

EE105: PWM Speed Control of a Motor

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3. Calculations:

(2)

Compute power in and power out.

For power in, using the voltage and the current.

For power out, $(\tau_m \cdot \omega_m)$.

At the end, they should be identical.

(3)

Show the bond graph and the state space model in matrix form.

(4)

Compute A , ζ , ω_n , ω_d , σ .

(5)

Determine the value of the inertia J . Please be careful about the unit of J .

Compute the decay rate.

Compute time constant.

How long will it take the system to reach steady state?

(6)

Compute the DC gain. (Note: The DC gain value is for pulse width of 100%, the value decrease with the pulse width.)

(7)

Compute $|G(j\omega)|$ for $f = 100\text{Hz}$. (Please review the relationship between ω and f .)

Attenuation for different frequencies. Please following the code instruction below.

```
% State-space model description
R = 0.1;      % ohms
L = 0.1;      % H
K = 0.01;     % unitless
J = 0.001;    % kg.m^2/s^2

A = [-R/L -K/L; K/J 0];
B = [1/L; 0];
C = [0 1];
D = 0;
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$$\frac{\text{kg m}^2}{\text{s}^2 \text{ A}}$$

$$e_m(t) = K \omega_m(t)$$

$$i_m(t) = \frac{T_m(t)}{K}$$

$$P_m(t) = e_m(t) i_m(t) \\ = (\cancel{K} \omega_m(t)) \left(\frac{P_m(t)}{\cancel{K}} \right)$$

$$P_m(t) = \omega_m(t) T_m(t)$$

$$P_{in} = P_{out}$$

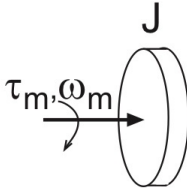


Diagram illustrating the reduction of a complex expression to a simpler form, showing the application of the distributive law of multiplication over addition.

The diagram shows a sequence of transformations:

$$\begin{aligned}
 & \text{Se: } u \rightarrow \frac{u}{L_m} \xrightarrow{L_m} 1 \xrightarrow{R_m L_m} 1 \xrightarrow{K_m} G \cdot Y \xrightarrow{K_m} I : J \\
 & \text{Se: } u \xrightarrow{L_m} 1 \xrightarrow{R_m L_m} 1 \xrightarrow{K_m} G \cdot Y \xrightarrow{K_m} I : J \\
 & \text{Se: } u \xrightarrow{L_m} 1 \xrightarrow{R_m L_m} 1 \xrightarrow{K_m} G \cdot Y \xrightarrow{K_m} I : J
 \end{aligned}$$

The diagram includes several annotations and arrows indicating the steps of the reduction:

- $L: L_m$ and $R: R_m$ are written above the first two steps.
- $L: L_m$ and $R: R_m$ are written above the third and fourth steps.
- $L: L_m$ and $R: R_m$ are written above the fifth and sixth steps.
- $L: L_m$ and $R: R_m$ are written above the seventh and eighth steps.
- $L: L_m$ and $R: R_m$ are written above the ninth and tenth steps.
- $L: L_m$ and $R: R_m$ are written above the eleventh and twelfth steps.
- $L: L_m$ and $R: R_m$ are written above the thirteenth and fourteenth steps.
- $L: L_m$ and $R: R_m$ are written above the fifteenth and sixteenth steps.
- $L: L_m$ and $R: R_m$ are written above the seventeenth and eighteenth steps.
- $L: L_m$ and $R: R_m$ are written above the nineteenth and twentieth steps.
- $L: L_m$ and $R: R_m$ are written above the twenty-first and twenty-second steps.
- $L: L_m$ and $R: R_m$ are written above the twenty-third and twenty-fourth steps.
- $L: L_m$ and $R: R_m$ are written above the twenty-fifth and twenty-sixth steps.
- $L: L_m$ and $R: R_m$ are written above the twenty-seventh and twenty-eighth steps.
- $L: L_m$ and $R: R_m$ are written above the twenty-ninth and thirtieth steps.
- $L: L_m$ and $R: R_m$ are written above the thirty-first and thirty-second steps.
- $L: L_m$ and $R: R_m$ are written above the thirty-third and thirty-fourth steps.
- $L: L_m$ and $R: R_m$ are written above the thirty-fifth and thirty-sixth steps.
- $L: L_m$ and $R: R_m$ are written above the thirty-seventh and thirty-eighth steps.
- $L: L_m$ and $R: R_m$ are written above the thirty-ninth and fortieth steps.
- $L: L_m$ and $R: R_m$ are written above the forty-first and forty-second steps.
- $L: L_m$ and $R: R_m$ are written above the forty-third and forty-fourth steps.
- $L: L_m$ and $R: R_m$ are written above the forty-fifth and forty-sixth steps.
- $L: L_m$ and $R: R_m$ are written above the forty-seventh and forty-eighth steps.
- $L: L_m$ and $R: R_m$ are written above the forty-ninth and fiftieth steps.
- $L: L_m$ and $R: R_m$ are written above the fifty-first and fifty-second steps.
- $L: L_m$ and $R: R_m$ are written above the fifty-third and fifty-fourth steps.
- $L: L_m$ and $R: R_m$ are written above the fifty-fifth and fifty-sixth steps.
- $L: L_m$ and $R: R_m$ are written above the fifty-seventh and fifty-eighth steps.
- $L: L_m$ and $R: R_m$ are written above the fifty-ninth and sixtieth steps.
- $L: L_m$ and $R: R_m$ are written above the sixty-first and sixty-second steps.
- $L: L_m$ and $R: R_m$ are written above the sixty-third and sixty-fourth steps.
- $L: L_m$ and $R: R_m$ are written above the sixty-fifth and sixty-sixth steps.
- $L: L_m$ and $R: R_m$ are written above the sixty-seventh and sixty-eighth steps.
- $L: L_m$ and $R: R_m$ are written above the sixty-ninth and seventieth steps.
- $L: L_m$ and $R: R_m$ are written above the seventy-first and seventy-second steps.
- $L: L_m$ and $R: R_m$ are written above the seventy-third and seventy-fourth steps.
- $L: L_m$ and $R: R_m$ are written above the seventy-fifth and seventy-sixth steps.
- $L: L_m$ and $R: R_m$ are written above the seventy-seventh and seventy-eighth steps.
- $L: L_m$ and $R: R_m$ are written above the seventy-ninth and eightieth steps.
- $L: L_m$ and $R: R_m$ are written above the eighty-first and eighty-second steps.
- $L: L_m$ and $R: R_m$ are written above the eighty-third and eighty-fourth steps.
- $L: L_m$ and $R: R_m$ are written above the eighty-fifth and eighty-sixth steps.
- $L: L_m$ and $R: R_m$ are written above the eighty-seventh and eighty-eighth steps.
- $L: L_m$ and $R: R_m$ are written above the eighty-ninth and ninetieth steps.
- $L: L_m$ and $R: R_m$ are written above the ninety-first and ninety-second steps.
- $L: L_m$ and $R: R_m$ are written above the ninety-third and ninety-fourth steps.
- $L: L_m$ and $R: R_m$ are written above the ninety-fifth and ninety-sixth steps.
- $L: L_m$ and $R: R_m$ are written above the ninety-seventh and ninety-eighth steps.
- $L: L_m$ and $R: R_m$ are written above the ninety-ninth and one hundredth steps.

$$e_1 = r f_2$$

$$e_2 = rf$$

$$x_i(t) = \begin{bmatrix} i_m \\ i_j \end{bmatrix} \in \mathbb{R}^{2 \times 1}$$

$$\begin{aligned} x_1(t) &= i_m \\ \dot{x}_1(t) &= \dot{i}_m \end{aligned}$$

$$\begin{aligned} \dot{x}_2(t) &= \dot{i}_j \\ \dot{x}_2(t) &= \dot{i}_j \\ &= \frac{1}{J} [KL_m] \\ &= \frac{K}{J} x_1(t) \end{aligned}$$

$$\dot{x}_i(t) = \begin{bmatrix} -\frac{R_m}{m} & -\frac{K}{m} \\ \frac{K}{J} & 0 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} \frac{1}{m} \\ 0 \end{bmatrix} u$$

$$= \frac{1}{m} [u - (R_m L_m + K \omega_m)]$$

$$= -\frac{R_m}{m} x_1(t) - \frac{K}{m} x_2(t) + \frac{1}{m} u$$

$$y(t) = [K \ 0] x + [0] D$$

4. The transfer function for the system is

$$\frac{\omega(s)}{U(s)} = \frac{\frac{K}{JL}}{s^2 + \frac{R}{L}s + \frac{K^2}{JL}} = G(s).$$

$$G(s) = \frac{A \omega_n^2}{s^2 + 2\zeta \omega_n s + \omega_n^2}$$

$$\omega = K$$

$$\omega_n^2 = \frac{K^2}{JL}$$

$$\omega_n = \sqrt{\frac{K^2}{JL}}$$

$$\omega_n = \frac{K}{\sqrt{JL}}$$

$$K \cdot \frac{1}{J} \cdot \frac{1}{L}$$

$$A \omega_n^2 = \frac{K}{JL}$$

$$A = \frac{K}{JL} \omega_n^2$$

$$2\zeta \omega_n s = \frac{R}{L} s$$

$$2\zeta \frac{K}{\sqrt{JL}} = \frac{R}{L}$$

$$\zeta = \left(\frac{R}{L} \frac{\sqrt{JL}}{K} \right) \frac{1}{2}$$

$$\zeta = \frac{R\sqrt{JL}}{2LK}$$

$$A \omega_n^2 = \frac{K}{JL}$$

$$A \left(\frac{K^2}{JL} \right) = \frac{K}{JL}$$

$$A = \frac{1}{K}$$

$$\begin{aligned} \omega_d &= \omega_n + \sqrt{1 - \zeta^2} \\ &= \frac{K}{\sqrt{JL}} + \sqrt{1 - \frac{R^2 J L}{4 L^2 K^2}} \end{aligned}$$

$$\sigma = \zeta \omega_n = \left(\frac{R \sqrt{JL}}{2LK} \right) \frac{K}{\sqrt{JL}}$$

$$\sigma = \frac{R}{2L}$$

(5)

compute the decay rate

$$\sigma = \frac{0.1 \Omega}{2(0.1 \text{H})} = \frac{0.1}{0.2} = \frac{1}{2}$$

undamped natural frequency

$$\omega_n = \frac{K}{\sqrt{JL}} \Rightarrow J = \left(\frac{K}{\omega_n} \right)^2 \frac{1}{L} \Rightarrow \left(0.01^2 \frac{\text{Kg}^2 \text{m}^2}{\cancel{\text{s}^4 \text{A}^2}} \right) \left(\frac{1 \cancel{\text{s}^2}}{\text{rad}^2} \right) \left(\frac{\cancel{\text{s}^2 \text{A}^2}}{\text{Kgme}} \right)$$

$$\frac{\text{Kg m}^2}{\text{rad}^2}$$

Time constant

$$\frac{1}{\sigma} = 2$$

steady state

(time constant) 4

$$= 8 \text{ s}$$

$$j = \frac{(0.01)^2}{0.1} \frac{\text{Kg m}^2}{\text{rad}^2}$$

$$= 0.001$$

6) The DC gain is amplified 100 times

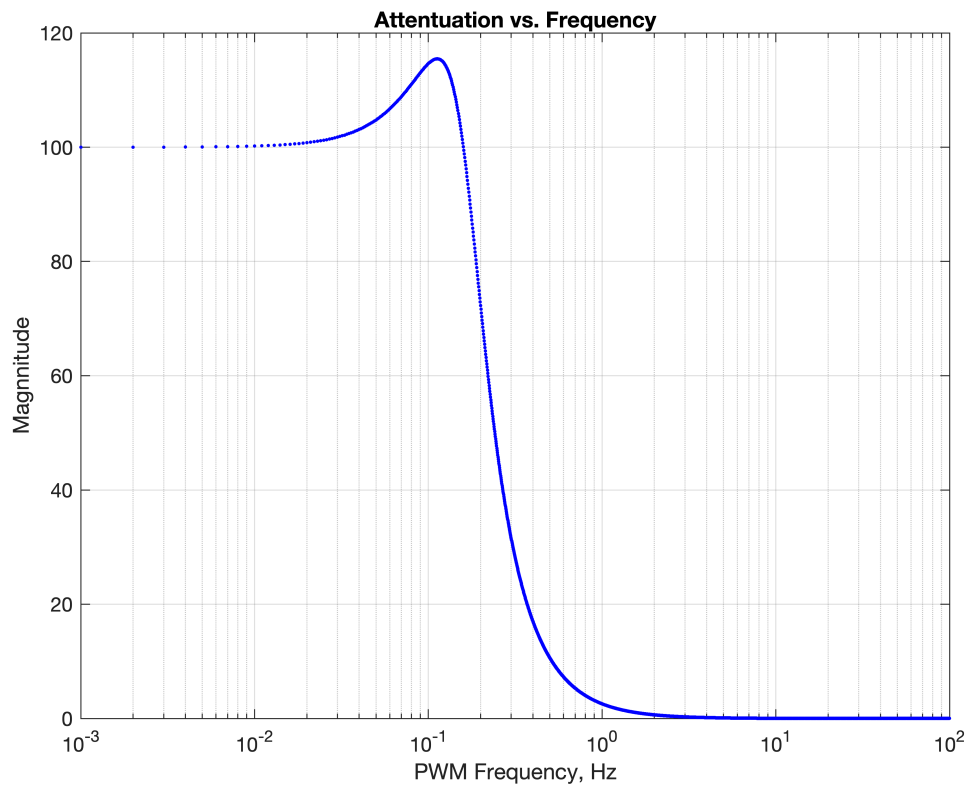
$$A = \frac{1}{K} = \frac{1}{0.01} = 100$$

$$7. |G(j\omega)| = \sqrt{\frac{\omega^2}{\omega^2 + 1}} = \sqrt{\frac{[2\pi(100)]^2}{2\pi(100)^2 + 1}} = 1$$

```

% Decay rate
sig = 0.5;
% Time constant
Tau = 1/sig;
% DC Amplification
DC_amp = K/(J*L);
% Attenuation for different frequencies
num = K/(J*L); % coefficients of numerator of G(s)
den = [1 R/L (K^(2)/(J*L))]; % coefficients of denominator of G(s)
f = 0:0.001:100;
s = 1i*2*pi*(f);
Gs = abs(polyval(num,s)./polyval(den,s));
semilogx(f, Gs, 'b. ');
grid on;
xlabel('PWM Frequency, Hz');
ylabel('Magnnitude');
title('Attentuation vs. Frequency')

```

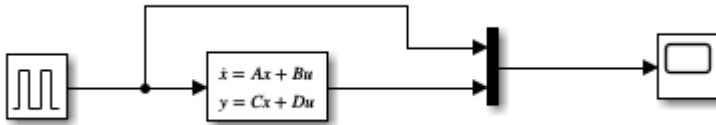


(8)

In terms of the graph you get above, does the system effectively keep the DC component and essentially remove all other components?

(The Attentuation vs. Frequency showing how the DC gain $G(s)$ changes with the frequency. The purpose of low pass filter is to remove the high frequency component, namely, the DC gain will get much smaller when the frequency increase.)

4. Simulation (Lab Assignment):

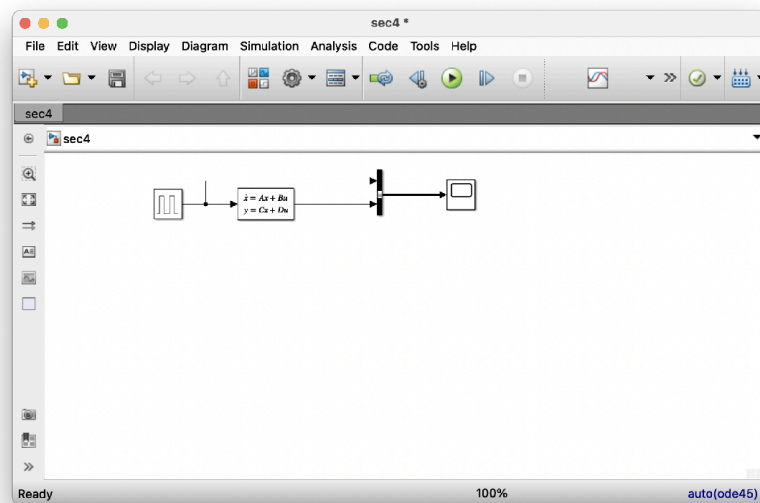


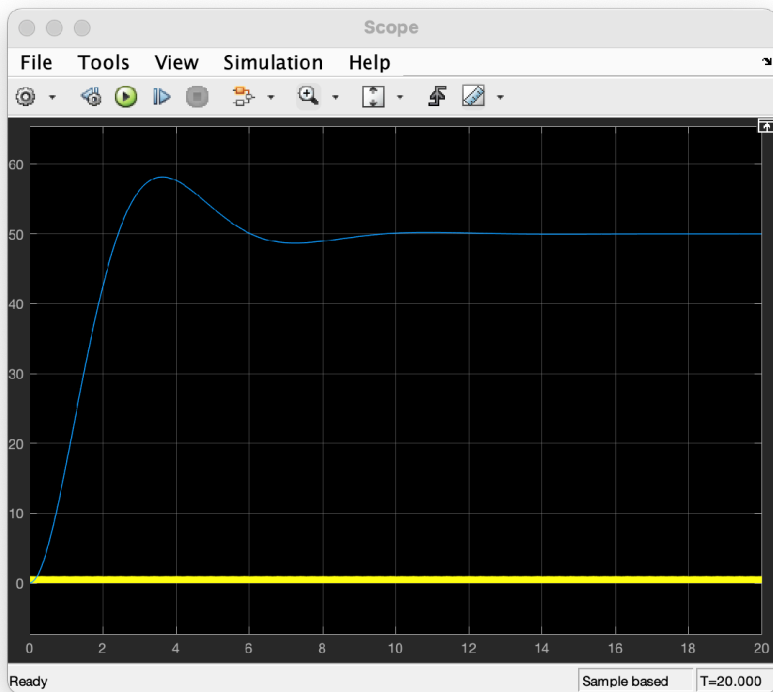
You have defined the matrix A,B,C,D above, just write it to the state-space block.

Set the stop time to 10 time constants.

(1) & (2)

Show the simulation result.



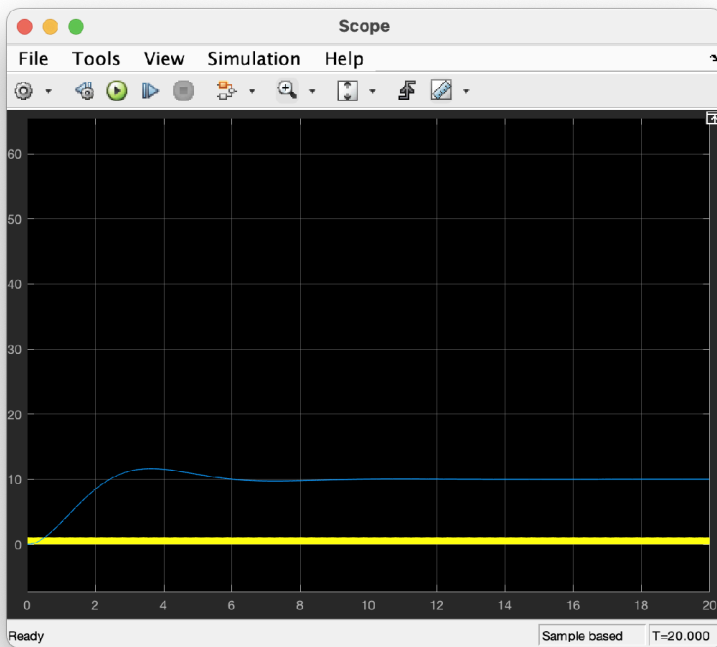


(You will see the steady-state is about 50. Because the DC component is $\frac{\tau}{T} B$.)

The steady state is at 50 because that is the percentage of pulse width modulation

(3) Change the pulse width to 10% and rerun the simulation.

Show the simulation result.



No requirement for the steady-state angular rate plot.

Again the steady state value is at 10 because that is the pulse width modulation.

5. Questions:

1.

Answer the question.

The single parameter that determines the DC gain is K because the DC gain is found via $1/K$

2.

Answer the question.

If I change the parameter neither the DC gain nor the decay rate will change.

3.

The parameter that would change the decay rate is R . We would need to make it larger in order to decrease time constant without changing the natural frequency

Answer the question.

Show the simulation result.

```
% State-space model description
R = 0.2;      % ohms
L = 0.1;      % H
K = 0.01;     % unitless
```



```

J = 0.001;    % kg.m^2/s^2

A = [-R/L -K/L; K/J 0];
B = [1/L; 0];
C = [0 1];
D = 0;

% Decay rate
sig = 0.5;
% Time constant
Tau = 1/sig;
% DC Amplification
DC_amp = K/(J*L);
% Attenuation for different frequencies
num = K/(J*L); % coefficients of numerator of G(s)
den = [1 R/L (K^(2)/(J*L))]; % coefficients of denominator of G(s)
f = 0:0.001:100;
s = 1i*2*pi*(f);
Gs = abs(polyval(num,s)./polyval(den,s));
semilogx(f, Gs, 'b. ');
grid on;
xlabel('PWM Frequency, Hz');
ylabel('Magnnitude');
title('Attentuation vs. Frequency')

```

