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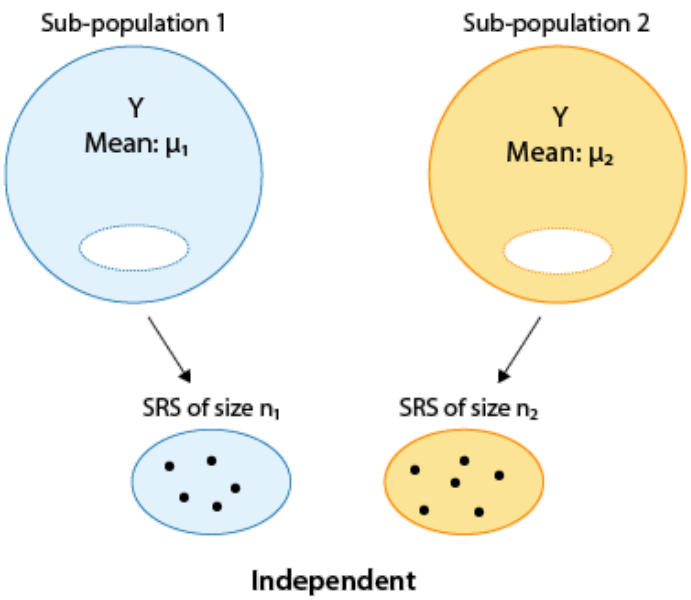
Two Independent Samples: Hypotheses

Learning Objective: In a given context, carry out the inferential method for comparing groups and draw the appropriate conclusions.

Learning Objective: Specify the null and alternative hypotheses for comparing groups.

The Two-Sample t-Test

Here again is the general situation which requires us to use the two-sample t-test:



Our goal is to compare the means μ_1 and μ_2 based on the two independent samples.

Step 1: Stating the Hypotheses

The hypotheses represent our goal, comparing the means: μ_1 and μ_2 .

The null hypothesis has the form:

- $H_o : \mu_1 - \mu_2 = 0$ (which is the same as $H_o : \mu_1 = \mu_2$)

The alternative hypothesis takes one of the following three forms (depending on the context):

- $H_a : \mu_1 - \mu_2 < 0$ (which is the same as $H_a : \mu_1 < \mu_2$) (one-sided)
- $H_a : \mu_1 - \mu_2 > 0$ (which is the same as $H_a : \mu_1 > \mu_2$) (one-sided)
- $H_a : \mu_1 - \mu_2 \neq 0$ (which is the same as $H_a : \mu_1 \neq \mu_2$) (two-sided)

Note that the null hypothesis claims that there is no difference between the means, which can either represented as the difference is 0 (no difference), or as its (algebraically and conceptually) equivalent, $\mu_1 = \mu_2$ (the means are equal). Either way, conceptually, H_o claims that there is no relationship between the two relevant variables.

The first way of writing the hypotheses (using a difference between the means) will be easier to use when (in the future) we look for a difference that is not 0.

Each one of the three alternatives claims that there is a difference between the means. The two one-sided alternatives specify the nature of the difference; either negative, indicating that μ_1 is smaller than μ_2, or positive, indicating that μ_1 is larger than μ_2. The two-sided alternative, as usual, is more general and simply claims that a difference exists. As before, it should be clear from the context of the problem which of the three alternatives is appropriate.

Comment

Note that our parameter of interest in this case (the parameter about which we are making an inference) is the difference between the means $\mu_1 - \mu_2$, and that the null value is 0.

Example

Recall that the purpose of this survey was to examine whether the opinions of females and males **differ** with respect to the importance of looks vs. personality. The hypotheses in this case are therefore:

$$H_0 : \mu_1 - \mu_2 = 0$$
$$H_a : \mu_1 - \mu_2 \neq 0$$

where μ_1 represents the mean importance for females and μ_2 represents the mean importance for males.

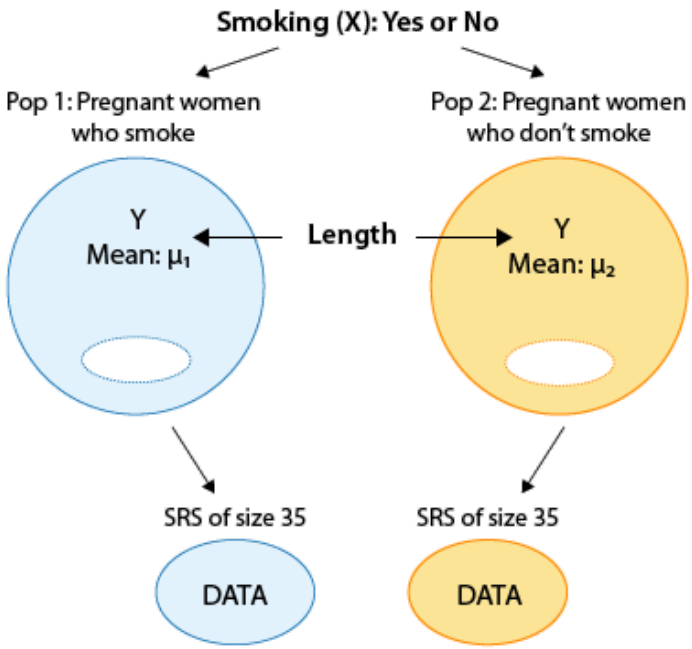
It is important to understand that conceptually, the two hypotheses claim:

H_o: Score (of looks vs. personality) is not related to gender

H_a: Score (of looks vs. personality) is related to gender

Scenario: Pregnancy and Smoking

In order to check the claim that the pregnancy length of women who smoke during pregnancy is shorter, on average, than the pregnancy length of women who do not smoke, a random sample of 35 pregnant women who smoke and a random sample of 35 pregnant women who do not smoke were chosen and their pregnancy lengths were recorded. Here is a figure of this example:



Did I Get This

1/1 point (graded)
What is the null hypothesis in this case?

- (●) $\mu_1 - \mu_2 = 0$ ✔
- ( ) $\mu_1 - \mu_2 < 0$
- ( ) $\mu_1 - \mu_2 > 0$
- ( ) $\mu_1 - \mu_2 \neq 0$

Answer
Correct:
Indeed, in the two-sample case, the null hypothesis is always H_o: $\mu_1 - \mu_2 = 0$ (which can also be written as H_o: $\mu_1 = \mu_2$).

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Did I Get This

1/1 point (graded)
Which of the following statements about the null hypothesis is correct?

- ( ) The null hypothesis in this case claims that pregnancy length **is** related to (or affected by) whether or not the woman smokes during pregnancy.
- (●) The null hypothesis in this case claims that pregnancy length **is not** related to (or affected by) whether or not the woman smokes during pregnancy. ✔

Answer
Correct:
Indeed, H_o: $\mu_1 - \mu_2 = 0$ claims that the mean pregnancy length of women who smoke is equal to that of women who do not smoke. In other words, the null hypothesis claims that pregnancy length is not affected by whether or not the woman smokes during pregnancy.

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Did I Get This

1/1 point (graded)
What is the alternative hypothesis in this case?

- ( ) $\mu_1 - \mu_2 = 0$

- ● $\mu_1 - \mu_2 < 0$ ✔
- ○ $\mu_1 - \mu_2 > 0$
- ○ $\mu_1 - \mu_2 \neq 0$

Answer
Correct:
Indeed, we want to test the claim that the mean pregnancy length of women who smoke (μ_1) is smaller than the mean pregnancy length of women who do not smoke (μ_2). Therefore, the alternative hypothesis in this case is H_a: μ_1 < μ_2 or H_a: $\mu_1 - \mu_2$ < 0.

Submit

Did I Get This

1/1 point (graded)
Which of the following statements about the alternative hypothesis is correct?

- ● The alternative hypothesis in this case claims that pregnancy length **is** related to (or affected by) whether or not the woman smokes during pregnancy. ✔
- ○ The alternative hypothesis in this case claims that pregnancy length **is not** related to (or affected by) whether or not the woman smokes during pregnancy.

Answer
Correct:
Indeed, H_a: μ_1 - μ2 < 0 claims that the mean pregnancy length of women who smoke is smaller than that of women who do not smoke. This means that pregnancy length is affected by whether or not the woman smokes during pregnancy.

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