School of Mathematics, Thapar Institute of Engineering & Technology, Patiala

UMA007 : Numerical Analysis
Assignment 7
Newton's Interpolation

1. Using Newton's divided difference interpolation, construct interpolating polynomials of degree one, two, and three for the following data. Approximate the specified value using each of the polynomials.

$$f(0.43) \text{ if } f(0) = 1, f(0.25) = 1.64872, f(0.5) = 2.71828, f(0.75) = 4.4816.$$

2. Show that the polynomial interpolating the following data has degree 3.

x	-2	-1	0	1	2	3
$f(x)$	1	4	11	16	13	-4

3. Let $f(x) = e^x$, show that $f[x_0, x_1, \ldots, x_m] > 0$ for all values of m and all distinct equally spaced nodes $\{x_0 < x_1 < \cdots < x_m\}$.

4. Verify that the polynomials $P(x) = 5x^3 - 27x^2 + 45x - 21$, $Q(x) = x^4 - 5x^3 + 8x^2 - 5x + 3$ interpolate the data

x	1	2	3	4
y	2	1	6	47

and explain why this does not violate the uniqueness part of the theorem on existence of polynomial interpolation.

5. The following data are given for a polynomial $P(x)$ of unknown degree.

x	0	1	2	3
$f(x)$	4	9	15	18

Determine the coefficient of x^3 in $P(x)$ if all fourth-order forward differences are 1.

6. The Newton's forward-difference formula is used to approximate $f(0.3)$ given the following data.

x	0	0.2	0.4	0.6
$f(x)$	15.0	21.0	30.0	51.0

Suppose it is discovered that $f(0.4)$ was understated by 10 and $f(0.6)$ was overstated by 5. By what amount should the approximation to $f(0.3)$ be changed?

7. Given that

x	1.0	1.5	2.0
$f(x)$	0.0	0.17609	0.30103

Find Newton's interpolating polynomial $P_2(x)$ with this data. If we add one more point, say $f(3.0) = 0.47712$ in the above data then find $P_3(x)$ in such a way that $P_3(x) = P_2(x) + R(x)$. Find $R(x)$ explicitly.

8. Suppose that $f(x) = \cos x$ to be approximated on $[0, 1]$ by an interpolating polynomial on $n + 1$ equally spaced points. What step size h ensure that linear interpolation gives an absolute error of at most 10^{-6} for all $x \in [0, 1]$.

9. Let $i_0, i_1, \cdots, i_n$ be a rearrangement of the integers $0, 1, \cdots, n$. Show that

$$f[x_{i_0}, x_{i_1}, \cdots, x_{i_n}] = f[x_0, x_1, \cdots, x_n].$$

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10. Construct the interpolating polynomial that fits the following data using Newton's forward and backward

x	0	0.1	0.2	0.3	0.4	0.5
$f(x)$	-1.5	-1.27	-0.98	-0.63	-0.22	0.25

difference interpolation. Hence find the values of $f(x)$ at $x = 0.15$ and 0.45.

11. A fourth-degree polynomial $P(x)$ satisfies $\Delta^4 P(0) = 24$, $\Delta^3 P(0) = 6$, and $\Delta^2 P(0) = 0$, where $\Delta P(x) = P(x+1) - P(x)$. Compute $\Delta^2 P(10)$.

12. Show that

$$f[x_0, x_1, \cdots, x_n, x] = \frac{f^{(n+1)}(\xi(x))}{(n+1)!}$$

for some $\xi(x)$.