Name: ______________________________

MATH F302 UX1 Differential Equations (Bueler) 19–21 February 2019

SAMPLE Midterm 1

Proctored. 90 minutes. 100 points total. No textbook or notes or calculator. When it makes sense to do so, please circle your final answer(s).

1. (*10 pts*) Show that $6y^{1/3} - x^2 = 7$ defines an implicit solution to the ODE

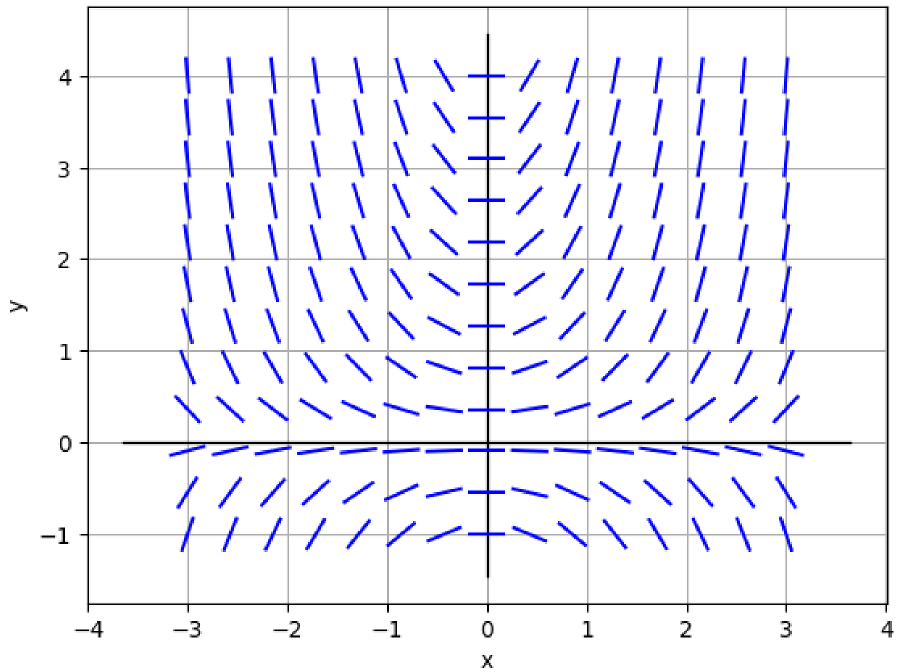
$$\frac{dy}{dx} = xy^{2/3}$$

Extra Credit. (*3 pts*) Note $6y^{1/3} - x^2 = 0$ also defines a solution to the DE in the above problem. Show this solution passes through $(0, 0)$. Then find another solution to the same DE which also passes through $(0, 0)$. Explain why this situation is *not* a violation of the theorem on existence and uniqueness for initial value problems.

2. **(a)** (*5 pts*) The ODE

$$\frac{dy}{dx} = xy$$

has the direction field shown below. Sketch the solution to this ODE which passes through $(x_0, y_0) = (0, 2)$.



(b) (*10 pts*) Solve the following initial value problem—*give an exact formula $y(x)$ for the solution!*—and then sketch the solution you find on the direction field above:

$$\frac{dy}{dx} = xy, \qquad y(1) = 1.$$

3. **(a)** (*5 pts*) Show that the following equation is exact:

$$\frac{t}{y}\,dy + (1 + \ln y)\,dt = 0.$$

(b) (*10 pts*) Solve the ordinary differential equation in part **(a)**. Write the solution as an explicit formula for $y(t)$.

4. (*15 pts*) Find the solution to the initial value problem:

$$\frac{dy}{dx} + 2y = e^{-x}, \qquad y(-1) = e.$$

5. (*15 pts*) Use Euler's method to approximate the solution to the initial value problem at the points $x = 0.1$ and $x = 0.2$, with steps of size $h = 0.1$:

$$\frac{dy}{dx} = x + y, \qquad y(0) = 1.$$

6. Recall that Newton's law of cooling is the DE

$$\frac{dT}{dt} = k(T_m - T) \tag{1}$$

where $T(t)$ is the temperature of the object, T_m is the ambient temperature, and k is a positive constant.

(a) (*5 pts*) Suppose that at $t = 0$ a glass of water at room temperature, say $70\,^\circ$F, is taken outside when it is $40\,^\circ$F. Suppose that for this glass of water, $k = 0.01$, and that we measure time in minutes. Including (1), write a concrete ODE IVP (i.e. initial value problem) for this situation.

(b) (*10 pts*) A more realistic situation is that though we know the ambient temperature—again assume it is $40\,^\circ$F—we don't know k for the glass of water. However, we measure that the water is at temperature $58\,^\circ$F after 3 minutes. Determine k. (*Hint.* You will need to solve (1). You can leave your answer as a formula for a specific number; a calculator would give a decimal value.)

7. (*15 pts*) Find the general solution

$$\frac{dz}{dt} = \frac{t-5}{z^2 - z}$$

(*Hint.* You may write the solution implicitly.)

SPACE AVAILABLE FOR COMPUTATIONS CLEARLY LABEL ANYTHING YOU WANT GRADED