Confidence Intervals for the Population Mean: Increasing Precision

Learning Objective: Explain what a confidence interval represents and determine how changes in sample size and confidence level affect the precision of the confidence interval.

Learning Objective: Find confidence intervals for the population mean and the population proportion (when certain conditions are met), and perform sample size calculations.

Let us now go back to the confidence interval for the mean, and more specifically, to the question that we posed at the beginning of the previous page:

Is there a way to increase the precision of the confidence interval (i.e., make it narrower) *without* compromising on the level of confidence?

Since the width of the confidence interval is a function of its margin of error, let's look closely at the margin of error of the confidence interval for the mean and see how it can be reduced:

$$z^* * \frac{\sigma}{\sqrt{n}}$$

Since z* controls the level of confidence, we can rephrase our question above in the following way:

Is there a way to reduce this margin of error other than by reducing z*?

If you look closely at the margin of error, you'll see that the answer is yes. We can do that by increasing the sample size n (since it appears in the denominator).

Many Students Wonder ...

Question: Since the margin of error is $z^* \cdot \frac{\sigma}{\sqrt{n}}$, isn't it true that another way to reduce the margin of error (for a fixed z*) is to reduce σ?

Answer: While it is true that strictly mathematically speaking the smaller the value of σ, the smaller the margin of error, practically speaking we have absolutely no control over the value of σ (i.e., we cannot make it larger or smaller). σ is the population standard deviation; it is a fixed value (which here we assume is known) that has an effect on the width of the confidence interval (since it appears in the margin of error), but is definitely not a value we can play with.

Let's look at an example first and then explain why increasing the sample size is a way to increase the precision of the confidence interval *without* compromising on the level of confidence.

Example

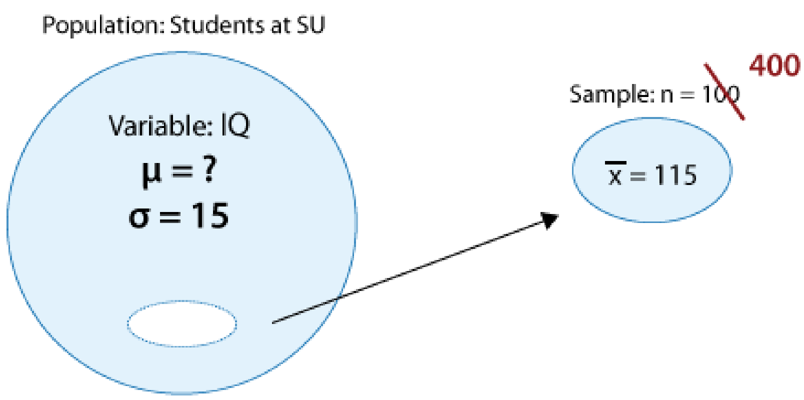
Recall the IQ example:

The IQ level of students at a particular university has an unknown mean (μ) and a known standard deviation of $\sigma = 15$. A simple random sample of 100 students is found to have the sample mean IQ $\overline{x} = 115$. A 95% confidence interval for μ in this case is:

$$\overline{x} \pm 2\frac{\sigma}{\sqrt{n}} = 115 \pm 2\left(\frac{15}{\sqrt{100}}\right) = 115 \pm 3.0 = (112,\ 118)$$

Note that the margin of error is m = 3, and therefore the width of the confidence interval is 6.

Now, what if we change the problem slightly by increasing the sample size, and assume that it was 400 instead of 100?



In this case, the 95% confidence interval for μ is:

$$\overline{x} \pm 2\frac{\sigma}{\sqrt{n}} = 115 \pm 2\left(\frac{15}{\sqrt{400}}\right) = 115 \pm 1.5 = (113.5,\ 116.5)$$

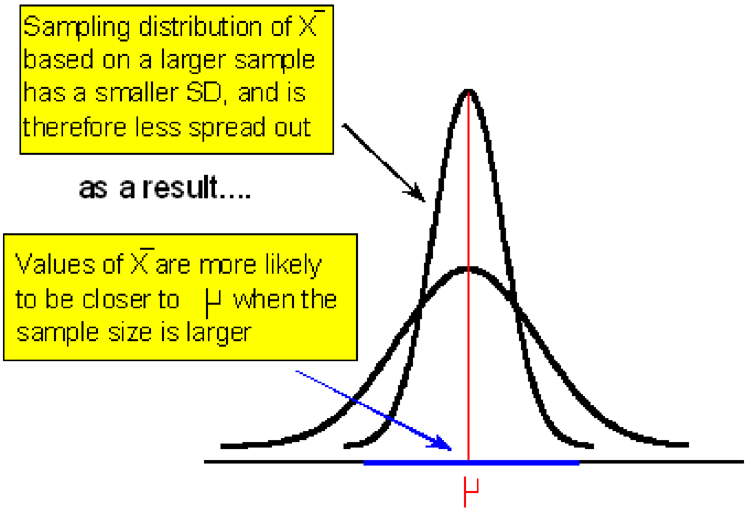
The margin of error here is only m = 1.5, and thus the width is only 3.

Note that for the same level of confidence (95%) we now have a narrower, and thus more precise, confidence interval.

Let's try to understand why a larger sample size will reduce the margin of error for a fixed level of confidence. There are three ways to explain it: mathematically, using probability theory, and intuitively.

We've already alluded to the mathematical explanation; the margin of error is $z^* * \frac{\sigma}{\sqrt{n}}$, and since n, the sample size, appears in the denominator, increasing n will reduce the margin of error.

As we saw in our discussion about point estimates, probability theory tells us that



This explains why with a larger sample size the margin of error (which represents how far apart we believe $\overline{x}$ might be from μ for a given level of confidence) is smaller.

On an intuitive level, if our estimate $\overline{x}$ is based on a larger sample (i.e., a larger fraction of the population), we have more faith in it, or it is more reliable, and therefore we need to account for less error around it.

Comment

While it is true that for a given level of confidence, increasing the sample size increases the precision of our interval estimation, in practice, increasing the sample size is not always possible. Consider a study in which there is a non-negligible cost involved for collecting data from each participant (an expensive medical procedure, for example). If the study has some budgetary constraints, which is usually the case, increasing the sample size from 100 to 400 is just not possible in terms of cost-effectiveness. Another instance in which increasing the sample size is impossible is when a larger sample is simply not available, even if we had the money to afford it. For example, consider a study on the effectiveness of a drug on curing a very rare disease among children. Since the disease is rare, there are a limited number of children who could be participants. This is the reality of statistics. Sometimes theory collides with reality, and you just do the best you can.

Did I Get This

1/1 point (graded)

A medical researcher wanted to estimate μ, the mean weight of newborns born to women over the age of 40.

The researcher chose a random sample of 100 pregnant women who were over 40, followed them through the pregnancy, and found that the mean weight of the 100 newborns was 3,035 grams. From past research, it is assumed that the weight of newborns has a standard deviation of $\sigma = 500$. The researcher calculated the 95% confidence interval for μ to be (2935, 3135).

If the researcher wanted to maintain the 95% level of confidence but report a confidence interval with a smaller margin of error, which of the following will achieve that?

- ○ Redo the study, but this time with a sample of size 64.
- ◉ Redo the study, but this time with a sample of size 225. ✔
- ○ Redo the study and choose a different sample of size 100.

Answer

Correct:
A larger sample size reduces the margin of error (for a given level of confidence). In particular, by using a sample size of 225 instead of 100, the margin of error will be reduced from 100 to roughly 67.

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Did I Get This

1/1 point (graded)

The mean score on the quantitative reasoning part of the GRE (Graduate Record Examination) of non-U.S. citizens has an unknown mean, , and an assumed standard deviation =8. Based on a random sample of non-U.S. citizens who took the GRE in 2014 the 95% confidence interval for was calculated to be (153.6, 158.6).

Suppose now that a different random sample of non-U.S. citizens who took the GRE in 2014 is chosen and that this sample is **larger** than the sample that produced the confidence interval above.

Which of the following is the **most likely** 95% confidence interval for μ based on the larger sample?

- (153.4, 159.2)
- (153.9, 158.9)
- (153.8, 158.4)
- (153.5, 158.1) ✔

Answer
Correct:
When the sample size increases, the confidence interval gets narrower, and this confidence interval is narrower (width = 4.6) than the confidence interval that is based on the smaller sample size (width = 5).

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