(№11)

$H_0: \quad p_0(x) = \begin{cases} 1, & x \in (0,1) \\ 0, & x \in (0,1) \end{cases}$

$H_1: \quad p_1(x) = \begin{cases} \frac{e^{1-x}}{e-1}, & x \in (0,1) \\ 0, & x \in (0,1) \end{cases}$

(a) $n=1$, α - дана

по Т Неймана Пирсона

$l = \frac{L_1}{L_0} \geq C$

$\frac{p_1(x)}{p_0(x)} = \frac{e^{1-x}}{e-1} \geq C$ (1)

$(1-x) \geq \tilde{C} \ln(e-1)$

$\hat{C} \geq x$

$P(x \leq \hat{C} | H_0) = \int_0^{\hat{C}} 1\,dx = \alpha \quad \hat{C} = \alpha$

$\Rightarrow G: x \leq \alpha$ - реш. оптим. задачи

Найдём ошибки:

$\alpha_1 = P(H_1|H_0) = P(x \in G | H_0) = P(x_1 \leq \alpha | H_0) = \alpha$

$\alpha_2 = P(H_0|H_1)$ - ?

$$W = P(x_n \in G \mid H_1) = P(x \leq \hat{c} \mid H_1) = \int_0^{\alpha} \frac{e}{e-1} e^{-x} dx =$$

$$= \frac{e}{e-1} e^{-x} \Big|_{\alpha}^{0} = \frac{e}{e-1}\left(1 - e^{-\alpha}\right)$$

$$\alpha_2 = 1 - W = 1 - \frac{e}{e-1}\left(1 - e^{-2}\right)$$

б) $n = 2,\ \alpha$

$$\ell = \frac{\prod_i p_{1i}}{\prod_i p_{0i}} = \frac{\left(\frac{e}{e-1}\right)^2 e^{-x_1 - x_2}}{1} \geq C$$

$$e^{-x_1 - x_2} \geq \tilde{C}$$

$$x_1 + x_2 \leq \hat{C}$$

$$P(x_1 + x_2 \leq \hat{c} \mid H_0) = \iint\limits_{x_1 + x_2 \leq \hat{c}} 1 \, dx_1 dx_2$$

$$P(x_1 + x_2 \leq \hat{c} \mid H_0) = \frac{\hat{c}^2}{2} = \alpha$$

$$\hat{c} = \sqrt{2\alpha}$$

$$\alpha_1 = \alpha$$

$$W = P(x_1 + x_2 \leq \hat{c} \mid H_1) = \int_0^{\hat{c}} dx_1 \int_0^{\hat{c} - x_1} \left(\frac{e}{e-1}\right)^2 e^{-x_1 - x_2} dx_2 =$$

$$= \frac{e^2}{(e-1)^2}\left(1 - e^{-\sqrt{2\alpha}} - \sqrt{2\alpha}\, e^{-\sqrt{2\alpha}}\right)$$

$$\alpha_2 = 1 - W$$

в) $\ln l = \sum_i \ln\left(\frac{p_1(x_i)}{p_0(x_i)}\right)$

$$\frac{\sum \eta_i - M[\eta_i]}{\sqrt{nD[\eta_i]}} \sim N(0,1)$$

в H_0:

$$M\left[\ln\frac{p_1}{p_2}\right] = M[\eta_i] = M\left[\ln\frac{e}{e-1}e^{-x_i}\right] = M\left[\ln\frac{e}{e-1} - x_i\right] =$$

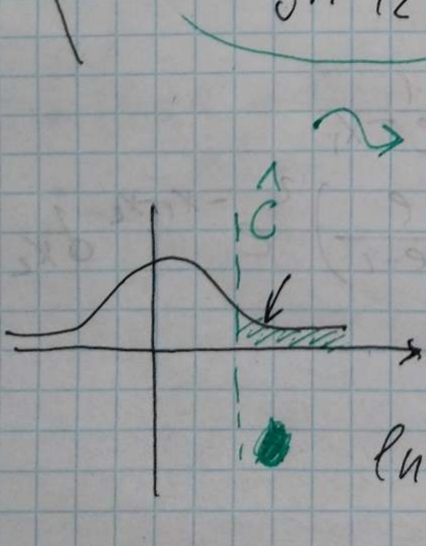
$$= \int_0^1 \left(\ln\frac{e}{e-1} - x_i\right)x\,dx = \ln\frac{e}{e-1} - \frac{1}{2}$$

$$D[\eta_i] = D\left[\ln\frac{e}{e-1} - x_i\right] = \int_0^1 \left(\frac{1}{2} - x_i\right)^2 dx =$$

$$= \frac{x_i^3}{3}\Big|_0^1 - \frac{x_i^2}{2}\Big|_0^1 + \frac{1}{4}\Big|_0^1 = \frac{1}{3} - \frac{1}{2} + \frac{1}{4} = \frac{1}{12}$$

- $P(\ln l \geq \ln C^* \mid H_0) =$

$$= P\Bigg(\underbrace{\frac{\ln l - n\left(\ln\frac{e}{e-1} - \frac{1}{2}\right)}{\sqrt{n\cdot\frac{1}{12}}}}_{\sim N(0,1)} \geq \underbrace{\frac{\ln C^* - n\left(\ln\frac{e}{e-1} - \frac{1}{2}\right)}{\sqrt{n\frac{1}{12}}}}_{\hat{C}}\Bigg) = \alpha$$



$\hat{C} = q_{1-\alpha}$

$$\ln C = \sqrt{\frac{n}{12}}\,q_{1-\alpha} + n\ln\frac{e}{e-1} - \frac{1}{2}$$

$G: \ln l \geq \ln C$

$$\ln l = \sum_{i=1}^{n} \eta_i = \sum_i \ln \frac{e}{e-1} e^{-x_i} =$$

$$= n \ln \frac{e}{e-1} - n\bar{x} \geq \sqrt{\frac{n}{12}}\, q_{1-\alpha} + n \ln \frac{e}{e-1} - \frac{n}{2}$$

$$\Rightarrow \bar{x} \leq \frac{1}{2} - \frac{q_{1-\alpha}}{\sqrt{12n}} \quad (G)$$

$$\alpha = \alpha_1$$

$$W = P\left(\bar{x} \leq \frac{1}{2} - \frac{q_{1-\alpha}}{\sqrt{12n}} \,\Big|\, H_1\right)$$

$$\frac{\bar{x} - M_\xi}{\sqrt{D_\xi}} \sqrt{n} \sim N(0,1)$$

в H_1: $M_\xi = \int_0^1 \frac{e}{e-1} x e^{-x} dx = \frac{e-2}{e-1}$

$$M[\xi^2] = \int_0^1 \frac{e}{e-1} x^2 e^{-x} dx = \frac{2e-5}{e-1}$$

$$D_\xi = M_{\xi^2} - M_\xi^2 = \frac{e^2 - 3e + 1}{(e-1)^2}$$

$$W = P\left(\frac{\bar{x} - \alpha_1}{\sqrt{M_\xi}} \sqrt{n} \leq \frac{\left(\frac{1}{2} - \frac{q_{1-\alpha}}{\sqrt{12n}}\right) - \alpha_1}{\sqrt{M_\xi}} \sqrt{n}\right) =$$

$$= \int_{-\infty}^{B} \frac{e^{-x^2/2}}{\sqrt{2\pi}} = 1 - \alpha_2$$

(2) $G: x_{min} < C$

$$P(x_{min} < C \mid H_0) = \alpha$$

$$F_{min} = 1 - (1 - F(y))^n = 1 - (1-x)^n \ (0,1) \text{ в } H_0$$

$$P(x_{min} \le C \mid H_0) = 1 - (1-C)^n = \alpha$$

$$C = 1 - \sqrt[n]{1-\alpha}$$

$$\alpha_1 = \alpha$$

$$F(x) = \int_0^x \frac{e}{e-1} e^{-t}\, dt = \frac{e}{e-1}(1 - e^{-x})$$

$$F_{min} = 1 - \left(1 - \frac{e}{e-1}(1-e^{-x})\right)^n = 1 - \left(\frac{e^{1-x} - 1}{e-1}\right)^n$$

$$W = P(x_{min} \in G \mid H_1) = 1 - \left(\frac{e^{1-1+\sqrt[n]{1-\alpha}} - 1}{e-1}\right)^n = 1 - \alpha_2$$

~~в H_1: $F(x)$~~ $$\alpha_2 = \left(\frac{e^{\sqrt[n]{1-\alpha}} - 1}{e-1}\right)^n$$

№12 H_0: кость сделана нечестно
H_1: кость сделана честно

4	3	1 и 2
8	4	по 12

$n = 2$

$$\ell = \frac{\prod P_1(x_i)}{\prod_i P_0(x_i)}$$

H_0	1	2	3	4
1	1/16	1/16	1/24	1/12
2	1/16	1/16	1/24	1/12
3	1/24	1/24	1/36	1/18
4	1/12	1/12	1/18	1/9

H_1	1	2	3	4
1	Везде 1/16			
2				
3				
4				

ℓ	1	2	3	4
1	1	1	1.5	0.75
2	1	1	1.5	0.75
3	1.5	1.5	2.25	1.125
4	0.75	0.75	1.125	0.5625

Хотим $\max P(\ell \geq C \mid H_0) \leq \alpha = 0.2$

$C = 2.25 \quad P = \frac{1}{36} < \alpha$

$C = 1.5 \quad P = \frac{4}{24} + \frac{1}{36} = \frac{7}{36} \approx 0.194 < \alpha$

$C = 1.125 \quad P = \frac{4 \cdot 1}{16} + \frac{4 \cdot 1}{24} + \frac{1}{36} + \frac{2 \cdot 1}{18} > 0.2$

0.66.

Если C брать меньше 1.5 P будем $> \alpha$

$\Rightarrow G: \ell \geq 1.5$

3 случая
$3 \times 3 \quad C = 2.25$
$1 \times 3 \quad C = 1.5$
$2 \times 3 \quad C = 1.5$

$W = P(\ell \geq 1.5 \mid H_1) = \frac{5}{16} = 0.3125 = 1 - \alpha_2$

$\boxed{N13}$ $\sigma^2 = 0.1$ $n = 25$

$s^2 = 0.2$

По $\boxed{T}$ Фишера $\Delta = \dfrac{(n-1)s^2}{\sigma^2} \sim \chi^2_{(n-1)}$

посчитали на вольфраме

$$p\text{-value} = P(\Delta \geq \tilde{\Delta} \mid H_0) = \int_{48}^{+\infty} p_{\chi^2_{n-1}}(t)\,dt = 0.05$$

$$\boxed{\tilde{\Delta} = \frac{24 \cdot 0.2}{0.1} = 48}$$

$G: \Delta \geq C$

$$P(\Delta \geq C \mid H_0) = \alpha = 0.05$$

$C \approx 36.4$ - посчитали численно

$$s^2 = \frac{\sigma^2}{n-1} \cdot C = 0.151$$

$G: s^2 \geq 0.151$

$$W = P(s^2 \geq 0.151 \mid H_1) =$$

$$= P\left(\frac{s^2 \cdot 24}{\sigma^2} \geq \frac{0.151 \cdot 24}{\sigma^2}\right) = \int_{0.151}^{+\infty} p_{\chi^2_{n-1}}(t)\,dt$$

№14 $\xi \sim N(a; \sigma_x^2)$; $\sigma_x^2 = 2$

$\eta \sim N(b; \sigma_y^2)$ $\sigma_y^2 = 1$

Выборки x_n, y_m - независимые

$x = \{-1.11, -6.10, 2.42\}$ $y = \{2.29, -2.91\}$

$H_0: a = b$ $H_1: a \neq b;\ a < b;\ a > b$

по $\sqrt{\ }$ Фишера

$\sqrt{n}\,\frac{\bar{x} - \xi}{\sigma} \sim N(0,1)$ $\bar{x} - a \sim N\left(0, \frac{\sigma_x^2}{n}\right)$

$\bar{y} - b \sim N\left(0, \frac{\sigma_y^2}{m}\right)$

$\cancel{\Delta = \bar{x} - \bar{y} - (a-b)}$

$\bar{x} - \bar{y} - (a-b) \sim N\left(0; \frac{\sigma_x^2}{n} + \frac{\sigma_y^2}{m}\right)$

В H_0: $\tilde{\Delta} = \frac{\bar{x} - \bar{y}}{\sqrt{\frac{\sigma_x^2}{n} + \frac{\sigma_x^2}{m}}} = \frac{-1.597 + 2.6}{1.414} \approx 0.7$

$H_1: a \neq b$

хвосты слева и справа

$p\text{-value} = 2\underline{P}(\Delta \geq 0.7 \mid H_0) = 2\frac{1}{\sqrt{2\pi}}\int_{0.7}^{+\infty} e^{-x/2}\,dx =$

$\approx 1.12 > \alpha$

нет оснований отв. H_0

$H_1: a > b, a < b$

$$p\text{-value} = P\left(\Delta \geq |\tilde{\Delta}| \,\middle|\, H_0\right) = \frac{1}{\sqrt{2\pi}} \int_{0.7}^{+\infty} \left| e^{-x^2/2} \right| dx \cong \underline{0.176} > \alpha$$

нет оснований отвергнуть H_0