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Introduction to Normal Random Variables: Standard Deviation Rule

Learning Objective: Find probabilities associated with the normal distribution.

The Standard Deviation Rule for Normal Random Variables

We began to get a feel for normal distributions in the Exploratory Data Analysis (EDA) section, when we introduced the Standard Deviation Rule (or the **68-95-99.7** rule) for how values in a normally-shaped **sample data set** behave relative to their mean ($\overline{x}$) and standard deviation (s). This is the same rule that dictates how the distribution of a normal **random variable** behaves relative to its mean μ and standard deviation σ. Now we use probability language and notation to describe the random variable's behavior. For example, in the EDA section, we would have said "68% of pregnancies in our data set fall within 1 standard deviation (s) of their mean ($\overline{x}$)." The analogous statement now would be "If X, the length of a randomly chosen pregnancy, is normal with mean (μ) and standard deviation (σ), then $0.68 = P(\mu - \sigma < X < \mu + \sigma)$."

In general, if X is a normal random variable, then the probability is

68% that X falls within 1 σ of μ , that is, in the interval $\mu \pm \sigma$

95% that X falls within 2 σ of μ , that is, in the interval $\mu \pm 2\sigma$

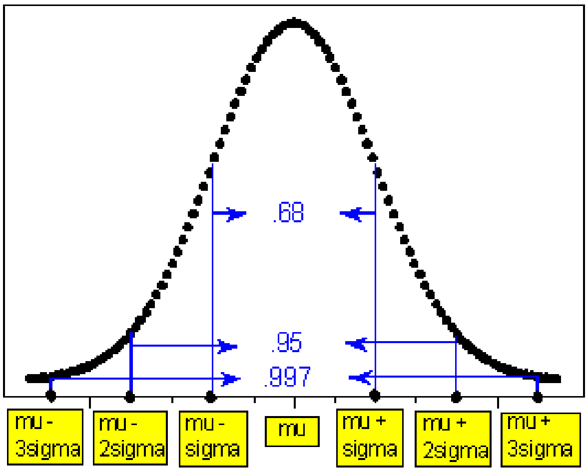
99.7% that X falls within 3 σ of μ , that is, in the interval$\mu \pm 3\sigma$

Using probability notation, we may write

$$0.68 = P(\mu - \sigma < X < \mu + \sigma)$$

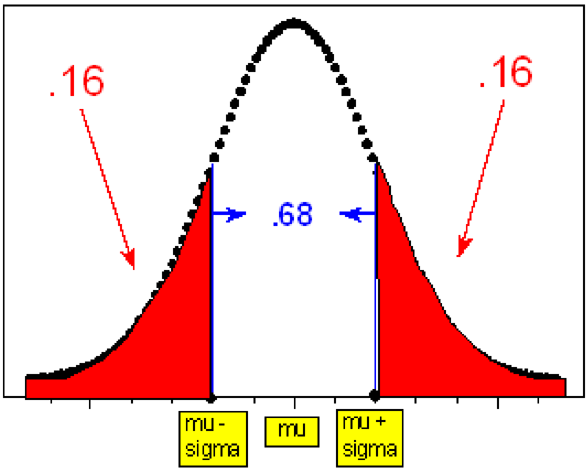
$$0.95 = P(\mu - 2\sigma < X < \mu + 2\sigma)$$

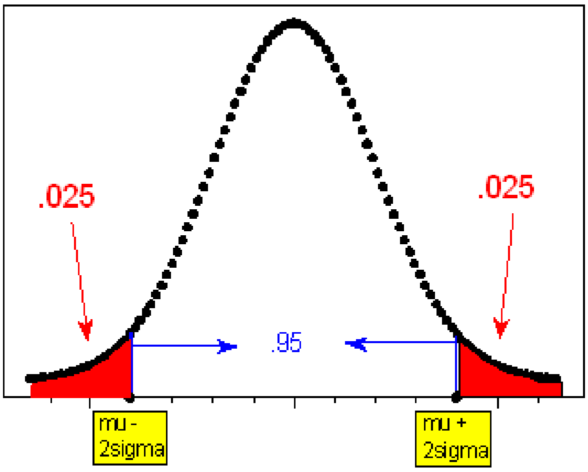
$$0.997 = P(\mu - 3\sigma < X < \mu + 3\sigma)$$

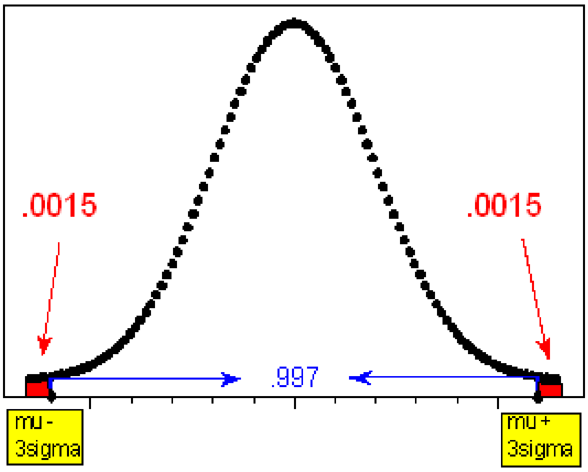


Comment

Notice that the information from the rule can be interpreted from the perspective of the tails of the normal curve: since 0.68 is the probability of being within 1 standard deviation of the mean, (1 - .68) / 2 = 0.16 is the probability of being further than 1 standard deviation below the mean (or further than 1 standard deviation above the mean). Likewise, (1 - .95) / 2 = 0.025 is the probability of being more than 2 standard deviations below (or above) the mean; (1 - .997) / 2 = 0.0015 is the probability of being more than 3 standard deviations below (or above) the mean. The three figures below illustrate this.

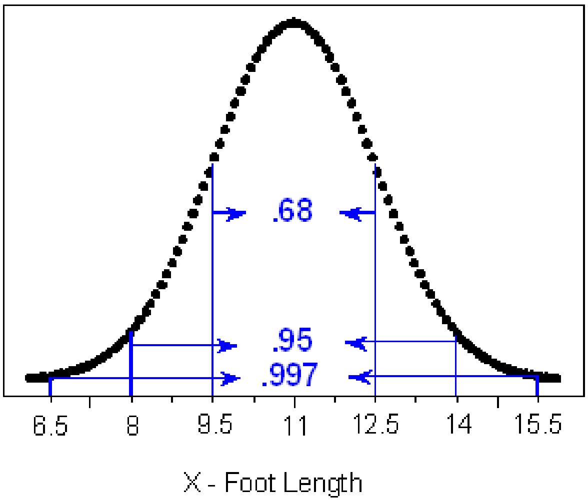






Example

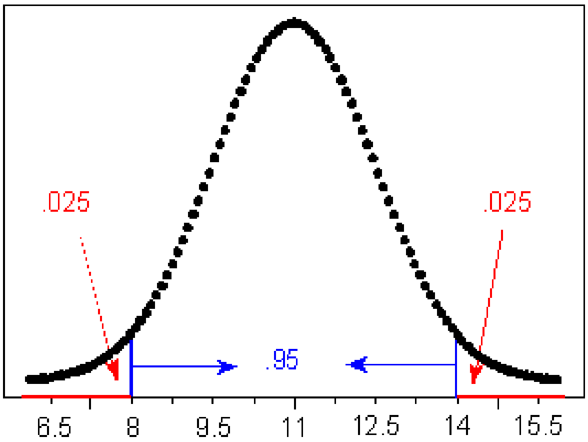
Suppose that foot length of a randomly chosen adult male is a normal random variable with mean $\mu = 11$ and standard deviation $\sigma = 1.5$. Then the Standard Deviation Rule lets us sketch the probability distribution of X as follows:



(a) What is the probability that a randomly chosen adult male will have a foot length between 8 and 14 inches? 0.95, or 95%.

(b) An adult male is almost guaranteed (.997 probability) to have a foot length between what two values? 6.5 and 15.5 inches.

(c) The probability is only 2.5% that an adult male will have a foot length greater than how many inches? 14.



Now you should try a few. (Use the figure that is just before **part (a)** to help you.)

Learn By Doing

1/1 point (graded)

How likely or unlikely is a male's foot length to be smaller than 9.5 inches? Not too unlikely, since the probability of being smaller than 9.5 is __________ , which is not a particularly low probability.

○ 0.025

○ 0.05

● 0.16 ✓

○ 0.32

○ 0.68

Answer
Correct:
Indeed, the probability that foot length is between 9.5 and 12.5 is 0.68, and therefore the remaining two tails together have probability 1 - .68 = 0.32. We conclude, then, that: P(X < 9.5) = 0.32 / 2 = 0.16.

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1/1 point (graded)
How likely or unlikely is a foot length longer than 15.5 inches? Extremely unlikely, since the probability of being longer than 15.5 is only __________ .

● 0.0015 ✓

○ 0.003

○ 0.015

○ 0.025

○ 0.05

Answer
Correct:
Indeed, the probability that foot length is between 6.5 and 15.5 is 0.997, and therefore the remaining two tails together have probability 1 - .997 = 0.003. We conclude, then, that: P(X > 15.5) = .003 / 2 = 0.0015.

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Learn By Doing

1/1 point (graded)
There is probability of 0.5 that a male's foot is shorter than __________ .

○ 9.5

● 11 ✓

○ 12.5

Answer

Correct:
Indeed, 11 is the value that divides the area under the curve into two halves, so that P(X < 11) = P(X > 11) = 0.5.

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Notice that there are two types of problems we may want to solve: those in which a particular interval of values of a normal random variable is given, and we are asked to find a probability, and those in which a probability is given and we are asked to identify what the normal random variable's values would be.

Scenario: Length of Human Pregnancies

Length (in days) of human pregnancies is a normal random variable (X) with mean 266, standard deviation 16.

(It would be useful to sketch this normal distribution yourself, marking its mean and the values that are 1, 2, and 3 standard deviations below and above the mean.

Click here to compare your figure to ours.

Did I Get This

1/1 point (graded)
The probability is 0.95 that a pregnancy will last between __________ and __________ days.

○ 218, 314

● 234, 298 ✓

○ 250, 282

○ 266, 314

Answer

Correct:
Indeed, with probability 0.95 a normal random variable will get values that are between 2 standard deviations below the mean (266 - 2 * 16 = 234) and 2 standard deviations above the mean (266 + 2 * 16 = 298).

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Did I Get This

1/1 point (graded)

The shortest 16% of pregnancies last less than how many days?

- ○ 218
- ○ 234
- ● 250 ✓
- ○ 266
- ○ 282
- ○ 298
- ○ 314

Answer

Correct:
Indeed 16% of pregnancies last less than 1 standard deviation below the mean, which in this case is 266 - 16 = 250.

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Did I Get This

1/1 point (graded)

What is the probability of a pregnancy lasting longer than 314 days?

- ● 0.0015 ✓

○ 0.025

○ 0.16

○ 0.68

○ 0.95

○ 0.997

Answer

Correct: Indeed, since P(218 < X < 314) = 0.997, P(X > 314) = (1 - .997) / 2 = 0.0015.

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Did I Get This

1/1 point (graded)

There is a probability of 0.5 that a pregnancy will last longer than how many days?

○ 218

○ 234

○ 250

● 266 ✓

○ 282

○ 298

○ 314

Answer

Correct:

Indeed, 266 is the value that divides the area under the curve into two halves, so that P(X < 266) = P(X > 266) = 0.5. A pregnancy is as likely to last less than 266 days as it is to last more than 266 days.

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