EE 141 DIGITAL SIGNAL PROCESSING

Lab 5: Discrete-time Processing of Continuous Signals

In this lab, we will process some artificially generated (and simple) continuous-time signals in discrete-time. Specifically, we will implement i) a differentiator, and ii) a delay element.

Question 1 (Differentiator): Let

$$x_c(t) = \cos(10\pi t) + \frac{1}{2}\cos(5\pi t) - \frac{1}{4}\cos(20\pi t) \tag{1}$$

and create $x[n]$ for $0 \leq n \leq 300$ by sampling $x_c(t)$ with a period of $T = 0.01$ secs. Observe that this sampling rate is more than adequate to prevent aliasing.

We have seen in class that we can implement a differentiator using the filter

$$h[n] = \begin{cases} 0 & n = 0 \\ \frac{(-1)^n}{nT} & n \neq 0 \end{cases}$$

in discrete-time. However, in practice, we will need to i) truncate this filter at some point (say beyond $n = \pm N$), and ii) shift it until it becomes causal. So, let us use

$$\hat{h}[n] = \begin{cases} 0 & n = N \\ \frac{(-1)^{n-N}}{(n-N)T} & 0 \leq n \leq 2N \text{ and } n \neq N \\ 0 & \text{otherwise} \end{cases} .$$

instead. The side effect of this shift will be that this filter differentiates $x(t)$ with an N-sample (i.e., NT seconds) delay, i.e.,

$$y_c(t) = \frac{d}{dt} x_c(t - NT) . \tag{2}$$

a) Let $N = 1$. Perform the convolution $\hat{y}[n] = x[n] \star \hat{h}[n]$ using the command `conv`.

b) Analytically calculate $y_c(t)$ using (1) and (2).

c) Compare $\hat{y}[n]$ with $y[n] = y_c(nT)$ by plotting $y[n]$ and $\hat{y}[n]$ on the same graph. Other than the "transient" effects you see on the plot of $\hat{y}[n]$ around the borders, do $y[n]$ and $\hat{y}[n]$ agree?

d) If they did not agree, your approximation $\hat{h}[n]$ must be too crude. Increase N and redo **a** through **c** until they $y[n]$ and $\hat{y}[n]$ more or less agree. (You should not have to increase N beyond 20).

Question 2 (Fractional sample delay): This time, take a simpler signal

$$x_c(t) = \cos(2\pi t)$$

and sample it with $T = 0.1$ secs, again for $0 \leq n \leq 300$.

We want to shift this signal by $\Delta = 0.03$ secs while staying in the discrete-time domain. In class, we have seen that one can accomplish this task by using

$$h[n] = \text{sinc}\left(\pi(n - \frac{\Delta}{T})\right) = \text{sinc}\left(\pi(n - 0.3)\right) .$$

As in Question 1, we have to modify this filter by truncating and shifting, i.e., use

$$\hat{h}[n] = \begin{cases} 0 & n < 0 \text{ or } n > 2N \\ \text{sinc}\left(\pi(n - N - 0.3)\right) & 0 \leq n \leq 2N \end{cases} .$$

Also as in Question 1, due to this modification, we expect to see a side effect of

$$y_c(t) = x_c(t - NT - \Delta)$$

Now, repeat the steps of Question 1, **a** through **d**.