Matched Pairs: Confidence Interval

Learning Objective: In a given context, carry out the inferential method for comparing groups and draw the appropriate conclusions.

Learning Objective: Specify the null and alternative hypotheses for comparing groups.

Confidence Interval for μ_d (Paired t Confidence Interval)

So far we've discussed the paired t-test, which checks whether there is enough evidence stored in the data to reject the claim that $\mu_d = 0$ in favor of one of the three possible alternatives.

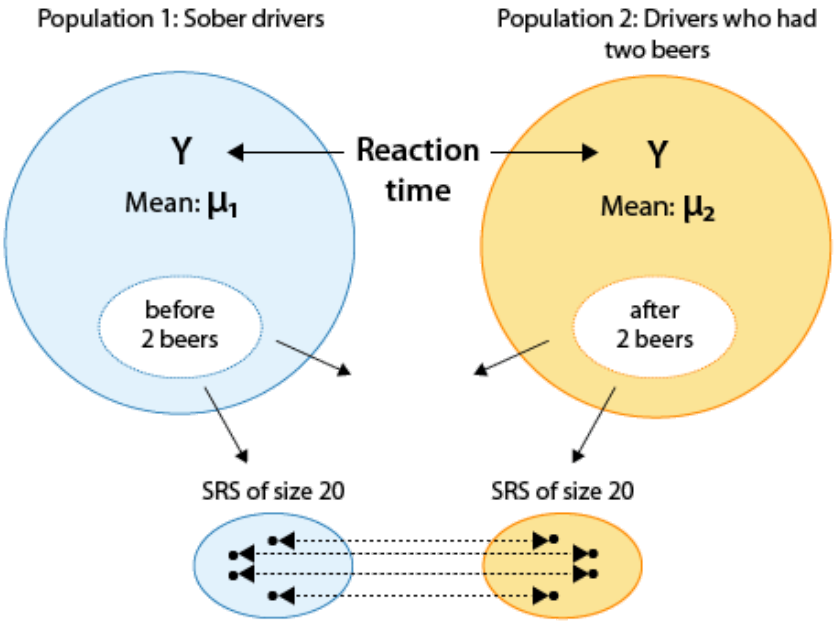
If we would like to estimate μ_d, the mean of the differences (response 1 - response 2), we can use the natural point estimate, $\overline{x_d}$, the sample mean of the differences, or preferably, use a 95% confidence interval, which will provide us with a set of plausible values for μ_d.

In particular, if the test has rejected $H_0 : \mu_d = 0$, a confidence interval for μ_d can be insightful, since it quantifies the effect that the categorical explanatory variable has on the response variable.

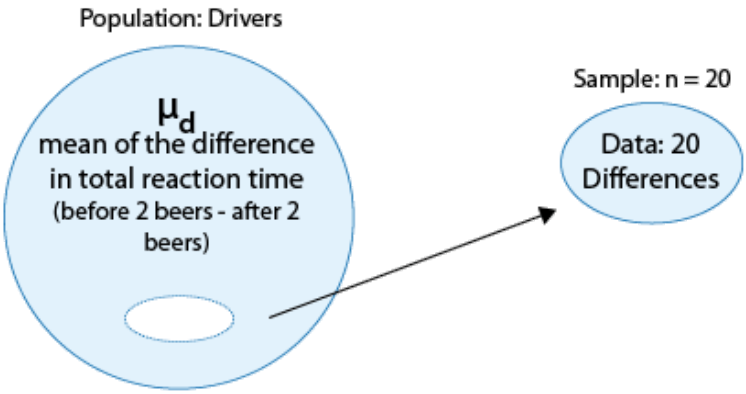
Comment: We will not go into the formula and calculation of the confidence interval, but rather ask our statistical software to do it for us, and focus on interpretation.

Example

Recall our leading example about whether drivers are impaired after having two beers:



which is reduced to inference about a single mean, the mean of the differences (before - after):



The p-value of our test, $H_0 : \mu_d = 0$ vs. $H_0 : \mu_d < 0$ was .009, and we therefore rejected H_o and concluded that the mean difference in total reaction time (before beer - after beer) was negative, or in other words, that drivers are impaired after having two beers. As a follow-up to this conclusion, it would be interesting to quantify the effect that two beers have on the driver, using the 95% confidence interval for μ_d.

Using statistical software, we find that the 95% confidence interval for μ_d, the mean of the differences (before - after), is roughly (-0.9, -0.1).

We can therefore say with 95% confidence that drinking two beers increases the total reaction time of the driver by between 0.1 and 0.9 of a second.

Comment

As we've seen in previous tests, as well as in the matched pairs case, the 95% confidence interval for μ_d can be used for testing in the two-sided case ($H_0 : \mu_d = 0$ vs. $H_a : \mu_d \neq 0$):

If the null value, 0, falls outside the confidence interval, H_o is rejected.

If the null value, 0, falls inside the confidence interval, H_o is not rejected.

Example

Let's go back to our twin study example, where we found a 95% confidence interval for μ_d of (-6.11322, 0.30072) and a p-value of 0.074.

We used the fact that the p-value is 0.074 to conclude that H_o can not be rejected (at the 0.05 significance level), and that whether or not a person was raised by his or her birth parents doesn't necessarily have an effect on intelligence (as measured by IQ scores). The last comment tells us that we can also use the confidence interval to reach the same conclusion, since 0 falls inside the confidence interval for μ_d. In other words, since 0 is a plausible value for μ_d we cannot reject H_o which claims that $\mu_d = 0$.

Scenario: Typing Speed for Different Word Processors

A publishing company wanted to test whether typing speed differs when using word processor A or word processor B. A random sample of 25 typists was selected and the typing speeds (in words per minute) were recorded for each secretary when using word processor A and then when using word processor B. (Which word processor was used first was determined for each typist by a coin flip).

Based on the collected data, a 95% confidence interval for μ_d, the mean difference (word processor A - word processor B) was found to be (2.5, 7.8).

The appropriate hypotheses for testing whether the typing speeds differ when using word processor A or word processor B is the two-sided test:

$$H_0 : \mu_d = 0 \qquad H_a : \mu_d \neq 0$$

Learn By Doing (1/1 point)

Based on this confidence interval for μd, what would be your conclusion (at the 0.05 significance level)? Explain.

Your Answer:

0 is not in the interval, so we can take it to mean that the p-value was low enough for Ho to be rejected, and Ha, accepted.

Our Answer:
Since 0 (our null value) falls outside the 95% confidence interval, we can reject Ho (at the 0.05 significance level), and conclude that the typing speeds differ when using processor A or B.

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Learn By Doing (1/1 point)

Interpret the 95% confidence interval in context. Make sure that your interpretation quantifies the effect that the type of word processor used (the explanatory variable, X) has on the typing speed (the response variable, Y).

Your Answer:

Since mean difference was positive, that means that processor A minus processor B yielded a positive difference -- meaning, processor A had higher typing speed on average, higher by 2.5-7.8 words per minute.

Sooooo processor A users had 2.5 - 7.8 WPM higher typing speed.

Our Answer:
We are 95% confident that on average, typists type between 2.5 and 7.8 more words per minute when using word processor A.

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