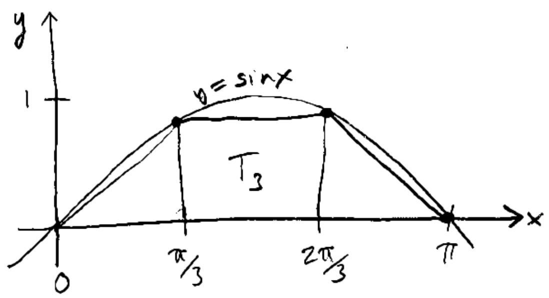
Solutions to Quiz # 5

1. (*I suggest first drawing a number line showing $a = 0$, $b = 4$, and the locations of the midpoints $\tilde{x}_i$.*)
Here $n = 4$ so $\Delta x = 1$ and $\tilde{x}_1 = 1/2$, $\tilde{x}_2 = 3/2$, $\tilde{x}_3 = 5/2$, $\tilde{x}_4 = 7/2$. Thus

$$\begin{aligned} M_4 &= \Delta x \left(f(\tilde{x}_1) + f(\tilde{x}_2) + f(\tilde{x}_3) + f(\tilde{x}_4) \right) \\ &= \frac{1}{2(1/2) + \cos(\pi/2)} + \frac{1}{2(3/2) + \cos(3\pi/2)} + \frac{1}{2(5/2) + \cos(5\pi/2)} + \frac{1}{2(7/2) + \cos(7\pi/2)} \\ &= \frac{1}{1+0} + \frac{1}{3+0} + \frac{1}{5+0} + \frac{1}{7+0} = \frac{1}{1} + \frac{1}{3} + \frac{1}{5} + \frac{1}{7} = \frac{176}{105} \end{aligned}$$

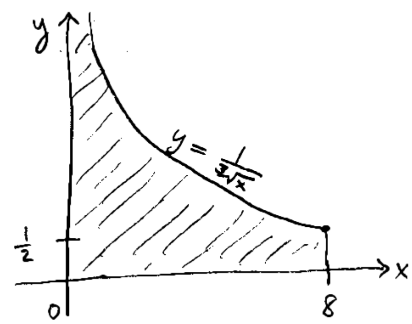
We have used the fact that $\cos\theta = 0$ if θ is an odd multiple of $\pi/2$.

2. T_3 is an underestimate.



3. (a)

$$\int_0^8 \frac{1}{\sqrt[3]{x}}\,dx = \lim_{t\to 0^+} \int_t^8 \frac{1}{\sqrt[3]{x}}\,dx$$



3. (b)

$$\int_0^8 \frac{1}{\sqrt[3]{x}}\,dx = \lim_{t\to 0^+} \int_t^8 x^{-1/3}\,dx = \lim_{t\to 0^+} \left[\frac{3}{2} x^{2/3}\right]_t^8 = \frac{3}{2} \lim_{t\to 0^+} \left[4 - t^{2/3}\right] = \frac{3}{2}\,[4 - 0] = 6$$

4. Use $u = x^3$:

$$\int_1^\infty x^2 e^{-x^3}\,dx = \lim_{t\to\infty} \int_1^t x^2 e^{-x^3}\,dx = \lim_{t\to\infty} \int_1^{t^3} e^{-u}\,\frac{du}{3} = \frac{1}{3} \lim_{t\to\infty} \left[-e^{-u}\right]_1^{t^3} = \frac{1}{3} \lim_{t\to\infty} \left[-e^{-t^3} + e^{-1}\right] = \frac{1}{3e}$$