Hypothesis Testing for the Population Mean: Confidence Intervals

Learning Objective: Apply the concepts of: sample size, statistical significance vs. practical importance, and the relationship between hypothesis testing and confidence intervals.

Relating Hypothesis Tests and Confidence Intervals

Just as we did for proportions, we may examine a confidence interval to decide whether a proposed value of the population mean is plausible.

Suppose we want to test $H_0 : \mu = \mu_0$ vs. $H_a : \mu \neq \mu_0$ using a significance level of $\alpha = 0.05$. An alternative way to perform this test is to find a 95% confidence interval for μ and make the following conclusions:

If μ_0 falls outside the confidence interval, reject H_o.

If μ_0 falls inside the confidence interval, do not reject H_o.

Example

We'll use example 2, in which the alternative was two-sided.

Recall that we want to check whether a medication conforms to a target concentration of a chemical ingredient by testing

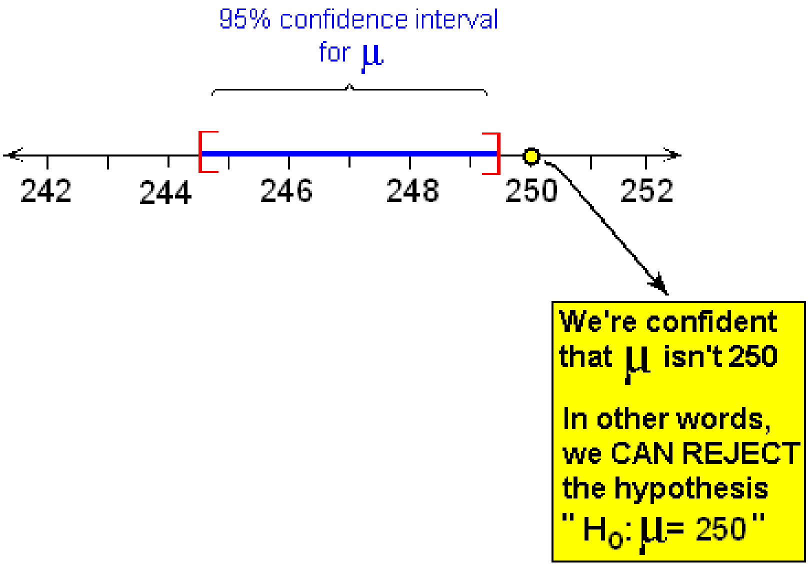
$H_0 : \mu = 250$

$H_a : \mu \neq 250$

We assume that $\sigma = 12$, and in a sample of size n = 100 we obtained a sample mean of $\overline{x} = 247$.

A 95% confidence interval for μ is $\overline{x} \pm 2\frac{\sigma}{\sqrt{n}} = 247 \pm 2\frac{12}{\sqrt{100}} = 247 \pm 2.4 = (244.6,\ 249.4)$.

Since the interval does not contain 250, we reject H_o and conclude that the alternative is true: the population mean concentration differs from 250.



Did I Get This

1/1 point (graded)

One of the following is an actual output from a statistical software package; the others were edited to be incorrect.

A.

One-Sample Z

```
Test of mu = 264 vs not = 264
The assumed standard deviation = 16


 N     Mean  SE Mean         95% CI            Z       P
35  260.000    2.704  (254.699, 265.301)   -2.22   0.047
```

B.

One-Sample Z

```
Test of mu = 267 vs not = 267
The assumed standard deviation = 16


 N     Mean  SE Mean          95% CI              Z      P
35  260.000    2.704  (254.699, 265.301)  -2.22  0.087
```

C.

One-Sample Z

```
Test of mu = 266 vs not = 266
The assumed standard deviation = 16


 N     Mean  SE Mean          95% CI              Z      P
35  260.000    2.704  (254.699, 265.301)  -2.22  0.027
```

Which of the above is the correct complete output? Use the relationship between the two-sided z-test and the 95% confidence interval for μ to guide your thinking.

- A
- B
- C ✔

Answer

Correct:

We are testing H_o: $\mu = 266$ vs. H_a: $\mu \neq 266$. The 95% confidence interval does not cover the null value, 266, which means that at the .05 level H_o can be rejected. This is also confirmed by the p-value being 0.027, which at the 0.05 level indicates that H_o can be rejected. This is the correct answer, because this is the only output out of the three where the conclusion we make using the confidence interval and the conclusion we make using the p-value agree.

Submit

Comment

Beyond using the confidence interval as a quick way to carry out the two-sided test, the confidence interval can provide insight into the actual value of the population mean if H_o is rejected. In the concentration level example, H_o was rejected, and all we could conclude about the mean concentration level of the entire shipment, μ, was that it was not 250. The 95% confidence interval for μ (244.6, 249.4) gives us an idea of what plausible values for μ would be. In particular, we can conclude that since the confidence interval lies below 250, at least a large portion of the shipment contains medication that is ineffective.

We are done with the case where the population standard deviation, σ, is known. We now move on to the more common case where σ is unknown.