Homework 5.4: Replacement problems

The Exercises in this section are not well-aligned to the text! Here are replacements.

Use the comparison test to determine whether the series converge or diverge.

1. $\displaystyle\sum_{n=1}^{\infty} a_n$ where $a_n = \dfrac{2}{n(n+1)}$

2. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{2(n+1)}$

3. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{2n-1}$

4. $\displaystyle\sum_{n=2}^{\infty} \frac{1}{(n \ln n)^2}$

5. $\displaystyle\sum_{n=1}^{\infty} \frac{n!}{(n+2)!}$

6. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n!}$

7. $\displaystyle\sum_{n=1}^{\infty} \frac{\sin^2 n}{n^2}$

Use the limit comparison test to determine whether the series converge or diverge.

8. $\displaystyle\sum_{n=0}^{\infty} \frac{1}{n+3}$

9. $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n^2 - \sqrt{n}}$

10. $\displaystyle\sum_{n=1}^{\infty} \frac{3^n}{5^n + 4^n}$

11. $\displaystyle\sum_{n=1}^{\infty} \frac{\ln n}{n^3}$

12. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{4^n - 3^n}$

13. Does $\displaystyle\sum_{n=2}^{\infty} \frac{1}{(\ln n)^p}$ converge if p is large enough? If so, for which p?

14. For which p does the series $\sum_{n=1}^{\infty} \frac{2^{pn}}{3^n}$ converge? Why?

15. For which $p > 0$ does the series $\sum_{n=1}^{\infty} \frac{n^p}{2^n}$ converge? Why?

16. Suppose that $a_n > 0$ and that $\displaystyle\sum_{n=1}^{\infty} a_n$ converges. Suppose also that b_n is an arbitrary sequence of zeros and ones. Does $\displaystyle\sum_{n=1}^{\infty} a_n b_n$ necessarily converge? Explain using a convergence test.

17. Show that if $a_n \geq 0$ and $\displaystyle\sum_{n=1}^{\infty} a_n^2$ converges, then $\displaystyle\sum_{n=1}^{\infty} \sin^2(a_n)$ converges. (*Hint. Use a convergence test, and the fact that* $|\sin\theta| \leq |\theta|$.)

18. Let d_n be an infinite sequence of digits, meaning d_n takes values in $\{0, 1, \ldots, 9\}$. What is the largest possible value of $x = \displaystyle\sum_{n=1}^{\infty} \frac{d_n}{10^n}$ that converges?