Hypothesis Testing for the Population Proportion p: Hypotheses

Learning Objective: In a given context, specify the null and alternative hypotheses for the population proportion and mean.

Recall that there are basically 4 steps in the process of hypothesis testing:

1. State the null and alternative hypotheses.

2. Collect relevant data from a random sample and summarize them (using a test statistic).

3. Find the p-value, the probability of observing data like those observed assuming that H_o is true.

4. Based on the p-value, decide whether we have enough evidence to reject H_o (and accept H_a), and draw our conclusions in context.

We are now going to go through these steps as they apply to the hypothesis testing for the population proportion p. It should be noted that even though the details will be specific to this particular test, some of the ideas that we will add apply to hypothesis testing in general.

1. Stating the Hypotheses

Here again are the three set of hypotheses that are being tested in each of our three examples:

Example: 1

Has the proportion of defective products been reduced as a result of the repair?

H_o: $p = 0.20$ (No change; the repair did not help).

H_a: $p < 0.20$ (The repair was effective).

Example: 2

Is the proportion of marijuana users in the college higher than the national figure?

H_o: $p = 0.157$ (Same as among all college students in the country).

H_a: $p > 0.157$ (Higher than the national figure).

Example: 3

Did the proportion of U.S. adults who support the death penalty change between 2003 and a later poll?

H_o: $p = 0.64$ (No change from 2003).

H_a: $p \neq 0.64$ (Some change since 2003).

Note that the null hypothesis always takes the form:

H_o: p = some value

and the alternative hypothesis takes one of the following three forms:

H_a: p < that value (like in example 1) **or**

H_a: p > that value (like in example 2) **or**

H_a: p ≠ that value (like in example 3).

Note that it was quite clear from the context which form of the alternative hypothesis would be appropriate. The value that is specified in the null hypothesis is called the **null value**, and is generally denoted by p_o. We can say, therefore, that in general the null hypothesis about the population proportion (p) would take the form:

H_o: $p = p_o$

We write H_o: $p = p_o$ to say that we are making the hypothesis that the population proportion has the value of p_o. In other words, p is the unknown population proportion and p_o is the number we think p might be for the given situation.

The alternative hypothesis takes one of the following three forms (depending on the context):

H_a: $p < p_o$ **(one-sided)**

H_a: $p > p_o$ **(one-sided)**

H_a: $p \neq p_o$ **(two-sided)**

The first two possible forms of the alternatives (where the = sign in H_o is challenged by < or >) are called **one-sided alternatives**, and the third form of alternative (where the = sign in H_o is challenged by ≠) is called a **two-sided alternative.** To understand the intuition behind these names let's go back to our examples.

Example 3 (death penalty) is a case where we have a two-sided alternative:

H_o: $p = 0.64$ (No change from 2003).

H_a: $p \neq 0.64$ (Some change since 2003).

In this case, in order to reject H_o and accept H_a we will need to get a sample proportion of death penalty supporters which is very different from 0.64 **in either direction,** either much larger or much smaller than 0.64.

In example 2 (marijuana use) we have a one-sided alternative:

H_o: $p = 0.157$ (Same as among all college students in the country).

H_a: $p > 0.157$ (Higher than the national figure).

Here, in order to reject H_o and accept H_a we will need to get a sample proportion of marijuana users which is much **higher** than 0.157.

Similarly, in example 1 (defective products), where we are testing:

H_o: $p = 0.20$ (No change; the repair did not help).

H_a: $p < 0.20$ (The repair was effective).

in order to reject H_o and accept H_a, we will need to get a sample proportion of defective products which is much **smaller** than 0.20.

In each of the following examples, a test for the population proportion (p) is called for. You are asked to select the right null and alternative hypotheses.

Scenario: Online Credit Card Fraud

The UCLA Internet Report (February 2003) estimated that roughly 8.7% of Internet users are extremely concerned about credit card fraud when buying online. Has that figure changed since? To test this, a random sample of 100 Internet users was chosen, and when interviewed, 10 said that they were extremely worried about credit card fraud when buying online. Let p be the proportion of all Internet users who are concerned about credit card fraud.

Learn By Doing

1/1 point (graded)

What is the null hypothesis in this case?

- ( ) H_o: $p = 8.7$
- (•) H_o: $p = 0.087$ ✔
- ( ) H_o: $p = 0.10$
- ( ) H_o: $p \neq 0.087$
- ( ) H_o: $p > 0.087$

Answer

Correct:

Indeed, the null hypothesis represents the claim that there is "no change." In this case, the null hypothesis claims that the proportion of Internet users who are extremely concerned about credit card fraud has not changed since the report (when it was 0.087).

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Learn By Doing

1/1 point (graded)

What is the alternative hypothesis in this case?

- ( ) H_a: $p > 0.087$

H_a: $p < 0.087$

H_a: $p \neq 0.087$

H_a: $p = 0.087$

Answer

Correct:

Indeed, we want to test whether the proportion of Internet users who are concerned about credit fraud has changed since the report. The alternative hypothesis, therefore, in this case, is the two-sided alternative H_a: $p \neq .087$

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Scenario: Dial-up Internet Access

The UCLA Internet Report (February 2003) estimated that a proportion of roughly .75 of online homes are still using dial-up access, but claimed that the use of dial-up is declining. Is that really the case? To examine this, a follow-up study was conducted a year later in which out of a random sample of 1,308 households that had Internet access, 804 were connecting using a dial-up modem. Let p be the proportion of all U.S. Internet-using households that have dial-up access.

Learn By Doing

1/1 point (graded)

What is the null hypothesis in this case?

H_o: $p = 0.75$

H_o: $p = 0.615$

H_o: $p < 0.75$

H_o: $p > 0.615$

Answer

Correct:
Indeed, the null hypothesis represents the claim of "no change." In this case, the null hypothesis claims that the proportion of Internet households that use dial-up connections remained 0.75, as estimated by the UCLA Internet Report a year before.

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Learn By Doing

1/1 point (graded)
What is the alternative hypothesis in this case?

- H_a: $p > 0.75$
- H_a: $p < 0.75$ ✔
- H_a: $p \neq 0.75$
- H_a: $p < 0.615$

Answer
Correct:
Indeed, since we want to test whether the proportion of online households that have a dial-up connection has declined since the report was published, the appropriate alternative is H_a: $p < 0.75$.

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Scenario: Home Internet Access

According to the UCLA Internet Report (February 2003) the use of the Internet at home is growing steadily and it is estimated that roughly 59.3% of households in the United States have Internet access at home. Has that trend continued since the report was released? To study this, a random sample of 1,200 households from a big metropolitan area was chosen for a more recent study, and it was found that 972 had an Internet connection. Let p be the proportion of U.S. households that have internet access.

Learn By Doing (1/1 point)

Write down the null and alternative hypotheses in the space provided below:

Your Answer:

Ho = Still only 59.3% of households have internet access at home.
Ha = It has changed; either more or less than 59.3%

Our Answer:
According to the report, the proportion of homes in the United States is estimated at 0.593 in 2003, and we are interested in testing whether that proportion has increased since. Therefore:Ho: p = 0.593Ha: p > 0.593

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In each of the following examples, a test for the population proportion (p) is called for. You are asked to select the right null and alternative hypotheses.

Scenario: Shift Defect Rate

When shirts are made, there can occasionally be defects (such as improper stitching). But too many such defective shirts can be a sign of substandard manufacturing.

Suppose, in the past, your favorite department store has had only one defective shirt per 200 shirts (a prior defective rate of only .005). But you suspect that the store has recently switched to a substandard manufacturer. So you decide to test to see if their overall proportion of defective shirts today is higher.

Suppose that, in a random sample of 200 shirts from the store, you find that 27 of them are defective, for a sample proportion of defective shirts of .135. You want to test whether this is evidence that the store is "guilty" of substandard manufacturing, compared to their prior rate of defective shirts.

Did I Get This

1/1 point (graded)
What is the null hypothesis in this case?

- H_o: p = 27
- H_o: p ≠ 0.005
- H_o: p > 0.005

- H_o: $p = 0.135$
- H_o: $p = 0.005$ ✔

Answer

Correct:

The null hypothesis is always a formal statement of "nothing unusual," or, in this case, "innocence" on the part of the store (the store did not switch to a substandard manufacturer). In this case, the null hypothesis is the formal statement that p (the overall proportion of defective shirts at the store today) is still the same as the prior defect rate.

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Did I Get This

1/1 point (graded)

What is the alternative hypothesis in this case?

- H_a: $p = 0.135$
- H_a: $p = 0.005$
- H_a: $p < 0.005$
- H_a: $p > 0.005$ ✔
- H_a: $p \neq 0.005$

Answer

Correct:

The question of interest is whether p (the overall proportion of defective shirts at the store today) is higher than the prior defect rate.

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Scenario: Birthrate by Gender

It is a known medical fact that just slightly fewer females than males are born (although the reasons are not completely understood); the known "proper" baseline female birthrate is about 49% females.

In some cultures, male children are traditionally looked on more favorably than female children, and there is concern that the increasing availability of ultrasound may lead to pregnant mothers deciding to abort the fetus if it's not the culturally "desired" gender. If this is happening, then the proportion of females in those nations would be significantly lower than the proper baseline rate.

To test whether the proportion of females born in India is lower than the proper baseline female birthrate, a study investigates a random sample of 6,500 births from hospital files in India, and finds 44.8% females born among the sample.

Did I Get This

1/1 point (graded)

What is the null hypothesis in this case?

- H_o: p = 49.0
- H_o: p = 0.448
- H_o: p = 0.49 ✔
- H_o: p ≠ 0.49
- H_o: p < 0.49

Answer

Correct:

The null hypothesis is always a formal statement of "nothing unusual" or "no effect." In this case, the null hypothesis is the formal statement that p (the female birthrate in India) is the "proper" baseline rate.

Submit

Did I Get This

1/1 point (graded)

What is the alternative hypothesis in this case?

- H_a: p > 0.49
- H_a: p < 0.448

- [x] H_a: $p < 0.49$ ✔
- [ ] H_a: $p = 0.49$
- [ ] H_a: $p \neq 0.49$

Answer
Correct:
The question of interest is whether p (the female birthrate in India) is lower than the proper baseline female birthrate.

Submit

Scenario: Fair 6-Sided Die

A properly-balanced 6-sided game die should give a 1 in exactly 1/6 (16.7%) of all rolls. A casino wants to test its game die. If the die is not properly balanced one way or another, it could give either too many 1's or too few 1's, either of which could be bad.

The casino wants to use the proportion of 1's to test whether the die is out of balance. So the casino test-rolls the die 60 times and gets a 1 in 9 of the rolls (15%).

Did I Get This (1/1 point)

Write down the null and alternative hypotheses in the space provided below:

Your Answer:

Ho: p = 16.7%
Ha: p ≠ 16.7%

Our Answer:
The casino is testing p, the overall proportion of 1's this die would give (if it could be rolled forever). They want to test whether the die is perfectly balanced—i.e., they want to test whether p for this die is equal to the theoretical value of 0.167. The null hypothesis (the presumption of "no problem," or the "presumption of innocence" for the die) is Ho: $p = 0.167$. Alternatively, if the die is not properly balanced one way or another, then the proportion of '1's would be different from the theoretical value, so the alternative hypothesis is Ha: $p \neq 0.167$. (Note that the hypotheses should be formulated before

looking at the data, so the alternative hypothesis is properly "$p \neq 0.167$," rather than "$p < 0.167$.") So the hypotheses are: Null: $p = 0.167$ Alternative: $p \neq 0.167$

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