Assignment #6

(All Problems Due Friday 10/26/01.)

Section 3.5, # 23a.

Section 3.5, # 23b.

Section 3.5, # 23c.

Section 3.5, # 23d.

Section 3.5, # 24.

Section 3.5, # 25.

Section 3.6, # 30.

Additional IX. Suppose that $f: D \to \mathbf{R}^*$ is measurable. Suppose that $g: D \to \mathbf{R}^*$ has the property that for all $\epsilon > 0$, $m\{x \in D: |f(x) - g(x)| > \epsilon\} = 0$. Then g = f a.e. and g is measurable.

Additional X. (a) Show that if $f: D \to \mathbf{R}$ is measurable and $\{x \in D: f(x) = 0\} = \emptyset$ then $\frac{1}{f}$ is measurable.

- (b) Extend (a): If $f^{-1}(\{0\})$ is a set of measure zero, then $\frac{1}{f}$ is measurable. That is, show that any function $h: D \to \mathbf{R}^*$ such that $h(x) = \frac{1}{f(x)}$ for all x such that $f(x) \neq 0$ is measurable.
- (c) Show that if $f, g: D \to \mathbf{R}^*$ are measurable and $m(g^{-1}(\{0\})) = 0$ then $\frac{f}{g}$ is measurable.