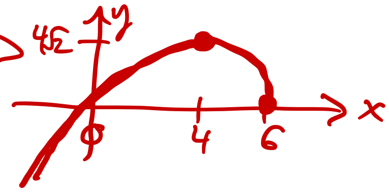
1.

$$F(x) = x\sqrt{6-x}$$



(a) What is the domain of $F(x)$? $(-\infty, 6]$

(b) Find the intervals of increase or decrease and critical numbers.
increase: $(-\infty, 4]$
decrease: $[4, 6]$
crit. numbers: $x = 4, 6$

(c) Find the intervals of concavity and the inflection points.
concave down $(-\infty, 6)$
never concave up
no inflection points

(d) Sketch the graph.

$$F'(x) = 1\cdot\sqrt{6-x} + x\cdot\tfrac{1}{2}(6-x)^{-1/2}(-1)$$

$$= (6-x)^{-1/2}\left((6-x) - \tfrac{x}{2}\right) \quad \leftarrow \text{factor lowest power of } (6-x)$$

$$= \frac{6-\frac{3}{2}x}{\sqrt{6-x}}$$

organize:

x	F	F'	F''
0	0		
4	$4\sqrt{2}$	0	
6	0	d.n.e.	d.n.e.

$$F''(x) = \frac{(-\frac{3}{2})(6-x)^{1/2} - (6-\frac{3}{2}x)\frac{1}{2}(6-x)^{-1/2}(-1)}{6-x}$$

$$= \frac{(6-x)^{-1/2}\left[-\frac{3}{2}(6-x) + \frac{1}{2}(6-\frac{3}{2}x)\right]}{6-x} = \frac{\frac{3}{4}x - 6}{(6-x)^{-3/2}}$$

2.

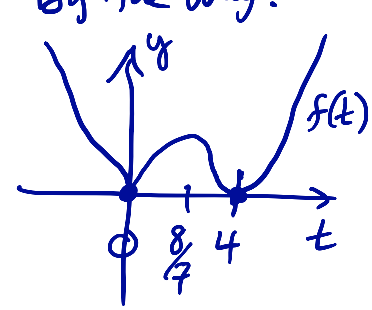
$$f(t) = t^{4/5}(t-4)^2$$

(a) What is the domain of $f(t)$? $(-\infty, \infty)$

(b) Find $f'(t)$. What is its domain? formula below; $(-\infty, 0)\cup(0,\infty)$

(c) Find all the critical numbers. $c = 0, 8/7, 4$

by the way:



$$f'(t) = \tfrac{4}{5}t^{-1/5}(t-4)^2 + t^{4/5}\,2(t-4)$$

factor lowest powers

$$= t^{-1/5}(t-4)\left[\tfrac{4}{5}(t-4) + 2t\right]$$

$$= \frac{(t-4)\left(\frac{14}{5}t - \frac{16}{5}\right)}{t^{1/5}}$$

$\therefore$ $f'(c) = 0$ at $c = 4, 8/7$

$f'(c)$ d.n.e. at $c = 0$

3.

$$g(x) = \frac{e^x}{1-e^x}$$

(a) What is the domain of $g(x)$? $x \neq 0$ or $(-\infty,0)\cup(0,\infty)$ $\therefore$ $x=0$ is vertical

(b) Find the horizontal and vertical asymptotes. ⟵ §4.4 *helps with this kind of limit* ∴ *you can do it anyway!*

$\lim_{x\to 0^-} g(x) = +\infty$, $\lim_{x\to\infty} g(x) = -1$, $\lim_{x\to-\infty} g(x) = 0$

$y=0$ is hor.
$y=-1$ is hor.

(c) Find the intervals of increase or decrease and critical numbers. no critical #s, increase: $(-\infty,0)\cup(0,\infty)$, never decreasing

(d) Find the intervals of concavity and the inflection points. concave up on $(-\infty,0)$, concave down on $(0,\infty)$, no infl. pts

(e) Sketch the graph.

$$g'(x) = \frac{e^x(1-e^x) - e^x(-e^x)}{(1-e^x)^2} = \frac{e^x}{(1-e^x)^2}$$

$\therefore g'(c)=0$ has no sol'ns and $g'(x)>0$ always

$$g''(x) = \frac{e^x(1-e^x)^2 - e^x 2(1-e^x)(-e^x)}{(1-e^x)^4} = \frac{e^x(1-e^x) + 2e^x e^x}{(1-e^x)^3}$$

$$= \frac{e^x(1+e^x)}{(1-e^x)^3}$$

$\therefore g''(c)=0$ has no solutions and $g'(x)>0$ if $x<0$, $g'(x)<0$ if $x>0$

x	g	g'	g''
-1	$+0.6$	$+$	$+$
0	d.n.e.	d.n.e.	d.n.e.
1	-1.6	$+$	$-$