Wrap-Up (Hypothesis Testing)

These sections covered the z-test for population proportion and both the z-test and t-test for the population mean. The following table summarizes when each of the tests are used:

Type of Hypothesis Test	Circumstances When Used
z-test for the Population Proportion	• Testing the Population Proportion(p) • Variable of interest is Categorical • Population Proportion is unknown
z-test for the Population Mean	• Testing the Population Mean (μ) • Variable of interest is Quantitative • Population standard deviation is known(σ)
t-test for the Population Mean	• Testing the Population Mean (μ) • Response Variable is Quantitative • Population standard deviation is unknown, so sample standard deviation is used (s) instead.

These sections are also loaded with very important ideas that apply to the general process of hypothesis testing. Thus, the following summary discusses each of the above named hypothesis tests within the context of the hypothesis testing process.

The process of hypothesis testing has four steps:

I. Stating the null and alternative hypotheses (Ho and Ha).

Type of Hypothesis Test	Null Hypothesis	Alternative Hypothesis
z-test for the Population Proportion	$H_0: p=p_0$	$H_a: p \neq p_0$ **or** $H_a: p<p_0$ **or** $H_a: p>p_0$

z-test for the Population Mean	H_0:$\mu=\mu_0$	H_a:$\mu\neq\mu_0$ **or** H_a:$\mu<\mu_0$ **or** H_a:$\mu>\mu_0$
t-test for the Population Mean	H_0:$\mu=\mu_0$	H_a:$\mu\neq\mu_0$ **or** H_a:$\mu<\mu_0$ **or** H_a:$\mu>\mu_0$

II. Obtaining a random sample (or at least one that can be considered random) and collecting data. Using the data:

- **Check that the conditions under which the test can be reliably used are met.**

For the **z-test for the Population Proportion**, we can reliably use the test is if the following conditions holds: $np_0 \geq 10$ *and* $n(1-p_0) \geq 10$

For the **z-test for the Population Mean** and the **t-test for the Population Mean**, the following table is a summary the conditions under which they can be reliably used, and which test to use when:

Situation	Sigma Known? Known	Sigma Known? Unknown
Large sample size (regardless of whether the population is normal or not)	**z-test**	**t-test** (z-test is a good approx.)
Small sample size, population Normal* (footnote)	**z-test**	**t-test**
Small sample size, population shape not Normal* or unknown (footnote)	**Neither z-test nor t-test**	

*by "population normal" we mean that either the population is known to be normal, or else that the population can be reasonably assumed to be normal as judged by the shape of the data histogram.

- **Summarize the data using a test statistic.**

The test statistic is a measure of the evidence in the data against the H_o. The larger the test statistic is in magnitude, the more evidence the data present against the H_o.

Hypothesis Test	Test Statistic
z-test for the Population Proportion	$z = \frac{\hat{p}-p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}}$

z-test for the Population Mean	$z = \frac{\bar{x}-\mu_0}{\frac{\sigma}{\sqrt{n}}}$
t-test for the Population Mean	$t = \frac{\bar{x}-\mu_0}{\frac{s}{\sqrt{n}}}$

III. Finding the p-value of the test.

The p-value is the probability of getting data like those observed (or even more extreme) assuming that the null hypothesis is true, and is calculated using the null distribution of the test statistic. The p-value is a measure of the evidence against H_o. The smaller the p-value, the more evidence the data present against H_o.

In this module, we learned how to compute the p-value for the two z-tests (z-test for the population proportion and the z-test for the population mean). However, for the t-test (and, actually, from this point on in the course), we will use software to find the p-value for us.

IV. Making conclusions.

Conclusions about the significance of the results:

If the p-value is small, the data present enough evidence to reject H_0 (and accept H_a).

If the p-value is not small, the data do not provide enough evidence to reject H_0.

To help guide our decision, we use the significance level as a cutoff for what is considered a small p-value. The significance cutoff is usually set at .05, but should not be considered inviolable.

Conclusions should always be made in the context of the problem.

Additional 'Big Ideas' about hypothesis Testing.

Note: *These ideas were already mentioned in the summary for hypothesis testing for the population proportion p, however it is worth repeating them and thus stress that these idea apply to hypothesis testing in general!*

Results that are based on a larger sample carry more weight, and therefore results that are not significant (do not provide evidence to reject H_0) may become significant if based on a larger sample size. As a result...

Even a very small and practically unimportant effect becomes statistically significant with a large enough sample size. The distinction between statistical significance and practical importance should therefore always be considered.

For given data, the p-value of the two-sided test is always twice as large as the p-value of the one-sided test. It is therefore harder to reject H_0 in the two-sided case than it is in the one-sided case in the sense that stronger evidence is required. Intuitively, the hunch or information that leads us to use the one-sided test can be regarded as a head-start toward the goal of rejecting H_0.

95% confidence intervals can be used in order to carry out **two-sided tests** (at the 0.05 significance level). If the null value is not included in the confidence interval (i.e., is not one of the plausible values for the parameter), we have enough evidence to reject H_0. Otherwise, we cannot reject H_0.

If the results are significant, it might be of interest to follow up the tests with a confidence interval in order to get insight into the actual value of the parameter of interest.
