

## EE110B Lab 4

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Consider a discrete-time LTI system governed by the following (causal)

$$y[n] = 1.8 \cos(\pi/4)y[n-1] - 0.81y[n-2] + x[n] - 2 \cos(3\pi/4)x[n-1] + x[n-2]$$

1) Determine an expression of the frequency response  $H(f)$  of this system

$$y[n] \xrightarrow{F} Y(f)$$

$$F[y[n]] = 1.8 \cos(\pi/4) Y(f) e^{-j2\pi f} - 0.81 Y(f) e^{-j4\pi f} + X(f) - 2 \cos(3\pi/4) X(f) e^{-j2\pi f} + X(f) e^{-j4\pi f}$$

$$Y(f) = 1.8 \cos(\pi/4) Y(f) e^{-j2\pi f} - 0.81 Y(f) e^{-j4\pi f} + X(f) - 2 \cos(3\pi/4) X(f) e^{-j2\pi f} + X(f) e^{-j4\pi f}$$

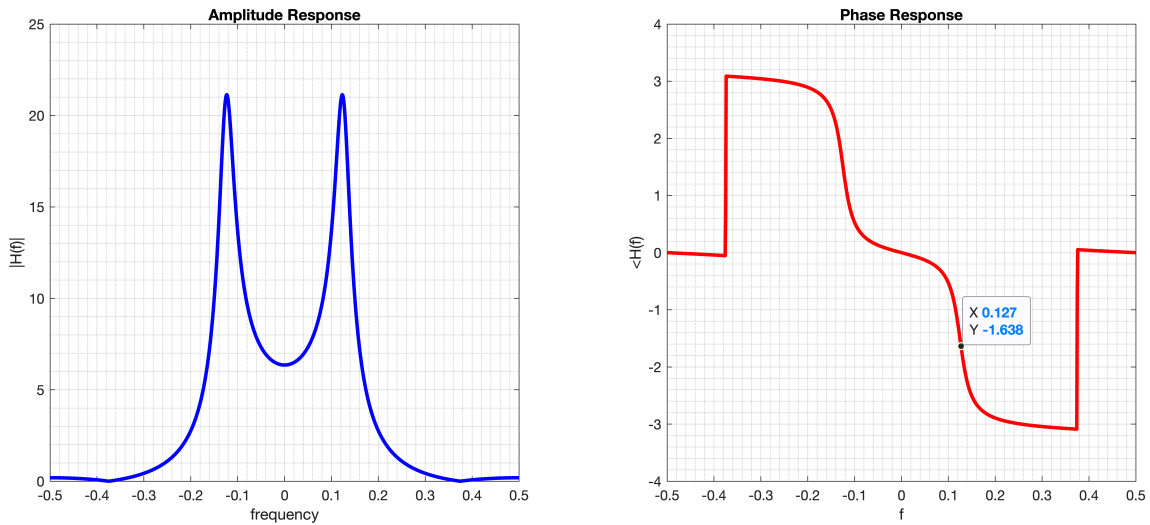
$$Y(f) - 1.8 \cos(\pi/4) Y(f) e^{-j2\pi f} + 0.81 Y(f) e^{-j4\pi f} = X(f) - 2 \cos(3\pi/4) X(f) e^{-j2\pi f} + X(f) e^{-j4\pi f}$$

$$(1 - 1.8 \cos(\pi/4) e^{-j2\pi f} + 0.81 e^{-j4\pi f}) Y(f) = (1 - 2 \cos(3\pi/4) e^{-j2\pi f} + e^{-j4\pi f}) X(f)$$

$$H(f) = \frac{Y(f)}{X(f)} = \frac{1 - 2 \cos(3\pi/4) e^{-j2\pi f} + e^{-j4\pi f}}{1 - 1.8 \cos(\pi/4) e^{-j2\pi f} + 0.81 e^{-j4\pi f}}$$

2) Plot the amplitude and phase responses i.e.,  $|H(f)|$  and  $\angle H(f)$ , versus  $-0.5 < f < 0.5$ . Discuss the features of the responses (such as the positions of peaks and valleys).

Part2()



The features of the peaks and valleys are as follows. Concerning the graph titled Amplitude Response, the peak represents the highest gain that we can get from the system using a periodic function as an input. Thus, at frequencies  $\pm 0.123$ , the highest gain is reached from the system. In signal processing, **phase response** is the relationship between the phase of a sinusoidal input and the output signal passing through any device that accepts input and produces an output signal, such as an amplifier or a filter. Thus, at frequencies  $\pm 0.371$  the system experiences the greatest phase shift (time delay) relative to the periodic input.

3)

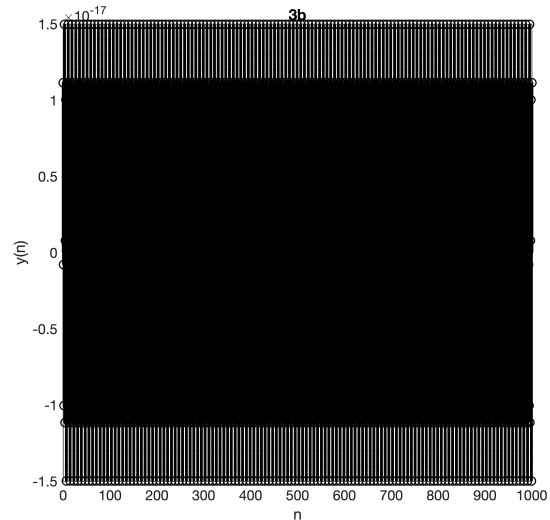
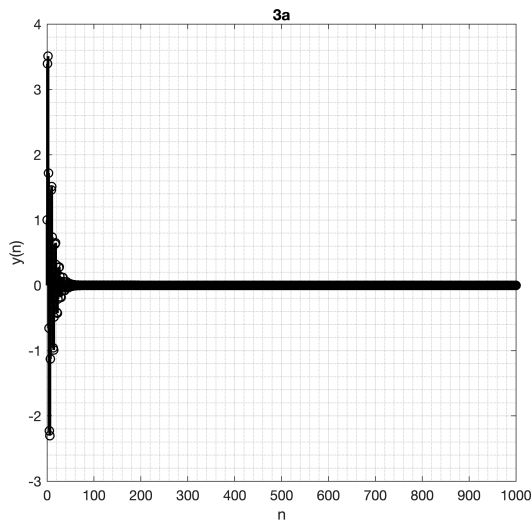
a) Assume  $y[-1] = y[-2] = 0$  and  $x[n] = \cos\left(\frac{3\pi}{4}n\right)u[n]$ . Apply the recursive formula

(1) to compute and plot  $y[n]$  for  $n \geq 0$ . Discuss your results

b) Now assume  $x[n] = \cos\left(\frac{3\pi}{4}n\right)$  (without the step function  $u[n]$ ). Compute and plot the output of the system,  $y[n]$  for  $n \geq 0$  using the following:

$$y[n] = \left|H\left(\frac{3}{8}\right)\right| \cos\left(\frac{3\pi}{4}n + \angle H\left(\frac{3}{8}\right)\right)$$

Part3()



**Compare these result. Are they close for large n? Do you know why?**

These results are close for large n. This is because the frequency chosen represents the peak of the phase response and a valley for the amplitude response. Therefore as n increases the output for both graphs tend to 0. This is consistent with the data from part 2.

**4)**

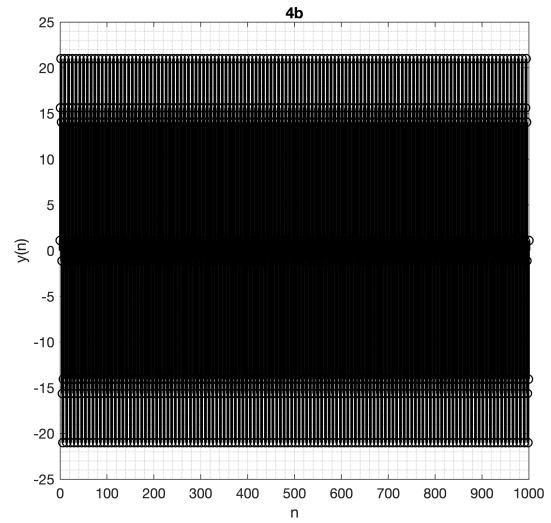
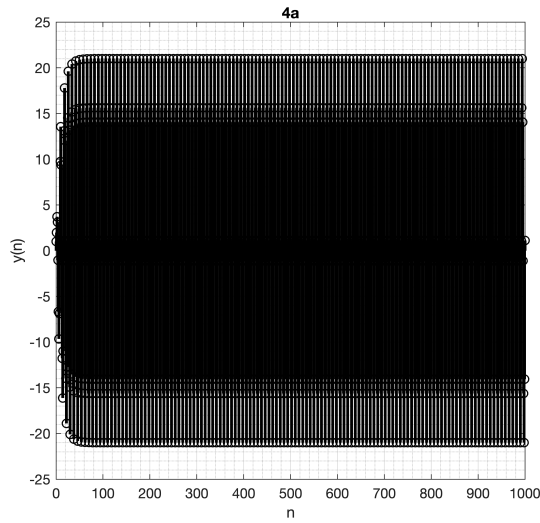
**a) Assume  $y[-1] = y[-2] = 0$  and  $x[n] = \cos\left(\frac{\pi}{4}n\right)u[n]$  Apply the recursive formula**

**(1) to compute and plot  $y[n]$  for  $n \geq 0$ . Discuss your results**

**b) Now assume  $x[n] = \cos\left(\frac{\pi}{4}n\right)$  (without the step function  $u[n]$ ). Compute and plot the output of the system,  $y[n]$  for  $n \geq 0$  using the following:**

$$y[n] = \left|H\left(\frac{1}{8}\right)\right| \cos\left(\frac{\pi}{4}n + \angle H\left(\frac{1}{8}\right)\right)$$

Part4()



**Compare this with the above result. Are they close for large  $n$ ? Do you know why?**

They are close for large  $n$ . This is because the frequency used matches the peak of the amplitude response and a valley for the phase response. Therefore as  $n$  increases both graphs tend to 20. This is consistent with the data in part 2.