EE 141: Digital Signal Processing

Lab 7: FIR Filter Design Using DFT and Projection Onto Convex Sets

Lab Section: 022

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Objective

Design a discrete-time linear phase FIR filter with the following specifications:

1. $0.98 \leq |H(e^{j\omega})| \leq 1 \text{ for } 0 \leq \omega \leq 0.25\pi$ and $1.75\pi \leq \omega \leq 2\pi$
2. $|H(e^{j\omega})| \leq 0.02$ for $0.5\pi \leq \omega \leq 1.5\pi$

using the iterative design technique learned in class. Specifically the 512-point DFT. This alters our design specifications in the following manner.

1. $0.98 \leq |H(k)| \leq 1 \text{ for } 0 \leq k \leq 64$ and $448 \leq k \leq 511$
2. $|H(k)| \leq 0.02$ for $128 \leq k \leq 384$

Our desire is a type 1 linear phase FIR filter $h[n]$ of some relatively low order N matching these requirements. Therefore, $H[k]$ will always be of the form

$$H[k] = e^{-j\frac{2\pi k}{512} * \frac{N}{2}} A[k] = e^{-j\frac{\pi kN}{512}} A[k]$$

Since $H[k]$ will always be in the above form, It suffices to apply the design requirements to $A[k]$

1. $0.98 \leq A(k) \leq 1 \text{ for } 0 \leq k \leq 64$ and $448 \leq k \leq 511$
2. $A(k) \leq 0.02$ for $128 \leq k \leq 384$

1) Set N = 10

```
% Order of the initial value
clear;
N = 20;
```

2) Initialize with $A[k] = 1$ for all $0 \le k \le 64$ and $448 \le k \le 511$, and $A[k] = 0$ for all other k.

```
% This is the pass band area where we convert omega to k
for k = 1:65
    A(k) = 1;
end

% This is the second pass band
for k = 449:512
    A(k) = 1;
end

% This is the stop band
for k = 66:448
    A(k)= 0;
end
```

3) Computer the 512-point interse DFT of $H[k] = e^{-j\frac{\pi kN}{512}} A[k]$ using the ifft command, and set it to $g[n]$

```
h_o = zeros(1, 512);
x = 0;
h_n = zeros(1, 512);
k = 0:511;
% DFT of h[n]
H_k = exp((-1i*pi*k*N)/512).*A;
% Inverse DFT
g_n = ifft(H_k);
```

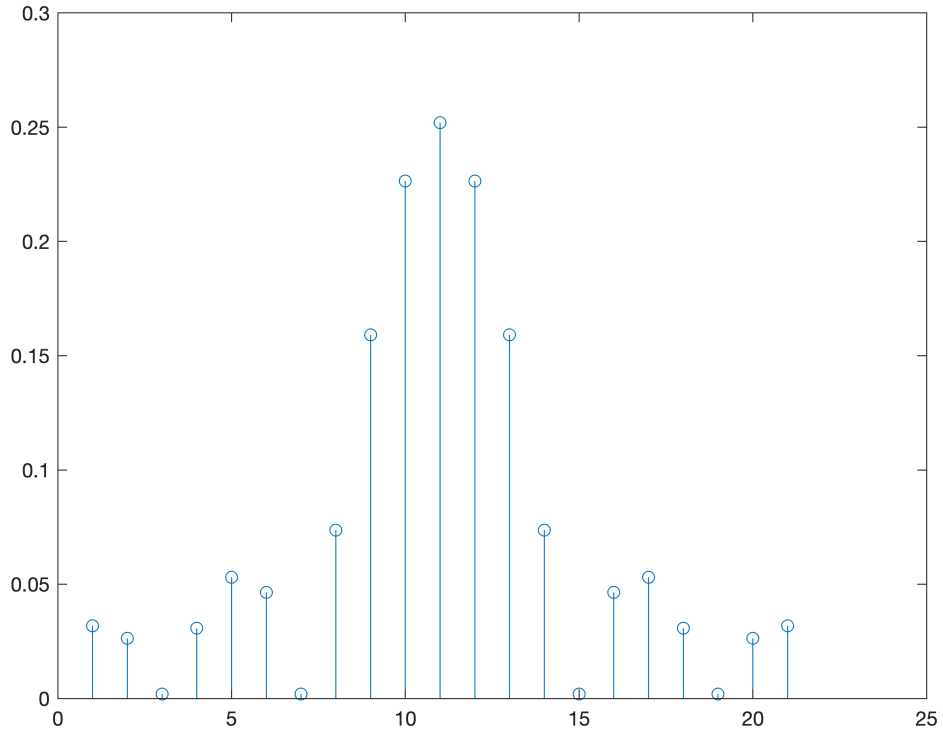
4) Set

$$h[n] = \begin{cases} g[n] & 0 \le n \le N \\ 0 & \text{otherwise} \end{cases}$$

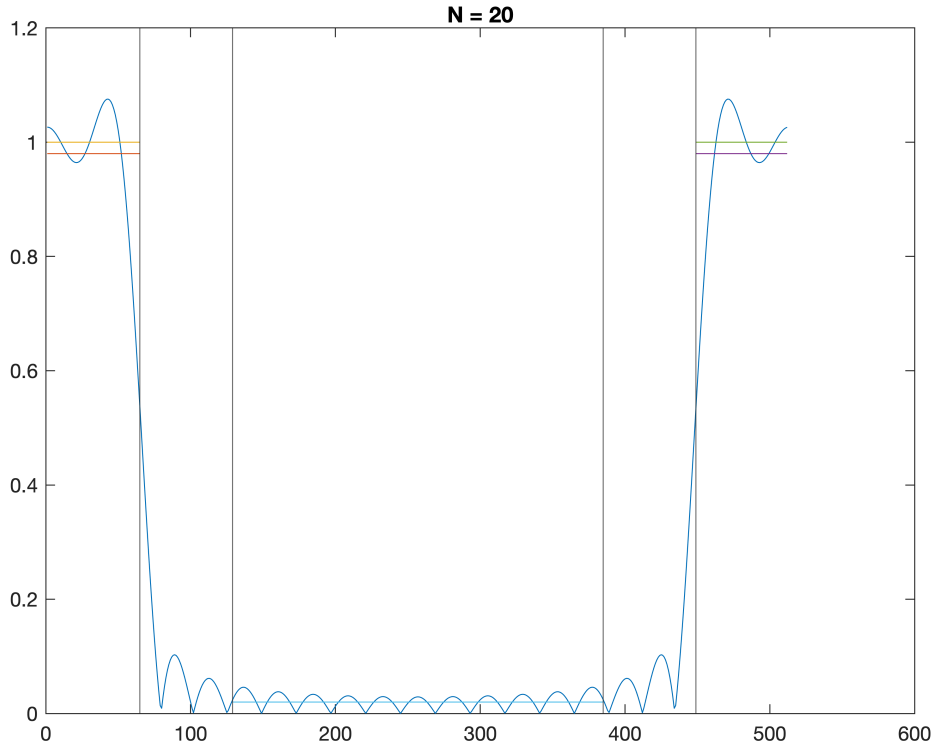
```
h_o = h_n;
h_n(1:N+1) = g_n(1:N+1);
```

5) Set = $H[k] = \text{DFT}\{h[n]\}$ using the fft command

```
% DFT
H_k = fft(h_n);
figure
stem(abs(h_n(1:N+1)))
```




```
figure
plot(abs(H_k))
hold on;
plot([1,65], [0.98,0.98]);
plot([1,65], [1,1]);
plot([449,512], [0.98,0.98]);
plot([449,512], [1,1]);
plot([129,385], [0.02, 0.02]);
xline(65);
xline(129);
xline(449);
xline(385);
title(sprintf('N = %d', N));
```



6) Set $B[k] = H[k]e^{j\frac{\pi kN}{512}}$

```
B_k = H_k.*exp((1i*pi*k*N)/512);
```

7) Clip $B[k]$ wherever it violates the specifications above and set the clipped version to $A[k]$. That is, for $0 \le k \le 64$ or $44 \le k \le 511$.

$$A[k] = \begin{cases} B[k] & 0.98 \le B[k] \le 1 \\ 1 & B[k] > 1 \\ 0.98 & B[K] < 0.98 \end{cases}$$

The internal $128 \le k \le 384$ should be treated similarly. For all remaining K values, $A[k] = B[k]$.

```
A = B_k;
for k = 1:65
    if B_k(k) > 1
        A(k) = 1;
    end
    if B_k(k) < 0.98
        A(k) = 0.98;
    end
end

for k = 449:512
```

```
    if B_k(k) > 1
        A(k) = 1;
    end
    if B_k(k) < 0.98
        A(k) = 0.98;
    end
end

for k = 129:385
    if B_k(k) > 0.02
        A(k) = 0.02;
    end
    if B_k(k) < -0.02
        A(k) = -0.02;
    end
end
```

For some reason my while loop is not working

```
while(1)
    k = 0:511;
    h_k = exp((-1i*pi*k*N)/512).*A;
    g_n = ifft(H_k);

    %iteratio of step 4
    h_o = h_n;
    h_n(1:N+1) = g_n(1:N+1);
    %distance between the original and the new h_n
    delta_h_n = sum((h_o - h_n).^2);
    if delta_h_n < 0.00000000000001
        break
    elseif delta_h_n < 0.001
        N = N+2;
        continue
    end

    % Iteration of step 5
    H_k = fft(h_n);
    x = x+1;
    if x == 21
        x = 0;
    end

    %Plots some of the filter
    if(mod(x,10)==1)
        figure
        stem(abs(h_n(1:N+1)))
        figure
        plot(abs(H_k))
        hold on;
        plot([1,65], [0.98,0.98]);
```

```
        plot([1,65], [1,1]);
        plot([449,512], [0.98,0.98]);
        plot([449,512], [1,1]);
        plot([129,385], [0.02, 0.02]);
        xline(65);
        xline(129);
        xline(449);
        xline(385);
        title(sprintf('N = %d', N));
    end

    % iteration of step 6
    B_k = H_k.*exp((1i*pi*k*N)/512);

    %iteration of step 7
    A = B_k;
    for k = 1:65
        if B_k(K) > 1
            A(k) = 1;
        end
        if B_k(k) < 0.98
            A(k) = 0.98;
        end
    end

    for k = 449:512
        if B_k(k) > 1
            A(K) = 1;
        end
        if B_k(k) < 0.98
            A(k) = 0.98;
        end
    end

    for k = 129:385
        if B_k(k) > 0.02
            A(k) = 0.02;
        end
        if B_k(k) < -0.02
            A(k) = -0.02;
        end
    end
end
```