Homework 3: Spherical Harmonics

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Requirements

1. Analyze how many residual low order components left in three scenarios.
2. You may interpretate the data in two ways: histogram and Spherical Harmonic coefficients.
3. Submit a report with your results and data interpretation in form of pdf, and your code used to process the data.

Theory: Spherical Harmonics

The spherical harmonics are the angular part of the solution to Laplace's equation in spherical coordinates. The spherical harmonics are a complete set of orthogonal functions on the sphere, and thus may be used to represent functions defined on the surface of a sphere, just as circular functions are used to represent functions on a circle through Fourier series.

$R_n^m(r,\theta,\phi) = r^n P_n^m(\cos\theta)e^{jm\phi}$

where P_n^m are the associated Legendre polynomials, and n and m are integers that satisfy $0 \leq m \leq n$. The spherical harmonics are orthonormal on the sphere, meaning that they are orthogonal with respect to the L^2 inner product on the sphere.

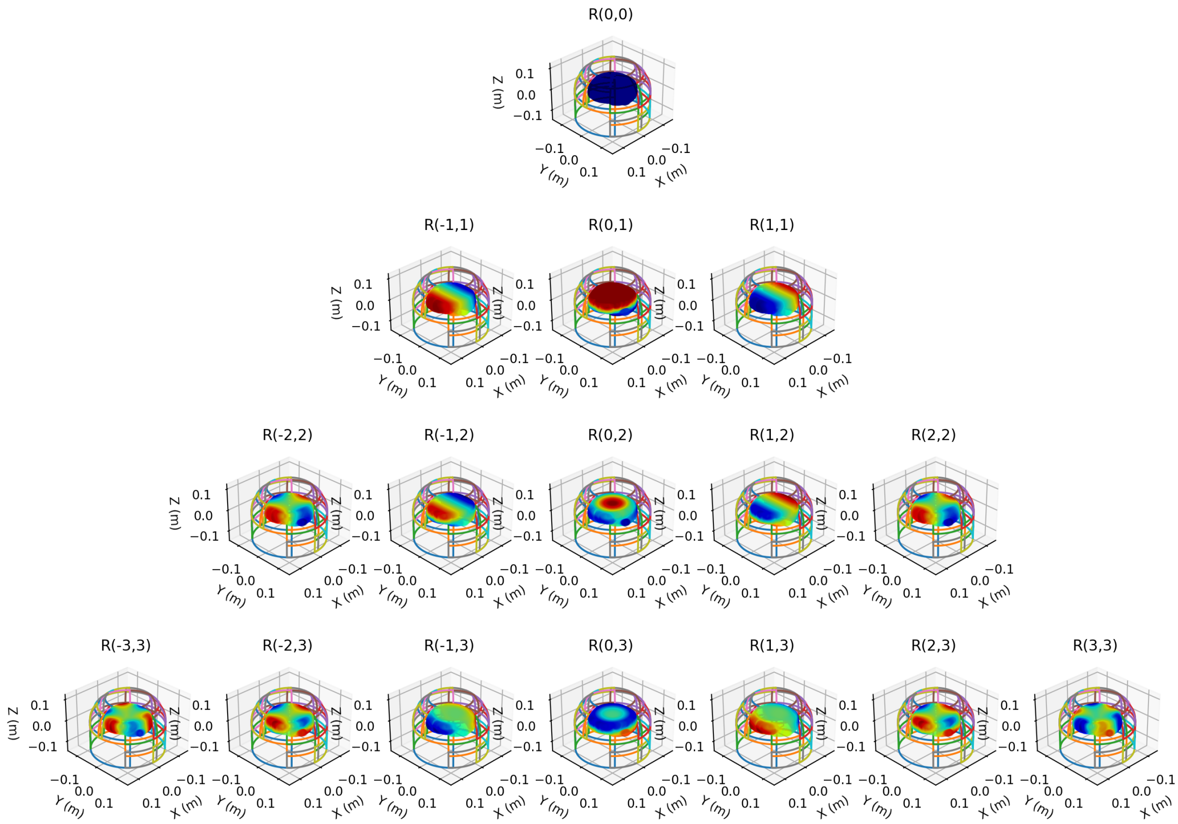
Methods and Materials

Data

The analysis is performed on the brain B0 map with and without system shimming. Homework 2 also produces the 29-ch multi-coil active shimming data with different loss functions (L1 norm, L2 norm, std). All the B0 maps are stored in .mat files and loaded into Python for analysis.

Spherical Harmonic Expansion

The spherical harmonic Expansion is done utilized scipy library. Here we expand the first 3 order of spherical harmonics to the B0 map. The basis functions for the spherical harmonics shown below:



Histogram Analysis

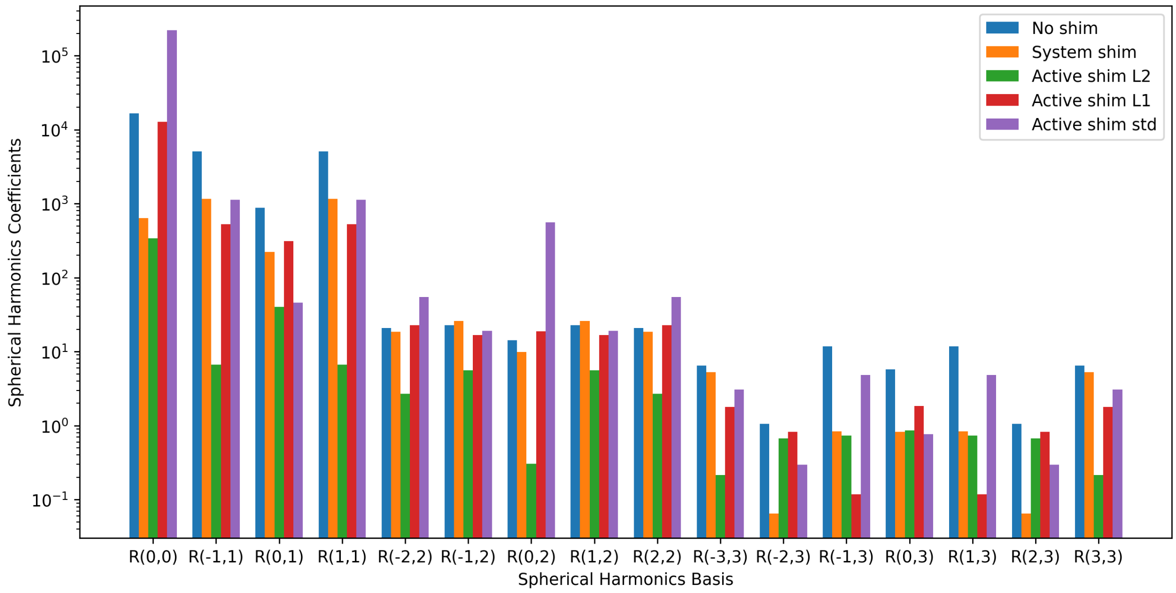
Although the histogram analysis has been done in the previous homework, we still present the histogram of the B0 map over here.

Code Availability

The code is available at https://github.com/bughht/MRI_System_Design

Results

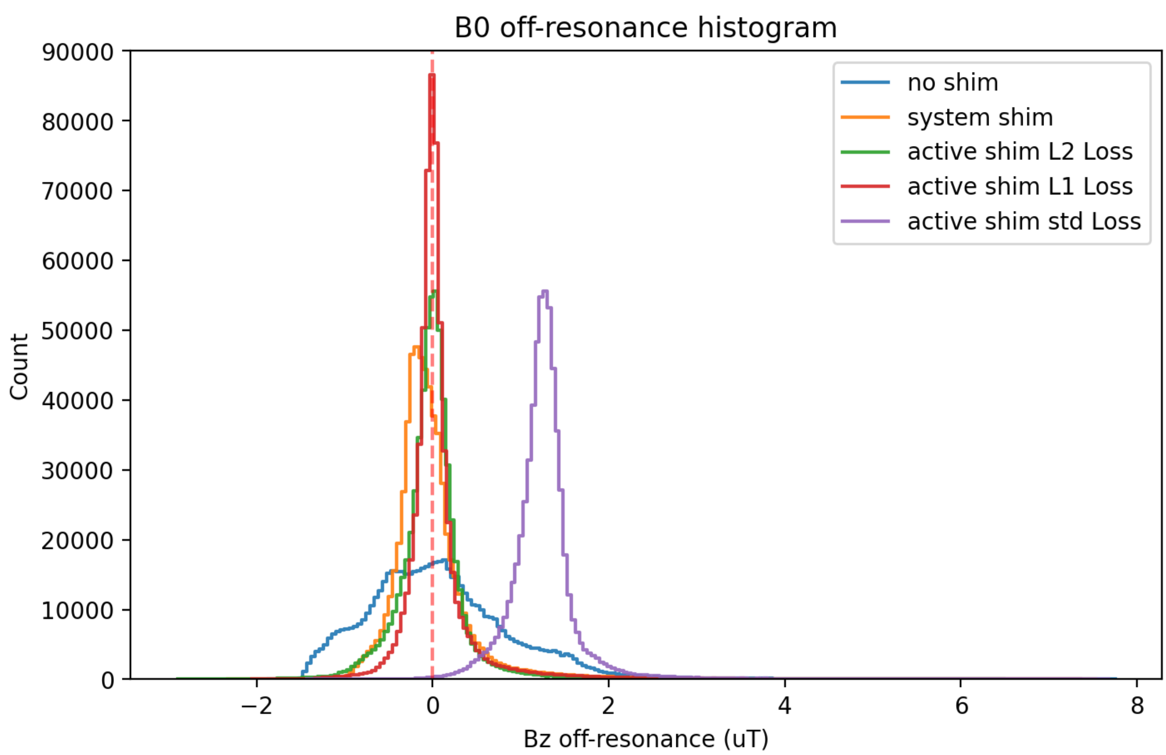
The spherical harmonic expansion of the B0 map with and without system shimming is shown in the bar plot and the table below. The x-axis represents the order of the spherical harmonics, and the y-axis represents the magnitude of the coefficients in the log scale. Here we can see that the B0 map without shim shows a relatively high magnitude for all orders of spherical harmonics. However, the B0 map with system shimming shows a significant reduction in zeroth order spherical harmonics, which indicates that the system shimming effectively removes the low-frequency components in the B0 map. With respect to the 29-channel active shimming, the L2 norm loss function shows the best performance in reducing the low order spherical harmonic components.



Shimming Method	R(0,0)	R(-1,1)	R(0,1)	R(1,1)	R(-2,2)	R(-1,2)	R(0,2)	R(1,2)	R(2,2)	R(-3,3)	R(-2,3)	R(-1,3)	R(0,3)	R(1,3)	R(2,3)	R(3,3)
No Shim	16579.010	5090.957	885.143	5090.957	20.755	22.654	14.172	22.654	20.755	6.519	1.057	11.846	5.758	11.846	1.057	6.519
System Shim	639.954	1158.936	222.990	1158.936	18.674	25.885	9.885	25.885	18.674	5.263	0.064	0.835	0.830	0.835	0.064	5.263
Active Shim L2	340.876	6.653	40.109	6.653	2.676	5.582	0.304	5.582	2.676	0.215	0.668	0.738	0.862	0.738	0.668	0.215
Active Shim L1	12724.295	526.471	310.119	526.471	22.922	16.701	18.800	16.701	22.922	1.794	0.821	0.118	1.830	0.118	0.821	1.794
Active Shim Std	221274.086	1135.238	45.682	1135.238	55.021	19.102	559.784	19.102	55.021	3.051	0.297	4.868	0.764	4.868	0.297	3.051

The following histograms illustrate the shimmed B0 field using different loss functions: L2 norm, L1 norm, and standard deviation. These are compared with the original field and the system shimmed field.

The field homogeneity shows marked improvement post-shimming. The L1 norm loss function outperforms the others in terms of peak sharpness at the on-resonance frequency, indicating minimal field offset. The L2 norm results are comparable to the system shimmed field. While the standard deviation loss function result exhibits a sharper peak, surpassing the L2 norm result, it is not centered at the on-resonance frequency, suggesting the introduction of an unwanted constant field offset.



Conclusion

In this homework, we've successfully analyzed the residual low-order components in the B0 map with and without system shimming. The spherical harmonic expansion shows that the system shimming effectively removes the low-frequency components in the B0 map. The 29-channel active shimming with the L2 norm loss function shows the best performance in reducing the low-order spherical harmonic components. The histogram analysis further confirms the effectiveness of the shimming methods, with the L1 norm loss function providing the sharpest peak at the on-resonance frequency. The L2 norm loss function shows comparable results to the system shimmed field, while the standard deviation loss function introduces an undesirable constant field offset.