Quickest Detection of Dynamic Events in Networks

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Summary

Introduction

- In the problem of quickest change detection (QCD), a stochastic system is observed sequentially.
- At some unknown time, a change occurs that changes the data generating process.

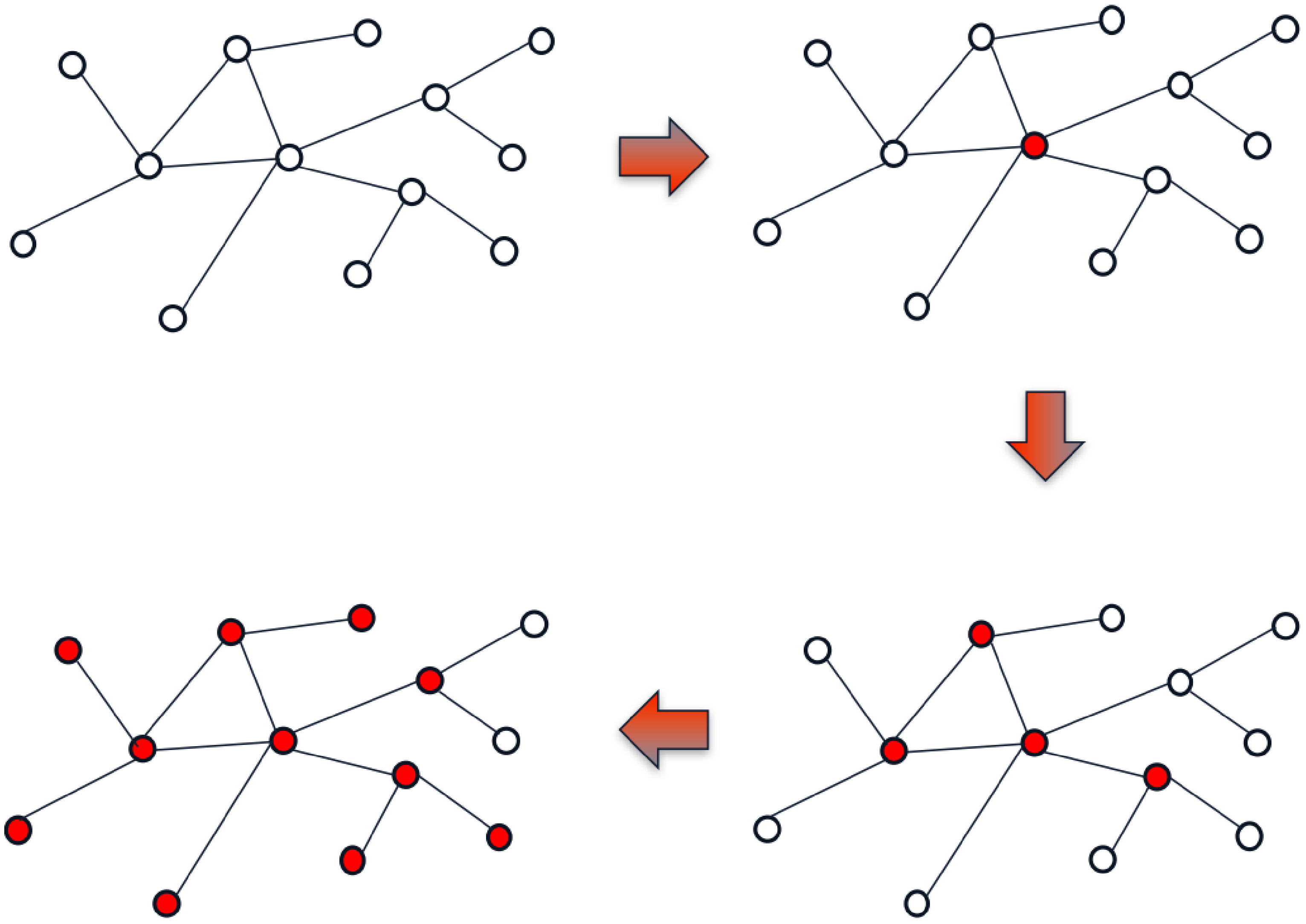

Figure 1: A dynamic event propagates in a network with time.

Introduction

- Observations are taken sequentially with time, and the objective is to detect the change as quickly as possible subject to **false alarm constraints**.
- E.g., fraud detection, environmental monitoring, quality control in online manufacturing systems.

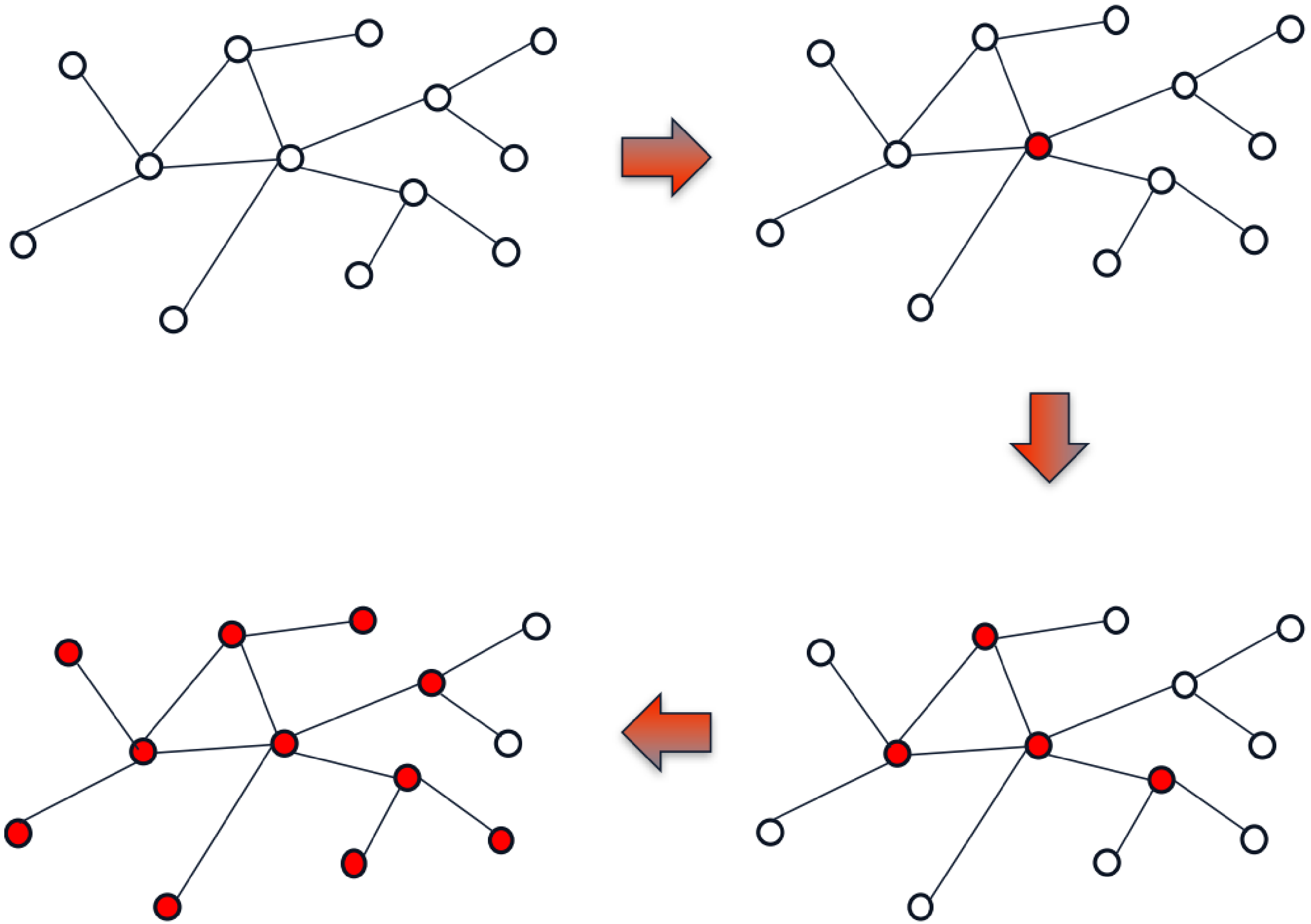

Figure 1: A dynamic event propagates in a network with time.

Introduction

- Many problems, like epidemic detection, have dynamic propagation in specific graph topologies, where propagation is important.
- This paper study QCD dynamically:
    1. Monitor L nodes. At some unknown time, an event occurs in the network that causes eventual changes in the observations of a connected subset of nodes.
    2. The event occurs at a connected subset of nodes and then dynamically propagates along the edges in the network, and the affected nodes form a connected sub-graph, the size of which grows with time.
    3. The propagation dynamics are assumed to be unknown, i.e., the set of nodes and the order in which they are affected are unknown.
    4. **We are interested in detecting a "significant" event, i.e., one that affects $\eta \geq 1$ nodes as quickly as possible, subject to false alarm constraints.**

Mathematical Contextualization

The Sequential Probability Ratio Test

The SPRT is based on considering the likelihood ratio as a function of the number of observations. The goal of the SPRT is to decide which hypothesis is correct as soon as possible (i.e., for the smallest value of k). To do this the SPRT requires two thresholds, $\gamma_1 > \gamma_0$. The SPRT "stops" as soon as $\Lambda_k \geq \gamma_1$, and we then decide H_1 is correct, or when $\Lambda_k \geq \gamma_0$, and we then decide H_0 is correct.

$$H_i \; : \; X_1, X_2, \ldots, X_n \overset{iid}{\sim} p_i \; , \; i = 0, 1 \; .$$

$$\Lambda_k \; := \; \prod_{i=1}^{k} \frac{p_1(X_i)}{p_0(X_i)} \; , \; k = 1, 2, \ldots$$

Mathematical Contextualization

Expected Stopping Time

Since Λ_k is a random variable, the stopping time of the SPRT is also random. Let K^* denote the random (integer) stopping time. We can calculate the expected value of K^* as follows.

$$\mathbb{E}_j[\log \Lambda_k] = \mathbb{E}_j\left[\sum_{i=1}^{k} \log \frac{p_1(X_i)}{p_0(X_i)}\right] = \sum_{i=1}^{k} \mathbb{E}_j\left[\log \frac{p_1(X_i)}{p_0(X_i)}\right] = \begin{cases} k\,D(p_1||p_0)\ , & j=1 \\ -k\,D(p_0||p_1)\ , & j=0 \end{cases}$$

Mathematical Contextualization

Expected Stopping Time

Now suppose that M is a positive integer-valued random variable, independent of X_1, $X_2, \ldots$.

$$\mathbb{E}_j[\log \Lambda_M] = \mathbb{E}_j\left[\mathbb{E}_j[\log \Lambda_M \mid M]\right] = \mathbb{E}_j\left[\sum_{i=1}^{M} \mathbb{E}_j\left[\log \frac{p_1(X_i)}{p_0(X_i)} \mid M\right]\right] = \begin{cases} \mathbb{E}_j[M]\, D(p_1 \| p_0)\ , & j = 1 \\ -\mathbb{E}_j[M]\, D(p_0 \| p_1)\ , & j = 0 \end{cases}$$

The stopping time K^* is random, but it is also a function of X_1, X_2,… so we cannot apply the simple conditioning argument used for M above. However, a more delicate argument shows that a similar result holds with M replaced with K^*(**Wald´s Identity**).

Mathematical Contextualization

Expected Stopping Time

Finally:

$$\mathbb{E}_j[\log \Lambda_{K^*}] = \begin{cases} \mathbb{E}_j[K^*]\, D(p_1||p_0)\ , & j = 1 \\ -\mathbb{E}_j[K^*]\, D(p_0||p_1)\ , & j = 0 \end{cases}$$

And that's how we calculate the Expected Stopping Time of the SPRT model.

Mathematical Formulation

Before an event occurs, node i ∈ { 1, 2, . . . , L } receives independent and identically distributed (i.i.d.) samples from distribution f_0. If an event occurs, and node i is affected by the event at an unknown time v_i, then it starts to receive i.i.d. samples from distribution f_1, i.e., v_i **is the change-point at node** i. If v_i = ∞ , node i will not be affected by the event ever. More specifically, if we denote the observation received by node i at time k by $X_i[k]$, then:

$$X_i[k] \sim \begin{cases} f_0, & \text{if } k < \nu_i, \\ f_1, & \text{if } k \geq \nu_i. \end{cases}$$

Let $\boldsymbol{v} = \{v_1, \ldots, v_i\}$, which is unknown in advance. Without loss of generality, we assume that $v_1 \leq v_2 \leq \cdots \leq v_L$

$$|C(\boldsymbol{\nu})| = \sum_{1 \leq i \leq L} \mathbb{1}_{\{\nu_i < \infty\}}$$

Mathematical Formulation

For any stopping time τ, to measure how frequently false alarms occur, we define the worst-case average run length:

$$\mathrm{WARL}(\tau) = \inf_{\boldsymbol{\nu}:|C(\boldsymbol{\nu})|<\eta} \mathbb{E}_{\boldsymbol{\nu}}[\tau].$$

Let $d_i = v_{i+1} - v_i$ denote the time it takes for the event to propagate from node i to node $i + 1$, for $1 \le i \le L - 1$. If di = 0, then node i and node $i + 1$ are affected simultaneously. Denote $D := \{ d_\eta, d_{\eta+1}, \ldots, d_{L-1}\}$.

$$J_{\boldsymbol{D}}(\tau) = \sup_{\nu_1 \le \cdots \le \nu_\eta < \infty} \mathbb{E}_{\boldsymbol{\nu}}[\tau - \nu_\eta | \tau \ge \nu_\eta].$$

The goal is to minimize the following quantity:

$$\inf_{\tau:\mathrm{WARL}(\tau)\ge\gamma} J_{\boldsymbol{D}}(\tau).$$

In other words, it is to find stopping rules so that for all possible scenarios with fewer than η affected nodes, the average run length to false alarm is at least γ.

Generalized log-likelihood ratio statistic:

$$\mathcal{H}_0[k] : \sum_{i=1}^{L} \mathbb{1}_{\{\nu_i \le k\}} < \eta,$$

$$\mathcal{H}_1[k] : \sum_{i=1}^{L} \mathbb{1}_{\{\nu_i \le k\}} \ge \eta.$$

$$W[k] = \log\left(\frac{\max\limits_{\boldsymbol{\nu}:\sum_{i=1}^{L} \mathbb{1}_{\{\nu_i \le k\}} \ge \eta} \mathbb{P}_{\boldsymbol{\nu}}(\boldsymbol{X}[1,k])}{\max\limits_{\boldsymbol{\nu}:\sum_{i=1}^{L} \mathbb{1}_{\{\nu_i \le k\}} < \eta} \mathbb{P}_{\boldsymbol{\nu}}(\boldsymbol{X}[1,k])}\right).$$

$$\tilde{\tau}(b) = \inf\{k \ge 1 : W[k] > b\},$$

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$$\begin{aligned}
&\log\left(\frac{\mathbb{P}_{\boldsymbol{\nu}}(\boldsymbol{X}[1,k])}{\mathbb{P}_{\infty}(\boldsymbol{X}[1,k])}\right) \\
&= \log\left(\prod_{i=1}^{L} \frac{\prod_{j=1}^{\min\{\nu_i-1,k\}} f_0(X_i[j]) \prod_{j=\nu_i}^{k} f_1(X_i[j])}{\prod_{j=1}^{k} f_0(X_i[j])}\right) \\
&= \sum_{i=1}^{L} \sum_{j=\nu_i}^{k} \log \frac{f_1(X_i[j])}{f_0(X_i[j])},
\end{aligned}$$

$$W_i[k] = \max_{1 \le \nu_i \le k} \sum_{j=\nu_i}^{k} \log \frac{f_1(X_i[j])}{f_0(X_i[j])},$$

$$W_{\mu(1)}[k] \ge W_{\mu(2)}[k] \ge \cdots \ge W_{\mu(L)}[k],$$

Generalized log-likelihood ratio statistic:

$$\mathcal{H}_0[k] : \sum_{i=1}^{L} \mathbb{1}_{\{\nu_i \leq k\}} < \eta,$$

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&= \sum_{i=1}^{L} \sum_{j=\nu_i}^{k} \log \frac{f_1(X_i[j])}{f_0(X_i[j])},
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$$W_i[k] = \max_{1 \le \nu_i \le k} \sum_{j=\nu_i}^{k} \log \frac{f_1(X_i[j])}{f_0(X_i[j])},$$

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$$W_i[k] = \max_{1 \le \nu_i \le k} \sum_{j=\nu_i}^{k} \log \frac{f_1(X_i[j])}{f_0(X_i[j])},$$

$$W_{\mu(1)}[k] \ge W_{\mu(2)}[k] \ge \cdots \ge W_{\mu(L)}[k],$$

Generalized log-likelihood ratio statistic

S-CuSum Algorithm

$$\tilde{\tau}(b) = \inf\{k \geq 1 : W[k] > b\},$$

is equivalent to

$$\hat{\tau}(b) = \inf\left\{k \geq 1 : \sum_{i=\eta}^{L} \left(W_{\mu(i)}[k]\right)^{+} \geq b\right\},$$

However, this new form can be implemented efficiently:
Which is O(L) at each time k.

$$W_i[k] = (W_i[k-1])^{+} + \log \frac{f_1(X_i[k])}{f_0(X_i[k])}.$$

Asymptotical Analysis

- It's a more complicated part of the paper, with proofs at its end.
- First, why do we need a asymptotical analysis?
- We want to find **stopping rules** so that for all possible scenarios with fewer than η affected nodes, the average run length to false alarm is at least γ. At the same time, among those **stopping rules** that satisfy the false alarm requirement, we want to find the one that minimizes the WADD for all propagation dynamics after η nodes are affected. No guarantee that the optimization problem can be solved, since we require the same stopping rule to simultaneously minimize the WADD for all propagation dynamics after η nodes are affected. A "uniformly" optimum solution can be found up to a first-order approximation in an appropriately defined asymptotic setting.

Asymptotical Analysis

$$\mathrm{WARL}(\tau) = \inf_{\boldsymbol{\nu}:|C(\boldsymbol{\nu})|<\eta} \mathbb{E}_{\boldsymbol{\nu}}[\tau].$$

$$J_{\boldsymbol{D}}(\tau) = \sup_{\nu_1 \le \cdots \le \nu_\eta < \infty} \mathbb{E}_{\boldsymbol{\nu}}[\tau - \nu_\eta | \tau \ge \nu_\eta].$$

$$\inf_{\tau:\mathrm{WARL}(\tau)\ge\gamma} J_{\boldsymbol{D}}(\tau).$$

Asymptotical Analysis

Theorem 7 (N-CuSum, Asymptotic Optimality)**.** *Let threshold $b \sim \log \gamma$ so that $WARL(\bar{\tau}(b)) \geq \gamma$. Assume that $d_\eta, d_{\eta+1}, \ldots, d_{L-1}$ and γ go to infinity as in (27), then the N-CuSum algorithm is asymptotically optimal:*

$$
\begin{aligned}
J_{\boldsymbol{D}}(\bar{\tau}(b)) &\sim \inf_{\tau:\mathrm{WARL}(\tau)\geq\gamma} J_{\boldsymbol{D}}(\tau) \\
&\sim \log \gamma \left(\sum_{i=1}^{h-1} \frac{c_i}{iI} + \frac{1 - \sum_{i=1}^{h-1} c_i}{hI} \right). \qquad (41)
\end{aligned}
$$

Proof. This result follows from Theorems 2, 5 and 6. □

Arbitrarily Connected Networks

- For an arbitrarily connected network, the induced sub-graph on some subset of nodes may not be connected. S-CuSum may raise alarm when it detects η nodes that are **not connected**.
- To exploit knowledge of the network structure, one can directly adapt the generalized log-likelihood ratio test, creating the N-CuSum algorithm.

Arbitrarily Connected Networks

- At each time step k, we update the local CuSum statistics $W_i[k], \forall 1 \leq i \leq L$.
- We then compare each local CuSum statistic to a threshold log b, and delete this node if its CuSum statistic is less than log b. The resulting graph is then denoted by G0[k].
- We run the BFS algorithm on $G_0[k]$ to recover all connected components of $G_0[k]$: $C_1[k]$, $C_2[k]$, ...
- The computational complexity for this step is at most $O(L + |E|)$. We then run the S-CuSum algorithm on each connected component, and use $\mathrm{SCuSum}_i[k]$ to denote the test statistic value of the S-CuSum algorithm on $C_i[k]$:

$$\text{S-CuSum}_i[k] = \min_{\substack{C' \subseteq C_i[k]: \\ |C'|=|C_i[k]|-\eta+1}} \sum_{i \in C'} (W_i[k])^+ .$$

- If any of these statistics crosses the threshold b, we stop and raise an alarm.

Results

First numerical Test:

- We consider a fully connected network with $L = 3$ and $\eta = 2$.
- We choose $f_0 = N(0,1)$ and $f_1 = N(1,1)$.
- We set $\nu\ =\ \{\,1, 40, 80\,\}$.
- Shows adaptability of S-Cusum.

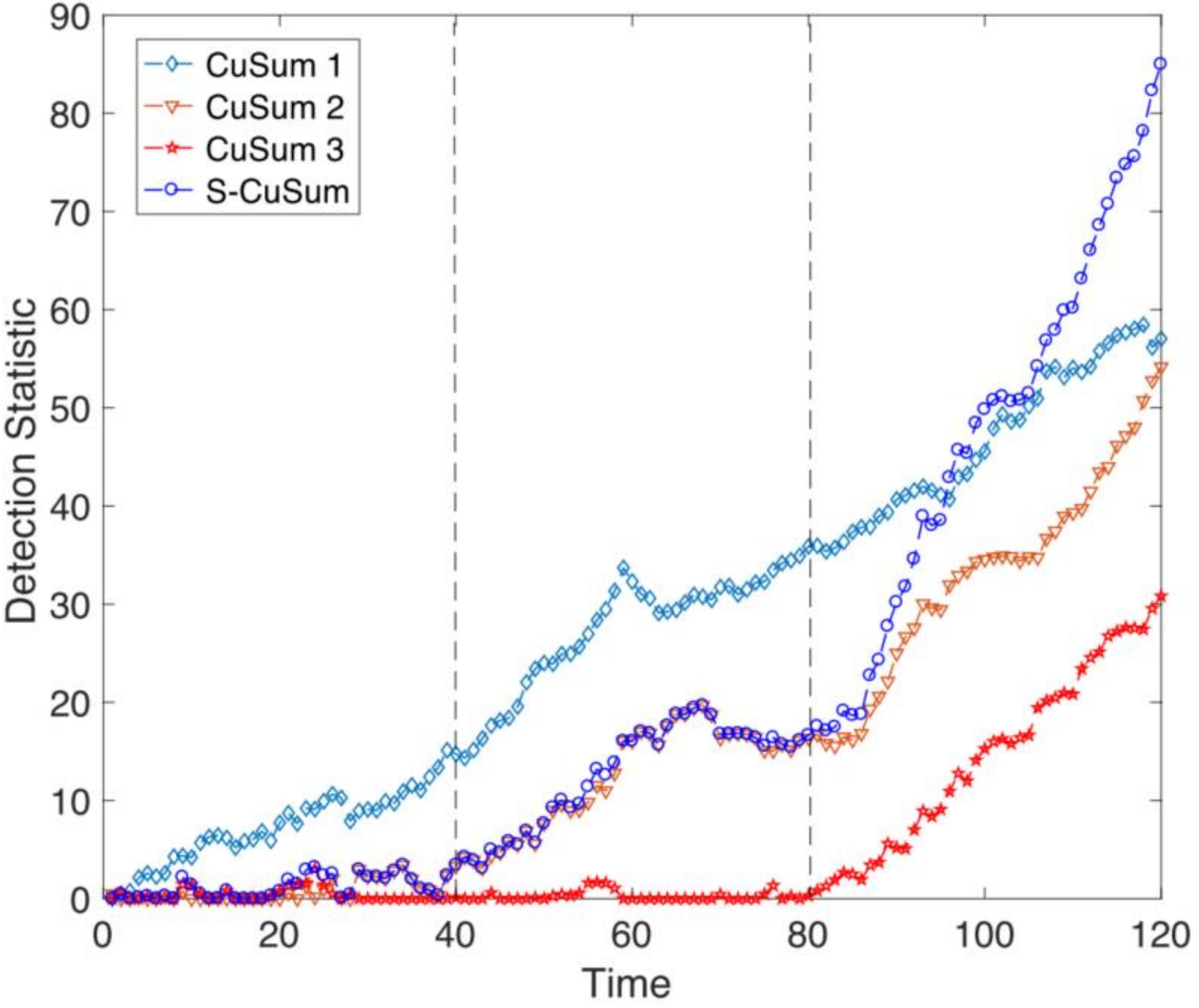

Figure 5: Sample evolution paths of all individual CuSums and the S-CuSum. CuSum i denotes the individual CuSum statistic at node i, for $i = 1, 2, 3$.

Second numerical Test:

- We consider a fully connected network with $L = 3$ and $\eta = 2$.
- We choose $f_0 = N(0,1)$ and $f_1 = N(0.4,1)$.
- $\nu = \{1, 1,41\}$ to simulate the average detection delay (WADD)
- $\nu = \{1, \infty, \infty\}$ to simulate the average run length to false alarm (WARL)
- Averaged over 1000 runs.

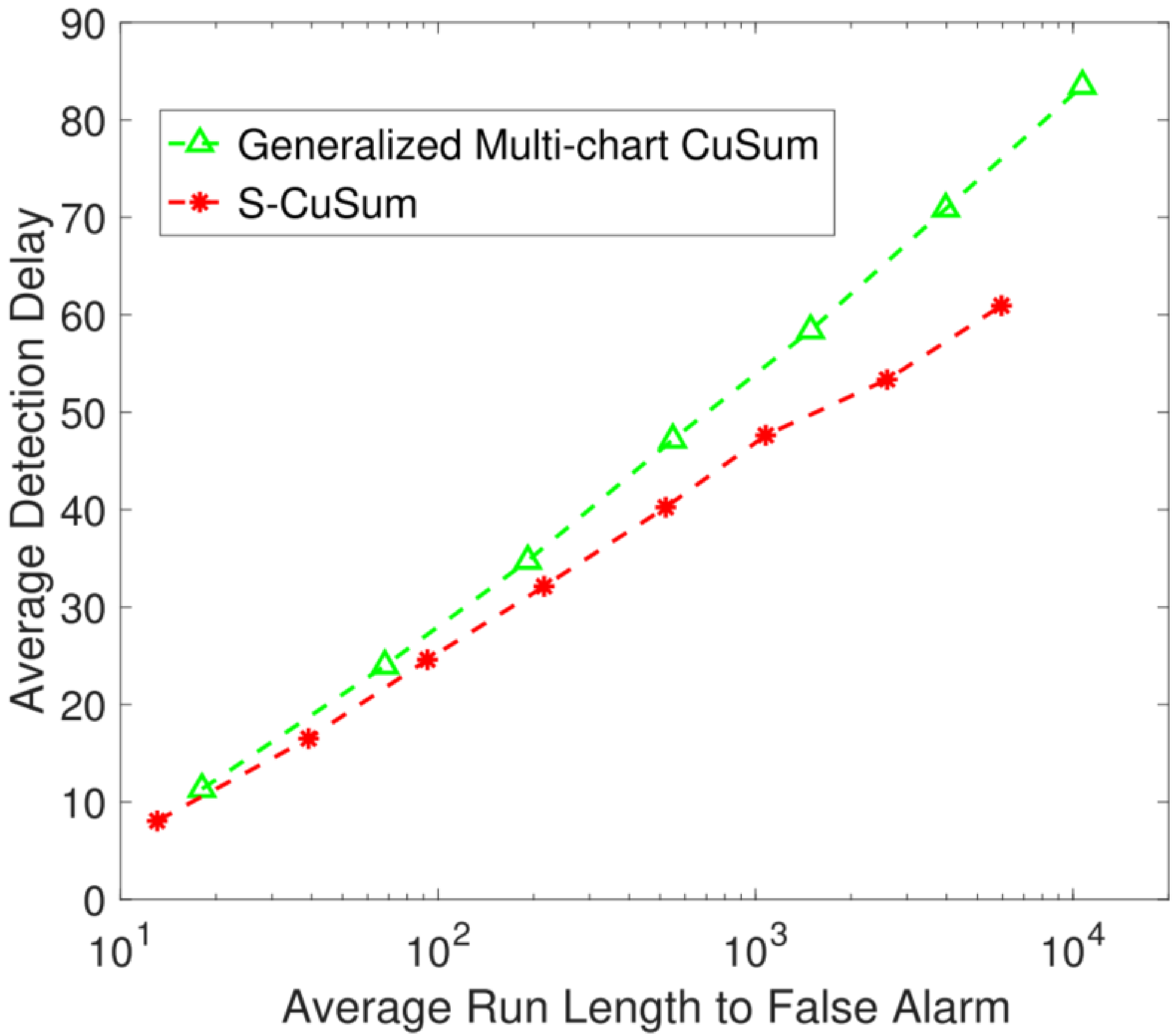

Figure 6: Comparison between the S-CuSum algorithm and the generalized multi-chart CuSum algorithm for a fully connected network.

Third numerical Test:

- Lattice network with 36 nodes and $\eta = 4$.
- We choose $f_0 = N(0,1)$ and $f_1 = N(1,1)$.
- To simulate the average run length to false alarm, we assume that nodes 14, 15, 16 are affected at time 1, and no other node is affected by the event.
- Averaged over 1000 runs.
- Compared to the S-CuSum algorithm, the N-CuSum algorithm has significantly reduced the WADD, **which is due to its effective exploitation of the network structure**.

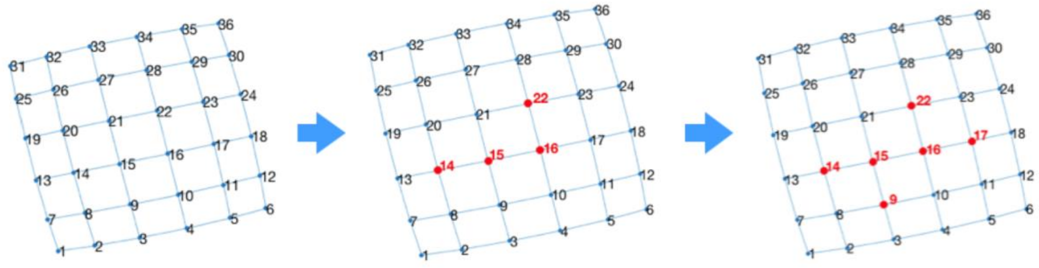

Figure 7: A dynamic event propagates in a lattice network.

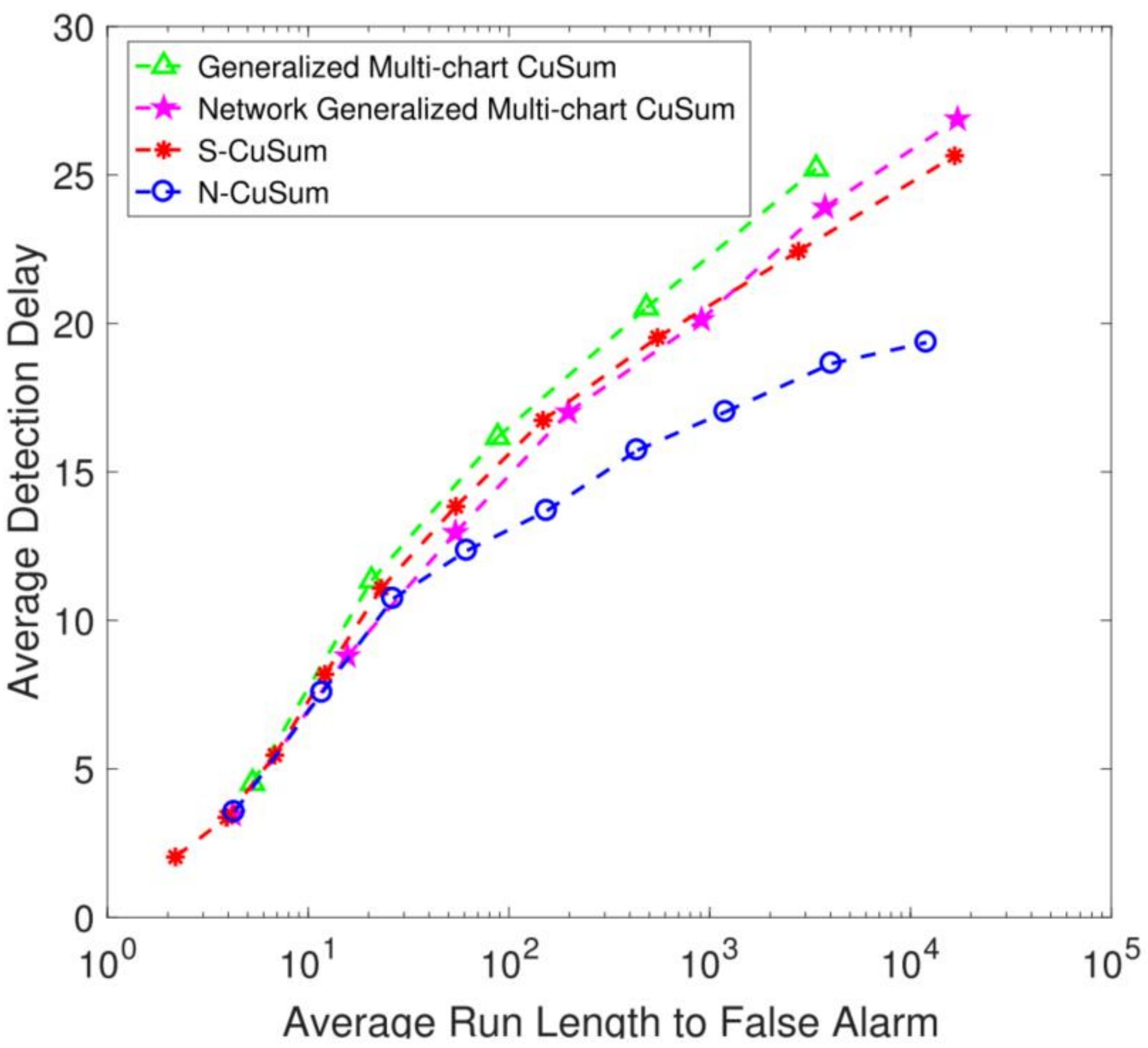

Figure 8: Comparison among the generalized multi-chart CuSum algorithm, the N-generalized multi-chart CuSum algorithm, the S-CuSum algorithm and the N-CuSum algorithm.

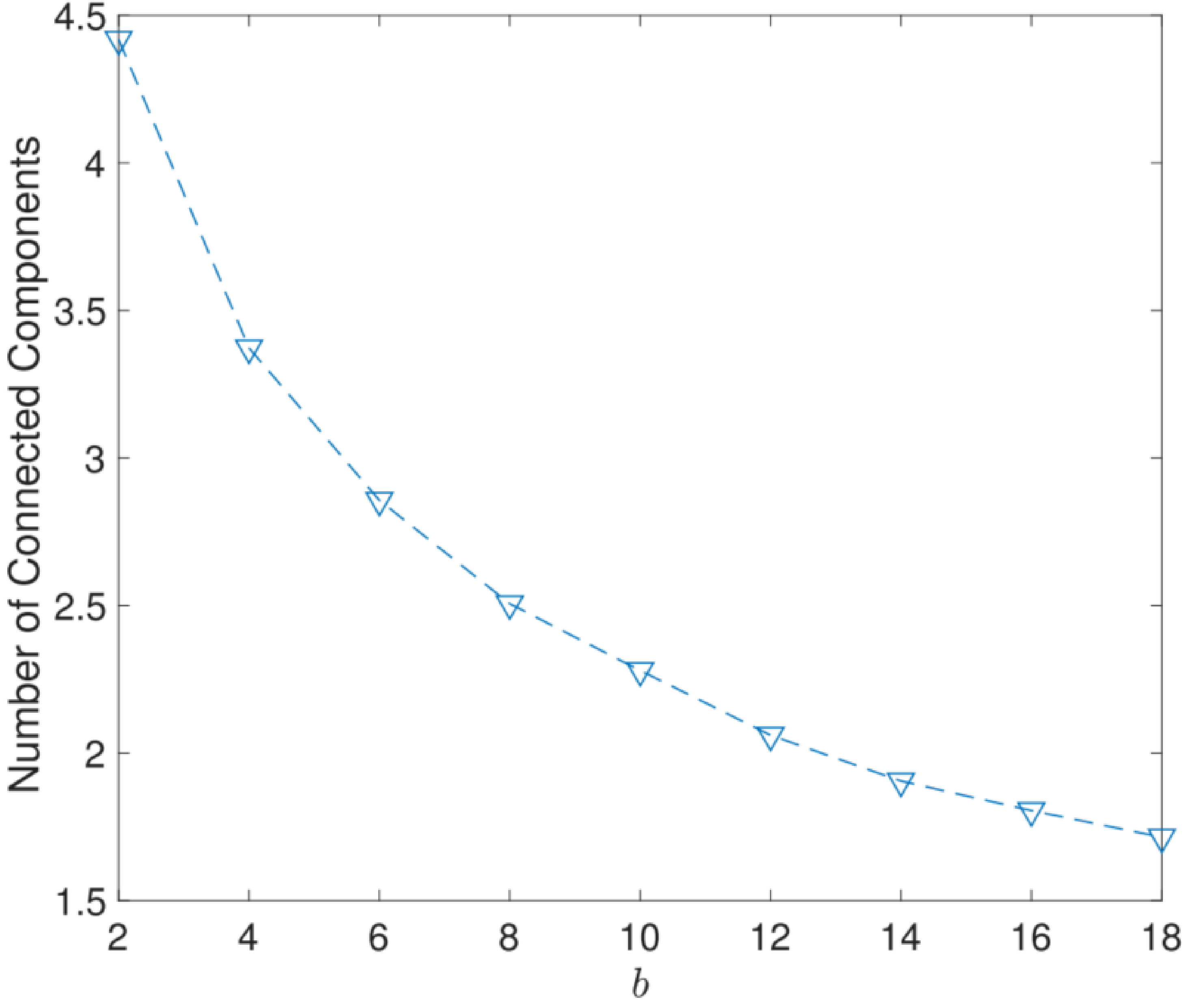

Figure 9: Number of connected components in $G'[k]$ when N-CuSum crosses the threshold.

Table 1: Comparison of time consumption

Generalized Multi-chart CuSum	Network Generalized Multi-chart CuSum	S-CuSum	N-CuSum
0.15	0.77	0.12	0.94

Discussion

- (+)The mathematics was clear enough such that It can be changed accordingly to problem specifics, e.g. sample distributions.
- (+)All the derivational details are shown – its understading steps being secure.
- (+/-)Quantity of math maybe didn't "deliver" proportional benefits.
- (-)No numerical test on real data.
- (-)Is there a problem? Meaning: detecting spread of a single event, possessing control via a fusion center, knowing the data generating distribution, etc. Which kind of application actually have these set of characterics?