1) units of $K = \frac{T_m(t)}{i_m t} = \frac{Nm}{A}$

$\frac{Kgm^2}{s^2 A}$

2) $e_m(t) = K\omega_m(t)$

$i_m(t) = \frac{T_m(t)}{K}$

$P_m(t) = e_m(t)\, i_m(t)$

$= (\cancel{K}\omega_m(t))\left(\frac{T_m(t)}{\cancel{K}}\right)$

$P_m(t) = \omega_m(t)\, T_m(t)$

$P_{in} = P_{out}$

3)

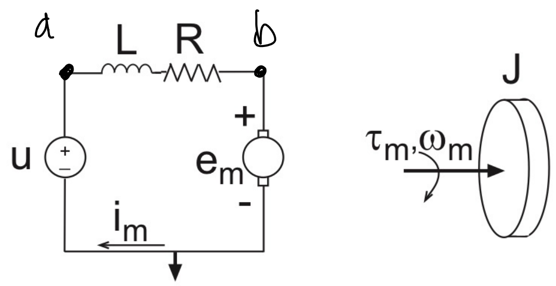
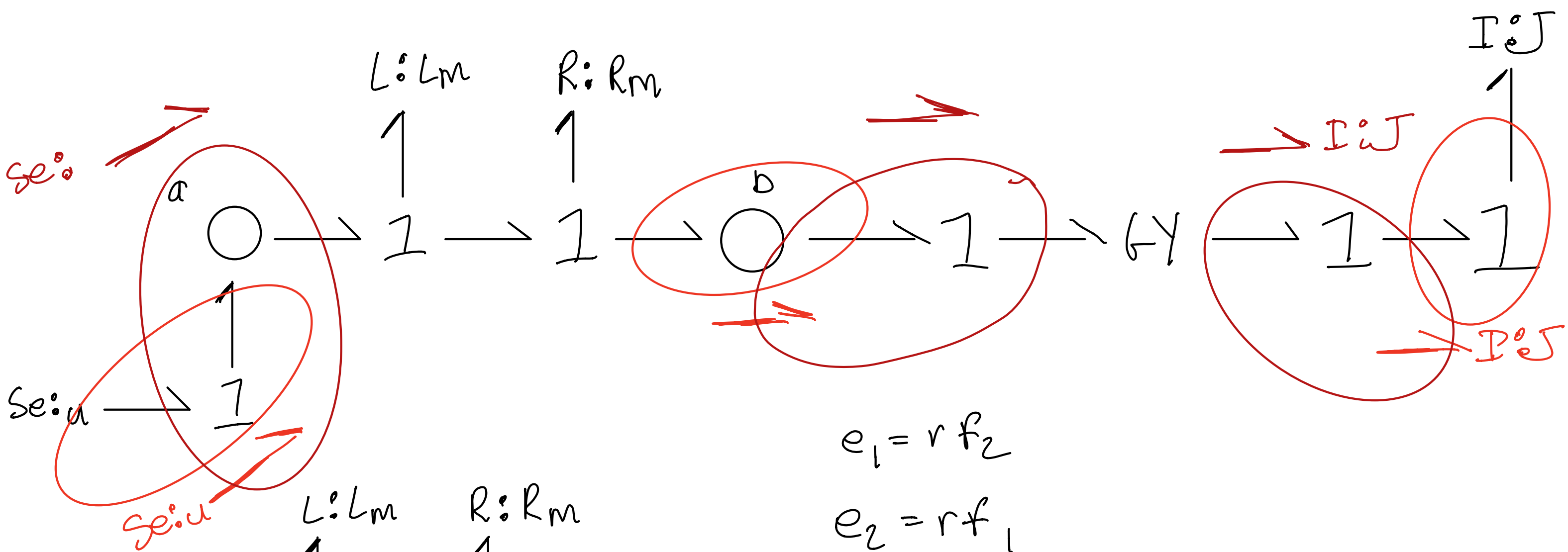
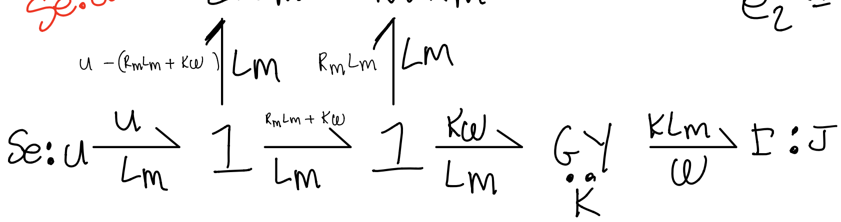


Figure 1: Voltage driven DC motor



$e_1 = r f_2$

$e_2 = r f_1$



$$x_1(t) = \begin{bmatrix} i_m \\ i_j \end{bmatrix} \in R^{2\times 1}$$

$$x_2(t) = i_j$$
$$\dot{x}_2(t) = \dot{i}_J = \frac{1}{J}\left[K L_m\right] = \frac{K}{J} x_1(t)$$

$$\dot{x}_1(t) = \begin{bmatrix} -\frac{R_m}{m} & -\frac{K}{m} \\ \frac{K}{J} & 0 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} \frac{1}{m} \\ 0 \end{bmatrix} u$$

$$x_1(t) = i_m$$
$$\dot{x}_1(t) = \dot{i}_m$$
$$= \frac{1}{m}\left[u - (R_m L_m + K\omega_m)\right]$$
$$= \frac{-R_m}{m} x_1(t) - \frac{K}{m} x_2(t) + \frac{1}{m} u$$

$$y(t) = [K \quad 0] x + [0] D$$

4. The transfer function for the system is

$$\frac{\omega(s)}{U(s)} = \frac{\frac{K}{JL}}{s^2 + \frac{R}{L}s + \frac{K^2}{JL}} = G(s).$$

$$G(s) = \frac{A\,\omega_n^2}{s^2 + 2\,\zeta\,\omega_n\,s + \omega_n^2}$$

$$\omega = K$$

$$\boxed{\begin{aligned} \omega_n^2 &= \frac{K^2}{JL} \\ \omega_n &= \sqrt{\frac{K^2}{JL}} \\ \omega_n &= \frac{K}{\sqrt{JL}} \end{aligned}}$$

$$K \cdot \frac{1}{J} \cdot \frac{1}{L}$$

$$A\omega_n^2 = \frac{K}{JL}$$
$$A = \frac{K}{JL}\omega_n^2$$

$$\boxed{\begin{aligned} A\omega_n^2 &= \frac{K}{JL} \\ A\left(\frac{K^2}{JL}\right) &= \frac{K}{JL} \\ A &= \frac{1}{K} \end{aligned}}$$

$$\boxed{\begin{aligned} 2\zeta\omega_n s &= \frac{R}{L}s \\ 2\zeta\frac{K}{\sqrt{JL}} &= \frac{R}{L} \\ \zeta &= \left(\frac{R}{L}\,\frac{\sqrt{JL}}{K}\right)\frac{1}{2} \\ \zeta &= \frac{R\sqrt{JL}}{2LK} \end{aligned}}$$

$$\boxed{\begin{aligned} \omega_d &= \omega_n + \sqrt{1-\zeta^2} \\ &= \frac{K}{\sqrt{JL}} + \sqrt{1 - \frac{R^2 JL}{4L^2K^2}} \end{aligned}}$$

$$\sigma = \zeta\omega_n = \left(\frac{R\sqrt{\cancel{JL}}}{2L\cancel{K}}\right)\frac{\cancel{K}}{\cancel{\sqrt{JL}}}$$

$$\sigma = \frac{R}{2L}$$

(5)

Compute the decay rate

$$\sigma = \frac{0.1\,\Omega}{2(0.1\,H)} = \frac{0.1}{0.2} = \frac{1}{2}$$

undamped natural frequency

$$\omega_n = \frac{K}{\sqrt{JL}} \Rightarrow J = \left(\frac{K}{\omega_n}\right)^2 \frac{1}{L} \Rightarrow \left(0.01^2 \frac{Kg^2 m^{\cancel{4}\,2}}{\cancel{s^4}\cancel{A^2}}\right)\left(\frac{1\,\cancel{s^2}}{rad^2}\right)\left(\frac{\cancel{s^2}\cancel{A^2}}{\cancel{Kg}\cancel{m^2}}\right)$$

$$\frac{Kgm^2}{rad^2}$$

$$j = \frac{(0.01)^2}{0.1}\ \frac{Kgm^2}{rad^2}$$

$$= 0.001$$

Time constant

$$\frac{1}{\sigma} = 2$$

steady state

(time constant) 4

$= 8s$

6) The DC gain is amplified 100 times

$$A = \frac{1}{K} = \frac{1}{0.01} = 100$$

7. $$|G(j\omega)| = \sqrt{\frac{\omega^2}{\omega^2 + 1}} = \sqrt{\frac{[2\pi(100)]^2}{2\pi(100)^2 + 1}} = 1$$