Logistic Regression cost function:

$$J(\theta) = \frac{1}{m}\sum_{i=1}^{m} Cost(h_\theta(x^{(i)}), y^{(i)})$$

$$Cost(h_\theta(x), y) = \begin{cases} -\log(h_\theta(x)) & \text{if } y=1 \\ -\log(1-h_\theta(x)) & \text{if } y=0 \end{cases}$$

Note ⇒ $y = 0$ or 1 always

$$Cost(h_\theta(x), y) = -y\log(h_\theta(x)) - (1-y)\log(1-h_\theta(x))$$

If $y=1 \Rightarrow Cost(h_\theta(x), y) = \boxed{-\log(h_\theta(x))} - 0$

$\boxed{1-y=0}$

If $y=0 \Rightarrow Cost(h_\theta(x), y) = 0 - (1-0)\log(1-h_\theta(x))$

$\boxed{y=0}$

$$Cost(h_\theta(x), y) = \boxed{-\log(1-h_\theta(x))}$$

∴, $J(\theta) = \frac{1}{m}\sum_{i=1}^{m} Cost(h_\theta(x^{(i)}), y^{(i)})$

$$= -\frac{1}{m}\left[\sum_{i=1}^{m}\left[y^{(i)}\log(h_\theta(x^{(i)})) + (1-y^{(i)})\log(1-h_\theta(x^{(i)}))\right]\right]$$

To fit Parameters θ: $\min_\theta J(\theta)$

To make a prediction given new x: Output $h_\theta(x) = \frac{1}{1+e^{-\theta^T x}}$

$P(y=1|x;\theta)$