

Matrix Norm Essentials

- Matrix norms have vector norm **properties** plus $\|AB\| \leq \|A\|\|B\|$.
- Only **four** norms to know: $\|\cdot\|_1$, $\|\cdot\|_2$, $\|\cdot\|_\infty$, and $\|\cdot\|_{\text{Frob}}$.
- Three have easy **formulas** ($1, \infty, \text{Frob}$).
- Three are **induced** from vector norms ($1, 2, \infty$).
- **Usually** $\rho(A) < \|A\|$ for all norms. But **always** $\rho(A) \leq \|A\|$ for any norm.
- $\|\cdot\|_2$ norm best for **hermitian**. One reason: if $A^* = A$ then $\rho(A) = \|A\|_2$.
- **Iteration** v, Av, A^2v, \dots converges if and only if $\rho(A) < 1$. Thus *if* $\|A\| < 1$ *then* convergence. Not conversely!