SOLUTIONS

Math F251: Section 2.6 Worksheet 3 February 2020

1. Compute the following limits or state that they do not exist. (*THINK before doing algebra!*)

(a) $\lim\limits_{x\to-\infty} \dfrac{2x-2}{x^2+1} \boxed{\frac{\infty}{\infty}} = \lim\limits_{x\to-\infty} \dfrac{(2x-2)\frac{1}{x^2}}{(x^2+1)\frac{1}{x^2}} = \lim\limits_{x\to-\infty} \dfrac{\frac{2}{x}-\frac{2}{x^2}}{1+\frac{1}{x^2}} = \dfrac{0-0}{1+0} = 0$

(b) $\lim\limits_{x\to0} \dfrac{2x-2}{x^2+1} = \dfrac{2\cdot 0-2}{0+1} = -2$

(c) $\lim\limits_{x\to\infty} \dfrac{x^2-2}{x^2+1} \boxed{\frac{\infty}{\infty}} = \lim\limits_{x\to\infty} \dfrac{x^2-2}{x^2+1}\,\dfrac{\frac{1}{x^2}}{\frac{1}{x^2}} = \lim\limits_{x\to\infty} \dfrac{1-\frac{2}{x^2}}{1+\frac{1}{x^2}} = \dfrac{1-0}{1+0} = 1$

(d) $\lim\limits_{x\to\infty} \dfrac{2}{x} - \dfrac{1}{\ln x} = 0 - 0 = 0$

(e) $\lim\limits_{x\to-\infty} \dfrac{x^2}{\sqrt{x^4+1}} \boxed{\frac{\infty}{\infty}} = \lim\limits_{x\to-\infty} \dfrac{x^2}{\sqrt{x^4+1}}\,\dfrac{\frac{1}{x^2}}{\frac{1}{x^2}} = \lim\limits_{x\to-\infty} \dfrac{1}{\sqrt{1+\frac{1}{x^4}}} = \dfrac{1}{\sqrt{1+0}} = 1$

(f) $\lim\limits_{x\to2} \dfrac{x^2-4}{2x^2-3x-2} \boxed{\frac{0}{0}} = \lim\limits_{x\to2} \dfrac{\cancel{(x-2)}(x+2)}{\cancel{(x-2)}(2x+1)} = \dfrac{2+2}{2\cdot2+1} = \dfrac{4}{5}$

(g) $\lim\limits_{t\to\infty} \sqrt{t^2+at} - \sqrt{t^2+bt} \boxed{\infty-\infty} = \lim\limits_{t\to\infty} \sqrt{t^2+at} - \sqrt{t^2+bt}\;\dfrac{\sqrt{t^2+at}+\sqrt{t^2+bt}}{\sqrt{t^2+at}+\sqrt{t^2+bt}}$

$= \lim\limits_{t\to\infty} \dfrac{(t^2+at)-(t^2+bt)}{\sqrt{t^2+at}+\sqrt{t^2+bt}} = \lim\limits_{t\to\infty} \dfrac{(a-b)t}{\sqrt{t^2+at}+\sqrt{t^2+bt}}\,\dfrac{\frac{1}{t}}{\frac{1}{t}} = \lim\limits_{t\to\infty} \dfrac{a-b}{\sqrt{1+\frac{a}{t}}+\sqrt{1+\frac{b}{t}}}$

$= \dfrac{a-b}{2}$

(h) $\lim\limits_{x\to\infty} \dfrac{1-e^x}{1+2e^x} \boxed{\frac{\infty}{\infty}} = \lim\limits_{x\to\infty} \dfrac{1-e^x}{1+2e^x}\,\dfrac{e^{-x}}{e^{-x}} = \lim\limits_{x\to\infty} \dfrac{e^{-x}-1}{e^{-x}+2}$

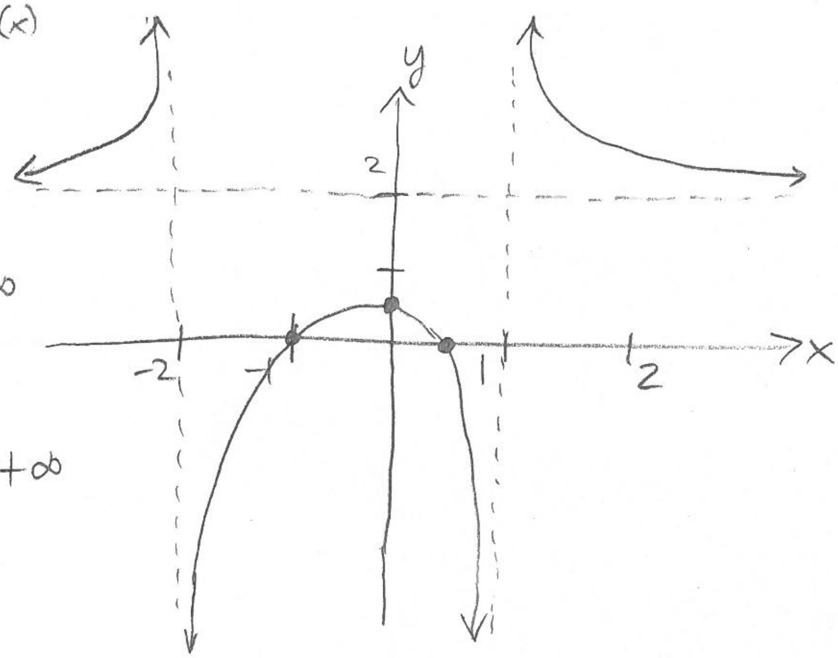
$= \dfrac{0-1}{0+2} = -\dfrac{1}{2}$

2. Find all the vertical and horizontal asymptotes of the graph

$$y = \frac{2x^2 + x - 1}{x^2 + x - 2},$$

and clearly state limits which justify these asymptotes. Also make a rough sketch of the graph. (*Confirm your work by graphing calculator or Desmos etc.?*)

$$y = \frac{(2x-1)(x+1)}{(x+2)(x-1)} = f(x)$$



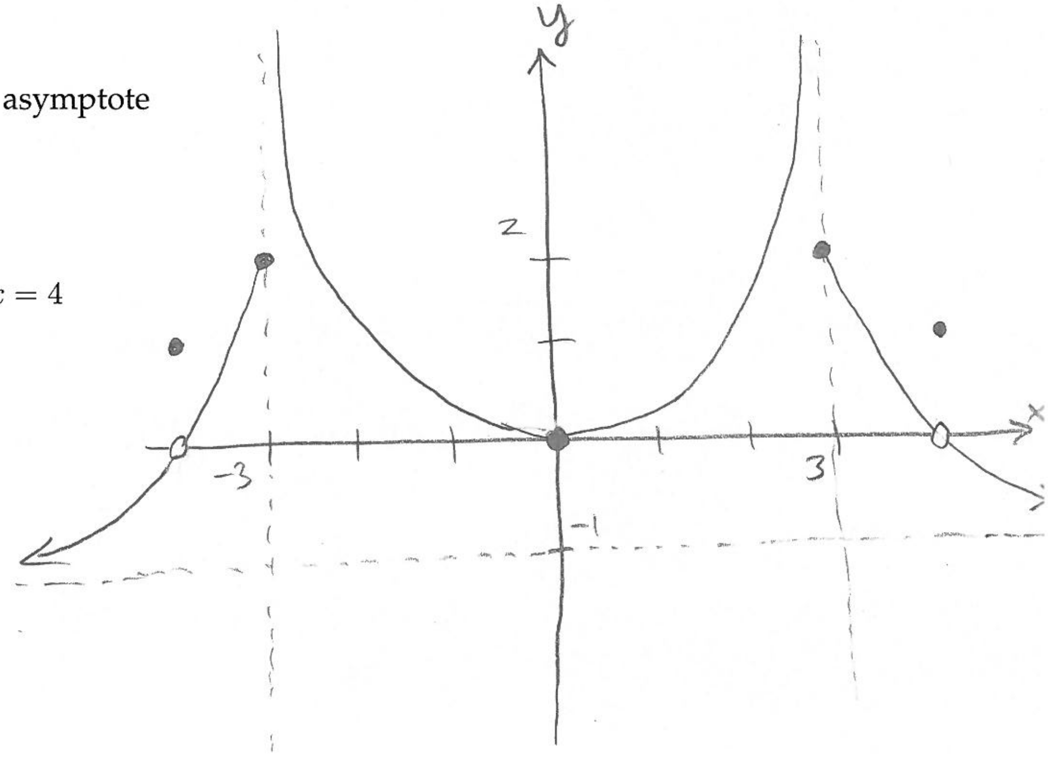
$\underline{y = 2 \text{ is hor.}}$ $\lim_{x \to \infty} f(x) = 2$

$\underline{x = -2 \text{ is vert.}}$ $\lim_{x \to -2^+} f(x) = -\infty$

$\underline{x = +1 \text{ is vert.}}$ $\lim_{x \to 1^+} f(x) = +\infty$

3. Sketch the graph of a function that satisfies all of the given conditions:

- f is even
- $f(0) = 0$
- $y = -1$ is a horizontal asymptote
- $\lim_{x \to 3+} = 2$
- $\lim_{x \to 3-} f(x) = \infty$
- f is discontinuous at $x = 4$



answers will vary!