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Confidence Intervals for Population Proportion p: Margin of Error

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Confidence Intervals for Population Proportion p: Margin of Error

Learning Objective: Explain what a confidence interval represents and determine how changes in sample size and confidence level affect the precision of the confidence interval.

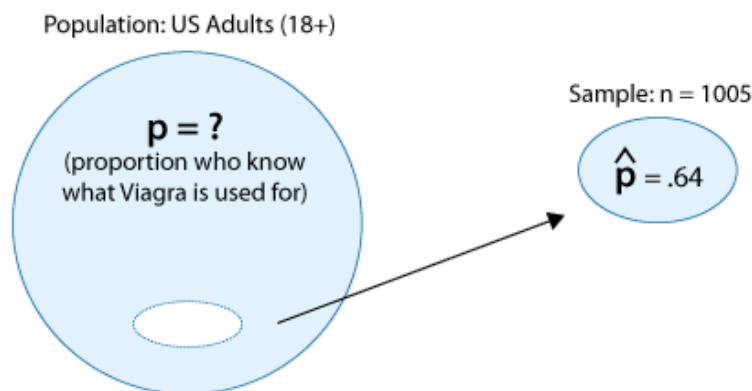
Learning Objective: Find confidence intervals for the population mean and the population proportion (when certain conditions are met), and perform sample size calculations.

Example

The drug Viagra became available in the U.S. in May, 1998, in the wake of an advertising campaign that was unprecedented in scope and intensity. A Gallup poll found that by the end of the first week in May, 643 out of a random sample of 1,005 adults were aware that Viagra was an impotency medication (based on "Viagra A Popular Hit," a Gallup poll analysis by Lydia Saad, May 1998).

Let's estimate the proportion p of all adults in the U.S. who by the end of the first week of May 1998 were already aware of Viagra and its purpose by setting up a 95% confidence interval for p .

We first need to calculate the sample proportion \hat{p} . Out of 1,005 sampled adults, 643 knew what Viagra is used for, so $\hat{p} = \frac{643}{1005} = 0.64$



Therefore,

A 95% confidence interval for p is

$$\hat{p} \pm 2\sqrt{\frac{\hat{p}(1-\hat{p})}{n}} = 0.64 \pm 2\sqrt{\frac{0.64(1-0.64)}{1005}} = 0.64 \pm 0.03 = (0.61, 0.67)$$

We can be 95% sure that the proportion of all U.S. adults who were already familiar with Viagra by that time was between 0.61 and 0.67 (or 61% and 67%).

The fact that the margin of error equals 0.03 says we can be 95% confident that unknown population proportion p is within 0.03 (3%) of the observed sample proportion 0.64 (64%). In other words, we are 95% confident that 64% is "off" by no more than 3%.

Did I Get This (1/1 point)

Researchers from Dartmouth Medical School conducted a study in 2003 to look at the connection between watching actors smoking in movies and smoking initiation among adolescents. In the study, 6,522 U.S. adolescents ages 10-14 who had never tried smoking were randomly selected. Of those who subsequently tried smoking for the first time, 38% did so because of exposure to smoking in the movies. Estimate the proportion of all U.S. adolescents ages 10-14 who started smoking because of seeing actors smoke in movies by constructing a 95% confidence interval.

Your Answer:

$$0.38 \pm \text{sqrt}(p^*(1-p)/n) = 0.38 \pm \text{sqrt}(0.38*0.62/6522) = (37.39\% \text{ to } 38.60\%)$$

Our Answer:

There is a known issue that the images in our answer do not appear. We are working on getting this fixed.

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Did I Get This (1/1 point)

Harris Interactive® conducted an online study from January 3 to 10, 2006 among a nationwide sample of 1,040 U.S. adults (ages 18 years and over), of whom 565 currently use organic foods in preparing meals. Figures for age by gender, race/ethnicity, education, region, and household income were weighted to reflect the total U.S. adult population. Propensity score weighting was used to adjust for respondents' propensity to be online. Construct a 99% confidence interval for the proportion of all U.S. adults who use organic foods in preparing meals.

Your Answer:

sample p = 54.33%

$0.5433 \pm 2 * \sqrt{54.33 * 45.67 / 1040} = (0.5124 \text{ to } 0.5742)$

Our Answer:

There is a known issue that the images we add our answer is not working. We are working to resolve the issue.

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Did I Get This

1/1 point (graded)

Based on the answer to the previous question, what is the margin of error of the confidence interval for the number of U.S. adults who use organic foods in preparing meals?

☐ 0.01

☒ 0.04 ✓

☐ 0.08

☐ 0.543

Answer

Correct:

The interval is (0.503, 0.583) and the value for the sample proportion is 0.543. The margin of error is the distance between the sample proportion and either endpoint of the interval. Both $0.543 - 0.503$ and $0.583 - 0.543 = 0.04$.

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We would like to share with you the methodology part of the poll release of the Viagra example, and show you that you now have the tools to understand how polls results are analyzed:

"The results are based on telephone interviews with a randomly selected national sample of 1,005 adults, 18 years and older, conducted May 8-10, 1998. For results based on samples of this size, one can say with 95 percent confidence that the error attributable to sampling and other random effects could be plus or minus 3 percentage points. In addition to sampling error, question wording and practical difficulties in conducting surveys can introduce error or bias into the findings of public opinion polls."

The purpose of the next activity is to provide guided practice in calculating and interpreting the confidence interval for the population proportion p , and drawing conclusions from it.

Scenario: Legalizing Marijuana Poll

A poll asked a random sample of 1,000 U.S. adults, "Do you think that the use of marijuana should be legalized?" 560 of those asked answered yes.

Learn By Doing (1/1 point)

Based on the poll's results, estimate p , the proportion of all U.S. adults who believe the use of marijuana should be legalized, with a 95% confidence interval.

Your Answer:

$p \pm 2 * \text{sqrt}(p(1-p)/n) = 0.53 \text{ to } 0.59$

Our Answer:

The sample proportion (\hat{p}) is $560/1000=.56$, and therefore a 95% confidence interval for p is: We are 95% confident that the proportion of U.S. adults who believe that marijuana should be legalized is between .53 and .59.

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Learn By Doing (1/1 point)

Give an interpretation of the margin of error in context.

Your Answer:

Our estimation can be at most 3% off from the actual population proportion

Our Answer:

The margin of error is .03 (or 3%). With 95% certainty, the sample proportion we got, 56%, is within 3% of (or, no more than 3% away from) the proportion of U.S. adults who believe that the use of marijuana should be legalized.

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Learn By Doing (1/1 point)

Do the results of this poll give evidence that the majority of U.S. adults believe that the use of marijuana should be legalized?

Your Answer:

Yes, though just a slight majority

Our Answer:

Yes. All of the values in our 95% confidence interval for p (.53, .59), which represents the set of plausible values for p , lies above .5, which provides evidence (at the 95% confidence level) that the majority of U.S. adults believe that the use of marijuana should be legalized. Click [here](#) to see a figure that explains this visually.

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Learn By Doing (1/1 point)

A similar poll was conducted 2 years ago, and reported the 95% confidence interval for p , the proportion of U.S. adults who believe the use of marijuana should be legalized, to be (0.48, 0.54). Do you think that the results of the current study (where the 95% confidence interval is (0.53, 0.59) provide evidence that the public opinion on the topic of legalization of marijuana has changed over the past two years?

Your Answer:

Things looking good for the legalization of marijuana

Edit: Apparently not, because the intersecting ranges mean that the value can still actually be the same. I would've guessed that having a different range is still an improvement; better than nothing?

Our Answer:

The answer is no, we do not have evidence to conclude that the public opinion has changed. Here is why. The two confidence intervals (0.48, 0.54) and (0.53, 0.59) overlap, which means that there are some plausible values for p that the two polls share (in particular, the values between 0.53 and 0.54), and we cannot rule out the possibility that p is one of these values.

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Two important results that we discussed at length when we talked about the confidence interval for μ also apply here:

1. There is a trade-off between level of confidence and the width (or precision) of the confidence interval. The more precision you would like the confidence interval for p to have, the more you have to pay by having a lower level of confidence.
2. Since n appears in the denominator of the margin of error of the confidence interval for p , for a fixed level of confidence, the larger the sample, the narrower, or more precise it is. This brings us naturally to our next point.

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