Probability Rules: Extensions

Learning Objective: Apply probability rules in order to find the likelihood of an event.

As you've seen, the last three rules that we've introduced (the Complement Rule, the Addition Rule for Disjoint Events, and the Multiplication Rule for Independent Events) are frequently used in solving problems. Before we move on to our next rule, here are two comments that will help you use these rules in broader types of problems and more effectively.

Comment

As we mentioned before, the Addition Rule can be extended to more than two disjoint events. Likewise, the Multiplication Rule can be extended to more than two independent events. So if A, B and C are three independent events, for example, then P(A and B and C) = P(A) * P(B) * P(C). These extensions are quite straightforward, as long as you remember that "or" requires us to add, while "and" requires us to multiply.

An example of a situation where more than two independent events naturally occur is when a random sample of more than two individuals is chosen from a large population.

Here is an example:

Example

Three people are chosen at random from a large population. What is the probability that all three have blood type B? We'll use the usual notation of B1, B2 and B3 for the events that persons 1, 2 and 3 have blood type B, respectively. We need to find P(B1 and B2 and B3). Let's solve this one together:

Learn By Doing

1/1 point (graded)

Are the events B1, B2, and B3 independent or not?

- (●) independent ✔
- ( ) not independent

Answer

Correct:

The probability that one person has blood type B does not affect the probability that either of the other two individuals has blood type B so the events are independent.

Submit

Learn By Doing

1/1 point (graded)

What feature of the three people makes the events independent or not?

- ( ) all have blood type B
- ( ) are all males
- (●) were chosen at random from a large population ✔

Answer

Correct:

When individuals are selected at random from a large population any event associated with one individual is independent of any event associated with the other individual.

Submit

Learn By Doing

1/1 point (graded)

Should we use the Addition or Multiplication Rule in this case?

- ( ) Addition Rule

- [x] Multiplication Rule ✔

Answer
Correct:
If A and B are two independent events, the Multiplication Rule is used when we try to find the probability for P(A and B).

Submit

Learn By Doing

1/1 point (graded)
What is P(B1 and B2 and B3)?

- [x] P(B1) * P(B2) * P(B3) = 0.1 * 0.1 * 0.1 = 0.001 ✔
- [ ] P(B1) + P(B2) + P(B3) = 0.1 + 0.1 + 0.1 = 0.3

Answer
Correct: This is the formula for the Multiplication Rule with three independent events.

Submit

Here is another example that might be quite surprising.

Example

A fair coin is tossed 10 times. Which of the following two outcomes is more likely?

(a) HHHHHHHHHH

(b) HTTHHTHTTH

Learn By Doing

1/1 point (graded)
Before we solve this, what does your intuition tell you?

(a) is more likely

(b) is more likely

(a) and (b) are equally likely

Answer
Correct: Read on to see the full solution.

Submit

In fact, they are equally likely. The 10 tosses are independent, so we'll use the Multiplication Rule for Independent Events:

P(HHHHHHHHHH) = P(H) * P(H) * ... *P(H) = 1/2 * 1/2 *... * 1/2 = $(1/2)^{10}$

P(HTTHHTHTTH) = P(H) * P(T) * ... * P(H) = 1/2 * 1/2 *... * 1/2 = $(1/2)^{10}$

Here is the idea:

My random experiment here is tossing a coin 10 times. You can imagine how huge the sample space is.

There are actually 1,024 possible outcomes to this experiment, all of which are equally likely. Therefore, while it is true that it is more likely to get an outcome that has 5 heads and 5 tails than an outcome that has only heads (since there is only one possible outcome of the latter kind, and many possible outcomes of the former), if I am comparing 2 **specific outcomes** as I do here, they are equally likely.

Did I Get This (1/1 point)

Recall: Three people are chosen at random. (Assume the choices are independent events). What is the probability that they all have the same blood type?

Your Answer:

P(A)^3 + P(B)^3 + P(AB)^3 + P(O)^3

Our Answer:

To get all three the same, either the first and the second and the third are type O, or the first and the second and the third are type A, or the first and the second and the third are type B, or the first and the second and the third are type AB. The probability is: (.44 * .44 * .44) + (.42 * .42 * .42) + (.10 * .10 * .10) + (.04 * .04 * .04) = .160336 About 16% of the time, three randomly chosen people would have the same blood type.

Resubmit Reset