Inventory Management

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Introduction

At the beginning of each month, the manager of a store of a given storage capacity must make the inventory of the stock of the single product it sells. Depending on the current level of the stock, he must decide, if necessary, to order additional stock to satisfy the demands. It is assumed that:

- Demands are random
- The cost of maintaining the stock at a given level is an increasing function of the level
- Stock outs induce penalties
- Stock surplus induce storage costs
- An order causes costs consisting of a fixed part and a variable part which increases
- With the amount of products ordered

Determine the renewal strategy that must be put in place to minimize the management costs on a given number of months.

Modeling

Starting from any month t, the storage already has a number of stocks on hand $\boldsymbol{S_t}$. If external demand $\boldsymbol{D_t}$ occurs, the manager needs to order new $\boldsymbol{a_t}$ number units of stock.

Assumtions:

1. The decision to order additional stock is made at the beginning of each month.

2. The demand for the product arrives throughout the month, but all orders are paid at the end of month.
3. If the demand is greater than inventory, customer can buy from other place. Which mean there is no backlogging of unfilled orders so that excess demand is lost.
4. The revenue, cost, and demand distribution do not vary from month to month.
5. The product is sold whole unit
6. The storage has capacity M units (M>=0)
7. The demand has a know time-homogenous probability distribution

$$P_j = P\{D_t = j\}, j = 0,1,2 \ldots$$

The inventory at decision epoch t+1, s_{t+1} is related to the decision epoch t, s_t and will be represented as following:

$$\boldsymbol{s_{t+1}} = \mathbf{max}\,\{\,\boldsymbol{s_t} + \boldsymbol{a_t} - \boldsymbol{D_t}, \mathbf{0}\} \equiv [\,\boldsymbol{s_t} + \boldsymbol{a_t} - \boldsymbol{D_t}]^{+} \qquad (1)$$

The inventory cannot be negative. Therefore, if $s_t + a_t - D_t < 0$ then the inventory level at subsequent decision epoch is **0**.

Let call the cost of odering **u** units in a month is O(u). This cost value is the cost at the present time of money**.** We assume it includes a fixed cost for placing order K>0 and a variable cost c(u) that increase with quantity ordered.

$$\mathbf{O(u)} = \begin{cases} \boldsymbol{K} + \boldsymbol{c}(\boldsymbol{u}) & if\ u > 0 \\ \mathbf{0} & otherwise \end{cases} \qquad (2)$$

The cost to maintain the inventory of **u** units is **h(u).** This is nondecreasing function. In finite horizon, the remaining inventory at the last decition epoch **N** has value g(u)= 0. If the demand is **j** units, and the inventory meets the demand, the manger receives revenue (reward) value $f\ (j)$. If j=0=> f (0)=0.

The revenue depends on the state of the system at next decision epoch as following

$$\boldsymbol{r_t}(\boldsymbol{s_t}, \boldsymbol{a_t}, \boldsymbol{s_{t+1}}) = -\mathbf{O}(\boldsymbol{a_t}) - \boldsymbol{h}(\boldsymbol{s_t} + \boldsymbol{a_t}) + \boldsymbol{f}(\boldsymbol{s_t} + \boldsymbol{a_t} - \boldsymbol{s_{t+1}}) \quad (3)$$

The expected revenue $F_t(u)$ (at start of month t) received when the inventory prior to receipt of customer order is **u** units. If the inventory **u** greater than the demand j
The present of value revenue is $f\ (j)$. This happens with probability P_j. Opposite, the present value of revenue is $f\ (u)$. This happens with probability $\boldsymbol{q_u = 1 - \sum_{j=0}^{u-1} P_j}$

$$F(u) = \sum_{j=0}^{u-1} f(j)P_j + f(u)q_u \qquad (4)$$

The formulation of our problem is as follows:

Decision Epoch

- T={1,2…N} $N \leq \infty$

States (the number of inventory on hand at the start of month t)

- S={0,1,2…M}

Actions (the number of additional stock need to order in month t to meet demand)

- $A_s = \{0,1,2,\dots,M-s\}$

Expected Reward (at start of month t)

- $r_t(s_{t,}a)$=$F_t(s_t + a_t) - O(a) - h(s+a), t = 1,2 \dots N-1$

The value of last inventory: $r_N(s) = g(s), t = N$

Transition Probabilities

$$P_t(j|s,a) = \begin{cases} 0 & if\ M \geq j > s+a \\ P_{s+a-j} & if\ M \geq s+a \geq j > 0 \\ q_{s+a} & if\ M \geq s+a\ and\ j = 0 \end{cases} \qquad (5)$$

The transition probabilities mean that, if the inventory on hand at the beginning of month **t** is **s** units and an order is made for **a** units, the inventory prior for the demand is **s+a.** an inventory level of j >0 at the start of month t+1 need a demand of **s+a-j** units in month **t**. this happens with probability P_{s+a-j}. Because backlogging is not permit (Assumtion 3), if the demand is in month **t** greater than **s+a** unit, then the inventory at the start of month t+1 is 0 units. This happens with probability $\boldsymbol{q_{s+a}}$.

The probability that the inventory level exceeds **s+a** units is 0 because demand is possitive.
Assumtion 2 means that, the inventory throughout a month is **s+a,** so that the total monthly holding cost is **h(s+a).**

Solving Model

We will use Backward Induction Algorithm to find the optimal policy for the model.
The input parameter of the model is not change during the running time (stationary).
Therefore we can represent the solutions of optimality equaltions as following:

$$u_t^*(s,a) = r(s,a) + \sum_{j\in s} P(j|s,a)u_{t+1}^*(j)$$

Algorithm optimal_MD
Begin
  $t \leftarrow N$
  For each s_t **Do**
    $u_t^*(s_t) \longleftarrow r_t(s_t)$
  While $(t > 1)$ **Do**
    $t \longleftarrow t - 1$
    For each s_t **Do**
      $u_t^*(s_t) \leftarrow \max_{a\in A_{st}} (r_t(s_t, a) + \sum_{j\in S} p_t(j \mid s_t, a)u_{t+1}^*(j))$
      $A^*_{st,t} \leftarrow \operatorname{argmax}_{a\in A_{st}} (r_t(s_t, a) + \sum_{j\in S} p_t(j \mid s_t, a)u_{t+1}^*(j))$
  Endwhile
End

Simulation

This simulation implemented and solved the model above with example data.

The soure code is in the same folder with this document. To run the simulation, open file **InventoryManagement.jar** or open source code project in Netbean IDE **(**need **java jdk** installed)

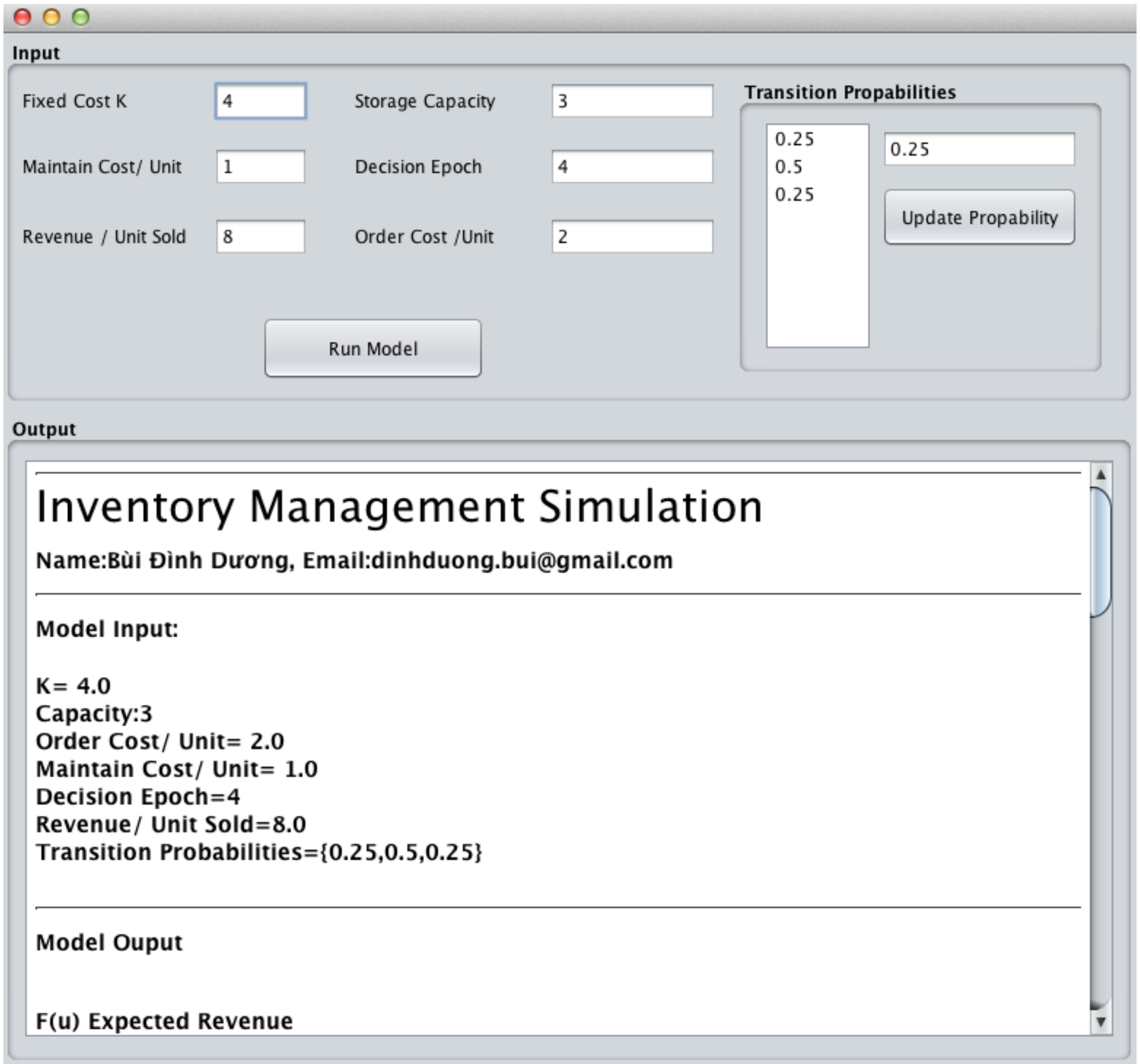

Figure 1 Model Simulation with Sample Input

X: denotes inavaiable actions.

Output

F(u) Expected Revenue

u	F(u)
0	0.0
1	6.0
2	8.0
3	8.0

Rt(s,a) Reward

	Rt(s,a)			
s/a	0	1	2	3
0	0.0	-1.0	-2.0	-5.0
1	5.0	0.0	-3.0	X
2	6.0	-1.0	X	X
3	5.0	X	X	X

Figure 2 Model Ouput with Expected Revenue, Reward

Output

P(j|s,a) Propability

	Pt(j|s,a)			
s+a/j	0	1	2	3
0	1.0	0.0	0.0	0.0
1	0.75	0.25	0.0	0.0
2	0.25	0.5	0.25	0.0
3	0.0	0.25	0.5	0.25

Figure 3 Ouput of Probabilities

Output

Decision Epoch T=3

s/a	0	1	2	3	U3*(s)	As,3*
0	0.0	-1.0	-2.0	-5.0	0.0	0.0
1	5.0	0.0	-3.0	X	5.0	0.0
2	6.0	-1.0	X	X	6.0	0.0
3	5.0	X	X	X	5.0	0.0

Decision Epoch T=2

s/a	0	1	2	3	U2*(s)	As,2*
0	0.0	0.25	2.0	0.5	2.0	2.0
1	6.25	4.0	2.5	X	6.25	0.0
2	10.0	4.5	X	X	10.0	0.0
3	10.5	X	X	X	10.5	0.0

Decision Epoch T=1

Figure 4 Output of model at each Decision Epoch

Output

3	10.5	X	X	X	10.5	0.0

Decision Epoch T=1

s/a	0	1	2	3	U1*(s)	As,1*
0	2.0	2.0625	4.125	4.1875	4.1875	3.0
1	8.0625	6.125	6.1875	X	8.0625	0.0
2	12.125	8.1875	X	X	12.125	0.0
3	14.1875	X	X	X	14.1875	0.0

Summary

s	d*1(s)	d*2(s)	d*3(s)	V*4(s)
0	3.0	2.0	0.0	4.1875
1	0.0	0.0	0.0	8.0625
2	0.0	0.0	0.0	12.125
3	0.0	0.0	0.0	14.1875

Figure 5 Model Summary of output

Explaination

Output

Decision Epoch T=4

s/a	0	1	2	3	U4*(s)	As,4*
0	0.0	0.0	0.0	0.0	0.0	0.0
1	0.0	0.0	0.0	0.0	0.0	0.0
2	0.0	0.0	0.0	0.0	0.0	0.0
3	0.0	0.0	0.0	0.0	0.0	0.0

Figure 6 Last decision epoch T=4

At the decision epoch 4 (last decision epoch), there is no action and therefore, no reward at all.

The algorithm give the optimal expected total revenue function V*4(s) and the optimal policy $\pi^* = (d_1^*(s), d_2^*(s), d_3^*(s))$ which is represented in the table **Summary** above.

The $v_4^*(s)$ represents the expected total reward using the optimal policy when the inventory from the beginning of month is s units.

At the start of month 1, the inventory is 0 units. The manager need to order additional 3 units, otherwise he does not.

If at the start of month 2, the inventory is 2 units, he needs to order 2 units, and otherwise he does not.

He does not need to order any additional units in month 3 for inventory.