Solutions to Quiz # 8

1. One approach is to notice $x^2 = t$ so $y = 1 - t = 1 - x^2$, a parabola. Correct Cartesian equations are either $y = 1 - x^2$ or $x = \sqrt{1-y}$. The sketch shows the part of the parabola from $(0,1)$ to $(2,-3)$.

2. We solve the equation $dy/dx = 1/2$ for t:

$$\frac{1}{2} = \frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{3t^2}{6t} = \frac{t}{2}$$

so $t = 1$. The point is $(4,0)$.

3. We use the point-slope form of the line. We have the point $(1,3)$. For the slope we compute

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{2t}{0 + \frac{1}{t}} = 2t^2.$$

However, to find a number for the slope we need to see that $t = 1$ at the point $(1,3)$ because $1 = 1 + \ln t$ and $3 = t^2 + 2$ are both solved by $t = 1$. Thus

$$m = \frac{dy}{dx}\bigg|_{t=1} = 2t^2\bigg|_{t=1} = 2$$

and so the line is $y - 3 = 2(x-1)$.

4. Because $x'(t) = 2t - 1$ and $y'(t) = 4t^3$,

$$L = \int_1^4 \sqrt{(2t-1)^2 + (4t^3)^2}\, dt = \int_1^4 \sqrt{1 - 4t + 4t^2 + 16t^6}\, dt$$

(*To my knowledge, no further exact progress is possible for this integral. Numerically:* $L \approx 255.38$.)

5.

$$\begin{aligned}
L &= \int_0^\pi \sqrt{(e^t\cos t - e^t \sin t)^2 + (e^t \sin t + e^t \cos t)^2}\, dt \\
&= \int_0^\pi e^t \sqrt{(\cos t - \sin t)^2 + (\sin t + \cos t)^2}\, dt \\
&= \int_0^\pi e^t \sqrt{\cos^2 t - 2\cos t \sin t + \sin^2 t + \sin^2 t + 2\cos t \sin t + \cos^2 t}\, dt \\
&= \int_0^\pi e^t \sqrt{2\cos^2 t + 2\sin^2 t}\, dt = \sqrt{2}\int_0^\pi e^t\, dt \\
&= \sqrt{2}\Big[e^t\Big]_0^\pi = \sqrt{2}(e^\pi - 1)
\end{aligned}$$