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Summary (Case C→Q)

We are now done with case C→Q. We learned that this case is further classified into sub-cases, depending on the number of groups that we are comparing (i.e., the number of categories that the explanatory variable has), and the design of the study (independent vs. dependent samples). For each of the three sub-cases that we covered, we learned the appropriate inferential method, and emphasized the idea behind the method, the conditions under which it can be safely used, how to carry it out using software, and the interpretation of the results.

The following table summarizes when each of the three sub-cases, covered in this module, are used:

Sub-Case of C→Q	Circumstances When Used
Two-Sample t-test	• Categorical explanatory variable with two categories • Comparing two population means based on two independent samples • Either normal populations or large sample size
Paired t-test (special case of the one sample t-test)	• Categorical explanatory variable with two categories • Comparing the two population means, when the samples are dependent on each other or "matched pairs." • Samples are dependent in the sense that every observation in one sample is linked to an observation in another sample. Examples of dependent samples include: 　○ same subjects measured twice 　○ Twins
ANOVA	• Categorical explanatory variable with more than two categories • Comparing more than two population means based on independent samples

The following summary discusses each of the above named sub-cases of C→Q within the context of the hypothesis testing process.

I. Stating the null and alternative hypotheses (H_0 and H_a).

Sub-Case of C→Q	Null Hypothesis	Alternative Hypothesis
Two-Sample t-test	$H_o: \mu_1 - \mu_2 = 0$ (or $H_o: \mu_1 = \mu_2$)	$H_a: \mu_1 - \mu_2 < 0$ (which is the same as $H_a: \mu_1 < \mu_2$) **or** $H_a: \mu_1 - \mu_2 > 0$ (which is the same as $H_a: \mu_1 > \mu_2$) **or** $H_a: \mu_1 - \mu_2 \neq 0$ (which is the same as $H_a: \mu_1 \neq \mu_2$)
Paired t-test	$H_o: \mu_d = 0$	$H_o: \mu_d < 0$ **or** $H_o: \mu_d > 0$ **or** $H_o: \mu_d \neq 0$
ANOVA	$H_0: \mu_1 = \mu_2 = ... = \mu_k$	$H_a:$ not all μ's are equal

II. Check Conditions, and Summarize the Data Using a Test Statistic

*Check that the conditions under which the test can be reliably used are met.

For the Two-Sample t-test, the conditions are:

1. Two samples are independent and random

2. One of the following two scenarios:

- Both populations are normal

- Populations are not normal, but large sample size (>30)

For the Paired t-test (as a special case of a one-sample t-test), the conditions are:

1. The sample of differences is random (or at least can be considered so in context).

2. We are in one of the three situations marked with a green check mark in the following table:

For an ANOVA, the conditions are:

1. The samples drawn from each of the populations being compared are independent.

2. The response variable varies normally within each of the populations being compared. As is often the case, we do not have to worry about this assumption for large sample sizes.

3. The populations all have the same standard deviation.

*Summarize the data using a test statistic.

Special Case of C→Q	Test Statistic
Two-Sample t-test	$t = \dfrac{(\overline{y_1} - \overline{y_2}) - 0}{\sqrt{\dfrac{s_1^2}{n_1} + \dfrac{s_2^2}{n_2}}}$

Paired t-test	$t = \dfrac{\overline{x_d} - 0}{\frac{s_d}{\sqrt{n}}}$
ANOVA	$F = \dfrac{\text{Variation Among the Sample Means}}{\text{Variation Within the Groups}}$

III. Finding the p-value of the test

Use statistical software to determine the p-value. The p-value is the probability of getting data like those observed (or even more extreme) assuming that the null hypothesis is true, and is calculated using the null distribution of the test statistic. The p-value is a measure of the evidence against H_0. The smaller the p-value, the more evidence the data present against H_0. The p-values for three C→Q tests are obtained from the output.

IV. Making conclusions.

-Conclusions about the significance of the results:

If the p-value is small, the data present enough evidence to reject H_0 (and accept H_a).

If the p-value is not small, the data do not provide enough evidence to reject H_0.

To help guide our decision, we use the significance level as a cutoff for what is considered a small p-value. The significance cutoff is usually set at .05, but should not be considered inviolable.

Conclusions should always be stated in the context of the problem.

Following the test...

For a two-sample t-test, a **95% confidence interval** for $\mu_1 - \mu_2$ can be very insightful after a test has rejected the null hypothesis, and can also be used for testing in the two-sided case.

For a paired t-test, a **95% confidence interval** for μ_d can be very insightful after a test has rejected the null hypothesis, and can also be used for testing in the two-sided case.

If the ANOVA F-test has rejected the null hypothesis, looking at the **confidence intervals** for the population means that are in the output can provide visual insight into why the H_0 was rejected (i.e., which of the means differ).