Review Guide for Midterm II on Friday, 22 November 2019

This in-class Midterm Exam is (again) *closed book* and *closed notes*. Bring only a writing implement. I encourage you to get together and work through this Review Guide collaboratively. If you come prepared it will be easy.

The Exam will cover the content listed below from Carothers, *Real Analysis*. It will only cover topics that have appeared on homework or in lecture, or ones which are closely-related. You are responsible for, and should be able to prove unless otherwise stated, all propositions (lemmas, theorems, and corollaries) in the following material. Likewise you should know all the definitions, unless otherwise stated. Note that some important ideas/definitions/results are stated in the Exercises.

- **Chapters 1–8:** The following topics are unavoidable even though they were on Midterm I: *real numbers, countability, metric spaces, norms, convergence, open/closed sets, continuity, completeness*, and *compactness*. However, on this exam I will not ask questions merely to test these concepts.

- **Chapter 10:** The definitions of pointwise and uniform convergence, and spaces $B(X)$ and $C(X)$ for X a metric space, are all fundamental. Consider examples and counterexamples in these spaces. Know Theorem 10.4 (uniform limit of continuous functions is continuous), Theorem 10.5, and Lemma 10.9 (Weierstrauss M-test). Of the "Applications" please know 10.10 on power series and 10.13 on a nowhere-differentiable function; I will not ask about Applications 10.11 or 10.12.
- **Chapter 11:** The Weierstrauss polynomial approximation Theorem 11.3, and associated tools (Lemmas/Theorems 11.1, 11.2, 11.4, 11.5), plus Application 11.6, are the important content. Both the statement and proof of Theorem 11.3 may be asked about, though I may remind you of Bernoulli polynomial facts on the exam. Material on trigonometric polynomials, Weierstrauss' second theorem, infinitely-differentiable functions, equicontinuity, and category can all be skipped.
- **David's notes on the Riemann integral:** *All* of these notes are fair game. Know the definitions of partitions, step functions, and the Riemann integral. Know basic facts: continuous functions are Riemann integrable, the integral is linear, and the integral is additive over intervals. (You may want to read Chapter 14, but Stieltjes ideas were not covered in lecture and will not be on the exam.)
- **Chapter 16:** Everything in Chapter 16 is fair game with the exception of the Vitali Covering Theorem 16.27. Everything else is important: reread the introductory material on pages 263–268, know the definition of outer measure m^*, know how to prove its major properties (Propositions/Corollaries 16.2–16.9), know the definition of measurable sets and basic properties (Lemmas/Corollary 16.14–16.17), and know countable additivity Theorem 16.18. Know why the set $\mathcal{M}$ of measurable sets is a

σ-algebra (esp. Corollary 16.20), and the definition of Lebesgue measure m. Understand material on the structure of measurable sets (Theorem 16.21–Corollary 16.26), including the idea that $\mathcal{M}$ is *complete* (page 284), but note that we have skipped "G_δ" and "F_σ" stuff. Look at the Cantor set construction in Chapter 2, as an example of a nontrivial (e.g. uncountable) measure zero set. Know how the nonmeasurable set N is defined and why it is nonmeasurable (Theorem 16.31), and about the other definitions of measurability (pages 292–293).

- **Chapter 17:** Everything in this Chapter is fair game, except for Theorem 17.4 and Corollary 17.5. Highlights include the definition of measurable functions, the categorization on bottom of page 298, the proof of Theorem 17.7, the definition of a simple function, the way ∞ is handled (pages 302–303), the fact that the set of measurable function is closed under (sequential) supremum/infimum/limit, the basic construction 17.14, and the approximation of simple and measurable functions by continuous functions.
- **Chapter 18:** Everything through Corollary 18.12 is fair game. In particular, know the definitions of the Lebesgue integral for simple functions, nonnegative measurable functions, and integrable (measurable) functions. Know the Monotone Convergence Theorem 18.7, and why/how a nonnegative function f defines a measure on $\mathcal{M}$ by $\mu_f(E) = \int_E f$.