1) a) $x_e = x - x_c$

$\dot{x}_e = \dot{x} - \cancel{\dot{x}_c}^{\,0} = \dot{x}$

b/c x_c is constant

$= -0.02x + 0.1F_1 x_c$

since $\dot{x}_c = -Kx_e$ when $K>0 \Rightarrow x_e \to 0$ as $t \to \infty$

$-0.02x + 0.1F_1 x_c = -Kx_e$

$= -K(x - x_c)$ where $-K = -0.02$

$$F_1 = \frac{-Kx + Kx_c + Kx}{0.1x_c} = \frac{Kx_c}{0.1x_c} = \frac{K}{0.1} = \frac{0.02}{0.1} = 0.2$$

$\therefore F_1 = 0.2$

b) $y = x$ angular velocity

take Laplace transform of

$sX + x(0) = -0.02X + 0.1F_1 x_c + \frac{d}{s}^{\to 0}$

Laplace Trans of x

$sX + 0.02X + \cancel{x(0)}^{\,0} = 0.1F_1 x_c + 0$

$X(s + 0.02) = 0.1F_1 x_c$

$$\frac{X}{x_c} = \frac{0.1F_1}{(s+0.02)} \quad \therefore s + 0.02 = 0$$

$\boxed{s = -0.02}$ pole

No, the pole is unaffected by parameter F_1

c) If settling time (T_{settle}) = 3 times the time constant (τ)

then $T_{settle} = 3\tau$

if $\dot{x} + \frac{1}{\tau}x = Au(t)$, and $\dot{x} = -0.02x + 0.1F_1 x_c$

$\therefore \frac{1}{\tau} = 0.02$

$$\tau = \frac{1}{0.02} = 50$$

$T_{settle} = 3\tau = 3(50) = 150$

d) $\dot{x}_e = (-0.02 + a) x_e + [-0.02 + a + (0.1 + b) F_1] x_c + d$

$$\mathcal{L}[\dot{x}_e = (-0.02 + a) x_e + [-0.02 + a + (0.1 + b) F_1] \overset{\text{const}}{x_c} + \overset{\text{const}}{d}]$$

$$sX_e - X_e(0) = (-0.02 + a) X_e + [-0.02 + a + (0.1 + b) F_1] \frac{1}{s} + \frac{d}{s}$$

$$sX_e - X_e(0) - (-0.02 + a) X_e = [-0.02 + a + (0.1 + b) F_1] \frac{1}{s} + \frac{d}{s}$$

$$X_e s(s + 0.02 - a) = [-0.02 + a + (0.1 + b) F_1] + d + sX_e(0)$$

$$X_e = \underbrace{\frac{[-0.02 + a + (0.1 + b) F_1]}{s(s + 0.02 - a)}}_{1} + \underbrace{\frac{d}{s(s + 0.02 - a)}}_{2} + \underbrace{\frac{X_e(0)}{(s + 0.02 - a)}}_{3}$$

utilizing Partial Fraction Expansion

1 $\frac{A}{s} + \frac{B}{s + 0.02 - a}$

$A(s + 0.02 - a) + B(s) = -0.02 + a + (0.1 + b)(\overset{\text{from part a}}{0.2})$

$= -0.02 + a + .02 + 0.2b$

$A(s + 0.02 - a) + B(s) = a + 0.2b$

if $s = 0$: $A(0 + 0.02 - a) + B(0) = a + 0.2b$

$A(0.02 - a) = a + 0.2b$

$$\boxed{A = \frac{a + 0.2b}{0.02 - a}}$$

if $s = -.02 + a$: $A(0) + B(-.02 + a) = a + 0.2b$

$$\boxed{B = \frac{a + 0.2b}{-.02 + a}}$$

2 $\frac{C}{s} + \frac{D}{s + 0.02 - a}$

$C(s + 0.02 - a) + D(s) = d$

if $s = 0$: $C(0.02 - a) + D(0) = d$

$$\boxed{C = d / 0.02 - a}$$

if $s = -0.02 + a$: $C(0) + D(-0.02 + a) = d$

$$\boxed{D = d / -0.02 + a}$$

inverse Laplace

$$xe = A + Be^{(-0.02+a)t} + C + De^{(-0.02+a)t} + xe(0)e^{(-0.02+a)t}$$

if $-0.02 + a < 0$ i.e $a < 0.02$

$e^{(-0.02+a)t} \to 0$ as $t \to \infty$

$\therefore$ $xe(t) = A + C$ as $t \to \infty$

$$= \frac{a + 0.26}{0.02 - a} + \frac{d}{0.02 - a}$$

$$= \frac{a + 0.26 + d}{(0.02 - a)}$$

2) a) $v(t) = P(x_c - x) + I\int_0^t (x_c - x)\,d\tau$ with $x(0) = 0$

$$\dot{x} = -0.02x + 0.1P(x_c - x) + 0.1I\int_0^t (x_c - x)\,d\tau$$

$$sX = -0.02X + 0.1\left(P + \frac{1}{s}I\right)(X_c - X)$$

	X_c	$-X$
$0.1Ps$	$0.1PsX_c$	$-0.1PsX$
$0.1I$	$0.1IX_c$	$-0.1IX$

$s*$ $s*$

$$s^2X + 0.02Xs = 0.1PsX_c - 0.1PsX + 0.1IX_c - 0.1IX$$

$$s^2X + 0.02Xs + 0.1PsX + 0.1IX = X_c(0.1(Ps + I))$$

$$X(s^2 + (0.02 + 0.1P)s + 0.1I) = X_c(0.1(Ps + I))$$

$$\therefore \frac{X}{X_c} = \frac{0.1(Ps + I)}{s^2 + (0.02 + 0.1P)s + 0.1I}$$

b) Find P & I

$s_1 = -0.1 + j0.05$

$s_2 = -0.1 - j0.05$

$as^2 + bs + c = 0$

where s_1 & s_2 are solutions of the equation

it holds that

① $s_1 + s_2 = -b/a$

② $s_1 s_2 = c/a$

① $-0.1 + j0.05 + -0.1 - j0.05 = -(0.02 + 0.1P)/1$

$$\frac{-0.2 + 0.02}{-0.1} = P$$

$\boxed{\therefore P = 1.8}$

② $-0.1 + j0.05$

	-0.1	$-j0.05$
-0.1	$.01$	$-j.005$
$-j0.05$	$+j.005$	$.0025$

$= .01 + 0 + .0025 = .0125$

$.0125 = 0.1I$

$\boxed{\therefore I = .025}$

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c) $T_{settle} = 3 \cdot \frac{1}{|\sigma|}$ where $\sigma = \text{Real}(s_1) = -0.1$

$\therefore T_{settle} = 3 * \frac{1}{|-0.1|} = 3 \times \frac{1}{0.1} = 30$

d) $\dot{x}_e = \dot{x} - \cancel{\dot{x}_c} \to 0$ b/c it's const.

$= \dot{x}$

$= (-0.02+a)x_e + (0.1+b)\left(-x_e P - I\int_0^t x_e \, d\tau\right) + d + (-0.02+a)x_c$

Take Laplace Trans.

$$s\mathcal{X}_e + x_e(0) = (-0.02+a)X_e - (0.1+b)PX_e - (0.1+b)I\frac{1}{s}X_e + \frac{d}{s} + \frac{(-0.02+a)}{s}$$

$$sX_e - (-0.02+a)X_e + (0.1+b)PX_e + (0.1+b)I\frac{1}{s}X_e = \frac{d}{s} + \frac{(-0.02+a)}{s} - x_e(0)$$

$s*$ $\qquad$ $s*$

$$s^2X_e - (-0.02+a)X_e s + (0.1+b)PX_e s + (0.1+b)IX_e = d + (-0.02+a) - sX_e(0)$$

$$X_e = \frac{\cancel{d + (-0.02+a) - s\,x_e(0)}}{s^2 - (-0.02+a)s + (0.1+b)Ps + (0.1+b)I}$$

$$s^2 - (-0.02+a)s + (0.1+b)Ps + (0.1+b)I = 0$$

s_1, s_2 Solutions

$$X_e = \frac{*}{(s-s_1)(s-s_2)} = \frac{A}{s-s_1} + \frac{B}{s-s_2}$$

1. if s_1, s_2 are $\mathbb{R}$

$X_e = A \cdot e^{s_1 t} + Be^{s_2 t}$

* if $s_1 - s_2 < 0$, then, $X_e \to 0$ as $t \to \infty$

2. If s_1 and s_2 are $\mathbb{C}$ conjugate pairs

$\left[\begin{array}{l} \text{e.g } s_1 = -0.1 + j0.05 \\ \quad\;\; s_2 = -0.1 - j0.05 \end{array}\right]$ sin

$X_e = Ae^{s_1 t} + Be^{s_2 t}$

$e^{(-0.1+j0.05)t} = e^{-0.1t}e^{j0.05t}$

$e^{j\theta} = \cos\theta + j\sin\theta$

$X_e = e^{-0.1t} \cdot 2\cos(0.05\theta$ $\qquad |\cos(0.05t)| \leq 1$

$X_e \to 0$ cost $\to \infty$ as long as $\sigma < 0$