REGULATIONS

- **Due date:** 19 December 2021, Sunday, 23:59 (Not subject to postpone)
- Electronically. You will be submitting your program source codes through text files as well as the report nameSurname.pdf by means of the AYBUZEM system.
- **Team:** There is **no** teaming up. The homework has to be done and turned in individually.
- **Cheating:** Source(s) and Receiver(s) will receive zero and be subject to disciplinary action.

DIRECT METHODS

Answer the following questions.

1. List the advantages of the direct methods over iterative methods
2. Consider the following linear system of equations

$$\begin{aligned} 2x_1 + 3x_2 + x_3 &= 0 \\ 20x_1 + 18x_2 + x_3 &= 3 \\ x_1 + abx_2 + x_3 &= 2 - ab \end{aligned}$$

where ab is the last two digits of your student ID. Apply LU Factorization method for solving the linear system. Show all steps clearly in your report.

Code Implementation

- You download an nxn matrix A from SuiteSparse Matrix Collection (Link: https://sparse.tamu.edu/).
- Write the properties of the matrix A such as dimension of A, the number of nonzeros, and name given by SuiteSparse in your report.
- You perform the following operation to make A diagonally dominant and symmetric.

$$A = A + A^T + I \tag{1}$$

where I is nxn identity matrix and A^T is the transpose of A

- Take the right hand side vector b as 1. That is, b is n dimensional vector whose values are 1.
- Write python code of the following *ULFactorization* function. This function takes A matrix, returns UL factorization of A where U is the upper triangular matrix and L is the lower triangular matrix. Do not use python built-in solvers.

def ULFactorization(A)

- **Hint:** First Compute LU factorization of A and then use permutation matrices (pivoting) to obtain UL Factorization.
- Write pseudocode of the UL algorithm you implement and explain all steps of the algorithm in your report.
- Compute the solution of the linear system $Ax = b$ using the UL algorithm you implement. Submit as direct.py. Do not use python built-in solvers such as lu.
- Compare your results with the results of the built-in *scipy.linalg.solve* function by making comments in your report.

ITERATIVE METHODS

Answer the following questions.

1. List advantages of the iterative methods over direct methods
2. Consider the following linear system of equations

$$\begin{aligned} 2x_1 + 3x_2 + x_3 &= 0 \\ 20x_1 + 18x_2 + x_3 &= 3 \\ x_1 + abx_2 + x_3 &= 2 - ab \end{aligned}$$

   where ab is the last two digits of your student ID. Apply Jacobi and Gauss-Seidel Method. Take the initial guess as $(0, 0, a - b)$ and perform 4 iterations. Show all steps clearly in your report. Compare the performance of the Jacobi and Gauss-Seidel Methods and which method has the better performance ?

Code Implementation

- You use the same matrix (A) (downloaded from SuiteSparse Matrix Collection) in Direct Methods part.
- You perform the following operations to make A diagonally dominant and symmetric

$$A = A + A^T + I \tag{2}$$

  whete A^T is the transpose of A and I is nxn identity matrix

- Take the right hand side vector b as 1. That is, b is n dimensional vector whose values are 1.
- Write python code of the following *jacobi* function. This function takes A matrix, right hand side vector b, and number of iterations. It returns the solution. Take zero vector as an initial guess. Submit it as jacobi.py. Note that you are expected to implement the algorithm mentioned in the lecture.

$$def \quad jacobi(A, b, numIter)$$

- compute the norm between real value and the value computed by Jacobi method using

$$numpy.linalg.norm((x_{jacobi} - x_{exact}), ord = 1)$$

  You can compute the exact solution (x_{exact}) by using built-in *scipy.linalg.solve* function.

- Plot a graphic showing the number of iterations versus the norm computed above in your report.

REPORT FORMAT

You download report.zip and open via Overleaf. Write all answers and comments in this report. Your report should include answer of the above questions clearly.

Submission: You submit a zip file including

- the2 folder including source codes
- your report, nameSurname.pdf