EE132 Automatic Control

Lab 4: Bank Angle Hold

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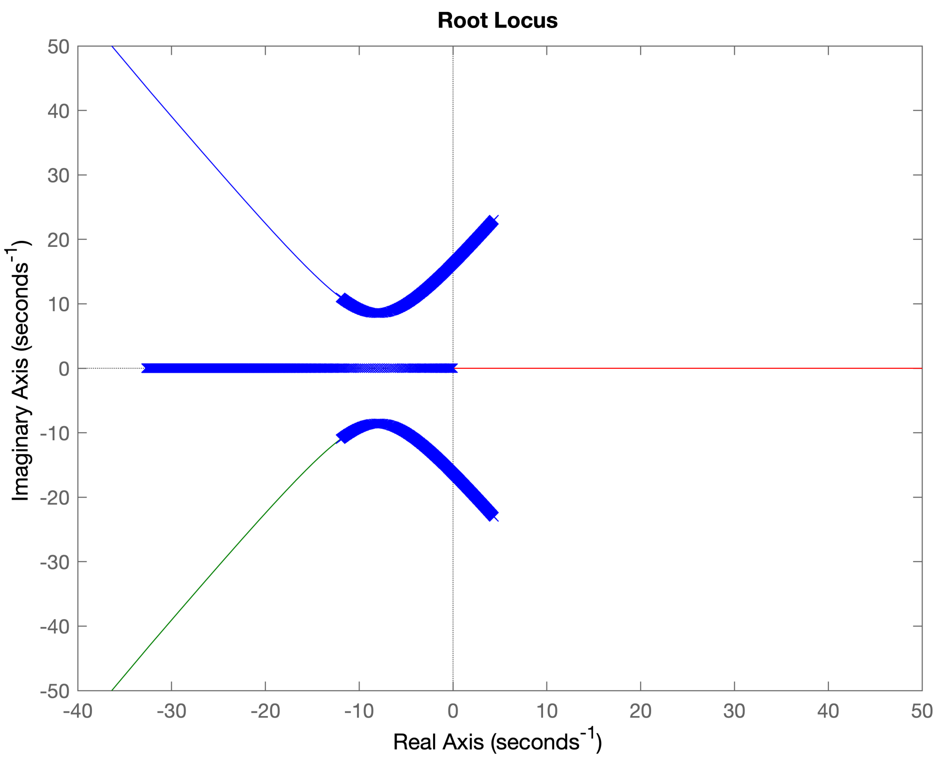
2) Calculate Poles

```
poles = roots([1 24.002 265.048 0.008])
```

```
poles = 3×1 complex
 -12.0010 +11.0011i
 -12.0010 -11.0011i
  -0.0000 + 0.0000i
```

3) Draw our own Root Locus

```
clear all
for k = 0:0.1:100
    p = roots([1 24.002 265.048 (0.53 + 180*k)]);
   for j = 1:length(p)
       hold on
        scatter(real(p(j)), imag(p(j)), 'bx')
    end
end
```



As can be seen from the root locus graph of the proportional controller, there exists both a constant damping ratio and mass overshoot. This shows that we have a constant damping ratio. We can also see that our system is marginally stable because there exists a pole on the imaginary axis. Moreover, this is a three pole system because we have three lines.

Use Matlab to find the values of k

```
K = 2;
poles = roots([1 24 265+(180*K)])
```

```
poles = 2×1 complex
  -12.0000 +21.9317i
  -12.0000 -21.9317i
```

```
wn = sqrt(poles(1)*poles(2))
```

```
wn = 25.0000
```

```
zeta = -(poles(1)+poles(2))/(2*wn)
```

```
zeta = 0.4800
```

```
Ts = 4.6/(wn*zeta)
```

```
Ts = 0.3833
```

```
Tp = pi/(wn*sqrt(1-zeta^2))
```

```
Tp = 0.1432
```

```
K = 30;
poles = roots([1 24.002 265.048 0.53+(180*K)])
```

```
poles = 3×1 complex
 -22.7757 + 0.0000i
  -0.6132 +15.3864i
  -0.6132 -15.3864i
```

```
wn = sqrt(poles(2)*poles(3))
```

```
wn = 15.3986
```

```
zeta = -(poles(2)+poles(3))/(2*wn)
```

```
zeta = 0.0398
```

```
Ts = 4.6/(wn*zeta)
```

```
Ts = 7.5022
```

```
Tp = pi/(wn*sqrt(1-zeta^2))
```

```
Tp = 0.2042
```

```
K = 15;
poles = roots([1 24.002 265.048 0.53+(180*K)])
```

```
poles = 3×1 complex
 -17.6535 + 0.0000i
  -3.1743 +11.9540i
  -3.1743 -11.9540i
```

```
wn = sqrt(poles(2)*poles(3))
```

```
wn = 12.3683
```

```
zeta = -(poles(2)+poles(3))/(2*wn)
```

```
zeta = 0.2566
```

```
Ts = 4.6/(wn*zeta)
```

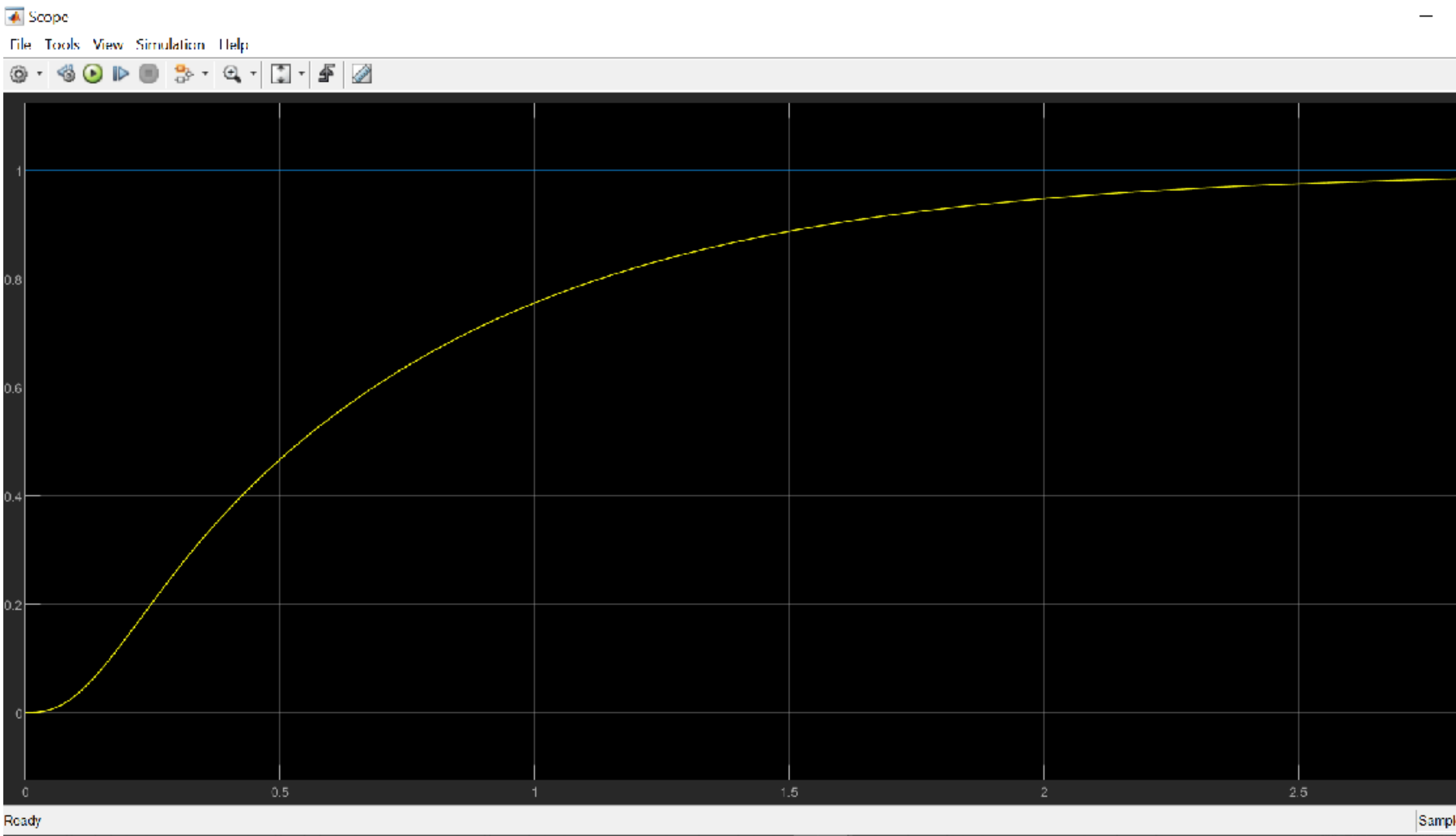
```
Ts = 1.4492
```

```
Tp = pi/(wn*sqrt(1-zeta^2))
```

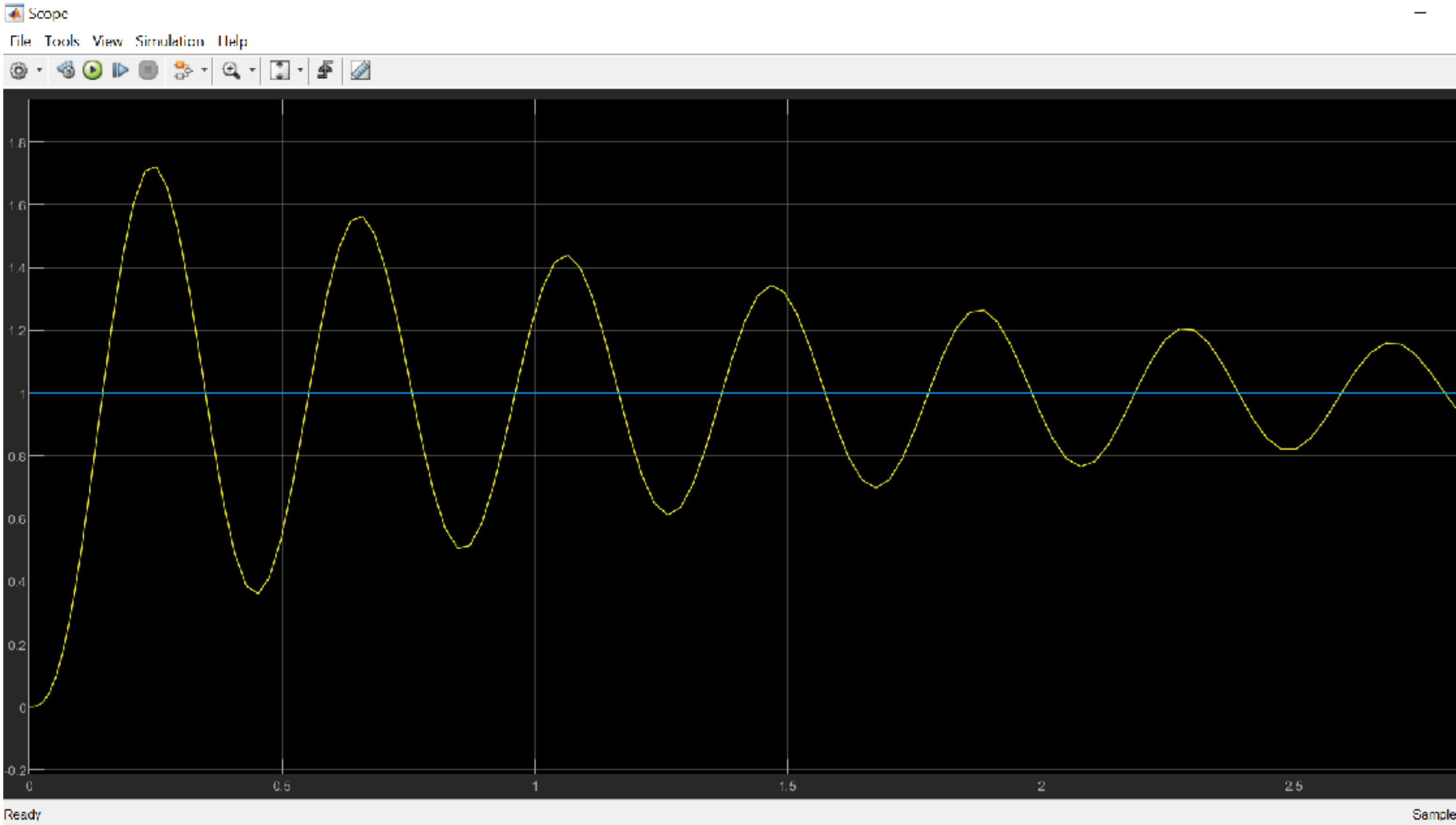
```
Tp = 0.2628
```

K	ω	ζ	Estimated T_s	Estimated T_p	Actual T_s	Actual
2	25	0.4800	0.3833	0.1432	X	X
30	15.3986	0.0398	7.5022	0.2042	7.084	0.242
15	12.3683	0.2566	1.4492	0.2680	1.4960	0.32

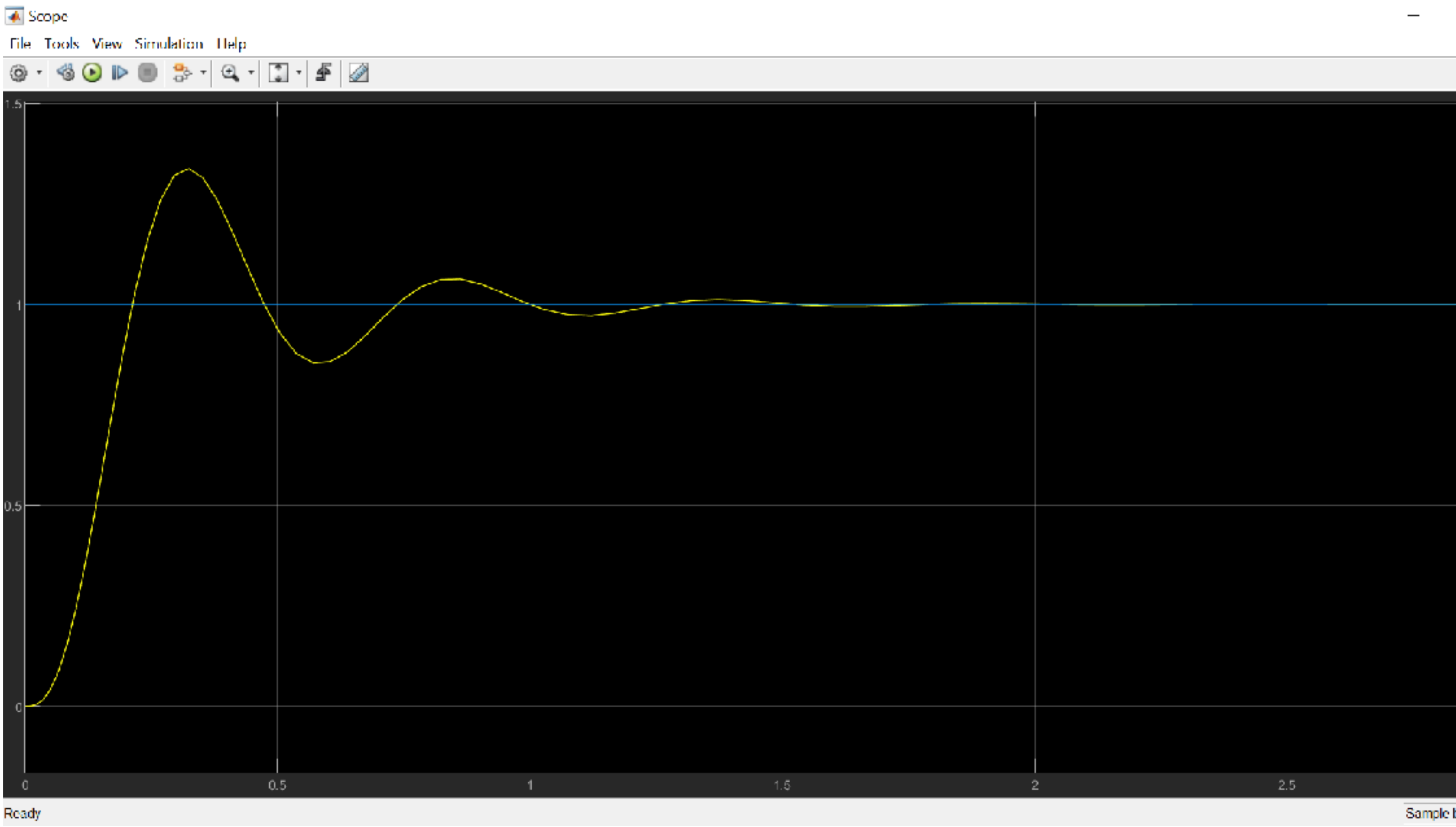
K=2



K=30


Scope
File Tools View Simulation Help
1.8
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
-0.2
0
0.5
1
1.5
2
2.5
Ready
Sample

K=15



Scope
File Tools View Simulation Help
1.5
1
0.5
0
0
0.5
1
1.5
2
2.5
Ready
Sample b

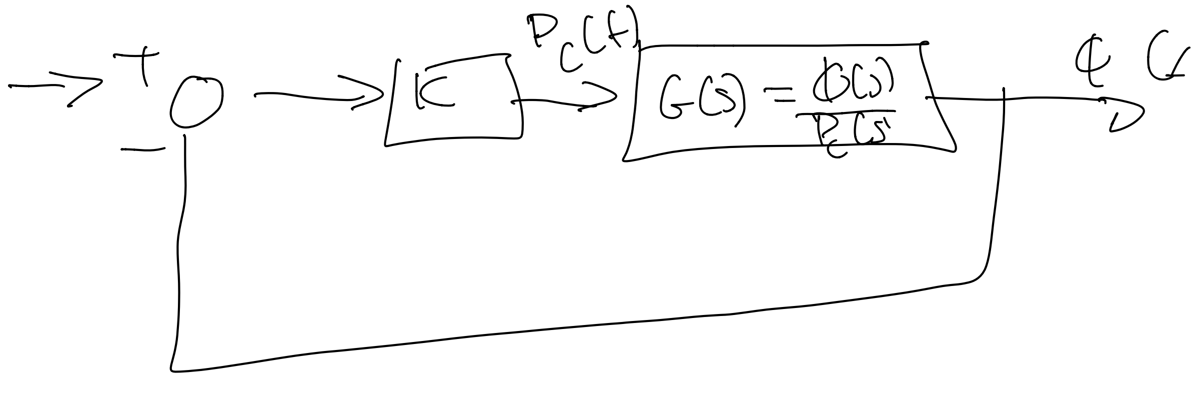
objective get Familar with root locus
use root locus to determine proportional controller

$$\frac{\phi(s)}{P(s)} = \text{Transfer function plant}$$

$$= \frac{180}{(s+0.002)(s^2+24s+265)}$$

$\phi(s)$ - bank angle in s domain
$P_c(s)$ - Commanded roll rate is s domain
$\phi(t)$, $P_c(t)$ - time domain

design proportion controller k



Specifications
① tracking error is less than 0.1%
② $t_p < 0.4s$
③ $t_s < 1.5s$

① Characteristic equat

$Q(s) = K \cdot G(s) \cdot (\Phi_c - \Phi)$ $G(s) = \frac{\Phi(s)}{P_c(s)}$

$\frac{\Phi(s)}{\Phi_c(s)} = \frac{K G(s)}{1 + K G(s)} = H(s)$

$1 + K G(s) = 0$

$1 + K \frac{180}{(s + 0.002)(s^2 + 24s + 265)}$

$\Rightarrow (s + 0.002)(s^2 + 24s + 265) + 180K = 0$

$s^3 + 24s^2 + 265s + 0.002s^2 + (0.002)(24s) + (0.002)(265) + 180K$

$s^3 + 24s^2 + 265s + 0.002s^2 + 0.048s + 0.53 + 180K$

$s^3 + 24s^2 + 0.002s^2 + 265s + 0.048s + 0.53 + 180K$

$s^3 + 24.002s^2 + 265.048s + (0.53 + 180K)$

② Routh Criteria

$$\begin{array}{c|ccc} s^3 & 1 & 265.048 & 0 \\ s^2 & 24.002 & (0.53 + 180K) & \\ s^1 & \left[265.048 - \frac{(0.53 + 180K)}{24.002}\right] & 0 & \\ s^0 & (0.53 + 180K) & & \end{array}$$

$b_1 = \frac{-1}{24.002} \begin{vmatrix} 1 & 265.048 \\ 24.002 & (0.53 + 180K) \end{vmatrix}$

$\frac{-1}{24.002}\left[(0.53 + 180K) - (24.002)(265.048)\right]$

$b_1 = \frac{-(0.53 + 180K)}{24.002} + 265.048$

$b_2 = \frac{-1}{24.002} \begin{bmatrix} 1 & 0 \\ 24.002 & 0 \end{bmatrix}$

$\frac{(24.002)(265.048) - 0.53 - 180K}{24.002}$

$= \frac{6361.15 - 180K}{24.002}$

$$c_1 = -\frac{1}{b_1}\begin{vmatrix} a_{n-1} & a_{n-3} \\ b_1 & b_2 \end{vmatrix}$$

$$= \frac{-24.002}{6361.15 - 180k}\begin{bmatrix} 24.002 & (0.53 + 180k) \\ \frac{6361.15 - 180k}{24.002} & 0 \end{bmatrix}$$

$$\frac{-24.002}{6361.15 - 180k}\left[0 - \frac{(6361.15 - 180k)(0.53 + 180k)}{24.002}\right]$$

$$c_1 = 0.53 + 180k$$

$$\frac{(6361.15 - 180k)(0.53 + 180k)}{24.002}$$

$$= (6361.15)(0.53) + (6361.15)(180k) - (180k)(0.53) - (180k)(180k)$$

$$= \frac{-180^2k^2 + 6360.62 \cdot 180k + (6361.15)(0.53)}{24.002}$$

$$=$$

$$= -1349.89k^2 + 47700.7k + 140.464$$

$$\frac{-24.002}{6361.15 - 180k}\left[\frac{-32400k^2 + 1.14491Ke^6 + 3371.41}{24.002}\right]$$

$$c_1 = \frac{32400k^2 - 1.14491Ke^6 - 3371.41}{6361.15 - 180k}$$

$$k > -0.00294 \quad k < 35.34$$

$$-1349.89k^2 + 47700.7k + 140.464$$

$\cancel{E}$ $0.53 + 180K > 0$

$180K > -0.53$

$K > \frac{-0.53}{180} \Rightarrow \boxed{K > -0.00294}$

$a_2 > 0$

$a_2 = 24.002$ ✓

$a_1 > \frac{a_0}{a_2}$

$0.53 + 180(-0.00$

(3) value of Kc
(the value of K when system is marginaly stable

that is K

$$\boxed{\begin{array}{c} 0 > K \geq -0.00294 \\ -0.00294 < K < 0 \end{array}}$$

Find the steady state response

Step 2

$as^3 + bs^2 + cs + d = 0$

$s^3 + 24.002s^2 + 265.048s + (0.53 + 180K)$

use matlab
poles = roots([a b c d])

Step 3 draw root locus

step 1) Find poles

step 4

	ω_n	ζ	
K_1			
K_2			
K_3			

K values	ω_n	ζ
2	25	.48
30		

Drop one pole

$s^2 + 24s + 265 + 180(2) = 0$

$(s^2 + 24s + 265) + 180K$

$s^2 + 24s + 265 + 180 \cdot 2$
$s^2 + 24s + (265 + 360)$
$s^2 + 24s + 625 = 0$

$K = 2$
$\cancel{\omega_n^2 = 360}$
$\cancel{\omega_n = \sqrt{360}}$
$\cancel{= 6\sqrt{10}}$

2ζ

$\omega_n^2 = 625$
$\omega_n = \sqrt{625}$
$= 25$

Steady state

$$\Phi_c(s) = \frac{1}{s}$$

$$\Phi(s) = \frac{1}{s} \cdot H(s)$$

Final Value Theorem

$$\Phi(\infty) = \lim_{s \to 0} s \cdot \Phi(s) = \lim_{s \to 0} H(s) = \frac{180K}{0.53 + 180K}$$

From specification 1

$$\phi(\infty) \in \left((1 - 0.1\%) u(t), \ (1 + 0.1\%) u(t) \right)$$

$$\Rightarrow 0.99 < \frac{180K}{0.53 + 180K} < 1.001\% \Rightarrow K$$

$$0.999 < \frac{180K}{0.53 + 180K}$$

$$(0.999)(0.53 + 180K) < 180K$$

$$(0.999)(0.53) + (0.999)(180K) < 180K$$

$$(0.999)(0.53) < 180K - (0.999)(180K)$$

$$(0.999)(0.53 < 0.18K$$

$$K > \frac{(0.999)(0.53)}{0.18} \Rightarrow K > 2.9415$$

$$180K < (1.001)(0.53 + 180K)$$

$$180K < (1.001)(0.53) + (1.001)(180K)$$

$$180K - (1.001)(180K) < 0.53053$$

$$-0.18K < 0.53053$$

$$K > \frac{-0.53053}{0.18}$$

$$K > -2.9415$$

$$\zeta = \frac{-(P_1 + P_2)}{2\omega_n} = \frac{24}{2(25)} = \frac{24}{50}$$

Pole $-12.00 + 21.9317i$

~~Pole~~ $-12.00 - 21.9317i$

$$T_p = \frac{\pi}{\omega_n\sqrt{1-\zeta^2}} = 0.4 \Rightarrow \omega_n\sqrt{1-\zeta^2} = \frac{\pi}{0.4}$$

$$T_s = \frac{4.6}{\zeta\omega_n} = 1.5$$

$$\omega_n = \frac{\pi}{\sqrt{1-\zeta^2}\,(0.4)}$$

$$\frac{4.6}{1.5\,\omega_n} = \zeta$$

$$\omega_n = \frac{\pi}{\sqrt{1-\left(\frac{4.6}{1.5\omega_n}\right)^2}\;0.4}$$

$$\sqrt{1-\left(\frac{4.6}{1.5\omega_n}\right)^2} = \frac{\pi}{0.4\,\omega_n}$$