

 Lagunita is retiring and will shut down at 12 noon Pacific Time on March 31, 2020. A few courses may be open for self-enrollment for a limited time. We will continue to offer courses on other online learning platforms; visit <http://online.stanford.edu>.

Course > Probability: Conditional Probability and Independence >

Conditional Probability and Independence Overview > Independence Check: Other Methods

 Bookmark this page

Independence Check: Other Methods

Learning Objective: Determine whether two events are independent or not.

Comment

Recall the pierced ears example. We checked the independence of the events M (being a male) and E (having pierced ears) by comparing $P(E)$ to $P(E | M)$.

An alternative method of checking for dependence would be to compare $P(E | M)$ with $P(E | \text{not } M)$ [same as $P(E | F)$]. In our case, $P(E | M) = 36/180 = 0.2$, while $P(E | \text{not } M) = 288/320 = 0.9$, and since the two are very different, we can say that the events E and M are not independent.

In general, another method for checking the independence of events A and B is to compare **$P(B | A)$ and $P(B | \text{not } A)$** . In other words, two events are independent if the probability of one event does not change whether we know that the other event has occurred or we know that the other event has not occurred. It can be shown that **$P(B | A)$ and $P(B | \text{not } A)$** would differ whenever $P(B)$ and $P(B | A)$ differ, so this is another perfectly legitimate way to establish dependence or independence.

Scenario: Smoke Alarms

Recall again the smoke alarms example.

A homeowner has smoke alarms installed in the dining room (adjacent to the kitchen) and an upstairs bedroom (above the kitchen). The two-way table below shows probabilities of smoke in the kitchen triggering the alarm in the dining room (D) or not, and in the bedroom (B) or not.

	B	not B	Total
D	0.38	0.57	0.95
not D	0.02	0.03	0.05
Total	0.40	0.60	1.00

Did I Get This

1/1 point (graded)

In order to check whether the events B and D are independent or not, (i.e., check whether the two smoke alarms detect smoke independently of each other) we need to compare:

☐ $P(D | B)$ and $P(\text{not } D | \text{not } B)$

☐ $P(D | B)$ and $P(\text{not } D | B)$

☒ $P(D | B)$ and $P(D | \text{not } B)$ ✓

☐ $P(D | B)$ and $P(B | D)$

Answer

Correct:

Indeed, to check whether two events are independent, we need to compare the conditional probability of one event given that the other event has occurred ($P(D | B)$) to the conditional probability of the event given that the other event has not occurred ($P(D | \text{not } B)$).

Submit

Did I Get This

1/1 point (graded)

Using the table we find that $P(D | B)$ is equal to:

☐ $0.38/0.95 = 0.40$

☒ $0.38/0.40 = 0.95$ ✓

☐ 0.38

Answer

Correct: Indeed, $P(D \mid B) = P(D \text{ and } B) / P(B) = 0.38/0.40 = 0.95$

Submit

Did I Get This

1/1 point (graded)

Using the table, we also find that $P(D \mid \text{not } B)$ is equal to:

☐ 0.57

☐ $0.57/0.95 = 0.60$

☒ $0.57/0.60 = 0.95$ ✓

Answer

Correct: Indeed, $P(D \mid \text{not } B) = P(D \text{ and not } B) / P(\text{not } B) = 0.57/0.60 = 0.95$

Submit

Did I Get This

1/1 point (graded)

From the answers to the previous questions, you can conclude that the events B and D are:

☒ independent ✓

☐ not independent

Answer

Correct: Since $P(D \mid B) = P(D \mid \text{not } B)$, the events are independent.

Submit

Before we establish a general rule for independence, let's consider an example that will illustrate another method that we can use to check whether two events are independent:

Example

A group of 100 college students were surveyed about their gender and whether they had decided on a major.

	Decided	Undecided	Total
Female	27	33	60
Male	18	22	40
Total	45	55	100

Offhand, we wouldn't necessarily have any compelling reason to expect that deciding on a major would depend on a student's gender. We can check for independence by comparing the overall probability of being decided to the probability of being decided given that a student is female:

$$P(D) = 45/100 = 0.45 \text{ and } P(D | F) = 27/60 = 0.45.$$

The fact that the two are equal tells us that, as we might expect, deciding on a major is independent of gender. Note from the comment that these must also equal $P(D | M)$, which is $18/40 = 0.45$.

Now let's approach the issue of independence in a different way: first, we may note that the overall probability of being decided is $45/100 = 0.45$.

	Decided	Undecided	Total
Female	27	33	60
Male	18	22	40
Total	45	55	100

And the overall probability of being female is $60/100 = 0.60$.

	Decided	Undecided	Total
Female	27	33	60
Male	18	22	40
Total	45	55	100

If being decided is independent of gender, then 45% of the 60% of the class who are female should have a decided major; in other words, the probability of being female and decided should equal the probability of being female multiplied by the probability of being decided. If the events F and D are independent, we should have $P(F \text{ and } D) = P(F) * P(D)$.

In fact, $P(F \text{ and } D) = 27/100 = 0.27 = P(F) * P(D) = 0.45 * 0.60$. This confirms our alternate verification of independence.

In general, another method for checking the independence of events A and B is to compare $P(A \text{ and } B)$ to $P(A) * P(B)$. If the two are equal, then A and B are independent, otherwise the two are not independent.

Let's summarize all the possible methods we've seen for checking the independence of events in one rule:

Two events A and B are independent if any one of the following hold:

$$P(B | A) = P(B)$$

$$P(A | B) = P(A)$$

$$P(B | A) = P(B | \text{not } A)$$

$$P(A \text{ and } B) = P(A) * P(B)$$

Comment

These various equalities turn out to be equivalent, so that if one equality holds, all are equal, and if one equality does not hold, all are not equal. (This is the case for the same reason that knowing one of the values $P(A \text{ and } B)$, $P(A \text{ and not } B)$, $P(\text{not } A \text{ and } B)$, or $P(\text{not } A \text{ and not } B)$, along with $P(A)$ and $P(B)$, allows you to determine the remaining cells of a two-way probability table.)

Therefore, in order to check whether events A and B are independent or not, it is sufficient to check only whether one of the four equalities holds—whichever is easiest for you.

The purpose of the next activity is to practice checking the independence of two events using the four different possible methods that we've provided, and see that all of them will lead us to the same conclusion, regardless of which of the four methods we use.

Learn By Doing (1/1 point)

Recall the delivery services example: It is vital that a certain document reach its destination within one day. To maximize the chances of on-time delivery, two copies of the document are sent using two services, service A and service B, and the following probability table summarizes the chances of on-

time delivery: Are the delivery times of the two services independent? In other words, are the events A and B independent? Common sense would say that there will be some degree of dependence between A and B, since the reasons that would cause one service to be delayed (like bad weather, airport delays, etc), would most likely also affect the other service, and cause it to be delayed too. Let's review the four possible methods that we can use to check whether events A and B are independent: Two events A and B are independent if any one of the following hold: $P(B | A) = P(B)$ $P(A | B) = P(A)$ $P(B | A) = P(B | \text{not } A)$ $P(A \text{ and } B) = P(A) * P(B)$ Use the four different methods to check whether events A and B are independent, and see that indeed all four are leading you to the same conclusion.

Your Answer:


i think the table was supposed to be here

Our Answer:


Our answer: • $P(B | A) = P(B \text{ and } A) / P(A) = .75/.90 = .833$ while $P(B) = .8$. Since the two are not equal, events A and B are not independent. • $P(A | B) = P(A \text{ and } B) / P(B) = .75/.80 = .9375$ while $P(A) = .9$. Since the two are not equal, events A and B are not independent. • $P(B | A) = P(B \text{ and } A) / P(A) = .75/.90 = .833$, while $P(B | \text{not } A) = P(B \text{ and not } A) / P(\text{not } A) = .05/.10 = .50$. Since the two are not equal, events A and B are not independent. • $P(A \text{ and } B) = .75$ while $P(A) * P(B) = .90 * .80 = .72$. Since the two are not equal, events A and B are not independent. Note that all four methods lead us to the same conclusion.

Resubmit

Reset

Open Learning Initiative 



Unless otherwise noted this work is licensed under a Creative Commons Attribution-NonCommercial-ShareAlike 4.0 International License .

© All Rights Reserved