# Lab 5: Ball Beam & Root Locus

Spring 2020

## 1 Objective

To use root locus methods to design a cascade compensator for the ball and beam system.

## 2 Equipment

Simulink

## 3 Background

Consider the ball and beam system shown in Fig. 1. The system variables are defined in Fig. 2. In an earlier lab, we derived the plant dynamics

$$\frac{X(s)}{\Theta(s)} = \frac{7}{s^2} = G(s) \tag{1}$$

where $X(s)$ and $\Theta(s)$ are the Laplace transforms of $x(t)$ and $\theta(t)$. In the earlier lab, you used both $x(t)$ and $\dot{x}(t)$ to compute the control signal $\theta(t)$. Therefore, that control approach requires two sensors: position and velocity. In this lab, our goal is to control the system using only the position signal $x(t)$ thereby reducing the implementation cost since only one sensor would be required.

Your task is to use the root locus design procedure to design a cascade compensator $K(s)$ for the plant $G(s)$ in a unity feedback loop

$$\frac{\Theta(s)}{E(s)} = K(s). \tag{2}$$

for the system in eqn. (1) to achieve the following specifications:

1. The settling time should satisfy $2.0 \leq T_s \leq 4.0$ seconds.
2. Denote the commanded value of $x(t)$ as $x_d(t)$. Denote the tracking error as $e(t) = x_d(t) - x(t)$. For step inputs, this (tracking) error should be zero in steady state.
3. At the beginning of any step change in the commanded value of $x_d(t)$, the initial change in the control signal $\theta$ should be less that 30° per 1.0 m of commanded change in position (e.g., for a 0.5 meter step change in position, the maximum angle should be 15°). Ultimately, this specification will place a constraint on the parameters of $K(s)$. The initial value theorem will be useful for determining the constraint. The initial value theorem states that

$$\theta(t)\Big|_{t=0^+} = \lim_{s\to\infty} s\Theta(s).$$

To succeed, you may have to use multiple lead networks, multiple lag networks, or none of one or the other.

## 4 Prelab (Complete this section prior to lab.)

1. Read Ch. 9 in Nise.
2. Draw a block diagram for the closed loop system. Clearly indicate the signals $x_d(t)$, $x(t)$, $e(t)$, and $\theta(t)$.
3. For the unity feedback control system with plant $G(s)$ and controller $K(s)$, find the transfer function from the reference input $X_d(s)$ to the control signal $\Theta(s)$.
4. Use the initial value theorem to determine what constraint Specification 3 places on the controller $K(s)$.
5. For the unity feedback control system with plant $G(s)$ and controller $K(s)$, find the transfer function from the reference input $X_d(s)$ to the output $X(s)$.
6. What constraint does Specification 1 place on the desired closed loop pole locations?

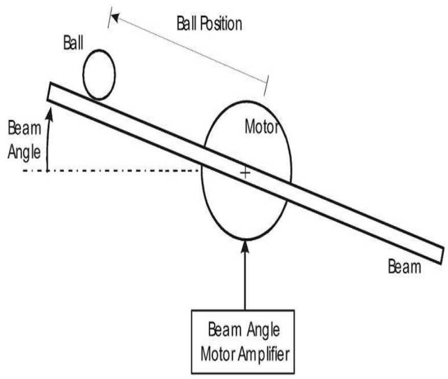


Figure 1: Ball beam system

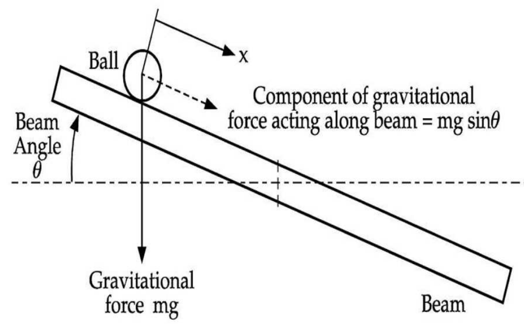


Figure 2: Schematic diagram for derivation of the ball dynamics

7. Look in the textbook index to find the discussion of 'dominant poles' and the effect of additional poles and zeros on the applicability of the relationships between transient specs and pole locations. You are expected to understand and apply those issues in your analysis of this lab.

8. What *type* is the system? Given the system type, what constraint does Specification 2 place on the forward path gain?

# 5 Experimental Procedure

1. Apply the root locus cascade compensation design procedure to the ball and beam system to design an implementable control (i.e., the order of the numerator cannot exceed that of the denominator) law that achieves the specifications.

2. As discussed in class, a prefilter can be used to remove the zero from the closed loop transfer function from $X_d(s)$ to the output $X(s)$. Try this. After designing the prefilter, reconsider the constraint that Specification 3 places on the design of $K(s)$.

# 6 Report

Your lab report should include:

- Your analysis of the specifications and the constraints that they impose on your design;
- Your analysis and computations involved in the choice of the parameters in $K(s)$;
- Root loci to support your design discussion;
- Discussion of the applicability of the design rules for second order transfer functions with dominant poles;
- A plot of the time response along with discussion explaining that the specification was achieved; and,
- A line clearly showing your complete control law. A reader of the lab report should be able to enter your controller into the sim and see the achieved performance.

No controller parameters can be picked by trial-and-error. Parameter values and controller structure (e.g., gains, poles, zeros) must be based on analysis.

There is no single correct answer. In fact every group should be able to design a unique controller.