a)

$$x_e = x - x_c \qquad\qquad x_c = 1 \cdot u(t)$$

$$\dot{x}_e = \dot{x} - \cancel{\dot{x}_c}$$

$$= -0.02x + 0.1F_1 x_c \qquad\qquad x_e = \text{tracking error}$$

Since $\dot{x}_e = -Kx_e$ when $K > 0$ $x_e \to 0$ as $t \to \infty$

$$-0.02x + 0.1F_1 x_c = -Kx_e$$

$$-0.02x + 0.1F_1 x_c = -K(x - x_c) \quad \text{where } -K = -0.02$$

$$-\cancel{Kx}$$

$$F_1 = \frac{-K(x - x_c) + 0.02x}{0.1x_c}$$

$$= \frac{\cancel{-Kx} + Kx_c + \cancel{Kx}}{0.1x_c} = \frac{K\cancel{x_c}}{0.1\cancel{x_c}} = \frac{K}{0.1}$$

$$= \frac{0.02}{0.1}$$

$$F_1 = 0.2$$

b) $\quad y = x \qquad\qquad x(0) = 0$

$$\dot{x} = -0.02x + 0.1F_1 x_c$$

$$sX(s) - x(0) = -0.02X(s) + \frac{0.1F_1 x_c}{s}$$

$$sX(s) + 0.02X(s) = \frac{1}{s}(0.1F_1 x_c)$$

$$X(s)[s + 0.02] = \frac{1}{s}(0.1F_1 x_c)$$

$$X(s) = \frac{0.1F_1 x_c}{s(s+0.02)}$$

pole $s = 0$

$s = -0.02$

No the poles are not affected by the parameter F_1

c) The settling time $t_s = 3\tau$ where τ is the time constant

if $\dot{x} + \frac{1}{\tau}x = Au(t)$

$\therefore \frac{1}{\tau} = 0.02$ y

$\tau = 50$

$T_s = 3\tau = 150$

d) $\dot{x}_e = (-0.02 + a)x_e + [-0.02 + a + (0.1 + b)F_1]x_c + d$

$sX(E) - \boxed{x_E(0)} = (-0.02 + a)X(E) + \frac{[-0.02 + a + (0.1 + b)F_1]}{s} + \frac{d}{s}$

$sX(E) - (-0.02 + a)X(E) = \frac{[-0.02 + a + (0.1 + b)F_1]}{s} + \frac{d}{s} + x_e(0)$

$X(E)(s + 0.02 - a) = \frac{[-0.02 + a + (0.1 + b)F_1]}{s} + \frac{d}{s} + x_e(0)$

$$X(E) = \frac{[-0.02 + a + (0.1 + b)F_1]}{s(s + 0.02 - a)} + \frac{d}{s(s + 0.02 - a)} + \frac{x_e(0)}{(s + 0.02 - a)}$$

$= \cancel{-0.02} + a + \cancel{0.1F_1} + F_1 b$

$$X(E) = \frac{a + F_1 b}{s(s + 0.02 - a)} + \frac{d}{s(s + 0.02 - a)} + \frac{x_e(0)}{(s + 0.02 - a)}$$

$\frac{A}{s} + \frac{B}{(s + 0.02 - a)} = a + F_1 b \Rightarrow a + 0.2b$

$A(s + 0.02 - a) + B(s) = a + 0.2b$

when $s = 0$

$\therefore A(0.02 - a) = a + 0.2b$

$A = \frac{a + 0.2b}{0.02 - a}$

when $s + 0.02 - a = 0$

$s = a - 0.02$

$\therefore$

$B(a - 0.02) = a + 0.2b$

$B = \frac{a + 0.2b}{a - 0.02}$

$$\frac{C}{s} + \frac{D}{(s+0.02-a)} = \frac{d}{s(s+0.02-a)}$$

$$C(s+0.02-a) + D(s) = d$$

when $s=0$

$$\therefore \quad C = \frac{d}{0.02-a}$$

when $s = a - 0.02$

$$D(a-0.02) = d$$

$$D = \frac{d}{a-0.02}$$

Laplace Transforms of Common Signals

Name	Time function, $f(t)$	Laplace tx., $F(s)$
Unit impulse	$\delta(t)$	1
Unit step	$1(t)$	$\frac{1}{s}$
Unit ramp	$t \cdot 1(t)$	$\frac{1}{s^2}$
nth order ramp	$t^n \cdot 1(t)$	$\frac{n!}{s^{n+1}}$
Sine	$\sin(bt)1(t)$	$\frac{b}{s^2+b^2}$
Cosine	$\cos(bt)1(t)$	$\frac{s}{s^2+b^2}$
Damped sine	$e^{-at}\sin(bt)1(t)$	$\frac{b}{(s+a)^2+b^2}$
Damped cosine	$e^{-at}\cos(bt)1(t)$	$\frac{s+a}{(s+a)^2+b^2}$
Diverging sine	$t\sin(bt)1(t)$	$\frac{2bs}{(s^2+b^2)^2}$
Diverging cosine	$t\cos(bt)1(t)$	$\frac{s^2-b^2}{(s^2+b^2)^2}$

$e^{-at} \quad \frac{1}{s+a}$

$te^{-at} \quad \frac{1}{(s+a)^2}$

$$X[E] = \frac{a+0.2b}{0.02-a} + \frac{a+0.2b}{a-0.02} + \frac{d}{0.02-a} + \frac{d}{a-0.02}$$

$$= A + Be^{(-0.02+a)t} + C + De^{(-0.02+a)t} + x_e(0)e^{(-0.02+a)t}$$

2a) $v(t) = P(x_c - x) + I\int_0^t (x_c - x)\,d\tau$ with $x(0) = 0$

$$\dot{x} = -0.02x + 0.1P(x_c - x) + 0.1\int_0^t (x_c - x)\,d\tau$$

$$sX = -0.02X + 0.1(P + 1/s\, I)(X_c - x)$$

$$s^2X + 0.02X(s) = 0.1PsX_c - 0.1PsX + 0.1IX_c - 0.1IX$$