Name:

_____/ 24

30 minutes maximum. 24 points possible; each part is worth 2 points. No aids (book, calculator, etc.) are permitted. Show all work and use proper notation for full credit. Answers should be in reasonably-simplified form.

1. [12 points] Compute the derivatives of the following functions.

a.
$$f(x) = \frac{e^x}{x^3}$$

$$C'(x) = \frac{e^x}{x^3}$$

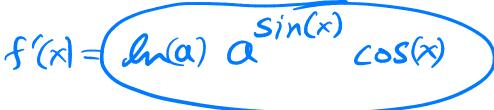
$$= \left(\frac{e^{x}}{x^{4}}(x-3)\right)$$

b.
$$f(x) = (\ln(x^2 + e^2))^5$$

$$f'(x) = 5(\ln(x^2 + e^2))^4 \frac{1}{x^2 + e^2} 2x$$

$$= \frac{10 \times (\ln(x^2 + e^2))^4}{x^2 + e^2}$$

c. $f(x) = a^{\sin(x)}$ where a is a constant, a > 1



Math 252: Quiz 1

13 January, 2022

d.
$$f(x) = \sec\left(\frac{x}{x+1}\right)$$

$$f'(x) = Sec(\frac{x}{x+1}) tam(\frac{x}{x+1}) \frac{x+1-x}{(x+1)^2}$$

$$= \left(\frac{Sec(\frac{x}{x+1}) tam(\frac{x}{x+1})}{(x+1)^2}\right)$$

$$= (x+1)^2$$

e.
$$f(x) = e^{\pi x + 1} + \sqrt{3} \tan(\pi x)$$

$$f'(x) = \pi e^{\pi x + 1} + \sqrt{3} \sec^2(\pi x) \pi$$

$$= \pi \left(e^{\pi x + 1} + \sqrt{3} \sec^2(\pi x)\right)$$

f. Find
$$\frac{dy}{dx}$$
 if $2x + y = \cos(xy)$. (You must solve for $\frac{dy}{dx}$.)

Find
$$\frac{dy}{dx}$$
 if $2x + y = \cos(xy)$. (You must solve for $\frac{dy}{dx}$.)

$$2 + \frac{dy}{dx} = -\sin(xy) \left(1 \cdot y + x \cdot \frac{dy}{dx} \right)$$

$$\frac{dy}{dx} \left(1 + x \sin(xy) \right) = -y \sin(xy) - 2$$

$$\frac{dy}{dx} = -\frac{y \sin(xy) + 2}{1 + x \sin(xy)}$$

Math 252: Quiz 1 13 January, 2022

2. [12 points] Compute the following definite integrals and antiderivatives (indefinite integrals). Remember that antiderivatives need a "+C".

a.
$$\int_{1}^{2} \frac{2+x^{3}}{x^{2}} dx = \int_{1}^{2} 2x^{-2} + x dx$$

$$= \left[-2x^{-1} + \frac{x^{2}}{2} \right]_{1}^{2} = \left(-\frac{2}{2} + 2 \right) - \left(-2 + \frac{1}{2} \right)$$

$$= -1 + 2 + 2 - \frac{1}{2} = \left(\frac{5}{2} \right)$$

$$= \int \frac{du/3}{\sqrt{5+e^{3x}}} dx$$

$$= \int \frac{du/3}{\sqrt{5+e^{3x}}} dx$$

$$= \int \frac{du/3}{\sqrt{5+e^{3x}}} dx$$

$$= \int \frac{du/3}{\sqrt{5+e^{3x}}} dx$$

$$=\frac{1}{3}\int u^{-1/2}du=\frac{1}{3}2u^{1/2}=\left(\frac{2}{3}(5+3x)^{1/2}+C\right)$$

c.
$$\int \frac{1}{x} + \sec(x) \tan(x) dx$$

$$= (\ln |x| + \operatorname{Sec}(x) + C)$$

Math 252: Quiz 1

d.
$$\int x\sqrt{2-x}\,dx$$

$$=-(6-u)\sqrt{u}\,du$$

$$= \int u^{3/2} - 2u^{1/2} du = \frac{2}{5}u^{5/2} - \frac{4}{3}u^{3/2} + C$$

$$= \int_{c^2}^{2} (2-x)^{5/2} - \frac{4}{3}(2-x)^{3/2} + C$$

e.
$$\int_0^2 e^x \cos(1+e^x) \, dx$$

$$= \int_{-\infty}^{\infty} |te^{2}| du$$

$$du = |+e^{x}|$$

$$du = e^{x}dx$$

$$= sin(u)$$

$$= sin(u)$$

$$= sin(lte^2) - sin(2)$$

$$f. \int \tan(x) \sec^2(x) \ dx$$

$$= \int u du$$

$$=\frac{u^2}{2}+c$$

$$= \begin{cases} u = tan(x) \\ du = sec^2(x) dx \end{cases}$$