EE 105: PWM Speed Control of a Motor

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1 Abstract

In a car, an internal combustion engine applies very high power and high frequency pulses of power, yet the vehicle velocity is very smooth. Similarly, electric motors can be effectively controlled by applying high-frequency pulses of power. This lab investigates certain engineering aspects of such systems.

2 Background

Figure 1 depicts the schematic of a DC motor with parameter K that is being used to spin an rotational inertia J. The DC motor model is

$$\begin{aligned} \tau_m(t) &= K\, i_m(t) && (1) \\ e_m(t) &= K\, \omega_m(t), && (2) \end{aligned}$$

where $e_m(t)$ is the motor back emf (i.e., voltage), $i_m(t)$ is the motor current, $\tau_m(t)$ is the torque applied by the motor to the inertia,and $\omega_m(t)$ is the angular rotation rate of the inertia. For this system, we will consider the applied voltage $u(t)$ to be the input and the rate of rotation of the inertia $\omega_m(t)$ to be the output.

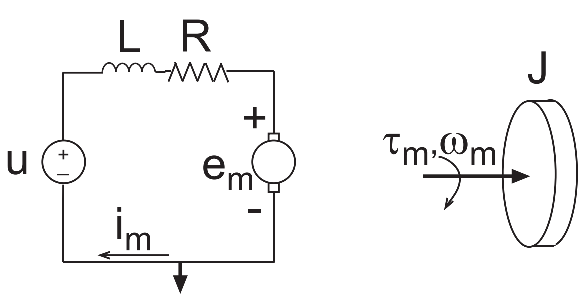

Figure 1: Voltage driven DC motor

The input u is a pulse-width-mudulate (PWM) signal. This means that $u(t)$ oscillates between zero and B volts, where B is a constant. The frequency f of oscillation is constant, but the percentage of each period that the pulse is high (i.e., B) is variable. The period of the oscillation is denoted by $T = \frac{1}{f}$ and $\tau < T$ is the pulse width.

By Fourier series analysis, the series of pulses can be decomposed into a DC component and sum of sinusoids

$$u(t) = \frac{\tau}{T}\, B + \sum_{i=1}^{\infty} a_i \sin\left(2\, i\, \pi\, f t + \phi_i\right).$$

The term $\frac{\tau}{T}\, B$ is called the DC component. The term with $i = 1$ is the fundamental component.

If the PWM is designed with the frequency f high enough and the system is a low-pass filter (LPF), then the LPF will remove essentially all of the sinusoidal components, leaving only the DC portion.

2.1 Second order systems

The standard form for the transfer function of a second-order low pass system is

$$G(s) = \frac{A\, \omega_n^2}{s^2 + 2\, \zeta\, \omega_n\, s + \omega_n^2}$$

where A, ζ, and ω_n are constant parameters and s is the Laplace variable. Any second order low pass system can be written in this form by proper definition of the three parameters.

The three parameters have physical meaning:

A: This is the DC gain. It specifies the amplification from the input to the output when the input is a constant.

ζ: This is the damping factor. It determines, for example, the amount of oscillation in a step response.

ω_n: This is the undamped natural frequency. The bandwidth is approximately the same as ω_n.

For a stable system, ζ and ω_n are both positive.

Two other important parameters can be determined from these parameters.

$\sigma = \zeta\omega_n$: This is the decay rate. The time constant is the reciprocal of the decay rate.

$\omega_d = \omega_n\sqrt{1-\zeta^2}$: This is the damped natural frequency.

When $0 < \zeta < 1$, the transfer function will have complex conjugate poles located at $-\sigma \pm j\omega_d$.

3 Calculations

1. The value of $K = 0.01$. Assuming ANSI standard units for the voltage, current, torque and angular rate, determine the units of the motor parameter K.

2. Using eqns. (1-2), confirm that the DC motor is power conserving (i.e., power in equals power out).

3. Use bond graphs to find a state space model for the coupled electrical-mechanical system.

4. The transfer function for the system is

$$\frac{\omega(s)}{U(s)} = \frac{\frac{K}{JL}}{s^2 + \frac{R}{L}s + \frac{K^2}{JL}} = G(s).$$

   Compare this to the standard form of a second order low pass filter.

   Determine the DC gain, damping factor, undamped natural frequency, damped natural frequency, and decay rate in terms of the systems parameters K, J, L, and R.

5. Let $R = 0.1\,\Omega$ and $L = 0.1\,H$, determine the value of the inertia J in $kg\,m^2/s^2$ such that the undamped natural frequency is 1.0 rad/s. Compute the decay rate and time constant. Based on the time constant, how long will it take the system to reach steady state.

6. How much is the DC component amplified?

7. Compute $|G(j\omega)|$ for $f = 100Hz$. If the pulse frequency is 100 Hz, by how much will the fundamental component of the Fourier series be attenuated. Will the other components (with $i > 1$) be attenuated more or less? Why?

8. Does the system effectively keep the DC component and essentially remove all other components?

4 Lab Assignment

1. In simulink, implement a signal flow diagram to simulate the system.
   - Use the R, L, K, and J values specified above.
   - For the input, use the 'pulse generator'. Double click it to specify the pulse frequency or period. Leave the amplitude set to 1. Start with the pulse width at 50%.
   - Set the maximum time step for the simulation to 0.005 seconds.
   - Connect a scope to display the angular rate variable. Change its settings so that there is no limit on the number of points that it saves.

2. Simulate the system for a time interval of at least 10 time constants. Confirm the following:
   - The system should have approximately reached steady state in about 4 time constants.
   - The DC gain should be correct.
   - Look at a time interval after about 10 time constants, to ensure that steady state is very nearly attained. If you zoom in to a high resolution for both axes near the end of the simulation, you should be able to see that there are still very small oscillations with frequency near 100 Hz, but that the amplitude should have been effectively reduced by the factor computed in the prelab.

3. Change the pulse width to 10% and rerun the simulation. Keep the same simulation duration (approximately 10 time constants). Record the steady-state angular rate. Repeat the simulation for pulse widths in 10% increments from 0-100%. When you have the steady-state angular rate for the eleven pulse widths: $i10\%$ for $i = 0, \ldots, 10$, plot the steady-state angular rate as a function of the pulse width. Is the pulse-width linearly related to the steady-state angular rate.

4. Note and discuss the steady-state current and power used for each pulse width in the previous step. Could you reasonable expect to pull these from a standard microprocessor output or would you need some type of higher power device?

5 Questions

1. What is the single parameter (R, L, K, J) that determines the DC gain?

2. If you change the parameter J, will the DC gain or decay rate change?

3. If you wanted to increase the decay rate (i.e., decrease the time constant) so that steady state would be achieved faster and you did not want to change the natural frequency, what is the single parameter that you would change? Select this parameter to change the decay rate to 1.0 and resimulate the system to confirm your answer.

6 Concluding Comments

This lab is motivated by past questions from students working on Senior Design projects in which they need to control the speed of a DC motor. Microcontrollers and FPGA's have circuitry to output a low power PWM signal. That signal can be connected through an amplifier or bridge to the input of a DC motor. In this manner, the Microcontroller or FPGA can constantly adjust the pulse width of the PWM signal to adjust the speed of the motor as desired for the task at hand. Such control systems are discussed in EE132.