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Course > EDA: Examining Distributions > One Quantitative Variable: Measures of Center > Mode, Mean, & Median

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# Mode, Mean, & Median

Learning Objective: Relate measures of center and spread to the shape of the distribution, and choose the appropriate measures in different contexts.

Intuitively speaking, the numerical measure of center is telling us what is a "typical value" of the distribution.

The three main numerical measures for the center of a distribution are the **mode**, the **mean** and the median. Each one of these measures is based on a completely different idea of describing the center of a distribution. We will first present each one of the measures, and then compare their properties.

#### Mode

So far, when we looked at the shape of the distribution, we identified the mode as the value where the distribution has a "peak" and saw examples when distributions have one mode (unimodal distributions) or two modes (bimodal distributions). In other words, so far we identified the mode visually from the histogram.

Technically, the mode is the most commonly occurring value in a distribution. For simple datasets where the frequency of each value is available or easily determined, the value that occurs with the highest frequency is the mode.

## **Example: Best Actress Oscar Winners**

We will continue with the Best Actress Oscar winners example (To see the full dataset, click here ๔.)

To find the most commonly occurring, or modal, age, it is helpful to list the ages in a frequency table, which give the following results.....

A g e	2 1	2 2	2 5	2	2 7	2 8	2 9	3	3		3	3 4	3 5	3	3 7	3 8	3 9	4	4 2	4	4 9	6	6 2	7	8
C t	1	1	1	2	1	1	3	1	1	2	6	2	2	2	1	1	1	3	2	2	2	2	1	1	1

The mode is 33, since it occurs the most times (6).

# **Example: World Cup Soccer**

Often we have large sets of data and use a frequency table to display the data more efficiently.

Data were collected from the last three World Cup soccer tournaments. A total of 192 games were played. The table below lists the number of goals scored per game (not including any goals scored in shootouts).

total # goals/game	frequency
0	17
1	45
2	51
3	37
4	25
5	11
6	3
7	2
8	1

We can see that the most frequently occurring value is 2 goal (which occurred 51 times). Therefore, the mode for this set of data is 2.

# Did I Get This

1/1 point (graded)

Here are the number of hours that 9 students spend on the computer on a typical day:

167558111215

What is the mode number of hours spent o	n the comp	uter?
--	------------	-------

O 5 🗸

**5.5** 

7.5

**8** 

#### **Answer**

Correct: Since 5 occurs twice in the sample, it is the most frequently occurring score.

Submit

# Did I Get This

1/1 point (graded)

What kind of distribution is formed by the data from the above 9 students?

o unimodal 🗸

multimodal

#### **Answer**

Correct: Since there is only one mode (5), it is a unimodal distribution.

Submit

#### Mean

The mean is the average of a set of observations (i.e., the sum of the observations divided by the number of observations). If the n observations are  $x_1, x_2, ..., x_n$ , their mean, which we denote by  $\overline{x}$  (and read  $\overline{x}$ ), is therefore:  $\overline{x} = \frac{x_1 + x_2 + ... + x_n}{n}$ 

# **Example: Best Actress Oscar Winners**

Again we will use the Best Actress Oscar winners example (To see the full dataset, click here ₫.)

34 34 27 37 42 41 36 32 41 33 31 74 33 49 38 61 21 41 26 80 42 29 33 36 45 49 39 34 26 25 33 35 35 28 30 29 61 32 33 45 29 62 22 44

The mean age of the 44 actresses is: 
$$\overline{x} = \frac{34 + 34 + 27 + ... + 62 + 22 + 44}{44} = \frac{1687}{44} = 38.3$$

Note that the mean gives a measure of center which is higher than our approximation of the center from looking at the histogram (which was 34). The reason for this will be clear soon.

# **Example: World Cup Soccer**

We will now continue with the data from the last three World Cup soccer tournaments. A total of 192 games were played. The table below lists the number of goals scored per game (not including any goals scored in shootouts).

total # goals/game	frequency
0	17
1	45
2	51
3	37
4	25
5	11
6	3
7	2
8	1

To find the mean number of goals scored per game, we would need to find the sum of all 192 numbers, then divide that sum by 192. Rather than add 192 numbers, we use the fact that the same numbers appear many times. For example, the number 0 appears 17 times, the number 1 appears 45 times, the number 2 appears 51 times, etc.

If we add up 17 zeros, we get 0. If we add up 45 ones, we get 45. If we add up 51 twos, we get 102. Repeated addition is multiplication.

Thus, the sum of the 192 numbers = 0(17) + 1(45) + 2(51) + 3(37) + 4(25) + 5(11) + 6(3) + 7(2) + 8(1) = 453.

The mean is 453 / 192 = 2.359.

This way of calculating a mean is sometimes referred to as a **weighted average**, since each value is "weighted" by its frequency. Note that, in this example, the values of 1, 2, and 3 are most heavily weighted.

#### Did I Get This

1/1 point (graded)

Here are the number of hours that 9 students spend on the computer on a typical day:

167558111215

Which of the following is the mean number of hours spent on the computer?

7.00
 7.78 ✓
 8.23
 9.14

#### **Answer**

Correct:

The mean involves the sum of all the observations (1+6+7+5+5+8+11+12+15=70) divided by the total number of observations (9), which equals 7.78.



## Did I Get This

1/1 point (graded)

A recent survey asked 90 students, How many hours do you spend on the computer in a typical day?

Of the 90 respondents, 3 said 1 hour, 5 said 2 hours, 15 said 3 hours, 25 said 4 hours, 20 said 5 hours, 15 said 6 hours, 5 said 7 hours, 1 said 8 hours, and 1 said 9 hours.

What is the average (mean) number of hours spent on the computer?

0.5	
<u> </u>	
<b>○</b> 4.44 <b>✓</b>	

#### **Answer**

Correct:

The average is found as follows: [3(1) + 5(2) + 15(3) + 25(4) + 20(5) + 15(6) + 5(7) + 1(8) + 1(9)]/90 = 4.44.



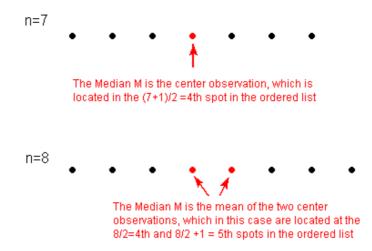
# Median

The median M is the midpoint of the distribution. It is the number such that half of the observations fall above, and half fall below. To find the median:

- Order the data from smallest to largest.
- Consider whether n, the number of observations, is even or odd.
  - If n is **odd**, the median M is the center observation in the ordered list. This observation is the one "sitting" in the (n + 1) / 2 spot in the ordered list.
  - If n is even, the median M is the mean of the two center observations in the ordered list.
    These two observations are the ones "sitting" in the n / 2 and n / 2 + 1 spots in the ordered list.

## Example: Median (1)

For a simple visualization of the location of the median, consider the following two simple cases of n = 7 and n = 8 ordered observations, with each observation represented by a solid circle:



# Example: Median (2)

To find the median age of the Best Actress Oscar winners, we first need to order the data. It would be useful, then, to use the stemplot, a diagram in which the data are already ordered.

Here n = 44 (an even number), so the median M, will be the mean of the two center observations. These are located at the n/2 = 44/2 = 22nd and n/2 + 1 = 44/2 + 1 = 23rd spots. Counting from the top, we find that:

- the 22nd ranked observation is 34
- the 23rd ranked observation is 35

Therefore, the median 
$$M=\frac{(34+35)}{2}=34.5$$
 
$$2 \mid 12$$
 
$$2 \mid 56678999$$
 
$$3 \mid 0122333333344$$
 
$$3 \mid 5566789$$

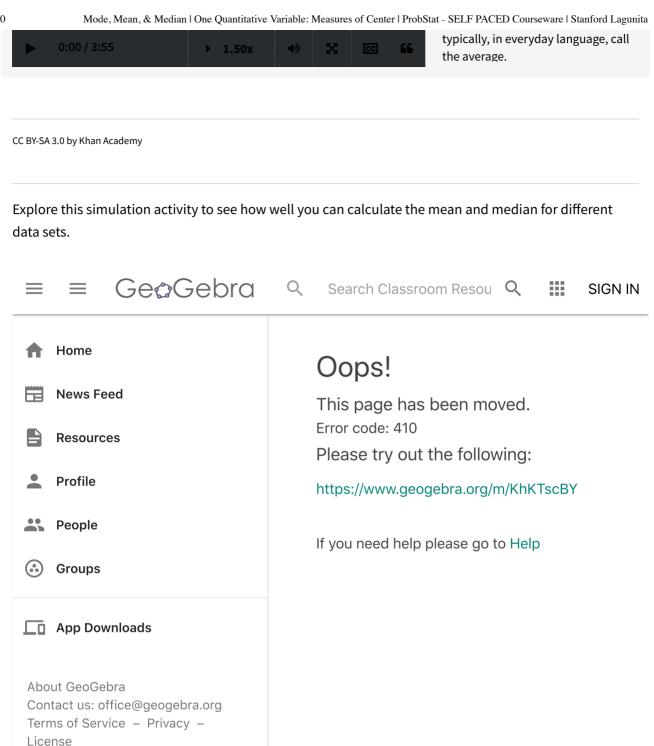
3|5566789 4|1112244 4|599 5|| 6|112 6| 7|4 7|

## Did I Get This

1/1 point (graded)

Here are the number of hours that 9 students spend on the computer on a typical day:

167585111215 What is the median number of hours spent on the computer? **5 6.5** 7 ✓ 7.5 0 8 **Answer** Correct: After you order the data, since n = 9, the median is the (9 + 1) / 2 = 5th observation in the ordered list, which in this case is 7. Submit View the video below for another look at how to calculate the mean, median, and mode Video Start of transcript. Skip to the end. Find the mean, median, and mode of the following sets of numbers. And they give us the numbers right over here. So if someone just says the mean, they're really referring to what we



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