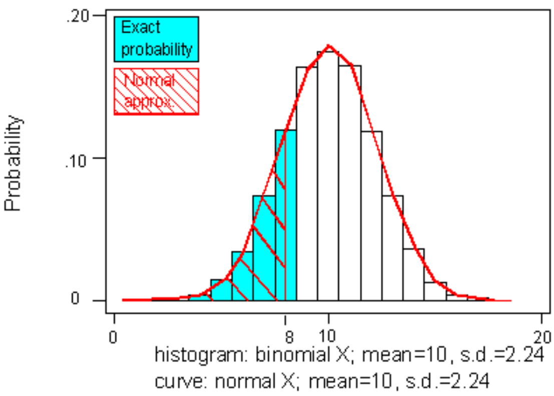
Normal Approximation to the Binomial: Continuity Correction

Learning Objective: Use the normal distribution as an approximation of the binomial distribution, when appropriate.

Actually, there is another important reason why the binomial approximation example from the middle of the previous page is not too good. The following comment will explain.

Comment

It is possible to improve the normal approximation to the binomial by adjusting for the discrepancy that arises when we make the shift from the areas of histogram rectangles to the area under a smooth curve. For example, if we want to find the binomial probability that X is less than **or equal to** 8, we are including the area of the entire rectangle over 8, which actually extends to 8.5. Our normal approximation only included the area up to 8. The figure below illustrates this:

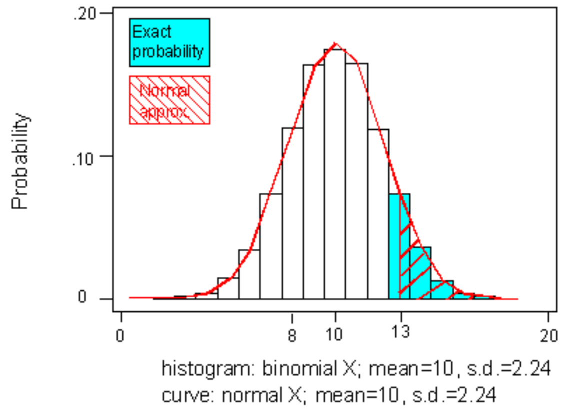


It can be improved upon by making the **continuity correction:**

in this case, we would have

$P\left(X_B \leq 8\right) \approx P\left(X_N \leq 8.5\right) = P\left(Z \leq \frac{8.5-10}{2.24}\right) = P\left(Z \leq -0.67\right) = 0.2514$, which is much closer to the actual binomial probability of 0.2517 than our original approximation (0.1867) was.

Similarly, suppose I wanted to answer: What is the probability that the student gets at least 13 questions right?



Here, to calculate the exact probability we are including the area of the entire rectangle over 13, which actually starts from 12.5. Our normal approximation only included the area from 13. The continuity correction in this case would be:

$$P\left(X_B \geq 13\right) \approx P\left(X_N \geq 12.5\right) = P\left(Z \geq \frac{12.5-10}{2.24}\right) = P\left(Z \geq 1.12\right) = \left(symmetry\right) = P\left(Z \leq -1.12\right) = \left(table\right) = 0.131$$

It turns out that the exact probability in this case (using software) is 0.1316, so the approximation is excellent.

The purpose of the next activity is to give you guided practice in solving word problems involving a binomial random variable, when the normal approximation is appropriate and is extremely helpful.

Scenario: Left-Handed College Students

Roughly 10% of all college students in the United States are left-handed. Most academic institutions, therefore, try to have at least a few left-handed chairs in each classroom. 225 students are about to enter a lecture hall that has 30 left-handed chairs for a lecture. What is the probability that this is not going to be enough; in other words, what is the probability that more than 30 (or at least 31) of the 225 students are left-handed?

Let's think about this situation.

Let X be the number of left-handed students (success) out of the 225 students (trials). X is therefore binomial with n = 225 and p = 0.1. We are asked to find P(X > 30) or P(X ≥ 31).

Clearly, doing this using the binomial distribution is out of the question.

Learn By Doing (1/1 point)

Explain why we can use the normal approximation in this case, and state which normal distribution you would use for the approximation.

Your Answer:

We can use normal approximation because it satisfies both rules of thumb, that n*p >= 10, and n*(1-p)>=10.

We'll use based on the formula: mean = 22.5, standard deviation = sqrt(np(1-p)) = 4.5.

Our Answer:

X is binomial with n = 225 and p = 0.1. The normal approximation is appropriate, since the rule of thumb is satisfied: np = 225 * 0.1 = 22.5 > 10, and also n(1 - p) = 225 * 0.9 = 202.5 > 10. We will approximate the binomial random variable X by the random variable Y having a normal distribution with mean μ = 225 * 0.1 = 22.5 and standard deviation σ = sqrt(225 * 0.1 * 0.9) = sqrt(20.25) = 4.5

Resubmit Reset

Learn By Doing (1/1 point)

Use the normal approximation to find P(X ≥ 31). For the approximation to be better, use the continuity correction as we did in the last example. In other words, rather than approximating P(X ≥ 31) by P(Y ≥ 31), approximate it by P(Y ≥ 30.5).

Your Answer:

(30.5-22.5)/4.5 = 1.78 --> 0.9625 --> 1-0.9625 = 0.0375

Our Answer:

P(X ≥ 31) ≈ (normal approximation + continuity correction) ≈ P(Y ≥ 30.5) = P(Z ≥ (30.5 - 22.5) / 4.5) = P(Z ≥ 1.78) = (symmetry) = P(Z ≤ -1.78) = (table) = 0.0375.

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