Behavior of Sample Mean: Introduction

Learning Objective: Apply the sampling distribution of the sample mean as summarized by the Central Limit Theorem (when appropriate). In particular, be able to identify unusual samples from a given population.

So far, we've discussed the behavior of the statistic $\hat{p}$, the sample proportion, relative to the parameter p, the population proportion (when the variable of interest is categorical). We are now moving on to explore the behavior of the statistic $\overline{X}$, the sample mean, relative to the parameter μ, the population mean (when the variable of interest is quantitative).

Behavior of Sample Mean $\overline{X}$

Example

Birth weights are recorded for all babies in a town. The mean birth weight is 3,500 grams, μ = 3,500 g. If we collect many random samples of 9 babies at a time, how do you think sample means will behave?

Here again, we are working with a random variable, since random samples will have means that vary unpredictably in the short run but exhibit patterns in the long run.

Based on our intuition and what we have learned about the behavior of sample proportions, we might expect the following about the distribution of sample means:

Center: Some sample means will be on the low side—say 3,000 grams or so—while others will be on the high side—say 4,000 grams or so. In repeated sampling, we might expect that the random samples will average out to the underlying population mean of 3,500 g. In other words, the mean of the sample means will be μ, just as the mean of sample proportions was p.

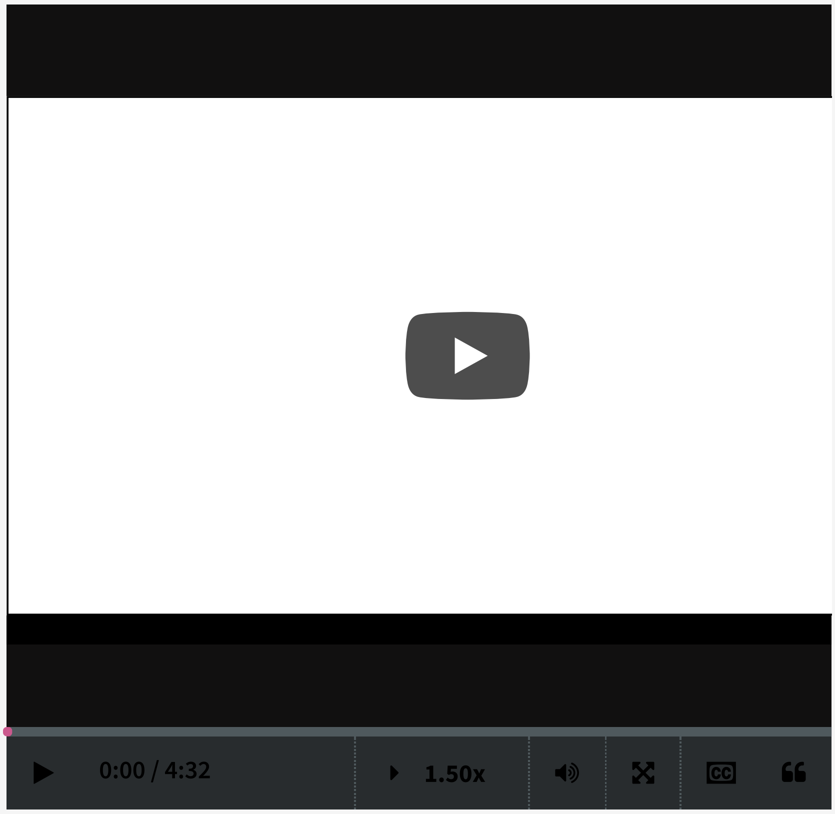
Spread: For large samples, we might expect that sample means will not stray too far from the population mean of 3,500. Sample means lower than 3,000 or higher than 4,000 might be surprising. For smaller samples, we would be less surprised by sample means that varied quite a bit from 3,500. In others words, we might expect greater variability in sample means for smaller samples. So sample size will again play a role in the spread of the distribution of sample measures, as we observed for sample proportions.

Shape: Sample means closest to 3,500 will be the most common, with sample means far from 3,500 in either direction progressively less likely. In other words, the shape of the distribution of sample means should bulge in the middle and taper at the ends with a shape that is somewhat normal. This, again, is what we saw when we looked at the sample proportions.

Comment

The **distribution** of the values of the sample mean ($\overline{x}$) in repeated **samples** is called the **sampling distribution of $\overline{x}$**.

Behavior of Sample Mean 1



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In this movie we will discuss the behavior of sample means. In particular,

we're going to investigate this question: What is the shape, center, and spread of the

distribution of sample means? To investigate this question we're going to

return to the familiar context of the previous example and look at

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In the simulation, when we are building a sampling distribution, what does each dot represent in the graph?

- ○ a baby
- ● a random sample of 9 babies ✔
- ○ thousands of babies

Answer

Correct: Each sample is represented in the sampling distribution with a dot at its $\bar{x}$ value.

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In the simulation, we collected thousands of random samples from the population of babies. The mean weight for individual babies is 3,500 grams. What is the mean of the sample means?

- ○ 500
- ○ 9
- ● 3,500 ✔
- ○ Unable to determine from the information provided

Answer

Correct:

The mean of the sample means is the population mean; therefore, the mean of the sample means or the sampling distribution of the mean is 3,500 grams.

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In the simulation, the standard deviation for the population is 500 grams. What is true about the standard deviation of sample means?

- larger than 500 grams
- smaller than 500 grams ✔
- about the same as 500 grams

Answer

Correct:

There is less variability in sample means than in individual measurements, so the standard deviation for sample means will be smaller than 500 grams.

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