Hypothesis Testing for the Population Mean: Hypotheses and z-score

Learning Objective: In a given context, specify the null and alternative hypotheses for the population proportion and mean.

Learning Objective: Carry out hypothesis testing for the population proportion and mean (when appropriate), and draw conclusions in context.

1. Stating the Hypotheses

The null and alternative hypotheses for the z-test for the population mean (μ) have exactly the same structure as the hypotheses for z-test for the population proportion (p):

- The null hypothesis has the form:

  $H_0 : \mu = \mu_0$

  (where μ_0 is the null value).

- The alternative hypothesis takes one of the following three forms (depending on the context):

  $H_a : \mu < \mu_0$ (one-sided)

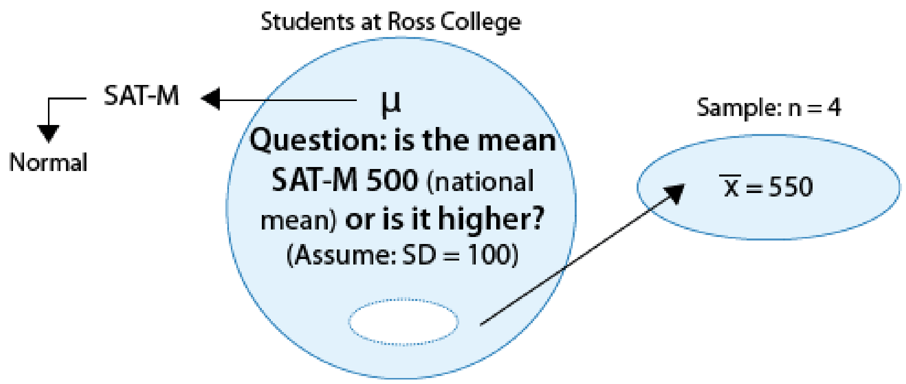
  $H_a : \mu > \mu_0$ (one-sided)

$H_a : \mu \neq \mu_0$ (two-sided)

Example: 1

In our example 1, based on a sample of 4 students from Ross College, we were testing whether the mean SAT-M of all of Ross College students is higher than the national mean (which, by construction, is 500).

Here is the figure that summarizes example 1:



Learn By Doing

1/1 point (graded)

What are the null and alternative hypotheses in this case?

- H_o: $\mu = 550$, and H_a: $\mu < 550$
- H_o: $\mu = 500$, and H_a: $\mu < 500$
- H_o: $\mu = 550$, and H_a: $\mu > 550$
- H_o: $\mu = 500$, and H_a: $\mu > 500$ ✔
- H_o: $\mu = 550$, and H_a: $\mu \neq 550$
- H_o: $\mu = 500$, and H_a: $\mu \neq 500$

Answer

Correct:

Indeed, we want to test whether the mean SAT-M of all the students in Ross College is the same as the national mean (500).

Submit

Did I Get This (1/1 point)

For the following scenario, give the null and alternative hypotheses and state in words what μ represents in your hypotheses.The National Assessment of Educational Progress (NAEP) is administered annually to 4th, 8th, and 12th graders in the United States. On the math assessment, a score above 275 is considered an indication that a student has the skills to balance a checkbook. In a random sample of 500 young men between the ages of 18 and 20, the mean NAEP math score is 272. Do we have evidence to support the claim that young men nationwide have a mean score below 275?

Your Answer:

Ho: μ = 275
Ha: μ > 275
μ is the estimated population mean, 275, i.e. score of young men nationwide for the NAEP.

Our Answer:
Ho: μ = 275, Ha: μ < 275, where μ is the mean NAEP score for young men in the United States.

Resubmit Reset

Did I Get This (1/1 point)

For the following scenario, give the null and alternative hypotheses and state in words what μ represents in your hypotheses.The National Center for Health Statistics reports that the systolic blood pressure for males 35 to 44 years of age has a mean of 128. In a study of business executives, a random sample of 100 executives has a mean systolic blood pressure of 134. Do the data suggest that the mean systolic blood pressure for business executives is higher than 128?

Your Answer:

Ho: μ = 128
Ha: μ > 128
μ signifies the systolic blood pressure of business executives

Our Answer:
Ho: μ = 128, Ha: μ > 128, where μ is the mean systolic blood pressure for business executives.

Resubmit Reset

Did I Get This (1/1 point)

For the following scenario, give the null and alternative hypotheses and state in words what μ represents in your hypotheses.An analytical chemistry lab is conducting quality control tests on a drug. A single dosage of the drug should contain 8 mg of active ingredient. Of course, there will be a small amount of variability due to imperfections in the production process, but the mean of all dosages produced should be 8 mg. In 20 random samples, the mean amount of active ingredient is 7.7 mg. Do the data suggest that the mean amount of active ingredient in all dosages produced is different from 8 mg?

Your Answer:

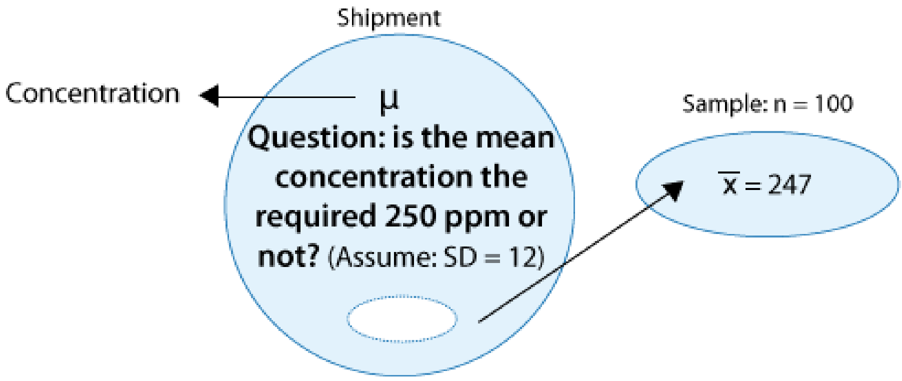
Ho: μ = 8
Ha: μ != 8
μ signifies mean amount of the active ingredient in the drugs produced

Our Answer:
Ho: μ = 8, Ha: μ≠ 8, where μ is the mean amount of active ingredient in all drug dosages produced.

Resubmit Reset

Example: 2

Here we want to test whether the mean concentration of a certain chemical in a large shipment of a certain prescription drug is the required 250 ppm:



The null and alternative hypotheses in this case are therefore:

$$H_0 : \mu = 250$$
$$H_a : \mu \neq 250$$

2. Collecting Data and Summarizing Them

Since our parameter of interest is the population mean (μ), once we collect the data, we find the sample mean ($\overline{x}$).

However, we already know that in hypothesis testing we go a step beyond calculating the relevant sample statistic and summarize the data with a test statistic.

Recall that in the z-test for the proportion, the test statistic is the z-score (standardized value) of the sample proportion, assuming that H_o is true. It should not be very surprising that in the z-test for the population mean, we do exactly the same thing.

The test statistic is the z-score (standardized value) of the sample mean ($\overline{x}$) assuming that H_o is true (in other words, assuming that $\mu = \mu_0$).

We rely once again on probability results—in this case, we refer to results about the sampling distribution of the sample mean ($\overline{X}$):

When we discussed probability models based on sampling distributions, we concluded that sample means behave as follows:

- Center: The mean of the sample means is μ, the population mean.
- Spread: The standard deviation of the sample means is $\frac{\sigma}{\sqrt{n}}$.
- Shape: The sample means are normally distributed if the variable is normally distributed in the population or the sample size is large enough to guarantee approximate normality. Recall that this last statement is the Central Limit Theorem. As a general guideline, we said that if n > 30, the Central Limit Theorem applies and we can use a normal curve as a probability model.

Based on this description of the sampling distribution of $\overline{X}$, we can define a test statistic that measures the distance between the hypothesized value of μ (denoted μ_o) and the sample mean (determined by the data) in standard deviation units.

The test statistic is: $z = \frac{\overline{x}-\mu_0}{\frac{\sigma}{\sqrt{n}}}$.

Comments

1. Note that our test statistic (because it is a z-score), tells us how far $\overline{x}$ is from the null value μ_0 measured in standard deviations. Since $\overline{x}$ represents the data and μ_0 represents the null hypothesis, the test statistic is a measure of how different our data are from what is claimed in the null hypothesis. The larger the test statistic, the more evidence we have against H_o, since what we saw in our data is very different from what H_o claims. This is an idea that we mentioned in the previous test as well.

2. As we established earlier, all inference procedures are based on probability. We are trying to determine if our sample results are likely or unlikely based on our assumptions about the population. This requires that we have a probability model that describes the long-term behavior of sample results

that are randomly collected from a population that fits our hypothesis. For this reason, the Central Limit Theorem gives us criteria for deciding if the z-test for the population mean can be used. We need to verify:

(i) The sample is random (or at least can be considered as random in context).

(ii) We are in one of the three situations marked with a green check mark in the following table:

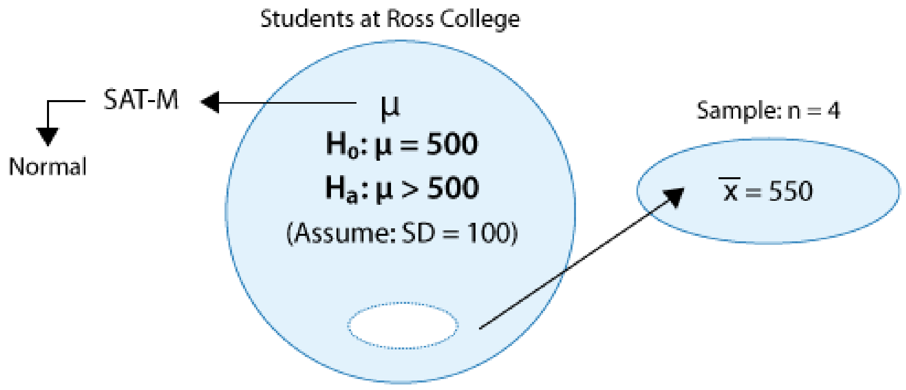
Conditions: z-test for a population mean	Small sample size	Large sample size
Variable varies normally in the population	✓	✓
Variable doesn't vary normally in the population	✗	✓

3. If the conditions are met, then $\overline{X}$ values vary normally, or at least close enough to normally to use a normal model to calculate probabilities. When $\overline{X}$ values are normal, then the z-scores will be normally distributed with a mean of 0 and a standard deviation of 1.

Let's go back to our examples.

Example: 1

Here is a summary of example 1:



Let's start by checking the conditions:

(i) The sample is random.

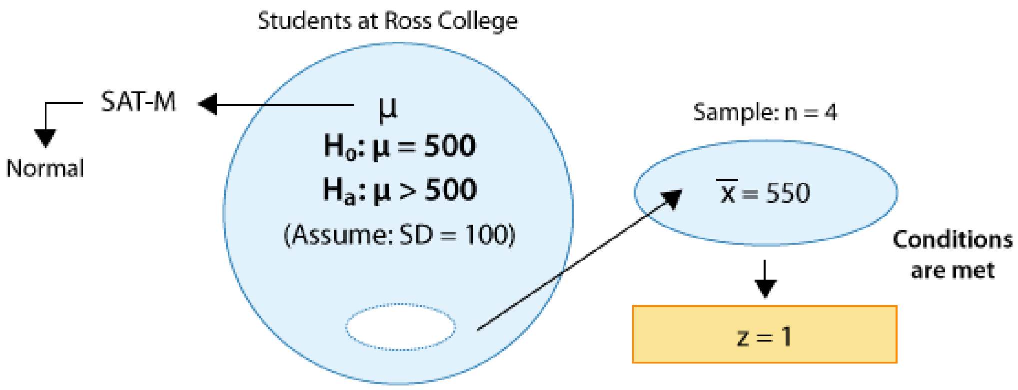
(ii) The variable of interest, SAT-M scores, is assumed to vary normally in the population, so the fact that the sample size is small (n = 4) is not a problem. Sample means will be normally distributed and we can use a normal probability model based on z-scores to determine probabilities.

Conditions: z-test for a population mean	Small sample size	Large sample size
Variable varies normally in the population	✓	✓
Variable doesn't vary normally in the population	✗	✓

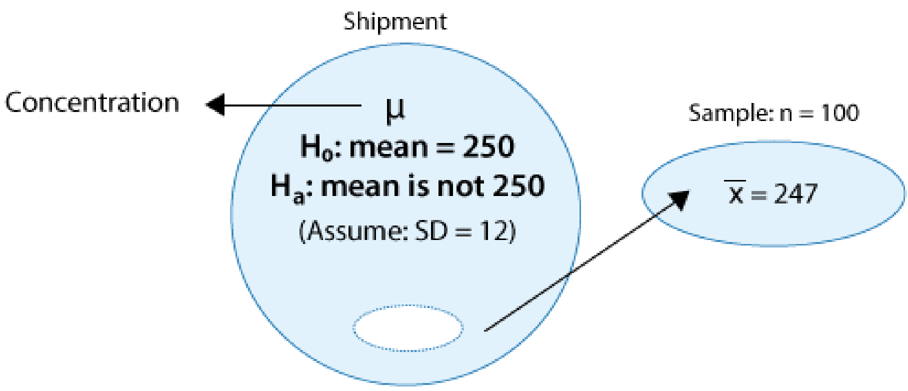
The sample mean is $\overline{x} = 550$, and so the test statistic is:

$$z = \frac{550-500}{\frac{100}{\sqrt{4}}} = 1$$

This means that our data (represented by the sample mean) are only 1 standard deviation above the null value (500). Clearly, this provides some evidence against H_0, but is this strong enough evidence to reject it? Probably not. This will be confirmed when we find the p-value. Here is an updated figure:



Example: 2



In this case, the conditions that allow us to carry out the z-test are met since:

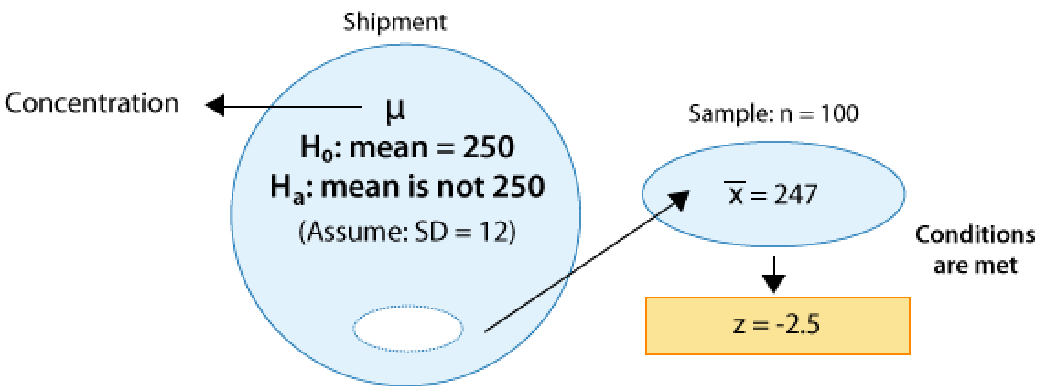
(i) The sample is random.

(ii) The sample size (n = 100) is large enough for the Central Limit Theorem to apply (note that in this case the large sample is essential since the concentration level is not known to vary normally).

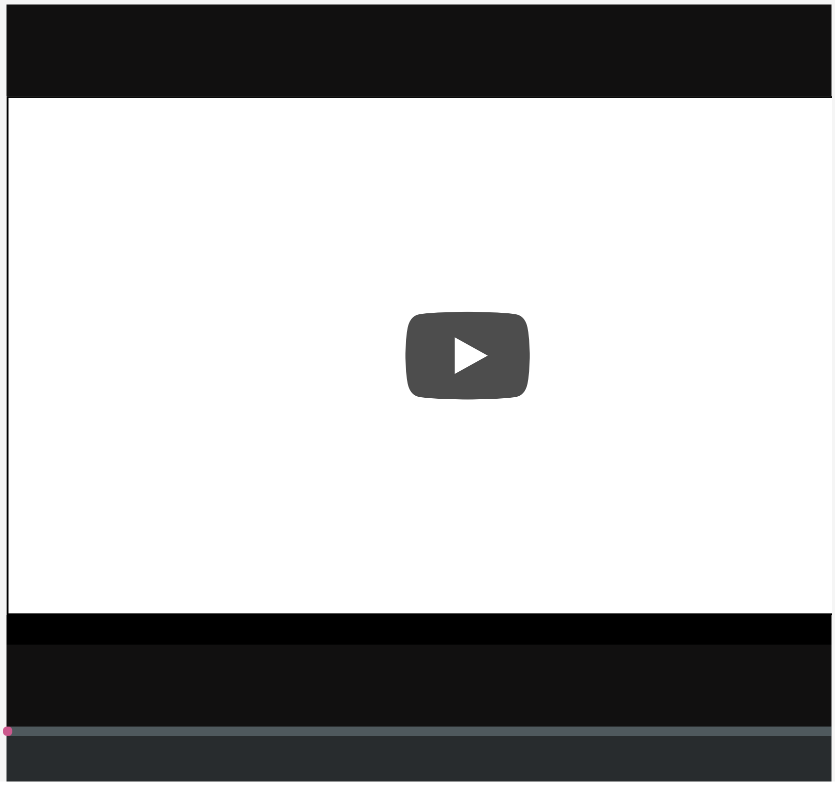
Conditions: z-test for a population mean	Small sample size	Large sample size
Variable varies normally in the population	✓	✓
Variable doesn't vary normally in the population	✗	✓

The z-statistic in this case is: $z = \frac{247-250}{\frac{12}{\sqrt{100}}} = -2.5$

Our data (represented by the sample mean concentration level—247) are 2.5 standard deviations below the null value. A difference of 2.5 standard deviations is considered quite strong evidence against H_o. (Essentially any difference that is above 2 standard deviations is considered quite large.) This will be confirmed when we find the p-value of the test. Here is an updated figure that represents the hypothesis testing process for this problem so far:



Test for Mean



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In this movie we're going to discuss this question: What conditions must be

met before we can conduct a hypothesis test with the mean? Now, as we established

earlier all inference procedures are based on probability, so when we ask this

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question, what we're really asking is

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