$$i(t)R = -\frac{1}{C}\int_{-\infty}^{\tau} i(\tau)\,d\tau$$

$$V_0(t) = R_\ell i_0 = \frac{1}{C}\int i_0(t)\,dt$$

$$R_\ell \frac{di_0}{dt} = -\frac{1}{C} i_0(t) \qquad \frac{dx(t)}{dt} = sX(s) - X_0$$

$$\cancel{\frac{1}{C} i_0(t) - R_\ell \frac{di_0}{dt} = 0}$$

$$R_\ell \frac{di_0}{dt} + \frac{1}{C} i_0(t) = 0$$

take the Laplace transform

$$R_\ell\,(sI_0(s) - I_i) + \frac{1}{sC} I_0(s) = 0$$

$$R_\ell s I_0(s) - R_\ell I_i + \frac{1}{sC} I_0(s) = 0$$

$$I_0(s)\left(R_\ell(s) + \frac{1}{sC}\right) - R_\ell I_i = 0$$

$$I_0(s) = \frac{R_\ell I_i}{R_\ell(s) + \frac{1}{sC}} \Rightarrow \frac{R_\ell I_i}{R_\ell s + \frac{1}{sC}} \Rightarrow \frac{R_\ell I_i}{\frac{CR_\ell s + 1}{sC}} \Rightarrow \frac{R_\ell sC}{sCR_\ell + 1}$$

$$I_0(s) = R_\ell C \cdot \frac{s}{sCR_\ell + 1}$$

Perform partial fraction decompositin

$$I_0(s) = 1 - \frac{1}{sCR_\ell + 1}$$

$$i(t) = \delta(t) - e^{\left(-\frac{1}{R_\ell C}\right)t} \cdot u(t)$$

$h_d(t)$ relates $R_\ell C$ in that the larger the capacitor or resistance, the quicker the circuit discharges

$$h_d(t) = I_i(t) = \delta(t) - e^{-t/R_\ell C} \cdot u(t)$$