SOLUTIONS

Math 615 NADE (Bueler) 15 February 2023 *Not turned in!*

1st-versus-2nd order equations, and singular perturbations

1. Solve by hand:

$$u'(x) = 0, \qquad u(0) = \alpha$$

$u(x) = c$

$\alpha = u(0) = c$

$u(x) = \alpha$

2. Solve by hand:

$$u'(x) = 0, \qquad u(0) = \alpha, \quad u(1) = \beta$$

no solution if $\alpha \neq \beta$

$\alpha = \beta$: $u(x) = \alpha$

3. Solve by hand:

$$u''(x) = 0, \qquad u(0) = \alpha, \quad u(1) = \beta$$

$u(x) = cx + d$

$\alpha = u(0) = d$

$\beta = u(1) = c + d$

$\rightarrow$ $u(x) = \alpha + (\beta - \alpha)x$

4. Solve by hand:

$$0.1u''(x) + u'(x) = 0, \qquad u(0) = \alpha, \quad u(1) = \beta$$

$u(x) = e^{rx} \implies 0.1r^2 + r = 0$

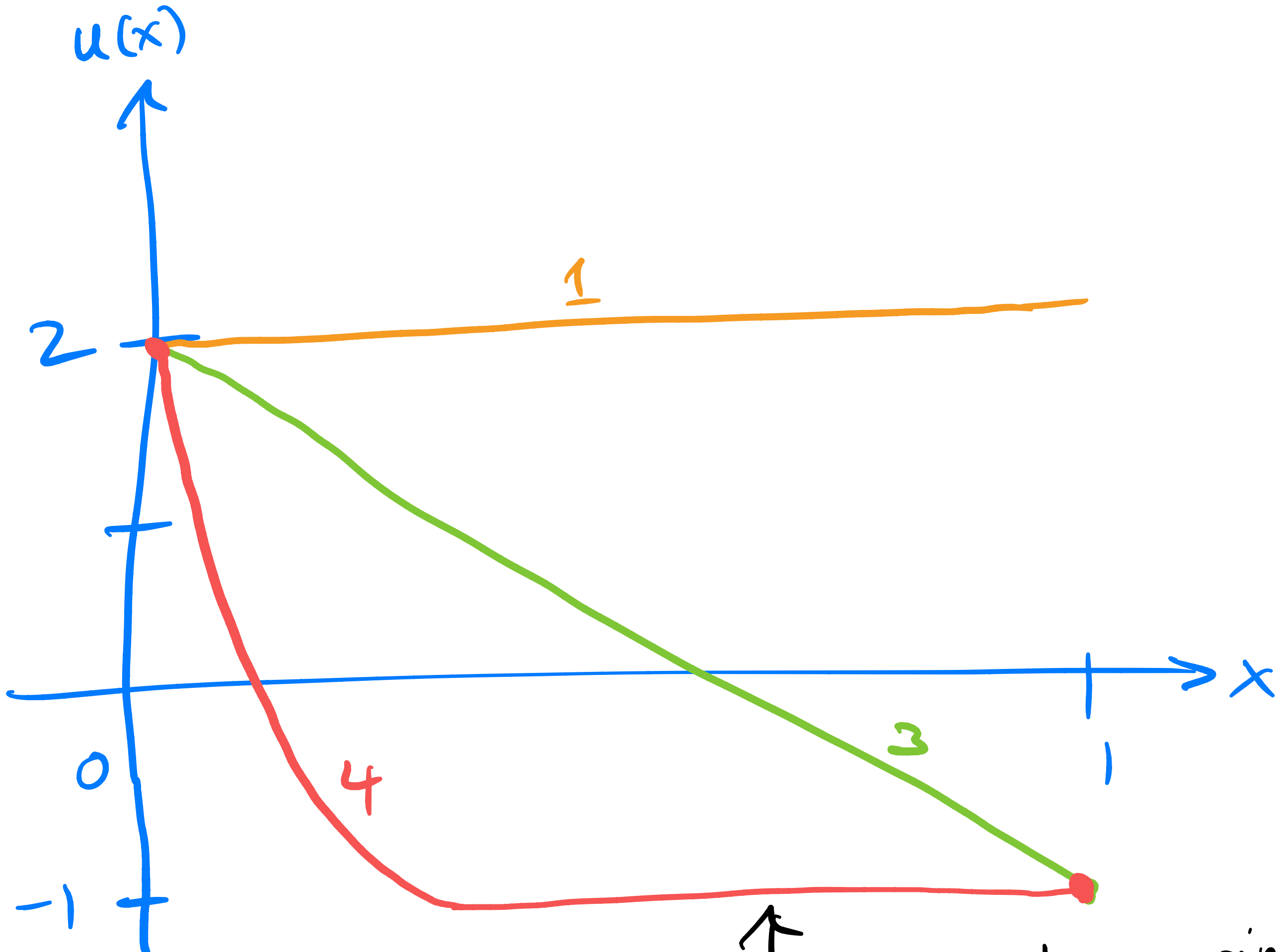
$r = 0, \quad r = -10$

$u(x) = c + de^{-10x}$

$\left.\begin{array}{l}\alpha = u(0) = c + d \\ \beta = u(1) = c + de^{-10}\end{array}\right\} \rightarrow d = \dfrac{\alpha - \beta}{1 - e^{-10}}, \quad c = \alpha - d$

$$u(x) = \left(\alpha - \frac{\alpha - \beta}{1 - e^{-10}}\right) + \left(\frac{\alpha - \beta}{1 - e^{-10}}\right)e^{-10x}$$

5. Sketch the graphs of all solutions from the previous page on the same axes, in the case where $\alpha = 2$ and $\beta = -1$. (*Make it big and label it clearly.*) Also sketch what happens in problem **4** if "0.1" is replaced by a much smaller $\epsilon > 0$; the ODE in question is $\epsilon u''(x) + u'(x) = 0$.

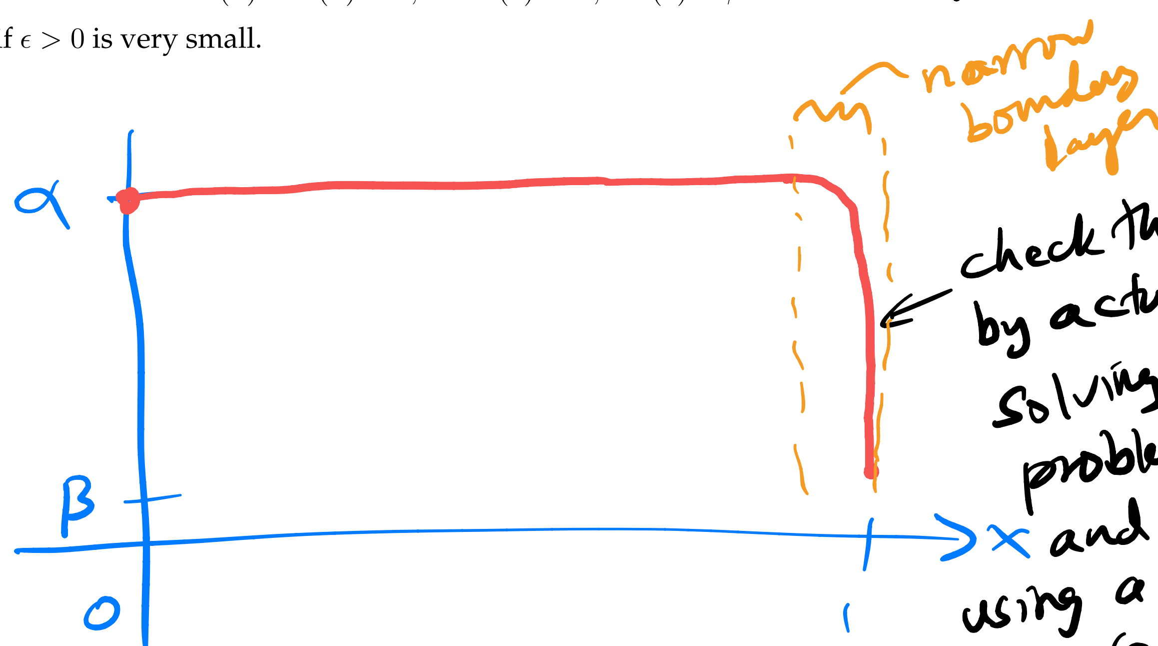


check this using a computer

6. Sketch what you think the solution of

$$\epsilon u''(x) - u'(x) = 0, \qquad u(0) = \alpha, \quad u(1) = \beta$$

will look like if $\epsilon > 0$ is very small.



check this by actually solving the problem, and then using a computer (e.g. $\varepsilon = 0.01, \alpha = 5, \beta = 1$)