Confidence Intervals for the Population Mean: Other Levels of Confidence

Learning Objective: Explain what a confidence interval represents and determine how changes in sample size and confidence level affect the precision of the confidence interval.

Learning Objective: Find confidence intervals for the population mean and the population proportion (when certain conditions are met), and perform sample size calculations.

Other Levels of Confidence

The most commonly used level of confidence is 95%. However, we may wish to increase our level of confidence and produce an interval that's almost certain to contain μ. Specifically, we may want to report an interval for which we are 99% confident that it contains the unknown population mean, rather than only 95%.

Using the same reasoning as in the last comment, in order to create a 99% confidence interval for μ, we should ask: There is a probability of .99 that any normal random variable takes values within how many standard deviations of its mean? The precise answer is 2.576, and therefore, a 99% confidence interval for μ is $\overline{x} \pm 2.576 * \frac{\sigma}{\sqrt{n}}$.

Another commonly used level of confidence is a 90% level of confidence. Since there is a probability of 0.90 that any normal random variable takes values within 1.645 standard deviations of its mean, the 90% confidence interval for μ is $\overline{x} \pm 1.645 * \frac{\sigma}{\sqrt{n}}$.

Example

Let's go back to our first example, the IQ example:

The IQ level of students at a particular university has an unknown mean (μ) and known standard deviation $\sigma = 15$. A simple random sample of 100 students is found to have a sample mean IQ $\overline{x} = 115$ Estimate μ with a 90%, 95%, and 99% confidence interval.

A 90% confidence interval for μ is

$$\overline{x} \pm 1.645\frac{\sigma}{\sqrt{n}} = 115 \pm 1.645\left(\frac{15}{\sqrt{100}}\right) = 115 \pm 2.5 = (112.5,\ 117.5).$$

A 95% confidence interval for μ is $\overline{x} \pm 2\frac{\sigma}{\sqrt{n}} = 115 \pm 2\left(\frac{15}{\sqrt{100}}\right) = 115 \pm 3.0 = (112,\ 118)$.

A 99% confidence interval for μ is

$$\overline{x} \pm 2.576\frac{\sigma}{\sqrt{n}} = 115 \pm 2.576\left(\frac{15}{\sqrt{100}}\right) = 115 \pm 4.0 = (111,\ 119).$$

The purpose of this next activity is to give you guided practice at calculating and interpreting confidence intervals, and drawing conclusions from them.

Did I Get This (1/1 point)

The Golden Retriever Club of America conducted a study of 64 golden retrievers, and found the average age at death in the sample to be 11.0 years old. Let’s assume the standard deviation of golden retriever lifespan is known to be 1.2 years (this is consistent with studies and with some other dog breeds). Give three confidence interval estimates for the unknown mean age at death for golden retrievers: first using 90% confidence, then 95%, and finally 99%. Please report your intervals in parenthesis notation, and please round your final values to the nearest tenth for simplicity. Be sure to notice the size of the intervals with the different confidence levels.

Your Answer:

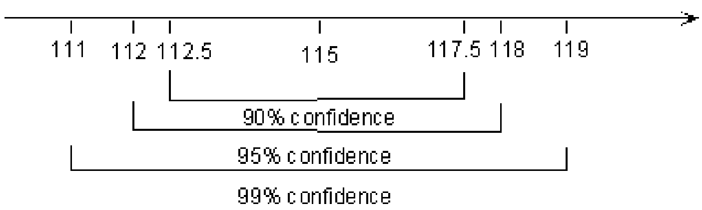
90% = 11 ± 1.645 * 1.2 / sqrt(64) = 10.76 to 11.25
95% = 11 ± 2 * 1.2 / sqrt(64) = 10.70 to 11.30
99% = 11 ± 2.576 * 1.2 / sqrt(64) = 10.61 to 11.39

Our Answer:

The 90% confidence estimate for μ is (10.8, 11.2). The 95% confidence estimate for μ is (10.7, 11.3). The 99% confidence estimate for μ is (10.6, 11.4).

Resubmit Reset

Note from the previous example and the previous "Did I Get This?" activity, that the more confidence I require, the wider the confidence interval for μ (pronounced and sometimes noted as "mu"). The 99% confidence interval is wider than the 95% confidence interval, which is wider than the 90% confidence interval.



Confidence Intervals for mu

This is not very surprising, given that in the 99% interval we multiply the standard deviation by 2.576, in the 95% by 2, and in the 90% only by 1.645. Beyond this numerical explanation, there is a very clear intuitive explanation and an important implication of this result.

Let's start with the intuitive explanation. The more certain I want to be that the interval contains the value of μ, the more plausible values the interval needs to include in order to account for that extra certainty. I am 95% certain that the value of μ is one of the values in the interval (112,118). In order to be 99% certain that one of the values in the interval is the value of μ, I need to include more values, and thus provide a wider confidence interval.

Visualizing the Relationship between Confidence and Width

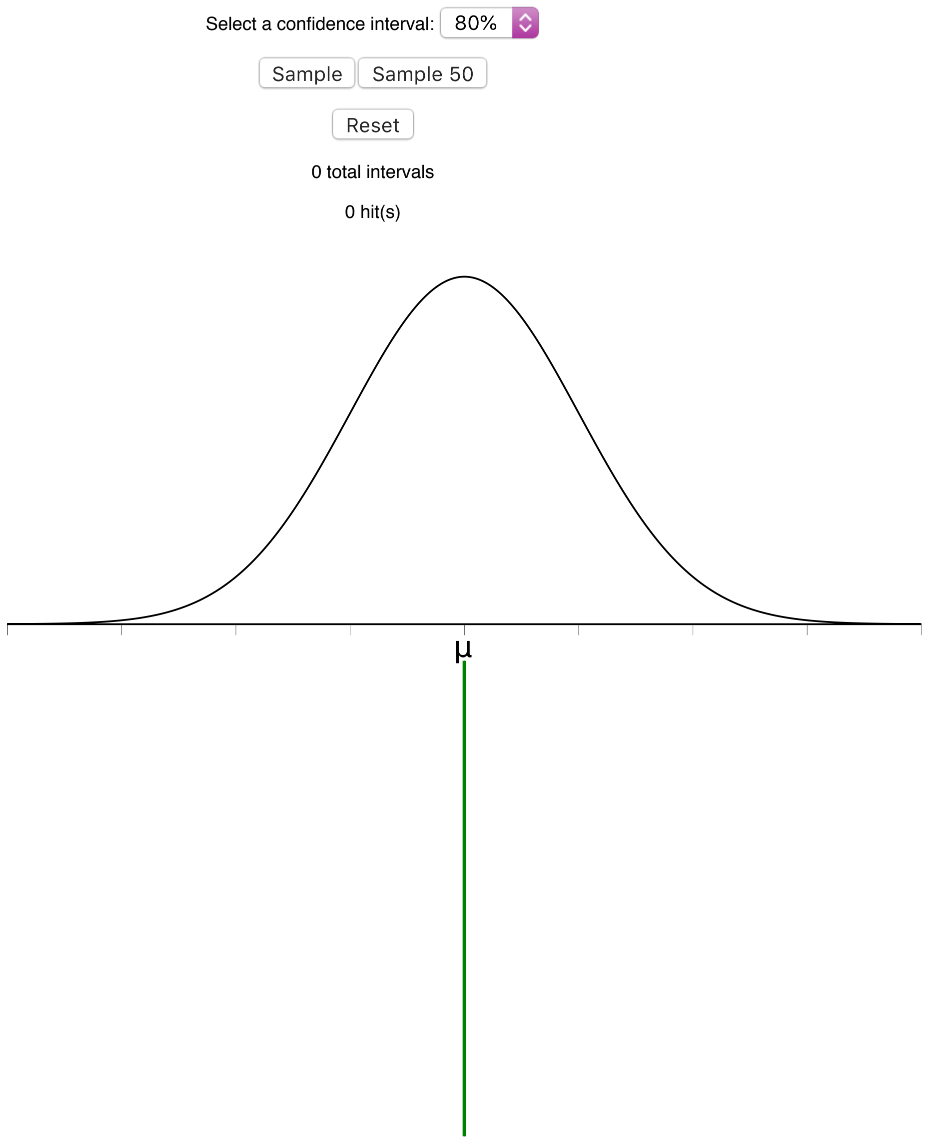
The purpose of this activity is to use a simulation to visually explore and reinforce the relationship between confidence and width, for confidence intervals. We will use the interactive simulation below to explore this concept.

To begin the simulation, make sure that the confidence level (on the very top) is set to 80%, and right below, click about 5 times on the **Sample** button. You have used the simulation to select 5 samples from the population; the simulation has automatically computed the sample means (indicated by the red dot in the center of intervals) and the corresponding 80% confidence intervals.

Now, change the confidence level to 90% and click on the **Sample** button 5 more times to simulate five 90% confidence intervals. Notice the change in the width of the confidence intervals compared to the 80% confidence intervals.

Now, change the confidence level to 95% and click on the **Sample** button 5 more times to simulate five 95% confidence intervals. Notice the change in the width of the confidence intervals compared to the 90% confidence intervals.

Finally, change the confidence level to 99% and click on the **Sample** button 5 more times to simulate five 99% confidence intervals. Notice the change in the width of the confidence intervals compared to the 95% confidence intervals.



Learn By Doing (1/1 point)

As the confidence level increased from 80% to 99%, what happened to the width of the intervals? {Did the intervals become wider, become narrower, or stay the same width?)

Your Answer:

Wider!

Our Answer:
As the confidence level increases, the intervals become wider.

Resubmit Reset

To continue the simulation, click on **Reset** and change the confidence level back to 80%. Click on the **Sample 50** button. You have used the simulation to simulate 50 samples from the population; the simulation has automatically computed the sample means (indicated by the red dot in the center of intervals) and the corresponding 80% confidence intervals.

Note that under the **Reset** button, you are told how many "hits" you have out of the 50 intervals. Recall from the applet activity on the previous page that *hits* means how many out of the 50 confidence intervals covered the unknown mean.

Click on **Sample 50** again for a total of 100 confidence intervals.

What percentage of the 100 confidence intervals covered? Write down this percentage in the textbox below.

Note that since we simulated 100 80% confidence intervals, the percentage of hits should be close to 80%.

Now change the confidence level to 90%, simulate 100 confidence intervals, and write the percentage of confidence intervals that cover in the textbox below.

Repeat for the 95% and 99% confidence levels.

Learn By Doing (1/1 point)

As the confidence level increased from 80% to 99%, what happened to the percentage of intervals that covered the population mean (μ)? Did the percentage increase, or decrease, or remain the same?

Your Answer:

Increase! Because larger size of intervals

Our Answer:
As the confidence level increased, the percent of intervals covering μ increased (i.e., more intervals contain μ when the intervals are wider).

Resubmit Reset

Learn By Doing (1/1 point)

With a 99% confidence level, what should be the long-run percentage of intervals that cover the population mean (μ), if you selected many thousands of samples?

Your Answer:

To 99%

Our Answer:
With a 99% confidence level, in the long run, 99% of the intervals should cover μ.

Resubmit Reset

In our example above, the **wider** 99% confidence interval (111, 119) gives us a **less precise** estimation about the value of μ than the narrower 90% confidence interval (112.5, 117.5), because the smaller interval 'narrows-in' on the plausible values of μ.

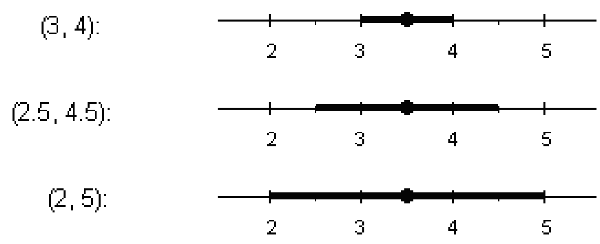
The important practical implication here is that researchers must decide whether they prefer to state their results with a higher level of confidence or produce a more precise interval. In other words,

There is a trade-off between the level of confidence and the precision with which the parameter is estimated.

The price we have to pay for a higher level of confidence is that the unknown population mean will be estimated with less precision (i.e., with a wider confidence interval). If we would like to estimate μ with more precision (i.e. a narrower confidence interval), we will need to sacrifice and report an interval with a lower level of confidence.

Scenario: Exercise Habits

In a recent study 1,115 males 25 to 35 years of age were randomly chosen and asked about their exercise habits. Based on the study results, the researchers estimated the mean time that a male 25 to 35 years of age spends exercising with 90%, 95%, and 99% confidence intervals. These were (not necessarily in the same order):



Did I Get This

1/1 point (graded)

For which of the three intervals do you have the **most confidence** that it captures the population mean (μ)?

- (3, 4)
- (2.5, 4.5)
- (2, 5) ✔

Answer

Correct:

The widest interval gives us the most confidence of capturing the population mean, because it covers more of the number line.

Submit

Did I Get This

1/1 point (graded)

The confidence interval in which you have the most confidence that it captures the population mean μ must be:

- the 90% confidence interval
- the 95% confidence interval

- [x] the 99% confidence interval ✔

Answer
Correct:
The confidence percentage measures our confidence that the interval captures the population mean, so the **widest** interval must be the one with the **largest confidence**.

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Did I Get This

1/1 point (graded)
Which of the three confidence intervals provides the most precise estimation?

- [x] (3, 4) ✔
- [ ] (2, 5)
- [ ] (2.5, 4.5)

Answer
Correct:
Indeed, since this is the narrowest confidence interval, it is the one that provides the most precise estimation of the unknown mean.

Submit

Did I Get This

1/1 point (graded)
The confidence interval that provides the most precise estimation, must be:

- [ ] the 99% confidence interval
- [ ] the 95% confidence interval
- [x] the 90% confidence interval ✔

Answer

Correct:
Indeed, since there is a trade-off between the level of confidence and precision, the interval that provides the most precise estimation must be the one with the lowest level of confidence.

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