Assignment #5

Due Tuesday, 8 October 2019, at the start of class

This Assignment is based on Chapter 7 material only.

Remember that when you turn in homework problems involving MATLAB/OCTAVE, the following two expectations always apply:

1. The commands that you used must be shown, along with the results.
2. Please strive to minimize use of paper while answering the question.

Do the following exercises:

CHAPTER 7

- Exercise A. **(a)** Consider the system

$$\begin{aligned} 2x_1 + 3x_2 - x_3 &= 5 \\ 4x_1 - 3x_2 + 2x_3 &= 1 \\ 2x_1 + x_2 + x_3 &= 3 \end{aligned}$$

Perform Gaussian elimination *by hand* to transform it into an upper triangular system. Then do back substitution *by hand*. (*Do not swap rows. Full credit requires that the elimination and substitution stages are clear.*)

(b) The system has abstract form $A\mathbf{x} = \mathbf{b}$. Enter A and $\mathbf{b}$ into MATLAB and confirm your by-hand solution by `x = A \ b`.

(c) Let M be the augmented matrix $M = \left[A\middle|b\right]$ and define

$$L_1 = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}, \qquad L_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -\frac{2}{9} & 1 \end{bmatrix}.$$

Confirm *by hand* that $L_2 L_1 M$ is the upper triangular system solved in **(a)**.

The computations in the remaining parts should use MATLAB.

(d) Compute $U = L_2 L_1 A$ and $L = (L_2 L_1)^{-1}$. Confirm that $LU = A$.

(e) In section 7.2 it says "…to solve $A\mathbf{x} = LU\mathbf{x} = \mathbf{b}$, one first solves the lower triangular system $L\mathbf{y} = \mathbf{b}$ (to obtain $\mathbf{y} = U\mathbf{x}$) and then the upper triangular system $U\mathbf{x} = \mathbf{y}$." Using U and L from part **(d)**, confirm that you get the same answer this way as in **(a)**.

- Exercise B. **(a)** Open a new m-file called `gebasic.m`. Type in the code from section 7.2, at the top of page 137, labeled "`Gaussian elimination without pivoting`". You don't need to type in the comments, but be careful with the loop indices and other details. Note that, for the code to run, the variables `n,A,b` must be defined and of the right size.

  (b) Consider the 4×4 linear system $A\mathbf{x} = \mathbf{b}$:

$$\begin{aligned} x_1 + 2x_2 + 3x_3 + 4x_4 &= 7 \\ 2x_1 + x_2 - x_4 &= -1 \\ x_1 + x_4 &= 4 \\ 2x_2 - 2x_3 &= -8 \end{aligned}$$

  Run `gebasic.m` on this example. Show the resulting A and $\mathbf{b}$. (*These will be different from the ones you typed in!*)

  (c) As a result of part **(b)**, the system is now upper triangular. Solve it using MATLAB. (*That is, use the transformed* `A` *and* `b` *and solve the system in an efficient way.*)

  (d) Confirm using `A \ b`, on the *original* A and $\mathbf{b}$ that you have the correct solution in part **(c)**.

  (e) Assuming `x` is what you computed in **(c)**, and A and $\mathbf{b}$ are the original data, confirm using `norm(A*x - b)` that **(c)** was correct.

- Exercise 2 on page 175.
- Exercise 3 on pages 175–176.
- Exercise 4 on page 176.
- Exercise 6 on page 176.
- Exercise 8 on page 176.