

1. Sketch the graph of a function that satisfies all of the given conditions:

- $\lim_{x \rightarrow \infty} f(x) = 3$
- $\lim_{x \rightarrow 2^-} f(x) = \infty$
- $\lim_{x \rightarrow 2^+} f(x) = -\infty$
- f is odd

2. Find all the vertical and horizontal asymptotes of the graph

$$y = \frac{2x^2 + x - 1}{x^2 + x - 2},$$

and clearly state limits which justify these asymptotes. (*Also make a rough sketch of the graph. You may be able to confirm your work by graphing calculator.*)

3. Show that f is continuous on $(-\infty, \infty)$, and sketch the graph:

$$f(x) = \begin{cases} \sin x & \text{if } x < \pi/4 \\ \cos x & \text{if } x \geq \pi/4 \end{cases}$$

4. Prove that the equation has at least one real root:

$$\ln x = 3 - 2x$$

(A calculator can help find an accurate approximation, but this is not required!)

5. *A challenge problem, but actually easy. It follows from the Intermediate Value Theorem. Start by sketching elevation versus time for each day, one on top of the other.*

A Tibetan monk leaves the monastery at 7:00 AM and takes his usual path to the top of the mountain, arriving at 7:00 PM and sleeping on top. The next morning he starts at 7:00 AM at the top and takes the same path back, arriving at the monastery at 7:00 AM. Show that there is a point on the path that the monk will cross at exactly the same time of day on both days.