Conditional Probability: Defined

Learning Objective: Explain the reasoning behind conditional probability, and how this reasoning is expressed by the definition of conditional probability.

Learning Objective: Find conditional probabilities and interpret them.

A good visual illustration of this conditional probability is provided by the two-way table:

Gender	Pierced	Not Pierced	Total
Male	36	144	180
Female	288	32	320
Total	324	176	500

which shows us that conditional probability is not very different from (and actually quite the same as) the conditional percents we calculated back in section 1.

Scenario: Piercings and Gender

Consider the piercing example, where the following two-way table is given,

Gender	Pierced	Not Pierced	Total
Male	36	144	180
Female	288	32	320
Total	324	176	500

Recall also that M represents the event of being a male ("not M" represents being a female), and E represents the event of having one or both ears pierced.

Did I Get This

1/1 point (graded)

Which of the following represents the probability that a randomly chosen female has pierced ears? In other words, what is the conditional probability that a randomly chosen student has pierced ears, given that this student is female?

- P (E | M)
- P (not E | M)
- P (not M | E)
- P (E | not M) ✔

Answer

Correct:

P(E | not M) represents the probability that a randomly chosen student has pierced ears (E) given that this student is a female (not M).

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Did I Get This

1/1 point (graded)

Which of the following represents the probability that a randomly chosen student is a non-pierced male?

- P(not E | M)

- P(E | not M)
- P(not E and M) ✔
- P(E and not M)

Answer
Correct:
Indeed, we are not looking for a conditional probability here. We are looking for the probability that a randomly chosen student is a non-pierced male.

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Did I Get This

1/1 point (graded)
P(not E | not M) represents the probability that:

- a randomly chosen male does not have pierced ears.
- a randomly chosen female does not have pierced ears. ✔
- a randomly chosen student is a non-pierced female.
- a randomly chosen non-pierced student is a female.

Answer
Correct:
P(not E | not M) conditions on the student being a female (not M). In other words, we choose only among the female students.

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Did I Get This

1/1 point (graded)
What is P(not E | not M), the probability that a randomly chosen female does not have pierced ears?

- 288/320

- ○ 32/176
- ● 32/320 ✔
- ○ 32/500

Answer
Correct:
When we condition on "not M," this means that we are restricting ourselves to only the 320 female students. Indeed, among them, 32 are not pierced, and therefore 32/320 is the right answer.

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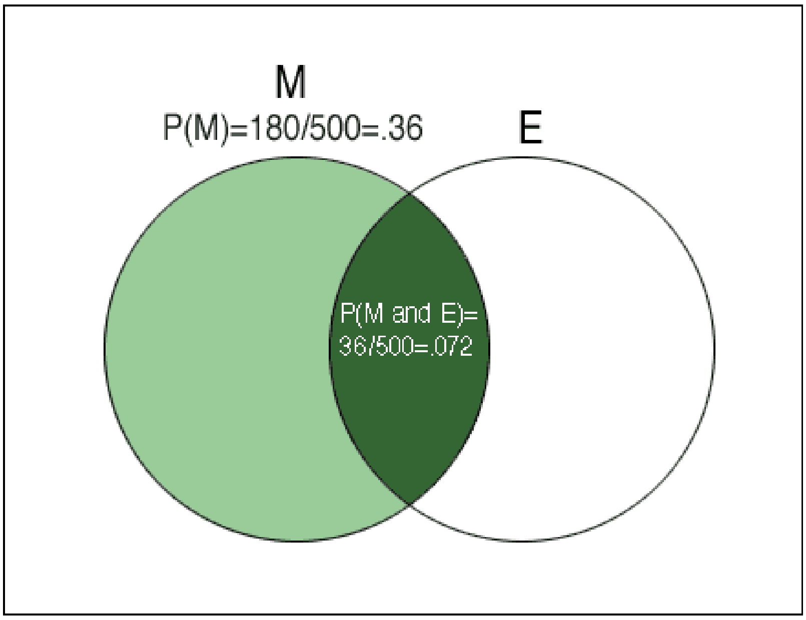
Another way to visualize conditional probability is using a Venn diagram:



In both the two-way table and the Venn diagram, the reduced sample space (comprised of only males) is shaded light green, and within this sample space, the event of interest (having ears pierced) is shaded darker green. The two-way table illustrates the idea via counts, while the Venn diagram converts the counts to probabilities, which are presented as regions rather than cells.

We may work with counts, as presented in the two-way table, to write

P(E | M) = 36/180.

Or we can work with probabilities, as presented in the Venn diagram, by writing

P(E | M) = (36/500) / (180/500).

We will want, however, to write our formal expression for conditional probabilities in terms of other, ordinary, probabilities and therefore the definition of conditional probability will grow out of the Venn diagram.

Notice that

P(E | M) = (36/500) / (180/500) = P(M and E) / P(M). Generalized, we have a formal definition of conditional probability:

conditional probability

(definition) The **conditional probability of event B, given event A,** is **P(B | A) = P(A and B) / P(A)**

Comments

1. Note that when we evaluate the conditional probability, we always divide by the probability of the given event. The probability of both goes in the numerator.

2. The above formula holds as long as P(A) > 0, since we cannot divide by 0. In other words, we should not seek the probability of an event given that an impossible event has occurred.

Let's see how we can use this formula in practice:

Example

On the "Information for the Patient" label of a certain antidepressant, it is claimed that based on some clinical trials, there is a 14% chance of experiencing sleeping problems known as insomnia (denote this event by I), there is a 26% chance of experiencing headache (denote this event by **H**), and there is a 5% chance of experiencing both side effects (**I and H**).

(a) Suppose that the patient experiences insomnia; what is the probability that the patient will also experience headache?

Since we know (or it is given) that the patient experienced insomnia, we are looking for P(H | I). According to the definition of conditional probability:

P(H | I) = P(H and I) / P(I) = 0.05/0.14 = 0.357.

(b) Suppose the drug induces headache in a patient; what is the probability that it also induces insomnia?

Here, we are given that the patient experienced headache, so we are looking for P(I | H).

Using the definition P(I | H) = P(I and H) / P(H) = 0.05/0.26 = 0.1923.

Comment

Note that the answers to (a) and (b) above are different. In general, P(A | B) does not equal P(B | A). We'll come back and illustrate this point later in this module.

The purpose of the following activity is to give you guided practice in using the definition of conditional probability, and teach you how the Complement Rule works with conditional probability.

Scenario: Delivery Services

Recall the delivery services example:

It is vital that a certain document reach its destination within one day. To maximize the chances of on-time delivery, two copies of the document are sent using two services, service A and service B, and the following probability table summarizes the chances of on-time delivery:

	B	not B	Total
A	0.75	0.15	0.90
not A	0.05	0.05	0.10
Total	0.80	0.20	1.00

Learn By Doing (1/1 point)

If the document has reached its destination on time through service A, what is the probability that it will also reach its destination through service B? When you answer, first write down the conditional probability we are looking for, and then find it using the definition of conditional probability: P(B | A) = P(A and B) / P(A).

Your Answer:

P(B|A) = 0.75/0.9 based on the two-way table

Our Answer:
We are given that the document has arrived on time using service A, and we are wondering what the probability is that it will also arrive on time using service B. We are therefore looking for P(B | A). Using

the definition of conditional probability and the probability table, we get that: P(B | A) = P(A and B) / P(A) = .75/.9 = 0.833

Resubmit Reset

Learn By Doing (1/1 point)

If service A has failed to deliver the document on time, what is the probability that it has arrived on time using service B? (Again, as before, first write down the conditional probability that you are asked to find, and then apply the definition of conditional probability to find it.)

Your Answer:

P(B|-A) = 0.05/0.10 = 0.5

Our Answer:
We are given that the document was not delivered on time using service A (not A), and we are wondering how likely is it that it was delivered on time using service B. We are therefore looking for P(B | not A). Using the definition of conditional probability and the probability table, we get that: P(B | not A) = P(not A and B) / P(not A) = .05/.1 = 0.50

Resubmit Reset

Learn By Doing (1/1 point)

If service A delivered the document on time, what is the probability that it was not delivered on time using service B?

Your Answer:

P(-B|A) = P(-B & A) / P(A) = 0.15 / 0.9 = 0.17

Our Answer:
We are given that the document was delivered on time using service A, and we are wondering how likely it is that it was not delivered on time using service B. We are therefore looking for P(not B | A). Using the definition of conditional probability and the probability table, we get that: P(not B | A) = P(A and not B) / P(A) = .15/.90 = 0.167 Let's summarize the results we got: P(B | A) = 0.833 P(B | not A) = 0.50 P(not B | A) = 0.167 Note that P(B | A) = 1 - P(not B | A), which tells us that P(not B | A) is the complement event of P(B | A). Students sometimes tend to get confused and think that the complement event of P(B

| A) is P(B | not A). Please note the distinction. The Complement Rule extends to conditional probabilities only when you condition on the same event.

Resubmit Reset

Scenario: Smoke Alarms

Recall the smoke alarms example from the previous module. A homeowner has smoke alarms installed in the dining room (adjacent to the kitchen) and an upstairs bedroom (above the kitchen). The two-way table below shows probabilities of smoke in the kitchen triggering the alarm in the dining room (D) or not, and in the bedroom (B) or not. Use this two-way table to answer the following:

	B	not B	Total
D	0.38	0.57	0.95
not D	0.02	0.03	0.05
Total	0.40	0.60	1.00

Did I Get This

1/1 point (graded)

The fraction 0.38/0.95 represents:

- P(D | B)
- P(B | D) ✔
- P(D and not B)
- P(D | not B)
- P(not B | D)

Answer

Correct: Indeed, P(B | D) = P(B and D) / P(D) = .38/.95

Submit

Did I Get This

1/1 point (graded)

The fraction 0.57/0.60 represents:

- P(D | B)
- P(B | D)
- P(D and not B)
- P(D | not B) ✔
- P(not B | D)

Answer

Correct: Indeed P(D | not B) = P(D and not B) / P(not B) = .57/.60

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Did I Get This

1/1 point (graded)

The probability 0.57 represents:

- P(D | B)
- P(B | D)
- P(D and not B) ✔
- P(D | not B)
- P(not B | D)

Answer

Correct: Indeed, lifting directly from the table, P(D and not B) = .57

Submit