IE 310 Fall 2024 Assignment

Due: 18.12.2024

In this assignment, you are expected to complete the following tasks.

1. **Generation of Instances:** Write a function that randomly generates a feasible instance of the transportation problem, based on its 4 arguments which are number of supply nodes, number of demand nodes, maximum possible cost (Maximum value c_{ij} can take) and lastly, maximum demand/supply amount (Maximum value a_i or b_j can take). Make sure the data are all integers.
2. **Utilizing Solvers:** Write a function that takes a transportation problem instance as input, formulates an LP model and solves it. Make sure that you define your decision variables as non-negative and continuous.
3. **Coding Revised Simplex:** Write a function that takes **any** linear programming instance as an input and solves it using the Revised Simplex Algorithm. You are free to utilize basic math and data structure libraries, such as NumPy, but you are forbidden to use any LP solver for this stage.
4. **Experiments:** Create multiple instances of the transportation problem by varying the problem size in terms of number of supply and demand nodes. Ensure that the maximum cost and the maximum supply/demand quantities remain consistent across all instances. Additionally, maintain an equal number of supply and demand nodes for each instance. Carry out experiments on these instances to evaluate the performance of your implemented Revised Simplex Algorithm against the solver. Try to come with conclusions on practical complexity. Check that optimal solutions found by your Revised Simplex Algorithm and the solver's match. Additionally, see if all the optimal solutions are integers or are there any fractional solutions or not. Comment on the result.

- You are free to use any programming language of your choice and any LP solver within that language. If in doubt, consider using Python and the Pulp package.
- You are expected submit a report for the assignment and your code. Your report should contain comments on your work and results. If you must include code in your report, do it minimally. Including excessive code in your reports will be penalized.
- The readability of your report and code is critical. Make sure that you use concise and unambiguous language. Use comments in your code. Convey the outputs and results in a legible way. Avoid repetition.
- You can either carry out the assignment individually or in groups of two people. It is up to you. If any resemblance is found in your codes or reports between your submission and another, both submissions will be counted as plagiarism. If you are working as a group, one submission is enough.

Transportation Problem

Transportation problem is a logistics problem, where there are facilities supplying a certain good and facilities demanding that same good. The objective is to minimize the total transportation cost, under the condition that all demand must be satisfied, while not exceeding the capacities of the supply nodes.

If the total demand and total supply do not match, a dummy node is added to the side with the deficit, with a capacity or demand equal to the difference between total supply and total demand, thus matching the total supply and demand quantities. A dummy supply node represents unsatisfied demand while a dummy demand node represents unused capacity. In your instance generation function, you can generate the supply/demand quantities as accordingly so that they match, eliminating the need to add a dummy node.

Sets:
I: Set of Supply Nodes
J: Set of Demand Nodes

Decision Variables:
x_{ij}: Amount of goods send from supply node *i* to demand node *j*

Parameters:
a_i: Capacity of Supply Node i
b_j: Required quantity of Demand Node *j*

Model:

$$\min z = \sum_{i \in I} \sum_{j \in J} c_{ij} x_{ij}$$

$$s.t. \sum_{j \in J} x_{ij} = a_i \qquad \forall i \in I \ \ (supply)$$

$$\sum_{i \in I} x_{ij} = b_j \qquad \forall j \in J \ \ (demand)$$

$$x_{ij} \geq 0 \quad \forall i \in I, \forall j \in J$$

For further information about the transportation problem, you may consult to the Chapter 9 of Hillier and Lieberman's *Introduction to Operations Research*.