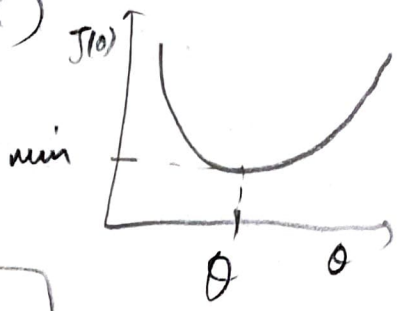
NORMAL EQⁿ : Method to solve for θ analytically

Intuition : If $1D$ $(\theta \in \mathbb{R})$

$$J(\theta) = a\theta^2 + b\theta + c$$



$$\frac{d}{d\theta} J(\theta) = 2a\theta + b \overset{\text{set}}{=} 0$$

Solve for, $\boxed{\theta = \frac{-b}{2a}}$

$\theta \in \mathbb{R}^{n+1}$ $J(\theta_0, \theta_1, \theta_2, \cdots, \theta_n) = \frac{1}{2m} \sum_{i=1}^{m} (h_\theta(x^{(i)}) - y^{(i)})^2$

$\frac{\partial}{\partial \theta_j} J(\theta) \overset{\text{set}}{=} 0$ (for every j)

then solve for $\theta_0, \theta_1, \theta_2, \cdots, \theta_n$

Eg) $m = 4$

x_0	Size (feet²) x_1	# bedrooms x_2	# floors x_3	Age x_4	Price y
1	2104	5	1	45	460
1	1416	3	2	40	232
1	1534	3	2	30	315
1	852	2	1	36	178

$m = 4$

$$X = \begin{bmatrix} 1 & 2104 & 5 & 1 & 45 \\ 1 & 1416 & 3 & 2 & 40 \\ 1 & 1534 & 3 & 2 & 30 \\ 1 & 852 & 2 & 1 & 36 \end{bmatrix}_{m \times (n+1)} \quad y = \begin{bmatrix} 460 \\ 232 \\ 315 \\ 178 \end{bmatrix}_m$$

$$\boxed{\theta = (X^T X)^{-1} X^T y}$$

normal eqⁿ