Assignment #5

Due Friday, 28 February 2020, at the start of class

At this point you should have read all of Chapter 8 of the textbook, and be comfortable with this material. Please read Chapter 9; it contains continuing details on the ℓ^p and L^p spaces which we have already been treating as examples.

One exercise below is identified with your initials. Please LaTeX this problem and send both the `.tex` and `.pdf` to me at `elbueler@alaska.edu` by the due date.

DO THE FOLLOWING EXERCISES from the textbook (Muscat, *Functional Analysis*, 2014):

- #5 in Exercises 8.14, pages 129–130. *Note "well-posed" is defined on page 128. You should assume that T is an operator, i.e. $T \in B(X, Y)$ for normed vector spaces, and that $y \neq 0$ and $y + \delta y \neq 0$.*
- #1 in Exercises 8.21, page 134.
- #3 in Exercises 9.4, page 143. $\longleftarrow$ **DD** *You are showing that ℓ^∞ is a commutative Banach algebra; see the definition on the first page of Chapter 13.*
- #4 in Exercises 9.4, page 143. $\longleftarrow$ **OS** *The word "embed" is defined on page 128. Note $B(\ell^1)$ is a noncommutative Banach algebra.*
- #7 in Exercises 9.4, page 143. $\longleftarrow$ **WV** *Start by recalling the definition of $d(x, M)$, namely when $x \in X$, $M \subset X$ is a subspace, and X is a normed vector space.*
- #3 in Exercises 9.7, page 146. *The convolution product is defined in the previous exercise.*
- #3 in Exercises 9.10, page 148.
- #2 in Exercises 9.15, page 153. *Hint: Make sure $\mathbf{x} \cdot \mathbf{y}$ is a sum of positive terms* and *make sure $\mathbf{x} \in \ell^p$.*
- #3 in Exercises 9.15, page 153. *Prove only the first inequality. (The second one is too obscure.)*
- #1 in Exercises 9.23, page 153.