(SOLUTIONS)

1. The rate of change of atmospheric pressure P with respect to altitude h is proportional to P. (This assumes the temperature is constant; let us assume that.)

(a) Write a differential equation corresponding to the first sentence above; use k for the constant of proportionality. Then write a formula for $P(h)$ in terms of $P(0)$, k, and h.

$$\frac{dP}{dh} = kP, \quad P(h) = P(0)e^{kh}$$

(b) At a temperature of $15\,°C$, the pressure is 101.3 kPa at sea level and the pressure is 87.14 kPa at $h = 1000$ m. From these facts, determine $P(0)$ and k.

$$P(0) = 101.3 \text{ kPa}$$

$$87.14 = P(1000) = 101.3\, e^{k(1000)} \iff k = \frac{1}{100} \ln\left(\frac{87.14}{101.3}\right)$$

$$k = -1.5057 \times 10^{-4} \quad \left[\text{units are } \tfrac{1}{m}\right]$$

(c) What is the pressure at the top of Denali, at an altitude of 6187 m? (*The problem in the book, #19 in §3.8, has an error. It calls it "Mount McKinley."*)

$$P(6187) = 101.3\, e^{(-1.5057\times10^{-4})(6187)}$$

$$= 39.9 \text{ kPa}$$

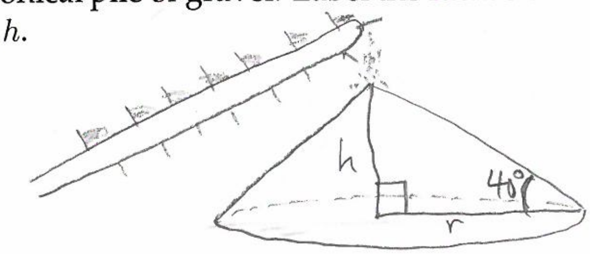
(d) At what altitude is the pressure 1/3 of what it is at sea level?

$$\frac{1}{3}\cancel{P(0)} = P(h) = \cancel{P(0)}\, e^{(-1.5057\times10^{-4})h}$$

$$h = \frac{1}{-1.5057\times10^{-4}} \ln\left(\frac{1}{3}\right) = 7296 \text{ m}$$

2. Gravel can be made by crushing rock and then running it through a screen for sorting. Typically the sorted gravel is piled into a cone by a conveyor belt. Because the gravel slides down the sides as the pile steepens, the sides alway have about the same angle (the *angle of repose*) and the pile keeps its shape as it grows.

(a) Draw a conveyor belt feeding a conical pile of gravel. Label the radius of the base of the cone as r and its height as h.



(b) The volume of a cone is

$$V = \frac{1}{3}\pi r^2 h.$$

As the pile grows, which of the variables in this equation depend on time?

V, r, h all depend on t:

$$V(t) = \frac{1}{3}\pi\, r(t)^2\, h(t)$$

(c) Compute dV/dt by differentiating the above equation, keeping in mind that the other variables are also functions of time.

$$\frac{dV}{dt} = \frac{1}{3}\pi\left(2r\frac{dr}{dt}h + r^2\frac{dh}{dt}\right)$$

$\frac{dV}{dt} = 5$

(d) If the conveyor belt is adding 5 m^3/min of gravel to the pile, and the angle of the sides of the pile is 40°, at what rate is the height increasing when the base has radius 20 m?

want $\frac{dh}{dt}$

$$\frac{h(t)}{r(t)} = \tan(40^\circ) \iff h(t) = 0.83910\, r(t)$$

$$\frac{dh}{dt} = 0.83910\frac{dr}{dt}$$

$$5 = \frac{1}{3}\pi\left(2\cdot 20\cdot\left(\frac{\frac{dh}{dt}}{0.83910}\right)\cdot(0.83910(20)) + 20^2\frac{dh}{dt}\right)$$

$$= \left(\frac{1}{3}\pi(800+400)\right)\frac{dh}{dt}$$

$$\frac{dh}{dt} = \frac{15}{1200\pi} = 0.00398\ \frac{\text{m}}{\text{min}}$$