Probability Rules: Multiplication Rule for Independent Events

Learning Objective: Apply probability rules in order to find the likelihood of an event.

Now that we understand the idea of independent events, we can finally get to rule 5. As mentioned before, Rule 5 actually has two versions, one for finding P(A and B) in the special case in which the events A and B are independent, and a more general version for use when the events are not necessarily independent. We will first present the version of rule 5 that is restricted to independent events, and in the next section we will revisit Rule 5 and present the more general version.

Rule 5: The Multiplication Rule for Independent Events

If A and B are two independent events, then P(A and B) = P(A) * P(B).

Comment

When dealing with probabilities, the word **"and"** will always be associated with the operation of **multiplication**; hence the name of this rule, "The Multiplication Rule."

Example

Recall the blood type example:

Blood Type	O	A	B	AB

Probability	0.44	0.42	0.10	0.04

Two people are selected simultaneously and at random from all people in the United States. What is the probability that both have blood type O?

Let O1= "person 1 has blood type O" and

O2= "person 2 has blood type O"

We need to find P(O1 and O2)

Since they were chosen simultaneously and at random, the blood type of one has no effect on the blood type of the other. Therefore, O1 and O2 are independent, and we may apply Rule 5:

P(O1 and O2) = P(O1) * P(O2) = 0.44 * 0.44 = 0.1936.

Did I Get This (1/1 point)

A 2011 poll by the Pew Research Center for People and the Press estimated that 62% of U.S. adults favor the death penalty for persons convicted of murder, 31% oppose it, with the remaining 7% undecided. What is the probability that two randomly chosen U.S. adults support the death penalty for persons convicted of murder?

Your Answer:

0.62 * 0.62

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Our Answer:
Let A be the event that the first person supports the death penalty. Let B be the event that the second person supports the death penalty. We want to find P(A and B). Since the two people are chosen at random from a large population, A and B are independent and we can use the Multiplication Rule for Independent Events. P(A and B) = P(A) * P(B) = 0.62 * 0.62 = 0.3844

Resubmit Reset

Learn By Doing (1/1 point)

In the 2000 presidential election, George Bush won 48% of the popular vote. In the 2004 presidential election, he won 51% of the popular vote. What is the probability that a randomly chosen voter voted for Bush in both elections?

Your Answer:

48% * 51%

Our Answer:

Let A be the event that the person voted for Bush in 2000. Let B be the event that the person voted for Bush in 2004. The question asks us to determine P(A and B). We might be tempted to use the Multiplication Rule for Independent Events and write P(A and B) = P(A)*P(B) = 0.48*0.51 = 0.2448, but this would be incorrect because these events are not independent. If an individual voted for Bush in 2000, it is likely that the individual voted for him in 2004. So we are unable to answer the question with the information given.

Resubmit Hint Reset

Did I Get This

1/1 point (graded)

Recall the estimate by the Pew Research Center that 62% of U.S. adults favor the death penalty for murder. The same report gave a much lower estimate for the percentage of U.S. college graduates supporting the death penalty in cases of murder. According to census data from 2000, roughly 28% of U.S. adults have a college degree.

What is the probability that a randomly selected U.S. adult has a college degree and favors the death penalty?

Let A be the event that a U.S. adult has a college degree. Let B be the event that this person supports the death penalty. We want to find P(A and B).

Which answer is correct, Answer 1 or 2?

- ○ Answer 1: P(A and B) = P(A)*P(B) = 0.62(0.28) = 0.1736 We can use the Multiplication Rule for Independent Events because the events are independent. Having a college degree does not determine a person's views on the death penalty.
- ◉ Answer 2: We do not have enough information to answer the question. We cannot use the Multiplication Rule for Independent Events because these events are dependent. Having a college degree affects the likelihood that a person supports the death penalty. ✔

Answer

Correct:
A and B are dependent events because if A occurs (we have selected a person with a college degree), then the probability of B is affected (a smaller chance that the person supports the death penalty.) So we cannot use the Multiplication Rule for Independent Events in this situation.

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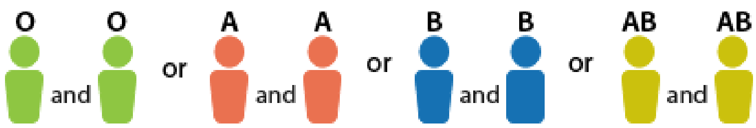
So far we have looked at examples where we have to consider and apply only one of the rules. The following example is a case where both the Addition Rule for Disjoint Events and the Multiplication Rule for Independent Events need to be applied in order to find the desired probability.

Example

Recall the blood types example:

Blood Type	O	A	B	AB
Probability	0.44	0.42	0.10	0.04

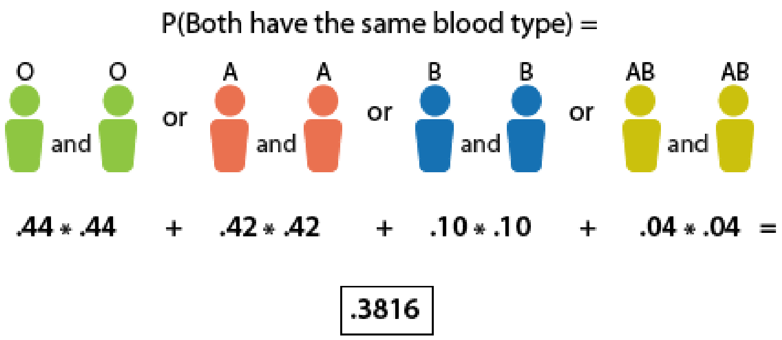
Two people are chosen simultaneously and at random. What is the probability that both have the same blood type? For both to have the same blood type there are four possibilities. Both have blood type O **or** both have blood type A **or** both have blood type B **or** both have blood type AB.



In other words, and using our regular notations,

P(same blood type) = P([O1 and O2] or [A1 and A2] or [B1 and B2] or [AB1 and AB2])

Since our four possibilities of both people having the same blood type are **disjoint**, using our **Addition Rule** we can add their probabilities (i.e., replace every "or" with +). Also, within each of the four possibilities, we can use the **Multiplication Rule** and replace "and" with * (using the same **independence** argument as the first example on this page). Our answer is therefore,



About 38% of the time, two randomly chosen U.S. people would have the same blood type. Note that in this example we used the Addition Rule and the Multiplication Rule one after the other, justifying along the way why it is appropriate to do so.

Did I Get This

1/1 point (graded)

According to the most updated data gathered by the American Association of Suicidology, 80% of suicides in the U.S. are committed by men.

Two suicide cases are selected at random. What is the probability that both suicides were committed by a person of the same gender?

- 0.64
- 0.68 ✔
- 1.6
- Cannot be determined since the outcomes are not independent
- Cannot be determined since the events are not disjoint

Answer

Correct: P(both suicides same gender) = P(both males) + P(both females) = (0.8)(0.8) + (0.2)(0.2) = 0.68

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Comment

The purpose of this comment is to point out the magnitude of P(A or B) and of P(A and B) relative to either one of the individual probabilities. Since probabilities are never negative, the probability of one event **or** another is always at least as large as either of the individual probabilities. Since probabilities are never more than 1, the probability of one event **and** another generally involves multiplying numbers that are less than 1, therefore can never be more than either of the individual probabilities.

Here is an example:

Example

Consider the event A that a randomly chosen person has blood type A. Modify it to a more general event—that a randomly chosen person has blood type A or B—and the probability increases. Modify it to a more specific (or restrictive) event—that not just one randomly chosen person has blood type A, but that out of two simultaneously randomly chosen people, person 1 will have type A and person 2 will have type B—and the probability decreases.

It is important to mention this in order to root out a common misconception. The word "and" is associated in our minds with "adding more stuff." Therefore, some students **incorrectly** think that P(A and B) should be larger than either one of the individual probabilities, while it is actually smaller, since it is a more specific (restrictive) event. Also, the word "or" is associated in our minds with "having to choose between" or "losing something," and therefore some students incorrectly think that P(A or B) should be smaller than either one of the individual probabilities, while it is actually larger, since it is a more general event.

Practically, you can use this comment to check yourself when solving problems. For example, if you solve a problem that involves "or," and the resulting probability is smaller than either one of the individual probabilities, then you know you have made a mistake somewhere.

Scenario: Eating Breakfast

Pick a student at random. Let B denote the event that the student ate breakfast this morning; let M denote the event that the student is male.

Did I Get This

1/1 point (graded)
One of the following choices is larger than the other two. Which is it?

- P(B)
- P(B or M)
- P(B and M)

Answer
Correct:
Indeed, the last comment tells us that: "B or M" is a more general event than "B" and therefore has a larger probability. "B and M" is a more specific event than "B" and therefore has a smaller probability.

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Did I Get This

1/1 point (graded)

One of the following choices is smaller than the other two. Which is it?

- ○ P(B)
- ○ P(B or M)
- ● P(B and M) ✔

Answer

Correct:

Indeed, the last comment tells us that: "B or M" is a more general event than "B" and therefore has a larger probability. "B and M" is a more specific event than "B" and therefore has a smaller probability.

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