Essential Information

Course Title	Functional Analysis
Instructor	Ed Bueler elbueler@alaska.edu
Class meeting	MWF 2:15–3:15 pm, Chapman 107
Section CRNs	in-person 901: 35367 online 701: 35378 (zoom link here)
Public website	bueler.github.io/fa
Canvas website	canvas.alaska.edu/courses/18441
Required text	D. Borthwick, *Spectral Theory*, Graduate Texts in Mathematics 284, Springer 2020

Description

Functional analysis is the theory of infinite-dimensional vector spaces and linear maps upon them. It is the mathematical home of partial differential equations, boundary value problems, quantum mechanics, the finite element method, field theories (electromagnetic and gravitational), fluid mechanics, and signal processing.

The UAF catalog description of the course is this: *Study of Banach and Hilbert spaces, and continuous linear maps between them. Linear functionals and the Hahn-Banach theorem. Applications of the Baire Category theorem. Compact operators, self adjoint operators, and their spectral properties. Weak topology and its applications.*

The above description is accurate, and all of these topics will arise. However, I will strongly emphasize some aspects and gloss others. This course will focus on: *Hilbert spaces including Sobolev spaces. Continuous and unbounded operators (linear maps) on Hilbert spaces. The spectral theory of compact and self-adjoint operators, including the Laplacian. Connections with finite-dimensional linear algebra and partial differential equations. Motivation from the axioms and basic concepts of quantum mechanics.*

Mathematically-inclined students from the sciences and engineering are encouraged to register, as are graduate students in mathematics looking for an elective with practical relevance. This course is particularly aimed at students interested in the mathematics of partial differential equations and of quantum mechanics. On the other hand, this will definitely be a graduate mathematics course, wherein almost every problem will be a proof!

Prerequisites

Officially: *MATH F314 Linear Algebra and MATH 401 Introduction to Real Analysis, or permission of instructor. Recommended: MATH F422 Introduction to Complex Analysis and MATH F641 Real Analysis.*

For graduate students with a background from another university, these prerequisites describe rigorous, though introductory, exposure to the analysis of real functions, plus a course in linear algebra. The basics of complex numbers are assumed. Quite a bit of mathematical maturity and motivation is assumed. The experienced difficulty of assigned work will depend on the student's comfort doing proofs, especially with vector spaces, sequences, real functions, convergence, continuity, and integrals.

The Hybrid Classroom

There are two sections: in-person (901; crn 35367) and online (701; crn 35378). They are treated as one course and occur synchronously. In this "hybrid" set-up, each lecture will be a recorded Zoom session generated from Chapman 107. (The link for the Zoom session is in Canvas. Recordings will be linked from inside Canvas only; they are not public.) I will try to treat all students the same regarding proctored assessments—see below—and participation during class time. Students have certain obligations to help make this work:

- **in-person students**: Please participate as energetically as you can. I prefer for in-person students to turn in their homework assignments on paper.
- **online students**: Please sign into the Zoom session, from Canvas, just before class starts. Please participate as energetically as you can, and, if possible, keep your camera on. Regarding in-class group work, check for worksheet PDFs from the public site before class starts. When you turn in homework assignments electronically, please generate clear, well-ordered, and combined PDFs. You will need to schedule proctoring for the Midterm Quizzes and Final Exam (see below), or attend in person on those days.

Schedule and Online Materials

The public course website includes a schedule and daily log. The schedule includes the due date of each homework Assignment, and the dates for the Midterm Quizzes and Final Exam. The daily log tracks the textbook sections actually covered during each lecture, and handouts or worksheets used during class. Please consult these frequently; they are subject to change but they will be kept up to date.

Most course materials (Homework assignments, worksheets, code examples, etc.) will be posted on the public website. Certain specific course materials will go on the Canvas site: student grades, homework solutions, and Quiz solutions.

Office Hours and Communication

My office hours are shown online at bueler.github.io/OffHrs.htm; I hold them in Chapman 306C. Students can also schedule meetings with me outside of these hours.

I will use Canvas to send announcements. If I need to contact you outside of class times, I will email via Canvas. Please set your email address in Canvas appropriately.

Evaluation and Grades

Homework	nearly weekly	50%
Midterm Quiz 1	in-class Wednesday February 28	15%
Midterm Quiz 2	in-class Wednesday April 3	15%
Final Exam	in-class Friday May 3	20%
total		100%

Scores may be adjusted based on the actual difficulty of the work and/or on average class performance, and adjustments will be applied to all students equally. The scores of the various parts will be summed and the final course grade will be assigned as follows.

A	93–100%	B-	79–81%	D+	65–67%
A-	90–92%	C+	76–78%	D	60–64%
B+	87–89%	C	68–75%	D-	57–59%
B	82–86%	C-	not given	F	$\leq 56\%$

These ranges are a guarantee and a lower bound. I reserve the right to increase your grade above these ranges based on the actual difficulty of the work and/or on average class performance. Any such increases will preserve grade ordering by weighted total score.

Homework

Homework is due at the start of class. **Late homework is not accepted.** If you have unavoidable circumstances which do not allow you to turn in an Assignment on time then please contact me (elbueler@alaska.edu) in advance.

The homework consists of proofs, rigorously-justified examples and counter-examples, and by-hand computations. LaTeX is encouraged for Homework, but definitely *not* required. Well-organized hand-written proofs are happily accepted!

Exams

There will be two in-class, hour-long Midterm Quizzes. These Quizzes will firm-up the basic concepts and definitions addressed more thoroughly in the Homework. Problems very similar to, e.g. shortened, Homework problems will appear on the in-class Midterm Quizzes.

The 2-hour, in-class Final Exam will require you to be familiar with, and prepared on, major themes and theorems in the course. I will describe the format of the Final in more detail as its date approaches.

Make-up Quizzes/Exam will be given only for documented circumstances, at my discretion. Department policy (below) does not allow me to move the time of the Final Exam.

Internet and AI usage, and other assistance, on Homework

You will not have access to AI tools, or any other tools except a writing implement, during the Quizzes and Final Exam. These assessments are proctored and on-paper.

Regarding Homework, you are encouraged to talk to other students about the problems, and to use other tools appropriately, but the work you turn in must be your own. Please do not copy proofs from online sources, from searching or generated via AI tools like ChatGPT. If I detect this then I have the right to give you a zero on that Assignment. Even when you get assistance, it goes without saying that your own thinking, as you do the homework, will have the greatest benefits. Fully understand the proofs you turn in, even if hints from other sources were used in generating them.

Rules and Policies

Incomplete Grade

Incomplete (I) will only be given in DMS courses in cases where the student has completed the majority (normally all but the last three weeks) of a course with a grade of C or better, but for personal reasons beyond his/her control has been unable to complete the course during the regular term. Negligence or indifference are not acceptable reasons for granting an incomplete grade.

Late Withdrawals

A withdrawal after the deadline from a DMS course will normally be granted only in cases where the student is performing satisfactorily (i.e., C or better) in a course, but has exceptional reasons, beyond his/her control, for being unable to complete the course. These exceptional reasons should be detailed in writing to the instructor, Department Chair and the Dean.

No Early Final Examinations

Final examinations for DMS courses shall not be held earlier than the date and time published in the official term schedule. Normally, a student will not be allowed to take a final exam early. Exceptions can be made by individual instructors, but should only be allowed in exceptional circumstances and in a manner which doesn't endanger the security of the exam.

Academic Dishonesty

Academic dishonesty, including cheating and plagiarism, will not be tolerated. It is a violation of the Student Code of Conduct and will be punished according to UAF procedures.

Student protections and service statement

Every qualified student is welcome in my classroom. As needed, I am happy to work with you, Disability Services, Veterans' Services, Rural Student Services, and so on, to find reasonable accommodations. Students at this University are protected against sexual harassment and discrimination (Title IX), and minors have additional protections. For more information on your rights as a student and the resources available to you to resolve problems, please go the following site: www.uaf.edu/handbook.

[syllabus version: January 15, 2024]