Hypothesis Testing for the Population Proportion p: z-test Conditions

Learning Objective: Carry out hypothesis testing for the population proportion and mean (when appropriate), and draw conclusions in context.

Comments

1. It should now be clear why this test is commonly known as **the z-test for the population proportion**. The name comes from the fact that it is based on a test statistic that is a **z-score.**

2. Recall fact 1 that we used for constructing the z-test statistic. Here is part of it again:

   When we take a **random** sample of size n from a population with population proportion p, the possible values of the sample proportion ($\hat{p}$) (**when certain conditions are met**) have approximately a normal distribution with a mean of ... and a standard deviation of ....

   This result provides the theoretical justification for constructing the test statistic the way we did, and therefore the assumptions under which this result holds (in bold, above) are the conditions that our data need to satisfy so that we can use this test. These two conditions are:

   a. The sample has to be random.

   b. The conditions under which the sampling distribution of $\hat{p}$ is normal are met. In other words:

$$n \cdot p_0 \geq 10$$
$$n \cdot (1 - p_0) \geq 10$$

3. Here we will pause to say more about condition (i.) above, the need for a random sample. In the Probability Unit we discussed sampling plans based on probability (such as a simple random sample, cluster, or stratified sampling) that produce a non-biased sample, which can be safely used in order to make inferences about a population. We noted in the Probability Unit that, in practice, other (non-random) sampling techniques are sometimes used when random sampling is not feasible. It is important though, when these techniques are used, to be aware of the type of bias that they introduce, and thus the limitations of the conclusions that can be drawn from them.

   For our purpose here, we will focus on one such practice, the situation in which a sample is not really chosen randomly, but in the context of the categorical variable that is being studied, the sample is regarded as random. For example, say that you are interested in the proportion of students at a certain college who suffer from seasonal allergies. For that purpose, the students in a large engineering class could be considered as a random sample, since there is nothing about being in an engineering class that makes you more or less likely to suffer from seasonal allergies. Technically, the engineering class is a convenience sample, but it is treated as a random sample in the context of this categorical variable. On the other hand, if you are interested in the proportion of students in the college who have math anxiety, then the class of engineering students clearly could not be viewed as a random sample, since engineering students probably have a much lower incidence of math anxiety than the college population overall.

Scenario: Bond Measure Vote

We are conducting a survey to determine if an upcoming bond measure will receive a majority vote in the county. The null hypothesis claims that $p = 0.50$, where p is the proportion of registered voters in the county who say they support the bond measure.

Learn By Doing

1/1 point (graded)

Which of the following samples is an appropriate random sample for testing this hypothesis?

- ○ A random sample of 100 county residents
- ○ A random sample of 100 registered voters who voted in the last statewide election
- ● A random sample of 100 registered voters from the county ✔
- ○ A random sample of 100 registered voters from the largest city in the county

Answer
Correct:
The population of interest is “registered voters in the county.” Here we are taking a random sample from the population.

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Scenario: Financial Situation Poll

In 2007, a Gallup poll estimated that 45% of U.S. adults rated their financial situation as “good.” Is the proportion different for this year? Which of the following samples could be used to test the null hypothesis p = 0.45?

Learn By Doing

1/1 point (graded)
Could the following sample be used to test the null hypothesis p = 0.45? Identify whether it is valid (OK to use to test the hypothesis) or not valid (should not be used to test the hypothesis).

100 adults randomly selected from people shopping at malls in each of 10 major U.S. cities.

- ○ valid
- ◉ not valid ✔

Answer
Correct:
This sample should not be treated as a random sample for this hypothesis test. People shopping at malls might have a more favorable view of their financial situation than the overall population of U.S. adults. Random selection from this group does not control for this bias.

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Learn By Doing

1/1 point (graded)

Could the following sample be used to test the null hypothesis p = 0.45? Identify whether it is valid (OK to use to test the hypothesis) or not valid (should not be used to test the hypothesis).

1,000 respondents randomly selected from the large pool of people who respond to an Internet survey posted on the CNN website.

- ○ valid
- ◉ not valid ✔

Answer
Correct:
People who voluntarily respond to Internet surveys typically have a strong opinion (that they are very interested in sharing.) This group will likely have a less favorable view of their financial situation than the population of U.S. adults. Random selection from this group will not control for this bias.

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Learn By Doing

1/1 point (graded)
Could the following sample be used to test the null hypothesis p = 0.45? Identify whether it is valid (OK to use to test the hypothesis) or not valid (should not be used to test the hypothesis).

250 adults selected randomly from active U.S. telephone or cell phone numbers in each of the four geographic regions of the U.S. (Northeast, Midwest, South, and West.)

- ◉ valid ✔
- ○ not valid

Answer
Correct:
Telephone surveys like this are commonly used by reputable polling organizations. This is a form of stratified sampling that controls for regional differences in financial circumstances. If the response rate is good, this sample can be viewed as a random sample that represents the population on this issue. Should we be concerned that the survey will not represent the poorest households without phone service? Studies conducted in 2008 by the National Center for Health Statistics concluded that approximately 1.9% of U.S. households have no telephone service. Using phone surveys is the best we can do, short of a full census, to gather information about adults in the United States.

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Scenario: Class Survey

We plan to poll 200 students enrolled in statistics at your college by distributing surveys during class. Which of the following hypotheses could be tested with the survey results?

Learn By Doing

1/1 point (graded)

Could the following hypothesis be used to test the survey results? Identify whether it is valid (OK to use to test the hypothesis) or not valid (should not be used to test the hypothesis).

The hypothesis that p = 0.60, where p is the proportion of students at your college who study the recommended “2 hours a week for each class unit.”

- ○ valid
- ◉ not valid ✔

Answer

Correct:

The study demands may be similar for students enrolled in statistics but different from students enrolled in other types of courses. To test a hypothesis about weekly study hours, we should sample randomly from all students at the college.

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Learn By Doing

1/1 point (graded)

Could the following hypothesis be used to test the survey results? Identify whether it is valid (OK to use to test the hypothesis) or not valid (should not be used to test the hypothesis).

The hypothesis that p = 0.60, where p is the proportion of students at your college who visit a dentist at least once a year.

- (●) valid ✔
- ( ) not valid

Answer
Correct:
When we consider whether or not a student visits a dentist, there is nothing that distinguishes students taking statistics from other students. So the 200 statistics students could be considered a random sample of all students at the college in this situation.

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Learn By Doing

1/1 point (graded)
Could the following hypothesis be used to test the survey results? Identify whether it is valid (OK to use to test the hypothesis) or not valid (should not be used to test the hypothesis).

The hypothesis that p = 0.60, where p is the proportion of students at your college who are receiving some form of financial aid.

- (●) valid ✔
- ( ) not valid

Answer
Correct:
When we consider whether or not a student gets financial aid, there is nothing that distinguishes students taking statistics from other students. So the 200 statistics students could be considered a random sample of all students at the college in this situation.

Submit

Learn By Doing

1/1 point (graded)
Could the following hypothesis be used to test the survey results? Identify whether it is valid (OK to use to test the hypothesis) or not valid (should not be used to test the hypothesis).

The hypothesis that p = 0.60, where p is the proportion of students at your college who spend more than $600 per semester on textbooks.

- ○ valid
- ● not valid ✔

Answer
Correct:
This sample should not be treated as a random sample in this situation. The courses students take will influence the amount they spend on textbooks. To test a hypothesis about textbook costs, we should sample randomly from all students at the college.

Submit

Let's check the conditions in our three examples.

Example: 1

1. The 400 products were chosen at random.

2. n = 400, $p_0 = 0.2$, and therefore:

   * $n \cdot p_0 = 80 \geq 10$
   * $n \cdot (1 - p_0) = 320 \geq 10$

Example: 2

1. The 100 students were chosen at random.

2. n = 100, $p_0 = 0.157$, and therefore:

   * $n \cdot p_0 = 15.7 \geq 10$
   * $n \cdot (1 - p_0) = 84.3 \geq 10$

Example: 3

1. The 1,000 U.S. adults were chosen at random.

2. n = 1,000, $p_0 = 0.64$, and therefore:

* $n \cdot p_0 = 640 \geq 10$
* $n \cdot (1 - p_0) = 360 \geq 10$

Learn By Doing

1/1 point (graded)

The UCLA Internet Report (February 2003) estimated that roughly 8.7% of Internet users are extremely concerned about credit card fraud when buying online. Has that figure changed since? To test this, a random sample of 100 Internet users was chosen. When interviewed, 10 said that they were extremely worried about credit card fraud when buying online. Let p be the proportion of all Internet users who are concerned about credit card fraud.

Which one of the following statements is correct about using the z-test for p?

- It is safe to use the z-test for p.
- It is not safe to use the z-test for p, since the sample is not a random sample from the entire population (or cannot be considered as one).
- It is not safe to use the z-test for p since n * p_o is not large enough. ✔
- It is not safe to use the z-test for p since n*(1 - p_o) is not large enough.

Answer

Correct: Indeed n * p_o = 100 * 0.087 = 8.7 is not large enough since it is less than 10.

Submit

Learn By Doing

1/1 point (graded)

The UCLA Internet Report (February 2003) estimated that a proportion of roughly 0.75 of Internet-using homes are still using dial-up access, but claimed that the use of dial-up is declining. Is that really the case? To examine this, a follow-up study was conducted a year later in which out of a random sample of 1,308 households that had an Internet connection, 804 were connecting using a dial-up modem. Let p be the proportion of all U.S. Internet-using households that have dial-up access.

Which one of the following statements is correct about using the z-test for p?

- (●) It is safe to use the z-test for p. ✔
- ( ) It is not safe to use the z-test for p, since the sample is not a random sample from the entire population (or cannot be considered as one).
- ( ) It is not safe to use the z-test for p since $n * p_o$ is not large enough.
- ( ) It is not safe to use the z-test for p since $n * (1 - p_o)$ is not large enough.

Answer
Correct:
Indeed, all the conditions are met: (a) the sample is random; (b) $n * p_o = 1{,}308 * 0.75 = 981 \geq 10$; and (c) $n * (1 - p_o) = 1{,}308 * 0.25 = 327 \geq 10$.

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Learn By Doing

1/1 point (graded)
According to the UCLA Internet Report (February 2003) the use of the Internet at home is growing steadily. The report estimated that roughly 59.3% of households in the Unites States have Internet access at home. Has that trend continued since the report was released? To study this, a random sample of 1,200 households from a big metropolitan area was chosen, and it was found that 972 had an Internet connection. Let p be the proportion of U.S. households that have Internet access.

Which one of the following statements is correct about using the z-test for p?

- ( ) It is safe to use the z-test for p.
- (●) It is not safe to use the z-test for p, since the sample is not a random sample from the entire population (or cannot be considered as one). ✔
- ( ) It is not safe to use the z-test for p since $n * p_o$ is not large enough.
- ( ) It is not safe to use the z-test for p since $n * (1 - p_o)$ is not large enough.

Answer
Correct:
Even though the sample is random, it is chosen from a specific big metropolitan area and not from the entire population of U.S. households.

Submit

Learn By Doing

1/1 point (graded)

A superintendent of a large school district claims that 80% of elementary school children in her district read at or above grade level (which is much higher than the national figure). To test the superintendent's claim, a random sample of 40 elementary school children from the school district is chosen, and it found that only 27 of read at or above grade level. Let p be the proportion of all school children in the school district who read at or above grade level.

Which one of the following statements is correct about using the z-test for p?

- It is safe to use the z-test for p.
- It is not safe to use the z-test for p, since the sample is not a random sample from the entire population (or cannot be considered as one).
- It is not safe to use the z-test for p since $n * p_o$ is not large enough.
- It is not safe to use the z-test for p since $n * (1 - p_o)$ is not large enough. ✔

Answer

Correct: Indeed $n * (1 - p_o) = 40 * (1 - 0.8) = 8$ is not large enough since it is less than 10.

Submit

Checking that our data satisfy the conditions under which the test can be reliably used is a very important part of the hypothesis testing process. So far we haven't explicitly included it in the 4-step process of hypothesis testing, but now that we are discussing a specific test, you can see how it fits into the process. We are therefore now going to amend our 4-step process of hypothesis testing to include this extremely important part of the process.

The Four Steps in Hypothesis Testing

1. State the appropriate null and alternative hypotheses, H_o and H_a.

2. Obtain a random sample, collect relevant data, and **check whether the data meet the conditions under which the test can be used**. If the conditions are met, summarize the data using a test statistic.

3. Find the p-value of the test.

4. Based on the p-value, decide whether or not the results are significant and **draw your conclusions in context.**

With respect to the z-test, the population proportion that we are currently discussing:

Step 1: Completed

Step 2: Completed

Step 3: This is what we will work on next.
