# Probability Rules: Addition Rule for Disjoint Events

**Learning Objective: Apply probability rules in order to find the likelihood of an event.**

Now that we understand the idea of disjoint events, we can finally get to rule 4. Rule 4 actually has two versions, one for finding P(A or B) in the special case when events A and B are disjoint, and a more general version for when the events are not necessarily disjoint. We will first present the version of rule 4 that is restricted to disjoint events, and later in the section (after rule 5) we will revisit rule 4 and present the more general version.

### Rule 4: The Addition Rule for Disjoint Events

**The Addition Rule for Disjoint Events: If A and B are disjoint events, then P(A or B) = P(A) + P(B).**

## Comment

When dealing with probabilities, the word "or" will always be associated with the operation of addition; hence the name of this rule, "The Addition Rule."

## Example

Recall the blood type example:

| Blood Type | O | A | B | AB |
|---|---|---|---|---|
| **Probability** | 0.44 | 0.42 | 0.10 | 0.04 |

Here is some additional information:

* A person with type **A** can donate blood to a person with type **A** or **AB.**

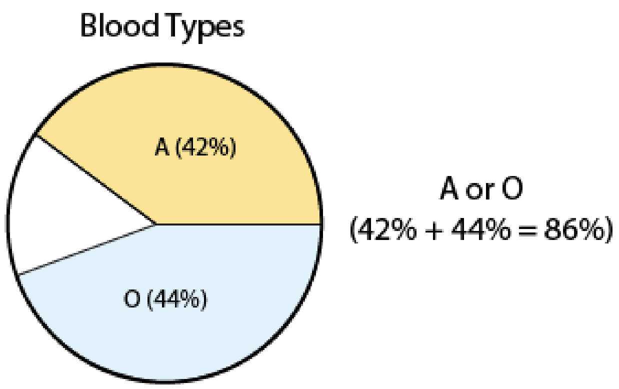
* A person with type **B** can donate blood to a person with type **B** or **AB.**

* A person with type **AB** can donate blood to a person with type **AB** only.

* A person with type **O** blood can donate to anyone.

What is the probability that a randomly chosen person is a potential donor for a person with blood type A?

From the information given, we know that being a potential donor for a person with blood type A means having blood type A or O. We therefore need to find P(A or O). Since the events A and O are disjoint, we can use the addition rule for disjoint events to get: P(A or O) = P(A) + P(O) = 0.42 + 0.44 = 0.86. It is easy to see why adding the probability actually makes sense. If 42% of the population has blood type A and 44% of the population has blood type O, then 42% + 44% = 86% of the population has either blood type A or O, and thus are potential donors to a person with blood type A. This reasoning about why the addition rule makes sense can be visualized using the pie chart below:




## Did I Get This

1/1 point (graded)

The probabilities in this table were calculated from data describing the highest level of educational attainment in 2005 for U.S. adults 25 years old or older. (Source: U.S. Census Bureau, *Current Population Survey*, March 2005)

| Highest Level of Education Attained | Probability |
|---|---|
| Below high school | 0.063 |
| Some high school | 0.085 |

| | |
|---|---|
| High school degree | 0.322 |
| Some college | 0.168 |
| College degree | 0.181 |
| Graduate or professional degree | 0.095 |

What is the probability that a randomly selected U.S. adult in this age category has not graduated from high school?

- 0.085
- 0.148 ✔
- It is impossible to tell since the events “no high school” and “some high school” are not disjoint.

**Answer**
Correct:
P(has not graduated from high school) = P(“no high school” or “some high school”) = 0.063 + 0.085 = 0.148. We can add these probabilities since the events “no high school” and “some high school” are disjoint. These events are disjoint because the table describes the highest level of educational attainment for an individual.

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## Scenario: Blood Type

So far we have introduced the addition rule for the special case in which the events being considered are disjoint. The purpose of this activity is to make you aware of the danger in wrongly using the addition rule for disjoint events in cases where the events are actually not disjoint. Consider the blood type example again.

Recall the blood type example:

| Blood Type | O | A | B | AB |
|---|---|---|---|---|
| **Probability** | 0.44 | 0.42 | 0.10 | 0.04 |

with the following additional information:

A person with type **A** can donate blood to a person with type **A** or **AB**.

A person with type **B** can donate blood to a person with type **B** or **AB**.

A person with type **AB** can donate blood to a person with type **AB** only.

A person with type **O** blood can donate to anyone.

Suppose that there are two patients who are each in need of a blood donation. Patient 1 has blood type A and patient 2 has blood type B. Consider the following events:

D1—a randomly chosen person can be a donor for patient 1.

D2—a randomly chosen person can be a donor for patient 2.

We are interested in finding the probability that a randomly chosen person can be a donor for patient 1 or patient 2. In other words, we are interested in finding P(D1 or D2).

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## Learn By Doing (1/1 point)

Find P(D1) and P(D2). Write your answers in the text box below.

**Your Answer:**

P(D1) = P(blood type A) + P(blood type O) = 0.42 + 0.44 = 0.86
P(D2) = P(type B) + P(type O) = 0.10 + 0.44 = 0.54

**Our Answer:**
To be a potential donor for patient 1, who has blood type A, a person must have blood type A or O. Therefore: P(D1) = P(A or O) = P(A) + P(O) = 0.42 + 0.44 = 0.86. Note that in this case we can safely use the addition rule for disjoint events since the events A and O are disjoint because we are presuming that a person cannot have more than one blood type. Similarly, to be a potential donor for patient 2, who has blood type B, a person must have blood type B or O. Therefore: P(D2) = P(B) + P(O) = 0.10 + 0.44 = 0.54.

Resubmit Reset

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## Learn By Doing (1/1 point)

Are events D1 and D2 disjoint or overlapping?

**Your Answer:**

Overlapping because both look at P(blood type O)

**Our Answer:**

Events D1 and D2 are not disjoint, they are overlapping, sharing the outcome O. In other words, D1 and D2 can occur at the same time if a donor has blood type O. Click here to see a visual representation.

Resubmit Reset

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## Learn By Doing (1/1 point)

Try to (wrongly) apply the addition rule for disjoint events to P(D1 or D2), and explain why the answer you got proves that the addition rule for disjoint events does not work in cases in which the events are not disjoint.

**Your Answer:**

Sum would've been 1.4 wherein probabilities should only be 1 at most.

**Our Answer:**

If we wrongly apply the addition rule for disjoint events in order to find P(D1 or D2) we'll get: P(D1 or D2) = P(D1) + P(D2) = 0.86 + 0.54 = 1.4 The answer is wrong for certain, since we got a probability that is greater than 1. This proves the danger of incorrectly applying the addition rule for disjoint events when it is not appropriate to do so.

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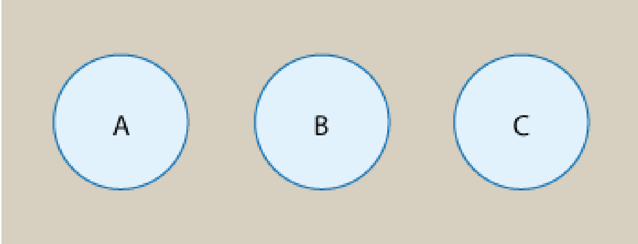
As we mentioned earlier, later on in this module we will establish a more general Addition Rule that applies even when two events are not disjoint.

## Comment

The Addition Rule for Disjoint Events can naturally be extended to more than two disjoint events. Let's take three, for example. If A, B and C are three disjoint events,




then P(A or B or C) = P(A) + P(B) + P(C). The rule is the same for any number of disjoint events.

## Learn By Doing (1/1 point)

The probabilities in this table were calculated from data describing North America's favorite car colors in 2003. (Source: DuPont Automotive as cited in money.cnn.com) ColorPercent Silver 20.2% White 18.4% Black 11.6% Med/Dark Gray 11.5% Light Brown 8.8% Med/Dark Blue 8.5% Medium Red 6.9% Med/Dark Green 5.3% Bright Red 3.8% Dark Red 0.9% What is the probability that a randomly chosen car will have one of the three most popular colors?

**Your Answer:**

20.2 + 18.4 + 11.6 = 50.2%

**Our Answer:**
Our answer: P(among the top three favorite colors) = P(Silver or White or Black). Since the three events, Silver, White and Black, are disjoint (since for the purposes of our example a car can have just one color), we can apply the Addition Rule for Disjoint Events (extended to three disjoint events) and get: P(among the top three colors) = P(Silver) + P(White) + P(Black) = 0.202 + 0.184 + 0.116 = 0.502. In other words, of the top 10 favorite car colors in the country, the three most popular colors comprised 50.2%.

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We are now done with the first version of the Addition Rule (the version restricted to disjoint events) and we are ready to move on to rule 5. As mentioned before, the general version of the Addition Rule will be presented after rule 5.