Problem 68. *The function* $f(x) = 1/x^2$ *is uniformly continuous on* $(1,2)$*, but it is not uniformly continuous on* $(0,1)$*.*

Proof. □

Problem 69. *We say that a function* $f : A \to \mathbb{R}$ *is* Lipschitz *if there exists* $M > 0$ *so that*

$$\frac{|f(x) - f(y)|}{|x - y|} \leq M$$

for all $x, y \in A$*. If* f *is Lipschitz then* f *is uniformly continuous.*

Proof. □

Problem 70. *Let* f *and* g *be functions defined on an interval* A*. Assume both are differentiable at some point* $c \in A$*, and suppose* $k \in \mathbb{R}$*. Then*

(i) $(f+g)'(c) = f'(c) + g'(c)$

(ii) $(kf)'(c) = kf'(c)$

Proof. □

Problem 71. *Let* $h(x) = 1/x$ *and* $\ell(x) = 1/x^2$*. For* $c \neq 0$*, we have*

$$h'(c) = -\frac{1}{c^2}, \qquad \ell'(c) = -\frac{2}{c^3}$$

Proof. □

Problem 72. *Let* f *and* g *be functions defined on an interval* A*. Assume both are differentiable at some point* $c \in A$*, and suppose* $g(c) \neq 0$*. Then*

$$\left(\frac{f}{g}\right)'(c) = \frac{f'(c)g(c) - f(c)g'(c)}{g(c)^2}.$$

Proof. □

Problem 73. *For* $a \in \mathbb{R}$*, let*

$$f_a(x) = \begin{cases} x^a, & \text{if } x > 0 \\ 0, & \text{if } x \leq 0 \end{cases}$$

(a) For which values of a *is* $f_a(x)$ *continuous at* $x = 0$*?*

(b) What is the derivative $f_a'(x)$*, and what is its domain? For which values of* a *is* $f_a(x)$ *differentiable at* $x = 0$*? When is the derivative function* $f_a'(x)$ *continuous?*