Confidence Intervals for Population Proportion p: When To Use

Learning Objective: Explain what a confidence interval represents and determine how changes in sample size and confidence level affect the precision of the confidence interval.

Learning Objective: Find confidence intervals for the population mean and the population proportion (when certain conditions are met), and perform sample size calculations.

When Is It Safe to Use These Methods?

As we mentioned before, one of the most important things to learn with any inference method is the conditions under which it is safe to use it.

As we did for the mean, the assumption we made in order to develop the methods in this unit was that the sampling distribution of the sample proportion, $\hat{p}$, is roughly normal. Recall from module 4 of the Probability unit that the conditions under which this happens are that $n \cdot p \geq 10$ and $n \cdot (1-p) \geq 10$. Since p is unknown, we will replace it with its estimate, the sample proportion, and set

$n \cdot \hat{p} \geq 10$ and $n \cdot (1-\hat{p}) \geq 10$

to be the conditions under which it is safe to use the methods we developed in this section.

Scenario: Ban on Assault Weapons

Background: The U.S. federal ban on assault weapons expired in September 2004, which meant that after 10 years (since the ban was instituted in 1994) there were certain types of guns that could be manufactured legally again. A poll asked a random sample of 1,200 eligible voters (among other questions) whether they were satisfied with the fact that the law had expired. Data are based on a poll conducted by NBC News/Wall Street Journal. We would like to estimate p, the proportion of U.S. eligible voters who were satisfied with the expiration of the law, with a 95% confidence interval.

Learn By Doing

1/1 point (graded)

Based on the sample size, what is the margin of error of this poll? Report your answer as a proportion (0.000 to 1.000) rounded to THREE decimal places.

0.029 ✔

0.029

Answer

Correct: $1 / \sqrt{(1{,}200)} = 0.029$

Submit

Learn By Doing

1/1 point (graded)

In analyzing these data, we found that 142 of the 1,200 sampled voters answered that they were satisfied (so p = 0.118 or about 12%). We calculated the 95% confidence interval for p to be (0.10, 0.14).

What is the margin of error for this question based on the 95% confidence interval for p?

- (●) 0.02 ✔
- ( ) 0.029
- ( ) 0.04

0.118

Answer
Correct: Since the width of the confidence interval is 0.04, the margin of error is 0.04 / 2 = 0.02.

Submit

Learn By Doing (1/1 point)

Why is there an apparent discrepancy between the margin of error calculated using the sample size and the margin of error calculated from the 95% confidence interval?

Your Answer:

0.029 was a conservative estimate; we assumed a sample probability of 0.5

Our Answer:
We found that the margin of error of this poll is roughly 2.9% using the sample size and a margin of error is 2% using the 95% confidence interval. This is because when we used the sample size we calculated a "conservative" margin of error. This margin of error is the margin of error *of the whole poll*. What it says is: Based on this sample size, the margin of error for *any* of the questions in this poll will be no more than 2.9% regardless of what the sample proportions are. In the particular question about the ban on assault weapons from the poll, the margin of error happened to be lower (2%).

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Let's Summarize

In general, a confidence interval for the unknown population proportion (p) is $\hat{p} \pm z^* \cdot \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$, where z* is 1.645 for 90% confidence, 2 for 95% confidence, and 2.576 for 99% confidence.

To obtain a desired margin of error (m) in a 95% confidence interval for unknown population proportion, a conservative sample size is $n = \frac{1}{m^2}$.

The margin of error of a poll is determined (conservatively) by $\frac{1}{\sqrt{n}}$.

The methods developed in this unit are safe to use as long as $n \cdot \hat{p} \geq 10$ and $n \cdot (1 - \hat{p}) \geq 10$.