Learn By Doing Activity

Connection Between Confidence Intervals and Sampling Distributions

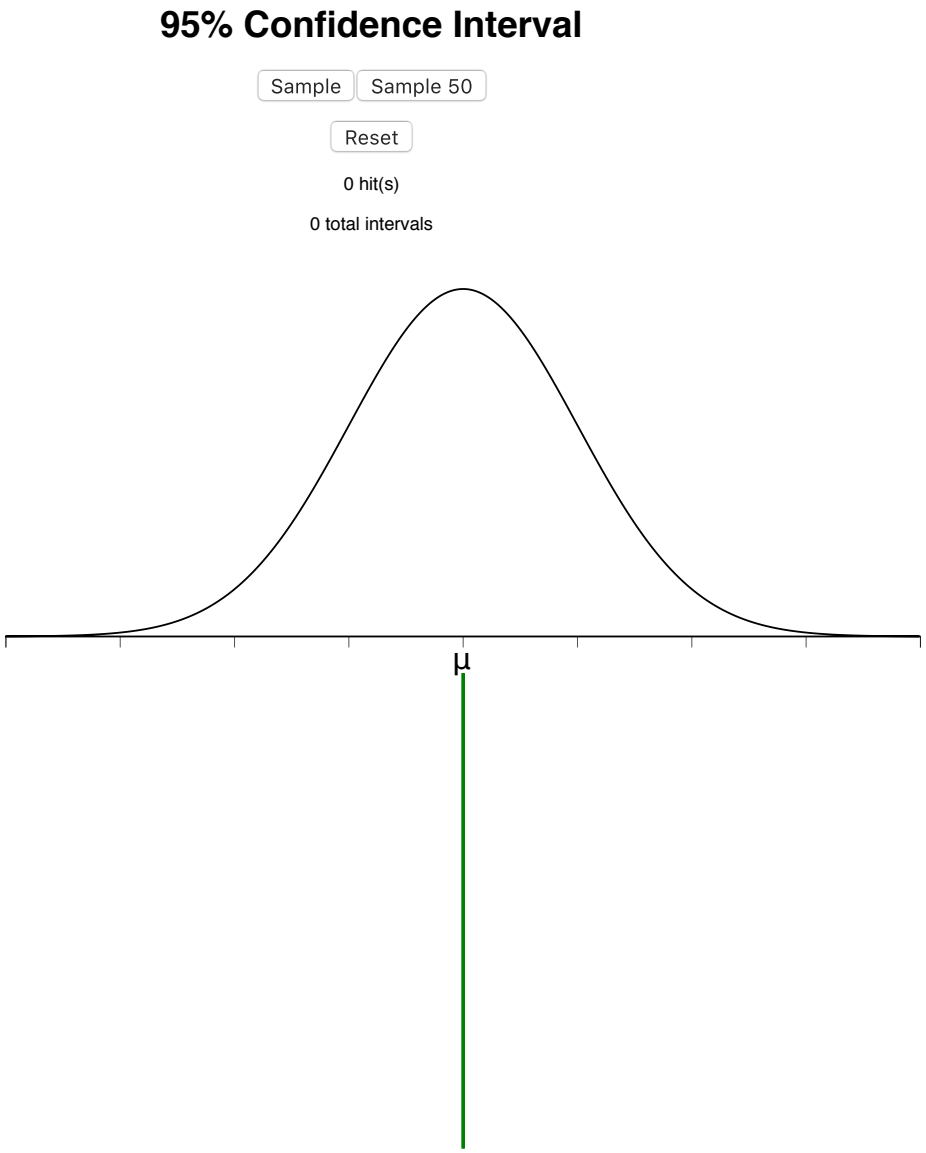
The purpose of this activity is to help give you a better understanding of the underlying **reasoning** behind the interpretation of confidence intervals. In particular, you will gain a deeper understanding of why we say that we are **"***95% confident* that the population mean is *covered* by the interval."

In the simulation below, you see a normal-shaped distribution, which represents the **sampling distribution of the mean** ($\bar{x}$) for random samples of a particular fixed sample size, from a population with a fixed standard deviation of σ.

The green line marks the value of the population mean, μ.

To begin the simulation, click the **"Sample"** button at the top of the simulation. You will see a line segment appear underneath the distribution; you should see that the line segment has a tiny red dot in the middle.

You have used the simulation to select a single sample from the population; the simulation has automatically computed the mean ($\bar{x}$) of your sample; your $\bar{x}$ value is represented by the little red dot in the middle of the line segment. The line segment represents a confidence interval. Notice that, by default, the simulation used a **95%** confidence interval for μ.



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Did your 95% confidence interval contain (or "cover") the population mean μ (the green line)?

Your Answer:

Yes

Our Answer:

Your answers may vary depending on the run of the simulation, but most likely it will be "yes." (If the interval did contain the population mean, the segment will be colored black. If the interval didn't contain the population mean, the segment will be colored red.)

Resubmit Reset

If your confidence interval *did* cover the population mean μ, then the simulation will have recorded 1 "hit" underneath the "Sample" buttons.

Now, click the Sample button to select another single sample.

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Was your second sample mean $\bar{x}$ (the new dot) the same value as your 1st sample mean? (i.e., is it in the same relative location along the axis?) Why is this result to be expected?

Your Answer:

No it was not the same value. These are random samples.

Our Answer:
No, the new $\bar{x}$ value won't be the same. This result is to be expected because there is a natural variation due to sampling. (This is a key idea in this course.)

Resubmit Reset

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A new 95% confidence interval has also been constructed (the new line segment, centered at the location of your second $\bar{x}$). Does the new interval cover the population mean μ?

Your Answer:

Yes

Our Answer:
Your answers may vary depending on the run of the simulation, but most likely it will be "yes." (If the interval did contain the population mean, the segment will be colored black. If the interval didn't contain the population mean, the segment will be colored red.)

Resubmit Reset

Notice, next to "Total Intervals" under the Reset button, the number of total selected samples has been tallied.

Now, click "**Sample 50**" repeatedly, until the simulation tallies a "total intervals" of around 1,000 samples. You will see that the simulation computes the "percent hit" for all the intervals.

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What percentage of the many 95% confidence intervals should cover the population mean μ?

Your Answer:

95%

Our Answer:
In the long run, 95% of the confidence intervals should cover μ. (That's precisely the meaning of the confidence percentage.)

Resubmit Reset

Now let's summarize some key ideas.

Based on what you've seen on the simulation (with the level set at 95%), decide which of the following statements are true and which are false.

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1/1 point (graded)
True or false? Each interval is centered at the population mean (μ).

- ○ True
- ● False ✔

Answer
Correct:
The population mean (μ) won't necessarily be at the **exact center** of the confidence intervals. On the simulation, the population mean was represented by the green line, and it wasn't always at the exact center of the intervals.

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1/1 point (graded)

True or false? Each interval is centered at the sample mean ($\bar{x}$).

- (•) True ✔
- ( ) False

Answer

Correct:

As you saw on the simulation, each interval was centered at a red dot which represented a sample mean ($\bar{x}$). In fact, a confidence interval is always made with the sample mean ($\bar{x}$) at the center.

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1/1 point (graded)

True or false? The population mean (μ) changes when different samples are selected.

- ( ) True
- (•) False ✔

Answer

Correct:

The population mean (μ) is a parameter, so it doesn't change. On the simulation, the population mean was indicated by the green line, which didn't change position along the axis.

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True or false? The sample mean ($\bar{x}$) changes when different samples are selected.

- (●) True ✔
- ( ) False

Answer
Correct:
The sample mean ($\bar{x}$) is a statistic that changes from sample to sample. On the simulation, each sample mean ($\bar{x}$) was represented by a red dot in the middle of each interval, and each red dot was in a different position along the axis.

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1/1 point (graded)
True or false? In the long run, only 95% of the intervals will contain (or "cover") the **sample** mean ($\bar{x}$).

- ( ) True
- (●) False ✔

Answer
Correct:
In fact, **100%** of the intervals will contain their sample mean, not only 95% of the intervals, because each confidence interval is **centered** at the sample mean ($\bar{x}$); each $\bar{x}$ was represented on the simulation by the red dot in the middle of each interval.

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1/1 point (graded)
True or false? In the long run, only 95% of the intervals will contain (or "cover") the **population** mean (μ).

- (●) True ✔
- ( ) False

Answer

Correct:
With the confidence level set at 95%, you saw that in the long run (if you selected many thousands of samples) 95% of the intervals would cover the green line, which represented the population mean (μ).

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1/1 point (graded)

Note that in actual scientific practice, we only select **one** single sample and therefore we can only see the **one** corresponding interval, out of the potential thousands that are displayed using the simulation. Give the best interpretation of a single 95% confidence interval as follows, based on what you've learned from the simulation.

We are 95% confident that our interval is one that covers which of the following?

- ○ The sample mean ($\bar{x}$)
- ◉ The population mean (μ) ✔

Answer
Correct:
The 95% confidence interval says that that in the long run (if you selected many thousands of samples) 95% of the intervals would cover the population mean (μ).

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