Probability Trees: Defined

Learning Objective: Use probability trees as a tool for finding probabilities.

So far, when two categorical variables are involved, we have displayed counts or probabilities for various events with two-way tables and with Venn diagrams. Another display tool, called a **probability tree**, is particularly useful for showing probabilities when the events occur in stages and conditional probabilities are involved.

Example

A sales representative tells his friend that the probability of landing a major contract by the end of the week, resulting in a large commission, is .4. If the commission comes through, the probability that he will indulge in a weekend vacation in Bermuda is .9. Even if the commission doesn't come through, he may still go to Bermuda, but only with probability .3.

First, let's identify the given probabilities for events involving **C** (the commission comes through) and **V** (the sales rep takes a Bermuda vacation):

P(C) = 0.4 [and so P(not C) = 0.6],

P(V | C) = 0.9 [and so P(not V | C) = 0.1], and

P(V | not C) = 0.3 [and so P(not V | not C) = 0.7.]

There are two stages in the problem. First, the sales rep will either get the commission or not.

Second, based on what happened in the first stage, the sales rep will either take the Bermuda vacation or not.

We follow exactly the same reasoning when we build the probability tree.

Probability Trees

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The first branch off represents the first stage - the sales representative either

gets the commission or does not get the commission. We are given that the

probability of getting the commission is 0.4 and therefore by the complement

rule the probability of not getting the commission must be 0.6. If

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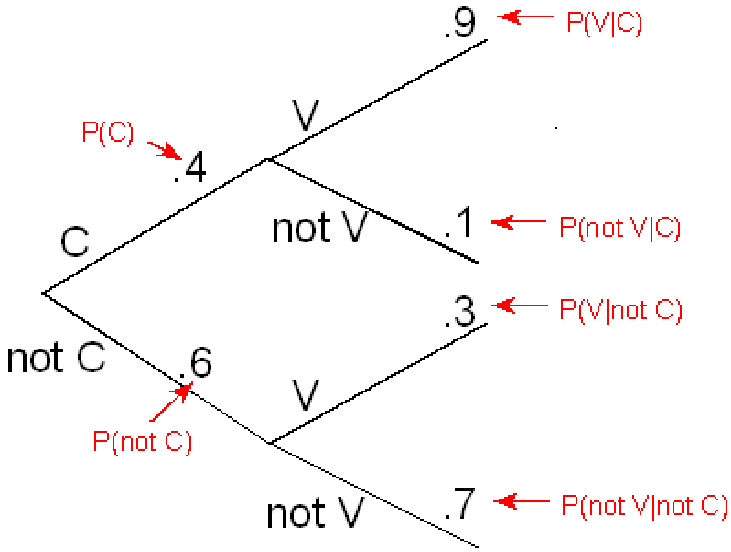
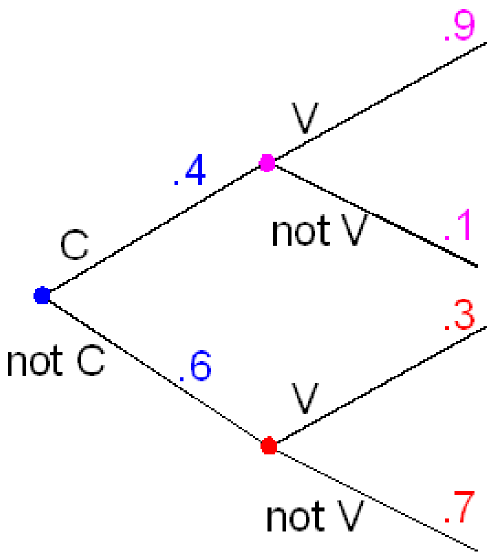
There are two important things to note here:

1. The probabilities in the **first branch-off are non-conditional probabilities** P(C) = 0.4, P(not C) = 0.6. However, the probabilities that appear in the **second branch-off are conditional probabilities.** The top two branches assume that C occurred: P(V | C) = 0.9, P(not V | C) = 0.1. The bottom two branches assume that not C occurred: P(V | not C) = 0.3, P(not V | not C) = 0.7



2. The second thing to note is that probabilities of branches that branch out from the same point always add up to one.



Did I Get This

1/1 point (graded)
Consider again the following example:

An overheating engine can quickly cause serious damage to a car, and therefore a dashboard red warning light is supposed to come on if that happens. In a certain model car, there is a 3% chance of the engine overheating (event H). The probability of the warning light showing up (event W) when it should (i.e., when the engine is really overheating) is 0.98, however, 1% of the time the warning light appears for no apparent reason (i.e., when the engine temperature is normal).

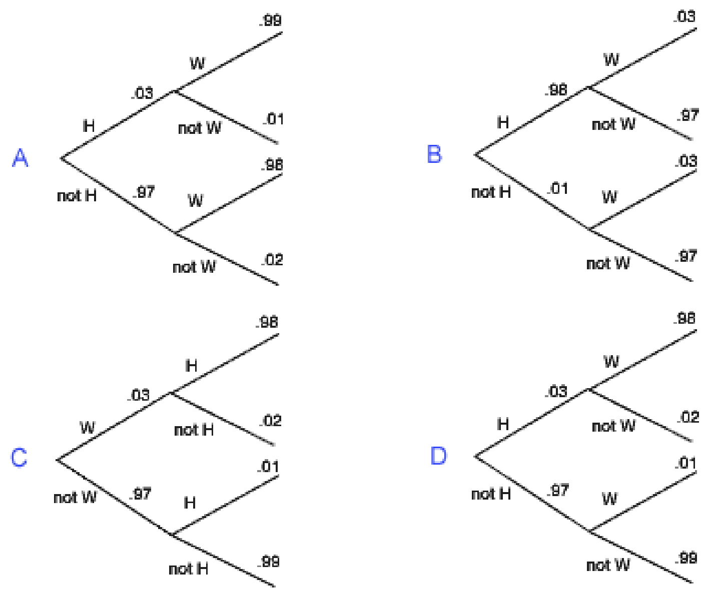
In an activity in the previous part we identified the information that this problem provides:

P(H) = 0.03

P(W | H) = 0.98

P(W | not H) = 0.01

Which of the following is a correct representation of the given information in a probability tree?



- A
- B
- C
- D ✔

Answer
Correct:
Note that the tree you chose depicts the fact that the natural order is that H happens first (engine overheats), and then W (warning light shows up). The probabilities of each branch are also correct, starting with the non-conditional probabilities for the first branch-off, and conditional probabilities for the second branch-off, making sure that the probabilities of branches branching off the same point add up to 1.

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