An approach to the kinematics and dynamics of a four-wheeled mecanum vehicles

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AN APPROACH TO THE KINEMATICS AND DYNAMICS OF A FOUR-WHEEL MECANUM VEHICLE

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Abstract: *The kinematics and dynamics of a mechanical system with Mecanum wheels is studied. A Mecanum wheel is a wheel with rollers attached to its circumference. Each roller rotates about an axis that forms an angle of 45° with the plane of the disk. Such a design provides additional kinematic advantages for the Mecanum wheels in comparison with the conventional wheels. Within the framework of non-holonomic mechanics, the equations of motion are derived. Using the kinematic and dynamic equations conclusions about the motion possibilities are formulated. The theoretical results are compared with experimental ones based on a four wheeled Mecanum vehicle (4WMV), made by NEXUS robot. In the future the research results will be used for the design of improved mobile systems for handicapped persons.*

Keywords: mecanum Wheel; modelling; kinematics; dynamics; nonholonomic systems.

1. INTRODUCTION

Mobility is an important property for people. Thus, its generation, conservation and, if necessary, its recovery is an important goal and a great challenge for life scientists and engineers.A very popular locomotion concept is based on wheels, so called wheeled locomotion systems. Wheels are a human invention and not found in nature. Locomotion based on wheels is still in the focus of engineers. But a wheel is no longer just a wheel in the classic sense. Different types of wheels are developed, which give this kind of locomotion systems a high maneuverability. This characteristic first of all is important for vehicles to solve its tasks. For example, the design of new wheelchairs, as one of the most commonly used assistive devices for enhancing personal mobility for physically disabled persons, includes new kinds of wheels and its topologies.

To this new class of wheels belong omnidirectional wheels. These are special designs for wheels with rollers distributed along the circumference of the wheel, seeFig. 1. When the angle between the wheel plane and the roller axis is $\pi/4$, the wheel is called a Mecanum wheel. It has previously appeared in some literature sources under the name “Swedish wheel” because this type of wheel is based on a patent from BENGT ILON, a Swedish inventor[1].

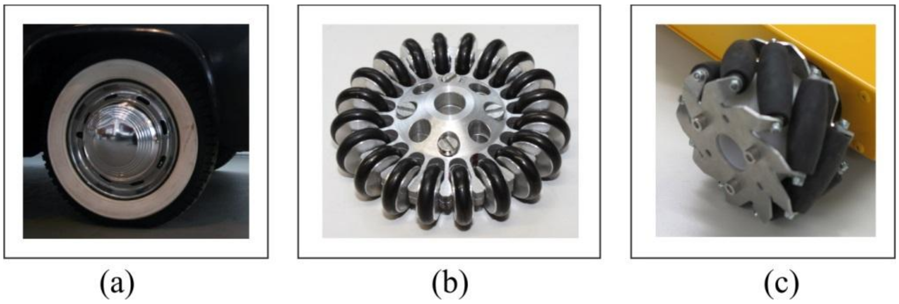

Fig. 1. Different kinds of wheel designs: (a) classical wheel, (b) classical omnidirectional wheel, (c) Mecanum wheel

ADASCALITE & DOROFTEI [2]discussed the most important applications of the Mecanum wheels in the field of mobility and industry. They presented the most common configurations of a Mecanum-wheeled vehicle as well as advantages and disadvantages and suggested solutions for overcoming negatives in performance. WAMPFLER, SALECKER & WITTENBURG [3] already in 1989 discussed the nonlinearity of Mecanum wheel kinematics. At the same time MUIR & NEUMANN [4] presented a paper about feedback control of an omnidirectional wheeled robot. They used a simplified kinematic analysis with a least-squares method for the so-called "actuated inverse solution", which leads directly to a linear kinematics. A great number of authors [5][6][7] and [8]control their Mecanum wheeled vehicles based on the strategy of MUIR & NEUMANN. They applied to the non-holonomic kinematics a certain kind of "holonomization" using pseudo-inverses matrices for solving the inverse kinematic problem.

There exists a gap in the literature regarding the consequent usage of the methods of non-holonomic mechanics for vehicles with omnidirectional wheels. Therefore, its application to a four wheeled Mecanum vehicles (4WMV) is in the focus of the presented paper. The aim is the description of the dynamical behavior of mobile robots with Mecanum wheels, moving on several non-standard trajectories.

2. BASIC PART
MECHANICAL MODEL

2.1. The kinematics of a Mecanum wheel in comparison with a classical wheel

Typically, the <u>classical wheel</u> is modeled as a thin disk with radius R rotating without slippage in the x-y-plane and with the wheel's plane perpendicular to the contact surface as shown in Fig. 2. From the kinematic side, rotation without slippage means that the instantaneous velocity of the contact point M between the wheel and the motion surface is zero, i.e. $\dot{\vec{r}}_M = \vec{0}$. Then the projections of the velocity of the contact point onto the direction lying in the wheel plane as well as onto the direction perpendicular to this plane are equal to zero,

$$\dot{\vec{r}}_M \cdot \vec{E}_x = \vec{0}\ , \quad \dot{\vec{r}}_M \cdot \vec{E}_y = \vec{0}. \tag{2.1}$$

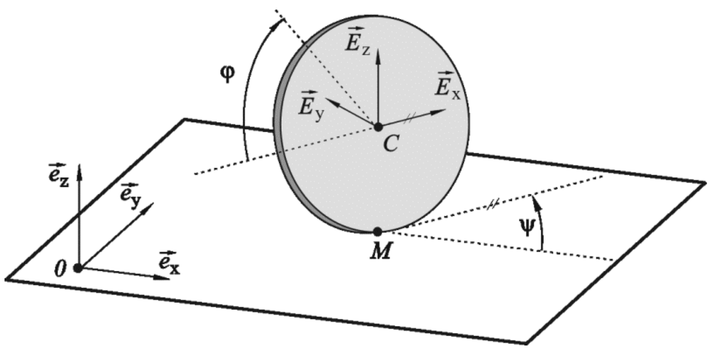

Fig. 2. The kinematics of a classical wheel in the plane

The position of the wheel can be defined using the coordinates (x_C, y_C, z_C) of the position of the wheel's center of mass C in a inertial coordinate system $\{0, \vec{e}_x, \vec{e}_y, \vec{e}_z\}$. Angle φ defines the rotation angle about the body-fixed unit vector $\vec{E}_y$ perpendicular to the wheel's plane and ψ is the angle between the wheel's plane and the unite vector $\vec{e}_x$. The non-holonomic constraints in this case are as follows

$$\dot{x}_C - R\dot{\varphi}\cos\psi = 0\,,\ \ \dot{y}_C - R\dot{\varphi}\sin\psi = 0\,, z = R\,. \tag{2.2}$$

Without taking into consideration the differential equations of motion, these constraints are not integrable.As mentioned in the introduction, the omnidirectional wheel is a wheel with a set of additional freely rotating rollers distributed along the external circumference of the wheel, see Fig. 3. The inclination of the rotation axes of these external rollers differs among designs. The first type of omnidirectional wheel in this paper is called "classical" omnidirectional wheel, in which the inclination angle of the rotation axes of the external rollers is 90°. See Fig. 3 for an overview of the classifications of omnidirectional wheels. In the following kinematic analysis, the inclination angle is denoted by δ to represent the general case of the omnidirectional wheel.

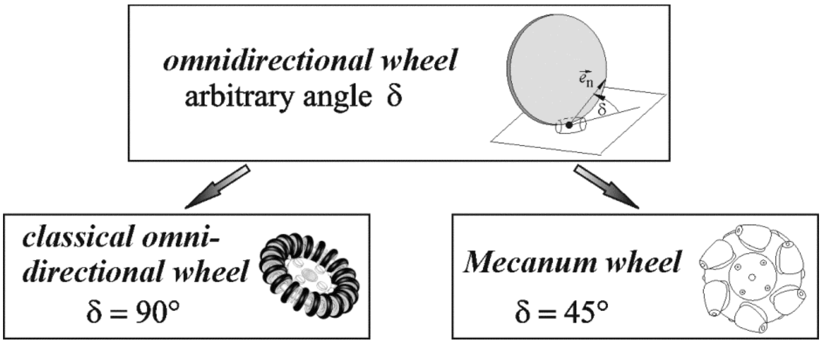

Fig. 3. Classification of omnidirectional wheels

The second type of an omnidirectional wheel is commonly known as the Mecanum wheel, in which the rotation axes of the external rollers are inclined by an angle $\delta = 45°$ to the wheel plane (Fig. 4).

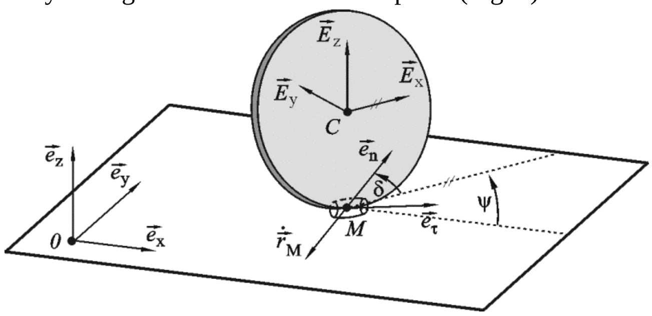

Fig. 4. A Mecanum wheel (left) and the mechanical model (right)

As a model of an omnidirectional wheel, we will consider a rolling disk of radius R centered at point C on a horizontal plane. The plane of the disk is always vertical. Let $\vec{e}_\tau$ denote the unit vector along the axes of the rollers and δ the angle between the plane of the wheel and the vector perpendicular to the roller axis (between vectors $\vec{e}_n$ and $\vec{E}_x$). Angle δ is constant. The kinematic constraint equation for an omnidirectional wheel implies that the projection of velocity vector $\dot{\vec{r}}_M$ of the point of contact M of the wheel with the plane onto the axis $\vec{e}_\tau$ is equal to zero as shown in Fig. 4. The kinematic constraint has the form

$$\dot{\vec{r}}_M \cdot \vec{e}_\tau = 0\,, \tag{2.3}$$

where velocity $\dot{\vec{r}}_M$ is defined by the equation

$$\dot{\vec{r}}_M = \dot{\vec{r}}_C + \vec{\omega} \times \overrightarrow{CM}\ , \tag{2.4}$$

and $\dot{\vec{r}}_C$ is the velocity of the center of mass C and $\vec{\omega}$ is the angular velocity of the disk. Let $\{O, \vec{e}_x, \vec{e}_y, \vec{e}_z\}$ be a fixed reference frame (inertial system) and let C be the origin of a movable reference frame (body-fixed frame) $\{C, \vec{E}_x, \vec{E}_y, \vec{E}_z\}$. Unit vectors $\vec{E}_x$ and $\vec{E}_y$ are parallel to the horizontal plane, vector $\vec{E}_z$ lies in the disk plane and vector $\vec{E}_y$ is perpendicular to this plane. Let φ be the angle of rotation of the disk about the axis passing through point C perpendicular to the plane of the disk and ψ the angle formed by the disk plane with a line parallel to vector $\vec{E}_x$ (the angle between vector $\vec{E}_x$ and vector $\vec{e}_x$) as shown in Fig. 4. Vectors $\vec{\omega}$ and $\overrightarrow{CM}$ are defined as

$$\vec{\omega} = \dot{\varphi}\vec{E}_y + \dot{\psi}\vec{E}_z\ , \qquad \overrightarrow{CM} = -R\vec{E}_z\ . \tag{2.5}$$

Let x_C, y_C and R be the coordinates of point C in the reference frame, then

$$\dot{\vec{r}}_C = (\dot{x}_C \cos\psi + \dot{y}_C \sin\psi)\vec{E}_x + (-\dot{x}_C \sin\psi + \dot{y}_C \cos\psi)\vec{E}_y\ , \qquad \vec{\omega} \times \overrightarrow{CM} = -R\dot{\varphi}\vec{E}_x\ . \tag{2.6}$$

Substituting equation (2.6) into equation (2.4) yields

$$\dot{\vec{r}}_M = (\dot{x}_C \cos\psi + \dot{y}_C \sin\psi - R\dot{\varphi})\vec{E}_x + (-\dot{x}_C \sin\psi + \dot{y}_C \cos\psi)\vec{E}_y. \tag{2.7}$$

For the mechanical configuration inFig. 4, the vector $\vec{e}_\tau$ is expressed as

$$\vec{e}_\tau = \sin\delta\, \vec{E}_x - \cos\delta\, \vec{E}_y. \tag{2.8}$$

Then, by substituting equations (2.7) and (2.8) into kinematic constraint equation (2.3) yields the following relation

$$\begin{aligned} \dot{\vec{r}}_M \cdot \vec{e}_\tau &= \dot{x}_C \cos\psi \sin\delta + \dot{y}_C \sin\psi \sin\delta - R\dot{\varphi}\sin\delta + \dot{x}_C \sin\psi \cos\delta - \dot{y}_C \cos\psi \cos\delta \\ &= \dot{x}_C \sin(\psi + \delta) - \dot{y}_C \cos(\psi + \delta) - R\dot{\varphi}\sin\delta = 0\ . \end{aligned} \tag{2.9}$$

Finally, kinematic equation (2.3) becomes:

$$\dot{x}_C \sin(\psi + \delta) - \dot{y}_C \cos(\psi + \delta) - R\dot{\varphi}\sin\delta = 0\ . \tag{2.10}$$

As mentioned before, the Mecanum wheel is a special case of an omnidirectional wheel, in which the inclination angle of the roller rotation axes is $\delta = \frac{\pi}{4}$. Therefore, the kinematic constraint in (2.10) becomes

$$\dot{x}_C \sin\left(\psi + \frac{\pi}{4}\right) - \dot{y}_C \cos\left(\psi + \frac{\pi}{4}\right) - R\,\dot{\varphi}\sin\left(\frac{\pi}{4}\right) = 0 \tag{2.11}$$

and finally the kinematic constraint on the Mecanum wheel is expressed as

$$\dot{x}_C(\cos\psi + \sin\psi) + \dot{y}_C(\sin\psi - \cos\psi) - R\dot{\varphi} = 0\ . \tag{2.12}$$

On the basis of the analysis of the kinematic constraints in equation (2.10) and in special case(2.12), in [12] it is shown, that if a mechanical system is equipped with n Mecanum wheels in such a way that:

(a) $n \geq 3$;

(b) Not all vectors $\vec{e}_{\tau_i}$ are parallel to each other;

(c) The points of contact of the wheels with the plane do not lie on one line then it is always possible to find control functions $\dot{\varphi}_i$ $(i = 1, \dots, n)$ that implement any prescribed motion of the system center of mass.

Now, we consider the 4WMV, shown in Fig. 5. The vectors $\vec{e}_x, \vec{e}_y, \vec{e}_z$ are the unit vectors of the inertial coordinate system, the vectors $\vec{E}_x, \vec{E}_y, \vec{E}_z$ are the unit vectors of the body-fixed coordinates. $O_1, O_2$are the middle points of the front and rear axles of the vehicle, respectively. C is the center of mass of the vehicle. During motion, the Mecanum wheel rotates by angular velocity $\dot{\varphi}_i$. The wheel radius R is constant for all four wheels. For the 4WMV it is assumed that the vehicle rotate about its center of mass C by angle ψ. The width of the vehicle is $2l$ and the distance between the vehicle center of mass C to the center of mass of the front axle O_1 is the distance ρ_1 and the distance between C and the center of mass of the rear axle O_2 is ρ_2.

By applying (2.12) to all four wheels respectively, we find the kinematic constraints for the 4WMV in the form

$$-\dot{x}_C(\cos\psi + \sin\psi) + \dot{y}_C(\cos\psi - \sin\psi) + \dot{\psi}(l + \rho_2) + R\dot{\varphi}_1 = 0\ , \tag{2.13}$$

$$\dot{x}_C(\cos\psi - \sin\psi) + \dot{y}_C(\cos\psi + \sin\psi) + \dot{\psi}(l + \rho_2) - R\dot{\varphi}_2 = 0\ , \tag{2.14}$$

$$\dot{x}_C(\cos\psi - \sin\psi) + \dot{y}_C(\cos\psi + \sin\psi) - \dot{\psi}(l + \rho_1) - R\dot{\varphi}_3 = 0\ , \tag{2.15}$$

$$-\dot{x}_C(\cos\psi + \sin\psi) + \dot{y}_C(\cos\psi - \sin\psi) - \dot{\psi}(l + \rho_1) + R\dot{\varphi}_4 = 0\ . \tag{2.16}$$

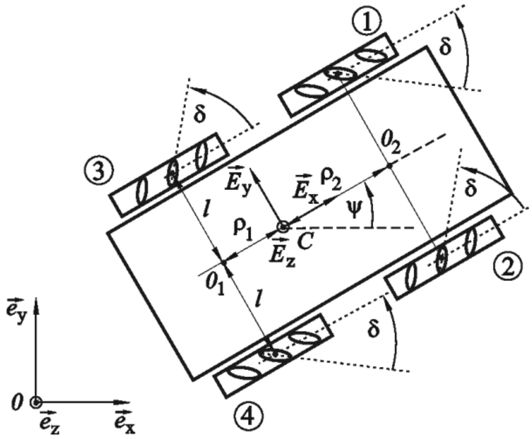

Fig. 5. A mobile robot with four Mecanum wheels (left) and the mechanical model with parameter $\delta = \frac{\pi}{4}$ (right)

Using only these kinematic equations some important information about the motion possibilities of the vehicle can be formulated. As an example, let us consider the following question. Is the motion of the vehicle kinematically possible, when a pair of wheels on the same axis does not rotate? And if yes, what is the respective trajectory?

We assume, that the wheels 3 and 4 do not rotate, i.e. $\dot{\varphi}_3 = \dot{\varphi}_4 = 0$.Then from the equations (2.15) and (2.16), we obtain

$$\dot{x}_C(\cos\psi - \sin\psi) + \dot{y}_C(\cos\psi + \sin\psi) = \dot{\psi}(l + \rho_1)\,, \tag{2.17}$$

$$-\dot{x}_C(\cos\psi + \sin\psi) + \dot{y}_C(\cos\psi - \sin\psi) = \dot{\psi}(l + \rho_1)\,. \tag{2.18}$$

From here

$$\dot{x}_C\cos\psi + \dot{y}_C\sin\psi = 0\,, \tag{2.19}$$

$$-\dot{x}_C\sin\psi + \dot{y}_C\cos\psi = \dot{\psi}(l + \rho_1)\,. \tag{2.20}$$

Using the equations (2.13) and (2.14), there is

$$\dot{\psi}(2l + \rho_1 + \rho_2) = -R\dot{\varphi}_1, \tag{2.21}$$

$$\dot{\psi}(2l + \rho_1 + \rho_2) = R\dot{\varphi}_2\,, \tag{2.22}$$

and finally, from here

$$\dot{\varphi}_1 = -\dot{\varphi}_2 = \dot{\varphi}\,. \tag{2.23}$$

That means if the wheel pair on one axis fixed is then the wheels on the other axis should rotate into opposite directions with the same angular velocity. The relation between the angular velocity of the vehicle and the wheels are given by

$$\dot{\psi} = \frac{R}{(2l + \rho_1 + \rho_2)}\dot{\varphi}\,. \tag{2.24}$$

From the equations (2.19) and (2.20), we find

$$\dot{x}_C = -\dot{\psi}(l + \rho_1)\sin\psi\,, \tag{2.25}$$

$$\dot{y}_C = \dot{\psi}(l + \rho_1)\cos\psi\,, \tag{2.26}$$

or

$$\frac{dx_C}{dt} = (l + \rho_1)\frac{d(\cos\psi)}{dt}\,, \tag{2.27}$$

$$\frac{dy_C}{dt} = (l + \rho_1)\frac{d(\sin\psi)}{dt}\,. \tag{2.28}$$

The integration of the system equations (2.27) and (2.28)with the initial conditions

$$x_C(0) = y_C(0) = \psi(0) = 0 \tag{2.29}$$

leads to the equations

$$x_C = (l + \rho_1)(\cos\psi - 1)\,, \tag{2.30}$$

$$y_C = (l + \rho_1)\sin\psi\,, \tag{2.31}$$

and then we find the relation between x_C and y_C in the form

$$(x_C + l + \rho_1)^2 + y_C{}^2 = (l + \rho_1)^2 \,. \tag{2.32}$$

In addition, it follows from the equations (2.25) and (2.26)that the velocity of the center of mass is

$$v = \sqrt{\dot{x}_C{}^2 + \dot{y}_C{}^2} = |\dot{\psi}|(l\ + \rho_1) = \frac{R(l + \rho_1)}{2l + \rho_1 + \rho_1}|\dot{\varphi}| \,. \tag{2.33}$$

Finally, we get the following information about the motion of the 4WMV. If the wheel pair on one axis (wheels 3 and 4, see Fig. 5) is fixed $\dot{\varphi}_3 = \dot{\varphi}_4 = 0$, then the wheels on the other axis (wheels 1 and 2) should rotate into opposite directions with the same angular velocity ($\dot{\varphi}_1 = -\dot{\varphi}_2 = \dot{\varphi}$). Thereby, the center of the mass of 4WMV describes a circle with radius $l + \rho_1$, as shown in Fig. 6. If the condition (2.29) is fulfilled at the beginning of motion, then the central point $M$of the circle has the coordinates $(x_M, y_M) = (-(l + \rho_1),0)$. The velocity of the center of mass on the circle is $v = \frac{R(l+\rho_1)}{2l+\rho_1+\rho_1}|\dot{\varphi}|$, thereby the vehicle rotates with the angular velocity $\dot{\psi} = \frac{R}{(2l\ +\rho_1+\rho_2)}\dot{\varphi}$ around its center of mass.

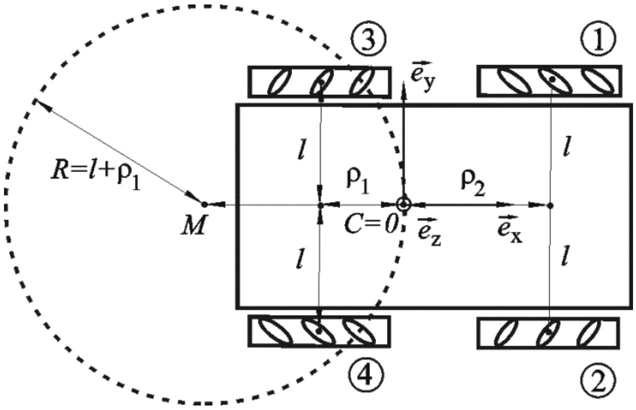

Fig. 6. The center of the mass of 4WMV describes a circle with radius $(l + \rho_1)$

2.2. Dynamic model of the 4WMV

The motion equations are derived from the framework of nonholonomic mechanics by using the LAGRANGE equations with multipliers. This equations state

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}^a}\right) - \frac{\partial T}{\partial q^a} = Q_a + \lambda_b f_a^b \,(a = 1,2, \dots, n; b = 1,2, \dots, r)\,, \tag{2.34}$$

where T is the general kinetic energy of the system, q^a are the generalized coordinates, n is the number of generalized coordinates, in this case $n = 7$ since $\underbrace{q^1 = x_C\,,\ q^2 = y_C,\ q^3 = \psi}_{vehicle}$ and $\underbrace{q^4 = \varphi_1,\ q^5 = \varphi_2,\ q^6 = \varphi_3,\ q^7 = \varphi_4}_{wheels}$, r is the number of nonholonomic kinematic constraints of the system (here $r = 4$), Q_a are the generalized forces (in this case driving moments), λ_b are the multipliers in the LAGRANGE equation. The f_a^b represents the coefficients in the constraint equations (2.13)-(2.16), which have the form

$$f_a^b(q^a) \cdot \dot{q}^a = 0,\ (a = 1,2, \dots ,7;\ b = 1,2, \dots ,4).$$

In the concrete case the equation (2.34) takes the form

$$\begin{aligned}\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{x}_C}\right) - \frac{\partial T}{\partial x_C} &= -\lambda_1(\cos\psi + \sin\psi) + \lambda_2(\cos\psi - \sin\psi) + \lambda_3(\cos\psi - \sin\psi) \\ &\quad + \lambda_4(\cos\psi + \sin\psi);\end{aligned} \tag{2.35}$$

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{y}_C}\right)-\frac{\partial T}{\partial y_C}=+\lambda_1(\cos\psi-\sin\psi)+\lambda_2(\cos\psi+\sin\psi)+\lambda_3(\cos\psi+\sin\psi) \\ +\lambda_4(\sin\psi-\cos\psi); \tag{2.36}$$

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{\psi}_C}\right)-\frac{\partial T}{\partial \psi_C}=\lambda_1(l\ +\rho_2)+\lambda_2(l\ +\rho_2)-\lambda_3(l\ +\rho_1)+\lambda_4(l\ +\rho_1); \tag{2.37}$$

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{\varphi}_1}\right)-\frac{\partial T}{\partial \varphi_1}=M_1+\lambda_1 R; \tag{2.38}$$

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{\varphi}_2}\right)-\frac{\partial T}{\partial \varphi_2}=M_2-\lambda_2 R; \tag{2.39}$$

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{\varphi}_3}\right)-\frac{\partial T}{\partial \varphi_3}=M_3-\lambda_3 R; \tag{2.40}$$

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{\varphi}_4}\right)-\frac{\partial T}{\partial \varphi_4}=M_4+\lambda_4 R. \tag{2.41}$$

The $M_i(i=1,2,3,4)$ are the torques applied to the respective wheels. The kinetic energy T is defined as the sum of the kinetic energies of the body and the wheels and is expressed as follows:

$$T=\frac{1}{2}m_0(\dot{x}_C^2+\dot{y}_C^2)+\frac{1}{2}J_0\dot{\psi}^2 \\ +\frac{1}{2}m_1\big[4(\dot{x}_C^2+\dot{y}_C^2)+(2\rho_1^2+2\rho_2^2+4l^2)\dot{\psi}^2+4\dot{x}_C\dot{\psi}\sin\psi\,(\rho_1-\rho_2) \\ +4\dot{y}_C\dot{\psi}\cos\psi\,(\rho_2-\rho_1)\big]+\frac{1}{2}J_1\dot{\varphi}_1^2+\frac{1}{2}J_1\dot{\varphi}_2^2+\frac{1}{2}J_1\dot{\varphi}_3^2+\frac{1}{2}J_1\dot{\varphi}_4^2+2J_2\dot{\psi}^2\,. \tag{2.42}$$

Here m_0 is the mass of the body, $m_1$is the mass of each of the wheels, J_0 is the mass moment of inertia of the body about the vertical axis passing though the center of mass, J_1 is the mass moment of inertia of each wheel about its axis of rotation, and J_2 is the moment of inertia of each wheel about the vertical axis passing through the center of the wheel.

Then, eliminating the multipliers λ_i $(i=1,2,3,4)$ in equations (2.35)-(2.41), we obtain

$$\ddot{x}_C\left[(m_0+4m_1)+\frac{4J_1}{R^2}\right]+\ddot{\psi}\left[2m_1\sin\psi\,(\rho_1-\rho_2)+\frac{2J_1}{R^2}\sin\psi\,(\rho_1-\rho_2)\right]= \\ -\frac{4J_1}{R^2}\dot{y}_C\dot{\psi}-2m_1\dot{\psi}^2\cos\psi\,(\rho_1-\rho_2)+\frac{(\cos\psi+\sin\psi)}{R}\cdot(M_1+M_4) \\ +\frac{(\cos\psi-\sin\psi)}{R}\cdot(M_2+M_3); \tag{2.43}$$

$$\ddot{y}_C\left[(m_0+4m_1)+\frac{4J_1}{R^2}\right]-\ddot{\psi}\left[2m_1\cos\psi\,(\rho_1-\rho_2)+\frac{2J_1}{R^2}\cos\psi\,(\rho_1-\rho_2)\right]= \\ \frac{4J_1}{R^2}\dot{x}_C\dot{\psi}-2m_1\dot{\psi}^2\sin\psi\,(\rho_1-\rho_2)-\frac{(\cos\psi-\sin\psi)}{R}\cdot(M_1+M_4) \\ +\frac{(\cos\psi+\sin\psi)}{R}\cdot(M_2+M_3); \tag{2.44}$$

$$\ddot{x}_C\left[2m_1\sin\psi\,(\rho_1-\rho_2)+\frac{2J_1}{R^2}\sin\psi\,(\rho_1-\rho_2)\right] \\ -\ddot{y}_C\left[2m_1\cos\psi\,(\rho_1-\rho_2)+\frac{2J_1}{R^2}\cos\psi\,(\rho_1-\rho_2)\right] \\ +\ddot{\psi}\left[J_C+\frac{2J_1}{R^2}\cdot[(l+\rho_2)^2+(l+\rho_1)^2]\right]=-\frac{2J_1}{R^2}\dot{x}_C\dot{\psi}\cos\psi\,(\rho_1-\rho_2) \\ -\frac{2J_1}{R^2}\dot{y}_C\dot{\psi}\sin\psi\,(\rho_1-\rho_2)+\frac{(l+\rho_2)}{R}\cdot(M_2-M_1)+\frac{(l+\rho_1)}{R}\cdot(M_4-M_3)\,. \tag{2.45}$$

As a result, for given torques $M_i(i=1,2,3,4)$, we have a system of three equations for three variables x_c, y_c and ψ. Then the angles φ_i $(i=1,2,3,4)$ of rotation of the wheels can be found from the kinematic constraints

(2.13)-(2.16). In the special case, where the center of mass of the system coincides with its geometric center (i.e. $\rho_1 = \rho_2 = \rho$) the equations (2.43), (2.44) and (2.45) can be simplified and become

$$\ddot{x}_C\left(m + \frac{4J_1}{R^2}\right) = -\frac{4J_1}{R^2}\dot{y}_C\dot{\psi} + \frac{(\cos\psi + \sin\psi)}{R}\cdot(M_1 + M_4) + \frac{(\cos\psi - \sin\psi)}{R}\cdot(M_2 + M_3)\,, \tag{2.46}$$

$$\ddot{y}_C\left(m + \frac{4J_1}{R^2}\right) = \frac{4J_1}{R^2}\dot{x}_C\dot{\psi} - \frac{(\cos\psi - \sin\psi)}{R}\cdot(M_1 + M_4) + \frac{(\cos\psi + \sin\psi)}{R}\cdot(M_2 + M_3)\,, \tag{2.47}$$

$$\ddot{\psi}\left[J_C + \frac{4J_1}{R^2}\cdot(l+\rho)^2\right] = \frac{(l+\rho)}{R}\cdot(M_2 - M_1 + M_4 - M_3)\,, \tag{2.48}$$

where $m = m_0 + 4m_1$ is the total mass of the system and $J_C = J_0 + 4[J_2 + m_1\cdot(l^2+\rho^2)]$ is the moment of inertia of the entire system relative to the center of mass.

3. NUMERICAL SIMULATIONS

For the numerical simulation the parameters of a NEXUS mobile robot with four Mecanum wheels (see Fig. 5 left) are used:

m_0 (kg)	20
m_1 (kg)	0.3
$\rho_1 + \rho_2$ (m)	0.4
l (m)	0.18
R (m)	0.05
$M_{i_{max}}$ (Nm)	0.9
$M_i(t)$ (Nm)	$\frac{M_{i_{max}}}{T}t$
Simulation time T	2 sec

The 4WMV can move along different motion trajectories. On the standard trajectories following the theory of, holonomization' the vehicle moves on straight lines forward, backward, left, right and left-forward/backward or right-forward/backward. Based on this theory complex trajectories in the plane consist necessarily of a set of these simple motion trajectories. The simulation of one standard trajectory for he case $\rho_1 = \rho_2$ is shown in Fig. 7 and the case of $\rho_1 \neq \rho_2$ is presented in Fig. 8.

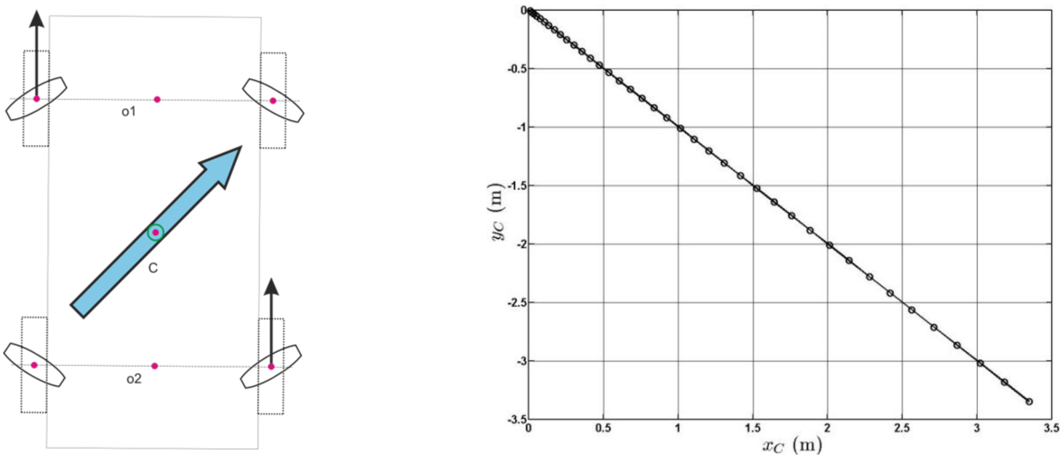

Fig. 7. Forward-right motion simulation for $\rho_1 = \rho_2$

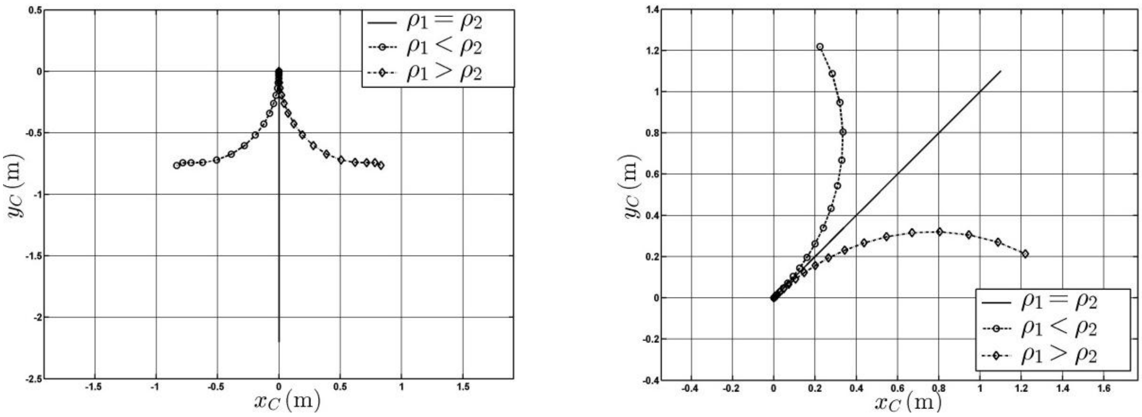

Fig. 8. Right and forward-right motion simulation for $\rho_1 = \rho_2$ and $\rho_1 \neq \rho_2$

With the equations, presented in this paper, it is also possible to discuss more realistic case, such as $\rho_1 \neq \rho_2$, i.e. the influence of the position of the center of mass can be seen.

The simulation of a complex trajectory, using the simulation environment of Simulink® 3D animation toolbox is shown in Fig. 9, left. A part of the real trajectory of a 4WMV is presented in Fig. 10.

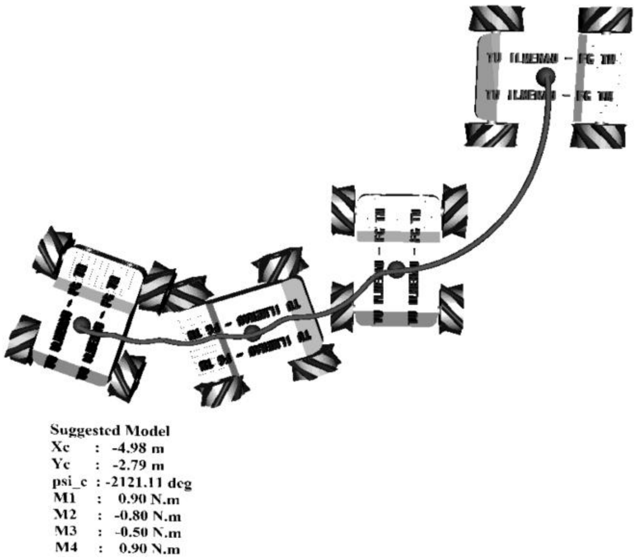

Fig. 9. Simulation of a complex trajectory

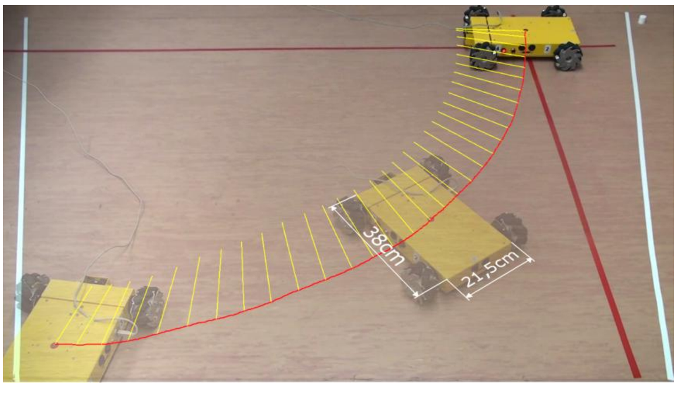

Fig. 10. The first part of the trajectory from Fig. 9 resulted from a motion sequence of a moving NEXUS mobile robot

4. SUMMARY AND OUTLOOK

The equations of motion for a vehicle with 4 mecanum wheels (4WMV) were derived estimated from a non-holonomic model for mecanum wheels. By comparing the non-holonomic model with an approximation model used in robotics (procedure of ‚holonomizarion') the authors found that both models lead to the same result in the particular cases for which the vehicle either moves translationally or rotates about its center of mass. Using only the constraint equations some important conclusions about the motion possibilities of the system are made. The results ofthe numerical integration of the equations of motion are compared with experimental results based on a mobile robot with four Mecanum wheels. The research results will be the first step in developing new concepts for assistant systems for disabled persons.

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ИССЛЕДОВАНИЕ КИНЕМАТИКИ И ДИНАМИКИ МЕХАНИЧЕСКОЙ СИСТЕМЫ С МЕКАНУМ-КОЛЕСАМИ

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Резюме: Рассматривается кинематика и динамика механической системы с меканум-колесами. Меканум-колесо – это колесо с роликами, расположенными по ободу колеса. Каждый ролик может вращаться вокруг оси, составляющей угол 45 градусов с плоскостью диска колеса. Такая конструкция обеспечивает меканум-колесам, по сравнению с обычными колесами, дополнительные кинематические возможности. В рамках неголономной механики получены уравнения движения рассматриваемой системы. На основании анализа кинематических и динамичеких уравнений сделаны заключения о возможностях различных видов движений. Результаты теоретических исследований сравниваются с результатами эксперимента, проведенными на базе четырехколесного робота с меканум-колесами фирмы NEXUS. Представленные научные разработки предполагается использовать для создания улучшенных средств передвижения инвалидов.