Solutions to Quiz # 11

1. **(a)** The series converges by the alternating series test. Here $b_n = n^2/(n^3+4)$ is positive, decreasing, and $b_n \to 0$.

(b) The series is alternating because

$$a_n = \frac{n\cos(\pi n)}{3^n} = (-1)^n \frac{n}{3^n}.$$

If we define $b_n = n/3^n$ then b_n is positive, decreasing, and $b_n \to 0$. Thus the series converges.

You can also use the ratio or root tests. In that case note $|a_n| = n/3^n$ and you will find $L = 1/3 < 1$.

2. The partial sum

$$s_{50} = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \dots + \frac{1}{49} - \frac{1}{50}$$

is an *underestimate* of the total sum. Note that the 50th term $a_{50} = -1/50$ is negative. If s is the sum of the series then we see that the partial sum s_{50} "hopped over" s, going left, because $s_{50} = s_{49} - 1/50$, so $s_{50} < s$.

3. **(a)** *Conditionally convergent*. The sum is convergent by the alternating series test because $a_n = (-1)^n b_n$ where $b_n = 1/\sqrt{n}$ is positive, goes to zero, and is decreasing. On the other hand the absolute sum is a p series with $p = 1/2 < 1$:

$$\sum_{n=1}^{\infty} |a_n| = \sum_{n=1}^{\infty} \frac{1}{\sqrt{n}} = \sum_{n=1}^{\infty} \frac{1}{n^{1/2}}.$$

This sum diverges.

(b) *Diverges*. Either the test for divergence or the root test can be applied. For the latter,

$$L = \lim_{n\to\infty} \sqrt[n]{|a_n|} = \lim_{n\to\infty} \sqrt[n]{\left(\frac{2n^2+1}{n^2+1}\right)^n} = \lim_{n\to\infty} \frac{2n^2+1}{n^2+1} = \frac{2}{1} = 2.$$

Since $L = 2 > 1$ the series diverges. (*The limit can be justified several ways. One method is two applications of L'Hopital's rule.*)

(c) *Converges absolutely*. Either the ratio or root test will work; we use the former:

$$L = \lim_{n\to\infty} \frac{|a_{n+1}|}{|a_n|} = \lim_{n\to\infty} \frac{\frac{(n+1)^{10}}{10^{n+2}}}{\frac{n^{10}}{10^{n+1}}} = \lim_{n\to\infty} \frac{(n+1)^{10}10^{n+1}}{n^{10}10^{n+2}} = \lim_{n\to\infty} \frac{(n+1)^{10}}{n^{10}10}$$

$$= \frac{1}{10} \lim_{n\to\infty} \left(1 + \frac{1}{n}\right)^{10} = \frac{1}{10}(1+0)^{10} = \frac{1}{10} < 1$$