$W = 2f_m\left(\frac{Kf}{f_m} + 1\right)$

$W = 2Kf + 2f_m$

$W = 2(K_f + f_m)$ ~~100 kHz~~

EE 115 Lab 4

10 kHz

Consider the FM signal

$$u_{FM}(t) = \cos(2\pi f_c t + 2\pi k_f \int_0^t m(\tau)d\tau) \tag{1}$$

where f_c is a (large) radio carrier frequency and $m(t)$ is a message signal. The bandwidth of the FM signal is approximately equal to

$BU = 2B_m(\beta+1)$

$$W = 2B_m(\beta + 1) \tag{2}$$

where B_m is the bandwidth of $m(t)$ and $\beta = \frac{\Delta f_{max}}{B_m} = \frac{k_f \max_t |m(t)|}{B_m}$ (FM modulation index).

If $m(t) = \cos(2\pi f_m t)$, we can let $B_m = f_m$ and $\beta = \frac{k_f}{f_m}$. Furthermore, it can be shown that the complex envelope of $u_{FM}(t)$ is

$$u(t) = e^{j\beta \sin(2\pi f_m t)} = \sum_{n=-\infty}^{\infty} J_n(\beta) e^{j2\pi f_m n t} \tag{3}$$

and its bandwidth is approximately equal to $B_u = K f_m = K B_m$ if $J_n(\beta)$ for $|n| > K$ can be all neglected.

β

Let

$$u_K(t) = \sum_{n=-K}^{K} J_n(\beta) e^{j2\pi f_m n T}. \tag{4}$$

Then the power of $u_K(t)$ is

$K=0$ $\Sigma=$

$$P_{u_K} = f_m \int_{-\frac{1}{2f_m}}^{\frac{1}{2f_m}} |u_K(t)|^2 dt = J_0^2(\beta) + 2\sum_{n=1}^{K} J_n^2(\beta). \tag{5}$$

$K=1$ $\Sigma=$

Since the power of $u(t) = e^{j\beta \sin(2\pi f_m t)}$ is

$$P_u = f_m \int_{-\frac{1}{2f_m}}^{\frac{1}{2f_m}} |u(t)|^2 dt = f_m \int_{-\frac{1}{2f_m}}^{\frac{1}{2f_m}} 1 \cdot dt = 1, \tag{6}$$

then $\lim_{K\to\infty} P_{u_K} = J_0^2(\beta) + 2\sum_{n=1}^{\infty} J_n^2(\beta) = 1$.

1) Compute and plot "P_{u_K} versus K" and "$1 - P_{u_K}$ versus K" for various β.
2) Explain why $1 - P_{u_K}$ is the power of the error function $u(t) - u_K(t)$. show on paper
3) Explain why the bandwidth of $u(t)$ is approximately equal to $B_u = f_m(\beta + 1)$.

two figures with multiply graphs

$\beta = 0:.1:1$

$Kf_m = f_m(\beta+1)$

$B_u = Kf_m = KB_m$

$B_m = f_m$

$2f_m(\beta+1)$ $2f_m\left(\frac{Kf}{B_m}+1\right)$

$2Kf + 2f_m$

Consider the FM signal

$$u_{FM}(t) = \cos(2\pi f_c t + 2\pi k_f \int_0^t m(\tau)d\tau) \quad (1)$$

☆ Question for Fahimeh

1) what is the complex envelope physically and why does it matter.

$u_{FM}(t) = \cos(2\pi f_c t + \theta(t))$

where $\theta(t) = 2\pi k_f \int_0^t m(\tau)d\tau$

f_c = (large) radio carrier frequency

$m(\tau)$ = message signal

Bandwidth of $U_{FM}(f) = W = 2B_m(\beta+1)$

$$= W = 2B_m\left(\frac{k_f \max_t |m(t)|}{B_m} + 1\right)$$

$$= W = 2k_f \max_t |m(t)| + 2B_m$$

$$= 2(k_f) + 2(f_m)$$

If $m(t) = \cos(2\pi f_m t)$

$$= 2(k_f + f_m)$$

then

$$\boxed{u_{FM}(t) = \cos\left(2\pi f_c t + 2\pi k_f \int_0^t \cos(2\pi f_m \tau)d\tau\right)}$$

This allows us to chang

$$\boxed{W = 2k_f + 2f_m}$$

Furthermore, it can be shown that the complex envelope of $U_{FM}(t)$ =

$$u_{FM}(t) = e^{j\beta\sin(2\pi f_m t)} = \sum_{n=-\infty}^{+\infty} J_n(\beta) e^{j2\pi f_m n t}$$

$\beta = \frac{k_f}{f_m}$ ☆

The Bandwidth of the series expansion for this signal is

$$B_u = K f_m = K B_m .$$

neglecting $J_n(\beta)$ for $|n| > K$ we can rewrite the fourier series expansion as

$$u_K(t) = \sum_{n=-K}^{K} J_n(\beta)\, e^{j2\pi f_m n T}$$

The Power of $u_K(t)$ is

$$P_{uK} = f_m \int_{-1/2f_m}^{1/2F_M} |u_K(t)|^2\, dt = J_0^2(\beta) + 2\sum_{n=1}^{K} J_n^2(\beta)$$

$$\boxed{u_{FM}(t) = \cos\left(2\pi f_c t + 2\pi K_p \int_0^t \cos(2\pi f_m \tau)\, d\tau\right)}$$

$$\theta(t) = 2\pi K_f \int_0^t \cos(2\pi f_m \tau)\, d\tau$$

$$= \frac{\cancel{2\pi} K_f}{\cancel{2\pi} f_m} \sin(2\pi f_m \tau) \Big|_0^t$$

$$= \frac{K_f}{f_m}\left(\sin(2\pi f_m t)\right) =$$

$$u_{FM}(t) = \cos\left(2\pi f_c t + \frac{K_f}{f_m}\sin(2\pi f_m t)\right)$$

where B_m = bandwidth of message $= f_m$ β = modulatio

β = Angle modulation index = $\frac{K_f}{f_m}$

f_m = largest frequency component in an arbitrary message

What I know

1) m(t) is a single tone message because it is in the form

$m(t) = A_m \cos(2\pi f_m t)$

* In this lab $A_m = 1$

2)

$$u_{FM}(t) = \cos(2\pi f_c t + \beta \sin(2\pi f_m t))$$

taking the complex envelope of the Angle modulated signal with respect to $2\pi f_c t = A_c e^{j\theta(t)}$

$$u(t) = e^{j\beta \sin(2\pi f_m t)}$$

$$= \sum_{n=-\infty}^{+\infty} J_n(\beta) e^{j2\pi f_m n t}$$

where $J_n = f_m \int_{-\frac{1}{2f_m}}^{1/2f_m} u(t) e^{-j2\pi k \cdot \frac{1}{T} t} \, dt$

$T = \frac{1}{f_m}$

$T/2 = \frac{1}{f_m} \div 2$

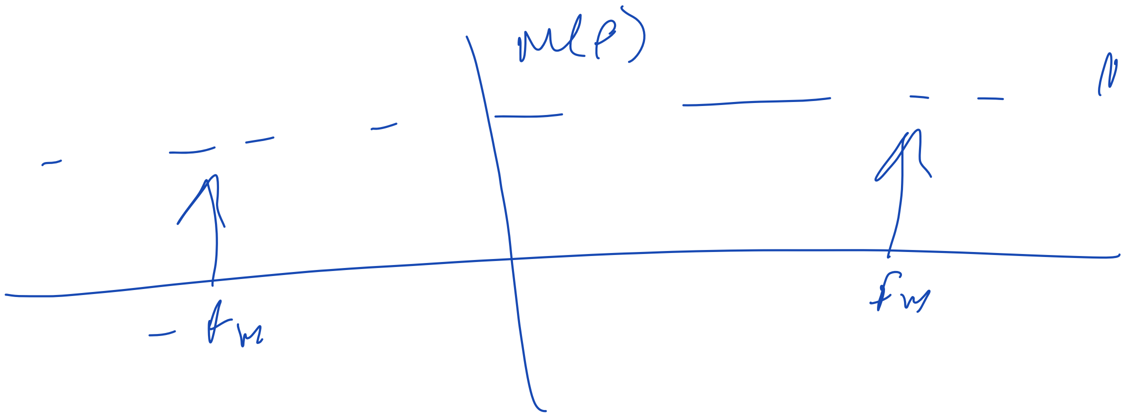
$\frac{1}{T} = f_m$

Notes

- power of f_m signal
- discussion of Bessel function
- message acts like frequency for whole signal
  - determined by K_f and maximum ~~of~~ of absolute value

  the maximum value $m(t)$ can have is 1?

- ~~delta~~ i $m(t)$ is delta from f_m and f_m



- properly choose K

$u_K(t)$ is the cropped version of u_K

- don't compute given in equation 5
- because its symmetric we can do twice the power J_0^2
- use Besselj in Matlab, order, value you're evalu

$J_n(B)$ Besselj(n, β)

order

Compute P_{uk}

$$u_{FM}(t) = \cos(2\pi f_c t + \theta(t))$$

where $\theta(t) = 2\pi K_f \int_0^t m(\tau)\, d\tau$

$$u_{FM}(t) = \cos\left(2\pi f_c t + 2\pi K_f \int_0^t \cos(2\pi f_m \tau)\, d\tau\right)$$

$$\theta(t) = 2\pi K_f \int_0^t \cos(2\pi f_m \tau)\, d\tau$$

$$= \frac{\cancel{2\pi} K_f}{\cancel{2\pi} f_m} \sin(2\pi f_m \tau)\Big|_0^t$$

$$= \frac{K_f}{f_m}\left(\sin(2\pi f_m t)\right) =$$

$$u_{FM}(t) = \cos\left(2\pi f_c t + \frac{K_f}{f_m}\sin(2\pi f_m t)\right)$$

$$u_{FM}(t) = \cos(2\pi f_c t + \beta \sin(2\pi f_m t))$$

The complex envelope of the Angle modulated message with respect to $2\pi f_c t$ is:

* $A = 1$ because $A_c = 1$

$$u(t) = e^{j\beta \sin(2\pi f_m t)}$$

$$= \sum_{n=-\infty}^{+\infty} J_n(\beta)\, e^{j2\pi f_m n t}$$

where $J_n = f_m \int_{-\frac{1}{2f_m}}^{1/2f_m} u(t)\, e^{-j2\pi k \cdot \frac{1}{T} t}\, df$

$$= f_m \int_{-1/2f_m}^{1/2f_m} e^{j\beta\sin(2\pi f_m t)} e^{-j2\pi K f_m t} \, dt$$

$$= f_m \int_{-1/2f_m}^{1/2f_m} e^{j\beta\sin(2\pi f_m t) - j2\pi K f_m t} \, dt$$

$$= f_m \int_{-1/2f_m}^{1/2f_m} e^{j(\beta\sin(2\pi f_m t) - 2\pi K f_m t)} \, dt$$

Using Euler's formula

$$e^{j\theta} = \cos(\theta) + j\sin(\theta)$$

$$\cos\left(\beta\sin(2\pi f_m t) - 2\pi K f_m t\right) + j\sin\left(\beta\sin(2\pi f_m t) - 2\pi K f_m t\right)$$

$$J_K = f_m \int_{-1/2f_m}^{1/2f_m} \cos\left(\beta\sin(2\pi f_m t) - 2\pi K f_m t\right) dt + f_m j \int_{-1/2f_m}^{1/2f_m} \sin\left(\beta\sin(2\pi f_m t) - 2\pi K f_m t\right) dt$$

$$u = 2\pi f_m t \; dt$$

$$du = 2\pi f_m \, dt$$

$$\frac{du}{2\pi f_m} = dt$$

$$J_K = \frac{1}{2\pi} \int_{-\pi}^{\pi} \cos\left(\beta\underline{\sin(u) - Ku}\right) du + \frac{j}{2\pi} \int_{-\pi}^{\pi} \sin\left(\beta\sin(u) - Ku\right) du \quad \boxed{J_K = J_n}$$

$$v = \beta\sin(u) - Ku \; du$$

$$J_K = -\frac{1}{2\pi}\cdot\frac{1}{B\cos(u)-K}\sin(B\sin(u)-Ku)\Big|_{-\pi}^{\pi}$$

$$-\frac{J}{2\pi}\cdot\frac{1}{B\cos(u)-K}\cos(B\sin(u)-Ku)\Big|_{-\pi}^{\pi}$$

$$=\frac{1}{2\pi}\left[\frac{1}{B-K}\sin(Ku)-\frac{1}{B-K}\sin(Ku)\right]$$

$$-\frac{J}{2\pi}\left[-\frac{1}{B-K}\cos(Ku)+\frac{1}{B-K}\cos(Ku)\right]$$

Now making $K=0$

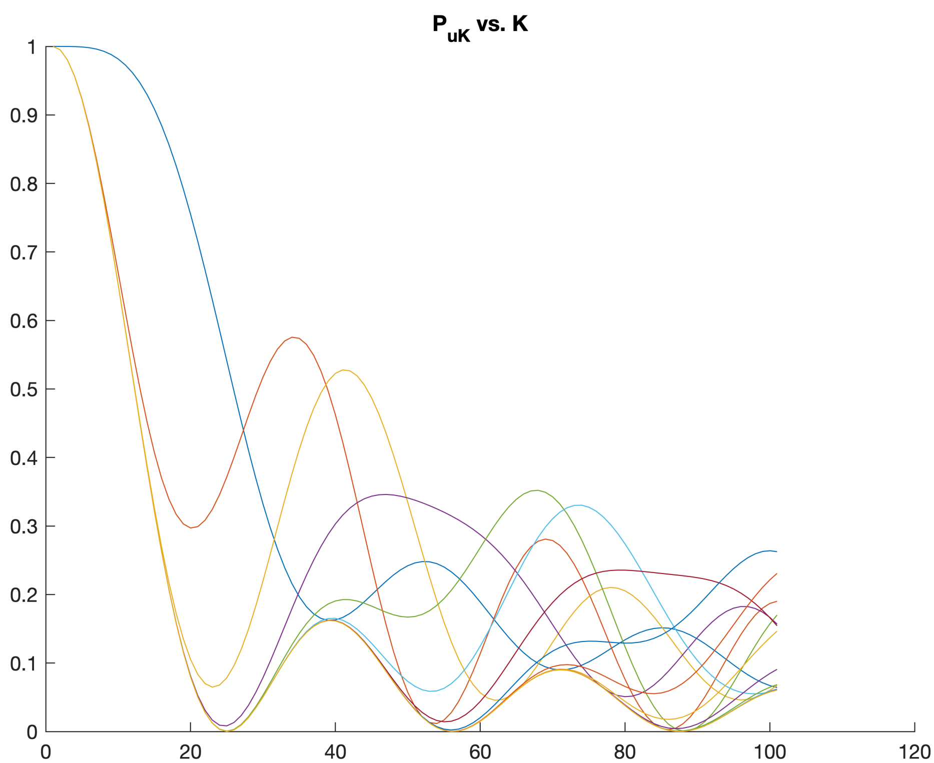
$$-\frac{1}{2\pi}\left[-\frac{1}{B}+\frac{1}{B}\right]$$

EE115 Lab 4

Buddy Ugwumba

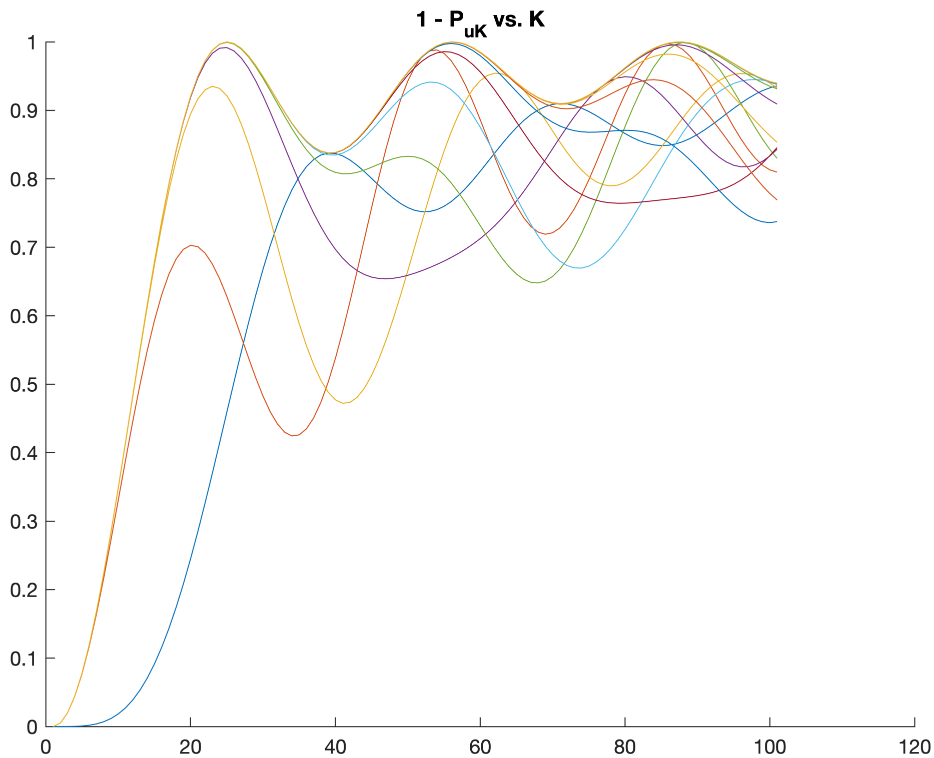
1. Compute and plot "P_{uK} versus K" and "1 - P_{uK} versus K" for various β

```
clf
%various beta
beta = 0:0.1:10;
K = 100;
n = 1:1:K+1;
uK_t = besselj(0,beta);
% For Loop
for i = n
    value = besselj(i,beta);
    P_uk = uK_t.^2 + 2*(value.^2);
    hold on
    plot(n, P_uk)
end
title('P_{uK} vs. K')
hold off
```




```
figure
```

```
for i = n
    value = besselj(i,beta);
    P_uk = uK_t.^2 + 2*(value.^2);
    P_uK = 1 - P_uk;
    hold on
    plot(n, P_uK)
end
title('1 - P_{uK} vs. K')
```



1. Explain why 1 - P_{uK} is the poower of the error function $u(t) - u_K(t)$

The total power of $u(t)$, which is the complex envelope of $u_{\mathrm{FM}}(t)$, is equal to 1. The limit, as K approaches infinity, of the fourier series expansion of $u_K(t)$ is also one. Therefore, the total power minus the power of various integer multiples of the same complex envelope (given the condition that we neglect all orders when $|n| > K$) gives us the error function $u(t) - u_K(t)$.

2. Explain why the bandwidth of $u(t)$ is approximately equal to $B_m = f_m(\beta + 1)$.

The bandwidth of $u(t) = B_u$ if $J_n(\beta)$ for $|n| > K$ can all be neglected. $B_u = \mathrm{K}\mathrm{f}_m = \mathrm{K}\mathrm{B}_m$. Performing algebra we get $B_m = f_m$. This leaves us with the equation $W = 2f_m(\beta + 1)$. Since we are only interested in the positive values of spectrum, we divide the bandwidth by 2. Therefore, the bandwidth of $u(t) = f_m(\beta + 1)$.