ASTRO 732 - FALL 2019

Homework Set 5

due date: November 30, 2019

Problem 5.1: Gauss-Laguerre Integration
The convolution of an exponential decay and a Gaussian resolution function is given by

$$f(t) = \int_0^\infty \frac{e^{\frac{-(t-t')^2}{2\sigma_t^2}}}{\sqrt{2\pi\sigma_t^2}} \frac{e^{\frac{-t'}{\tau}}}{\tau} dt'. \tag{1}$$

Evaluate this integral using Gauss-Laguerre quadrature for $\tau = 1$, $\sigma_t = 0.5$ and at 100 points for $t \in [-2, 6]$. Use N=5, 10, 15, 20, and 30 knots and compute both the relative and absolute error using the analytic solution

$$f(t) = \frac{1}{2\tau} \exp\left(\frac{\sigma_t^2}{2\tau^2} - \frac{t}{\tau}\right) \operatorname{erfc}\left(\frac{\sigma_t}{\sqrt{2}\tau} - \frac{t}{\sqrt{2}\sigma_t}\right). \tag{2}$$

Problem 5.2: Acceptance/Rejection Method Use the Acceptance-Rejection Method to draw random deviates from a probability distribution shaped like the first half-cycle of a sine function (properly normalized) on the interval $[0, \pi]$. I would like you to use two different functions for $f(x)$:

1. a constant $f(x)$ for the full interval,
2. any other non-constant function of your choice.

In each case you should compare the sine wave to a histogram your random deviates to show that you have faithfully sampled from the distribution. Finally, for each case show that the inefficiency factor (the number of trials you have to make divided by the number of good deviates that you get) is equal to the area under $f(x)$ on the interval $[0, \pi]$. Note that the function that you choose **must** have an efficiency factor that is smaller than that of the constant function.

Problem 5.4: Emcee Method Use the emcee python package to draw random deviates from a probability distribution shaped like the first half-cycle of a sine function (properly normalized) on the interval $[0, \pi]$.

Problem 5.5 (part A): Focussing the LMT Like all telescopes, the LMT must be focused several times a night. To do this we make a small map of a bright point-like source and then adjust the z-position of the secondary mirror until the point spread function (the telescope's response function) is round and maximized. In this problem you will focus the telescope using actual data taken on February 7, 2015.

The data is stored in the file maps.tar.gz on the class Moodle page. You can unpack that on a linux computer with the commands: gunzip maps.tar.gz and tar -xcv maps.tar. This will give you a set of 10 fits files (remember fits from homework #1) that you will read. The files with "signal" in the name are the maps of the source. The files with "weight" in the name are images with each pixel representing the weight ($1/\sigma^2$) of each pixel in the corresponding signal map.
Putting all this together, use the lmfit Levenberg-Marquardt fitting package to fit each image to a 2-d gaussian. Find the best fit amplitude in each case and fit the best fit amplitudes to the function

$$f(z) = a_0 + a_1 z + a_2 z^2. \tag{3}$$

where z is the z-position of the secondary as given in the table below. Use the results of this fit to determine the best-fit positon of the LMT's secondary mirror.

Observation #	**z-position (mm)**
35114	-3.0
35115	-2.0
35116	-1.0
35117	0.0
35118	1.0

Problem 5.5 (part b): Use either the procedure outlined in the class18 notes or the emcee python package to simulate the posterior probability distributions for all of the fitted parameters in one of the 2-d gaussian fits you did. Describe what you see.

Problem 5.6: Metropolis-Hastings MCMC (linear model): Write a simple Metropolis-Hastings MCMC to fit the data from linfit_data.npz to the linear function

$$f(\vec{x}|\vec{a}) = a_0 + a_1 \vec{x}. \tag{4}$$

Plot up the joint posterior probability distribution for the two parameters as well as marginalized posterior probabilities for each parameter. Clearly describe your process for obtaining a good acceptance rate through tuning the average step sizes via your proposal distributions. Overplot the joint 68.3% and 95% confidence intervals for the two parameters.

Problem 5.7: Metropolis-Hastings MCMC (nonlinear model): Use the same Metropolis Hastings code to fit the data from gaussfit_data.npz to the gaussian function

$$f(\vec{x}|\vec{a}) = a_0 + a_1 e^{-\frac{1}{2}\left(\frac{x-a_2}{a_3}\right)^2} \tag{5}$$

Overplot the joint 68.3% and 95% confidence intervals for all combinations of parameters.

Problem 5.8: MCMC Hammer (linear model): Repeat problem 5.6 using either the emcee package or by implementing your own slide-step sampler in place of the M-H sampler.

Make sure you clearly describe both your implementation as well as what checks you do to make sure that things have worked well.

Problem 5.9: MCMC Hammer (nonlinear model): Repeat problem 5.7 using either the emcee package or by implementing your own slide-step sampler in place of the M-H sampler. Make sure you clearly describe both your implementation as well as what checks you do to make sure that things have worked well.