Assignment #3

Due Monday 26 September at the start of class

Please read Chapter 2 and section 3.1, pages 10–29 in the textbook (Nocedal & Wright).

Exercise 2.7

Exercise 2.10

Exercise 3.1 *Note:* I have programmed back-tracking line search and it is posted online: bueler.github.io/M661F16/matlab/bt.m You can call it from your codes.

Problem P6. **(a)** Suppose $r : \mathbb{R} \to \mathbb{R}$ is a continuous function. Assume there is an initial *bracket* $[a, b]$ satisfying $a < b$ and $r(a)r(b) < 0$. First, show that there is a solution x^* to $r(x) = 0$ on the interval $a \le x \le b$. Next, assuming also that there is only one such solution x^*, show that the *bisection algorithm* below generates a sequence of brackets and that it terminates with x satisfying

$$|x - x^*| \le \frac{1}{2^k}|b - a|.$$

(*Hint:* A sketch is useful in your solution, but there must be rigor in the words you write.) Last, explain in a sentence or two why "bisection gets one bit per iteration".

```
function x = bisection(r,a,b,k)
    for j = 1:k
        c = (a + b) / 2;
        if r(c) * r(a) < 0
            b = c;
        else
            a = c;
        end
    end
    x = c;
```

(b) Suppose $r : \mathbb{R} \to \mathbb{R}$ is a twice-continuously-differentiable function. Assume that *Newton's method*, the algorithm below, converges to a solution x^* of the equation $r(x) = 0$ when starting from initial iterate x_0. Assume also that $r'(x^*) \ne 0$. Show that there is $M \ge 0$ and J such that if $j > J$ then the iterates x_j from Newton's method satisfy

$$|x_{j+1} - x^*| \le M|x_j - x^*|^2.$$

(*Hint*: Use Taylor's theorem. But also: look up this famous proof.) Explain in a sentence or two why "after Newton gets close, the number of correct digits in x_j starts to double per iteration".

```
function x = newton(r,dr,x0,k)
    x = x0;
    for j = 1:k
        x = x - r(x) / dr(x)    % compute x_{j+1} from x_j
    end
```