Assignment #1

Due Friday, 27 January 2023, at the start of class

Please read sections 1.1–1.4 and 2.1–2.6 from the textbook.[1] The Problems on this assignment are designed to encourage additional review of certain important topics. In fact, please find three representative textbooks:

- Find a **calculus book**.
- Find an introductory textbook on **ordinary differential equations** (ODEs).
- Find an introductory textbook on **linear algebra**.

You will need these references throughout the semester. In particular, for this assignment, please review these two topics:

Calculus book: Taylor's theorem with the remainder formula.[2]
ODEs book: The solution of linear homogeneous constant-coefficient ODEs.

Problem P1. Calculate $257^{1/8}$ to within 10^{-5} of the exact value *without* any computing machinery except a pencil or pen. Prove that your answer has this accuracy. (*You should use a computer to* check *your by-hand value! Hint:* $f(x) = x^{1/4}$.)

Problem P2. Assume f' is continuous. Derive the remainder formula

$$\int_0^a f(x)\,dx = af(0) + \frac{1}{2}a^2 f'(\nu) \tag{1}$$

for some (unknown) ν between zero and a. (*Hint: Start by showing* $f(x) = f(0) + f'(\xi)x$ *where* $\xi = \xi(x)$ *is some number between* 0 *and* x.) Use two sentences to explain the meaning of (1) as an approximation to the integral. That is, answer the question "What properties of $f(x)$ or a would make the left-endpoint rule $\int_0^a f(x)\,dx \approx af(0)$ inaccurate?"

Problem P3. Get started in the programming language of your choice.[3] Now work at the command line to compute a finite sum approximation to

$$\sum_{n=1}^{\infty} \frac{\sin n}{n^3+1}.$$

Compute at least three partial (finite) sums, with increasing numbers of terms N. Turn your command-line work into a function `mysum(N)`, defined in a file `mysum.m`,

[1] R. J. LeVeque, *Finite Difference Methods for Ordinary and Partial Differential Equations*, SIAM Press 2007

[2] Taylor's theorem may be best explained by an undergraduate **numerical analysis** textbook.

[3] MATLAB/OCTAVE or PYTHON or JULIA.

and show that it works. Turn in both the command line session and the code. (*Hint: These can be very brief.*) Do you think you are getting close to the infinite sum, and if so, why?

Problem P4. Solve, by hand,

$$y'' + y' - 6y = 0, \quad y(2) = 0, \quad y'(2) = -1, \tag{2}$$

for the solution $y(t)$. Then find $y(4)$. (*This is a* prediction *of the outcome at $t = 4$, given initial data at $t = 2$ and a precise "law" about how $y(t)$ evolves in time, namely the differential equation itself.*) Give a reasonable by-hand sketch on t, y axes which shows the initial values, the solution, and the value $y(4)$.

Problem P5. Using Euler's method for approximately solving ODEs, write your own program to solve initial value problem (2) to find $y(4)$. A first step is to convert the second-order ODE into a system of two first-order ODEs. Use a few different step sizes, decreasing as needed, so that you get apparent three-digit accuracy. (*Hint: You can use a black-box ODE solver to* check *your work, but this is not required.*)

Problem P6. Solve, by hand, the ODE boundary value problem

$$y'' + 2y' - 3y = 0, \quad y(0) = \alpha, \quad y(\tau) = \beta, \tag{3}$$

for the solution $y(t)$. Note that α, β, τ are the data of the problem, so the solution will have these parameters in it.