

$$\textcircled{1} \quad \therefore, z^T z = \sum_i z_i^2$$

$$\frac{1}{2} (X\theta - \vec{y})^T (X\theta - \vec{y}) = \frac{1}{2} \sum_{i=1}^m (h_\theta(x^{(i)}) - y^{(i)})^2$$

$$= J(\theta)$$

$$\therefore, \nabla_{A^T \text{tr} A B A^T C} = B^T A^T C^T + B A^T C$$

$$\nabla_\theta J(\theta) = \nabla_\theta \frac{1}{2} (X\theta - \vec{y})^T (X\theta - \vec{y})$$

$$= \frac{1}{2} \nabla_\theta (\theta^T X^T - \vec{y}^T) (X\theta - \vec{y})$$

$$= \frac{1}{2} \nabla_\theta (\theta^T X^T X \theta - \theta^T X^T \vec{y} - \vec{y}^T X \theta + \vec{y}^T \vec{y})$$

$$= \frac{1}{2} \nabla_\theta \text{tr} (\theta^T X^T X \theta - \theta^T X^T \vec{y} - \vec{y}^T X \theta + \vec{y}^T \vec{y})$$

$$= \frac{1}{2} \nabla_\theta (\text{tr} \theta^T X^T X \theta - 2 \text{tr} \vec{y}^T X \theta) \quad ??$$

$$= \frac{1}{2} (X^T X \theta + X^T X \theta - 2 X^T \vec{y})$$

$$= X^T X \theta - X^T \vec{y}$$

$$\nabla_\theta J(\theta) \stackrel{\text{set}}{=} \vec{0} \text{ for } \theta \text{ st } J(\theta) \text{ is min}$$

$$\nabla_\theta J(\theta) = X^T X \theta - X^T \vec{y} = 0$$

$$X^T X \theta - X^T \vec{y} = 0$$

$$\boxed{\theta = X^T \vec{y} (X^T X)^{-1}}$$