EE 141: Digital Signal Processing

Lab 6: Filter Design Using Butterworth Filters

Lab Section: 022

Name: Buddy Ugwumba

SID: 862063029

Objective:

Design discrete-time lowpass filters using the continuous-time Butterworh filter design. Both the impulse invariance approach and the bilinear transformation approach will be explored in this lab

a) Chose some sampling period T and a high enough order N such that the continuous-time Butterworth filter with magnitude

$$|H_c(j\Omega)| = \frac{1}{\sqrt{1+\left(\frac{\Omega}{\Omega_c}\right)^{2N}}}$$

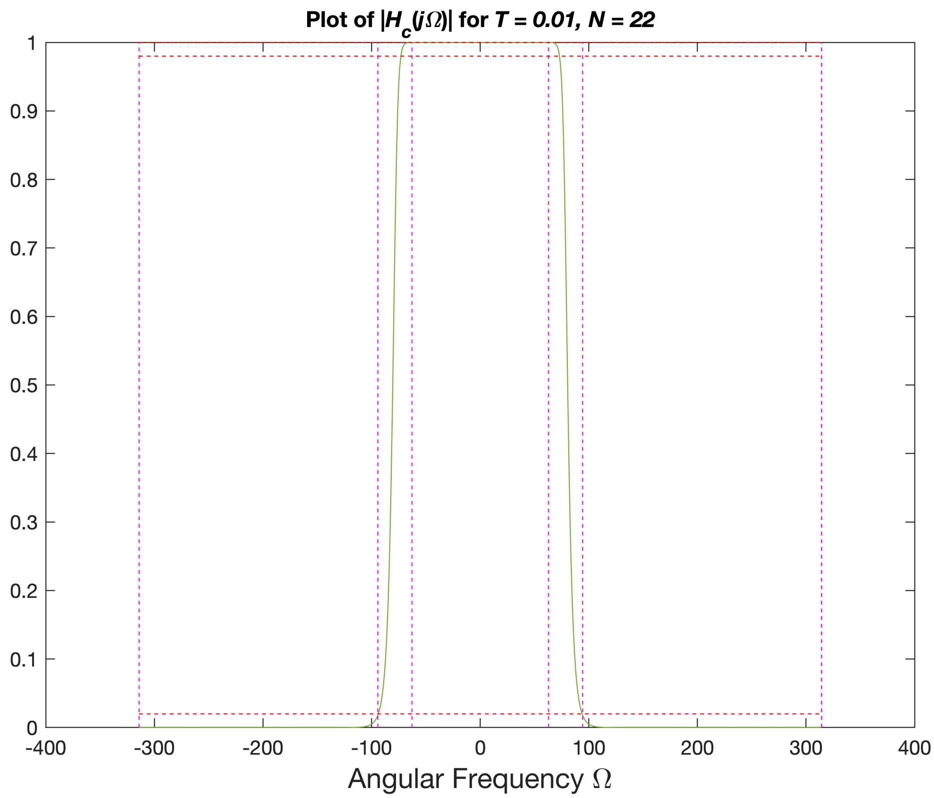
with $\Omega_c = \frac{0.25\pi}{T}$ satisfies

- $0.98 \le |H_c(j\Omega)| \le 1$ for all $-\frac{0.25\pi}{T} \le \Omega \le \frac{0.25\pi}{T}$
- $|H_c(j\Omega)| \le 0.02$ for all $-\frac{\pi}{T} \le \Omega \le \frac{0.3\pi}{T}$ and $\frac{0.3\pi}{T} \le \Omega \le \frac{\pi}{T}$

```
clear all close all clc
N = 22;
T =0.01;
omegaC = 0.25*pi/T;
omega = -pi/T:pi/T;

h = 1./sqrt(1+(omega./omegaC).^(2*N));
plot(omega,h);
hold on
y1 = get(gca, 'ylim');
plot([-0.2*pi./T -0.2*pi/T], y1, '--m')
plot([0.2*pi./T 0.2*pi/T], y1, '--m')
plot([-pi./T -pi/T], y1, '--m')
plot([-0.3*pi./T -0.3*pi/T], y1, '--m')
plot([pi./T pi/T], y1, '--m')
```

```
plot([0.3*pi./T 0.3*pi/T], y1, '--m')
yline1 = 0.98;
yline2 = 1;
yline3 = 0.02;
line([-pi/T,pi/T], [yline1, yline1], 'Color', 'red', 'LineStyle', '--')
line([-pi/T,pi/T], [yline2, yline2], 'Color', 'red', 'LineStyle', '--')
line([-pi/T,pi/T], [yline3, yline3], 'Color', 'red', 'LineStyle', '--')
title('Plot of |{\itH_c}({\itj\Omega})| for \itT = 0.01, \itN = 22')
xlabel('Angular Frequency \Omega', 'FontSize',14)
```



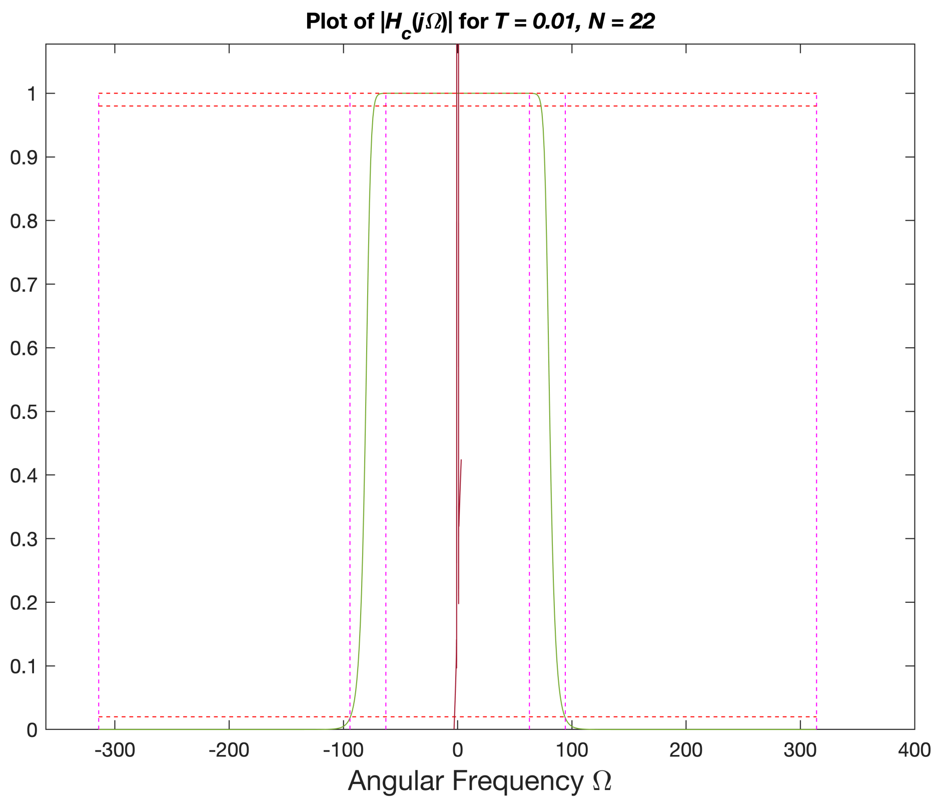
b)

```
clear all close all clc
N = 22;
T =0.01;
omegaC = 0.25*pi/T;
omega = -pi:0.01:pi;
NN = 1:22;
sk = omegaC*exp(j*pi*(2*NN + N-1)/(2*N));
b = poly(sk);
[Ak, ssk, k] = residue(omegaC^N, b);
```

c) Plot $|H_d(e^{j\omega})|$ for $-\pi \leq \omega \leq \pi$ b y sufficiently sampling the frequency axis (step size of 0.01 would do) On top of your plot, draw lines indication the specifications above. Are they satisfied? Discuss why or why not?

```
sum  = 0;
for k = 1:length(omega)
    for i = 1:22
        sum = sum + T*(Ak(i))/(1-exp(ssk(i)*T)*exp(-j*omega(k)));
    end
    Hd(k) = sum;
end

plot(omega, abs(Hd));
hold on
```



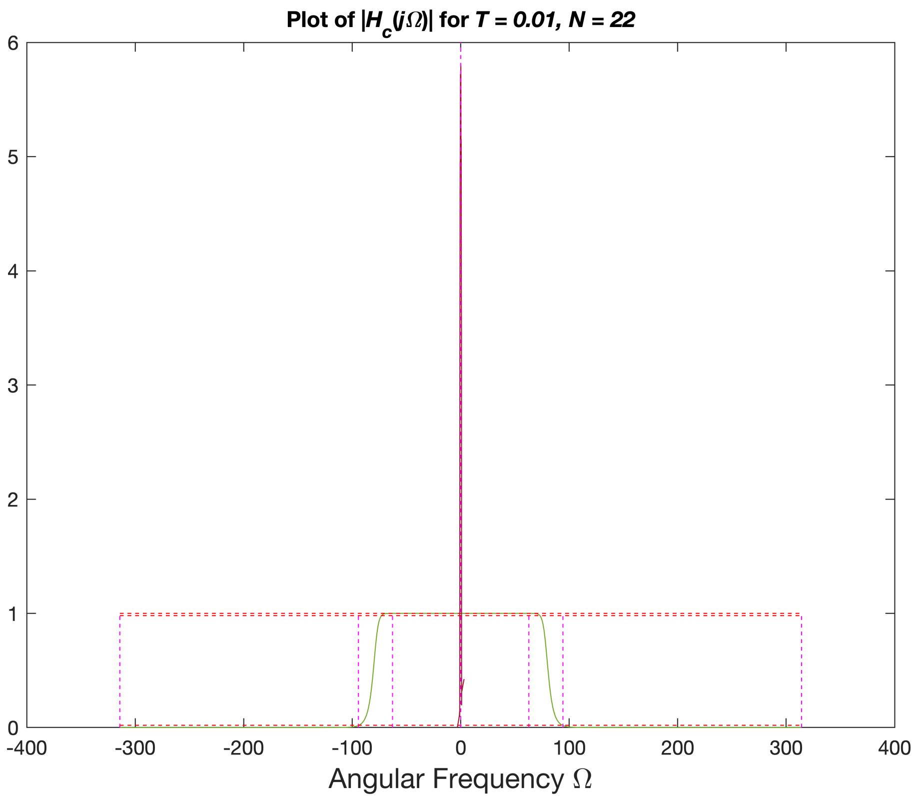
By checking the stop band and passband ranges, we can see from the graph the filter we designed meets our specificiations.

2 a)

```
clear all close all clc
N = 22;
T =0.01;
omegaC = 0.4;
omega = tan(-pi/T):tan(pi/T);

y1 = get(gca, 'ylim');
plot([tan(-0.2*pi./T) tan(-0.2*pi/T)], y1, '--m')
plot([tan(0.2*pi./T) tan(0.2*pi/T)], y1, '--m')
```

```
plot([tan(-pi./T) tan(-pi/T)], y1, '--m')
plot([tan(-0.3*pi./T) tan(-0.3*pi/T)], y1, '--m')
plot([tan(pi./T) tan(pi/T)], y1, '--m')
plot([tan(0.3*pi./T) tan(0.3*pi/T)], y1, '--m')
title('Plot of |{\itH_c}({\itj\Omega})| for \itT = 0.01, \itN = 22')
xlabel('Angular Frequency \Omega', 'FontSize',14)
```



Yes, The specifications are met perfectly as shown on the graph. Using the bilinear transformation, the specifications match with the Butterworth filter designed.