5.1.1. Nhân chuỗi ma trận: O(n^3)

$$m[i,j]=\begin{cases}0, \text{ if } i=j\\ \min\{m[i,k]+m[k+1,j]+p_{i-1}p_kp_j\}, \text{ if } i<j, i\le k<j\end{cases}$$

- $p_0=30; p_1=35; p_2=15; p_3=5; p_4=10; p_5=20; p_6=25$
- Compute tables m and s:

$$m[1,5]=\min\begin{cases}m[1,1]+m[2,5]+p_0p_1p_5=0+7125+30.35.20=28125\\ m[1,2]+m[3,5]+p_0p_2p_5=15750+2500+30.15.20=27250\\ m[1,3]+m[4,5]+p_0p_3p_5=7875+1000+30.5.20=\mathbf{11875}\\ m[1,4]+m[5,5]+p_0p_4p_5=9375+0+30.10.20=15375\end{cases}$$

$\rightarrow k=3 \rightarrow s[1,5]=3$

$$m[2,6]=\min\begin{cases}m[2,2]+m[3,6]+p_1p_2p_6=0+5375+35.15.25=18500\\ m[2,3]+m[4,6]+p_1p_3p_6=2625+3500+35.5.25=\mathbf{10500}\\ m[2,4]+m[5,6]+p_1p_4p_6=4375+5000+35.10.25=18125\\ m[2,5]+m[6,6]+p_1p_5p_6=7125+0+35.20.25=24625\end{cases}$$

$\rightarrow k=3 \rightarrow s[2,6]=3$

Table s

	1	2	3	4	5	6
6	3	3	3	5	5	-
5	3	3	3	4	-	
4	3	3	3	-		
3	1	2	-			
2	1	-				
1	-					

How do we derive the optimal parenthesization from table s?

Result:

(A1 (A2 A3)) ((A4 A5) A6)

5.1.2. Chuỗi con chung dài nhất (LCS)

$$c[i,j]=\begin{cases}0, \text{ if } i=0\vee j=0\\ c[i-1,j-1]+1, \text{ if } i,j>0\wedge x_i=y_j\\ \max(c[i,j-1],c[i-1,j]), \text{ if } i,j>0\wedge x_i=y_j\end{cases}$$

$$C_{LCS}=2mn+m+n=O(mn)$$

	y_j	B	D	C	A	B	A
x_i	0	0	0	0	0	0	0
A	0	0 ↑	0 ↑	0 ↑	1 ↖	1 ←	1 ↖
B	0	1 ↖	1 ←	1 ←	1 ↑	2 ↖	2 ←
C	0	1 ↑	1 ↑	2 ↖	2 ←	2 ↑	2 ↑
B	0	1 ↖	1 ↑	2 ↑	2 ↑	3 ↖	3 ←
D	0	1 ↑	2 ↖	2 ↑	2 ↑	3 ↑	3 ↑
A	0	1 ↑	2 ↑	2 ↑	3 ↖	3 ↑	4 ↖
B	0	1 ↖	2 ↑	2 ↑	3 ↑	4 ↖	4 ↑

Tables

c[0..7, 0..6]

b[1..7, 1..6]

LCS =

<B, C, B, A>

5.1.3. Cái túi (0-1)

$$\cos t[i]=\begin{cases}\max\{\text{cost}[i],\text{cost}[i-size[j]]+value[j]\} \text{ if } i-size[j]\ge 0\\ \text{unchange if } i-size[j]<0\end{cases}$$

$$C_{knapsack}=2MN+M=O(MN)$$

Không giới hạn

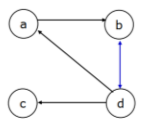
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
A	size	=	3		val	=	4										
cost	0	0	4	4	4	8	8	8	12	12	12	16	16	16	20	20	20
best	-	-	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A
B	size	=	4		val	=	5										
cost	0	0	4	5	5	8	9	10	12	13	14	16	17	18	20	21	22
best	-	-	A	B	B	A	B	B	A	B	B	A	B	B	A	B	B
C	size	=	7		val	=	10										
cost	0	0	4	5	5	8	10	10	12	14	15	16	18	20	20	22	24
best	-	-	A	B	B	A	C	B	A	C	C	A	C	C	A	C	C
D	size	=	8		val	=	11										
cost	0	0	4	5	5	8	10	11	12	14	15	16	18	20	21	22	24
best	-	-	A	B	B	A	C	D	A	C	C	A	C	C	D	C	C
E	size	=	9		val	=	13										
cost	0	0	4	5	5	8	10	11	13	14	15	17	18	20	21	23	24
best	-	-	A	B	B	A	C	D	E	C	C	E	C	C	D	E	C

Giới hạn mỗi loại 1 cái

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
A	size	=	3		val	=	4										
cost	0	0	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4
best	-	-	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A
B	size	=	4		val	=	5										
cost	0	0	4	5	5	5	9	9	9	9	9	9	9	9	9	9	9
best	-	-	A	B	B	B	B	B	B	B	B	B	B	B	B	B	B
C	size	=	7		val	=	10										
cost	0	0	4	5	5	5	10	10	10	14	15	15	15	19	19	19	19
best	-	-	A	B	B	B	C	C	C	C	C	C	C	C	C	C	C
D	size	=	8		val	=	11										
cost	0	0	4	5	5	5	10	11	11	14	15	16	16	19	21	21	21
best	-	-	A	B	B	B	C	D	D	C	C	D	D	C	D	D	D
E	size	=	9		val	=	13										
cost	0	0	4	5	5	5	10	11	13	14	15	17	18	19	21	23	24
best	-	-	A	B	B	B	C	D	E	C	C	E	E	C	D	E	E

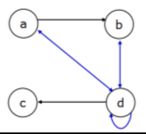
5.1.4. Warshall - bao đóng truyền $O(|V^3|)$

Given a directed graph, what is its transitive closure?



A^1 =

	a	b	c	d
a	0	1	0	0
b	0	0	0	1
c	0	0	0	0
d	1	1	1	0



A^2 =

	a	b	c	d
a	0	1	0	1
b	0	0	0	1
c	0	0	0	0
d	1	1	1	1

5.1.5. Floyd – bao đóng truyền $O(|V^3|)$

A^2

	a	b	c	d
a	0	∞	3	∞
b	2	0	5	∞
c	9	7	0	1
d	6	∞	9	0

P^2

	a	b	c	d
a	0	0	0	0
b	0	0	1	0
c	2	0	0	0
d	0	0	1	0

A^3

	a	b	c	d
a	0	10	3	4
b	2	0	5	6
c	9	7	0	1
d	6	16	9	0

P^3

	a	b	c	d
a	0	3	0	3
b	0	0	1	3
c	2	0	0	0
d	0	3	1	0

What is the shortest path from *c* to *a*?
→ path(c,a)
→ path(c,d), *d*, path(d,a)
→ *c, d, a*

A^4

	a	b	c	d
a	0	10	3	4
b	2	0	5	6
c	7	7	0	1
d	6	16	9	0

P^4

	a	b	c	d
a	0	3	0	3
b	0	0	1	3
c	4	0	0	0
d	0	3	1	0

5.2.1. Bài toán chọn hoạt động

Sắp thứ tự thời gian kết thúc các hoạt động, chọn hoạt động 1, chọn các hoạt động tiếp theo thỏa thời gian bắt đầu sau (>=) khi hoạt động trước kết thúc. Không sắp xếp: *O(n)*. Sắp xếp: *O(nlog₂n)*

5.2.2. Cái túi dạng phân số

- With the knapsack M = 50 and the following items:
  - W[1] = 10, V[1] = 60, W[2] = 20, V[2] = 100, W[3] = 30, V[3] = 120, W[4] = 40, V[4] = 120
- What is the combination of items that makes the total value of the knapsack the highest?

$$\frac{V[1]}{W[1]}=6,\quad \frac{V[2]}{W[2]}=5,\quad \frac{V[3]}{W[3]}=4,\quad \frac{V[4]}{W[4]}=3$$

$\rightarrow$ Pick item 1, remaining capacity $= M - W[1] = 40$

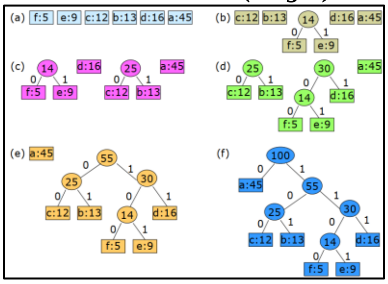
$\rightarrow$ Pick item 2, remaining capacity $= 40 - W[2] = 20$

$\rightarrow$ Pick $\left(\frac{2}{3}\right) *$ item 3, remaining capacity $= 20-\left(\frac{2}{3}\right)*W[3]=0$

$\rightarrow X = \{1, 1, 2/3, 0\}$

Không sắp xếp: *O(n)* - Sắp xếp: *O(nlog₂n)*

5.2.3. Mã Huffman *O(nlog₂n)*



- Given the following characters and their frequencies in textual files.

Character	a	b	c	d	e	f
Frequency	45	13	12	16	9	5

Huffman code:
a = 0
b = 101
c = 100
d = 111
e = 1101
f = 1100

Averaged code length = (1*45 + 3*13 + 3*12 + 3*16 + 4*9 + 4*5)/100 = 2.24 bits/character

Fixed-length code:
code length = $\lceil \log_2 6 \rceil$ = 3 bits/character
Requires: 200,000*3 = 600,000 bits

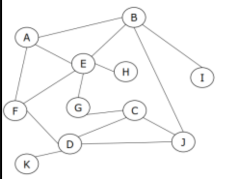
Huffman code:
Requires: 200,000*2.24 = 448,000 bits

Saved: (3-2.24)/3 = 25.33%

15

5.2.4. Tô màu đồ thị: m*O(n^2)

- Given a graph as follows. Conduct graph coloring with Welsh and Powell's heuristic. How many colors have been used?



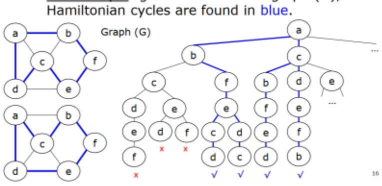
sort
E: 5
B: 4
D: 4
A: 3
C: 3
F: 3
J: 3
G: 2
H: 1
I: 1
K: 1

coloring
Color E, D, I with blue
Color B, C, F, H, K with green
Color A, J, G with pink

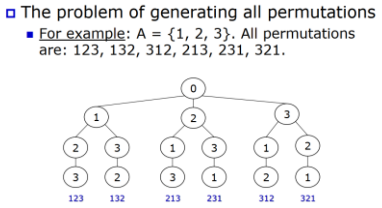
13

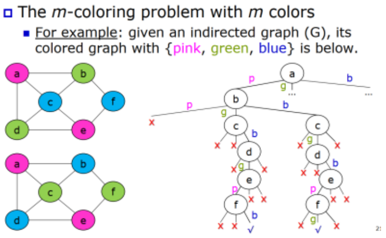
6.1. GT quay lui: O(a^n)

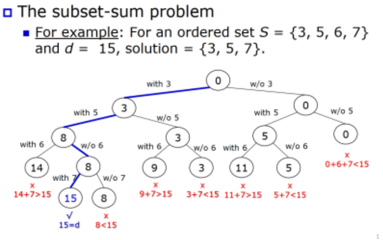
- The problem of finding a Hamiltonian cycle in an indirected graph
  - For example: given an indirected graph (G), one Hamiltonian cycles are found in blue.



16





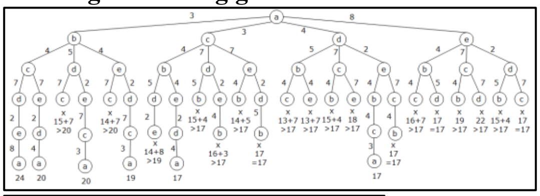


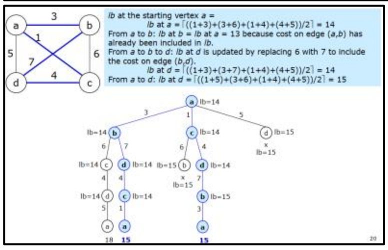
6.1.1. Quân mã

	y = 1	y = 2	y = 3	y = 4	y = 5
x = 1		**3** (x-2,y-1)		**2** (x-2,y+1)	
x = 2	**4** (x-1,y-2)				**1** (x-1,y+2)
x = 3			⊕ (x,y)		
x = 4	**5** (x+1,y-2)				**8** (x+1,y+2)
x = 5		**6** (x+2,y-1)		**7** (x+2,y+1)	

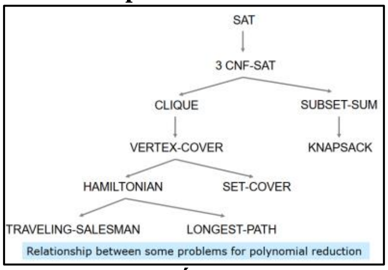
6.1.2. Tám con hậu: Đặt quân hậu theo cột, nếu không có lời giải thì quay lui dời xuống hàng dưới.

6.2.1. Người thương gia du hành





7. NP Complete



Relationship between some problems for polynomial reduction

8.1 Giải thuật xấp xỉ

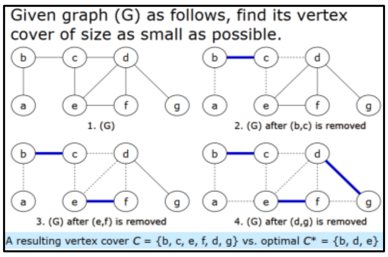
Performance bound, càng nhỏ càng tốt:

- Bài toán tối thiểu: $c(i)/c^*(i)$
- Bài toán tối đa: $c^*(i)/c(i)$

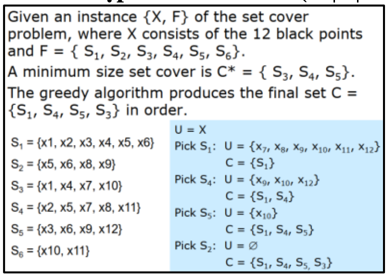
Cận tỉ số: $\max(c(i)/c^*(i), c^*(i)/c(i)) \le p(n)$, >=1, =1 là tối ưu

Cận sai số tương đối: $|c(i)-c^*(i)|/c^*(i) \le \varepsilon(n)$, >=0, =0 là tối ưu

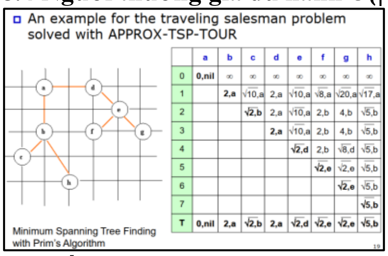
8.2 Phủ đỉnh Ratio bound = 2



8.3 Phủ tập Ratio bound = $(\ln|X| + 1) = H(\max\{|S|: S \in F\})$



8.4 Người thương gia du hành $O(|V|^2)$**:** Ratio bound = 2



8.5 Xếp lịch công tác: Chọn task có thời gian dài nhất đưa vô processor trống. $|F^*(I)-F(I)|/F^*(I) \le 1/3 - 1/(3^*processsor)$

8.6 Đóng thùng: First fit (FF), Best fit (BF), First fit Decreasing (FFD), Best fit Decreasing (BFD)