1 LINEAR INTERPOLATION

The General Form of Linear Interpolation for this question is:

$$f_4(x) = b_0 + b_1(x - x_0) + b_2(x - x_0)(x - x_1) + b_3(x - x_0)(x - x_1)(x - x_3) + b_4(x - x_0)(x - x_1)(x - x_3)(x - x_4)$$

Solution is:

x	$f[x1]$	$f[,]$	$f[,,]$	$f[,,,]$	$f[,,,,]$
0	1	$f[x_1, x_0]$	$f[x_2, x_1, x_0]$	$f[x_3, x_2, x_1, x_0]$	$f[x_4, x_3, x_2, x_1]$
1	9	$f[x_2, x_1]$	$f[x_3, x_2, x_1]$	$f[x_4, x_3, x_2, x_1]$	
2	23	$f[x_3, x_2]$	$f[x_4, x_3, x_2]$		
4	93	$f[x_4, x_3]$			
6	259				

$f[x_1, x_0] = \frac{9-1}{1-0} = 8$, $\quad f[x_2, x_1] = \frac{23-9}{2-1} = 14$

$f[x_3, x_2] = \frac{93-23}{4-2} = 35$, $\quad f[x_4, x_3] = \frac{259-93}{6-4} = 83$

$f[x_2, x_1, x_0] = \frac{f[x_2,x_1]-f[x1,x_0]}{2-0} = 3$, $\quad f[x_3, x_2, x1] = \frac{f[x_3,x_2]-f[x_2,x_1]}{4-1} = 7$,

$f[x_4, x_3, x_2] = \frac{f[x_4,x_3]-f[x_3,x_2]}{6-2} = 12$, $\quad f[x_3, x_2, x_1, x_0] = \frac{f[x_3,x_2,x_1]-f[x_2,x_1,x_0]}{4-0} = 1$

$f[x_4, x_3, x_2, x_1] = \frac{f[x_4,x_3,x_2,x_1]-f[x_3,x_2,x_1,x_0]}{6-0} = 0$,

$$f(x) = 1 + 8(x - 0) + 3(x - 0)(x - 1) + 1(x - 0)(x - 1)(x - 2) + 0(x - 0)(x - 1)(x - 2)(x - 4)$$

$$f(x) = 1 + 8x + 3x^2 - 3x + x^3 - 3x + x^3 - 3x^2 + 2x$$

So the function is : $f(x) = 1 + 7x + x^3$

- What is the value at points 1.2, 10.5 and 17.3 for data1?

  At point 1.2 value in **myInterplationFunc** is **5.2** and result of **interp1d** is **5.2**
  At point 10.5 value in **myInterplationFunc** is **3.5** and result of **interp1d** is **3.5**
  At point 17.3 value in **myInterplationFunc** is **2.099** and result of **interp1d** is **2.099**

- What is the value for data2 at points 10.3, 20.2 and 29.7?

  At point 10.3 value in **myInterplationFunc** is **10.20** and result of **interp1d** is **10.20**
  At point 20.2 value in **myInterplationFunc** is **30.2** and result of **interp1d** is **30.2**
  At point 29.7 value in **myInterplationFunc** is **4.85074** and result of **interp1d** is **4.85074**

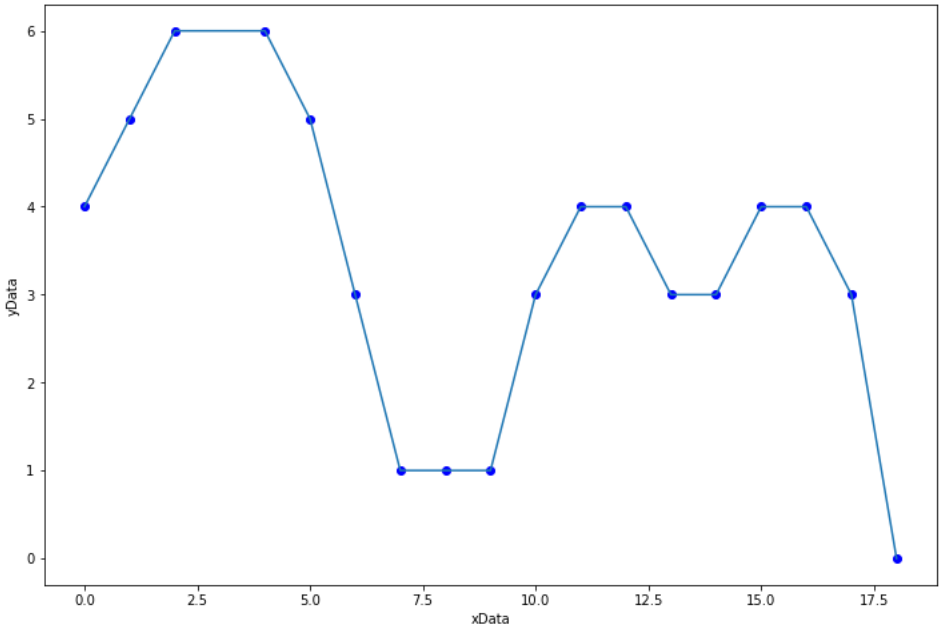
- What is the value for data3 at points -0.82, 0.40 and 0.91?

  At point -0.82 value in **myInterplationFunc** is **0.05619** and result of **interp1d** is **0.05619**
  At point 0.40 value in **myInterplationFunc** is **0.201028** and result of **interp1d** is **0.201028**
  At point 0.91 value in **myInterplationFunc** is **0.04608** and result of **interp1d** is **0.04608**

- What is the value for data4 at points -0.51, 0.72, 0.97?

  At point -0.51 value in **myInterplationFunc** is **0.13347** and result of **interp1d** is **0.13347**
  At point 0.72 value in **myInterplationFunc** is **0.071726** and result of **interp1d** is **0.071726**
  At point 0.97 value in **myInterplationFunc** is **0.04077** and result of **interp1d** is **0.04077**

When we plot the interpolation function which is returning function of python built-in interp1d for data1.txt the output will be look like in the below figure



Results

- The results are same with built-in **interp1d** because I implement Linear Interpolation which is 1D also interp1d function is 1D and Linear too, so we can expect the same values for functions.

Pseudocode

1. define a function read data from file and split them to x,y 1darray data
2. define myInterpolationFunc takes x,y and interpolationPoint as a parameter
3. find a interval in xdata according to the given interPolationPoint
4. if interpolationPoint bigger or equal to xData[i] then get the initial interval as a value of i(index) and make it n (n = i)
5. find b_0 using index of n and it is yData[n]
6. calculate b_1 with divided difference equation
7. define y as a linear equation and calculate it with b values and interpolationPoint
8. return y

2 NUMERICAL INTEGRATION

- Trapozoidal rule for n = 4

So this is our equation:

$$\int_{1}^{2} e^{-x} + x\, dx$$

if we convert equation to apply Trapozoidal rule it will be like this:

$$\int_{1}^{1,25} f(x)\, dx + \int_{1,25}^{1,5} f(x) + \int_{1,5}^{1,75} f(x)\, dx + \int_{1,75}^{2} f(x), dx$$

So now we can apply Trapozoidal rule:

$$\int_{a}^{b} f(x)\, dx = (b-a)[\frac{f(a)+f(b)}{2}]$$

We know that $f(x) = \frac{e^{-x}+x}{x}$ so

$$\int_{1}^{1.25} f(x)\, dx = (1.25-1)[\frac{f(1)+f(1.25)}{2}] = (0.25) * 1.2985416393297973$$

$$\int_{1.25}^{1.5} f(x)\, dx = (1.5-1.25)[\frac{f(1.25)+f(1.5)}{2}] = (0.25) * 1.1889786387935528$$

$$\int_{1.5}^{1.75} f(x)\, dx = (1.75-1.5)[\frac{f(1.75)+f(1.5)}{2}] = (0.25) * 1.1240264181781752$$

$$\int_{1.75}^{2} f(x)\, dx = (2-1.75)[\frac{f(1.75)+f(2)}{2}] = (0.25) * 1.0834835189378518$$

= 0.32463541 + 0.29724466 + 0.281006605 + 0.27087088 = 1.173757555

When we sum of all we get = 1.173757555

- Simpson 1/3 rule by taking n = 2

This time we convert equation like this:

$$\int_{1}^{1.5} f(x)\, dx + \int_{1.5}^{2} f(x), dx$$

Our formula for Simpson 1/3 rule is:

$x = (\frac{b-a}{6})[f(a) + 4f(\frac{a+b}{2}) + f(b)]$ so let's apply this to our function

$$\int_{1}^{1.5} f(x)\, dx = \frac{(1.5-1)}{6}[f(1) + 4f(\frac{1+1.5}{2}) + f(1.5)] =$$

(0.083333333 * (1.3678794411714423) + 4 * (1.2583164406351979) + (1.1487534400989532))

$$+$$

$$\int_{1.5}^{2} f(x)\,dx = \frac{(2-1.5)}{6}[f(1.5) + 4f(\frac{2+1.5}{2}) + f(2)] =$$

(0.083333333 * (1.1487534400989532) + 4 * (1.1082105408586298) + (1.0676676416183064))

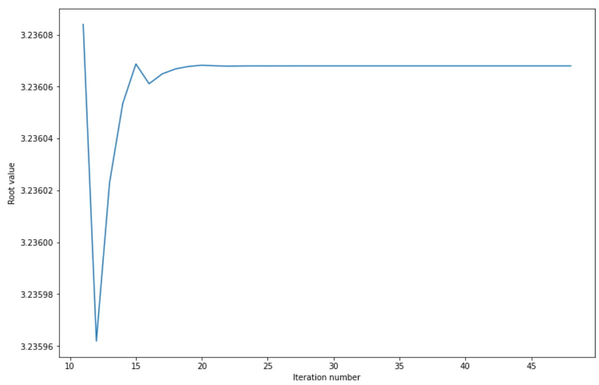
So after applying the rule we get = 0.629158218 + 0.554105268 = 1.183263486

3 ROOT FINDING

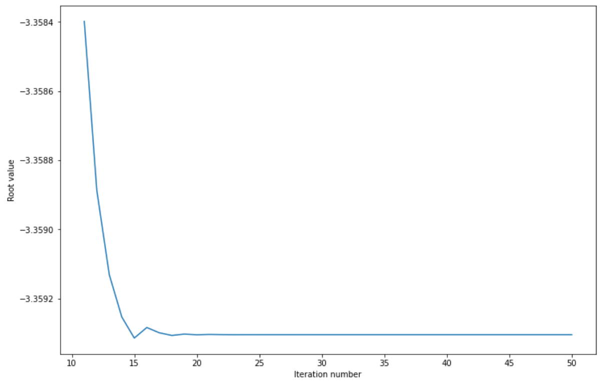
Pseudocode of Bisection Method

1. define functions to find root of it
2. define bisection function and create dictionary to keep root and iteration number pairs
3. Check if the sign of interval points are negative
4. if not return None
5. else find middle point(m) of interval [a,b] m = a+b/2
6. if the sign of f(m)*f(b) are negative, replace a with m
7. else if the sign of f(m)*f(a) are negative, replace b with m
8. else if f(m) == 0 it means finding root then insert iteration number as a key and m value then return m,dictionary
9. increase n one and insert iteration number and m to dictionary
10. exit loop when desiring iteration bigger then iteration number and return m,dictionary
11. plot dictionay keys as a iteration number and roots are value

Plotting number of iterations that changes between 10 and 50 $f_1(x) = \frac{1}{2}x - (x+1)^{\frac{1}{3}}$ function with given [3,4] interval



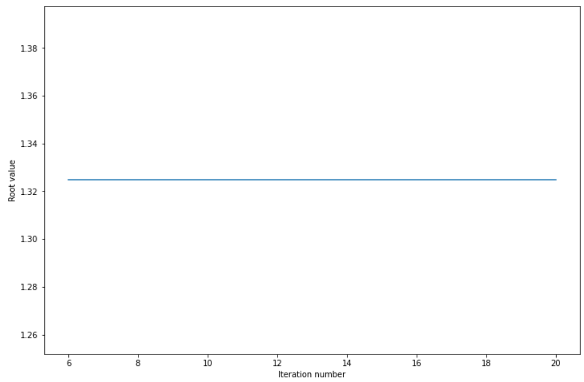
Plotting number of iterations that changes between 10 and 50 $f_2(x) = x^3 + 5x^2 + 7x + 5$ function with given [-4,0] interval



Pseudocode of Newton-Raphson Method

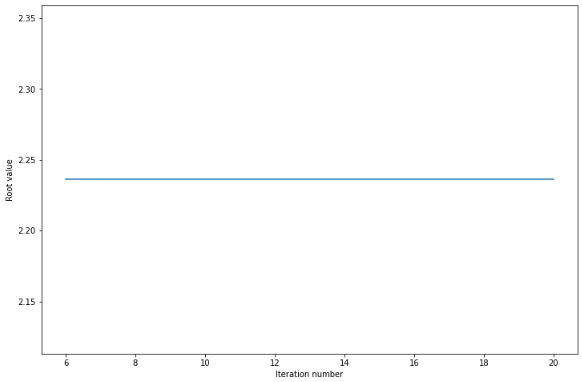
1. define functions to find root of it
2. define newtonRaphson function and create dictionary to keep root and iteration number pairs
3. define lambda functions which gives derivative of defined functions
4. apply formula of newton-raphson which take x_0 to parameter and return x_{next}(next value of x)
5. if $f(x_{next}) = 0$ then we know that found exact solution and return x_{next} , dictionary
6. else update x_0 replace with x_{next}
7. exit loop when desiring iteration bigger then iteration number and return x_{next},dictionary
8. plot dictionay keys as a iteration number and roots are value

Plotting number of iterations that changes between 5 and 20, $f_1(x) = x^3 - x - 1$ function with given $x_0 = 1$



We want to find out square root of $\sqrt{5}$ so we need to set $f(x) = 0$ when x = $\sqrt{5}$. Basically we are trying to find which function's root is $\sqrt{5}$ so our function will be $f(x) = x^2 - 5$.

Plotting number of iterations that changes between 5 and 20, $f_2(x) = x^2 - 5$ function with given $x_0 = 1$



4 SOLUTION OF ORDINARY DIFFERENTIAL EQUATIONS

1. Use Euler Method to obtain an approximation to y(0.1)

   Firstly, Euler formula is : $y(x_i + h) = y(x_i) + y^{'}(x_i)h$

   Secondly, we define x_i and h values

   $\frac{d_y}{d_x}$ = y x , y(0) = 0 , h = 0.1

   y(0 + 0.1) = y(0) + $y^{'}$(0) * (0.1)

   $y^{'} = y - x \; y^{'}(0) = y(0) - x(0) = 0$

   y(0.1) = 0 + $y^{'}(0) * (0.1)$ = 0 + 0*0 = 0

   So $y(x_i + h) = y(x_i) + y^{'}(x_i) * h$, $y(0.1) = 0, h = 0.1$

   $y^{'}(0 + 0.1)$ = y(0) + $y^{'}(0) * 0.1$

   $y^{'}(0.1)$ = 0 + $y^{'}(0) * 0.1$

   $y^{'} = y - x \rightarrow y^{'}(0)$ = $y(0) - x(0)$ = 0 - 0 = 0

   $y^{'}(0.1)$ = 0 + $y^{'}(0) * 0.1$ $\rightarrow$y$^{'}(0.1) = 0 + 0 * 0 = 0$

2. Use Fourth Order Runge-Kutta to obtain an approximation to y(0.1) h = 0.1 $\frac{dy}{dx}$ = y-x y(0)=0

   y(0)=0 is given so first iteration will be $\rightarrow y(0 + h) = y(0.1)$ as we can see one iteration is enough for us.

   First Iteration:

   $y(x_i + h) = y(x_i) + 1/6[k_1 + 2k_2 + 2k_3 + k_4]h$

   $x_i = 0 \quad and \quad h = 0.1 \quad , \quad y_i = 0[y(0) = 0]$

$k_1 = f(x_i, y_i) = f(0,0) = 0 \quad where \quad [y^{'} = f(x,y) = y - x]$

$k_1 = 0$

$k_2 = f(x_i + h/2, y_i + \frac{k1*h}{2}) = f(0 + \frac{0.1*0}{2}, 0 + \frac{0*0.1}{2})$

$k_2 = f(0.05, 0) = 0 - 0.05 = -0.05$

$k_3 = f(x_i + h/2, y_i + \frac{k2*h}{2}) = f(0 + \frac{0.1}{2}, 0 + \frac{-0.05*(0.1)}{2})$

$k_3 = f(0.05, 0.0025) = -0.0025 - 0.05 =$-0.0525

$k_4 = f(x_i + h, y_i + k_3 * h) = f(0 + 0.1, 0 + (0.0525 * 0.1)) = f(0.1, -0.00525) = -0.00525 - 0.1 =$ -0.10525

$k_4 =$-0.10525

$y(0 + 0.1) = y(0.1) = y(0) + \frac{1}{6}[0 + 2*(-0.05) + 2*(-0.0525) + (-0.10525)]*(0.1)y(0.1) = 0 + \frac{1}{6}[0 + 0 + 0.1 + 0.005]*0.1$

$y(0.1) = -0.00517083$

3. **Pseudocode of Runge-Kutta**
   - Define function to calculate the derivative of given function
   - Define myRangeKutta function which takes derivative of function and xi,yi,h ,desired value x
   - find k1,k2,k3,k4 with given formula of Runge Kutta order four
   - calculate y_{next} with using k values
   - update y_i with replace y_{next} ,for use it next iteration
   - update x_i with sum x_i and h ,for use it next iteration
   - do this while desired value bigger than x_i and exit when x_i bigger or equal of desired value which mean found a value
   - return y_{next}

4. **Pseudocode of Euler**
   - Define function to calculate the derivative of given function
   - Define myEuler function which takes derivative of function, h,xi,yi and desired value x
   - apply Euler formula which sum of yi with derivative of (xi,yi) * h
   - update x_i with sum x_i and h ,for use it next iteration
   - do this while desired value bigger or equal with x_i
   - return y_i