Review Guide for In-Class Midterm Exam 2 (which is on Friday, 7 November 2025)

Midterm Exam 2 is *closed book* and *closed notes*. (No technology is allowed. Please bring nothing but a writing implement.) Exam questions will be of these types: prove propositions, state definitions and axioms, state theorems, and give examples with certain properties. You are expected to use reasonable and common notation; it is often wise to define your terms for clarity.

The Exam will cover sections 2.1, 2.3, 2.4, 2.5, 2.6, 2.7, and 3.2 of the textbook.[1] However, earlier material will inevitably arise, with completeness of the real numbers, the definition of supremum/infimum, and convergence of sequences all certainly needed.

This Review Guide list *specific material that might appear on the exam.* Material which is *significantly* different from what is listed below will *not* appear. The Exam will be built on topics that have appeared on homework and in lecture, and closely related things. I will *not* ask you to "state theorem 2.4.2," or anything like that which would require remembering locations in the book, instead asking you to "state the Monotone Convergence Theorem." However, numbers are listed below for ease of locating.

Strongly recommended: Get together with other students and work through this Review Guide. Be honest with yourself about what you can easily prove, versus what you should think harder about and/or practice. Talk it through and learn!

Definitions. Be able to state and use all of the definitions listed on the Review Guide for Midterm Exam 1. In addition, be able to state and use all of these definitions:

- $V_\epsilon(a) = (a-\epsilon, a+\epsilon)$ ϵ-neighborhood of a
- a sequence is increasing, decreasing, or monotone
- partial sum of an infinite series
- convergence of an infinite series (*is* convergence of the sequence of partial sums)
- harmonic series $\displaystyle\sum_{n=1}^{\infty}\frac{1}{n}$ and p-series $\displaystyle\sum_{n=1}^{\infty}\frac{1}{n^p}$
- arithmetic mean $(\frac{1}{2}(a+b))$ and geometric mean $(\sqrt{ab})$
- subsequence of a sequence
- Cauchy sequence
- geometric sequence
- alternating series
- absolutely-convergent and conditionally-convergent series
- open set
- arbitrary union and intersection of a collection of sets (see Problem 49, A7)
- limit point of a set

[1] S. Abbott, *Understanding Analysis*, 2nd edition, Springer Press 2015

- closed set
- isolated point of a set
- closure of a set

Theorems. Be able to state, apply, and prove[2] these theorems.

- algebraic limit theorem (Thm 2.3.3) ← **proof not expected**
- order limit theorem (Thm 2.3.4) ← **proof not expected**
- monotone convergence theorem (Thm 2.4.2)
- p-series converge if and only if $p > 1$ (Cor 2.4.7) ← **proof not expected**
- subsequences of a convergent sequence converge (Thm 2.5.2)
- Bolzano-Weierstrauss theorem (Thm 2.5.5) ← **proof not expected**
- convergent sequences are Cauchy sequences (Thm 2.6.2)
- Cauchy sequences are bounded (Lem 2.6.3)
- Cauchy criterion (Thm 2.6.4)
- (easy) algebraic limit theorem for series (Thm 2.7.1)
- Cauchy criterion for series (Thm 2.7.2)
- divergence test: if $\sum a_n$ converges then $a_n \to 0$ (Thm 2.7.3)
- comparison test (Thm 2.7.4)
- absolute convergence test (Thm 2.7.6)
- alternating series test (Thm 2.7.7) ← **proof not expected**
- absolute convergence allows rearrangement (Thm 2.7.10) ← **proof not expected**
- arbitrary unions and finite intersections of open sets are open (Thm 3.2.3)
- De Morgan's laws for arbitrary unions and intersections (see Problem 49, A7)
- a limit point of A is a limit of a sequence from A (Thm 3.2.5)
- a set is closed if and only if every Cauchy sequence converges (Thm 3.2.8)
- every real number is the limit of a rational sequence (Thm 3.2.10)
- the closure is a closed set (Thm 3.2.12)
- a set F is closed if and only if F^c is open (Thm 3.2.13)
- arbitrary intersections and finite unions of closed sets are closed (Thm 3.2.14)

Examples. Every numbered Example in the identified textbook sections is fair game, as are closely-related examples. Every question on homework Assignments 5–7 which mentions an example sequence or series or set, or is of the form "provide an example (with these properties)," is fair game, as are closely-related examples.

[2]Except if I say **proof not expected**, of course. Even though I won't ask you to prove these during the in-class exam, please read and understand these proofs!