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Probability Distribution: Table of Outcomes

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Probability Distribution: Table of Outcomes

Learning Objective: Find the probability distribution of discrete random variables, and use it to find the probability of events of interest.

The probability distribution for two flips of a coin was simple enough to construct at once. For more complicated random experiments, it is common to first construct a table of all the outcomes in S and their probabilities, then use the addition principle to condense that information into the actual probability distribution table.

Example: Flipping a Coin Three Times

A coin is tossed three times. Let the random variable X be the number of tails. Find the probability distribution of X . We'll follow the same reasoning we used in the previous example:

First, we specify the 8 possible outcomes in S , along with the number and the probability of that outcome. (Because they are all equally likely, each has probability $1/8$. Alternatively, by the multiplication principle, each particular sequence of three coin faces has probability $1/2 * 1/2 * 1/2 = 1/8$.)

Outcome	Probability
HHH	$1/2 * 1/2 * 1/2 = 1/8$
HHT	$1/8$
HTH	$1/8$
THH	$1/8$
HTT	$1/8$
THT	$1/8$
TTH	$1/8$
TTT	$1/8$

Next, we figure out what the value of X is (number of tails) for each possible outcome.

Outcome	Probability	X
HHH	$1/8$	0
HHT	$1/8$	1
HTH	$1/8$	1
THH	$1/8$	1
HTT	$1/8$	2
THT	$1/8$	2
TTH	$1/8$	2
TTT	$1/8$	3

Next, we use the addition principle to assert that

$$P(X = 1) = P(\text{HHT or HTH or THH}) = P(\text{HHT}) + P(\text{HTH}) + P(\text{THH}) = 1/8 + 1/8 + 1/8 = 3/8.$$

$$\text{Similarly, } P(X = 2) = P(\text{HTT or THT or TTH}) = 3/8.$$

Outcome	Probability	X	
HHH	$1/8$	0	→ $1/8$
HHT	$1/8$	1	→
HTH	$1/8$	1	→ $1/8 + 1/8 + 1/8 = 3/8$
THH	$1/8$	1	→
HTT	$1/8$	2	→
THT	$1/8$	2	→ $1/8 + 1/8 + 1/8 = 3/8$
TTH	$1/8$	2	→
TTT	$1/8$	3	→ $1/8$

The resulting probability distribution is:

X	0	1	2	3
P(X=x)	$1/8$	$3/8$	$3/8$	$1/8$

The purpose of the next activity is to give you guided practice in finding the probability distribution of a discrete random variable.

Scenario: Number of Children

A young couple decides to try to have children until they have a boy. For financial reasons, the couple decides that they are going to stop trying when they have three children, whether they have a boy or not. (We are assuming that having a boy or a girl is equally likely, and that the child's gender in each birth is independent of the gender in the other births.)

Let the random variable X be the number of children the couple has.

Our goal is to find the probability distribution of X . In other words, we would like to create a table that lists all the possible values of X and the corresponding probabilities. We'll follow the same steps we followed in the two examples we solved.

Learn By Doing (1/1 point)

What is the sample space S in this case? In other words, what are all the possible outcomes in this case? (Use B for a boy and G for a girl).

Your Answer:

B, GB, GGB, GGG

Our Answer:

There are four possible outcomes in this case: B—the first child is a boy, and the couple stops having children. GB—the first child is a girl and the second is a boy, and then the couple stops having children. GGB—the first and second child are girls and the third is a boy, and the couple stops having children. GGG—all three children are girls but the couple stops having children for financial reasons.

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Learn By Doing (1/1 point)

Using simple principles of probability, find the probability of each of the outcomes listed in the previous question.

Your Answer:

P(X=B) = 0.5
P(X=GB) = 0.5 * 0.5
P(X=GGB) = 0.5 * 0.5 * 0.5
P(X=GGG) = 0.5 * 0.5 * 0.5

Our Answer:

P(B) = 1/2 P(GB) = 1/2 * 1/2 = 1/4 (using the multiplication principle for independent events) P(GGB) = 1/2 * 1/2 * 1/2 = 1/8 (using the multiplication principle for independent events) P(GGG) = 1/2 * 1/2 * 1/2 = 1/8 (using the multiplication principle for independent events)

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Learn By Doing (1/1 point)

Write down the value of the random variable X that is associated with each outcome.

Your Answer:

B: X=1
GB: X=2
GGB, GGG: X=3

Our Answer:

For B, X = 1 (the couple has just one child) For GB, X = 2 (the couple has two children) For GGB, X = 3 (the couple has three children) For GGG, X = 3 (the couple has three children)

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Learn By Doing (1/1 point)

Using what you found in the question above, summarize the probability distribution of X in a table.

Your Answer:

x	1	2	3
----- ----- ----- -----			
P(X=x)	1/4	1/2	1/4

Our Answer:

Note that P(X = 3) = 1/4 since by the addition principle (for disjoint events): P(X = 3) = P(GGB) + P(GGG) = 1/8 + 1/8 = 1/4

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