So You're Going Outside (SYGO) Coronavirus Risk Estimator Derivation

Uploaded 2 days

$$P\left(\substack{\text{you're infected}\\\text{from going outside}}\right) = 1 - P\left(\substack{\text{you're uninfected}\\\text{from going outside}}\right) \quad (1)$$

$$\simeq 1 - \underbrace{P\left(\substack{\text{you're uninfected}\\\text{from facetouches}}\right)}_{\substack{\text{infected surfaces, \&}\\\text{infected ppl breathing on you.}}} + \underbrace{P\left(\substack{\text{you're infected}\\\text{by airborne virus}}\right)}_{\text{assumed miniscule.}} \quad (2)$$

$$P\left(\substack{\text{you're uninfected}\\\text{from facetouches}}\right) = \prod_{\substack{\text{num}\\\text{facetouches}}} \left[\overset{=1}{\cancel{P\left(\substack{\text{you're}\\\text{uninfected}} \middle| \substack{\text{clean}\\\text{hands}}\right)}} P\left(\substack{\text{clean}\\\text{hands}}\right) + \underbrace{P\left(\substack{\text{you're}\\\text{uninfected}} \middle| \substack{\text{virus}\\\text{hands}}\right)}_{p_{u\text{-}vh}} P\left(\substack{\text{virus}\\\text{hands}}\right) \right] \quad (3)$$

Let $p_{u\text{-}vh}$ = the probability that touching your face does not infect you, given that you have virus on your hands.
N_F = the number of times you touched your face.
Also note $P\left(\substack{\text{clean}\\\text{hands}}\right) = 1 - P\left(\substack{\text{virus}\\\text{hands}}\right)$. Then:

$$P\left(\substack{\text{you're uninfected}\\\text{from facetouches}}\right) = \left[P\left(\substack{\text{clean}\\\text{hands}}\right) + p_{u\text{-}vh}\left(1 - P\left(\substack{\text{clean}\\\text{hands}}\right)\right)\right]^{N_F} \quad \boxed{(4)}$$

This... seems like unnecessary complexity. Lets simplify: assume you touch your face once, with a transmission probability $p_{i\text{-}vh}$ = probability you're infected given that you have virus on your hands. Next, look at the end of your excursion – so, they're when $P\left(\substack{\text{clean}\\\text{hands}}\right)$ is lowest. This lets us calculate a "worst case" $P_{wc}\left(\substack{\text{clean}\\\text{hands}}\right)$:

$$P_{wc}\left(\substack{\text{clean}\\\text{hands}}\right) = \prod_{\substack{\text{all}\\\text{opportunities}\\\text{for contaminatn}}} P\left(\substack{\text{clean}\\\text{hands}} \middle| \substack{\text{contamination}\\\text{opportunity}}\right) \quad (5)$$

Let's break this down into the two distinct kinds of contamination opportunities: surface-touching and "breath events" – one unit of a person breathing on you, e.g. through a cough, talking, etc.

$$P_{wc}\left(\substack{\text{clean}\\\text{hands}}\right) = \left[\prod_{\substack{\text{surfaces}\\\text{touched}}} P\left(\substack{\text{clean}\\\text{hands}} \middle| \substack{\text{surface}\\\text{touched}}\right)\right]\left[\prod_{\substack{\text{breath}\\\text{events}}} P\left(\substack{\text{clean}\\\text{hands}} \middle| \substack{\text{breath}\\\text{event}}\right)\right] \quad (6)$$

note: in eq. (?!) I re-order this to product over distinct people first, and breath events second

Let N_S = number of surfaces touched
N_B = number of breath events

$$P_{wc}\left(\substack{\text{clean}\\\text{hands}}\right) = \left[\underbrace{P\left(\substack{\text{clean}\\\text{hands}} \middle| \substack{\text{surface}\\\text{touched}}\right)}_{1}\right]^{N_S}\left[\underbrace{P\left(\substack{\text{clean}\\\text{hands}} \middle| \substack{\text{breath}\\\text{event}}\right)}_{2}\right]^{N_B} \quad (7)$$

Let's model 1 first, and come back to 2 later.

$$P\left(\substack{\text{clean}\\\text{hands}} \middle| \substack{\text{surface}\\\text{touched}}\right) = \underbrace{P\left(\substack{\text{clean}\\\text{hands}} \middle| \substack{\text{virus}\\\text{surface}}\right)}_{p_{hu\text{-}vs}} P\left(\substack{\text{virus}\\\text{surface}}\right) + \overset{=1}{\cancel{P\left(\substack{\text{clean}\\\text{hands}} \middle| \substack{\text{clean}\\\text{surface}}\right)}} P\left(\substack{\text{clean}\\\text{surface}}\right)$$

APPROXIMATES: "contamination" status of a surface as a binary.

Let $p_{hu\text{-}vs}$ = probability that your hands are uninfected given that you touched a contaminated surface.
p_{vs} = the probability the surface is contaminated with virus.

Then:

$$P\left(\substack{\text{clean}\\\text{hands}} \middle| \substack{\text{surface}\\\text{touched}}\right) = p_{hu\text{-}vs}\, p_{vs} + (1 - p_{vs}) = 1 - p_{vs}(1 - p_{hu\text{-}vs}) \quad (8)$$

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LEGIBILITY NOTE:

Over the course of these four pages, the names change a bit, though the underlying mathematical structures don't.

The two main changes:
1. Renaming the three components. I keep "surface-based" transmission the same name, but change "breathing-based" to "warm-body-based" and "inhalation-based" to "wildcard." Why? By "breathing-based" I had initially been modeling "cough --> your clothes --> your hand --> your face" as the main transmission vector, and I assumed that "inhalation based" (the virus flies directly into your nose) was rare enough to be the non-zero remaining fluke. But what I was /really/ getting at with the latter two categories was "scales with diffusion from nearby people" and "doesn't." My calculations don't care about infection mechanics: "virus lands on your clothes if it's close enough" vs. "you inhale it if it's close enough." They're just based on the existence of a transmission mode subject to diffusion effects where 6 feet is effective protection.
2. For this same reason, I de-emphasize "facetouches" as the vector, focusing only on the split between "surface-based," "warm-body (diffusion)-based," and wildcard transmission modes. Why? The mechanics of how the diffusion actually infects you are still unclear. But, the fact that diffusion modulates infection probability (via distance, masks, etc.) is well established.

Subbing (8) back into part of (7) we get:

$$\left[P\left(\begin{smallmatrix}\text{clean}\\ \text{hands}\end{smallmatrix} \middle| \begin{smallmatrix}\text{surface}\\ \text{touched}\end{smallmatrix}\right)\right]^{N_s} = \left[1 - p_{vs}\left(1 - p_{ha\text{-}vs}\right)\right]^{N_s} \tag{9}$$

* binomial approximation *

$$\approx 1 - N_s p_{vs}\left(1 - p_{ha\text{-}vs}\right) \tag{10}$$

Let's expand p_{vs}.

Let N_p = the number of people who touched the surface since it was last sanitized.

p_i = probability the person is infected

APPROXIMATE: - every touch contaminates the surface (by an infected person)

- virus lasts until sanitizer kills it. no premature death

$$p_{vs} = 1 - p_{clean} = 1 - (1 - p_i)^{N_p} \tag{11}$$

* binomial approx *

$$p_{vs} \approx 1 - 1 + N_p p_i$$

$$p_{vs} \approx N_p p_i \tag{12}$$

Plugging (12) into (10):

$$\left[P\left(\begin{smallmatrix}\text{clean}\\ \text{hands}\end{smallmatrix} \middle| \begin{smallmatrix}\text{surface}\\ \text{touched}\end{smallmatrix}\right)\right]^{N_s} \approx 1 - \left(1 - p_{ha\text{-}vs}\right) N_s N_p p_i \tag*{$\boxed{(13)}$}$$

& for kicks, (11) into (9) to get the exact version:

" "
$$= \left[1 - \left(1 - (1 - p_i)^{N_p}\right)\left(1 - p_{ha\text{-}vs}\right)\right]^{N_s} \tag{14}$$

... clearly the binomial-approx'd version is nicer, ha.

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Next up: let's model breathing-transmission.
Let N_p = num distinct people you encounter. Then:

$$P\left(\text{clean hands} \mid \text{breath events}\right) = \prod_{\text{people}} \left[P\left(\text{clean hands} \mid \text{person infected}\right) p_i + \cancel{P\left(\text{clean hands} \mid \text{person uninf}\right)}^{=1} (1-p_i) \right]$$

$$= \left[1 - p_i + p_i P\left(\text{clean hands} \mid \text{person infected}\right) \right]^{N_p} = \left[1 - \left(1 - P\left(\text{clean hands} \mid \text{person inf}\right)\right)_{p_i} \right]^{N_p} = \left[1 - {}_{p_i} P\left(\text{virus hands} \mid \text{person inf}\right) \right]^{N_p}$$

binomial approx

$$P\left(\text{clean hands} \mid \text{breath events}\right) \approx 1 - N_p \underbrace{P\left(\text{virus hands} \mid \text{person inf}\right)} p_i \approx 1 - \sum_{\text{all people}} p_i P\left(\text{virus hands} \mid \text{person inf}\right) \qquad (15)$$

Cool! We've accounted for distinctness of people. Now time to account for breathing.
Let N_b = number of breathing events
For 1 person:

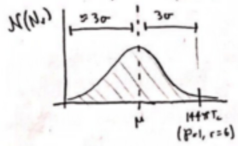
$$P\left(\text{virus hands} \mid \text{person inf}\right) = 1 - P\left(\text{clean hands} \mid \text{person inf}\right) \qquad (16)$$

$$\text{" "} = 1 - \left[\underbrace{P\left(\text{virus doesn't land on you} \mid \text{breathing event, infected person}\right)} \right]^{N_b} \qquad (17)$$

Let r = your distance from the person
T_c = "contamination density threshold," in units of $\frac{\text{"\# viral particles"}}{m^2}$
γ = "projectiness." Defined as $=1$ for a cough, to match the 6 Foot social distancing recommendation. Unitless, but it scales "# viral particles" so $\gamma=0.1$ for mask-wearing, and $\gamma=0.3$ for talking seem reasonable.

ASSUME: # viral particles per breath event is emitted in a normal distribution from which "6 feet" provides 3 z scores of safety against exceeding T_c.

Find: $- P(N_v < 4\pi\gamma r^2 T_c)$
$- \mu, \sigma$ for N_v.



so $\mu = 3\sigma$ and $3\sigma = 144\pi T_c - \mu$

$2\mu = 144\pi T_c$

$\mu = 72\pi T_c,\ \sigma = 24\pi T_c$ ✓ units check out: $\frac{\#\text{particles}}{\cancel{m^2}} \cdot \cancel{m^2}$

COOL! so let

$$N_v = \mathcal{N}(\underbrace{72\pi \cancel{T_c}}_{\mu}, \underbrace{24\pi \cancel{T_c}}_{\sigma}) \qquad \boxed{(18)}$$

"=1" let's take units that rescale to eliminate this constant factor

then:

$$P\left(\text{virus doesn't land on you} \mid \text{breath event, infected person}\right) = P(N_v < 4\pi\gamma r^2 \cancel{T_c}) \qquad (19)$$

... this looks annoying but is actually readily simulatable with the Gaussian CDF.
Backfilling (19) → (17) → (15):

$$P\left(\text{you get virus hands} \mid \text{person inf}\right) = 1 - \left(P(N_v < 4\pi\gamma r^2 \cancel{T_c})\right)^{N_b}$$

$$P\left(\text{clean hands} \mid \text{all breath events}\right) = 1 - \sum_{N_p} p_i \left(1 - \left(P(N_v < 4\pi\gamma r^2 \cancel{T_c})\right)^{N_b}\right)$$

$$P\left(\text{clean hands} \mid \text{all breath events}\right) = 1 - N_p p_i + p_i \sum_{N_p} \left(P\left(N_v < 4\pi\gamma r^2 \cancel{T_c}\right)\right)^{N_b} \qquad \boxed{(20)}$$

actually, rescale:

$$\frac{\mu = 72\pi T_c \quad \sigma = 24\pi T_c \quad x = 4\pi\gamma r^2 T_c}{4\pi T_c}$$

$\rightarrow \mu = 18,\ \sigma = 6,\ x = \gamma r^2$

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privacy