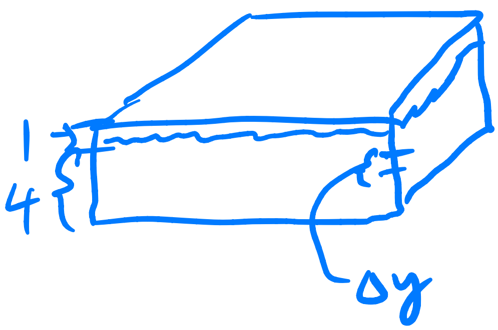
Worksheet: Integral applications

Do these calculations with a group, if possible.

A. (*§2.5 #250*) How much work is required to pump-out a swimming pool if the area of the base is 800 ft^2, the water is 4 ft deep, and the top of the pool is 1 foot above the water level? (*Assume that the density of water is* $62\,lb/ft^3$.)



a thin layer has volume

$$\Delta V = 800\,\Delta y \quad [ft^3]$$

and <u>weight</u>

$$\Delta M = 62(800\,\Delta y) \quad [lb]$$

total work:

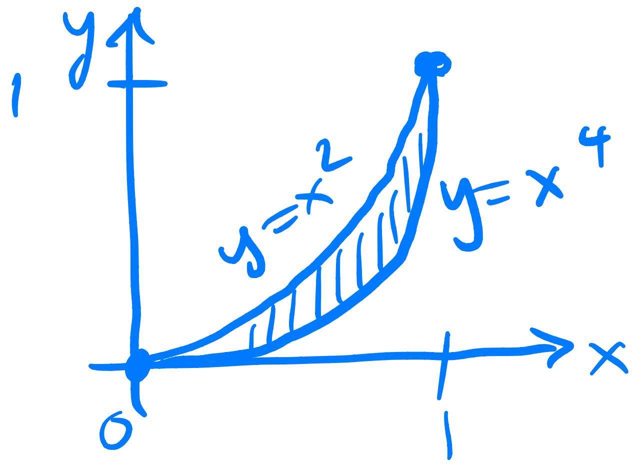
$$W = \int_0^4 (5-y)\,62(800)\,dy$$

$$= 49600 \int_0^4 5-y\,dy$$

$$= 49600 \left[5y - \frac{y^2}{2}\right]_0^4$$

$$= 49600\,(20-8) = 595{,}200 \text{ ft-lb}$$

B. (§2.6 #279) Find the center of mass $(\bar{x}, \bar{y})$ of the region bounded by $y = x^2$ and $y = x^4$ in the first quadrant. Start by sketching the region.



assume $\rho = 1$
if not stated
(it cancels anyway!)

$$m = \int_0^1 x^2 - x^4 \, dx = \left[\frac{x^3}{3} - \frac{x^5}{5}\right]_0^1 = \frac{1}{3} - \frac{1}{5} = \frac{2}{15}$$

$$M_y = \int_0^1 x(x^2 - x^4)\, dx = \int_0^1 x^3 - x^5 \, dx = \frac{1}{4} - \frac{1}{6} = \frac{1}{12}$$

$$M_x = \frac{1}{2}\int_0^1 (x^2 - x^4)^2 \, dx = \frac{1}{2}\int_0^1 x^4 - 2x^6 + x^8 \, dx = \frac{1}{2}\left(\frac{1}{5} - \frac{2}{7} + \frac{1}{9}\right) = \frac{1}{2}\left(\frac{63 - 2\cdot 45 + 35}{315}\right) = \frac{4}{315}$$

$$\bar{x} = \frac{M_y}{m} = \frac{1}{12}\,\frac{15}{2} = \frac{15}{24}, \quad \bar{y} = \frac{\cancel{4}^{\,2}}{\cancel{315}_{\,21}}\,\frac{\cancel{15}^{\,1}}{\cancel{2}} = \frac{2}{21}$$