A^* means $A-\{0\}$.

1 Let G be a group of order 30. Show that a 3-sylow subgroup or a 5-sylow subgroup must be normal in G.

2 Prove or disprove: Suppose group G is the internal direct product of A and B. Further, suppose also that G is the internal direct product of A and C. It follows that B=C.

3 Let n be fixed and suppose $\sigma \in S_n$. Show that σ is even if and only if every conjugate of σ is even.

4 Suppose H is the homomorphic image of an abelian group. Show that H is abelian.

5 Suppose H is a subring of $\mathbb{Z}$. Show that there exists an x in $\mathbb{Z}$ where $H=\langle x \rangle$. Prove this from first principles.

6 Let R be a ring containing a unit element. It may or may not be the case that R is commutative. Suppose the only right-ideals contained in R are $\{0\}$ and R. Show that R is a division ring.

7 Suppose J is an integral domain. Let $A = J \times J^*$. For (a,b) and (x,y) in A let $(a,b)\sim(x,y)$ if $ay=bx$. Show that $\sim$ is an equivalence relation on A.

8 You don't need to show work or proofs on this problem. Do not just state the name of the set. State the name of the operations or operation you are using. If the operation has no name, define it.

Give an example of:

(a) a group which has no nontrivial subgroups

(b) a nonabelian group

(c) a ring which is not an integral domain

{d) an integral domain which is not a field

(e) a euclidean ring

(f) a module over the real numbers

(g) an ideal in $\mathbb{Z}$ that is not maximal

(h) a nontrivial ideal in $\mathbb{Z}$ that is maximal.

DO TWO OF THE REMAINING PROBLEMS

9. Let G be a group of order 30. Show that a 3-sylow subgroup or a 5-sylow subgroup must be normal in G.

10. Show that every group of order 35 is abelian.

11. (a) How many elements in S_7 commute with (1,2,3,4)?

(b) How many elements in S_7 are conjugates of (12)(345)?

(c) How many elements in S_7 are conjugates of (123)(456)?

12 List, up to isomorphism, all abelian groups of orders 32 and 36.