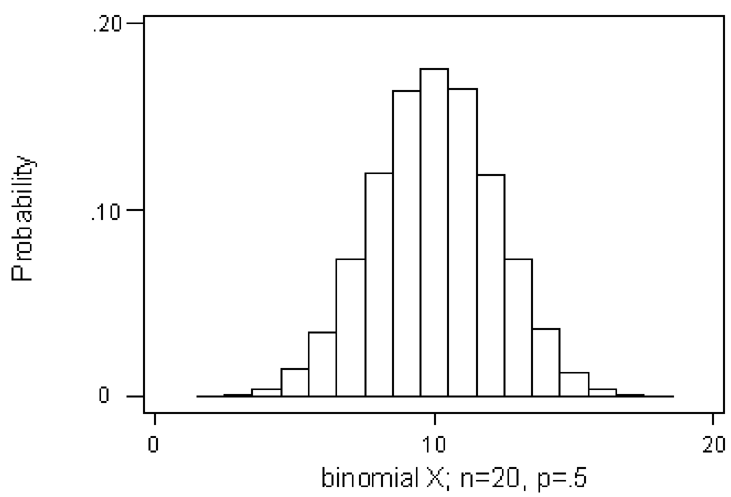
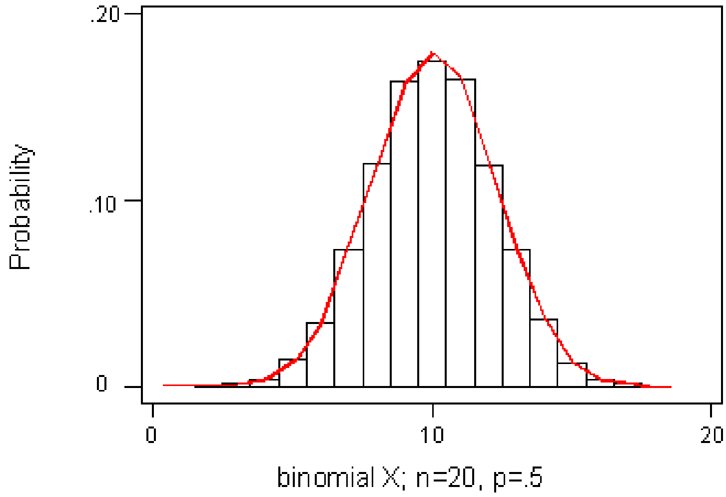
Normal Approximation to the Binomial: Rule of Thumb

Learning Objective: Use the normal distribution as an approximation of the binomial distribution, when appropriate.

Consider the appearance of the probability histogram for the distribution of X:



Clearly, the shape of the distribution of X for n = 20, p = 0.5 has a normal appearance: symmetric, bulging at the middle, and tapering at the ends. The following figure should help you visualize this:



This suggests a method of approximating binomial probabilities:

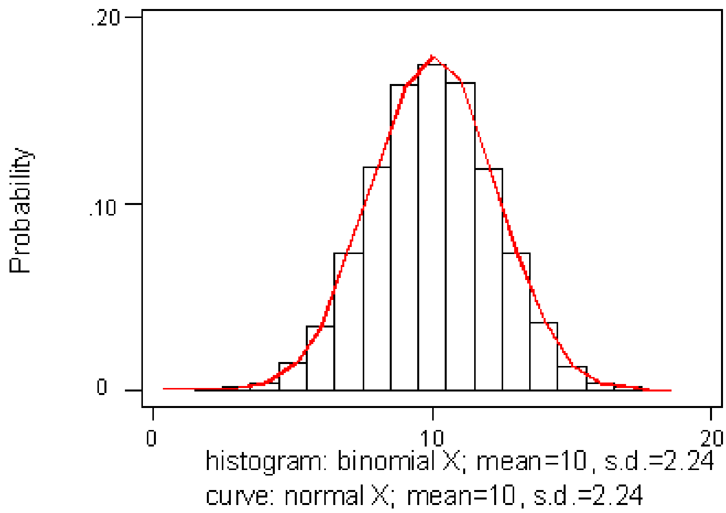
Estimate the binomial probability of X_B taking a value over a certain interval with the probability that a normal random variable X_N takes a value over the same interval, where X_N has the same mean and standard deviation as X_B, namely $\mu = np, \sigma = \sqrt{np(1-p)}$

Example

Suppose a student answers 20 true/false questions completely at random. Use a normal approximation to estimate the probability of getting no more than 8 correct. The number (X) correct is a binomial random variable that represents the number of successes in 20 trials when the probability of success for each trial is 0.5. X has a mean and standard deviation of:

$$\mu = np = 20\left(0.5\right) = 10, \sigma = \sqrt{np(1-p)} = \sqrt{20(0.5)(1-0.5)} = 2.24$$

and so we approximate the binomial X with a normal random variable having the same mean and standard deviation:



Then we solve in the usual way using normal tables:

$$P\left(X_B \le 8\right) \approx P\left(X_N \le 8\right) = P\left(Z \le \frac{8-10}{2.24}\right) = P\left(Z \le -0.89\right) = 0.1867$$

Unfortunately, the approximated probability, .1867, is quite a bit different from the actual probability, 0.2517. However, this example constitutes something of a "worst-case scenario" according to the usual criteria for use of a normal approximation.

Rule of Thumb

Probabilities for a binomial random variable X with n and p may be approximated by those for a normal random variable having the same mean and standard deviation as long as the sample size n is large enough relative to the proportions of successes and failures, p and 1 - p. Our Rule of Thumb will be to require that

$np \ge 10$ and $n(1-p) \ge 10$

Example

May we use a normal approximation for a binomial X with n = 20 and p = 0.5? In this case, np = 20(.5) = 10 and n(1 - p) = 20(1 - .5) = 10. The criteria are just barely satisfied, and so we should not expect the approximation to be especially good.

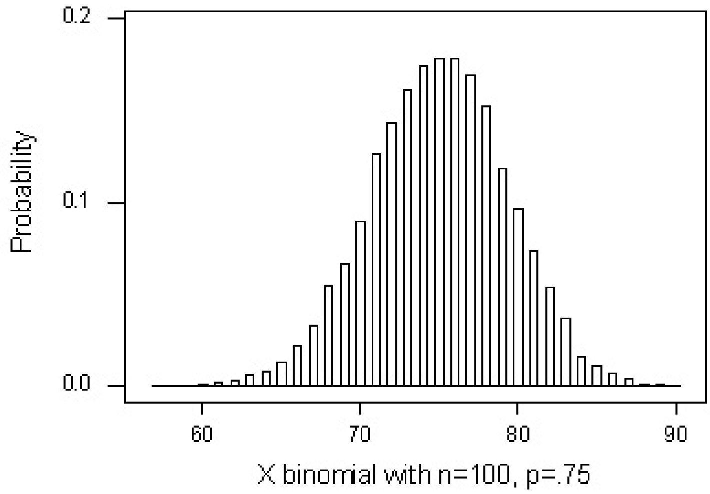
The purpose of the next activity is to give you practice at deciding whether the normal approximation is appropriate for a given binomial random variable. You'll get to practice checking the rule of thumb ($np \ge 10$ and $n(1-p) \ge 10$), but also get a visual sense of when the normal approximation is appropriate.

Learn By Doing

1/1 point (graded)

Below is a histogram representing the probability distribution of a binomial random variable (below the histogram you can see which binomial distribution it is.) Decide whether the normal approximation is appropriate by checking the rule of thumb.

Is the normal approximation appropriate?



- [x] normal approximation is appropriate ✔
- [ ] normal approximation is not appropriate

Answer
Correct:
Indeed, the rule of thumb is satisfied, since np = 100 * 0.75 = 75 > 10 and n(1 - p) = 100 * 0.25 = 25 > 10. Also, visually, it is quite clear that the normal approximation would be very good in this case. The distribution looks essentially normal.

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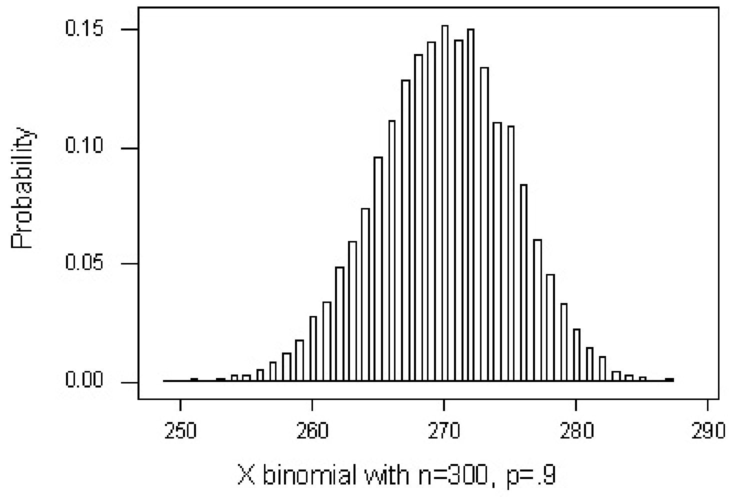

- [x] normal approximation is appropriate ✔
- [ ] normal approximation is not appropriate

Answer
Correct:
Indeed, the rule of thumb is satisfied, since np = 300 * 0.9 = 270 > 10 and n(1 - p) = 300 * 0.1 = 30 > 10. Also, visually, it is quite clear that the normal approximation would be very good in this case. The distribution looks essentially normal.

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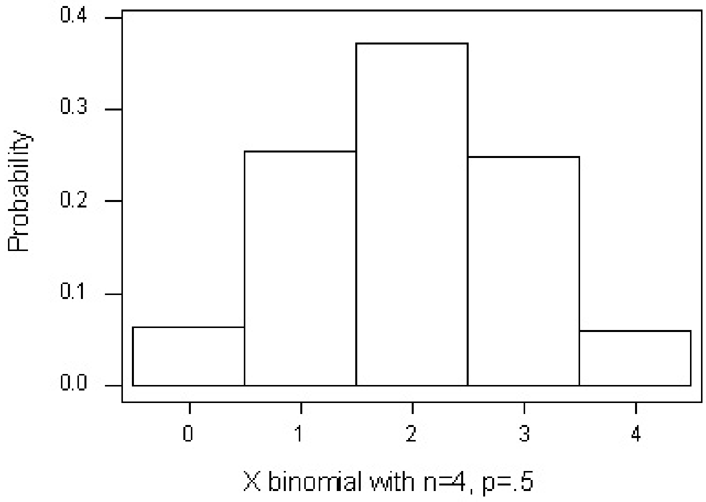

- ( ) normal approximation is appropriate
- (•) normal approximation is not appropriate ✔

Answer
Correct:
Indeed, np = n(1 - p) = 4 * 0.5 = 2, so the rule of thumb is not satisfied. Visually, although the distribution is symmetric and maybe remotely resembles the normal distribution, it is not "fine" enough for the normal approximation to be appropriate.

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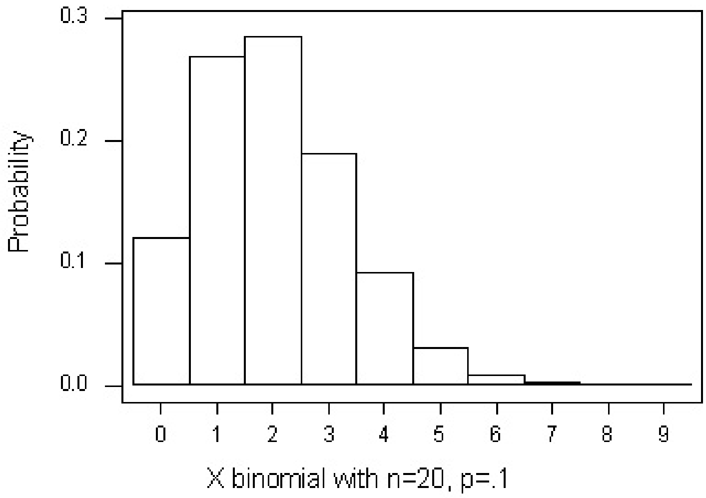
Is the normal approximation appropriate?



- [ ] normal approximation is appropriate
- [x] normal approximation is not appropriate ✔

Answer
Correct:
Indeed, np = 20 * 0.1 = 2 < 10 so the rule of thumb is not satisfied. Visually, it is quite clear that the normal approximation is not appropriate in this case, since the distribution is skewed to the right.

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Learn By Doing (1/1 point)

Recall that when appropriate, a binomial random variable can be approximated by a normal random variable that has the same mean and standard deviation as the binomial random variable. In other words, when appropriate, a binomial random variable with n trials and probability of success p, can be approximated by a normal distribution with mean $\mu = np$ and standard deviation $\sigma = \text{sqrt}(\ np\ (1 - p)\ )$. For those binomial distributions in questions 1-4 of the previous exercise for which the normal approximation is appropriate, write down which normal distribution you would use to approximate them.

Your Answer:

For each one, I can get the mean and standard deviations using the given n and p values.

Our Answer:

For example 1: X is binomial with n = 100 and p = 0.75, and would therefore be approximated by a normal random variable having mean μ = 100 * 0.75 = 75 and standard deviation σ = sqrt(100 * 0.75 * 0.25) = sqrt(18.75) = 4.33. Note that if you look at the histogram, this makes sense. The distribution is indeed centered at 75, and extends approximately 3 standard deviations (3 * 4.33 = 13) on each side of the mean (as we know normal distributions do). For example 2: X is binomial with n = 300 and p = .9, 0and would therefore be approximated by a normal random variable having mean μ = 300 * 0.9 = 270 and standard deviation σ = sqrt(300 * 0.9 * 0.1) = sqrt(27) = 5.2. Note that if you look at the histogram, this makes sense. The distribution is indeed centered at 270, and extends approximately 3 standard deviations (3 * 5.2 = 15.6) on each side of the mean (as we know normal distributions do).

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