

1. Sketch the graph $y = f(x)$ of a function which has all of the following properties; do not worry about any *formula* for $f(x)$:

(a) $f(0) = 3$

(b) $\lim_{x \rightarrow 0} f(x) = 0$

(c) $\lim_{x \rightarrow -2^-} f(x) = 1$

(d) $\lim_{x \rightarrow -2^+} f(x) = -1$

(e) $\lim_{x \rightarrow 5^-} f(x)$ d.n.e.

(f) $\lim_{x \rightarrow 5^+} f(x) = -\infty$

(g) the domain of f is $(-\infty, \infty)$

2. Evaluate the limit, if it exists:

$$\lim_{h \rightarrow 0} \frac{(2+h)^3 - 8}{h} =$$

3. Evaluate the limit, if it exists:

$$\lim_{u \rightarrow 2} \frac{\sqrt{4u+1} - 3}{u-2} =$$

4. Evaluate the limit, if it exists:

$$\lim_{t \rightarrow 0} \left(\frac{1}{t\sqrt{1+t}} - \frac{1}{t} \right) =$$

5. Evaluate the limit, if it exists:

$$\lim_{x \rightarrow 3} \frac{\frac{1}{x} - \frac{1}{3}}{x - 3} =$$

6. Evaluate the limits, if they exist, and otherwise explain why they do not:

(a)

$$\lim_{x \rightarrow 0^-} \left(\frac{1}{x} - \frac{1}{|x|} \right) =$$

(b)

$$\lim_{x \rightarrow 0^+} \left(\frac{1}{x} - \frac{1}{|x|} \right) =$$

7. *Challenge problem.* Consider the following function:

$$f(x) = \begin{cases} 1, & \text{if } x \text{ is rational} \\ 0, & \text{if } x \text{ is irrational} \end{cases}$$

Evaluate the limit $\lim_{x \rightarrow 0} f(x)$ if it exists. If it does not exist, explain why.