin{aligned} f'(x) &= 6x^2 - 6x - 12 \\ &= 6(x^2 - x - 2) \\ &= 6(x - 2)(x + 1) = 0 \\ x &= 2, x = -1 \end{aligned}$$

$x$	$f(x)$
-2	-3
-1	8
2	$16 - 12 - 24 + 1 = -19$
3	-8

abs. max.

abs. min.

5. Find the absolute maximum and minimum values of  $f$  on the given interval:

$$f(x) = x^{-2} \ln x, \quad \left[\frac{1}{2}, 4\right]$$

$$\begin{aligned} f'(x) &= -2x^{-3} \ln x + x^{-2} \cdot \frac{1}{x} \\ &= \frac{-2 \ln x}{x^3} + \frac{1}{x^3} = \frac{-2 \ln x + 1}{x^3} = 0 \end{aligned}$$

$$-2 \ln x + 1 = 0$$

$$\ln x = \frac{1}{2}$$

$$x = e^{1/2} \approx 1.5$$

$x$	$f(x)$
$\frac{1}{2}$	$4 \cdot \ln\left(\frac{1}{2}\right) = -4 \ln 2$
$e^{1/2}$	$e^{-1} \cdot \frac{1}{2} = \frac{1}{2e}$
4	$\frac{\ln(4)}{16} = \frac{\ln(2)}{8}$

abs. min.

abs. max.

6. Find the critical numbers of the function:

$$h(p) = \frac{p-1}{p^2+4}$$

$$h'(p) = \frac{1 \cdot (p^2+4) - (p-1)2p}{(p^2+4)^2} = \frac{p^2+4-2p^2+2p}{(p^2+4)^2} = 0$$

$$\Leftrightarrow -p^2 + 2p + 4 = 0$$

$$\Leftrightarrow p = \frac{-2 \pm \sqrt{4+16}}{-2} = \frac{-2 \pm 2\sqrt{5}}{-2} = 1 \pm \sqrt{5}$$

never 0

Critical numbers are  
 $C = 1 + \sqrt{5}, 1 - \sqrt{5}$