2.2 Separable Equations

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D. Zill, *A First Course in Differential Equations with Modeling Applications*, 11th ed.

a *separable* differential equation can be put in the form

$$\frac{dy}{dx} = g(x)h(y)$$

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- **Example.**

$$\frac{dy}{dx} = \frac{y\cos(x)}{1+y^2}$$

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$$\frac{dy}{dx} = g(x)h(y)$$

- **Not an example.**

$$\frac{dy}{dx} = \cos(x) + y$$

- **Also not an example.**

$$\frac{dy}{dx} = \sin(x + y^2)$$

a *separable* differential equation can be put in the form

$$\frac{dy}{dx} = g(x)h(y)$$

OR in the form

$$p(y)\frac{dy}{dx} = g(x)$$

where $p(y) = 1/h(y)$

how to solve separable equations?
answer. clear denominators in

$$p(y)\frac{dy}{dx} = g(x)$$

and integrate both sides of

$$p(y)\,dy = g(x)\,dx$$

to get

$$\int p(y)\,dy = \int g(x)\,dx$$

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this works because of the chain rule

- **Example.**

$$\frac{dy}{dx} = xy^2$$

some familiar equations are separable

- **Example.**

$$\frac{dy}{dx} = -5y$$

what if there are initial conditions?

- **Example.** Find $z(t)$ if

$$z' = \frac{e^{-z}}{t}$$

and if $z(4) = 1$

you may end up only knowing the solution implicitly

- **Example.**

$$\frac{dy}{dx} = \frac{x(1-x)}{y(2+y)}$$

expectations. you must *read* section 2.2 to learn about some pitfalls, including

- making sure you find all solutions
- how to write solutions if you can't do the integrals by hand

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and *doing exercises* is essential