SOLUTIONS



**1.** Find an equation of the tangent line to the curve at the given point.

$y = \sqrt{1+x^3}, \quad (2,3)$

$$\frac{dy}{dx} = \frac{1}{2}(1+x^3)^{-1/2}(3x^2)$$
$$= \frac{3}{2}x^2(1+x^3)^{-1/2}$$

$$m = \left.\frac{dy}{dx}\right|_{x=2} = \frac{3}{2}\cdot 2^2 \cdot (1+2^3)^{-1/2}$$
$$= 6\cdot 9^{-1/2} = \frac{6}{3} = 2$$

$$\boxed{y-3=2(x-2)}$$

**2.** If $F(x) = f(g(x))$, and if $f(-2) = 8$, $f'(-2) = 4$, $f'(5) = 3$, $g(5) = -2$, and $g'(5) = 6$, find $F'(5)$.

$$F'(x) = f'(g(x))\,g'(x)$$
$$F'(5) = f'(g(5))\,g'(5)$$
$$= f'(-2)\cdot 6 = 4\cdot 6 = \boxed{24}$$

**3.** Find the 49th derivative of $f(x) = x\,e^{-x}$.

$$f'(x) = 1\cdot e^{-x} + x\cdot e^{-x}(-1) = (1-x)e^{-x}$$
$$f''(x) = (0-1)e^{-x} + (1-x)e^{-x}(-1) = (-1-1+x)e^{-x} = (-2+x)e^{-x}$$
$$f'''(x) = (0+1)e^{-x} + (-2+x)e^{-x}(-1) = (1+2-x)e^{-x} = (3-x)e^{-x}$$

[see pattern now!]

$$f^{(4)}(x) = (-4+x)e^{-x}$$
$$\vdots$$

$$f^{(49)}(x) = \boxed{(49-x)e^{-x}}$$

**4.** Find the derivative of the function. You do not need to simplify your answer.

(a) $y = \left(x + \frac{1}{x}\right)^7$

$$\frac{dy}{dx} = \boxed{7\left(x + x^{-1}\right)^6 \left(1 - x^{-2}\right)}$$

(b) $f(\theta) = \cos(\theta^2)$

$$f'(\theta) = -\sin(\theta^2) \cdot 2\theta = \boxed{-2\theta \sin(\theta^2)}$$

(c) $g(t) = 2^{t^3}$

$$g'(t) = 2^{(t^3)} \ln 2 \cdot (3t^2) = \boxed{3 \ln 2\, t^2\, 2^{t^3}}$$

(d) $y = \sqrt{x + \sqrt{x + \sqrt{x}}}$ $= \left(x + \left(x + x^{\frac{1}{2}}\right)^{\frac{1}{2}}\right)^{\frac{1}{2}}$

$$\frac{dy}{dx} = \boxed{\frac{1}{2}\left(x + \left(x + x^{\frac{1}{2}}\right)^{\frac{1}{2}}\right)^{-\frac{1}{2}} \cdot \left(1 + \frac{1}{2}\left(x + x^{\frac{1}{2}}\right)^{-\frac{1}{2}} \left(1 + \frac{1}{2}x^{-\frac{1}{2}}\right)\right)}$$