Solutions to Quiz # 4

1. (*7.2 #21.*) Separate $\sec x \tan x$ for du and use $u = \sec x$:

$$\int \tan x \sec^3 x\, dx = \int \sec^2 x\, \sec x \tan x\, dx = \int u^2\, du = \frac{1}{3}u^3 + C = \frac{1}{3}\sec^3 x + C$$

2. (*7.2 #4.*) Separate $\sin x$ for du, use $\sin^2 x = 1 - \cos^2 x$, use $u = \cos x$, and convert limits to u:

$$\int_0^{\pi/2} \sin^5 x\, dx = \int_0^{\pi/2} \left(\sin^2 x\right)^2 \sin x\, dx = \int_0^{\pi/2} \left(1 - \cos^2 x\right)^2 \sin x\, dx$$

$$= \int_1^0 (1-u^2)^2(-du) = \int_0^1 1 - 2u^2 + u^4\, du = \left[u - \frac{2}{3}u^3 + \frac{1}{5}u^5\right]_0^1 = 1 - \frac{2}{3} + \frac{1}{5} = \frac{8}{15}$$

3. (*7.3 #12, but as indefinite integral.*) Substitute $t = 2\tan\theta$, use memorized $\int \sec\theta\, d\theta$, and use triangle to return to old variable:

$$\int \frac{dt}{\sqrt{4+t^2}} = \int \frac{2\sec^2\theta\, d\theta}{\sqrt{4 + 4\tan^2\theta}} = \int \frac{2\sec^2\theta\, d\theta}{2\sec\theta} = \int \sec\theta\, d\theta = \ln|\sec\theta + \tan\theta| + C = \ln\left|\frac{\sqrt{4+t^2}}{2} + \frac{t}{2}\right| + C$$

4. (*7.3 #4.*) Substitute $x = 3\sin\theta$, use $\sin^2\theta = \frac{1}{2}(1 - \cos(2\theta))$, and use triangle to return:

$$\int \frac{x^2}{\sqrt{9-x^2}}\, dx = \int \frac{9\sin^2\theta}{3\cos\theta} 3\cos\theta\, d\theta = 9\int \sin^2\theta\, d\theta = \frac{9}{2}\int 1 - \cos(2\theta)\, d\theta = \frac{9}{2}\left(\theta - \frac{1}{2}\sin(2\theta)\right) + C$$

$$= \frac{9}{2}(\theta - \sin\theta\cos\theta) + C = \frac{9}{2}\left(\arcsin\left(\frac{x}{3}\right) - \frac{x}{3}\frac{\sqrt{9-x^2}}{3}\right) + C = \frac{9}{2}\arcsin\left(\frac{x}{3}\right) - \frac{1}{2}x\sqrt{9-x^2} + C$$