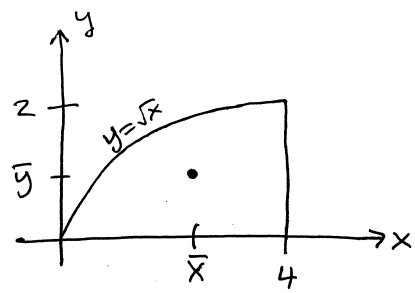
Solutions to Quiz # 7

1. **(a)** See sketch at right.



(b) One needs to do 3 integrals:

$$A = \int_0^4 \sqrt{x}\,dx = \frac{2}{3}(4^{3/2} - 0) = \frac{16}{3}$$
$$\bar{x} = \frac{1}{A}\int_0^4 x\sqrt{x}\,dx = \frac{3}{16}\frac{2}{5}(4^{5/2} - 0) = \frac{12}{5}$$
$$\bar{y} = \frac{1}{A}\int_0^4 \frac{1}{2}\sqrt{x}\,\sqrt{x}\,dx = \frac{3}{32}\frac{1}{2}(4^2 - 0) = \frac{3}{4}$$

Note that $(\bar{x}, \bar{y}) = (12/5, 3/4)$ is not surprising, given the sketch.

2. The function is a p.d.f. because $f(x) \ge 0$ and the integral is one:

$$\int_{-\infty}^{\infty} f(x)\,dx = \int_0^{\infty} xe^{-x}\,dx = \lim_{t\to\infty}\int_0^t xe^{-x}\,dx = \lim_{t\to\infty}\left(\left[-xe^{-x}\right]_0^t + \int_0^t e^{-x}\,dx\right)$$
$$= \lim_{t\to\infty} -te^{-t} - (e^{-t} - 1) = -0 - 0 + 1 = 1$$

The integration by parts used $u = x, dv = e^{-x}\,dx$.

3. **(a)** (*Write something like the following.*) The area under the graph from $x = 0$ to $x = 10$ is one because the area of the left triangle is $A_l = \frac{1}{2}(0.2)6 = 0.6$ and the area of the right triangle is $A_r = \frac{1}{2}(0.2)4 = 0.4$ and thus $A_l + A_r = 1$.

(b) One way to do this is to observe that the remaining probability is another triangle:

$$P(0 \le X \le 8) = 1 - P(8 \le X \le 10) = 1 - \frac{1}{2}(0.1)2 = 1 - 0.1 = 0.9$$

4.

$$\mu = \int_{-\infty}^{\infty} xf(x)\,dx = \int_0^{10} x\,\frac{1}{50}x\,dx = \frac{1}{50}\int_0^{10} x^2\,dx = \frac{1}{50}\,\frac{10^3 - 0}{3} = \frac{20}{3}$$