

Name: _____

SOLNS

_____/ 24

30 minutes maximum. 24 points possible; each part is worth 2 points. No aids (book, calculator, etc.) are permitted. Show all work and use proper notation for full credit. Answers should be in reasonably-simplified form.

1. [12 points] Compute the derivatives of the following functions.

a. $f(x) = \frac{e^x}{x^3}$

$$f'(x) = \frac{e^x x^3 - e^x 3x^2}{x^6}$$

$$= \frac{e^x}{x^4} (x - 3)$$

b. $f(x) = (\ln(x^2 + e^2))^5$

$$f'(x) = 5(\ln(x^2 + e^2))^4 \frac{1}{x^2 + e^2} 2x$$

$$= \frac{10x (\ln(x^2 + e^2))^4}{x^2 + e^2}$$

c. $f(x) = a^{\sin(x)}$ where a is a constant, $a > 1$

$$f'(x) = \ln(a) a^{\sin(x)} \cos(x)$$

d. $f(x) = \sec\left(\frac{x}{x+1}\right)$

$$f'(x) = \sec\left(\frac{x}{x+1}\right) \tan\left(\frac{x}{x+1}\right) \frac{x+1-x}{(x+1)^2}$$

$$= \frac{\sec\left(\frac{x}{x+1}\right) \tan\left(\frac{x}{x+1}\right)}{(x+1)^2}$$

e. $f(x) = e^{\pi x+1} + \sqrt{3} \tan(\pi x)$

$$f'(x) = \pi e^{\pi x+1} + \sqrt{3} \sec^2(\pi x) \pi$$

$$= \pi (e^{\pi x+1} + \sqrt{3} \sec^2(\pi x))$$

f. Find $\frac{dy}{dx}$ if $2x + y = \cos(xy)$. (You must solve for $\frac{dy}{dx}$.)

$$2 + \frac{dy}{dx} = -\sin(xy) (1 \cdot y + x \cdot \frac{dy}{dx})$$

$$\frac{dy}{dx} (1 + x \sin(xy)) = -y \sin(xy) - 2$$

$$\frac{dy}{dx} = - \frac{y \sin(xy) + 2}{1 + x \sin(xy)}$$

2. [12 points] Compute the following definite integrals and antiderivatives (indefinite integrals). Remember that antiderivatives need a "+C".

$$\begin{aligned} \text{a. } \int_1^2 \frac{2+x^3}{x^2} dx &= \int_1^2 2x^{-2} + x dx \\ &= \left[2x^{-1} + \frac{x^2}{2} \right]_1^2 = \left(-\frac{2}{2} + 2 \right) - \left(-2 + \frac{1}{2} \right) \\ &= -1 + 2 + 2 - \frac{1}{2} = \left(\frac{5}{2} \right) \end{aligned}$$

$$\begin{aligned} \text{b. } \int \frac{e^{3x}}{\sqrt{5+e^{3x}}} dx & \quad \left\{ \begin{array}{l} u = 5 + e^{3x} \\ du = 3e^{3x} dx \\ \frac{du}{3} = e^{3x} dx \end{array} \right. \\ &= \int \frac{du/3}{\sqrt{u}} \\ &= \frac{1}{3} \int u^{-1/2} du = \frac{1}{3} 2u^{1/2} = \left(\frac{2}{3} (5+3x)^{1/2} + C \right) \end{aligned}$$

$$\begin{aligned} \text{c. } \int \frac{1}{x} + \sec(x) \tan(x) dx \\ &= \ln|x| + \sec(x) + C \end{aligned}$$

d. $\int x\sqrt{2-x} dx$

$$= \int (2-u)\sqrt{u} du \quad \leftarrow \begin{cases} u=2-x \Leftrightarrow x=2-u \\ du=-dx \\ -du=dx \end{cases}$$

$$= \int u^{3/2} - 2u^{1/2} du = \frac{2}{5} u^{5/2} - \frac{4}{3} u^{3/2} + C$$

$$= \left(\frac{2}{5} (2-x)^{5/2} - \frac{4}{3} (2-x)^{3/2} + C \right)$$

e. $\int_0^2 e^x \cos(1+e^x) dx$

$$= \int_2^{1+e^2} \cos(u) du$$

$$\leftarrow \begin{cases} u=1+e^x \\ du=e^x dx \end{cases}$$

$$= \sin(u) \Big|_2^{1+e^2} = \sin(1+e^2) - \sin(2)$$

f. $\int \tan(x) \sec^2(x) dx$

$$= \int u du$$

$$\leftarrow \begin{cases} u=\tan(x) \\ du=\sec^2(x) dx \end{cases}$$

$$= \frac{u^2}{2} + C = \frac{1}{2} (\tan(x))^2 + C$$