

Symbolic execution

Software Analysis

Topic 7

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Today's menu

Symbolic execution

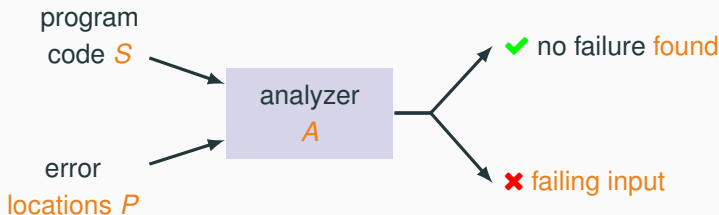
- Classic symbolic execution

- Dynamic symbolic execution

Challenges, tools, and applications

- A brief demo of Klee

Symbolic execution: the very idea



Symbolic execution:

- analyzes **real** program **code**
- analyzes **reachability** properties (equivalent to assertions), which offers a good flexibility
- is **automatic**
- is **unsound** because it may not analyze all possible inputs
- is **complete** because every failure comes with a concrete input that triggers it

Static vs. dynamic, reloaded

Static:

- without executing the software
- on generic/abstract inputs
- based on symbolic constraints
- typically sound and incomplete

Dynamic:

- while executing the software
- on specific/concrete inputs
- based on concrete states
- typically unsound and complete

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symbolic execution



“Symbolic execution” is a technique that executes programs on symbolic values, so that the output is expressed as a function of symbolic input.

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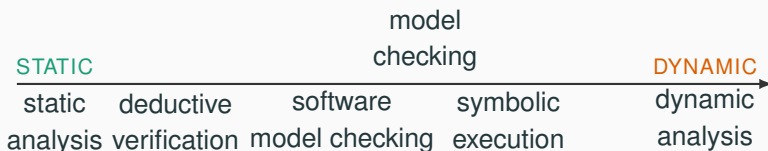
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symbolic execution



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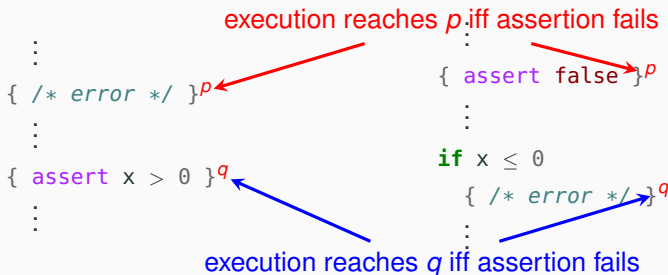


Reachability

Reachability properties are of the form:
does **some** execution of program S **reach** program point p in S ?

Often the program points are location of **error**, and hence reachability tries to verify that the error locations **cannot** be reached.

Reachability properties have the same **expressiveness** as **assertions**:



Symbolic execution

Symbolic execution

Classic symbolic execution

Symbolic state and path condition

Symbolic execution **executes** programs on **symbolic values**.

Symbolic state and path condition

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The **state** keeps track of the (symbolic) **value** of every variable:

```
procedure max(x, y: Integer):  
    (z: Integer)
```

State:

| | | |
|---|---|---|
| x | y | z |
|---|---|---|

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procedure max(x, y: Integer):  
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    z := x
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procedure max(x, y: Integer):
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    (z: Integer)
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    z := x
```

```
    if (z < y)
```

State:

| | x | y | z |
|-------|-------|-------|-------|
| then: | x_0 | y_0 | x_0 |
| else: | x | y | z |
| | x_0 | y_0 | x_0 |

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- a **path condition** π records the conditions of each path

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```
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```

```
    if (z < y)
```

State:

| | x | y | z | π |
|-------|-------|-------|-------|----------------|
| then: | x_0 | y_0 | x_0 | $x_0 < y_0$ |
| else: | x | y | z | π |
| | x_0 | y_0 | x_0 | $x_0 \geq y_0$ |

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- execution continues along every path that is **feasible**:
(the conjunction of) its path conditions is **satisfiable**

procedure **max**(x, y : **Integer**):

(z : **Integer**)

$z := x$

if ($z < y$)

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State:

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procedure **max**(x, y : **Integer**):

(z : **Integer**)

$z := x$

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assert $z \geq x \wedge z \geq y$

State:

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| x_0 | y_0 | x_0 | $x_0 \geq y_0$ |

Final states

When symbolic execution hits an **exit point** p :

```
procedure max(x, y: Integer):  
    (z: Integer)  
    z := x  
    if (z < y)  
        z := y  
    assert z ≥ x ∧ z ≥ y
```

Final states

When symbolic execution hits an **exit point** p :

normal exit point: π is **satisfiable**.

Any satisfying assignment to π 's variables gives a **concrete input** that reaches p in a concrete execution.

```
procedure max( $x, y$ : Integer):  
    ( $z$ : Integer)
```

```
     $z := x$ 
```

```
    if ( $z < y$ )
```

```
         $z := y$ 
```

```
    assert  $z \geq x \wedge z \geq y$ 
```

Normal exit:

$$\frac{x \quad y \quad z \quad \pi}{x_0 \quad y_0 \quad y_0 \quad \begin{array}{l} x_0 < y_0, \\ y_0 \geq x_0 \wedge y_0 \geq y_0 \end{array}}$$

satisfying assignment: $x_0 = 1, y_0 = 3$

Final states

When symbolic execution hits an **exit point** p :

normal exit point: π is **satisfiable**.

Any satisfying assignment to π 's variables gives a **concrete input** that reaches p in a concrete execution.

error exit point : π should be **unsatisfiable**.

Any satisfying assignment to π 's variables gives a concrete input that triggers a **failure** at p .

procedure $\text{max}(x, y: \text{Integer})$:

$z := x$

if $(z < y)$

$z := y$

assert $z \geq x \wedge z \geq y$

$(z: \text{Integer})$

Error exit:

| x | y | z | π |
|-------|-------|-------|----------------------------|
| <hr/> | | | |
| x_0 | y_0 | x_0 | $x_0 \geq y_0,$ |
| | | | $x_0 < x_0 \vee x_0 < y_0$ |

there is **no satisfying** assignment

Execution trees

All execution paths are collected in an **execution tree**, where final nodes are marked as **normal** ✓ or **error** ✗.

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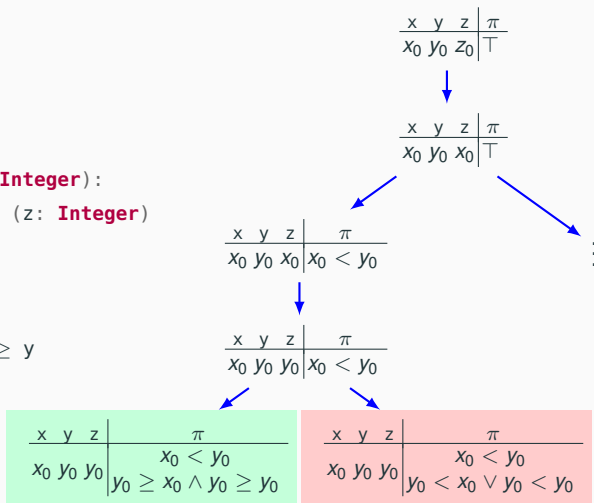
        z := y

```

```

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```



What does this program do?

```
procedure swap(x, y: Integer):  
    (X, Y: Integer)  
ensure X = old(y)  $\wedge$  Y = old(x)  
    X := x + y  
    Y := X - y  
    X := X - Y  
assert (X = y  $\wedge$  Y = x)
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$$\frac{x \quad y \quad X \quad Y}{x_0 \quad y_0} \bigg| \frac{\pi}{\top}$$

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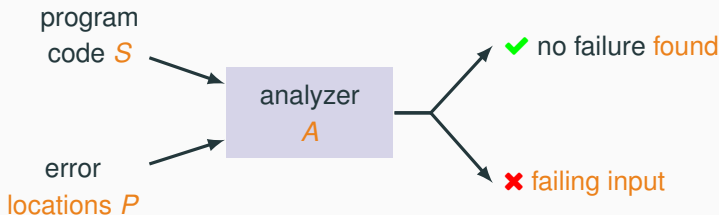


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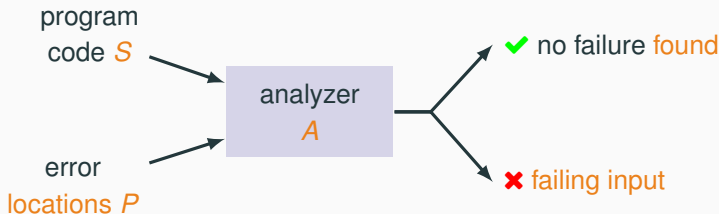
Completeness of symbolic execution



Whenever symbolic execution reaches an **assertion** it tries to solve a **constraint**. This is equivalent to solving a **reachability** problem:

A program is **correct** iff all its **error** locations are **unreachable**

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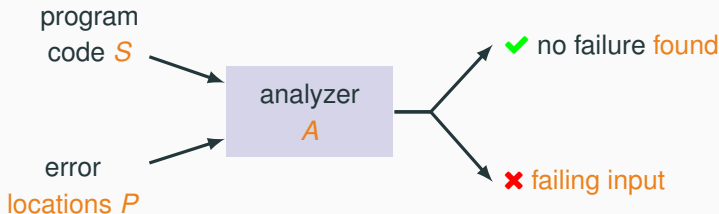


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A program is **correct** iff all path **constraints** of **error** are **unsatisfiable**

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complete: error location e is reachable

\implies path constraint π_e is satisfiable

\implies assignment to π_e is failure-inducing input

Soundness of symbolic execution

A program is **correct** iff all its **error** locations are **unreachable**.
that is,

A program is **correct** iff all path **constraints** of **error** are **unsatisfiable**.

What happens when **none** of the **failing locations** have satisfiable path constraints?

Soundness of symbolic execution

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that is,

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What happens when **none** of the **failing locations** have satisfiable path constraints?

- There may be **other paths** that the symbolic execution could not explore in reasonable time.
- A path constraints may be **too complex** for the constraint solver, which **cannot find** a satisfying assignment even if one **exists**.

Since the state-space exploration performed by symbolic execution is incomplete (it has limitations), symbolic execution is **unsound** as a **verification** technique.

Sources of unsoundness

Unsoundness of symbolic execution may originate in **language features** or **state-space size**:

complex constraints involving nonlinear arithmetic ($v * v \% 50$), bit-wise operations ($v << 1$), or strings ("**foo**" + "**bar**"), about which the constraint solver cannot reason

external code whose source is not available (for example, calls to a pre-compiled library), whose behavior the constraint solver does not know

large or infinite state spaces (for example, involving recursion and loops)

If we can **guarantee** that all execution paths have been enumerated, and the constraint solver has a **complete search procedure** for the kinds of path constraints that were generated, then symbolic execution's search becomes exhaustive (and sound).

In all other cases, it is just a thorough **test-case generation** process.

Unsoundness: external code

```
procedure m2(x, y: Integer):  
  z := bb(y) // black-box function  
  if z = x  
    z := y + 10  
  assert x ≤ z
```

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$$\frac{x \ y \ z}{x_0 \ y_0 \ \text{bb}(y_0)} \mid \frac{\pi}{\top}$$

$z = x$

$\neg (z = x)$

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$$\frac{x \ y \ z}{x_0 \ y_0 \ \text{bb}(y_0)} \mid \frac{\pi}{\text{bb}(y_0) \neq x_0}$$



$$\frac{x \ y \ z}{x_0 \ y_0 \ y_0 + 10} \mid \frac{\pi}{\text{bb}(y_0) = x_0}$$

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The constraint solver cannot find a satisfying assignment to path conditions involving $\text{bb}(y_0)$ because it does not know anything about how function bb behaves. Thus, those paths remain **unexplored**.

Symbolic execution

Dynamic symbolic execution

Symbolic execution's coming of age

Symbolic execution was first presented in two 1976 papers:

A PROGRAM TESTING SYSTEM*

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However, it hasn't become used in practice until about 30 years later:

Execution Generated Test Cases: How to Make Systems Code Crash Itself

Cristian Cadar and Dawson Engler*

DART: Directed Automated Random Testing

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Computer Science Department
University of Illinois at Urbana-Champaign
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Symbolic execution has finally become practical thanks to:

- spectacular progress in constraint solving – the same SMT solvers that power much of deductive verification
- the combination of dynamic and symbolic execution – which can alleviate several of the shortcomings of classical symbolic execution

Dynamic symbolic execution

The basic idea of **dynamic symbolic execution** is performing symbolic execution **alongside** a normal **dynamic** execution (on concrete inputs).

The **state** of dynamic symbolic execution combines a **symbolic** part and a **concrete** part, which are used as needed to make progress in the state-space exploration.

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concrete to symbolic: whenever a **concrete** computation **terminates**: **negate** one component of the **symbolic** path constraint, and solve it to get inputs exploring a **new path**

In the following example of procedure m , we start from a (random) concrete input, and then solve path constraints to explore new paths.

Dynamic symbolic execution: example

```
procedure m(x, y: Integer):
```

```
  z := 2*y
```

```
  if z = x
```

```
    z := y + 10
```

```
    assert x ≤ z
```


Dynamic symbolic execution: example

| x | y | z | π |
|-------|-------|---|--------|
| x_0 | y_0 | | \top |
| 22 | 7 | | |

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```

Dynamic symbolic execution: example

| x | y | z | π |
|-------|-------|---|--------|
| x_0 | y_0 | | \top |
| 22 | 7 | | |



| x | y | z | π |
|-------|-------|--------|--------|
| x_0 | y_0 | $2y_0$ | \top |
| 22 | 7 | 14 | |

```
procedure m(x, y: Integer):
```

```
  z := 2*y
```

```
  if z = x
```

```
    z := y + 10
```

```
    assert x ≤ z
```

Dynamic symbolic execution: example

```
procedure m(x, y: Integer):
```

```
  z := 2*y
```

```
  if z = x
```

```
    z := y + 10
```

```
    assert x ≤ z
```

| x | y | z | π |
|-------|-------|---|---|
| x_0 | y_0 | | ⊤ |
| 22 | 7 | | |



| x | y | z | π |
|-------|-------|--------|---|
| x_0 | y_0 | $2y_0$ | ⊤ |
| 22 | 7 | 14 | |



$\neg (z = x)$

| x | y | z | π |
|-------|-------|--------|-----------------|
| x_0 | y_0 | $2y_0$ | $2y_0 \neq x_0$ |
| 22 | 7 | 14 | |

Dynamic symbolic execution: example

```
procedure m(x, y: Integer):
```

```
  z := 2*y
```

```
  if z = x
```

```
    z := y + 10
```

```
    assert x ≤ z
```

| x | y | z | π |
|-------|-------|---|---|
| x_0 | y_0 | | ⊤ |
| 22 | 7 | | |



| x | y | z | π |
|-------|-------|--------|---|
| x_0 | y_0 | $2y_0$ | ⊤ |
| 22 | 7 | 14 | |



$\neg (z = x)$

| x | y | z | π |
|-------|-------|--------|-----------------|
| x_0 | y_0 | $2y_0$ | $2y_0 \neq x_0$ |
| 22 | 7 | 14 | |

solve: $\neg(2y_0 \neq x_0)$

Dynamic symbolic execution: example

| x | y | z | π |
|-------|-------|---|--------|
| x_0 | y_0 | | \top |
| 2 | 1 | | |

```
procedure m(x, y: Integer):
```


```
  z := 2*y
```

```
  if z = x
```

```
    z := y + 10
```

```
    assert x ≤ z
```

solve: $\neg(2y_0 \neq x_0)$



Dynamic symbolic execution: example

| x | y | z | π |
|-------|-------|---|--------|
| x_0 | y_0 | | \top |
| 2 | 1 | | |



| x | y | z | π |
|-------|-------|--------|--------|
| x_0 | y_0 | $2y_0$ | \top |
| 2 | 1 | 2 | |

```
procedure m(x, y: Integer):
```

```
  z := 2*y
```

```
  if z = x
```

```
    z := y + 10
```

```
    assert x ≤ z
```

Dynamic symbolic execution: example

```
procedure m(x, y: Integer):
```

```
  z := 2*y
```

```
  if z = x
```

```
    z := y + 10
```

```
    assert x ≤ z
```

| x | y | z | π |
|----------------|----------------|---|---|
| x ₀ | y ₀ | | ⊤ |
| 2 | 1 | | |



| x | y | z | π |
|----------------|----------------|-----------------|---|
| x ₀ | y ₀ | 2y ₀ | ⊤ |
| 2 | 1 | 2 | |

z = x



| x | y | z | π |
|----------------|----------------|-----------------|----------------------------------|
| x ₀ | y ₀ | 2y ₀ | 2y ₀ = x ₀ |
| 2 | 1 | 2 | |

Dynamic symbolic execution: example

```
procedure m(x, y: Integer):
```

```
  z := 2*y
```

```
  if z = x
```

```
    z := y + 10
```

```
    assert x ≤ z
```

| x | y | z | π |
|----------------|----------------|---|---|
| x ₀ | y ₀ | | ⊤ |
| 2 | 1 | | |



| x | y | z | π |
|----------------|----------------|-----------------|---|
| x ₀ | y ₀ | 2y ₀ | ⊤ |
| 2 | 1 | 2 | |

z = x



| x | y | z | π |
|----------------|----------------|-----------------|----------------------------------|
| x ₀ | y ₀ | 2y ₀ | 2y ₀ = x ₀ |
| 2 | 1 | 2 | |



| x | y | z | π |
|----------------|----------------|---------------------|----------------------------------|
| x ₀ | y ₀ | y ₀ + 10 | 2y ₀ = x ₀ |
| 2 | 1 | 11 | |

Dynamic symbolic execution: example

procedure `m`(`x`, `y`: **Integer**):

`z` := `2*y`

if `z` = `x`

`z` := `y` + 10

assert `x` ≤ `z`

| <code>x</code> | <code>y</code> | <code>z</code> | π |
|----------------|----------------|----------------|--------|
| x_0 | y_0 | | \top |
| 2 | 1 | | |



| <code>x</code> | <code>y</code> | <code>z</code> | π |
|----------------|----------------|----------------|--------|
| x_0 | y_0 | $2y_0$ | \top |
| 2 | 1 | 2 | |

`z` = `x`

| <code>x</code> | <code>y</code> | <code>z</code> | π |
|----------------|----------------|----------------|--------------|
| x_0 | y_0 | $2y_0$ | $2y_0 = x_0$ |
| 2 | 1 | 2 | |



| <code>x</code> | <code>y</code> | <code>z</code> | π |
|----------------|----------------|----------------|--------------|
| x_0 | y_0 | $y_0 + 10$ | $2y_0 = x_0$ |
| 2 | 1 | 11 | |

`x` ≤ `z`

| <code>x</code> | <code>y</code> | <code>z</code> | π |
|----------------|----------------|----------------|--------------------------------------|
| x_0 | y_0 | $y_0 + 10$ | $2y_0 = x_0,$ $x_0 \leq y_0 + 10$ |
| 2 | 1 | 11 | |

Dynamic symbolic execution: example

procedure `m`(`x`, `y`: **Integer**):

`z` := `2*y`

if `z` = `x`

`z` := `y` + 10

assert `x` ≤ `z`

| <code>x</code> | <code>y</code> | <code>z</code> | π |
|----------------|----------------|----------------|--------|
| x_0 | y_0 | | \top |
| 2 | 1 | | |



| <code>x</code> | <code>y</code> | <code>z</code> | π |
|----------------|----------------|----------------|--------|
| x_0 | y_0 | $2y_0$ | \top |
| 2 | 1 | 2 | |

`z` = `x` ↗

| <code>x</code> | <code>y</code> | <code>z</code> | π |
|----------------|----------------|----------------|--------------|
| x_0 | y_0 | $2y_0$ | $2y_0 = x_0$ |
| 2 | 1 | 2 | |



| <code>x</code> | <code>y</code> | <code>z</code> | π |
|----------------|----------------|----------------|--------------|
| x_0 | y_0 | $y_0 + 10$ | $2y_0 = x_0$ |
| 2 | 1 | 11 | |

`x` ≤ `z` ↗

| <code>x</code> | <code>y</code> | <code>z</code> | π |
|----------------|----------------|----------------|--------------------------------------|
| x_0 | y_0 | $y_0 + 10$ | $2y_0 = x_0,$ $x_0 \leq y_0 + 10$ |
| 2 | 1 | 11 | |

solve: $2y_0 = x_0 \wedge \neg(x_0 \leq y_0 + 10)$

Dynamic symbolic execution: example

```
procedure m(x, y: Integer):
```

```
  z := 2*y
```

```
  if z = x
```

```
    z := y + 10
```

```
    assert x ≤ z
```

| x | y | z | π |
|-------|-------|---|--------|
| x_0 | y_0 | | \top |
| 30 | 15 | | |

solve: $2y_0 = x_0 \wedge \neg(x_0 \leq y_0 + 10)$

Dynamic symbolic execution: example

| x | y | z | π |
|-------|-------|---|--------|
| x_0 | y_0 | | \top |
| 30 | 15 | | |



| x | y | z | π |
|-------|-------|--------|--------|
| x_0 | y_0 | $2y_0$ | \top |
| 30 | 15 | 30 | |

```
procedure m(x, y: Integer):
```

```
  z := 2*y
```

```
  if z = x
```

```
    z := y + 10
```

```
    assert x ≤ z
```

Dynamic symbolic execution: example

```
procedure m(x, y: Integer):
```

```
  z := 2*y
```

```
  if z = x
```

```
    z := y + 10
```

```
    assert x ≤ z
```

| x | y | z | π |
|-------|-------|---|--------|
| x_0 | y_0 | | \top |
| 30 | 15 | | |



| x | y | z | π |
|-------|-------|--------|--------|
| x_0 | y_0 | $2y_0$ | \top |
| 30 | 15 | 30 | |

$z = x$



| x | y | z | π |
|-------|-------|--------|--------------|
| x_0 | y_0 | $2y_0$ | $2y_0 = x_0$ |
| 30 | 15 | 30 | |

Dynamic symbolic execution: example

procedure **m**(**x**, **y**: **Integer**):

z := 2*y

if **z** = **x**

z := **y** + 10

assert **x** ≤ **z**

| x | y | z | π |
|----------------|----------------|---|---|
| x ₀ | y ₀ | | ⊤ |
| 30 | 15 | | |



| x | y | z | π |
|----------------|----------------|-----------------|---|
| x ₀ | y ₀ | 2y ₀ | ⊤ |
| 30 | 15 | 30 | |

z = **x**



| x | y | z | π |
|----------------|----------------|-----------------|----------------------------------|
| x ₀ | y ₀ | 2y ₀ | 2y ₀ = x ₀ |
| 30 | 15 | 30 | |



| x | y | z | π |
|----------------|----------------|---------------------|----------------------------------|
| x ₀ | y ₀ | y ₀ + 10 | 2y ₀ = x ₀ |
| 30 | 15 | 25 | |

Dynamic symbolic execution: example

procedure `m`(`x`, `y`: **Integer**):

`z` := 2*y

if `z` = `x`

`z` := `y` + 10

assert `x` ≤ `z`

| x | y | z | π |
|-------|-------|---|--------|
| x_0 | y_0 | | \top |
| 30 | 15 | | |



| x | y | z | π |
|-------|-------|--------|--------|
| x_0 | y_0 | $2y_0$ | \top |
| 30 | 15 | 30 | |

`z` = `x`

| x | y | z | π |
|-------|-------|--------|--------------|
| x_0 | y_0 | $2y_0$ | $2y_0 = x_0$ |
| 30 | 15 | 30 | |



| x | y | z | π |
|-------|-------|------------|--------------|
| x_0 | y_0 | $y_0 + 10$ | $2y_0 = x_0$ |
| 30 | 15 | 25 | |

$\neg (x \leq z)$

| x | y | z | π |
|-------|-------|------------|-----------------------------------|
| x_0 | y_0 | $y_0 + 10$ | $2y_0 = x_0,$ $x_0 > y_0 + 10$ |
| 30 | 15 | 25 | |

Dynamic symbolic execution: example

procedure `m`(`x`, `y`: **Integer**):

`z` := 2*y

if `z` = `x`

`z` := `y` + 10

assert `x` ≤ `z`

| <code>x</code> | <code>y</code> | <code>z</code> | π |
|----------------|----------------|----------------|--------|
| x_0 | y_0 | | \top |
| 30 | 15 | | |



| <code>x</code> | <code>y</code> | <code>z</code> | π |
|----------------|----------------|----------------|--------|
| x_0 | y_0 | $2y_0$ | \top |
| 30 | 15 | 30 | |

`z` = `x`

\neg (`z` = `x`)

| <code>x</code> | <code>y</code> | <code>z</code> | π |
|----------------|----------------|----------------|--------------|
| x_0 | y_0 | $2y_0$ | $2y_0 = x_0$ |
| 30 | 15 | 30 | |

| <code>x</code> | <code>y</code> | <code>z</code> | π |
|----------------|----------------|----------------|-----------------|
| x_0 | y_0 | $2y_0$ | $2y_0 \neq x_0$ |
| 22 | 7 | 14 | |



| <code>x</code> | <code>y</code> | <code>z</code> | π |
|----------------|----------------|----------------|--------------|
| x_0 | y_0 | $y_0 + 10$ | $2y_0 = x_0$ |
| 30 | 15 | 25 | |

`x` ≤ `z`

\neg (`x` ≤ `z`)

| <code>x</code> | <code>y</code> | <code>z</code> | π |
|----------------|----------------|----------------|--------------------------------------|
| x_0 | y_0 | $y_0 + 10$ | $2y_0 = x_0,$ $x_0 \leq y_0 + 10$ |
| 2 | 1 | 11 | |

| <code>x</code> | <code>y</code> | <code>z</code> | π |
|----------------|----------------|----------------|-----------------------------------|
| x_0 | y_0 | $y_0 + 10$ | $2y_0 = x_0,$ $x_0 > y_0 + 10$ |
| 30 | 15 | 25 | |

Concolic vs. execution generated

The same fundamental ideas of dynamic symbolic execution are implemented in two slightly different ways:

concolic (concrete + symbolic) execution starts from (concrete) test-cases, and uses symbolic execution to extend the test-case generation capabilities

execution-generated testing starts from symbolic exploration, and concretizes parts of the state selectively when a symbolic representation is not available (for example, when executing pre-compile code)

Concolic vs. execution generated

The same fundamental ideas of dynamic symbolic execution are implemented in two slightly different ways:

concolic (concrete + symbolic) execution starts from (concrete) test-cases, and uses symbolic execution to extend the test-case generation capabilities

execution-generated testing starts from symbolic exploration, and concretizes parts of the state selectively when a symbolic representation is not available (for example, when executing pre-compile code)

In practice, “concolic execution” and “dynamic symbolic execution” are often used as **synonyms**.

In our presentation, we need not emphasize these (fuzzy) differences, and hence will also use the two terms as synonyms.

Dynamic symbolic execution: explore external code

Let's see how **concretization** can explore exhaustively code that includes an **external function**.

Dynamic symbolic execution: explore external code

```
procedure m2(x, y: Integer):  
  z := bb(y)  // black-box function  
  if z = x  
    z := y + 10  
  assert x ≤ z
```

| x | y | z | π |
|-------|-------|---|--------|
| x_0 | y_0 | | \top |
| 22 | 7 | | |

Dynamic symbolic execution: explore external code

```
procedure m2(x, y: Integer):  
  z := bb(y)  // black-box function  
  if z = x  
    z := y + 10  
  assert x ≤ z
```

| x | y | z | π |
|----------------|----------------|---|---|
| x ₀ | y ₀ | | ⊤ |
| 22 | 7 | | |

↓

| x | y | z | π |
|----------------|----------------|---------------------|---|
| x ₀ | y ₀ | bb(y ₀) | ⊤ |
| 22 | 7 | 14 | |

Dynamic symbolic execution: explore external code

```
procedure m2(x, y: Integer):  
  z := bb(y) // black-box function  
  if z = x  
    z := y + 10  
  assert x ≤ z
```

| x | y | z | π |
|-------|-------|---|--------|
| x_0 | y_0 | | \top |
| 22 | 7 | | |



| x | y | z | π |
|-------|-------|-----------|--------|
| x_0 | y_0 | $bb(y_0)$ | \top |
| 22 | 7 | 14 | |

 $\neg (z = x)$

| x | y | z | π |
|-------|-------|-----------|--------------------|
| x_0 | y_0 | $bb(y_0)$ | $bb(y_0) \neq x_0$ |
| 22 | 7 | 14 | |

Dynamic symbolic execution: explore external code

```
procedure m2(x, y: Integer):  
  z := bb(y) // black-box function  
  if z = x  
    z := y + 10  
  assert x ≤ z
```

| x | y | z | π |
|-------|-------|---|---|
| x_0 | y_0 | | ⊤ |
| 22 | 7 | | |



| x | y | z | π |
|-------|-------|-----------|---|
| x_0 | y_0 | $bb(y_0)$ | ⊤ |
| 22 | 7 | 14 | |

↘ $\neg (z = x)$

| x | y | z | π |
|-------|-------|-----------|--------------------|
| x_0 | y_0 | $bb(y_0)$ | $bb(y_0) \neq x_0$ |
| 22 | 7 | 14 | |

concretize: $\neg(bb(y_0) \neq x_0)$

Dynamic symbolic execution: explore external code

```
procedure m2(x, y: Integer):  
  z := bb(y)  // black-box function  
  if z = x  
    z := y + 10  
  assert x ≤ z
```

| x | y | z | π |
|-------|-------|---|--------|
| x_0 | y_0 | | \top |
| 14 | 7 | | |

concretize: $\neg(\text{bb}(y_0) \neq x_0)$
solve: $\neg(14 \neq x_0)$

Dynamic symbolic execution: explore external code

```
procedure m2(x, y: Integer):  
  z := bb(y)  // black-box function  
  if z = x  
    z := y + 10  
  assert x ≤ z
```

| x | y | z | π |
|----------------|----------------|---|---|
| x ₀ | y ₀ | | ⊤ |
| 14 | 7 | | |

↓

| x | y | z | π |
|----------------|----------------|---------------------|---|
| x ₀ | y ₀ | bb(y ₀) | ⊤ |
| 14 | 7 | 14 | |

Dynamic symbolic execution: explore external code

```
procedure m2(x, y: Integer):  
  z := bb(y)  // black-box function  
  if z = x  
    z := y + 10  
  assert x ≤ z
```

| x | y | z | π |
|-------|-------|---|--------|
| x_0 | y_0 | | \top |
| 14 | 7 | | |



| x | y | z | π |
|-------|-------|-----------|--------|
| x_0 | y_0 | $bb(y_0)$ | \top |
| 14 | 7 | 14 | |

$z = x$

| x | y | z | π |
|-------|-------|-----------|-----------------|
| x_0 | y_0 | $bb(y_0)$ | $bb(y_0) = x_0$ |
| 14 | 7 | 14 | |

Dynamic symbolic execution: explore external code

```
procedure m2(x, y: Integer):  
  z := bb(y)  // black-box function  
  if z = x  
    z := y + 10  
  assert x ≤ z
```

| x | y | z | π |
|-------|-------|---|--------|
| x_0 | y_0 | | \top |
| 14 | 7 | | |



| x | y | z | π |
|-------|-------|-----------|--------|
| x_0 | y_0 | $bb(y_0)$ | \top |
| 14 | 7 | 14 | |

$z = x$

| x | y | z | π |
|-------|-------|-----------|-----------------|
| x_0 | y_0 | $bb(y_0)$ | $bb(y_0) = x_0$ |
| 14 | 7 | 14 | |



| x | y | z | π |
|-------|-------|------------|-----------------|
| x_0 | y_0 | $y_0 + 10$ | $bb(y_0) = x_0$ |
| 14 | 7 | 17 | |

Dynamic symbolic execution: explore external code

```
procedure m2(x, y: Integer):  
  z := bb(y)  // black-box function  
  if z = x  
    z := y + 10  
  assert x ≤ z
```

| x | y | z | π |
|-------|-------|---|--------|
| x_0 | y_0 | | \top |
| 14 | 7 | | |



| x | y | z | π |
|-------|-------|-----------|--------|
| x_0 | y_0 | $bb(y_0)$ | \top |
| 14 | 7 | 14 | |

$z = x$

| x | y | z | π |
|-------|-------|-----------|-----------------|
| x_0 | y_0 | $bb(y_0)$ | $bb(y_0) = x_0$ |
| 14 | 7 | 14 | |



| x | y | z | π |
|-------|-------|------------|-----------------|
| x_0 | y_0 | $y_0 + 10$ | $bb(y_0) = x_0$ |
| 14 | 7 | 17 | |

$x \leq z$

| x | y | z | π |
|-------|-------|------------|---------------------|
| x_0 | y_0 | $y_0 + 10$ | $bb(y_0) = x_0,$ |
| 14 | 7 | 17 | $x_0 \leq y_0 + 10$ |

Dynamic symbolic execution: explore external code

```

procedure m2(x, y: Integer):
  z := bb(y)  // black-box function
if z = x
    z := y + 10
  assert x ≤ z
  
```

| x | y | z | π |
|-------|-------|---|--------|
| x_0 | y_0 | | \top |
| 14 | 7 | | |



| x | y | z | π |
|-------|-------|-----------|--------|
| x_0 | y_0 | $bb(y_0)$ | \top |
| 14 | 7 | 14 | |

$z = x$

| x | y | z | π |
|-------|-------|-----------|-----------------|
| x_0 | y_0 | $bb(y_0)$ | $bb(y_0) = x_0$ |
| 14 | 7 | 14 | |



| x | y | z | π |
|-------|-------|------------|-----------------|
| x_0 | y_0 | $y_0 + 10$ | $bb(y_0) = x_0$ |
| 14 | 7 | 17 | |

$x \leq z$

| x | y | z | π |
|-------|-------|------------|---------------------|
| x_0 | y_0 | $y_0 + 10$ | $bb(y_0) = x_0,$ |
| 14 | 7 | 17 | $x_0 \leq y_0 + 10$ |

concretize: $bb(y_0) = x_0 \wedge \neg(x_0 \leq y_0 + 10)$

Dynamic symbolic execution: explore external code

```
procedure m2(x, y: Integer):  
  z := bb(y)  // black-box function  
  if z = x  
    z := y + 10  
  assert x ≤ z
```

| x | y | z | π |
|-------|-------|---|--------|
| x_0 | y_0 | | \top |
| 28 | 14 | | |

solve: $\neg(x_0 \leq 17 + 10)$

Dynamic symbolic execution: explore external code

```
procedure m2(x, y: Integer):  
  z := bb(y) // black-box function  
  if z = x  
    z := y + 10  
  assert x ≤ z
```

| x | y | z | π |
|-------|-------|---|--------|
| x_0 | y_0 | | \top |
| 28 | 14 | | |



| x | y | z | π |
|-------|-------|------------------|--------|
| x_0 | y_0 | $\text{bb}(y_0)$ | \top |
| 28 | 14 | 28 | |

Dynamic symbolic execution: explore external code

```
procedure m2(x, y: Integer):  
  z := bb(y)  // black-box function  
  if z = x  
    z := y + 10  
  assert x ≤ z
```

| x | y | z | π |
|-------|-------|---|--------|
| x_0 | y_0 | | \top |
| 28 | 14 | | |



| x | y | z | π |
|-------|-------|-----------|--------|
| x_0 | y_0 | $bb(y_0)$ | \top |
| 28 | 14 | 28 | |

$z = x$

| x | y | z | π |
|-------|-------|-----------|-----------------|
| x_0 | y_0 | $bb(y_0)$ | $bb(y_0) = x_0$ |
| 28 | 14 | 28 | |

Dynamic symbolic execution: explore external code

```
procedure m2(x, y: Integer):  
  z := bb(y)  // black-box function  
  if z = x  
    z := y + 10  
  assert x ≤ z
```

| x | y | z | π |
|-------|-------|---|--------|
| x_0 | y_0 | | \top |
| 28 | 14 | | |



| x | y | z | π |
|-------|-------|-----------|--------|
| x_0 | y_0 | $bb(y_0)$ | \top |
| 28 | 14 | 28 | |

$z = x$

| x | y | z | π |
|-------|-------|-----------|-----------------|
| x_0 | y_0 | $bb(y_0)$ | $bb(y_0) = x_0$ |
| 28 | 14 | 28 | |



| x | y | z | π |
|-------|-------|------------|-----------------|
| x_0 | y_0 | $y_0 + 10$ | $bb(y_0) = x_0$ |
| 28 | 14 | 24 | |

Dynamic symbolic execution: explore external code

```
procedure m2(x, y: Integer):  
  z := bb(y)  // black-box function  
  if z = x  
    z := y + 10  
  assert x ≤ z
```

| x | y | z | π |
|-------|-------|---|--------|
| x_0 | y_0 | | \top |
| 28 | 14 | | |



| x | y | z | π |
|-------|-------|-----------|--------|
| x_0 | y_0 | $bb(y_0)$ | \top |
| 28 | 14 | 28 | |

$z = x$

| x | y | z | π |
|-------|-------|-----------|-----------------|
| x_0 | y_0 | $bb(y_0)$ | $bb(y_0) = x_0$ |
| 28 | 14 | 28 | |



| x | y | z | π |
|-------|-------|------------|-----------------|
| x_0 | y_0 | $y_0 + 10$ | $bb(y_0) = x_0$ |
| 28 | 14 | 24 | |

$\neg (x \leq z)$

| x | y | z | π |
|-------|-------|------------|--------------------------------------|
| x_0 | y_0 | $y_0 + 10$ | $bb(y_0) = x_0,$ $x_0 > y_0 + 10$ |
| 28 | 14 | 24 | |

Dynamic symbolic execution: explore external code

```

procedure m2(x, y: Integer):
    z := bb(y)  // black-box function
    if z = x
        z := y + 10
    assert x ≤ z
    
```

| x | y | z | π |
|-------|-------|---|--------|
| x_0 | y_0 | | \top |
| 28 | 14 | | |



| x | y | z | π |
|-------|-------|-----------|--------|
| x_0 | y_0 | $bb(y_0)$ | \top |
| 28 | 14 | 28 | |

$z = x$

$\neg (z = x)$

| x | y | z | π |
|-------|-------|-----------|-----------------|
| x_0 | y_0 | $bb(y_0)$ | $bb(y_0) = x_0$ |
| 28 | 14 | 28 | |

| x | y | z | π |
|-------|-------|-----------|--------------------|
| x_0 | y_0 | $bb(y_0)$ | $bb(y_0) \neq x_0$ |
| 22 | 7 | 14 | |



| x | y | z | π |
|-------|-------|------------|-----------------|
| x_0 | y_0 | $y_0 + 10$ | $bb(y_0) = x_0$ |
| 28 | 14 | 24 | |

$x \leq z$

$\neg (x \leq z)$

| x | y | z | π |
|-------|-------|------------|---|
| x_0 | y_0 | $y_0 + 10$ | $bb(y_0) = x_0,$ $x_0 \leq y_0 + 10$ |
| 14 | 7 | 17 | |

| x | y | z | π |
|-------|-------|------------|--------------------------------------|
| x_0 | y_0 | $y_0 + 10$ | $bb(y_0) = x_0,$ $x_0 > y_0 + 10$ |
| 28 | 14 | 24 | |

Challenges, tools, and applications

Path explosion

The main challenge to symbolic execution is **path explosion**, which limits the technique's **scalability**.

Path explosion has two different forms:

- **large but finite** search space – generated by complex nested conditionals
- **infinite** search space – generated by loops or recursion with symbolic bounds

Solutions to address path explosion depend on whether we are interested in trying to achieve a **complete** exploration (**verification**), or we just want to find as many **bugs** as possible (**testing**).

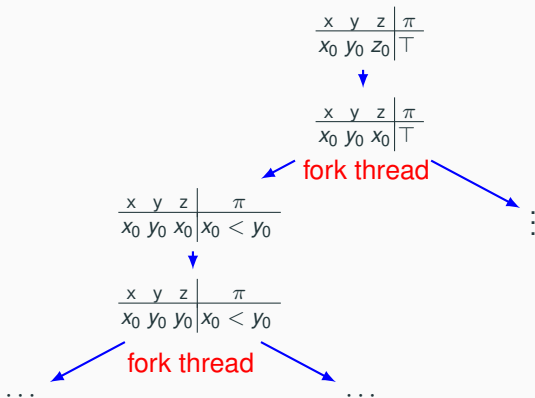
testing solutions are typically **best effort**

verification solutions try not to give up **soundness** when possible

Parallelization

Using **parallelization** is a straightforward way to search larger state spaces.

Parallelizing symbolic execution is particularly easy because **no coordination** is necessary between different branches in an execution tree. The challenge is deciding **when to fork** new threads in a way that work is **balanced** between threads.



Search heuristics

If we do not insist that all paths be explored, it becomes important to **prioritize** more interesting paths – such as those that are more likely to show an **error**.

To this end, we can use heuristics to decide how to explore the execution tree:

coverage metrics favor exploring branches or statements that have not been explored yet

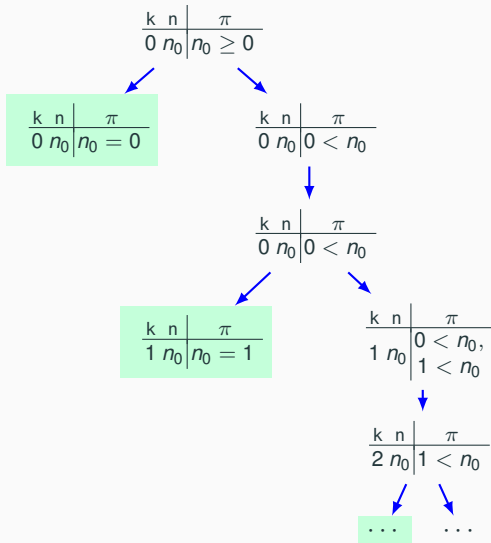
random search reduces bias in a search, thus helping reach special cases that systematic search may neglect

evolutionary search uses a fitness function to direct the search towards trying to maximize fitness

These heuristics are all **unsound** but can be effective to **find bugs** more quickly or in more complex programs.

Loops

Loops with symbolic bounds determine **infinite** execution trees.



```
procedure to_n(n: Integer):  
  require n ≥ 0  
  var k: Integer := 0  
  while k < n  
    k := k + 1  
  assert k = n
```


Loop summarization

If we can infer a **loop invariant**, we can **summarize** the input/output behavior of a loop symbolically.

```
procedure to_n(n: Integer):  
  require n ≥ 0  
  var k: Integer := 0  
  while k < n  
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  assert k = n
```

Since $k \leq n_0 = n$ is a loop invariant, the path condition after the loop is always $k \leq n_0 = n \wedge \neg(k < n)$, that is $k = n_0$.

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$$\frac{k \ n \mid \pi}{0 \ n_0 \mid n_0 \geq 0}$$

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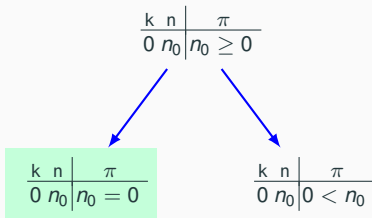
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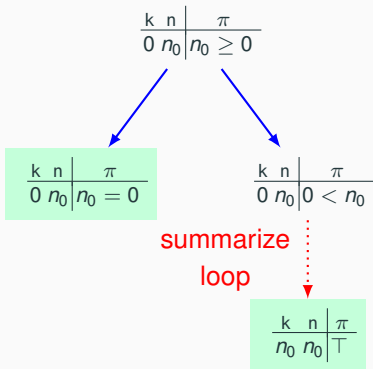


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Loop summarization: example

```
procedure loop(n: Integer):  
  require n ≥ 0  
  var k: Integer := 0  
  while n > 0  
    assert k ≠ 200  
    k := k + 1  
    n := n - 1  
  assert k ≠ 100
```

Using the loop invariant:

$$n + k = n_0 \wedge n \geq 0$$

we can summarize all
possible termination
conditions.

Loop summarization: example

$$\frac{k \ n \mid \pi}{0 \ n_0 \mid n_0 \geq 0}$$

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procedure loop(n: Integer):  
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Loop summarization: example

$$\frac{k \quad n}{0 \quad n_0} \mid \frac{\pi}{n_0 \geq 0}$$



$$\frac{k \quad n}{200 \quad n_0 - 200} \mid \frac{\pi}{n_0 > 200}$$

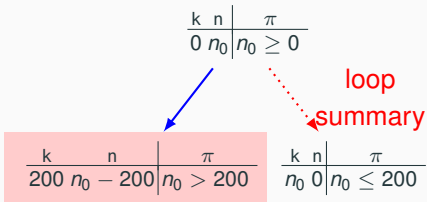
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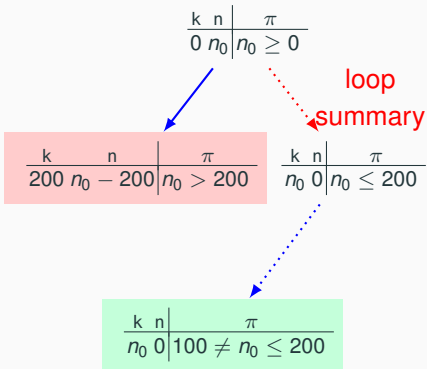
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Loop summarization: example



```
procedure loop( $n$ : Integer):
```

```
  require  $n \geq 0$ 
```

```
  var  $k$ : Integer := 0
```

```
  while  $n > 0$ 
```

```
    assert  $k \neq 200$ 
```

```
     $k := k + 1$ 
```

```
     $n := n - 1$ 
```

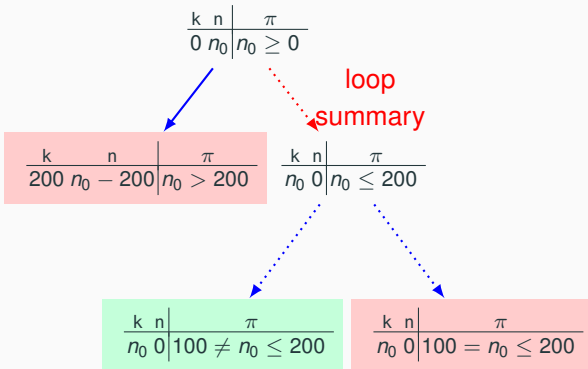
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```

```
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```

```
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```

```
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```

```
    k := k + 1
```

```
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```

```
  assert k ≠ 100
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State abstraction

An unsound way of handling loops, recursion, or very large state spaces is to **abstract** the symbolic state in a way that we effectively **merge** different states together.

This generally **loses precision** in the search, and hence we may miss errors, but helps scalability.

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Abstraction is often used to represent **data structures** such as arrays and lists in a manageable way.

- a list may be abstracted by keeping track of whether it is **null**, empty, or non-empty.
- an array may be abstracted by keeping track of its content **ignoring** the order

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- an array may be abstracted by keeping track of its content **ignoring** the order

Let's see abstraction on a simpler example where we abstract an integer variable by only keeping track of whether it is **zero** or **positive**.

Integer abstraction

```
procedure to_n(n: Integer):  
  require n ≥ 0  
  var k: Integer := 0  
  while k < n  
    k := k + 1  
  assert k = n
```

In symbolic execution, abstracting the state generally leads to an **under approximation** because it coalesces paths that are distinct in the concrete program.

Integer abstraction

$$k = 0 \wedge n = n_0 \geq 0$$

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procedure to_n(n: Integer):  
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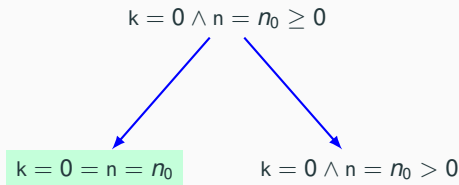


$$k = 0 = n = n_0$$

```
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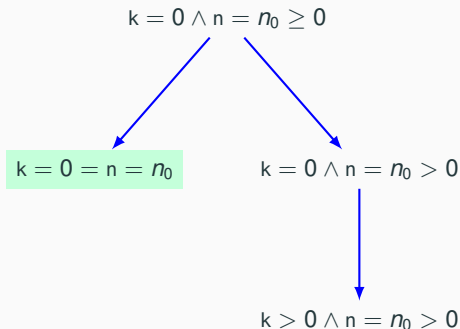
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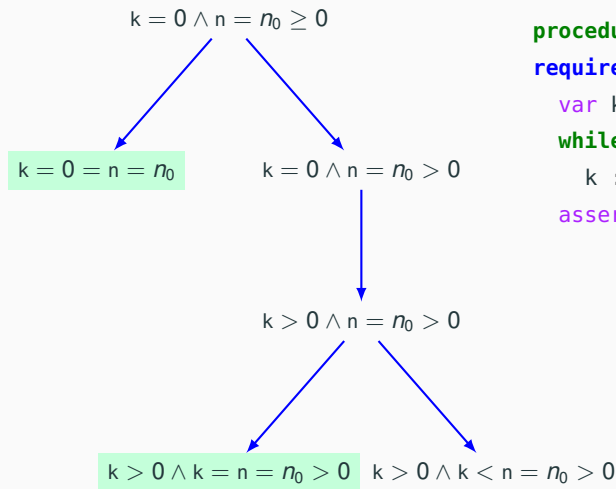
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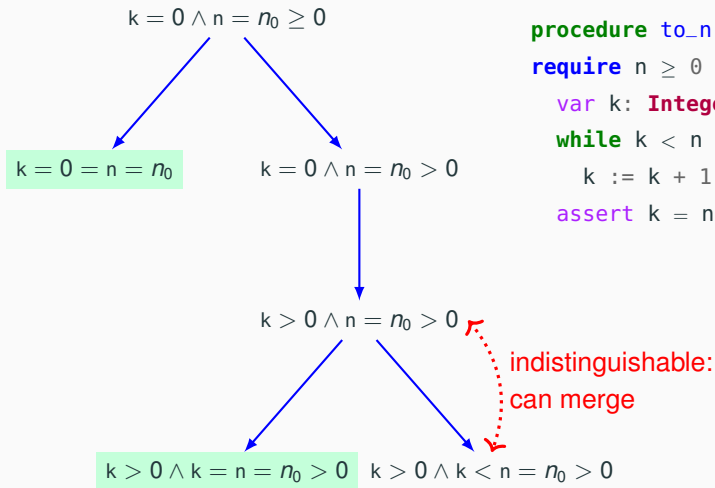
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```

In symbolic execution, abstracting the state generally leads to an **under approximation** because it coalesces paths that are distinct in the concrete program.

Memory modeling

Many aspects of symbolic memory representation affect **precision** and **scalability** of symbolic execution's state space exploration:

- **numeric** representation: machine vs. mathematical integers, floating point vs. reals
- **string** representation: tracking string variable content vs. only checking for equality
- **arrays**, **lists**, and other common data structures: detailed representation, content but no ordering, null vs. non-null
- **references**, pointers: memory footprint modeling, aliasing information, or only equality checking

The precision vs. scalability **trade-off** impacts on **bug-finding** effectiveness in subtle ways: a very precise exploration may find **more kinds** of bugs, but it may also be inapplicable in practice if it **doesn't scale**

Constraint solving

Constraint solving remains a major **bottleneck** of symbolic execution.

While constraint solvers' performance keep getting better, **optimizations** in the way constraints are built, checked, and modified are key to improving the scalability of symbolic execution.

elimination of constraints that are not relevant to the current branch

incremental checking relies on **caching** of constraint checking results, so that the constraint solver is called only on the **new parts** of a constraint. Choosing a suitable representation of constraints helps detect similarities that support caching

Constraint solving

Example: suppose the state at an exit point is:

| π | x | y | z |
|---|---|---|---|
| $(x + y) > 10 \wedge (z > 0) \wedge (y < 12)$ | 3 | 8 | 3 |

and we want to explore the branch with path condition

$$(x + y) > 10 \wedge (z > 0) \wedge \neg(y < 12)$$

The constraint involving z does not affect the satisfiability of the new $\neg(y < 12)$. Hence, we invoke the constraint solver on:

$$(x + y) > 10 \wedge \neg(y < 12)$$

and reuse the previous value of z to complete the concrete state:

| π | x | y | z |
|---|---|----|---|
| $(x + y) > 10 \wedge (z > 0) \wedge \neg(y < 12)$ | 2 | 14 | 3 |

Symbolic execution tools

Some (mostly open source) symbolic executors (normally based on **dynamic** symbolic execution):

CUTE/jCUTE (successors to DART) combine random testing and symbolic execution techniques to generate **test cases** that explore a program's paths (including concurrent executions) as exhaustively as possible

CREST is an extensible version of CUTE, which supports combination of different **path-selection** heuristics

KLEE (successor to EXE) is a symbolic execution engine for LLVM code, geared towards bit-level modeling of **systems** C code (including environment models of system calls)

JPF (Java PathFinder) is a model checker for Java source code supporting concurrency and symbolic execution (Symbolic PathFinder) of **structured data** (such as linked lists and trees)

Case studies

Notable **case studies** have demonstrated symbolic execution tools' capabilities of **finding complex critical bugs**.

Case studies

KLEE has found several critical bugs in Unix systems software (GNU Coreutils, ext2/3 file systems, network servers, kernel code, system libraries, and so on).

With bit-level modeling of data and suitable abstractions, symbolic execution engines can be used as **white-box fuzzers** to generate **structured input** that triggers failures in communication protocols.

- KLEE has been used to generate **packets** that trigger failures in Apple's Bonjour communication protocol
- Microsoft's **SAGE/SAGAN** tools generated **image** and **text** file inputs that trigger many vulnerabilities in Windows applications. These tools run over the clock at Microsoft on new releases and existing software, constituting one of the largest-case deployment of software analysis tools

The symbolic-execution engine of Java PathFinder has been used extensively at NASA to test more effectively and more quickly **control software**.

Challenges, tools, and applications

A brief demo of Klee

Klee is a powerful dynamic-symbolic execution engine for C, whose implementation is based on LLVM.

Symbolic input

Klee is a powerful dynamic-symbolic execution engine for C, whose implementation is based on LLVM.

To analyze a program with Klee, we explicitly choose which variables are to be treated as **symbolic inputs**; everything else takes on concrete values.

```
#include <klee/klee.h>
```

```
int max(int x, int y)
{
    int max;
    if (x > y)
        max = x;
    else
        max = y;
    return max;
}
```

```
// driver of 'max', which defines symbolic input
int main()
{
    int x, y;
    klee_make_symbolic(&x, sizeof(x), "x");
    klee_make_symbolic(&y, sizeof(y), "y");
    return max(x, y);
}
```

Input generation

Running Klee on `max.c` simply generates inputs for each program path, which will be stored in subdirectory `klee-last`.

```
# Run shell with Klee Docker image distro
> cd <DIRECTORY WHERE max.c IS>
> docker run -it -v "$(pwd):/home/klee/examples" klee/klee:2.0
# Compile max.c to LLVM bytecode
>> cd examples
>> clang -I ../klee_src/include -emit-llvm -c -g -O0 \
        -Xclang -disable-O0-optnone max.c
# Run Klee
>> klee max.bc
```

Test cases

Klee generates two tests for `max.c`: one where $x \leq y$ and one where $x > y$.

In test 1: $x = y = 0$

```
>> ktest-tool test000001.ktest
num objects: 2
object 0: name: 'x'
object 0: size: 4
object 0: data: b'\x00\x00\x00\x00'
object 0: hex : 0x00000000
object 0: int : 0
object 1: name: 'y'
object 1: size: 4
object 1: data: b'\x00\x00\x00\x00'
object 1: hex : 0x00000000
object 1: int : 0
```

In test 2: $x = 1 > y = 0$

```
>> ktest-tool test000002.ktest
num objects: 2
object 0: name: 'x'
object 0: size: 4
object 0: data: b'\x01\x00\x00\x00'
object 0: hex : 0x01000000
object 0: int : 1
object 1: name: 'y'
object 1: size: 4
object 1: data: b'\x00\x00\x00\x00'
object 1: hex : 0x00000000
object 1: int : 0
```


Symbolic input constraints

Klee finds an out-of-bound memory access in `maxa.c`, which happens when `n` takes a value that is larger than the actual size of array `a`.

```
KLEE: ERROR: maxa.c:25: memory error: out of bound pointer
```

```
>> ls klee-last/*.err
```

```
klee-last/test000002.ptr.err
```

```
>> ktest-tool klee-last/test000002.ktest
```

```
ktest file : 'klee-last/test000002.ktest'
```

```
args       : ['maxa.bc']
```

```
num objects: 2
```

```
object 0: name: 'a'
```

```
object 0: size: 12
```

```
object 0: data: b'\x00\x00\x00\x00\x00\x00\x00\x00\x00\x00\x00\x00'
```

```
object 1: name: 'n'
```

```
object 1: size: 4
```

```
object 1: int : 2147483647
```

Symbolic input constraints

Klee finds an out-of-bound memory access in `maxa.c`, which happens when `n` takes a value that is larger than the actual size of array `a`.

To fix these kinds of spurious errors, we can constraint the symbolic input using `klee_assume`:

```
klee_assume(0 <= n && n <= N);
```

Assumptions, combined with the symbolic representation of memory as bit strings, make Klee suitable to analyze low-level code and complex, structured input.

Concrete error-triggering input

On the **buggy** example `negpow.c`, which we also analyzed using CPAchecker, Klee quickly finds a concrete input that triggers an **error** (even though generating all paths may not terminate because of the unbounded loop): $x = -2147483648 < 0$ and $y = 1$.

```
>> ktest-tool klee-last/test000001.ktest
num objects: 2
object 0: name: 'x'
object 0: size: 4
object 0: data: b'\x00\x00\x00\x80'
object 0: hex : 0x00000080
object 0: int  : -2147483648
object 1: name: 'y'
object 1: size: 4
object 1: data: b'\x01\x00\x00\x00'
object 1: hex  : 0x01000000
object 1: int  : 1
```

Solving a maze using Klee

A fun example of Klee's constraint solving capabilities (by Felipe Manzano).

```
>> gcc -o maze maze.original.c
```

```
>> ./maze
```

Program the player moves with a sequence of 'w', 's', 'a' and 'd'
Try to reach the price(#!)

```
+--+---+---+
```

```
|X|      |#|
```

```
| | --+ | |
```

```
| |   | | |
```

```
| +- - | | |
```

```
|     |   |
```

```
+-----+---+
```

The **goal** of the game is to give a sequence of letters *w* (up), *s* (down), *a* (left), and *d* (right) that, when applied, go from the initial *x* to the goal *#*. We **lose** if we give an incomplete sequence, or one that tries to cross walls.

Instrumenting the maze code

Make the **input** sequence
(an array) **symbolic**:

```
// read(0,program,ITERS);  
    // use symbolic input  
klee_make_symbolic(program,  
    sizeof(char)*ITERS,  
    "program");
```

```
>> klee maze.symbolic.bc
```

```
[...]
```

```
>> ls klee-last/*.err
```

```
klee-last/test000139.assert.err
```

```
>> ktest-tool klee-last/test000139.ktest
```

```
object 0: text: sddwdddsddw.....
```

Add an **assert false** when **winning**:

```
printf ("You win!\n");  
    // mark winning sequence as "error"  
klee_assert(0);
```

Instrumenting the maze code

Make the **input** sequence
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```
// read(0,program,ITERS);  
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klee_make_symbolic(program,  
    sizeof(char)*ITERS,  
    "program");
```

```
>> klee maze.symbolic.bc
```

```
[...]
```

```
>> ls klee-last/*.err
```

```
klee-last/test000139.assert.err
```

```
>> ktest-tool klee-last/test000139.ktest
```

```
object 0: text: sddwdddsddw.....
```

Add an **assert false** when **winning**:

```
printf ("You win!\n");  
    // mark winning sequence as "error"  
klee_assert(0);
```

By using this input sequence, we find out that the maze program does not forbid crossing walls **everywhere**!

Instrumenting the maze code

Make the **input** sequence
(an array) **symbolic**:

```
// read(0,program,ITERS);  
    // use symbolic input  
klee_make_symbolic(program,  
    sizeof(char)*ITERS,  
    "program");
```

```
>> klee maze.symbolic.bc
```

```
[...]
```

```
>> ls klee-last/*.err
```

```
klee-last/test000139.assert.err
```

```
>> ktest-tool klee-last/test000139.ktest
```

```
object 0: text: sddwdddsddw.....
```

Add an **assert false** when **winning**:

```
printf ("You win!\n");  
    // mark winning sequence as "error"  
klee_assert(0);
```

By using this input sequence, we find out that the maze program does not forbid crossing walls **everywhere**!

To find out **all four** input sequences that lead to the goal run:

```
klee --emit-all-errors maze.symbolic.bc
```

Summary

Symbolic execution: techniques

Symbolic execution is a systematic path-exploration technique based on **executing** a program with **symbolic inputs**.

Symbolic execution **techniques** are mostly **best effort** and rely on **constraint solvers** to determine which paths are feasible.

soundness/completeness: **unsound** and complete – symbolic execution primarily focuses on finding concrete **inputs** that trigger **bugs**

complexity: according to the kinds of **constraints** that are used, and hence by how **accurately** program **features** are modeled

automation: fully **automated**

expressiveness: any **reachability** properties that are expressible in code (assertions)

Symbolic execution: tools and practice

Symbolic execution **tools** focus on test-case generation with high coverage, and on (white-box) fuzzing to generate failure-inducing inputs. They can often handle complex code, including systems code and concurrency.

In notable **case studies**, symbolic execution was used to detect vulnerabilities in Unix systems and communication software, in image filters, and in document converters.

Main outstanding **challenges**:

- **scalability** by optimizing the usage of constraint solving
- **thoroughness** of the search, which has to be traded-off against scalability
- handling **complex** language **features** (loops and recursion, memory modeling, external code) in a practical way

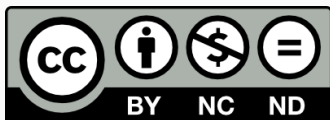
Two **surveys** on dynamic symbolic execution describe the state of the art and include many references to tools and techniques:

- Cadar and Sen: Symbolic execution for software testing: three decades later, Communications of the ACM, 56(2), 2013
- Baldoni et al.: A survey of symbolic execution techniques, ACM Computing Surveys, 51(3), 2018

Some of the examples in this class are based on these two surveys, as well as on slides by Antonio **Filieri** – which in turn were based on Corina **Păsăreanu**'s tutorial at Marktoberdorf 2012.

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