The General Multiplication Rule: Applications

Learning Objective: Use the General Multiplication Rule to find the probability that two events occur (P(A and B)).

Here, again, is the General Multiplication Rule:

For any two events A and B, P(A and B) = P(A) * P(B | A)

Comments

1. Note that although the motivation for this rule was to find P(A and B) when A and B are not independent, this rule is general in the sense that if A and B happen to be **independent**, then P(B | A) = P(B) is true, and we're back to Rule 5—the Multiplication Rule for Independent Events: P(A and B) = P(A) * P(B).

2. The General Multiplication Rule is just the definition of conditional probability in disguise. Recall the definition of conditional probability: **P(B | A) = P(A and B) / P(A)** Let's isolate P(A and B) by multiplying both sides of the equation by P(A), and we get: **P(A and B) = P(A) * P(B | A)**. That's it ... this is the General Multiplication Rule.

3. The General Multiplication Rule is useful when two events, A and B, occur in stages, first A and then B (like the selection of the two cards in the previous example). Thinking about it this way makes the General Multiplication Rule very intuitive. For both A and B to occur you first need A to occur (which happens with probability P(A)), **and** then you need B to occur, knowing that A has already occurred (which happens with probability P(B | A)).

Scenario: Selecting Coins

A woman's pocket contains 2 quarters and 2 nickels; she randomly extracts one of the coins, and without replacing it picks a second coin.

Did I Get This

1/1 point (graded)

Are the events Q1 = "first pick was a quarter" and Q2 = "second pick was a quarter" independent?

- ○ yes
- ● no ✔

Answer

Correct:

Indeed, Since the selection is done without replacement, Q1 and Q2 are not independent. In particular, the probability that the second selection is a quarter depends on whether the first selection was a quarter or not. P(Q2 | Q1) = 1/3, while P(Q2 | not Q1) = 2/3.

Submit

Did I Get This

1/1 point (graded)

What is the probability of getting a quarter both times, P(Q1 and Q2)?

- ● 2/4 * 1/3 = 1/6 ✔
- ○ 2/4 * 1/2 = 1/4
- ○ 2/4 * 2/3 = 1/3
- ○ 2/4 + 1/3 = 5/6

Answer

Correct:

Using the General Multiplication Rule, P(Q1 and Q2) = P(Q1) * P(Q2 | Q1). For the first pick, there are 2 quarters out of 4, so you are right that P(Q1) = 2/4. Given that the first coin was a quarter, there are 3 coins left, only 1 of which is a quarter, so P(Q2 | Q1) = 1/3. The answer is therefore (as you selected) 2/4 * 1/3 = 1/6.

Submit

Did I Get This

1/1 point (graded)

What is the probability of getting a quarter and then a nickel, P(Q1 and N2)?

- 2/4 * 1/3 = 1/6
- 2/4 * 1/2 = 1/4
- 2/4 * 2/3 = 1/3 ✔
- 2/4 + 1/3 = 5/6

Answer

Correct:

Using the General Multiplication Rule, P(Q1 and N2) = P(Q1) * P(N2 | Q1). For the first pick there are 2 quarters out of 4, so you are right that P(Q1) = 2/4. Given that the first coin was a quarter, there are 3 coins left, 2 of which are nickels, so P(N2 | Q1) = 2/3. The answer is therefore (as you selected) 2/4 * 2/3 = 1/3.

Submit

Let's look at another, more realistic example:

Example

In a certain region, one in every thousand people (0.001) of all individuals are infected by the HIV virus that causes AIDS. Tests for presence of the virus are fairly accurate but not perfect. If someone actually has HIV, the probability of testing positive is 0.95. Let **H** denote the event of having HIV, and **T** the event of testing positive.

(a) Express the information that is given in the problem in terms of the events H and T.

"one in every thousand people (0.001) of all individuals are infected with HIV" → **P(H) = 0.001**

"If someone actually has HIV, the probability of testing positive is 0.95" → **P(T | H) = 0.95**

(b) Use the General Multiplication Rule to find the probability that someone chosen at random from the population has HIV and tests positive.

P(H and T) = P(H) * P(T | H) = 0.001*0.95 = 0.00095.

(c) If someone has HIV, what is the probability of testing negative? Here we need to find P(not T | H).

Recall from an activity earlier in this module that the Complement Rule works with conditional probabilities as long as we condition on the same event, therefore: **P(not T | H)** = 1 - P(T | H) = 1 - 0.95 = 0.05.

The purpose of the next activity is to give you guided practice in expressing information in terms of conditional probabilities, and in using the General Multiplication Rule.

Scenario: Overheating Engine

An overheating engine can quickly cause serious damage to a car, and therefore a dashboard red warning light is supposed to come on if that happens.

In a certain model car, there is a 3% chance that the engine will overheat (event H).

The probability of the warning light showing up (event W) when it should (i.e., when the engine is really overheating) is 0.98. However, 1% of the time the warning light appears for no apparent reason (i.e., when the engine temperature is normal).

Learn By Doing (1/1 point)

Express all the information that is given in the problem in terms of probabilities involving the events H and W (note that some of the information involves conditional probabilities).

Your Answer:

P(H) = 0.03
P(W|H) = 0.98
P(W|not H) = 0.01

Our Answer:

We are given the following: • There is a 3% chance that the engine is overheating, therefore P(H) = 0.03. • The probability of the warning light showing up when it should is 0.98. Another way to say this is: Given that the engine is overheating, the probability of getting a warning light is 0.98. This, therefore, translates to: P(W | H) = 0.98. • 1% of the time, the warning light appears for no apparent reason. Another way to say this is: Given that the engine is not overheating, the probability of getting the warning light is 0.01. This, therefore, translates to: P(W | not H) = 0.01.

Resubmit Reset

Learn By Doing (1/1 point)

Use the General Multiplication Rule to find the probability that the engine is overheating and a warning shows up.

Your Answer:

P(H and W) = P(H) * P(W|H) = 0.03 * 0.98 = 0.0294

Our Answer:

We need to find: P(H and W). Using the General Multiplication Rule: P(H and W) = P(H) * P(W | H) = 0.03 * 0.98 = 0.0294.

Resubmit Reset

Learn By Doing (1/1 point)

Use the General Multiplication Rule to find the probability of the event that the engine is not overheating and a warning light shows up.

Your Answer:

P(not H and W) = P(not H) * P(W|not H) = 0.97 * 0.01 = 0.0097

Our Answer:

We need to find P(not H and W). Using the General Multiplication Rule: P(not H and W) = P(not H) * P(W | not H) Since P(H) = 0.03, P(not H) = 0.97, therefore P(not H and W) = 0.97 * 0.01 = 0.0097.

Resubmit Reset

Learn By Doing (1/1 point)

How likely is the dangerous event that no warning light shows up when the engine is overheating? In other words, given that the engine is overheating, how likely is it that we will not get a warning?

Your Answer:

P(not W | H) = 1 - P(W|H) = 1-0.98 = 0.02

Our Answer:

Here we need to find P(not W | H). Recall that the Complement Rule "works" with conditional probabilities as long as we condition on the same event. And therefore: P(not W | H) = 1 - P(W | H) = 1 - 0.98 = 0.02

Resubmit Reset