Homework 5.4: Replacement problems

The Exercises in this section are not well-aligned to the text! Here are replacements.

Use the comparison test to determine whether the series converge or diverge.

1.
$$\sum_{n=1}^{\infty} a_n \text{ where } a_n = \frac{2}{n(n+1)}$$

2.
$$\sum_{n=1}^{\infty} \frac{1}{2(n+1)}$$

3.
$$\sum_{n=1}^{\infty} \frac{1}{2n-1}$$

4.
$$\sum_{n=2}^{\infty} \frac{1}{(n \ln n)^2}$$

5.
$$\sum_{n=1}^{\infty} \frac{n!}{(n+2)!}$$

6.
$$\sum_{n=1}^{\infty} \frac{1}{n!}$$

7.
$$\sum_{n=1}^{\infty} \frac{\sin^2 n}{n^2}$$

Use the limit comparison test to determine whether the series converge or diverge.

8.
$$\sum_{n=0}^{\infty} \frac{1}{n+3}$$

9.
$$\sum_{n=2}^{\infty} \frac{1}{n^2 - \sqrt{n}}$$

10.
$$\sum_{n=1}^{\infty} \frac{3^n}{5^n + 4^n}$$

$$11. \quad \sum_{n=1}^{\infty} \frac{\ln n}{n^3}$$

12.
$$\sum_{n=1}^{\infty} \frac{1}{4^n - 3^n}$$

- **13.** Does $\sum_{n=2}^{\infty} \frac{1}{(\ln n)^p}$ converge if p is large enough? If so, for which p?
- **14.** For which p does the series $\sum_{n=1}^{\infty} \frac{2^{pn}}{3^n}$ converge? Why?
- **15.** For which p > 0 does the series $\sum_{n=1}^{\infty} \frac{n^p}{2^n}$ converge? Why?
- **16.** Suppose that $a_n > 0$ and that $\sum_{n=1}^{\infty} a_n$ converges. Suppose also that b_n is an arbitrary sequence of zeros and ones. Does $\sum_{n=1}^{\infty} a_n b_n$ necessarily converge? Explain using a convergence test.
- 17. Show that if $a_n \ge 0$ and $\sum_{n=1}^{\infty} a_n^2$ converges, then $\sum_{n=1}^{\infty} \sin^2(a_n)$ converges. (*Hint.* Use a convergence test, and the fact that $|\sin \theta| \le |\theta|$.)
- 18. Let d_n be an infinite sequence of digits, meaning d_n takes values in $\{0,1,\ldots,9\}$. What is the largest possible value of $x=\sum_{n=1}^{\infty}\frac{d_n}{10^n}$ that converges?