Problem 6.

$$\bigcap_{n=1}^{\infty}(0, 1/n) = \emptyset.$$

Proof. □

Problem 7. *Given a function f and a subset A of its domain, consider the image $f(A) = \{f(x) : x \in A\}$.*

(a) An example of a function f, and two subsets A, B of the domain of f, for which $f(A \cap B) \neq f(A) \cap f(B)$ is

(b) If A, B are subsets of the domain of f then $f(A \cup B)$ IS RELATED IN SOME WAY TO $f(A) \cup f(B)$.

Proof. □

Problem 8. *If $a \in \mathbb{R}$ is an upper bound for $A \subset \mathbb{R}$, and if a is also an element of A, then $a = \sup A$.*

Proof. □

Problem 9. *(a) Let $A = \{m/n : \ m, n \in \mathbb{N}$ with $m < n\}$. Then $\inf A =$ and $\sup A =$.*

(b) Let $B = \{(-1)^m/n : n, m \in \mathbb{N}\}$. Then $\inf B =$ and $\sup B =$.

(c) Let $C = \{n/(3n+1) : n \in \mathbb{N}\}$. Then $\inf C =$ and $\sup C =$.

(d) Let $D = \{m/(m+n) : m, n \in \mathbb{N}\}$. Then $\inf D =$ and $\sup D =$.

Problem 10. *(a) If A and B are nonempty, bounded, and satisfy $A \subseteq B$ then $\sup A \leq \sup B$.*

(b) If $\sup A < \inf B$ for nonempty sets A and B, then there exists $c \in \mathbb{R}$ such that $a < c < b$ for all $a \in A$ and $b \in B$.

(c) If there exists $c \in \mathbb{R}$ satisfying $a < c < b$ for all $a \in A$ and $b \in B$ then $\sup A < \inf B$.

Problem 11. *Denote the irrational numbers by $\mathbb{I} = \mathbb{R} \setminus \mathbb{Q}$.*

(a) If $a, b \in \mathbb{Q}$ then $ab \in \mathbb{Q}$ and $a + b \in \mathbb{Q}$.

Proof. □

(b) If $a \in \mathbb{Q}$ and $t \in \mathbb{I}$ then $a + t \in \mathbb{I}$. If also $a \neq 0$ then $at \in \mathbb{I}$.

Proof. □

(c) Suppose $s, t \in \mathbb{I}$. Then PROPOSITION ABOUT WHETHER st AND $s + t$ ARE EITHER RATIONAL OR IRRATIONAL IN GENERAL.