Review Guide for In-Class Midterm Exam I on *Monday, 3 October 2016*

The first Midterm Exam will cover Chapters 1, 2, 3, 4, 5, and 6 in Sutherland, *Introduction to Metric & Topological Spaces*, 2nd edition. See below for excluded material that will *not* be on the Exam. The Exam is *closed-book and closed-notes*. The problems will be of these types: state definitions, prove propositions which follow reasonably directly from the definitions or from known facts (below), give examples with certain properties, or describe or illustrate (sketch) concepts/examples.

> *I encourage all students to get together and work through this Review Guide. Be honest with yourself about what you do and don't know, and talk it through with your friends!*
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> *This first in-class Exam requires you to become comfortable with the definitions. The questions will be straightforwardly-stated, such as "define* bounded set *for a metric space" or "prove that if $f : \mathbb{R} \to \mathbb{R}$ is continuous then $|f|$ is continuous" or similar. I am aware that you will have no book or notes during the exam, and so I will never ask you to "prove theorem 4.23" or anything like that.*

Excluded material. The following material will *not* be on the exam:

- pages 18–19, Chapter 4, on least upper bound (sup) and greatest lower bound (inf)
- page 44, Chapter 5, Examples 5.11 and 5.12
- pages 64–65, Chapter 6, on limit points
- pages 69–72, Chapter 6, on equivalent and Lipschitz-equivalent metrics and isometry

Definitions. Be able to state and use the definition:

- on page 5:
$$\bigcup_{i\in I} A_i, \qquad \bigcap_{i\in I} A_i, \qquad A \times B$$
- graph of f (page 6)
- composition of functions (page 6)
- injective (one-to-one), surjective (onto), bijective (page 6)
- image $f(A)$ and preimage (inverse image) $f^{-1}(C)$ (page 9)
- f invertible (page 13) and inverse function f^{-1} (page 14)
- real sequence (s_n) converges to $l \in \mathbb{R}$ (page 21)
- $\lim_{x\to a} f(x) = L$ for $f : \mathbb{R} \to \mathbb{R}$ (page 25)
- for $f : \mathbb{R} \to \mathbb{R}$ (page 28) or $f : \mathbb{R}^n \to \mathbb{R}$ (page 38): continuous at a, continuous
- metric $d : X \times X \to \mathbb{R}$ (page 38) and metric space (X, d) (page 39)
- discrete metric on any X (page 41)
- d_1, d_2, d_∞ on $\mathbb{R}^n$ (pages 40 and 42) and on $X \times Y$ (page 43)
- (A, d_A), a metric subspace for $A \subseteq A$ (page 43)
- for (X, d) a metric space:
  - $S \subseteq X$ bounded (page 50) and $f : S \to X$ bounded (page 50)
  - open ball $B_r(x_0)$ (page 51)

  - $U \subseteq X$ open in X (page 54)
  - $f : X \to Y$ continous (page 55)
  - $V \subseteq X$ closed in X (page 61)
- for $A \subseteq X$, where (X, d) is a metric space:
  - closure $\bar{A}$ (page 63)
  - A is dense in X (page 63)
  - interior $\mathring{A}$ (page 66)
  - boundary ∂A (page 67)
- for (X, d) a metric space:
  - sequence (x_n) converges to x (for $x, x_n \in X$; page 68)
  - sequence (x_n) is Cauchy (for $x_n \in X$; page 68)

Propositions. Understand and remember these as facts. You can use these facts as needed in proving other propositions, but mention it if so (e.g. "because a function is invertible if and only if it is bijective"). Be *able* to prove each of these, if requested, unless it is otherwise noted.

- De Morgan's laws (page 5)
- proposition 3.7 (page 11)
- propositions 3.18 and 3.19 (pages 13–14)
- proposition 4.6, corollary 4.7, remark 4.8 (pages 19–20) [***Proofs will not be requested.***]
- theorem 4.18 (page 24) [***Proof will not be requested.***]
- proposition 4.20 (page 25)
- propositions 4.31, 4.32, 4.33 (pages 30–32)
- lemma 5.15 (page 46) [***Proof will not be requested.***]
- propositions 5.17, 5.18, 5.19 (pages 49)
- proposition 5.26 (page 51)
- proposition 5.37 (page 55)
- propositions 5.39, 5.41 (pages 56–57)
- propositions 6.3, 6.4, 6.5 (pages 61–62)
- proposition 6.11 (page 63) [***Proofs of difficult parts will not be requested.***]
- proposition 6.21 (page 66) [***Proofs of difficult parts will not be requested.***]
- proposition 6.24 (page 67)
- proposition 6.28 (page 68)

Examples, counter-examples, and sketches. Be able to give an example, sketch, or Venn diagram as appropriate.

- images and preimages (pages 9–13)
- graphs and inverse functions (pages 10–15)
- limit of a real sequence (page 22) or real function (pages 25–26)
- continuity of a real function (page 29)
- triangle inequality in Euclidean space (page 39)
- open balls in $\mathbb{R}^2$ for metrics d_1, d_2, d_∞ (pages 42 and 52)