Probability Trees: Applications

Learning Objective: Use probability trees as a tool for finding probabilities.

Here is a more practical example:

Example

Polygraph (lie-detector) tests are often routinely administered to employees or prospective employees in sensitive positions. A National Research Council study in 2002, headed by Stephen Fienberg from CMU, found that lie detector results are "better than chance, but well below perfection." Typically, the test may conclude someone is a spy 80% of the time when he or she actually is a spy, but 16% of the time the test will conclude someone is a spy when he or she is not.

Let us assume that 1 in 1,000, or 0.001, of the employees in a certain highly classified workplace are actual spies.

Let **S** be the event of being a spy, and **D** be the event of the polygraph detecting the employee to be a spy.

Let's first express the information using probability notations involving events S and D.

We are given:

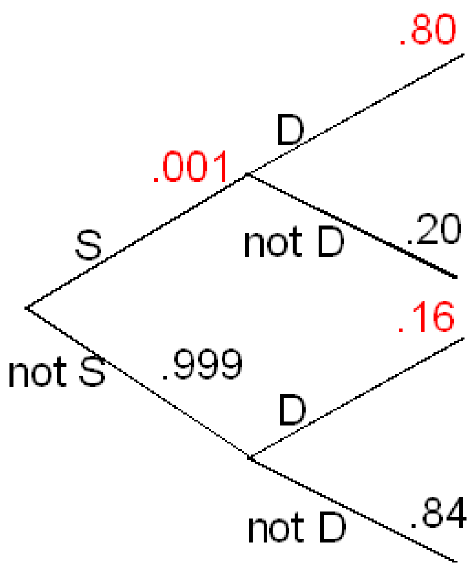
* 1 in 1,000, or 0.001, of the employees are actual spies. → **P(S) = 0.001**

* the test may conclude someone is a spy 80% of the time when he or she actually is a spy → **P(D | S) = 0.80**

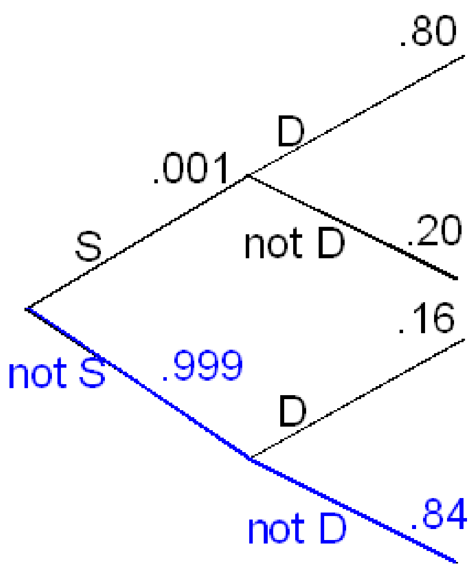
* 16% of the time, the test will conclude someone is a spy when he or she is not --> **P(D | not S) = 0.16**

(a) Let's create a tree diagram for this problem, starting, as usual, with the event for which a non-conditional probability is given, S. It also makes sense that we start with S, since the natural order is that first a person becomes a spy, and then he/she is either detected or not.

Note that marked in red are the probabilities that are given, and the rest are completed using the Complement Rule as explained before.

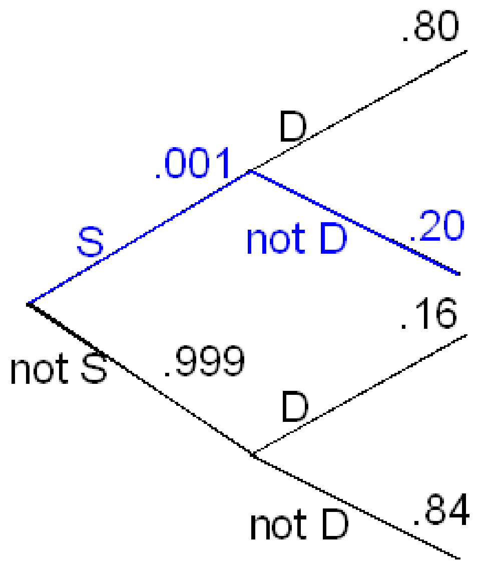


(b) What is the probability that a randomly chosen employee is not a spy, and the test does not detect the employee as one? In other words what is P(not S and not D)?



P(not S and not D) = P(not S) * P(not D | not S) = 0.999 * 0.84 = 0.83916

(c) What is the probability that a randomly chosen employee **is** a spy, and the test does **not** detect the employee as one? [This would be an incorrect conclusion.] In other words, what is P(S and not D)



P(S and not D) = P(S) * P(not D | S) = 0.001 * 0.20 = 0.0002

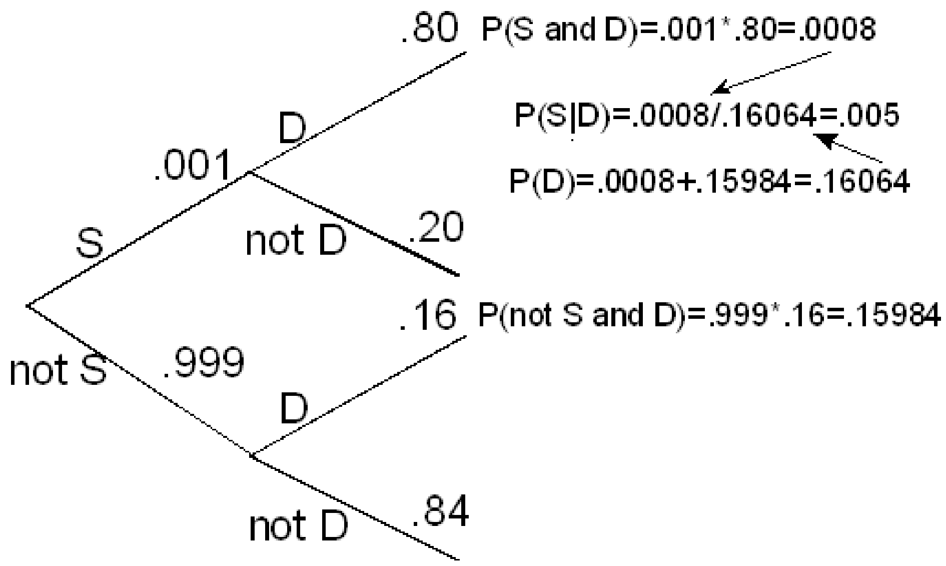
(d) Suppose the polygraph detects a spy; are you convinced that the employee is actually a spy? Find the probability of an employee actually being a spy, given that the test claims he or she is. In other words, find P(S | D).

Applying Bayes' Rule, we have

P(S | D) = P(S) * P(D | S) / [P(S) * P(D | S) + P(not S) * P(D | not S)]

=0.001 * 0.80 / [0.001 *0 .80 + 0.999 * 0.16] = 0.0008/0.16064 = 0.005

The study's conclusion, that more accurate tests than the traditional polygraph are sorely needed, is supported by our answer to part (d): if someone is detected as being a spy, the probability is only 0.005, or half of one percent, that he or she actually is one.



Comment

This example helps to highlight how different P(B | A) may be from P(A | B): the probability of being detected, given that an employee is a spy, is P(D | S) = 0.80. In contrast, the probability of being a spy, given that an employee has been detected by the polygraph, is P(S | D) = 0.005.

The purpose of the next activity is to give you guided practice in using the information displayed in probability trees in order to answer real-life problems.

Scenario: Overheating Engine

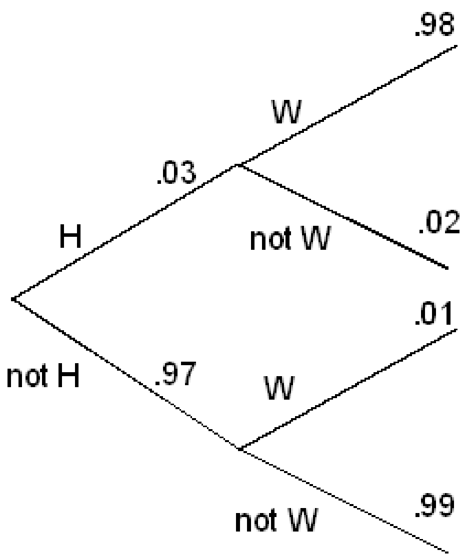
Let's consider the engine overheating example again, where H is the event that the engine overheats, and W is the event that a warning light turns on. We are given that:

P(H) = 0.03

P(W | H) = 0.98

P(W | not H) = 0.01

and in a previous activity we displayed the information using a probability tree:



Learn By Doing (1/1 point)

What is the probability that the warning light shows up, P(W)? (Recall from previous examples that you need to consider two possibilities here, since a W branch can be reached in two ways. Either the engine is overheated and the warning light is on, or the engine is not overheated and the warning light is on.)

Your Answer:

P(W) = P(W|H) + P(W|not H) = 0.03*0.98 + 0.01*0.97 = 0.0391

Our Answer:
The two possibilities that we need to consider here are: P(W) = P(H and W) + P(not H and W) = 0.03 * 0.98 + 0.97 * 0.01 = 0.0294 + 0.0097= 0.0391

Resubmit Reset

Learn By Doing (1/1 point)

When a driver notices that the warning light is on, how worried does he or she need to be? In other words, given that the warning light is on, how likely is it that the engine is really overheating? Use the definition of conditional probability, and the information you obtained in the previous question, to find P(H | W).

Your Answer:

P(H|W) = P(H and W) / P(W) = P(W|H) * P(H) / P(W) = 0.98 * 0.03 / 0.0391 = 0.75

Our Answer:
By the definition of conditional probability, P(H | W) = P(H and W) / P(W). In the first question we found that P(H and W) = .0294, and that P(W) = .0391. Therefore: P(H | W) = .0294/.0391 = .752. This means that when the warning light is on, there is about a 75% chance that the engine is indeed overheating, and therefore it is advisable for the driver to stop the car and let the engine cool down.

Resubmit Reset
