Probability and Intuition

Learning Objective: Relate the probability of an event to the likelihood of this event occurring.

Now that we understand how probability fits into the Big Picture as a key element behind statistical inference, we are ready to learn more about it. Our first goal is to introduce some fundamental terminology (the language) and notation that is used when discussing probability. Before we do that, though, let's start with two fun examples that explain the reasons for the careful treatment that we give probability in this course.

Often, relying only on our intuition is not sufficient to determine probability, so we need some tools to work with, which is exactly what we study in this section.

Here are two examples:

Example: The Let's Make a Deal Paradox

Let's Make a Deal was a popular television game show, which first aired in the 1960s. The *Let's Make a Deal* paradox is named after that show. In the show, the contestant had to choose between three doors. One of the doors had a big prize behind it such as a car or a lot of cash, and the other two were empty. (Actually, for entertainment's sake, each of the other two doors had some stupid gift behind it, like a goat or a chicken, but we'll refer to them here as empty.)

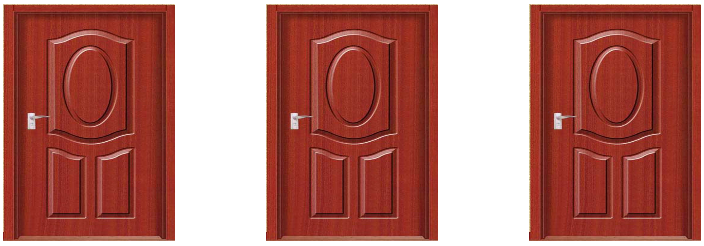
The contestant had to choose one of the three doors, but instead of revealing the chosen door, the host revealed one of the two unchosen doors to be empty. At this point in the game, there were two unopened doors: the door that the contestant had originally chosen and the remaining unchosen door. One of them had the prize behind it.

The contestant was given the option either to *stay* with the door that he or she had initially chosen or *switch* to the other door.

What do you think the contestant should do, stay or switch? What do you think is the probability that you will win the big prize if you stay? What about if you switch?

In order for you to gain a feel for this game, try playing it using this interactive simulation:

Let's Make a Deal Demonstration



Pick a door

	Wins	Losses
Switching:	0	0
Staying:	0	0

How it works:

In a popular game show, contestants are asked to choose one of three doors. Behind one is a fabulous prize! Behind the others are gag gifts. When you choose a door, the game show host shows you a gag gift behind one of the two doors not chosen. You are given the option of switching to the one remaining door or staying with your original choice. Which is the better strategy: switch or stay? You choose doors by clicking on a door. A gag gift (represented by a donkey) is then revealed behind one of the doors you did not select. Click on your original door to stay, or click on the other unopened door to switch. Then all the doors are opened. Did you win? The table keeps track of your wins and losses using each strategy.

Learn By Doing

1/1 point (graded)

Now that you are more familiar with the game, what do you think that the contestant should do?

- ○ Stay with the door that he or she had originally picked.
- ◉ Switch to the other unchosen door. ✔
- ○ Choose whichever door he or she wants to. It doesn't make a difference—each of the two doors has a 50 percent chance of having the prize behind it.

Answer

Correct: Switching is indeed the best strategy. Can you figure out why this is the case? Read on for an explanation.

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The intuition of most people is that each of the two doors is equally likely to contain the prize—that there is a 50-50 chance of winning with either selection. This, however, is not the case. Actually, there is a 67% chance—or a probability of 2/3 (2 out of 3)—of winning by switching, and only a 33% chance—or a probability of 1/3 (1 out of 3)—of winning by staying with the door that was originally chosen. This means that a contestant is twice as likely to win if he or she switches to the unchosen door. Isn't this a bit counterintuitive and confusing? Most people think so when they are first faced with this problem. We will now try to explain this paradox to you in two different ways:

The "Let's Make a Deal" Paradox Part 1

0:00 / 1:13 1.50x

Start of transcript. Skip to the end.

Initially when you're asked to choose a door, each one of the three doors is

equally likely to have the prize behind it with probability one-third. Let's say you

choose door A. Now let's divide the three doors into two groups. There is a

probability of one-third that the prize is behind the door that you chose, A, and a

probability of two-thirds that the prize is behind one of the other unchosen doors,

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If you are still not convinced (or even if you are), here is a different way of explaining the paradox:

The "Let's Make a Deal" Paradox Part 2

Start of transcript. Skip to the end.

When you, the contestant, start the game there are three possible cases. The prize,

indicated by the dollar sign, is either behind door C, behind door B, or behind

door A. Since you obviously have no idea which of the three cases you're faced

with, you choose one of the three doors at random. Say you choose door A.

0:00 / 1:44 1.50x

If you're in case one where you chose door A and the prize is behind door C,

the host will obviously reveal the empty

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Explore the interactive activity below which demonstrates a different twist on the *Let's Make a Deal* example above.

If the *Let's Make a Deal* example still did not persuade you that probability is not always intuitive, the next example should definitely do the trick.

Example: The Birthday Problem

Suppose that you are at a party with 59 other people (for a total of 60). What are the chances (or, what is the probability) that at least 2 of the 60 guests share the same birthday?

To clarify, by "share the same birthday," we mean that 2 people were born on the same date, not necessarily in the same year. Also, for the sake of simplicity, ignore leap years, and assume that there are 365 days in each year.

Learn By Doing

1/1 point (graded)

Use intuition to guess the answer to the following question: What is the probability that at least 2 of the 60 party guests share a birthday?

- ○ Quite small—somewhere between a 1% chance and a 10% chance.
- ○ About 1/6 (one out of 6), or a 17% chance, since there are 60 people and 365 days (60/365 is roughly 1/6).
- ○ About a 50% chance.
- ○ Quite high—somewhere around a 75%-80% chance.
- ◉ Very high—a more than 99% chance. ✔

Answer
Correct: This is the right answer. Isn't it bizarre?

Submit

Indeed, there is a 99.4% chance that at least 2 of the 60 guests share the same birthday. In other words, it is *almost certain* that at least 2 of the guests share the same birthday. This is very counterintuitive.

Unlike the *Let's Make a Deal* example, for this scenario, we don't really have a good step-by-step explanation that will give you insight into this surprising answer. Later in this section, we will revisit this example and explain the solution.

From these two examples, you have seen that your original hunches cannot always be counted upon to give you correct predictions of probabilities.

In general, *probability is not always intuitive*.

Even though these two examples are definitely from the "harder" end of the complexity spectrum, hopefully they have motivated you to learn more about probability. We will need to further expand and extend our understanding of probability. Eventually we will need to develop a more formal approach to probability, but we will begin with an informal discussion of what probability is.