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Hypothesis Testing for the Population Proportion p: Summary

Learning Objective: Carry out hypothesis testing for the population proportion and mean (when appropriate), and draw conclusions in context.

Let's Summarize

We have now completed going through the four steps of hypothesis testing, and in particular, we learned how they are applied to the z-test for the population proportion. Let's briefly summarize:

Step 1

State the null and alternative hypotheses:

$$H_0 : p = p_0$$

$$H_a : p \begin{Bmatrix} < \\ > \\ \neq \end{Bmatrix} p_0$$

where the choice of the appropriate alternative (out of the three) is usually quite clear from the context of the problem.

Step 2

Obtain data from a sample and:

(i) Check whether the data satisfy the conditions which allow you to use this test.

- Random sample (or at least a sample that can be considered random in context)
- $n \cdot p_0 \geq 10, n \cdot (1 - p_0) \geq 10$

(ii) Calculate the sample proportion $\hat{p}$, and summarize the data using the test statistic:

$$z = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}}.$$

(**Recall:** This standardized test statistic represents how many standard deviations above or below p_0 our sample proportion $\hat{p}$ is.)

Step 3

Find the p-value of the test either by using software or by using the test statistic as follows:

* for H_a:$p < p_0$:$P(Z \leq z)$

* for H_a:$p > p_0$:$P(Z \geq z)$

* for H_a:$p \neq p_0$:$2P(Z \geq |z|)$

Step 4

Reach a conclusion first regarding the significance of the results, and then determine what it means in the context of the problem. Recall that:

If the p-value is small (in particular, smaller than the significance level, which is usually 0.05), the results are significant (in the sense that there is a significant difference between what was observed in the sample and what was claimed in H_o), and so we reject H_o. If the p-value is not small, we do not have enough statistical evidence to reject H_o, and so we continue to believe that H_o **may** be true. (Remember, in hypothesis testing we never "accept" H_o).

Scenario: Airplane Drinking Water

Background: This activity is based on the results of a recent study on the safety of airplane drinking water that was conducted by the U.S. Environmental Protection Agency (EPA). A study found that out of a random sample of 316 airplanes tested, 40 had coliform bacteria in the drinking water drawn from restrooms and kitchens. As a benchmark comparison, in 2003 the EPA found that about 3.5% of the U.S. population have coliform bacteria-infected drinking water. The question of interest is whether, based on the results of this study, we can conclude that drinking water on airplanes is more contaminated than drinking water in general.

Learn By Doing

1/1 point (graded)

What is the null hypothesis in this case?

● H_0: $p = 0.035$ ✓

○ H_0: $p \neq 0.035$

○ H_0: $p > 0.035$

○ H_0: $p = 0.127$

Answer

Correct:

The null hypothesis is always a formal statement of "nothing unusual" or "no effect." In this case, the null hypothesis is the formal statement that p (the proportion of contaminated drinking water in the U.S.) is the "proper" baseline rate.

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Learn By Doing

1/1 point (graded)

What is the alternative hypothesis in this case?

○ H_a: $p < 0.035$

○ H_a: $p \neq 0.035$

● H_a: $p > 0.035$ ✓

○ H_a: $p \neq 0.127$

Answer

Correct:

The question of interest is whether p (the overall proportion of contaminated drinking water in airplanes) is higher than the proportion of contaminated water in the U.S. population.

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Learn By Doing (1/1 point)

Based on the collected data, is it safe to use the z-test for p in this scenario? Explain.

Your Answer:

yes because it was a random sample. Both n*p and n*(1-p) are also > 10.

Our Answer:

Let's check the conditions. • The sample of airplanes is random. • n * p0 = 316 * 0.035 = 11.06 > 10. • n * (1 − p0) = 316 * 0.965 = 304.94 > 10. Yes, it is safe to use the test.

Resubmit Reset

Learn By Doing

1/1 point (graded)

Given the following information:

n = 316

$\hat{p} = 40/316 = 0.127$

$p_0 = 0.035$

What is the value of the z statistic? Round your answer to ONE decimal place.

8.9 ✓

8.9

Answer

Correct: $z = (0.127 - 0.035) / \sqrt{((0.035 * (1 - 0.035))/316)} = 8.9$

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Learn By Doing (1/1 point)

For the above z statistic, we calculated a p-value nearly 0. Interpret what that means, and draw your conclusions.

Your Answer:

There's almost 0% probability of observing p=0.127, which is 8.9 standard deviations above p0, assuming Ho is true. So we vehemently reject it, and accept Ha, that it indeed is dirty af.

Our Answer:
A p-value that is so close to 0 tells us that it would be almost impossible to get a sample proportion of 12.5% (or larger) of contaminated drinking water had the true proportion been 3.5%. In other words, the airline industry cannot claim that this just happened to be a "bad" sample that occurred by chance. A p-value that is essentially 0 tells us that it is highly unlikely that such a sample happened just by chance. Our conclusion is therefore that we have an extremely strong reason to reject Ho and conclude that the proportion of contaminated drinking water on airplanes is higher than the proportion in general. On a technical level, the p-value is smaller than any significance level that we are going to set, so Ho can be rejected.

Resubmit Reset

What's Next?

Before we move on to the next test, we use the z-test for proportions to bring up and illustrate some very important issues regarding hypothesis testing.