Steel Beam Moment Strength

Flexural design strength of a steel wide-flange beam section.

Assumptions

[ASSUME] AISC 14th Edition controls design

[ASSUME] Beam web is unstiffened

[ASSUME] Beam design is not controlled by deflection requirements

Inputs

Beam ultimate moment demand;	$M_u = 30\ \mathrm{kip-ft}$	
Beam unbraced length;	$L_b = 20\ \mathrm{ft}$	
Steel yield strength;	$F_y = 50\ \mathrm{ksi}$	
Modulus of elasticity;	$E = 29000\ \mathrm{ksi}$	
Lateral-torsional buckling modification factor;	$C_b = 1$	[AISC F1(3)]

Section Properties

$$section = W12X26$$

$$S_x = 33.4\ \mathrm{in}^3$$

$$Z_x = 37.2\ \mathrm{in}^3$$

$$r_y = 1.51\ \mathrm{in}$$

$$r_{ts} = 1.75\ \mathrm{in}$$

$$J = 0.3\ \mathrm{in}^4$$

$$h_o = 11.8\ \mathrm{in}$$

$$b_f/2t_f = 8.54$$

$$h/t_w = 47.2$$

1. Beam Flexural Capacity

Flexural resistance factor

$\phi_b = 0.9$ [AISC F1(1)]

1.1. Section Compactness

$$\lambda_{pf} = 0.38 \cdot \sqrt{\frac{E}{F_y}} = 0.38 \ \cdot \sqrt{\frac{29000 \text{ ksi}}{50 \text{ ksi}}}$$ [AISC Table B4.1b(10)]

$$\therefore \lambda_{pf} = 9.152$$

$$\begin{aligned} Check\, b_f/2t_f \ &\leq \ \lambda_{pf} \\ 8.54 \ &\leq \ 9.152 \\ &\therefore CompactFlange \end{aligned}$$

$$\lambda_{pw} = 3.76 \cdot \sqrt{\frac{E}{F_y}} = 3.76 \ \cdot \sqrt{\frac{29000 \text{ ksi}}{50 \text{ ksi}}}$$ [AISC Table B4.1b(15)]

$$\therefore \lambda_{pw} = 90.55$$

$$\begin{aligned} Check\, h/t_w \ &\leq \ \lambda_{pw} \\ 47.2 \ &\leq \ 90.55 \\ &\therefore CompactWeb \end{aligned}$$

1.2. Plastic Moment Strength

Nominal plastic moment strength

$$M_p = \frac{F_y \cdot Z_x}{12 \text{ in/ft}} = \frac{50 \text{ ksi} \cdot 37.2 \text{ in}^3}{12 \text{ in/ft}}$$ [AISC Eq. F2-1]

$$\therefore M_p = 155 \text{ kip} - \text{ft}$$

1.3. Yielding Strength

$$M_{ny} = \frac{F_y \cdot Z_x}{12\ \text{in/ft}} = \frac{50\ \text{ksi} \cdot 37.2\ \text{in}^3}{12\ \text{in/ft}}$$

[AISC Eq. F2-1]

$$\therefore M_{ny} = 155\ \text{kip} - \text{ft}$$

1.4. Lateral-Torsional Buckling

$$L_p = \frac{1.76 \cdot r_y \cdot \sqrt{\frac{E}{F_y}}}{12\ \text{in/ft}}$$

$$= \frac{1.76\ \cdot 1.51\ \text{in} \cdot \sqrt{\frac{29000\ \text{ksi}}{50\ \text{ksi}}}}{12\ \text{in/ft}}$$

[AISC Eq. F2-5]

$$\therefore L_p = 5.334\ \text{ft}$$

$$c = 1$$

[AISC Eq. F2-8a]

$$L_r = \frac{\frac{1.95 \cdot r_{ts}}{12\ \text{in/ft}} \cdot E}{0.7 \cdot F_y} \cdot \sqrt{\frac{J \cdot c}{S_x \cdot h_o} + \sqrt{\left(\frac{J \cdot c}{S_x \cdot h_o}\right)^2 + 6.76 \cdot \left(\frac{0.7 \cdot F_y}{E}\right)^2}}$$

$$= \frac{\frac{1.95\ \cdot 1.75\ \text{in}}{12\ \text{in/ft}} \cdot 29000\ \text{ksi}}{0.7\ \cdot 50\ \text{ksi}} \cdot \sqrt{\frac{0.3\ \text{in}^4 \cdot 1}{33.4\ \text{in}^3 \cdot 11.8\ \text{in}} + \sqrt{\left(\frac{0.3\ \text{in}^4 \cdot 1}{33.4\ \text{in}^3 \cdot 11.8\ \text{in}}\right)^2 + 6.76\ \cdot \left(\frac{0.7\ \cdot 50\ \text{ksi}}{29000\ \text{ksi}}\right)^2}}$$

[AISC Eq. F2-6]

$$\therefore L_r = 14.88\ \text{ft}$$

$$\rightarrow L_b\ >\ L_r$$

$$F_{cr} = \frac{C_b \cdot (\pi)^2 \cdot E}{\left(\frac{L_b \cdot 12\ \text{in/ft}}{r_{ts}}\right)^2} + \sqrt{1 + \frac{0.078 \cdot J \cdot c}{S_x \cdot h_o} \cdot \left(\frac{L_b \cdot 12\ \text{in/ft}}{r_{ts}}\right)^2}$$

$$= \frac{1\ \cdot (3.142\ )^2\ \cdot 29000\ \text{ksi}}{\left(\frac{20\ \text{ft} \cdot 12\ \text{in/ft}}{1.75\ \text{in}}\right)^2} + \sqrt{1\ + \frac{0.078\ \cdot 0.3\ \text{in}^4 \cdot 1}{33.4\ \text{in}^3 \cdot 11.8\ \text{in}} \cdot \left(\frac{20\ \text{ft} \cdot 12\ \text{in/ft}}{1.75\ \text{in}}\right)^2}$$

[AISC Eq. F2-4]

$$\therefore F_{cr} = 16.67\ \text{ksi}$$

$$M_{ncr} = \frac{F_{cr} \cdot S_x}{12 \text{ in/ft}} = \frac{16.67 \text{ ksi} \cdot 33.4 \text{ in}^3}{12 \text{ in/ft}}$$

[AISC F2.2(c)]

$$\therefore M_{ncr} = 46.41 \text{ kip} - \text{ft}$$

$$M_{nltb} = \min\left(M_{ncr}, M_p\right) = \min\left(46.41 \text{ kip} - \text{ft}, 155 \text{ kip} - \text{ft}\right)$$

[AISC Eq. F2-3]

$$\therefore M_{nltb} = 46.41 \text{ kip} - \text{ft}$$

1.5. Controlling Strength

Design flexural strength of the section

$$\phi M_n = \phi_b \cdot \min\left(M_{ny}, M_{nltb}\right) = 0.9 \ \cdot \min\left(155 \text{ kip} - \text{ft}, 46.41 \text{ kip} - \text{ft}\right)$$

$$\therefore \phi M_n = 41.77 \text{ kip} - \text{ft}$$

$$Check\ M_u \ \leq \ \phi M_n$$

$$30 \text{ kip} - \text{ft} \ \leq \ 41.77 \text{ kip} - \text{ft}$$

$$\therefore OK$$