EE132 Automatic Control

Lab 5: Ball Beam & Root Locus

Buddy Ugwumba, Chris Hwang, Nick Andrade

Objective:

Use Root Locus method to design a controller or compensator for a ball and beam system. Our plant is the ball and beam system represented by the following transfer function:

$$G(s) = \frac{x(s)}{\theta(s)} = \frac{7}{s^2}$$ <---- Double integrator system

where: $x(s), x(t), \theta(s), \theta(t)$ are the respective position and angle of the ball and beam system in the s- and t-domain. With this is mind, we would like to design a controller that genreates $\theta(t)$ and we will use that controller to control the double integrator.

Controller Objective

- in unity feedback loop
- design a controller: $K(s) = \frac{\theta(s)}{E(s)}$ which will take the error signal as input
- Satisfy the following condiditions:

1) $2 \leq T_s \leq 4$

2) Tracking error must be zero

3) The desired position is a step function with the inital value of $\theta(t) \leq 30$. Then we will use the initial value theorem to determine the initial value.

Use root locus methods to design a cascade compensatore for the ball and beam system.

2) root locus

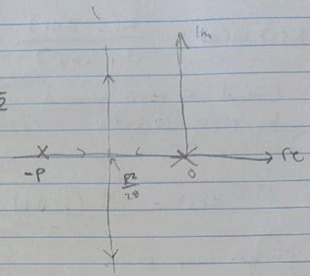
breakout point: $P_d = -\frac{P}{2}$

$1 + K(s)G(s) = 0$

or

$\begin{cases} |K(s)G(s)| = 1 \\ \angle K(s)G(s) = 180° \end{cases}$

$K_d = \frac{P^2}{28}$



3) Specification 1

$-2.3 \le \sigma \le -1.15$

choose complex poles for the closed loop system

$-2.3 \le -P/2 \le -1.15 \Rightarrow \boxed{4.6 \ge P \ge 2.3}$

4) Specification 3

$K < 0.5236$

for complex pole, $K > K_d$

$\frac{P^2}{28} < K < 0.5236 \Leftarrow \frac{P^2}{28} < 0.5236$

5) $2.3 \le P \le 3.8289$

pick P

6) After choosing P, then we can pick K such that

$\frac{P^2}{28} < K < 0.5236$

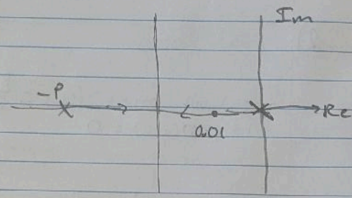
7) $K(s) = \frac{Ks}{s+P_K}$

8) replace s with s + 0.01

$K(s) = \frac{K(s+0.01)}{s+P}$

$$K(s)\,G(s) = \frac{k(s+0.01)}{s^2(s+p)}$$

Closed loop zero should be close to zero



$$K(s) = \frac{0.5\,(s+0.01)}{s+3}$$

Closed loop poles

$-1.4950 \pm 1.1113j$

$\underbrace{-0.0101}$ cancelled by the zero at -0.01

a) Verify the controller design in simulink

Report

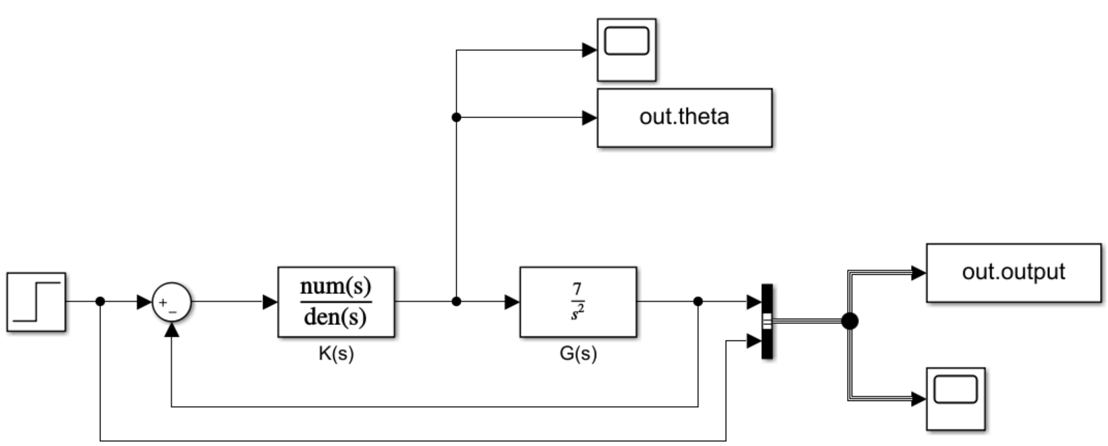
① design procedure, include your root locus

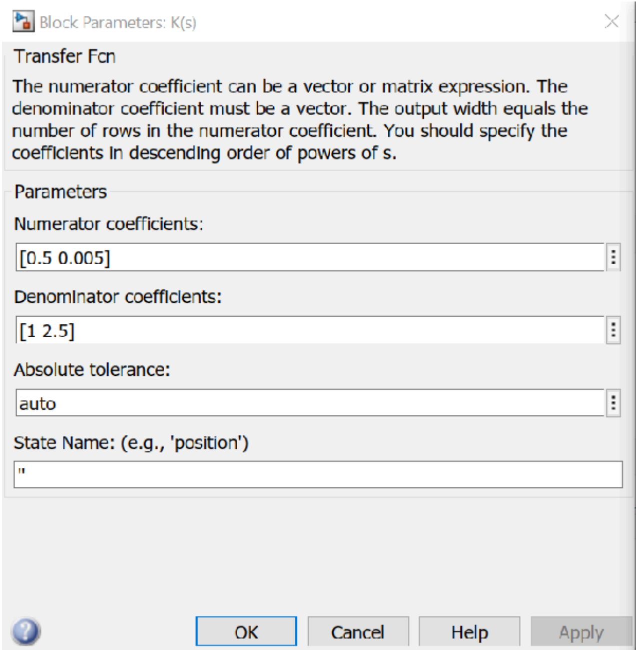
② controller

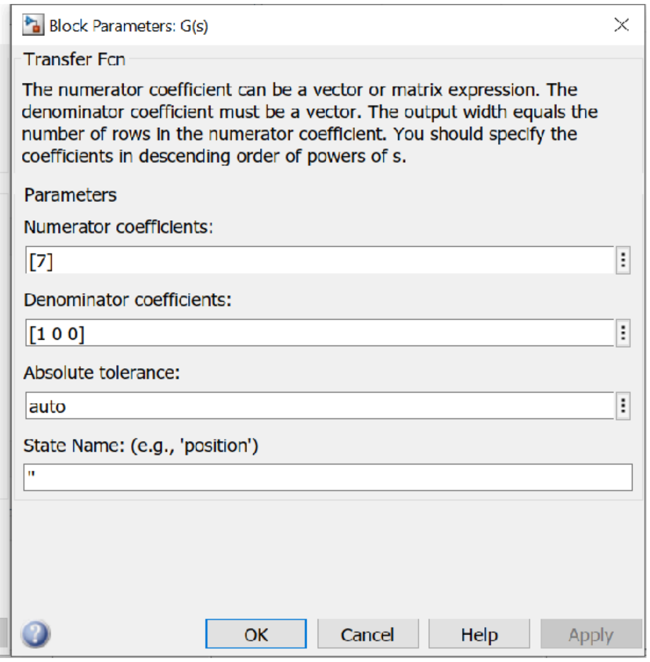
③ Output and control signals plots and explain that the specifications are achieved

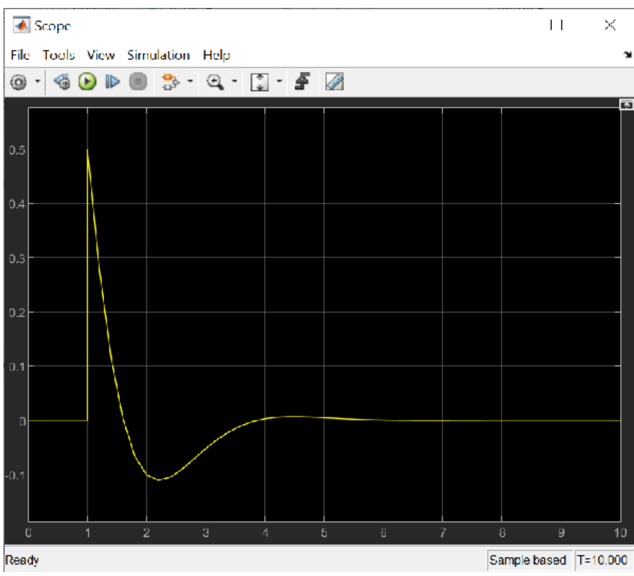
k = 0.5

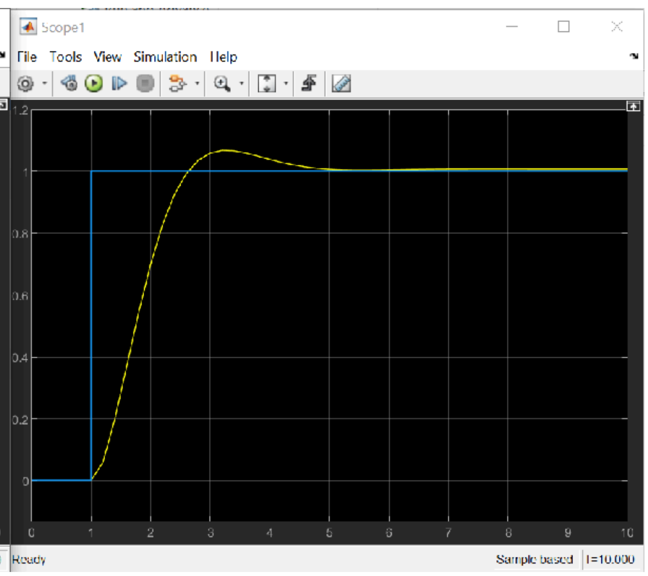
p = 2.5











Regarding the figure on the right, we can see that our desired value for both K and p were correct because our controller meets the design specificiations outlined in the lab; e.g. the settling time must be greater than or equal to 2 and less than or equal to 4. Moreover, we can see that the tracking error is none as t approaches infinity. Regarding the image on the right, we can see that our design specifications were met as well because the angle of the ball beam system is not greater than 30 degrees. Our calculations showed that pi/6 is about 0.5 rads.

```
tf([3.5 0.035],[1 2.5 3.5 0.035])
```

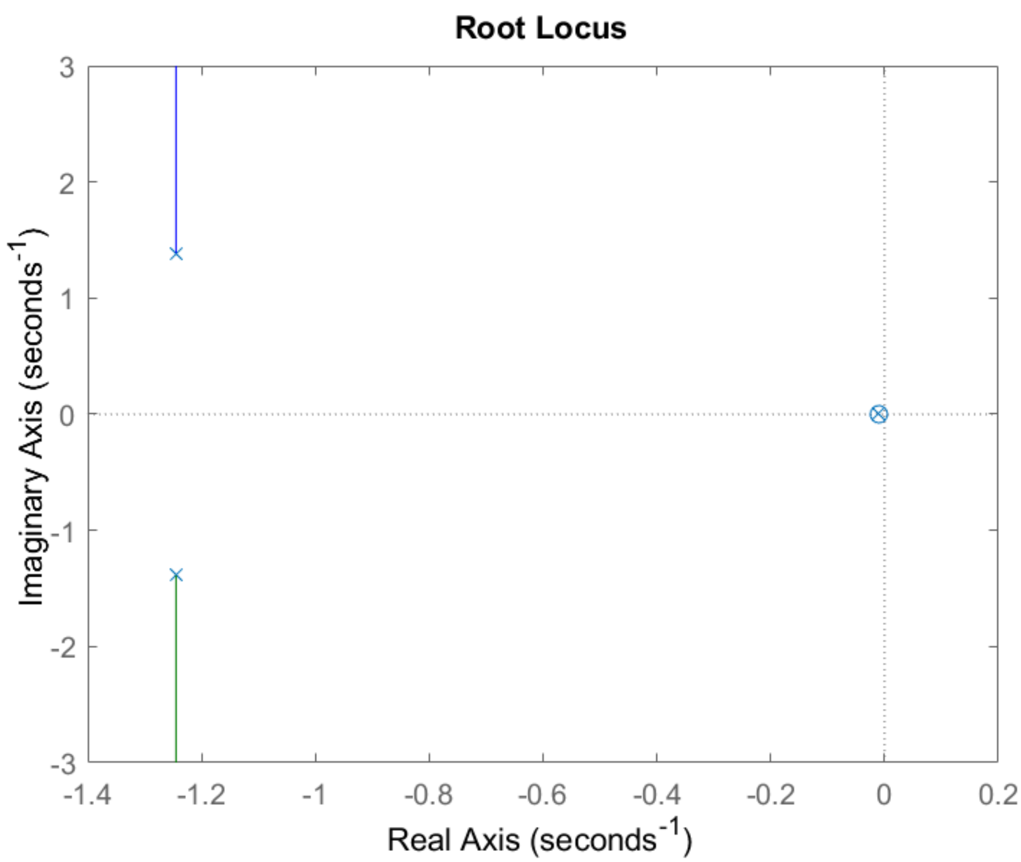
```
ans =

          3.5 s + 0.035
  -----------------------------
  s^3 + 2.5 s^2 + 3.5 s + 0.035
```

```
Continuous-time transfer function.
```

```
stepinfo(sys)
```

```
ans = struct with fields:
        RiseTime: 1.0771
    SettlingTime: 3.4019
     SettlingMin: 0.9106
     SettlingMax: 1.0672
       Overshoot: 6.7200
      Undershoot: 0
            Peak: 1.0672
        PeakTime: 2.2564
```

```
rlocus(sys)
```

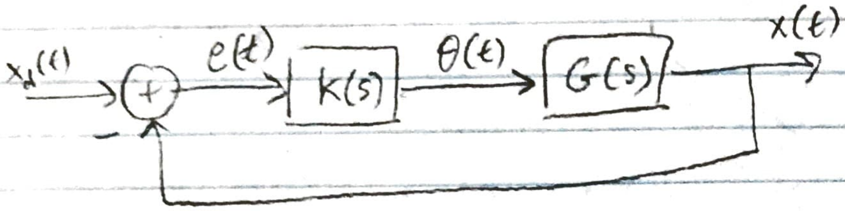


Concerning the root locus image we can see that out controller is stable because both the poles and zeros are strictly on the LHP. The complex conjugate pairs of poles tell us that this second order system is type 1.

Christopher Hwang
SID: 862133666

Pre-lab 5

2) $e(t) = x_d(t) - x(t)$



3) $$\begin{cases} E(s) = X_d(s) - X(s) \\ \theta(s) = K(s) \cdot E(s) \end{cases}$$

so

$$\boxed{\frac{\theta(s)}{X_d(s)} = \frac{K(s)}{1 + K(s)G(s)}}$$

4) $$\theta(t)\Big|_{t=0} < 30^\circ \text{ or } \frac{\pi}{6} \text{ rad}$$ ← Specification 3

$$\theta(s) = \frac{K(s)}{1 + K(s)G(s)} \cdot X_d(s) = \frac{1}{s} \frac{K(s)}{1 + K(s)G(s)}$$

$$G(s) = \frac{7}{s^2}$$

$$K(s) = \frac{K \prod_{i=1}^{m} (s - z_i)}{\prod_{j=1}^{m} (s - p_j)}$$

$$\theta(t)\Big|_{t=0} = \lim_{s \to \infty} s \cdot \theta(s) = \lim_{s \to \infty} \frac{K(s)}{1 + K(s)G(s)}$$
$$= K$$

so $\boxed{K < \frac{\pi}{6}}$

5) $$\frac{X(s)}{X_d(s)} = \frac{\theta(s) \cdot G(s)}{X_d(s)}$$

$$\frac{X(s)}{X_d(s)} = \frac{K(s) \cdot G(s)}{1 + K(s)G(s)}$$

6) Specification $2 \leq T_s \leq 4$

$$T_s = \frac{4.6}{\sigma}$$

$$\sigma = \frac{4.6}{T_s}$$

$$\Rightarrow \boxed{1.15 \leq \sigma \leq 2.3}$$

7) dominant poles

Compare the real parts of poles since the real parts determine the speed of the response decrease.

poles $-\delta\omega_n \pm j\omega_n\sqrt{1-\delta^2}$

$-a_r$

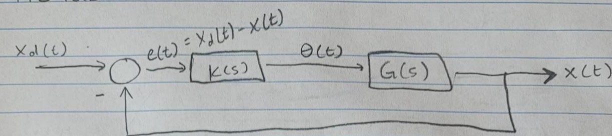
step response $C(s) = \frac{A}{s} + \frac{B(\delta\omega_n + s) + C\omega_d}{(s+\delta\omega_n)^2 + \omega_d^2} + \frac{D}{s+a_r}$

8) $K(s)G(s)$ should be type 1. Specification 2 is met. There would a 0 tracking error.

Nicolas
Andrade

Pre-lab

2)



3) $\begin{cases} E(s) = X_d(s) - X(s) \\ \Theta(s) = K(s) * E(s) \end{cases}$

$\Rightarrow \dfrac{\Theta(s)}{X_d(s)} = \dfrac{K(s)}{1 + K(s)G(s)}$

4) In specification 3:

$\Theta(t)\big|_{t=0} < 30°$ or $\dfrac{\pi}{6}$ rad

$\Theta(s) = \dfrac{K(s)}{1+K(s)G(s)} * X_d(s) = \dfrac{1}{s} * \dfrac{K(s)}{1+K(s)G(s)}$

$\begin{cases} G(s) = \dfrac{7}{s^2} \\ K(s) = \dfrac{K \prod_{i=1}^{M} (s - z_i)}{\prod_{j=1}^{N} (s - p_j)} \end{cases}$ z_i, p_j - zeros, poles
K - control gain

$\theta(t)\big|_{t=0} = \lim_{s\to\infty} s * \Theta(s) = \lim_{s\to\infty} \dfrac{K(s)}{1+K(s)G(s)}$

$= K$

$\therefore K < \dfrac{\pi}{6}$

5) $\dfrac{X(s)}{X_d(s)} = \dfrac{\Theta(s)G(s)}{X_d(s)}$

$= \dfrac{K(s)G(s)}{1 + K(s)G(s)}$

pole > zero reduces damping
inc damp factor

For lag compensator When dom. poles go closer to zero

move poles behind zero reduces ss error

6) from specification 1

$2 \le T_s \le 4\,s$

$T_s = \frac{4.6}{\sigma}$ $\quad \sigma$ - real part of pole / dominant pole

$\Rightarrow 1.15 \le \sigma \le 2.3$

7) dominant poles:

compare the real parts of poles since the real part determines how fast the response decreases

ex) poles: $-\delta\omega_n \pm j\omega_n\sqrt{1-\delta^2}$

$-a_r$

step response:

$$C(s) = \frac{A}{s} + \frac{B(s+\delta\omega_n) + C\omega_d}{(s+\delta\omega_n)^2 + \omega_d^2} + \frac{D}{s+a_r}$$

$$c(t) = Au(t) + e^{-\delta\omega_n t}(B\cos\omega_d t + C\sin\omega_d t) + De^{-a_r t}$$

if $a_r >> \delta\omega_n$

8) If Type1 or Type2 specification 1 is met

$K(s)\,G(s)$ should be at least type 1

control design:

$$K(s) = \frac{Ks}{s+p}$$

$$K(s)G(s) = 7K / s(s+p) \qquad \begin{matrix} K>0 \\ p>0 \end{matrix}$$

In practice, zero and pole cancellation is not allowed unless the pole is strictly in LHP. Also, since the poles at zero indicates the type of system, we usually do not cancel the pole at zero

$x(t) \longrightarrow x_d(t)$ as $t \to \infty$

(3) $x_d(t)$ is a step functio

the initial value of $\theta(t)\big|_{t=0} < 30^\circ$

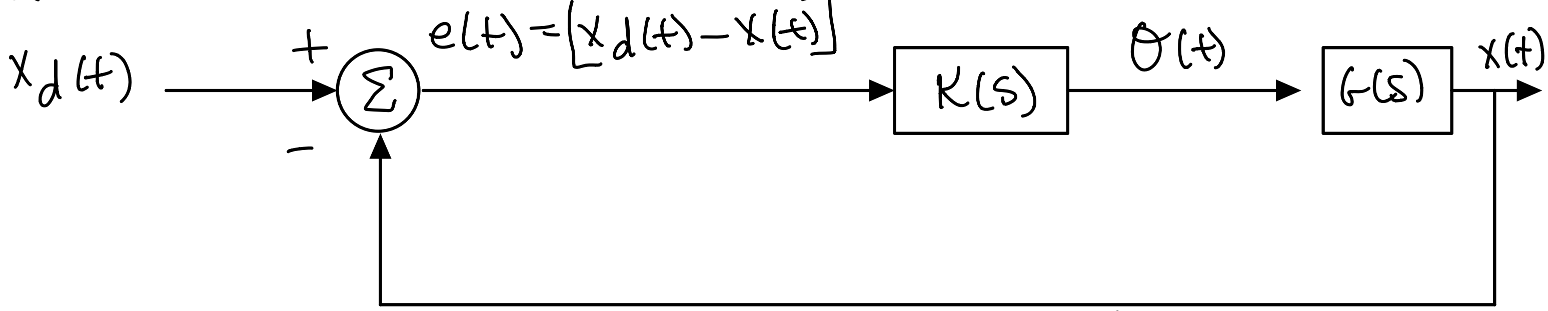
then we use initial value theorem to determine the initial value

$$\theta(t)\big|_{t\to 0} = \lim_{s \to 0} s\,(\theta(s))$$

To find constraints on the control input

Prelab

2.



3) $$\frac{K(s)\,G(s)}{1 + K(s)\,G(s)} = T(s)$$

? what happend to the G(s) because we're not finding the output

$$\begin{cases} E(s) = X_d(s) - X(s) \\ \theta(s) = K(s)\,G(s) \end{cases} \Rightarrow \frac{\theta(s)}{X_d(s)} = \frac{K(s)}{1 + K(s)\,G(s)}$$

4.)

$$\theta(t)\big|_{t=0} < 30^\circ \text{ or } \frac{\pi}{6}$$

$$\theta(s) = \frac{K(s)}{1+K(s)G(s)} \cdot X_d(s) = \frac{1}{s} \cdot \frac{K(s)}{1+K(s)G(s)}$$

$$G(s) = \frac{7}{s^2} \qquad = \frac{1}{s} \cdot \frac{K(s)}{1+K(s)\frac{7}{s^2}}$$

$$= K$$

Thus $K < \frac{\pi}{6}$

5.

$$\frac{X(s)}{X_d(s)} = \frac{\theta(s)\,G(s)}{X_d(s)}$$

$$= \frac{K(s)\,G(s)}{1+K(s)\,G(s)}$$

6) From specifications

$$2 \leq T_s \leq 4s$$

Use constant of dominant pole to estimate settly time

$T_s = \frac{4.6}{\sigma}$ σ - real part of pole

$$1.15 \leq \theta \leq 2.3$$

7) compare the real parts of pole since the real parts determine how fas the response decrease

8) Should be at least type 7