SOLUTIONS

1. Evaluate the integral by making the given substitution.

(a) $u = \sin\theta$:

$$\int \sin^2\theta \cos\theta \, d\theta =$$

$u = \sin\theta$

$du = \cos\theta \, d\theta$

$= \int u^2 \, du$

$= \frac{1}{3}u^3 + C = \frac{1}{3}\sin^3\theta + C$

(b) $u = x^4 - 5$:

$$\int \frac{x^3}{x^4 - 5} \, dx =$$

$u = x^4 - 5$

$du = 4x^3 \, dx$

$\frac{du}{4} = x^3 \, dx$

$= \int \frac{du/4}{u}$

$= \frac{1}{4}\int \frac{du}{u} = \frac{1}{4}\ln|u| + C$

$= \frac{1}{4}\ln|x^4 - 5| + C$

2. Evaluate the indefinite integral by substitution. What should you choose as u?:

$$\int e^x\sqrt{1 + e^x} \, dx =$$

$u = 1 + e^x$

$du = e^x \, dx$

$= \int \sqrt{u} \, du = \int u^{1/2} \, du$

$= \frac{2}{3}u^{3/2} + C$

$= \frac{2}{3}(1 + e^x)^{3/2} + C$

3. Evaluate the indefinite integrals:

(a)

$$\int 5^t \sin(5^t)\,dt = \int \sin(u)\,\frac{du}{\ln 5}$$

$u = 5^t$

$du = (\ln 5)5^t\,dt$

$\frac{du}{\ln 5} = 5^t\,dt$

$$= \frac{1}{\ln 5}(-\cos(u)) + C$$

$$= -\frac{1}{\ln 5}\cos(5^t) + C$$

(b)

$$\int \frac{x}{1+x^4}\,dx =$$

$u = x^2$

$du = 2x\,dx$

$\frac{du}{2} = x\,dx$

$$= \int \frac{du/2}{1+u^2} = \frac{1}{2}\int \frac{du}{1+u^2}$$

$$= \frac{1}{2}\arctan(u) + C = \frac{1}{2}\arctan(x^2) + C$$

4. Evaluate the definite integrals:

(a)

$$\int_0^1 (3t-1)^{50}\,dt = \int_{-1}^{2} u^{50} \cdot \frac{du}{3}$$

$u = 3t-1$

$du = 3\,dt$

$\frac{du}{3} = dt$

$$= \frac{1}{3}\left.\frac{u^{51}}{51}\right]_{-1}^{2} = \frac{2^{51}+1}{153}$$

(b)

$$\int_0^{\pi/2} \cos x\,\sin(\sin(x))\,dx =$$

$u = \sin(x)$

$du = \cos(x)\,dx$

$$= \int_0^1 \sin(u)\,du = -\cos(u)\Big]_0^1$$

$$= -\cos(1) + \cos(0) = 1 - \cos(1)$$