EE132 Automatic Control

Lab 2: Frequency Response

Objective

Design of a proportional and derivative controller; implemented through simulink for a ball and beam system.

Buddy's Pre - Lab for 5 and 6

For $\omega = 0$

```
omega = 0;
% Magnitude
G_s = (0.98)/((1i*omega)^2 + 1.4*(1i*omega) + 0.98);
% First Column
abs(G_s)
```

```
ans = 1
```

```
% Second Column
R_s = (0.1*omega)/((1i*omega)^2 + omega^2);
R_s2 = (0.2*omega)/((1i*omega)^2 + omega^2);
X_s = G_s*R_s;
X_s2 = G_s*R_s2;
rad2deg(angle(X_s) - angle(R_s))
```

```
ans = NaN
```

```
rad2deg(angle(X_s2) - angle(R_s2))
```

```
ans = NaN
```

```
% Angle
G_s_angle = ((1.372)*(1i*omega)^2)/((9.8)*(1i*omega)^2 + 13.72*(1i*omega) + 9.604);
% Third Column
abs(G_s_angle)
```

```
ans = 0
```

```
% Last Column
theta_s = G_s_angle*R_s;
theta_s2 = G_s_angle*R_s2;
rad2deg(angle(theta_s) - angle(R_s))
```

```
ans = NaN
```

```
rad2deg(angle(theta_s2) - angle(R_s2))
```

```
ans = NaN
```

For $\omega = 0.05$

```
omega = 0.05;
% Magnitude
G_s = (0.98)/((1i*omega)^2 + 1.4*(1i*omega) + 0.98);
% First Column
abs(G_s)
```

```
ans = 1.0000
```

```
% Second Column
R_s = (0.1*omega)/((1i*omega)^2 + omega^2);
R_s2 = (0.2*omega)/((1i*omega)^2 + omega^2);
X_s = G_s*R_s;
X_s2 = G_s*R_s2;
rad2deg(angle(X_s) - angle(R_s))
```

```
ans = -45
```

```
rad2deg(angle(X_s2) - angle(R_s2))
```

```
ans = -45
```

```
% Angle
G_s_angle = ((1.372)*(1i*omega)^2)/((9.8)*(1i*omega)^2 + 13.72*(1i*omega) + 9.604);
% Third Column
abs(G_s_angle)
```

```
ans = 3.5714e-04
```

```
% Last Column
theta_s = G_s_angle*R_s;
theta_s2 = G_s_angle*R_s2;
rad2deg(angle(theta_s) - angle(R_s))
```

```
ans = 135
```

```
rad2deg(angle(theta_s2) - angle(R_s2))
```

```
ans = 135
```

For $\omega = 0.5$

```
omega = 0.5;
% Magnitude
G_s = (0.98)/((1i*omega)^2 + 1.4*(1i*omega) + 0.98);
% First Column
```

```
abs(G_s)
```

```
ans = 0.9690
```

```
% Second Column
R_s = (0.1*omega)/((1i*omega)^2 + omega^2);
R_s2 = (0.2*omega)/((1i*omega)^2 + omega^2);
X_s = G_s*R_s;
X_s2 = G_s*R_s2;
rad2deg(angle(X_s) - angle(R_s))
```

```
ans = -45
```

```
rad2deg(angle(X_s2) - angle(R_s2))
```

```
ans = -45
```

```
% Angle
G_s_angle = ((1.372)*(1i*omega)^2)/((9.8)*(1i*omega)^2 + 13.72*(1i*omega) + 9.604);
% Third Column
abs(G_s_angle)
```

```
ans = 0.0346
```

```
% Last Column
theta_s = G_s_angle*R_s;
theta_s2 = G_s_angle*R_s2;
rad2deg(angle(theta_s) - angle(R_s))
```

```
ans = 135
```

```
rad2deg(angle(theta_s2) - angle(R_s2))
```

```
ans = 135
```

For $\omega = 5$

```
omega = 5;
% Magnitude
G_s = (0.98)/((1i*omega)^2 + 1.4*(1i*omega) + 0.98);
% First Column
abs(G_s)
```

```
ans = 0.0392
```

```
% Second Column
R_s = (0.1*omega)/((1i*omega)^2 + omega^2);
R_s2 = (0.2*omega)/((1i*omega)^2 + omega^2);
X_s = G_s*R_s;
X_s2 = G_s*R_s2;
rad2deg(angle(X_s) - angle(R_s))
```

```
ans = -135
```

```
rad2deg(angle(X_s2) - angle(R_s2))
```

```
ans = -135
```

```
% Angle
G_s_angle = ((1.372)*(1i*omega)^2)/((9.8)*(1i*omega)^2 + 13.72*(1i*omega) + 9.604);
% Third Column
abs(G_s_angle)
```

```
ans = 0.1399
```

```
% Last Column
theta_s = G_s_angle*R_s;
theta_s2 = G_s_angle*R_s2;
rad2deg(angle(theta_s) - angle(R_s))
```

```
ans = 45
```

```
rad2deg(angle(theta_s2) - angle(R_s2))
```

```
ans = 45
```

For $\omega = 50$

```
omega = 50;
% Magnitude
G_s = (0.98)/((1i*omega)^2 + 1.4*(1i*omega) + 0.98);
% First Column
abs(G_s)
```

```
ans = 3.9200e-04
```

```
% Second Column
R_s = (0.1*omega)/((1i*omega)^2 + omega^2);
R_s2 = (0.2*omega)/((1i*omega)^2 + omega^2);
X_s = G_s*R_s;
X_s2 = G_s*R_s2;
rad2deg(angle(X_s) - angle(R_s))
```

```
ans = -135
```

```
rad2deg(angle(X_s2) - angle(R_s2))
```

```
ans = -135
```

```
% Angle
G_s_angle = ((1.372)*(1i*omega)^2)/((9.8)*(1i*omega)^2 + 13.72*(1i*omega) + 9.604);
% Third Column
abs(G_s_angle)
```

```
ans = 0.1400
```

```
% Last Column
theta_s = G_s_angle*R_s;
theta_s2 = G_s_angle*R_s2;
rad2deg(angle(theta_s) - angle(R_s))
```

```
ans = 45
```

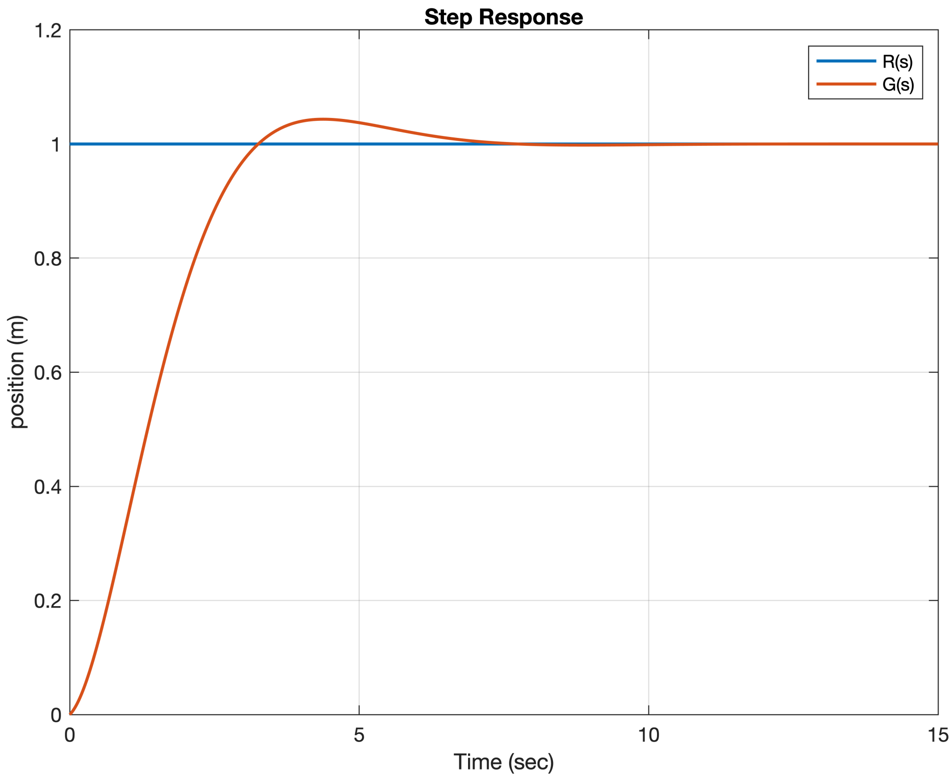
```
rad2deg(angle(theta_s2) - angle(R_s2))
```

```
ans = 45
```

Experimental Procedure

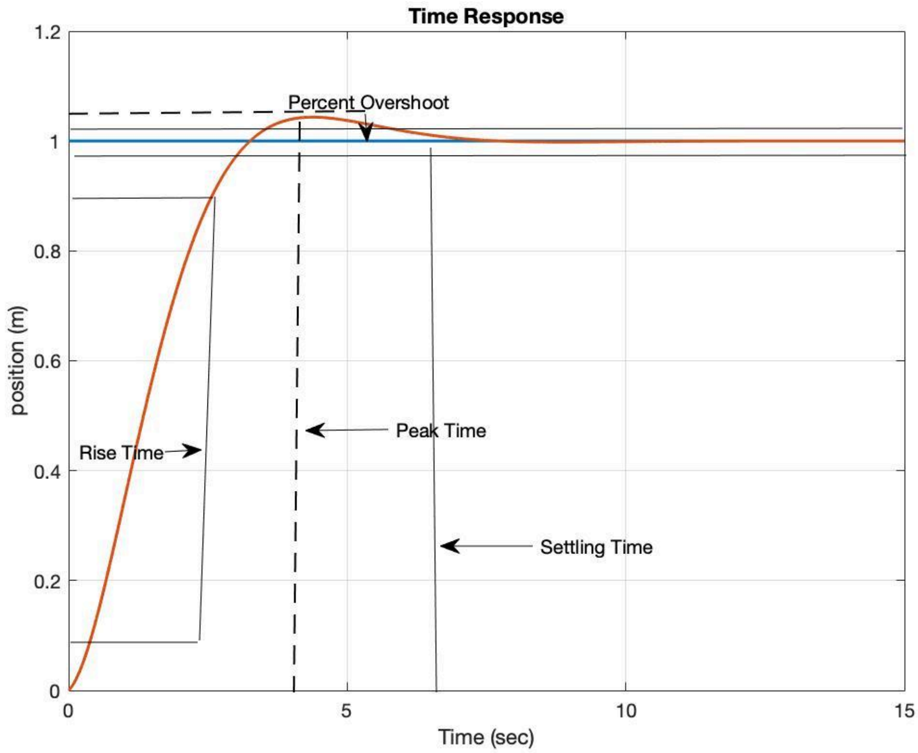
2.

```
close all
t = Data.time;
theta = Data.signals.values(:,1);
x = Data.signals.values(:,2);
r = Data.signals.values(:,3);
figure(1)
plot(t,r,'LineWidth',1.5)
hold on
plot(t,x,'LineWidth',1.5)
grid on
title('Step Response')
xlabel('Time (sec)')
ylabel('position (m)')
legend('R(s)', 'G(s)')
```



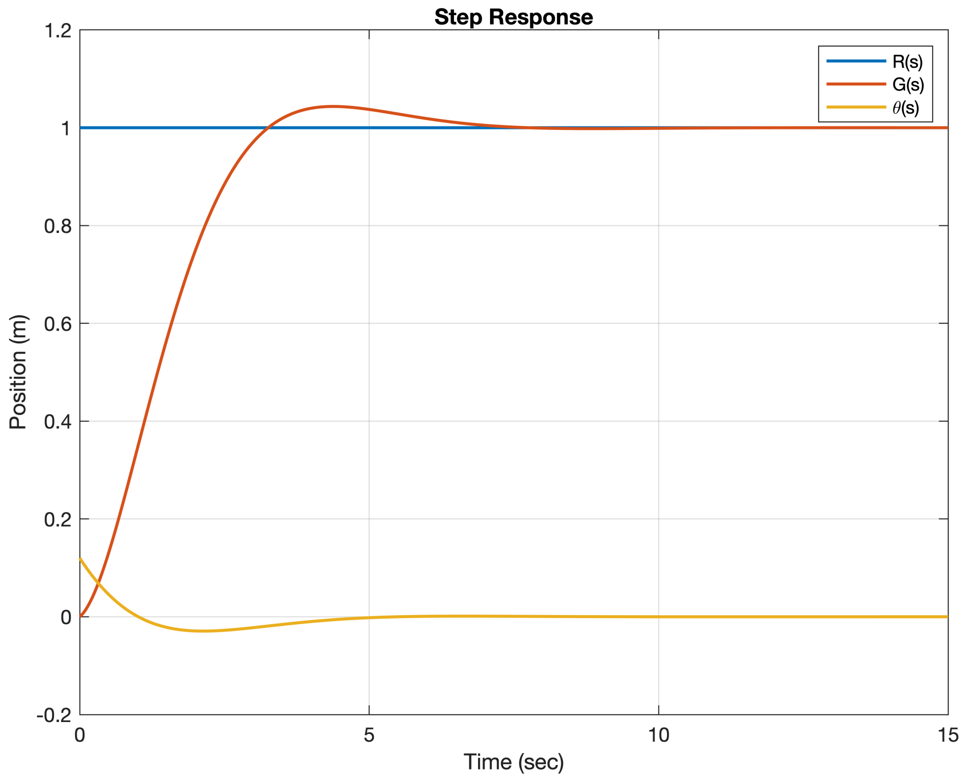

```
S =  stepinfo(x,t,1)
```

```
S = struct with fields:
        RiseTime: 2.1580
    SettlingTime: 5.9196
     SettlingMin: 0.9000
     SettlingMax: 1.0435
       Overshoot: 4.3457
      Undershoot: 0
            Peak: 1.0435
        PeakTime: 4.3782
```



As can be seen in the figures above along with its step information, the results match those from our pre-lab.

```
figure(2)
plot(t,r,'LineWidth', 1.5)
hold on
plot(t,x, 'LineWidth',1.5)
hold on
plot(t, theta, 'linewidth',1.5)
grid on
title('Step Response')
xlabel('Time (sec)')
ylabel('Position (m)')
legend('R(s)', 'G(s)', '\theta(s)')
```

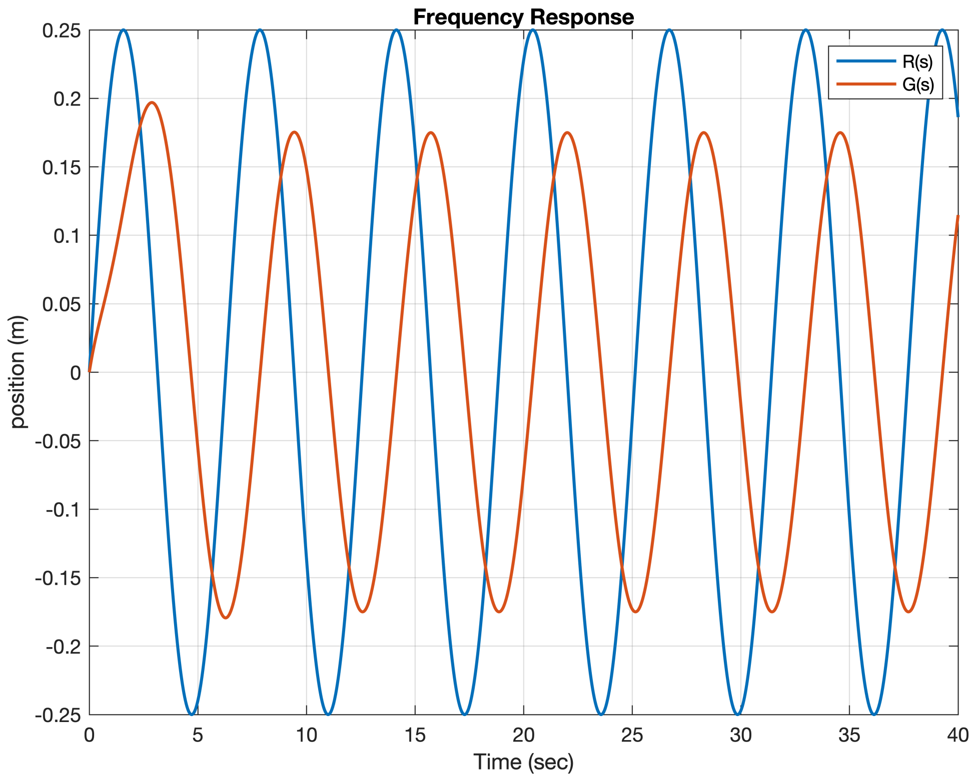


We can assume that as the magnitude of the initial angle of the beam increases , the settling time of the beam will increase. Also, we can see that the ball's rotational inertia becomes stable after the beam settles.

3.

for w = 0.05

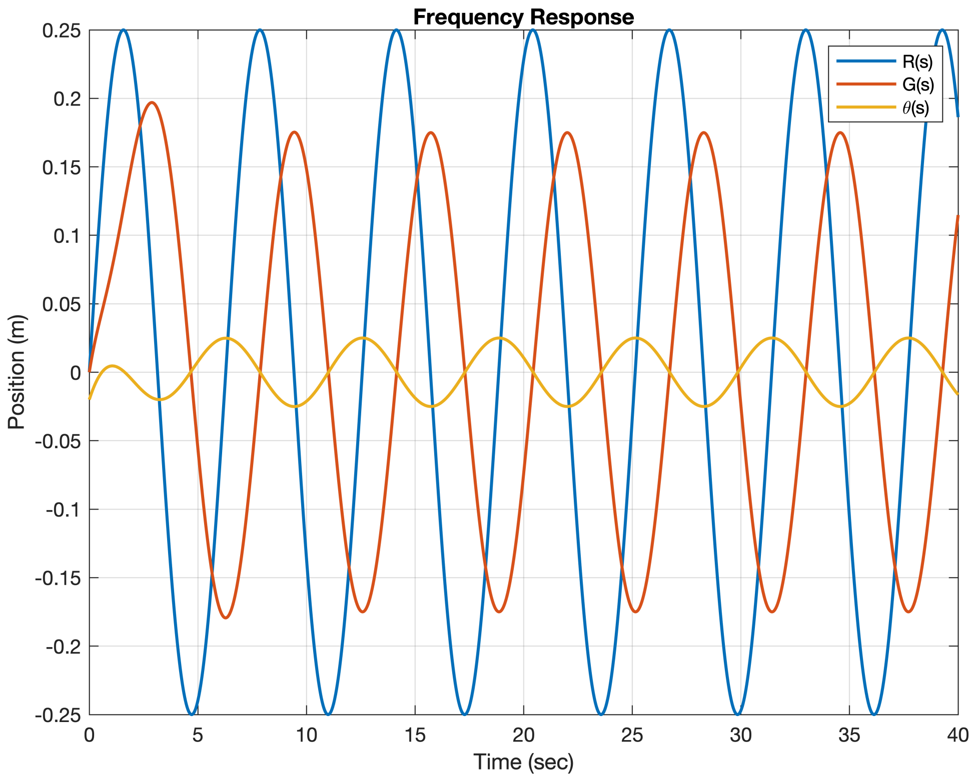
```
close all
t = Data.time;
theta = Data.signals.values(:,1);
x = Data.signals.values(:,2);
r = Data.signals.values(:,3);
figure(1)
plot(t,r,'LineWidth',1.5)
hold on
plot(t,x,'LineWidth',1.5)
grid on
xlabel('Time (sec)')
ylabel('position (m)')
title('Frequency Response')
legend('R(s)', 'G(s)')
```



This system is not stable. the final value as the transfer function approaches infitie does not converge. It oscillates between +- 0.01. Even though this is in stype with the rhythm of the input, it's not good. We can also see a much greater overshoot.

We can see that with a sinusoidal input

```
figure(2)
plot(t,r,'LineWidth', 1.5)
hold on
plot(t,x, 'LineWidth',1.5)
hold on
plot(t, theta, 'linewidth',1.5)
grid on
xlabel('Time (sec)')
ylabel('Position (m)')
title('Frequency Response')
legend('R(s)', 'G(s)', '\theta(s)')
```



As we can see in this figure, the system becomes unstable when a sinusodial input is provided. The oscillation of the beam angle is much greateer than the oscilation of the position of the ball.

Questions:

1. You can use the transfer function (or frequency response) to quickly and easily compute the steady state responses of $\theta(t)$ and $x(t)$ when $r(t) = A\sin(\omega t + \phi)$ with a stable system and a sinusodial input our frequency response results in another sinusoid with a magnitude and phase determined by our transfer function . Therefore, when we set s = jw and take the absolute value of the frequency response we get the magnitude. If we set s = 0 we get the dc gain. To get the phase we take the angular difference when s = jw between the beam angle and the position of the ball for the system and the reference input
2. When ω is the very large, the magnitude of $\theta(t)$ increases
3. When ω is the very large, the magnitude of $x(t)$ decreases.
4. As a controller designer, the aspects of the system that I can change to increase this range of frequencies (bandwidth), then I would know that bandwidth is inversely proportional to the closed loop settling time. Therefore, I would ensure a highly underdamped damping factor to ensure a small overshoot which will shorten the settling time and increase bandwidth. Thus, for this specific controller we should increase D and decrease P.
5. The practical aspects of the system that will limit the range of frequencies that can be accurately tracked are the pole positions which correlate to the inverse sine of the damping factor. These in turn are affected by the mass of the ball and the position of the beam, etc.

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Prelab 2

1)

$$G(s) = \frac{X(s)}{R(s)}$$

$$\ddot{x} = \frac{g}{1.4}\theta = \frac{g}{1.4}\left(P(r-x) - D\dot{x}\right)$$

$$s^2 X = \frac{g}{1.4}\left(P(r-x) - DsX\right)$$

Zero initial condition:
$x(0) = 0$
$\dot{x}(0) = 0$

$$\boxed{G(s) = \frac{\frac{g}{1.4}P}{s^2 + \frac{gD}{1.4}s + \frac{g}{1.4}P}}$$

2) $s_1 = -0.7 + j0.7$, $s_2 = -0.7 - j0.7$

$s^2 + \frac{gD}{1.4}s + \frac{g}{1.4}P = 0$, s_1, s_2 are solutions

$as^2 + bs + c = 0$ $\qquad a = 1$, $-b = \frac{gD}{1.4}$

$$\begin{cases} s_1 + s_2 = \frac{-b}{a} \\ s_1 s_2 = \frac{c}{a} \end{cases}$$

$-0.7 + j0.7 + -0.7 - j0.7 = \frac{-b}{a}$

$-1.4 = -\frac{gD}{1.4}$

$\boxed{D = -0.2}$

$(-0.7 + j0.7)(-0.7 - j0.7) = \frac{gP}{1.4}$

$0.49 + 0.49 = \frac{gP}{1.4}$

$0.98 = \frac{gP}{1.4}$

$\boxed{P = 0.14}$

Chris

3)

$$G(s) = \frac{\omega_n^2}{s^2 + 2\delta\omega_n s + \omega_n^2} = \frac{\frac{9}{1.4}P}{s^2 + \frac{9D}{1.4}s + \frac{9}{1.4}P}$$

$$\begin{cases} \omega_n^2 = \frac{9}{1.4}P \\ \frac{9D}{1.4} = 2\delta\omega_n \end{cases} \Rightarrow \begin{cases} \omega_n = \sqrt{\frac{9.8}{1.4}0.14} = 0.990 \\ \delta = \frac{9.8(0.2)}{2.8\,\omega_n} = 0.707 \end{cases}$$

$$T_r = \frac{1.8}{\omega_n} = \frac{1.8}{0.990}$$

$$\boxed{T_r = 1.818 \text{ sec}}$$

$$T_p = \frac{\pi}{\omega_n\sqrt{1-\delta^2}} = \frac{\pi}{0.990\sqrt{1-(0.707)^2}} = \frac{\pi}{0.990\sqrt{1-\frac{1}{2}}}$$

$$\boxed{T_p = 4.488 \text{ sec}}$$

$$M_p = e^{-\delta\pi/\sqrt{1-\delta^2}} = e^{-0.707\pi/\sqrt{1-(0.707)^2}}$$

$$T_s = \frac{4.6}{\delta\omega_n} = \frac{4.6}{0.7}$$

$$\boxed{T_s = 6.571 \text{ sec}}$$

$$\boxed{M_p = 0.0432}$$

Chris

4) $G(s) = \dfrac{\theta(s)}{R(s)} = \dfrac{\frac{1.4}{\theta} s^2 X(s)}{R(s)} = \frac{1.4}{9} s^2 \cdot G(s)$

$\ddot{X} = \frac{9}{1.4}\theta \rightarrow s^2 x(s) = \frac{9}{1.4}\theta(s)$ $\boxed{\theta(s) = \frac{1.4}{9} s^2 X(s)}$

There are zeros but the locations are both at 0 which makes the transfer function very unstable.

5) $r(t) = 0.1\sin(\omega t)$

ω, rad/s	$\lvert X(j\omega)\rvert / \lvert R(j\omega)\rvert$	$\angle(X(j\omega) - R(j\omega))$, deg	$\lvert\theta(j\omega)\rvert / \lvert R(j\omega)\rvert$	$\angle(\theta(j\omega) - R(j\omega))$, deg
0	1	NaN	0	NaN
0.05	1	$-45°$	3.57×10^{-4}	$135°$
0.5	0.9690	$-45°$	0.0428	$135°$
5	0.0392	$-135°$	0.1399	$45°$
50	3.92×10^{-4}	$-135°$	0.14	$45°$

6) $r(t) = 0.2\sin(\omega t)$

ω, rad/s	$\lvert X(j\omega)\rvert / \lvert R(j\omega)\rvert$	$\angle(X(j\omega) - R(j\omega))$, deg	$\lvert\theta(j\omega)\rvert / \lvert R(j\omega)\rvert$	$\angle(\theta(j\omega) - R(j\omega))$, deg
0	1	NaN	0	NaN
0.05	1	$-45°$	3.57×10^{-4}	$135°$
0.5	0.9690	$-45°$	0.0428	$135°$
5	0.0392	$-135°$	0.1399	$45°$
50	3.92×10^{-4}	$-135°$	0.14	$45°$

Nick

1

Nicolas Andrade
#861280099

EE132 Lab 2 (Pre-lab)

1) $G(s) = \frac{X(s)}{R(s)}$ $\ddot{x} = \frac{g}{1.4}\theta = \frac{g}{1.4}\left[P(r-x) - D\dot{x}\right]$

$s^2 X = \frac{g}{1.4}\left[P(R-X) - DsX\right]$

$G(s) = \frac{(g/1.4)P}{s^2 + \frac{gD}{1.4}s + \frac{g}{1.4}P} = \frac{.98}{s^2 + 1.4s + .98}$

2) $S_1 = -0.7 + j0.7$ $S_2 = -0.7 - j0.7$

$s^2 + \frac{gD}{1.4}s + \frac{g}{1.4}P = 0$ $\therefore S_1$ and S_2 are solutions

$as^2 + bs + c = 0$

$\begin{cases} S_1 + S_2 = -b/a \\ S_1 \cdot S_2 = c/a \end{cases} \Rightarrow$

$-0.7 + j0.7 + -0.7 - j0.7 = -\frac{gD}{1.4}/1$

$-1.4 = \frac{-gD}{1.4}$

$D = +(1.4)^2/(9.8)$

$\boxed{D = 0.2}$

	-0.7	$j0.7$
-0.7	$.49$	$-j.49$
$-j0.7$	$j.49$	$.49$

$\therefore .98 = \frac{g}{1.4}P$ $\therefore P = \frac{.98(1.4)}{g} = .14$

$\boxed{\therefore P = .14}$

3) $G(s) = \frac{W_n^2}{s^2 + 2\zeta W_n s + W_n^2} = \frac{(g/1.4)P}{s^2 + \frac{gD}{1.4}s + \frac{g}{1.4}P}$

$W_n^2 = \frac{g}{1.4}P$ $\boxed{\therefore W_n = .98}$ $\downarrow W_n = \sqrt{\downarrow \#}$

$\frac{gD}{1.4} = 2\zeta W_n$ $\boxed{\therefore \zeta = \frac{gD}{1.4(2W_n)} = .7142}$

$\therefore T_r = 1.8/\omega_n = 1.8/.98 = 1.8367$

$\therefore T_s = 4.6/\zeta W_n = 4.6/((.7142)(.98)) = 6.571$

$\therefore T_p = \pi/\left[(W_n\sqrt{1-\zeta^2})\right] = \pi/\left[.98\sqrt{(1-(.7142)^2}\right] = 4.5805$

$\therefore M_p = e^{-\zeta\pi/\sqrt{1-\zeta^2}} = e^{-\pi(.7142)/\sqrt{1-\zeta^2}} = .04050$

Nick

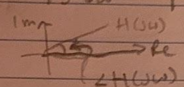
4) $G_1(s) = \dfrac{\theta(s)}{R(s)} = \dfrac{\frac{1.4}{g} s^2 X(s)}{R(s)}$

$\ddot{x} = \frac{g}{1.4}\theta \rightarrow s^2 X(s) = \frac{g}{1.4}\theta(s) \quad \therefore \Theta(s) = \frac{1.4}{g} s^2 X(s)$

Yes, located at zero, but the system is unstable

5) $H(s) \Rightarrow H(jw) = H(s)\big|_{s=jw} = |H(jw)| * e^{j\angle H(jw)}$

$H(jw) = a + jb$



$u_0 \sin(wt) \longrightarrow \boxed{H(s)} \longrightarrow u_0 |H(jw)| * \sin[wt \oplus \angle H(jw)]$

w, rad/s	$\lvert X(jw)\rvert / \lvert R(jw)\rvert$	$\angle(X(jw) - R(jw))$	$\lvert\theta(jw)\rvert / \lvert R(jw)\rvert$	$\angle(\theta(jw) - R(jw))$
0	1	NaN	0	NaN
0.05	1	$-45°$	3.57×10^{-4}	$135°$
0.5	0.9690	$-45°$	0.0428	$135°$
5	0.0392	$-135°$	0.1399	$45°$
50	3.92×10^{-4}	$-135°$	0.14	$45°$

6)

w, rad/s	$\lvert X(jw)\rvert / \lvert R(jw)\rvert$	$\angle(X(jw) - R(jw))$	$\lvert\theta(jw)\rvert / \lvert R(jw)\rvert$	$\angle\theta(jw) - R(jw)$
0	1	NaN	0	NaN
0.05	1	$-45°$	3.57×10^{-4}	$135°$
0.5	0.9690	$-45°$	0.0428	$135°$
5	0.0392	$-135°$	0.1399	$45°$
50	3.92×10^{-4}	$-135°$	0.14	$45°$

4 Prelab (Complete this section prior to lab.)

Bud

1. Find the transfer function from the reference input r to the output x. Call this transfer function $\frac{X(s)}{R(s)} = G(s)$, which should be a second order system with no finite zeros.

$\theta(t) = P(r(t) - x(t)) - D(\dot{x}(t))$

$$\theta(t) = P[r(t)] - P[x(t)] - D[\dot{x}(t)]$$
$$\theta(s) = PR(s) - PX(s) - DsX(s)$$
$$\Theta(s) = PR(s) - (P + Ds)X(s)$$
$$\Theta(s) - PR(s) = -(P + Ds)X(s)$$
$$X(s) = \frac{\Theta(s) - PR(s)}{-(P + Ds)}$$

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① In this lab we will design a proportional and derivative controller. Also known as PD

② Implement the PD controller in simulink for a ball and beam system

③ Analyze the frequency response of the system

Background

Ball and Beam system

- The model for this system can be expressed as

$$\ddot{x} = \frac{M}{\left(m + \frac{J}{L^2}\right)} g\theta \qquad J = 0.4mL^2$$

$$= \frac{g}{1.4}\theta$$ → where θ is the beam angle — controller input

x – position of the ball
- output state of the system

g – gravity due to acceleration
- constant

$\theta(t)$ such that output, $x(t)$ will track a desired value $x_d(t)$ Bud

PD controller

$$\theta(t) = \overbrace{P(r-x)}^{\text{porportional part}} - \underbrace{D\dot{x}}_{\text{derivative part}}$$

where P & D are the design parameters

r = refference signal; the desired position for the ball and beam system; position of ball

---

In this lab we will chose P and D such that the closed loop system is stable is stable

$x \to r$ (x - tracks r) as $t \to \infty$ and transvent response has special shape

4 Prelab (Complete this section prior to lab.)

1. Find the transfer function from the reference input r to the output x. Call this transfer function $\frac{X(s)}{R(s)} = G(s)$, which should be a second order system with no finite zeros.

$$\ddot{x} = \frac{g}{1.4}\theta = \frac{g}{1.4}(P(r-x) - D\dot{x}]$$

$$\ddot{x} = \frac{g}{1.4}P(r-x) - \frac{g}{1.4}D\dot{x}$$

$$s^2X(s) - sX(0) - \dot{x}(0) = \frac{g}{1.4}P(R-X)\frac{1}{s} - \frac{g}{1.4}DsX(s) - x(0)$$

In this we assume zero initial conditions

$\dot{x}(0) = 0$

$x(0) = 0$

$$s^2X(s) = \frac{gP(r-x)}{1.4s} - \frac{gDsX(s)}{1.4} \Rightarrow$$

$$s^2X(s) + \frac{gDsX(s)}{1.4} = \frac{gP(r-x)}{1.4s}$$

* ask TA how we get that

* TA's work

$$s^2X = \frac{g}{1.4}[P(R-X) - Ds X]$$

$$G(s) = \frac{\frac{g}{1.4}P}{s^2 + \frac{gD}{1.4}s + \frac{g}{1.4}P}$$

Bud

$$X(s)\left(s^2 + \frac{gDs}{1.4}\right) = \frac{gP(r-x)}{1.4s}$$

$$X(s)\left(\frac{1.4s^2 + gDs}{1.4}\right) = \frac{gP(r-x)}{1.4s}$$

$$X(s) = \frac{gP(r-x)}{s(1.4s^2 + gDs)}$$

$s_1 = -0.7 + j0.7$

$s_2 = -0.7 - j0.7$

this was retrieved by setting the denominator to 0 ; finding poles

2. Assume that $g = 9.8m/s^2$. Select the controller parameters P and D so that the closed loop system has poles located at $s = -0.7 \pm 0.7j$.

$$s^2 + \frac{gD}{1.4}s + \frac{g}{1.4}P$$

s_1 and s_2 are the solutions

takes this form

$$as^2 + bs + c = 0$$

if s_1 and s_2 are the solutions then it holds that

$$\begin{cases} s_1 + s_2 = -\dfrac{b}{a} \\ s_1 * s_2 = \dfrac{c}{a} \end{cases} \Rightarrow P \,\&\, D$$

$bs = \frac{gD}{1.4}s$

$b = \frac{gD}{1.4}$

$D = \frac{b(1.4)}{g}$

$c = \frac{g}{1.4}P$

$P = \frac{c(1.4)}{g}$

Via the above relationship, we can determine the values for the parameters P and D

$s_1 + s_2 = -\frac{b}{a}$

$-b = (s_1 + s_2)a$

$b = -(s_1 + s_2)a$

$c = (s_1 s_2)a$

$$2\zeta\omega_n s = \frac{gD}{1.4}s$$

Buel

$$D = \frac{[(s_1+s_2)a](1.4)}{g}$$

$$\omega_1^2 = \frac{g}{1.4}P$$

$$\omega_n = \sqrt{\frac{gP}{1.4}}$$

$$= -\frac{(-0.7 + \cancel{0.7j} + -0.7 - \cancel{0.7j})(1.4)}{9.8\,m/s^2}$$

$$= -\frac{(-1.4)(1.4)}{9.8\,m/s^2}$$

$$\boxed{D = 0.2}$$

$$P = \frac{[(s_1 s_2)a]1.4}{g} = \frac{(-0.7+0.7j)(-0.7-0.7j)(1)(1.4)}{9.8\,m/s^2}$$

$$= 0.14$$

$$\boxed{P = 0.14}$$

$$\cancel{= \frac{[(-0.7+0.7j) - (-0.7-0.7j)]1.4}{9.8\,m/s^2}}$$

$$\cancel{= \frac{(1.4j)(1.4)}{9.8\,m/s^2} = 0.2}$$

* my value for P is wrong

3. For these pole locations, if the input were a unit step, predict the rise time T_r ($T_r = 1.8/\omega_n$), settling time T_s ($T_s = 4.6/\zeta\omega_n$), peak time T_p ($T_p = \pi/(\omega_n\sqrt{1-\zeta^2})$), and overshoot M_p ($M_p = e^{-\zeta\pi/\sqrt{1-\zeta^2}}$).

$$T_r = \frac{1.8}{\omega_n} = 1.8 \Big/ \sqrt{\frac{g}{1.4}P} \Rightarrow$$

$$\boxed{T_r = 1.8s}$$

$$\omega_n^2 = \frac{gP}{1.4}$$

$$\omega_n = \sqrt{\frac{g}{1.4}P}$$

$$= 0.989949$$

$$T_s = \frac{4.6}{\zeta\omega_n} = \frac{4.6}{\frac{gD}{2(1.4)}}$$

$$2\zeta\omega_n = \frac{gD}{1.4}s$$

$$\zeta\omega_n = \frac{gD}{(1.4)2}$$

$$= \frac{(4.6)(2.8)}{gD}$$

$$\boxed{T_s = 6.57143s}$$

$$\zeta = \frac{gD}{2(1.4)\omega_n} = \frac{gD}{2(1.4)\sqrt{\frac{gP}{1.4}}} = 0.707107$$

Bud

$$t_p = \frac{\pi}{\sqrt{\frac{gP}{1.4}}\left(\sqrt{1-\left(\frac{gD}{2(1.4)\sqrt{\frac{gP}{1.4}}}\right)^2}\right)}$$

$$= 4.49007$$

$$M_p = e^{-\frac{0.707107\pi}{\sqrt{1-(0.707107)^2}}} \times 100$$

$$= 4.32139$$

4. Find the transfer function G_1 from the reference input r to the control input θ. Are there any zeros in this closed loop transfer function? If so, where are they located?

$$G(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

$$\omega_n^2 = 0.98$$

$$2\zeta\omega_n s = 1.4$$

$$\boxed{G(s) = \frac{0.98}{s^2 + 1.4s + 0.98}}$$

$$G(s) = \frac{\theta(s)}{R(s)} = \frac{\frac{1.4}{g}s^2 X(s)}{R(s)} = \frac{1.4}{g}s^2 \cdot G(s)$$

$$\ddot{x} = \frac{g}{1.4}\theta \rightarrow s X(s) = \frac{g}{1.4}\theta(s) \qquad \theta(s) = \frac{1.4}{g}s^2 X(s)$$

5. Assume that the reference input is $r(t) = 0.1\sin(\omega t)$. For $\omega = 0, 0.05, 0.50, 5.00, 50$ rad/s, assume that all transients have decayed away, determine the magnitude and phase shift of $\theta(t)$ and $x(t)$ relative to $r(t)$. Use the concept of frequency response. Do not use partial fractions. List the magnitude and phase shift of $\theta(t)$ and $x(t)$ for different ω in a table as shown in Table below.

ω, rad/s	$\lvert X(j\omega)\rvert/\lvert R(j\omega)\rvert$	$\angle(X(j\omega) - R(j\omega))$, deg	$\lvert\Theta(j\omega)\rvert/\lvert R(j\omega)\rvert$	$\angle(\Theta(j\omega) - R(j\omega))$, deg
0				
0.05				
0.5				
5				
50				

$$G(j\omega) = G(s)\Big|_{s=j\omega}$$

$$\sin(\omega t)\cdot 1(t) = \frac{b}{s^2+b^2}$$

$$= \frac{\omega}{s^2+\omega^2}$$

$$r(t) = 0.1\sin(\omega t)$$

$$\mathcal{L}\{r(t)\} = \mathcal{L}\{0.1\sin(\omega t)\}$$

$$R(s) = 0.1\frac{\omega}{s^2+\omega^2} = \frac{0.1\omega}{s^2+\omega^2}$$

$s^2+\omega$ $\quad$ $s^2+\omega$ $\quad$ Bud

$$G(s) = \frac{1.4}{g}s^2 \cdot G(s)$$
$$= \frac{1.4s^2}{g} \cdot \frac{0.98}{s^2+1.4s+0.98} = \boxed{\frac{1.372s^2}{9.8s^2+13.72s+1.372}}$$

6. Assume that the reference input is $r(t) = 0.2\sin(\omega t)$, For $\omega = 0, 0.05, 0.50, 5.00, 50$ rad/s, assume that all transients have decayed away, determine the magnitude and phase shift of $\theta(t)$ and $x(t)$ relative to $r(t)$. Use the concept of frequency response. Do not use partial fractions. List the magnitude and phase shift of $\theta(t)$ and $x(t)$ for different ω in a table as you did for question IV.5).

$$s^2X(s) = \frac{g}{1.4}P(R-X) - \frac{gDsX}{1.4}$$
$$s^2X(s) + \frac{gDsX}{1.4} = \frac{g}{1.4}P(R-X)$$
$$X(s)\left[s^2+\frac{gDs}{1.4}\right] = \frac{gP(R-X)}{1.4}$$
$$X(s) = \frac{\frac{gP(R-X)}{1.4}}{s^2+\frac{gDs}{1.4}}$$

$$\theta(t) = Pr(t) - Px(t) - PD(\dot{x}(t))$$
$$\theta(s) = PR(s) - PX(s) - PDsX(s) - X(0)$$
$$\theta(s) = PR(s) - PX(s) - PDsX(s)$$

τ_r = the time it takes between 10% and 90% of the final value

Bud

$$r(t) = 0.1\sin(\omega t)$$

$$R(s) = 0.1\frac{\omega}{s^2+\omega^2}$$

$$\theta(s) = R(s)$$

$$\frac{1.4s^2}{g}X(s) = R(s)$$

$$\frac{1.4s^2}{g}X(s) = 0.1\frac{\omega}{s^2+\omega^2}$$

$$\frac{G(s)}{R(s)}$$

$$X(s) =$$

$$G(s) = \frac{\theta(s)}{R(s)} \qquad \frac{\frac{1.4}{g}s^2 X(s)}{R(s)} = \frac{\frac{1.4s^2}{g}\cancel{\frac{G(s)}{R(s)}}}{\cancel{R(s)}} \quad \cancel{G(s)R(s)}$$

$$= \frac{1.4s^2 G(s)}{(R(s))^2}$$

$$G(s) = \frac{X(s)}{R(s)}$$

$$X(s) = G(s)R(s)$$

$$= \frac{1.4s^2}{g}G(s)$$

$$\boxed{G(s) = \frac{0.98}{s^2+1.4s+0.98}}$$

$$\frac{(0.98)1.4s^2}{g(s^2+1.4s+0.98)}$$