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Behavior of Sample Mean: Examples

Learning Objective: Apply the sampling distribution of the sample mean as summarized by the Central Limit Theorem (when appropriate). In particular, be able to identify unusual samples from a given population.

Before we work some examples. Let's compare and contrast what we now know about the sampling distributions for sample means and sample proportions.

			Sampling Distribution		
Variable	**Parameter**	**Statistic**	**Center**	**Spread**	**Shape**
Categorical (example: left-handed or not)	p = population proportion	$\hat{p}$ = sample proportion	p	$\sqrt{\frac{p(1-p)}{n}}$	Normal IF np ≥ 10 and n(1 - p) ≥ 10
Quantitative (example: age)	μ = population mean, σ = population standard deviation	$\overline{x}$ = sample mean	μ	$\frac{\sigma}{\sqrt{n}}$	Normal if n > 30 (always normal if population is normal)

Scenario: Pell Grant Awards

Recall our earlier scenario: The Federal Pell Grant Program provides need-based grants to low-income undergraduate and certain postbaccalaureate students to promote access to postsecondary education. According to the National Postsecondary Student Aid Study conducted by the U.S. Department of Education in 2008, the average Pell grant award for 2007-2008 was $2,600. Assume that the standard deviation in Pell grants awards was $500.

Learn By Doing

1/1 point (graded)

If we randomly sample 36 Pell grant recipients, would you be surprised if the mean grant amount for the sample was $2,940? Pick the correct response that gives the best reason.

● Yes, $2,940 would be surprising because this sample result is more than 3 standard deviations from the overall mean grant amount of $2,600. ✓

○ Yes, $2,940 would be surprising because this sample result is $340 greater than the overall mean grant amount of $2,600.

○ No, $2,940 would not be surprising because this sample result is within 2 standard deviations of the overall mean grant amount of $2,600.

○ No, $2,940 would not be surprising because this sample result is only $340 greater than the overall mean grant amount of $2,600, and we expect there to be variability in sample means.

Answer
Correct:
For n=36, sample means are approximately normal, so we can use the Standard Deviation Rule. Three standard deviations above 2,600 is 2,600 + 3(500/6)) = 2,850). So $2,940 is more than 3 standard deviations above $2,600, thus this sample mean would be surprising.

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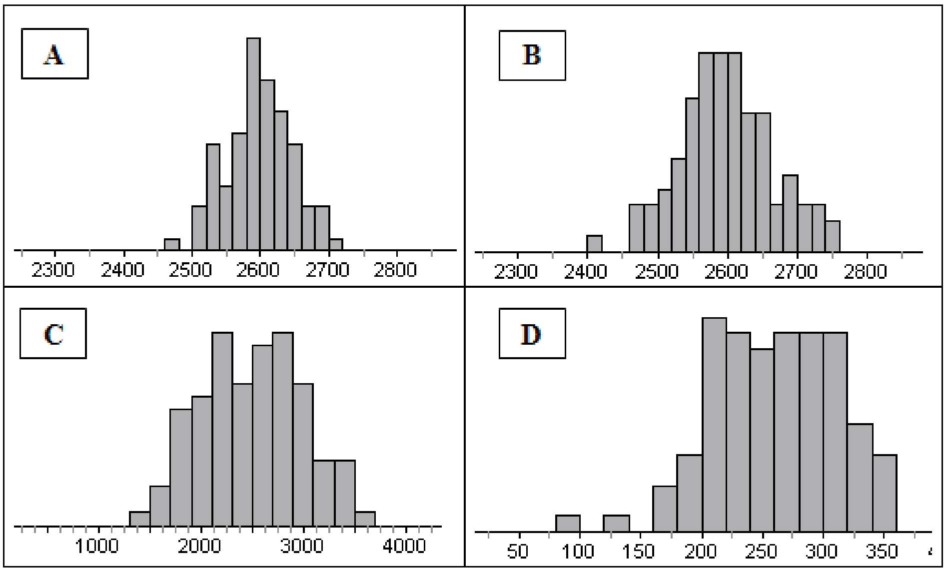
Learn By Doing

1/1 point (graded)

Which distribution is a plausible representation of 100 samples, with each sample containing 50 randomly selected students?



○ A

● B ✓

○ C

○ D

Answer
Correct:
The sampling distribution will have a mean of 2,600 and a standard deviation of 70. Using the Standard Deviation Rule, approximately 68% of the values should be between 2,530 and 2,670 (within 1 standard deviation of the mean.). Graph B looks like it could fit this description.

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Example

Household size in the United States has a mean of 2.6 people and standard deviation of 1.4 people.

(a) What is the probability that a randomly chosen household has more than 3 people?

A normal approximation should not be used here, because the distribution of household sizes would be considerably skewed to the right. We do not have enough information to solve this problem.

(b) What is the probability that the mean size of a random sample of 10 households is more than 3?

By anyone's standards, 10 is a small sample size. The Central Limit Theorem does not guarantee sample mean coming from a skewed population to be approximately normal unless the sample size is large.

(c) What is the probability that the mean size of a random sample of 100 households is more than 3?

Note: To review how to determine probabilities for z scores, please refer to the Standard Normal Table section of the Random Variables module.

Now we may invoke the Central Limit Theorem: even though the distribution of household size X is skewed, the distribution of sample mean household size $\overline{X}$ is approximately normal for a large sample size such as 100. Its mean is the same as the population mean, 2.6, and its standard deviation is the population standard deviation divided by the square root of the sample size: $\frac{\sigma}{\sqrt{n}} = \frac{1.4}{\sqrt{100}} = \mathbf{0.14.}$ The z-score for 3 is $\frac{3-2.6}{\frac{1.4}{\sqrt{100}}} = \frac{0.4}{0.14} = \mathbf{2.86}$ The probability of the mean household size in a sample of 100 being more than 3 is therefore P($\overline{X}$ > 3) = P(Z > 2.86) = P(Z < -2.86) = 0.0021.

Households of more than 3 people are, of course, quite common, but it would be extremely unusual for the mean size of a sample of 100 households to be more than 3.

The purpose of the next activity is to give guided practice in finding the sampling distribution of the sample mean ($\overline{X}$), and use it to learn about the likelihood of getting certain values of $\overline{X}$.

Scenario: Annual Teacher Salary

The annual salary of teachers in a certain state X has a mean of $54,000 and standard deviation of σ = $5,000.

Learn By Doing (1/1 point)

What is the probability that the mean annual salary of a random sample of 5 teachers from this state is more than $60,000? Find this probability or explain why you cannot.

Your Answer:

5 teachers are too few because salary distribution is typically skewed to the right. So, we would've needed an n of at least 30.

Our Answer:
Recall from the Exploratory Data Analysis unit that salary distribution is typically skewed to the right. Since 5 is a small sample size, and the Central Limit Theorem does not guarantee that the sample mean coming from a skewed population is approximately normal unless the sample size is larger, we thus do not have enough information to solve the problem.

Resubmit Reset

Learn By Doing

1/1 point (graded)
We are trying to determine the probability that the mean annual salary of a random sample of 64 teachers from this state is less than $52,000.

What is the mean of the sampling distribution of sample of means?

○ $52,000

● $54,000 ✓

Answer
Correct:
According to the Central Limit Theorem, the mean has approximately a normal distribution with the same mean as the population; therefore, $54,000 is the mean of the distribution of the sample means.

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Learn By Doing

1/1 point (graded)
We are trying to determine the probability that the mean annual salary of a random sample of 64 teachers from this state is less than $52,000.

What is the the standard deviation for the sampling distribution of means?

● 625 ✓

○ 5,000

Answer
Correct:
According to the Central Limit Theorem, then, the mean has approximately a normal distribution with the same mean as the population, $54,000, and a standard deviation of: σ/square root(n) = 5000/square root(64) = 625.

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Learn By Doing

1/1 point (graded)
We are trying to determine the probability that the mean annual salary of a random sample of 64 teachers from this state is less than $52,000.

What is the z score for solving this problem?

○ 0.4

○ -4

○ 3.2

● -3.2 ✓

Answer
Correct: The z-score of 52,000 is: (52,000 - 54,000)/5000/sqrt(64).

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Learn By Doing

1/1 point (graded)
Given the z-score from the problem above, what is the probability of that the mean annual salary of a random sample of 64 teachers from this state is less than $52,000?

● 0.0007 ✓

○ 0.9993

○ 0.3446

○ 0.6554

Answer
Correct:
The probability of the z score of -3.21 using the Normal Table is 0.0007 or P(<52,000) = P(z ,-3.2) = (table) = 0.0007. Thus, we find that while it is probably quite common to find teachers in this state with an annual salary that is less than $52,000, it would be extremely unusual for the mean salary of a sample of 64 teachers to be less than $52,000.

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Scenario: SAT Math Scores

Scores on the math portion of the SAT (SAT-M) in a recent year have followed a normal distribution with mean μ = 507 and standard deviation σ = 111.

Did I Get This (1/1 point)

What is the probability that the mean SAT-M score of a random sample of 4 students who took the test that year is more than 600? Explain why you can solve this problem, even though the sample size (n = 4) is very low.

Your Answer:

(600 - 507)/(500/sqrt(4)) = 93/250 = 0.372 z score

We can solve it because we know that sample distribution was normal.

Our Answer:
Since the scores on the SAT-M in the population follow a normal distribution, the sample mean automatically also follows a normal distribution, for any sample size. Therefore, the mean has a normal distribution with the same mean as the population, 507, and standard deviation The z-score of 600 is therefore: And therefore, We find that while it is very common to find students who score above

600 on the SAT-M, it would be quite unlikely (4.65% chance) for the mean score of a sample of 4 students to be above 600.

Resubmit Reset
