3.1 Linear Models
a lesson for MATH F302 Differential Equations

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January 30, 2019

for textbook: D. Zill, *A First Course in Differential Equations with Modeling Applications*, 11th ed.

linear models

- not new material?
  - we got a good start back in §1.3
- this section requires ability to solve these types of DEs:
  1. the easiest DE: $y' = ky$
  2. separable linear equations from §2.2: $y' = g(x)y$ or $y' = ay + b$
  3. general linear equations from §2.3: $y' + P(x)y = f(x)$
- . . . which should be easy at this point
- the method in §2.3 handles all first-order *linear* equations, but the other methods are usually quicker for forms **1** or **2**
- these slides simply contain 5 exercises from §3.1 in this order:

  # 4, 42, 17, 36, 37

4. *The population of bacteria in a culture grows at a rate proportional to the number of bacteria present at time t. After 3 hours it is observed that 400 bacteria are present. After 10 hours 2000 bacteria are present. What was the initial number of bacteria?*

42. Fluctuating Population *The differential equation $dP/dt = (k \cos t)p$, where k is a positive constant, is a mathematical model for a population $P(t)$ that undergoes yearly seasonal fluctuations. Solve the equation subject to $P(0) = P_0$. Use a graphing utility to graph the solution for different choices of P_0 [and k].*
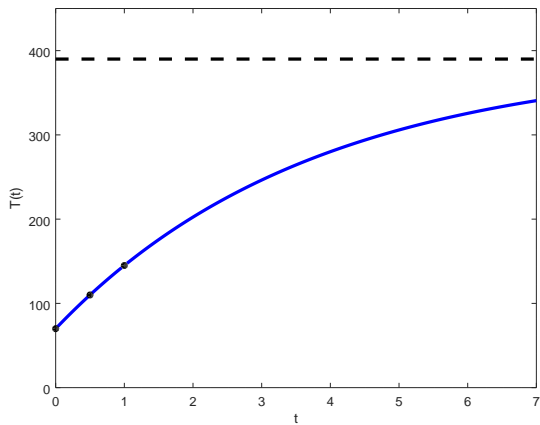
exercise 42, cont.
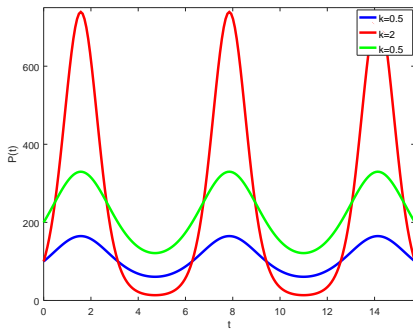
17. *A thermometer reading* $70°F$ *is placed in an oven preheated to a constant temperature. Through a glass window in the oven door, an observer records that the thermometer reads* $110°F$ *after* $\frac{1}{2}$ *minute and* $145°F$ *after 1 minute. How hot is the oven?*
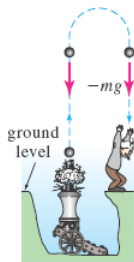
36. How High?—No Air Resistance *Suppose a small cannonball weighing 16 pounds is shot vertically upward, as shown in the Figure, with an initial velocity $v_0 = 300 ft/s$. The answer to "How high does the cannonball go?" depends on if we take air resistance into account.*

(a) *Suppose air resistance is ignored. If the positive direction is upward then a model for the height $s(t)$ of the cannonball is given by $m\, d^2s/dt^2 = -mg$ or equivalently $d^2s/dt^2 = -g$. Since $ds/dt = v(t)$ the last differential equation is the same as $dv/dt = -g$. We take $g = 32 ft/s^2$. Find the velocity $v(t)$ of the cannonball at time t.*

(b) *Use the result in part **(a)** to determine the height $s(t)$ of the cannonball, measured from ground level. Find the maximum height attained by the cannonball.*

37. How High?—Linear Air Resistance *Repeat exercise 36, but this time assume that air resistance is proportional to instantaneous velocity. In particular, suppose that the constant of proportionality is $k = 0.25$.[1] It stands to reason that the maximum height must be less than computed in exercise 36. Show this.*

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[1]Value for k changed to make effect more obvious.

exercise 37, cont.

exercise 37, cont. cont.
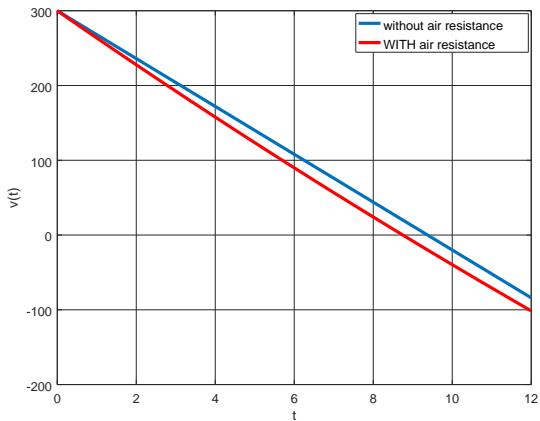
expectations

- to learn this material, just watching this video is *not* enough!
- also:
    - *explore* "found online" videos at
      bueler.github.io/math302/week5.html
    - *read* section 3.1 in the textbook
        - for quizzes and exams you must be able to handle the <u>examples</u> like 1-6 in this section
        - . . . and <u>exercises</u> like those in WebAssign
        - one topic you will not be responsible for: "series circuits" and other electical examples
    - *do* the WebAssign exercises for section 3.1
    - *start* Mini-Project 2, which uses a model like in section 3.1