SOLUTIONS

Worksheet: Various trigonometric integrals

Compute these integrals with a group, if possible!

A. $\int \tan(4x)\,dx =$ $\int \frac{\sin(4x)}{\cos(4x)}dx = \int \frac{-du/4}{u} = -\frac{1}{4}\int \frac{du}{u}$

$u = \cos(4x)$

$-du/4 = \sin(4x)dx$

$= -\frac{1}{4}\ln|\cos(4x)| + C$

B. $\int \sec^2 x \tan^3 x\,dx =$ $\int u^3\,du = \frac{1}{4}u^4 + C = \frac{1}{4}(\tan x)^4 + C$

$u = \tan x$

$du = \sec^2 x\,dx$

C. $\int_0^\pi \sin(4x)\cos(3x)\,dx =$ $\frac{1}{2}\int_0^{2\pi} \sin(4x+3x) + \sin(4x-3x)\,dx$

$[\sin a\cos b = \frac{1}{2}(\sin(a+b) + \sin(a-b))]$

$= \frac{1}{2}\int_0^{2\pi} \sin(7x) + \sin(x)\,dx = \frac{1}{2}\left[\frac{-\cos(7x)}{7} - \cos x\right]_0^{2\pi}$

$= \frac{1}{2}\left[\frac{-\cos(14\pi)+\cos(0)}{7} - (\cos(2\pi) - \cos(0))\right]$

$= 0$

D. $\int \tan^4 t\,dt =$

$[\tan^2 t = \sec^2 t - 1]$

$= \int \tan^2 t(\sec^2 t - 1)\,dt = \int \tan^2 t \sec^2 t\,dt - \int \tan^2 t\,dt$

E. $\int \sec(2x)\,dx =$

$= \int \sec(2x)\frac{\sec(2x)+\tan(2x)}{\sec(2x)+\tan(2x)}dx = \int \frac{\sec^2(2x)+\sec(2x)\tan(2x)}{\sec(2x)+\tan(2x)}dx$ cont.

$= \int \frac{du/2}{u} = \frac{1}{2}\ln|\sec(2x)+\tan(2x)| + C$

$[u = \sec(2x)+\tan(2x)]$

D. cont

$$= \int \tan^2 t \sec^2 t \, dt - \int \tan^2 t \, dt$$

$\tan^2 t = \sec^2 t - 1$

$$= \int u^2 \, du - \int \sec^2 t - 1 \, dt$$

$(u = \tan t, \; du = \sec^2 t \, dt)$

$$= \frac{1}{3}(\tan t)^3 - \tan t + t + C$$