Solutions to Quiz # 9

1. (a) Converges. Note the square root is continuous and the inside limit can be found by L'Hopital's Rule (or algebra):

$$\lim_{n\to\infty} a_n = \lim_{n\to\infty} \sqrt{\frac{1+4n^2}{1+n^2}} = \sqrt{\lim_{n\to\infty} \frac{1+4n^2}{1+n^2}} = \sqrt{\lim_{n\to\infty} \frac{8n}{2n}} = \sqrt{\lim_{n\to\infty} 4} = \sqrt{4} = 2$$

(b) Diverges; in the last limit the numerator goes to infinity but the denominator goes to one:

$$\lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{n^4}{n^3-2n} = \lim_{n\to\infty} \frac{n^3}{n^2-2}\frac{1/n^2}{1/n^2} = \lim_{n\to\infty} \frac{n}{1-2/n^2} = +\infty$$

(c) Converges because the numerator is bounded while the denominator goes to infinity:

$$\lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{(-1)^n}{2\sqrt{n}} = 0$$

2.

$$\begin{aligned}
s_1 &= \frac{1}{1^2+1} = \frac{1}{2} \\
s_2 &= \frac{1}{1^2+1} + \frac{1}{2^2+1} = \frac{7}{10} \\
s_3 &= \frac{1}{1^2+1} + \frac{1}{2^2+1} + \frac{1}{3^2+1} = \frac{4}{5}
\end{aligned}$$

3. (a) Here $a = 10$ and $r = -1/5$. Since $|r| < 1$ the geometric series converges:

$$10 - 2 + 0.4 - 0.08 + \dots = \sum_{n=1}^{\infty} 10 \left(-\frac{1}{5}\right)^{n-1} = \frac{10}{1-(-1/5)} = \frac{10}{6/5} = \frac{25}{3}$$

(b) Here $a = 5/\pi$ and $r = 1/\pi$. Since $|r| < 1$ the geometric series converges:

$$\sum_{n=1}^{\infty} \frac{5}{\pi^n} = \sum_{n=1}^{\infty} \frac{5}{\pi}\left(\frac{1}{\pi}\right)^{n-1} = \frac{5/\pi}{1-(1/\pi)} = \frac{5}{\pi-1}$$