№4 $\xi \sim R(\Theta; 2\Theta)$

Дана выборка $\vec{X}_n$

~~а) Оценки~~ а) Оценки

~~$\tilde{\Theta}_1$ - ОММ~~ $F(x) = P(\xi < x) = \int^{x} \frac{1}{\Theta} dx = \frac{1}{\Theta}(x - \Theta)\ (\Theta; 2\Theta)$

$$M\xi = \int_{\Theta}^{2\Theta} x \frac{1}{\Theta} dx = \frac{1}{\Theta}\left(\frac{4\Theta^2}{2} - \frac{\Theta^2}{2}\right) = \frac{3}{2}\Theta$$

$$M\xi^2 = \int_{\Theta}^{2\Theta} x^2 \frac{1}{\Theta} dx = \frac{1}{\Theta}\left\{\frac{8\Theta^3}{3} - \frac{\Theta^3}{3}\right\} = \frac{7}{3}\Theta^2$$

$$D\xi = M\xi^2 - M^2\xi = \frac{7}{3}\Theta^2 - \frac{9}{4}\Theta^2 = \frac{\Theta^2}{12}$$

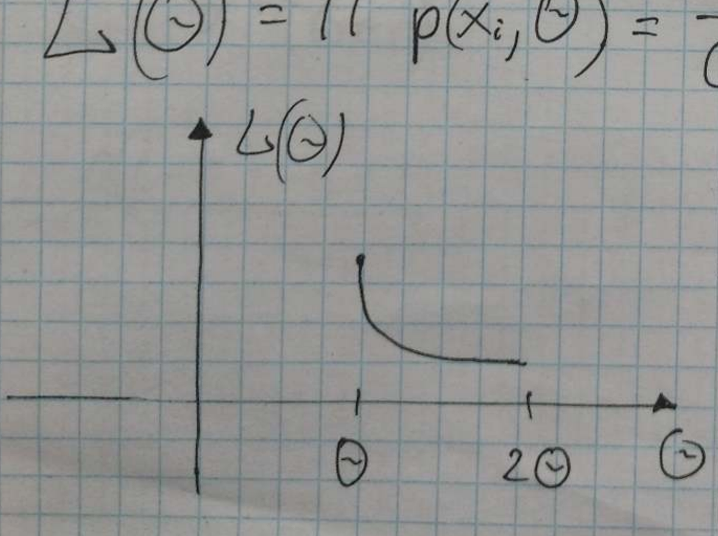
ОММ $- \tilde{\Theta}_1$

$\Theta = f(\alpha_1)$

$\alpha_1 = M\xi = \frac{3}{2}\Theta \rightarrow \boxed{\tilde{\Theta}_1 = \frac{2}{3}\overline{X}}$

ОМП $- \tilde{\Theta}_2$

$L(\Theta) = \prod p(x_i, \Theta) = \frac{1}{\Theta^n}(\Theta; 2\Theta)$ - ф-ия правдоподобия



Чем большее $L(\Theta)$ тем более вероятно событие, а мы находимся в самом вероятном событии

Т.к. Если $x_i \notin [\Theta; 2\Theta]$ $L(\Theta)=0$

то $x_{max}, x_{min} \in [\Theta; 2\Theta]$

$\rightarrow \frac{x_{max}}{2} \le \Theta \le x_{min}$

берём $\frac{x_{max}}{2}$ ~~т.к.~~ т.к. $L\left(\frac{x_{max}}{2}\right) \to max$

$$\boxed{\tilde{\Theta}_2 = \frac{x_{max}}{2}}$$

$$\tilde{\Theta}_3 = \frac{1}{5}\left\{x_{min} + 2x_{max}\right\}$$

б) $\boxed{\tilde{\Theta}_1}$ $M[\tilde{\Theta}_1] = \frac{2}{3} M[\bar{x}] = \frac{2}{3}\frac{1}{n}\sum_{i=1}^{n} M_\xi = \Theta$ – несмещ

$$D[\tilde{\Theta}_1] = D\left[\frac{2}{3}\bar{x}\right] = \frac{4}{9n^2}\sum D_\xi = \frac{4}{9n^2} n\frac{\Theta^2}{12} = \frac{1}{27n}\Theta^2$$

$\xrightarrow{n\to\infty} 0$ – сост по дост условию

$\boxed{\tilde{\Theta}_2}$ $M[\tilde{\Theta}_2] = \frac{1}{2}\int_{\Theta}^{2\Theta} (F(y))^{n-1} p(y)\, dy = \frac{n}{2\Theta^n}\int_{\Theta}^{2\Theta} y(y-\Theta)^{n-1} dy =$

$= \boxed{\text{Замена } y-\Theta=t} = \frac{n}{2\Theta^n}\int_0^{\Theta}(t+\Theta)t^{n-1}dt =$

$$= \frac{1}{2\Theta^n}\int_0^{\Theta}(t+\Theta)\,d(t^n) = \frac{\Theta^{n+1}}{\Theta^n}\left(t^n(t+\Theta)\Big|_0^{\Theta} - \int_0^{\Theta} t^n dt\right) =$$

$= \Theta - \frac{\Theta}{2(n+1)} = \frac{\Theta(2n+1)}{2(n+1)}$ – смещ.

$$\tilde{\Theta}_2' = \frac{2(n+1)}{2n+1}$$

$$D[\tilde{\Theta}_2'] = \left(\frac{n+1}{2n+1}\right)^2 D[x_{max}]\,4$$

$$M x_{max}^2 = \int_{\Theta}^{2\Theta} y^2 n \left(\frac{y-\Theta}{\Theta}\right)^{n-1} \frac{1}{\Theta} dy = \{\text{замена}\} = \int_0^{\Theta} (t+\Theta)^2 n \frac{t^{n-1}}{\Theta^n} dt =$$

$$= \boxed{u = \frac{t}{\Theta}} = \int_0^1 \Theta^2 (u+1)^2 n u^{n-1} du = \text{вкус} = \Theta^2 \frac{2(2n^2+4n+1)}{n^2+3n+2}$$

$$D[\tilde{\Theta}_2'] = \left(\frac{n+1}{2n+1}\right)^2 \left(2\Theta^2 \frac{2n^2+4n+1}{n^2+3n+2} - \Theta^2 \left(\frac{2n+1}{n+1}\right)^2\right) =$$

$$= \frac{n\,\Theta^2}{(2n+1)^2(n+2)} \xrightarrow[n\to\infty]{} 0$$

<u>сост по дост условию</u>

$$\boxed{\tilde{\Theta}_3} \quad M\tilde{\Theta}_3 = \frac{1}{5} M[x_{min} + 2x_{max}] = \frac{1}{5}[M_{x_{min}} + 2M_{2x_{max}}]$$

$$M_{x_{min}} = \int_{\Theta}^{2\Theta} y \frac{1}{\Theta} n \left(1 - \frac{y-\Theta}{\Theta}\right)^{n-1} dy = \frac{n+2}{n+1}\Theta$$

$$M[\tilde{\Theta}_3] = \frac{1}{5}\frac{n+2}{n+1}\Theta + \frac{2}{5}\Theta\frac{2n+1}{n+1} = \frac{5n+4}{5n+5}\Theta - \text{смещ}$$

$$\tilde{\Theta}_3' = \frac{5n+5}{5n+4}\tilde{\Theta}_3$$

$$D[\tilde{\Theta}_3'] = D_{x_{min}} + 4D_{x_{max}} + 2cov(x_{min}; x_{max})$$

$$M[x_{min}^2] = \int_{\Theta}^{2\Theta} y^2 \frac{1}{\Theta} n\left(1 - \frac{y-\Theta}{\Theta}\right)^{n-1} dy = \boxed{Bxyx} = \frac{n(n+5)+8}{(n+1)(n+2)}$$

$$D[x_{min}] = \frac{n}{(n+1)^2(n+2)} \Theta^2$$

$$M[x_{min} \cdot x_{max}] = \iint_{-\infty}^{+\infty} xy\, p(x,y)\, dx dy = \iint_{z \geq y} xy\, p(x,y)\, dx dy =$$

$$= \int_{\Theta}^{2\Theta} dz \int_{\Theta}^{2\Theta} yz\, n(n-1)\left(\frac{z-y}{\Theta}\right)^{n-2} \frac{1}{\Theta^2} dy = \Theta^2 \frac{2n+5}{n+2}$$

$$cov(x_{min}, x_{max}) = \frac{\Theta^2(2n+5)}{n+2} - \Theta^2 \frac{n+2}{n+1} \frac{2n+1}{n+1} = \frac{\Theta^2}{(n+1)^2(n+2)}$$

$$D[\tilde{\Theta}_3'] = \frac{\Theta^2}{(5n+4)(n+2)} \xrightarrow[n\to\infty]{} 0$$

<u>сост. по дост. условию</u>

c) $D[\tilde{\Theta}_1] = \frac{1}{27n}\Theta^2 \sim \frac{1}{n}$

$$D[\tilde{\Theta}_2'] = \frac{n\,\Theta^2}{(n+2)(2n+1)^2} \sim \frac{1}{4n^2}$$

$$D[\tilde{\Theta}_3'] = \frac{\Theta^2}{(5n+4)(n+2)} \sim \frac{1}{5n^2} \to min$$

$\to \tilde{\Theta}_3'$ - наиболее эффективная

d) Точный доверительный интервал Θ

$$P\left[q_1 < f(\vec{x}_n; h) < q_2\right] = \beta = 0,95$$

$$f(\vec{x}_n; h) \sim q(t)$$

Ищем ф-ию наугад

$$f(\vec{x}, \Theta) =$$

e) Ассимптотический доверительный интервал

$f(\vec{x};\Theta) \rightsquigarrow q(t)$

$$\frac{f(\tilde{\alpha}) - f(\alpha)}{\sigma(\alpha)}\sqrt{n} \rightsquigarrow N(0,1)$$

$\sigma(\alpha) = \sqrt{\nabla^T f(\alpha)\ K\ \nabla f(\alpha)}$ $K_{ij} = \alpha_{i+j} - \alpha_i \alpha_j$

$\tilde{\Theta} = f(\alpha_1) = \frac{2}{3}\bar{x} = \frac{2}{3}\tilde{\alpha}_1 = \tilde{\Theta}$

$f(\alpha) = \frac{2}{3}\alpha_1 = \Theta$

$\nabla f(\alpha) = \frac{2}{3}$

$K = K_{11} = \alpha_2 - \alpha_1^2$ $\sigma(\alpha) = \frac{2}{3}\sqrt{\alpha_2 - \alpha_1^2}$

$\alpha_k \xrightarrow{P} \alpha_k$ пот. о насл. довании (т.к. $\tilde{\sigma} \xrightarrow{P} \sigma$)

$$\frac{\tilde{\Theta} - \Theta}{\frac{2}{3}\sqrt{\tilde{\alpha}_2 - \tilde{\alpha}_1^2}}\sqrt{n} \rightsquigarrow N(0,1)$$

$\beta = 0{,}95$

$p(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}$ $p(t_1) = -1{,}96$ $t_1 = 0{,}025$

$p(t_2) = 1{,}96$ $t_2 = 0{,}975$

$$\frac{-1{,}96\,\frac{2}{3}\sqrt{\hat{\alpha}_2-\alpha_1^2}}{\sqrt{n}}+\frac{2}{3}\hat{\alpha}_1<\theta<\frac{1{,}96\,\frac{2}{3}\sqrt{\hat{\alpha}_2-\hat{\alpha}_1^2}}{\sqrt{n}}+\frac{2}{3}\hat{\alpha}_1$$

f) g) h) – см код.