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Prelab 1

1) A) $V(t) = F_1 x_c(t)$

$$\lim_{t \to \infty} x(t) - x_c(t) = 0$$

$$x_e = x - x_c$$

$$\dot{x}_e = \dot{x} - \dot{x}_c = \dot{x}$$

$$= -0.02x + 0.1 F_1 x_c$$

$$\dot{x}_e = -k x_e, \quad k > 0 \quad \Rightarrow x_e \to 0 \text{ as } t \to \infty$$

let $-0.02x + 0.1F_1 x_c = -k x_e$

$$= -k(x - x_c)$$

$$-k = -0.02$$

$$\Rightarrow -k = 0.1F_1$$

$$-0.02 = 0.1F_1$$

$$\boxed{F_1 = 0.2}$$

B) $y = x$

$$sX - x(0) = -0.02X + 0.1F_1 X_c + \frac{d}{s}$$

$$sX + 0.02X = 0.1F_1 X_c$$

$$X(s + 0.02) = 0.1F_1 X_c$$

$$\frac{X}{X_c} = \frac{0.1F_1}{(s + 0.02)}$$

$$\boxed{s = -0.02 \text{ pole}}$$

No, the pole location affected by the choice of the F_1 paramter

c) T_s = settling time = 3τ

τ = time constant

$$\dot{x} + \frac{1}{\tau}x = Au(t) \quad \text{and} \quad \dot{x} = -0.02X + 0.1F_1X_c$$

$$\frac{1}{\tau} = 0.02$$

$$\tau = \frac{1}{0.02} = 50$$

$$T_s = 3\tau = 3(50)$$

$$\boxed{T_s = 150 \text{ sec}}$$

D) $$\dot{X}_e = (-0.02 + a)X_e + (-0.02 + a + (0.1 + b)F_1]X_c + d$$

taking the Laplace transform

$$sX_e - X_e(0) = (-0.02 + a)X_e + [-0.02 + a + (0.1 + b)F_1]\frac{1}{s} + \frac{d}{s}$$

$$X_e = \frac{(-0.02 + a + (0.1 + b)F_1)}{s(s + 0.02 - a)} + \frac{d}{s(s + 0.02 - a)} + \frac{X_e(0)}{(s + 0.02 - a)}$$

$$\frac{A}{s} + \frac{B}{s + 0.02 - a} \qquad A(s + 0.02 - a) + B(s) = -0.02 + a + (0.1 + b)(0.2)$$

$$= -0.02 + a + 0.02 + 0.2b$$

$$A(s + 0.02 - a) + B(s) = a + 0.2b$$

$$s = 0 \mid A(0.02 - a) = a + 0.2b$$

$$A = \frac{a + 0.2b}{0.02 - a}$$

$$s = -0.02 + a \mid B(-0.02 + a) \qquad B = \frac{a + 0.2b}{-0.02 + a}$$

$$\frac{C}{s} + \frac{D}{s+0.02-a} \qquad C(s+0.02-a) + D(s) = d$$

$$s=0 \;\Big|\; C(0.02-a) = d$$

$$C = \frac{d}{0.02-a}$$

$$s=-0.02+a \;\Big|\; D(-0.02+a) = d$$

$$D = \frac{d}{-0.02+a}$$

$$X_e = A + Be^{(-0.02+a)t} + C + De^{(-0.02+a)t} + X_e(0)e^{(-0.02+a)t}$$

$$-0.02+a < 0$$

$$e^{(-0.02+a)t} \to 0 \quad \text{as} \quad t \to \infty$$

$$X_e(t) = A + C$$

$$\boxed{X_e(t) = \frac{a+0.2b+d}{(0.02-a)}}$$

2)A) $v(t) = P(X_c - X) + I\int_0^t (X_c - X)\,d\tau$

$\dot{x} = -0.02x + 0.1P(X_c - X) + 0.1I\int_0^t (x_c - x)\,d\tau$

$sX = -0.02x + 0.1\left(P + \frac{1}{s}I\right)(X_c - X)$

$s^2X + 0.02sX = 0.1PsX_c - 0.1PsX + 0.1IX_c - 0.1Ix$

$$\frac{X}{X_c} = \frac{0.1(PS + I)}{s^2 + (0.02 + 0.1P)s + 0.1I}$$

B) $s_1 = -0.1 + j0.05$ $\quad as^2 + bs + c = 0$

$s_2 = -0.1 - j0.05$

$s_1 + s_2 = \frac{-b}{a}$

$-0.1 + j0.05 + 0.1 - j0.05$ $\quad s_1 s_2 = \frac{c}{a}$

$= -(0.02 + 0.1P)$

$P = \frac{-0.2 + 0.02}{-0.1}$

$(-0.1 + j0.05)(-0.1 - j0.05)$

$\boxed{P = 1.8}$

$= 0.01 + 0.0025 = 0.0125$

$0.0125 = 0.1I$

$\boxed{I = 0.125}$

c) $T_s = 3 \times \dfrac{1}{|\sigma|}$ $\quad \sigma = -0.1$

$= 3 \times \dfrac{1}{|-0.1|}$

$\boxed{T_s = 30 \text{ sec}}$

D)

$\dot{X}_e = \dot{X} - \dot{X}_c$

$= \dot{X}$

$= (-0.02 + a) X_e + (0.1 + b)\left(-X_e P - I \int_0^t X_e \, d\tau\right) + d + (-0.02 + a) X_c$

Using Laplace transform

$s X_e - (-0.02 + a) X_e + (0.1 + b) P X_e + (0.1 + b) I \frac{1}{s} X_e = \frac{d}{s} + \frac{(-0.02 + a)}{s} - X_e(0)$

$s^2 X_e - (-0.02 + a) X_e s + (0.1 + b) P X_e s + (0.1 + b) I X_e = d + (-0.02 + a) - s X_e(0)$

$s^2 - (-0.02 + a) s + (0.1 + b) P s + (0.1 + b) I = 0$

$X_e = \dfrac{A}{s - s_1} + \dfrac{B}{s - s_2}$ $\quad$ if s_1, s_2 are $\mathbb{R}$

$X_e = A e^{s_1 t} + B e^{s_2 t}$

$s_1 = -0.1 + j\,0.05$

$s_2 = -0.1 - j\,0.05$

$e^{j\theta} = \cos\theta + j \sin\theta$

$\boxed{X_e = e^{-0.1t} \cdot 2\cos(0.05\theta)}$

$|\cos(0.05\theta)| \le 1$

$X_e \to 0$ cost $\to \infty$

as long as $\sigma < 0$