EE132 Automatic Control

Lab 1: Open vs Closed Loop

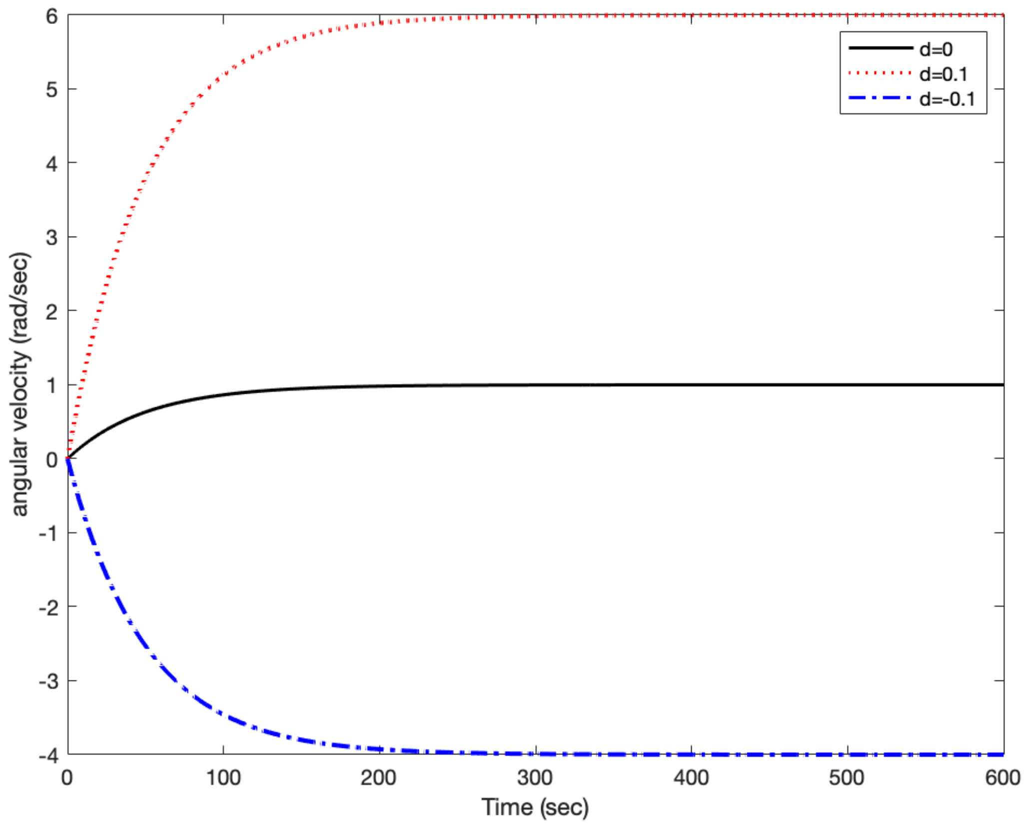
1 Objective

Comparison of performance for both open- and closed-loop controllers applied to a simple plant in the presence of disturbances or modeling error

Prelab

Simulation Procedure.

```
close all
figure(1)
plot(Data.time, Data.signals.values, 'k', 'linewidth', 1.5)
hold on
plot(Data.time, Data.signals.values, 'r:', 'Linewidth', 1.5)
hold on
plot(Data.time, Data.signals.values, 'b-.', 'Linewidth', 1.5)
xlabel('Time (sec)')
ylabel('angular velocity (rad/sec)')
legend('d=0', 'd=0.1', 'd=-0.1')
```



The settling time calculated in the pre-lab matches our simulation graph results. When disturbances increase in the positive direction, the angular velocity also increases. When the disturbance is increased in the negative direction the angular velocity decreases as well. Thus our controller is sensitive to disturbances.

```
close all
figure(1)
plot(Data.time, Data.signals.values, 'k', 'linewidth', 1.5)
hold on
plot(Data.time, Data.signals.values, 'r:', 'Linewidth', 1.5)
hold on
plot(Data.time, Data.signals.values, 'b-.', 'Linewidth', 1.5)
xlabel('Time (sec)')
ylabel('angular velocity (rad/sec)')
legend('d=0', 'd=0.1', 'd=-0.1')
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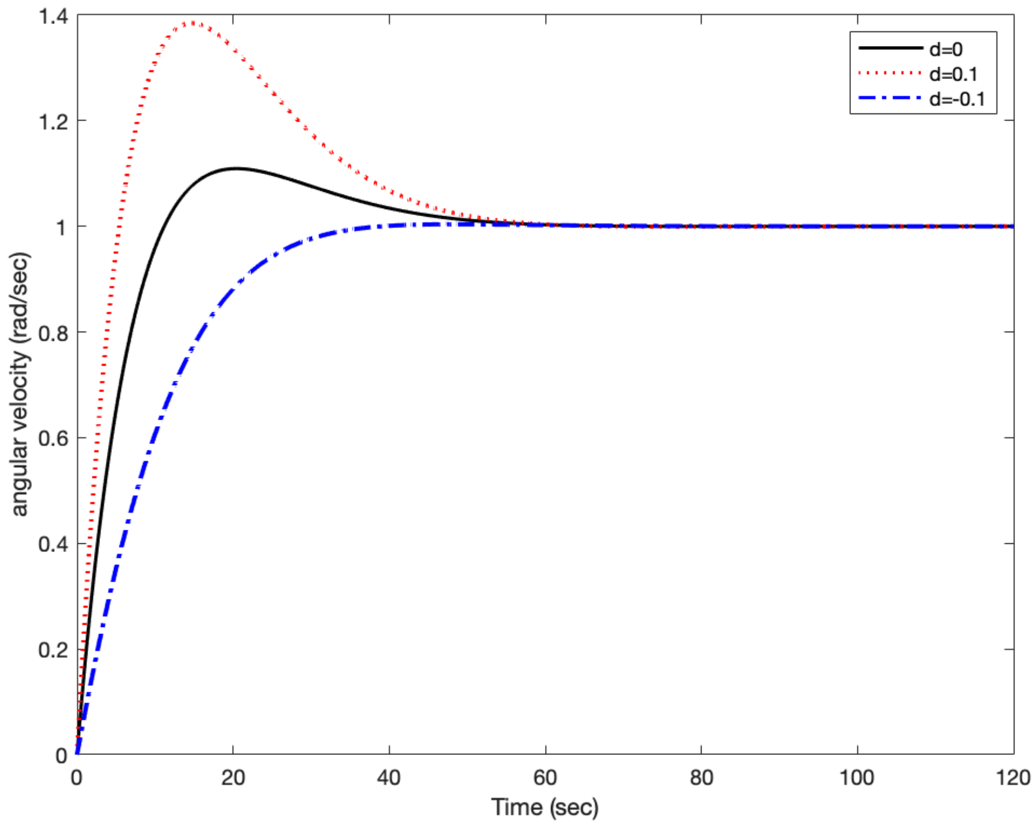

When the disturbance is increased in the positive direction, the percent overshoot increases; the systems return to stability faster. Our peak time decreases with an increase in the disturbance. When the dampening is increased in the negative direction the system reaches a steady-state too slowly.

Nick

1) a) $x_e = x - x_c$

$\dot{x}_e = \dot{x} - \cancel{\dot{x}_c}^{\,0} = \dot{x}$

b/c x_c is constant

$= -0.02x + 0.1F_1 x_c$

since $\dot{x}_e = -Kx_e$ when $K>0$ $\Rightarrow$ $x_e \to 0$ as $t \to \infty$

$-0.02x + 0.1F_1 x_c = -Kx_e$

$= -K(x - x_c)$ where $-K = -0.02$

$$F_1 = \frac{-Kx + Kx_c + Kx}{0.1x_c} = \frac{Kx_c}{0.1x_c} = \frac{K}{0.1} = \frac{0.02}{0.1} = 0.2$$

$\therefore F_1 = 0.2$

b) $y = x$ angular velocity

take Laplace transform of

$sX + x(0) = -0.02X + 0.1F_1 X_c + \frac{d}{s}$ (with $d \to 0$)

Laplace Trans of x

$sX + 0.02X + \cancel{x(0)}^{\,0} = 0.1F_1 X_c + 0$

$X(s + 0.02) = 0.1F_1 X_c$

$$\frac{X}{X_c} = \frac{0.1F_1}{(s+0.02)} \quad \therefore \quad s + 0.02 = 0$$

$\boxed{s = -0.02}$ ← pole

No, the pole is unaffected by parameter F_1

c) If settling time (T_{settle}) = 3 times the time constant (τ)

then $T_{settle} = 3\tau$

if $\dot{x} + \frac{1}{\tau}x = Au(t)$, and $\dot{X} = -0.02X + 0.1F_1 X_c$

$\therefore \frac{1}{\tau} = 0.02$

$$\tau = \frac{1}{0.02} = 50$$

$T_{settle} = 3\tau = 3(50) = 150$

Nick

d) $\dot{x}_e = (-0.02 + a)x_e + [-0.02 + a + (0.1 + b)F_i]x_c + d$

$$\mathcal{L}\left[\dot{x}_e = (-0.02+a)x_e + [-0.02+a+(0.1+b)F_i]\overset{\text{const}}{x_c} + \overset{\text{const}}{d}\right]$$

$$sX_e - X_e(0) = (-0.02+a)X_e + [-0.02+a+(0.1+b)F_i]\frac{1}{s} + \frac{d}{s}$$

$$sX_e - X_e(0) - (-0.02+a)X_e = [-0.02+a+(0.1+b)F_i]\frac{1}{s} + \frac{d}{s}$$

$$X_e s(s+0.02-a) = [-0.02+a+(0.1+b)F_i] + d + sX_e(0)$$

$$X_e = \underbrace{\frac{[-0.02+a+(0.1+b)F_i]}{s(s+0.02-a)}}_{①} + \underbrace{\frac{d}{s(s+0.02-a)}}_{②} + \underbrace{\frac{X_e(0)}{(s+0.02-a)}}_{③}$$

utilizing Partial Fraction Expansion

① $\frac{A}{s} + \frac{B}{s+0.02-a}$

$$A(s+0.02-a) + B(s) = -0.02 + a + (0.1+b)(\overset{\text{from part a}}{0.2})$$
$$= -0.02 + a + .02 + 0.2b$$

$$A(s+0.02-a) + B(s) = a + 0.2b$$

if $s=0$: $A(0+0.02-a) + B(0) = a + 0.2b$

$$A(0.02-a) = a + 0.2b$$

$$\boxed{A = \frac{a+0.2b}{0.02-a}}$$

if $s=-.02+a$: $A(0) + B(-.02+a) = a + 0.2b$

$$\boxed{B = \frac{a+0.2b}{-.02+a}}$$

② $\frac{C}{s} + \frac{D}{s+0.02-a}$

$$C(s+0.02-a) + D(s) = d$$

if $s=0$: $C(0.02-a) + D(0) = d$

$$\boxed{C = d/0.02-a}$$

if $s=-0.02+a$: $C(0) + D(-0.02+a) = d$

$$\boxed{D = d/-0.02+a}$$

Nick

inverse Laplace

$xe = A + Be^{(-0.02+a)t} + C + De^{(-0.02+a)t} + xe(0)e^{(-0.02+a)t}$

if $-0.02 + a < 0$ i.e $a < 0.02$

$e^{(-0.02+a)t} \to 0$ as $t \to \infty$

$\therefore Xe(t) = A + C$ as $t \to \infty$

$= \frac{a+0.26}{0.02-a} + \frac{d}{0.02-a}$

$= \frac{a + 0.26 + d}{(0.02 - a)}$

2) a) $V(t) = P(X_c - X) + I\int_0^t (X_c - X)\,d\tau$ with $X(0) = 0$

$\dot{X} = -0.02X + 0.1P(X_c - X) + 0.1I\int_0^t (X_c - X)\,d\tau$

$SX = -0.02X + 0.1(P + \frac{1}{S}I)(X_c - X)$

$X_c \quad -X$

$0.1PS \quad 0.1PSX_c \quad -0.1PSX$

$0.1I \quad 0.1IX_c \quad -0.1IX$

$S*$ $\quad S*$

$S^2X + 0.02XS = 0.1PSX_c - 0.1PSX + 0.1IX_c - 0.1IX$

$S^2X + 0.02XS + 0.1PSX + 0.1IX = X_c(0.1(PS + I))$

$X(S^2 + (0.02 + 0.1P)S + 0.1I) = X_c(0.1(PS + I))$

$\therefore \frac{X}{X_c} = \frac{0.1(PS + I)}{S^2 + (0.02 + 0.1P)S + 0.1I}$

b) Find P & I

$S_1 = -0.1 + j0.05$

$S_2 = -0.1 - j0.05$

$aS^2 + bS + C = 0$

where S_1 & S_2 are solutions of the equation

it holds that

① $S_1 + S_2 = -b/a$

② $S_1 S_2 = c/a$

① $-0.1 + j0.05 + -0.1 - j0.05 = -(0.02 + 0.1P)/1$

$\frac{-0.2 + 0.02}{-0.1} = P$

$\boxed{\therefore P = 1.8}$

② $(-0.1 + j0.05)(-0.1 - j0.05)$

	-0.1	$+j0.05$
-0.1	$.01$	$-j.005$
$-j0.05$	$+j.005$	$.0025$

$= .01 + 0 + .0025 = .0125$

$.0125 = 0.1I$

$\boxed{\therefore I = .025}$

Nick

gorillaglue9214

c) $T_{settle} = 3 \cdot \frac{1}{|\sigma|}$ where $\sigma = Real(s_1) = -0.1$

$\therefore T_{settle} = 3 * \frac{1}{|-0.1|} = 3 \times \frac{1}{0.1} = 30$

d) $\dot{x}_e = \dot{x} - \cancel{\dot{x}_c} \nearrow 0$ b/c its const.

$= \dot{x}$

$= (-0.02+a)x_e + (0.1+b)\left(-x_eP - I\int_0^t x_e \, d\tau\right) + d + (-0.02+a)x_c$

Take Laplace Trans.

$sX_e + x_e(0) = (-0.02+a)X_e - (0.1+b)PX_e - (0.1+b)I\frac{1}{s}X_e + \frac{d}{s} + \frac{(-0.02+a)}{s}$

$sX_e - (-0.02+a)X_e + (0.1+b)PX_e + (0.1+b)I\frac{1}{s}X_e = \frac{d}{s} + \frac{(-0.02+a)}{s} - x_e(0)$

s^* $\quad$ s^*

$s^2X_e - (-0.02+a)X_es + (0.1+b)PX_es + (0.1+b)IX_e = d + (-0.02+a) - sX_e(0)$

$$X_e = \frac{d + (-0.02+a) - sX_e(0)}{s^2 - (-0.02+a)s + (0.1+b)Ps + (0.1+b)I}$$

$s^2 - (-0.02+a)s + (0.1+b)Ps + (0.1+b)I = 0$

s_1, s_2 Solutions

$$X_e = \frac{*}{(s-s_1)(s-s_2)} = \frac{A}{s-s_1} + \frac{B}{s-s_2}$$

① if s_1, s_2 are $\mathbb{R}$

$X_e = A \cdot e^{s_1t} + Be^{s_2t}$

* if $s_1, s_2 < 0$, then, $X_e \to 0$ as $t \to \infty$

② If s_1 and s_2 are $\mathbb{C}$ conjugate pairs

e.g. $s_1 = -0.1 + j0.05$ $\quad$ sin

$s_2 = -0.1 - j0.05$

$X_e = Ae^{s_1t} + Be^{s_2t}$

$e^{(-0.1+j0.05)t} = e^{-0.1t}e^{j0.05t}$

$e^{j\theta} = \cos\theta + j\sin\theta$

$X_e = e^{-0.1t} \cdot 2\cos(0.05\theta)$ $\quad$ $|\cos(0.05t)| \leq 1$

$X_e \to 0$ $\cos t \to \infty$

as long as $\sigma < 0$

Christopher Hwang
SID: [redacted]

Prelab 1

1) A) $V(t) = F_1 x_c(t)$

$$\lim_{t \to \infty} x(t) - x_c(t) = 0$$

$$x_e = x - x_c$$

$$\dot{x}_e = \dot{x} - \dot{x}_c = \dot{x}$$

$$= -0.02x + 0.1F_1 x_c$$

$$\dot{x}_e = -k x_e, \quad k > 0 \quad \Rightarrow x_e \to 0 \text{ as } t \to \infty$$

let $-0.02x + 0.1F_1 x_c = -k x_e$

$$= -k(x - x_c)$$

$$-k = -0.02$$

$$\Rightarrow -k = 0.1F_1$$

$$-0.02 = 0.1F_1$$

$$\boxed{F_1 = 0.2}$$

B) $y = x$

$$sX - x(0) = -0.02X + 0.1F_1 X_c + \frac{d}{s}$$

$$sX + 0.02X = 0.1F_1 X_c$$

$$X(s + 0.02) = 0.1F_1 X_c$$

$$\frac{X}{X_c} = \frac{0.1F_1}{(s + 0.02)}$$

$$\boxed{s = -0.02 \text{ pole}}$$

No, the pole location affected by the choice of the F_1 paramter

Chris

c) T_s = settling time $= 3\tau$

τ = time constant

$\dot{x} + \frac{1}{\tau}x = Au(t)$ and $\dot{x} = -0.02X + 0.1F_1X_c$

$\frac{1}{\tau} = 0.02$

$\tau = \frac{1}{0.02} = 50$

$T_s = 3\tau = 3(50)$

$\boxed{T_s = 150 \text{ sec}}$

D) $\dot{X}_e = (-0.02 + a)X_e + (-0.02 + a + (0.1 + b)F_1]X_c + d$

taking the Laplace transform

$sX_e - X_e(0) = (-0.02 + a)X_e + (-0.02 + a + (0.1 + b)F_1]\frac{1}{s} + \frac{d}{s}$

$X_e = \frac{(-0.02 + a + (0.1 + b)F_1)}{s(s + 0.02 - a)} + \frac{d}{s(s + 0.02 - a)} + \frac{X_e(0)}{(s + 0.02 - a)}$

$\frac{A}{s} + \frac{B}{s + 0.02 - a}$ $\quad A(s + 0.02 - a) + B(s) = -0.02 + a + (0.1 + b)(0.2)$

$= -0.02 + a + 0.02 + 0.2b$

$A(s + 0.02 - a) + B(s) = a + 0.2b$

$s = 0 \mid A(0.02 - a) = a + 0.2b$

$A = \frac{a + 0.2b}{0.02 - a}$

$s = -0.02 + a \mid B(-0.02 + a)$ $\quad B = \frac{a + 0.2b}{-0.02 + a}$

Chris

$$\frac{C}{s} + \frac{D}{s+0.02-a} \qquad C(s+0.02-a) + D(s) = d$$

$$s=0 \;\Big|\; C(0.02-a) = d$$

$$C = \frac{d}{0.02-a}$$

$$s = -0.02+a \;\Big|\; D(-0.02+a) = d$$

$$D = \frac{d}{-0.02+a}$$

$$X_e = A + Be^{(-0.02+a)t} + C + De^{(-0.02+a)t} + X_e(0)e^{(-0.02+a)t}$$

$$-0.02+a < 0$$

$$e^{(-0.02+a)t} \to 0 \quad \text{as} \quad t \to \infty$$

$$X_e(t) = A + C$$

$$\boxed{X_e(t) = \frac{a+0.2b+d}{(0.02-a)}}$$

Chris

2)A) $v(t) = P(X_c - X) + I\int_0^t (X_c - X)\,d\tau$

$\dot{x} = -0.02x + 0.1P(X_c - X) + 0.1I\int_0^t (x_c - x)\,d\tau$

$sX = -0.02X + 0.1\left(P + \frac{1}{s}I\right)(X_c - X)$

$s^2X + 0.02sX = 0.1PsX_c - 0.1PsX + 0.1IX_c - 0.1IX$

$$\frac{X}{X_c} = \frac{0.1(PS + I)}{s^2 + (0.02 + 0.1P)s + 0.1I}$$

B) $s_1 = -0.1 + j0.05$ $\qquad as^2 + bs + c = 0$

$s_2 = -0.1 - j0.05$

$s_1 + s_2 = \frac{-b}{a}$

$-0.1 + j0.05 + 0.1 - j0.05$ $\qquad s_1 s_2 = \frac{c}{a}$

$= -(0.02 + 0.1P)$

$P = \frac{-0.2 + 0.02}{-0.1}$

$\boxed{P = 1.8}$

$(-0.1 + j0.05)(-0.1 - j0.05)$

$= 0.01 + 0.0025 = 0.0125$

$0.0125 = 0.1I$

$\boxed{I = 0.125}$

c) $T_s = 3 \times \dfrac{1}{|\sigma|}$ $\qquad \sigma = -0.1$

$$= 3 \times \frac{1}{|-0.1|}$$

$$\boxed{T_s = 30 \text{ sec}}$$

D)

$$\dot{X}_e = \dot{X} - \dot{X}_c$$

$$= \dot{X}$$

$$= (-0.02 + a) X_e + (0.1 + b)\left(-X_e P - I \int_0^t X_e \, d\tau\right) + d + (-0.02 + a) X_c$$

Using Laplace transform

$$s X_e - (-0.02 + a) X_e + (0.1 + b) P X_e + (0.1 + b) I \frac{1}{s} X_e = \frac{d}{s} + \frac{(-0.02 + a)}{s} - X_e(0)$$

$$s^2 X_e - (-0.02 + a) X_e s + (0.1 + b) P X_e s + (0.1 + b) I X_e = d + (-0.02 + a) - s X_e(0)$$

$$s^2 - (-0.02 + a) s + (0.1 + b) P s + (0.1 + b) I = 0$$

$$X_e = \frac{A}{s - s_1} + \frac{B}{s - s_2}$$

if s_1, s_2 are $\mathbb{R}$
$X_e = A e^{s_1 t} + B e^{s_2 t}$

$s_1 = -0.1 + j\,0.05$
$s_2 = -0.1 - j\,0.05$

$e^{j\theta} = \cos\theta + j \sin\theta$

$$\boxed{X_e = e^{-0.1t} \cdot 2\cos(0.05\theta)}$$

$|\cos(0.05\theta)| \leq 1$

$X_e \to 0$ cost $\to \infty$
as long as $\sigma < 0$

a) Buddy

$x_e = x - x_c$ $\qquad$ $x_c = 1 \cdot u(t)$

$\dot{x}_e = \dot{x} - \cancel{\dot{x}_c}$

$= -0.02x + 0.1F_1 x_c$ $\qquad$ x_e = tracking error

Since $\dot{x}_e = -Kx_e$ when $K > 0$ $x_e \to 0$ as $t \to \infty$

$-0.02x + 0.1F_1 x_c = -Kx_e$

$-0.02x + 0.1F_1 x_c = -K(x - x_c)$ where $-K = -0.02$

$-\cancel{Kx}$

$$F_1 = \frac{-K(x - x_c) + 0.02x}{0.1x_c}$$

$$= \frac{\cancel{-Kx} + Kx_c + \cancel{Kx}}{0.1x_c} = \frac{K\cancel{x_c}}{0.1\cancel{x_c}} = \frac{K}{0.1}$$

$$= \frac{0.02}{0.1}$$

$F_1 = 0.2$

b) $y = x$ $\qquad$ $x(0) = 0$

$\dot{x} = -0.02x + 0.1F_1 x_c$

$sX(s) - x(0) = -0.02X(s) + \frac{0.1F_1 x_c}{s}$

$sX(s) + 0.02X(s) = \frac{1}{s}(0.1F_1 x_c)$

$X(s)[s + 0.02] = \frac{1}{s}(0.1F_1 x_c)$

$$X(s) = \frac{0.1F_1 x_c}{s(s+0.02)}$$

pole $s = 0$

$s = -0.02$

No the poles are not affected by the parameter F_1

Buddy

c) The settling time $t_s = 3\tau$ where τ is the time constant

if $\dot{x} + \frac{1}{\tau}x = Au(t)$

$$\therefore \frac{1}{\tau} = 0.02$$

y

$$\tau = 50$$

$$T_s = 3\tau = 150$$

d) $\dot{x}_e = (-0.02 + a)x_e + [-0.02 + a + (0.1 + b)F_1]x_c + d$

$$sX(E) - x_E(0) = (-0.02 + a)X(E) + \frac{[-0.02 + a + (0.1 + b)F_1]}{s} + \frac{d}{s}$$

$$sX(E) - (-0.02 + a)X(E) = \frac{[-0.02 + a + (0.1 + b)F_1]}{s} + \frac{d}{s} + x_e(0)$$

$$X(E)(s + 0.02 - a) = \frac{[-0.02 + a + (0.1 + b)F_1]}{s} + \frac{d}{s} + x_e(0)$$

$$X(E) = \frac{[-0.02 + a + (0.1 + b)F_1]}{s(s + 0.02 - a)} + \frac{d}{s(s + 0.02 - a)} + \frac{x_e(0)}{(s + 0.02 - a)}$$

$$= \cancel{-0.02} + a + \cancel{0.1F_1} + F_1 b$$

$$X(E) = \frac{a + F_1 b}{s(s + 0.02 - a)} + \frac{d}{s(s + 0.02 - a)} + \frac{x_e(0)}{(s + 0.02 - a)}$$

$$\frac{A}{s} + \frac{B}{(s + 0.02 - a)} = a + F_1 b \Rightarrow a + 0.2b$$

$$A(s + 0.02 - a) + B(s) = a + 0.2b$$

when $s = 0$

$$\therefore A(0.02 - a) = a + 0.2b$$

$$A = \frac{a + 0.2b}{0.02 - a}$$

when $s + 0.02 - a = 0$

$$s = a - 0.02$$

$$\therefore B(a - 0.02) = a + 0.2b$$

$$B = \frac{a + 0.2b}{a - 0.02}$$

Buddy

$$\frac{C}{s} + \frac{D}{(s+0.02-a)} = \frac{d}{s(s+0.02-a)}$$

$$C(s+0.02-a) + D(s) = d$$

when $s=0$ $\therefore$ $C = \dfrac{d}{0.02-a}$

when $s = a - 0.02$

$$D(a-0.02) = d$$

$$D = \frac{d}{a-0.02}$$

Laplace Transforms of Common Signals

Name	Time function, $f(t)$	Laplace tx., $F(s)$
Unit impulse	$\delta(t)$	1
Unit step	$1(t)$	$\frac{1}{s}$
Unit ramp	$t \cdot 1(t)$	$\frac{1}{s^2}$
nth order ramp	$t^n \cdot 1(t)$	$\frac{n!}{s^{n+1}}$
Sine	$\sin(bt)1(t)$	$\frac{b}{s^2+b^2}$
Cosine	$\cos(bt)1(t)$	$\frac{s}{s^2+b^2}$
Damped sine	$e^{-at}\sin(bt)1(t)$	$\frac{b}{(s+a)^2+b^2}$
Damped cosine	$e^{-at}\cos(bt)1(t)$	$\frac{s+a}{(s+a)^2+b^2}$
Diverging sine	$t\sin(bt)1(t)$	$\frac{2bs}{(s^2+b^2)^2}$
Diverging cosine	$t\cos(bt)1(t)$	$\frac{s^2-b^2}{(s^2+b^2)^2}$

e^{-at} $\quad \frac{1}{s+a}$

te^{-at} $\quad \frac{1}{(s+a)^2}$

$$X[E] = \frac{a+0.2b}{0.02-a} + \frac{a+0.2b}{a-0.02} + \frac{d}{0.02-a} + \frac{d}{a-0.02}$$

$$= A + Be^{(-0.02+a)t} + C + De^{(-0.02+a)t} + x_e(0)e^{(-0.02+a)t}$$

2a) $V(t) = P(x_c - x) + I\int_0^t (x_c - x)\,d\tau$ with $x(0) = 0$

$$\dot{x} = -0.02x + 0.1P(x_c - x) + 0.1\int_0^t (x_c - x)\,d\tau$$

$$sX = -0.02X + 0.1(P + 1/sI)(X_c - X)$$

$$s^2X + 0.02X(s) = 0.1PsX_c - 0.1PsX + 0.1IX_c - 0.1IX$$