Solutions to Quiz # 3

1. (*I was wrong to ask for the answer in Newtons. It should be in Joules = Newton-meters.*) The linear weight density of the rope is $\frac{4\,\text{kg}}{60\,\text{m}} = \frac{1}{15}\frac{\text{kg}}{\text{m}}$. The weight of a small piece of rope of length dy is $dF = \left(\frac{1}{15}\frac{\text{kg}}{\text{m}}\right)\left(10\frac{\text{m}}{\text{s}^2}\right)(dy\ \text{m}) = \frac{2}{3}dy$ Newtons. A piece of rope at position y must be lifted $60-y$ meters (if the bottom of the rope is at $y=0$). The total work to coil it into your hand is

$$W = \int_0^{60} \frac{2}{3}dy(60-y) = \frac{2}{3}\int_0^{60}(60-y)\,dy = \frac{2}{3}\left[60y - \frac{y^2}{2}\right]_0^{60} = \frac{2}{3}\left[60^2 - \frac{60^2}{2}\right] = 1200\ \text{J}$$

2.

$$f_{\text{ave}} = \frac{1}{7-3}\int_3^7 \frac{1}{x}\,dx = \frac{1}{4}\left[\ln x\right]_3^7 = \frac{1}{4}(\ln 7 - \ln 3)$$

3. Do integration-by-parts with $u = x$ and $dv = e^x\,dx$:

$$\int_{-1}^{1} x\,e^x\,dx = \left[x\,e^x\right]_{-1}^{1} - \int_{-1}^{1} e^x\,dx = \left[e^1 + e^{-1}\right] - \left[e^x\right]_{-1}^{1} = e + \frac{1}{e} - \left(e - \frac{1}{e}\right) = \frac{2}{e}$$

4. Do integration-by-parts with $u = \arctan x$ and $dv = dx$. Then use substitution $u = 1 + x^2$:

$$\begin{aligned}\int \arctan x\,dx &= x\arctan x - \int \frac{x}{1+x^2}\,dx = x\arctan x - \int \frac{du/2}{u} \\ &= x\arctan x - \frac{1}{2}\ln|u| + C = x\arctan x - \frac{1}{2}\ln(1+x^2) + C\end{aligned}$$