Lagunita is retiring and will shut down at 12 noon Pacific Time on March 31, 2020. A few courses may be open for self-enrollment for a limited time. We will continue to offer courses on other online learning platforms; visit http://online.stanford.edu.

Mean of a Random Variable: Examples

Learning Objective: Find the mean and variance of a discrete random variable, and apply these concepts to solve real-world problems.

Here is another example:

Example: Life Insurance #1

Suppose you work for an insurance company, and you sell a $100,000 whole-life insurance policy at an annual premium of $1,200. (This means that the person who bought this policy pays $1,200 per year so that in the event that he or she dies, the policy beneficiaries will get $100,000). Actuarial tables show that the probability of death during the next year for a person of your customer's age, sex, health, etc. is .005. Let the random variable X be the company's gain from such a policy.

What is the expected or mean gain (amount of money made by the company) for a policy of this type?

In other words, we need to find μ_X .

Since this is a whole-life policy, there are two possibilities here; either the customer dies this year (which you are given will happen with probability 0.005), or the customer does not die this year (which, by the complement rule, must be 0.995).

In both cases, the company gets the $1,200 premium. If the customer lives, the company just gains the $1,200, but if the customer dies, the company needs to pay $100,000 to the customer's beneficiaries. Therefore, here is the probability distribution of X:

	live	die
X	+1200	1200-100,000
P(X=x)	.995	.005

Their average, or expected, gain overall is

μ_X = 1200(0.995) + (1200 - 100,000)(0.005) = 700 dollars.

Example: Life Insurance #2

Suppose that five years have passed and your actuarial tables indicate that the probability of death during the next year for a person of your customer's current age has gone up to 0.0075. Obviously, this change in probability should be reflected in the annual premium (since it is slightly more risky for the insurance company to insure the customer).

What should the annual premium be (instead of $1,200) if the company wants to keep the same expected gain?

Now we substitute 0.0075 for 0.005, replace 1,200 with an unknown new premium N, and set the mean gain equal to 700, as it was before:

	live	die
X	N	N-100,000
P(X=x)	.9925	.0075

We need to solve:

- 700 = (N)(0.9925) + (N - 100,000)(0.0075)

Using some algebra:

- 700 = N - 750

Finally:

- N = 1450

In order to keep the same expected gain of $700, the company should increase that customer's premium to $1,450.

The purpose of this next activity is to give you guided practice in solving practical problems whose solution is based on the mean of random variables.

Scenario: Fire Insurance

Suppose that you work for an insurance company and you sell a $100,000 fire insurance policy at an annual premium of $1,350. Experience has shown that:

- The probability of total loss (due to fire) to a house in that area and of the size of your customer's house is .002 (in which case the insurance company will pay the full $100,000 to the customer).
- The probability of 50% damage (due to fire) to a house in that area and of the size of your customer's house is .008 (in which case the insurance company will pay only $50,000 to the customer).

For simplicity, we'll ignore any other partial losses.

Let the random variable X be the insurance company's annual gain from such a policy (i.e., the amount of money made by the insurance company from such a policy).

Learn By Doing (1/1 point)

Find the probability distribution of X. In other words, list the possible values that X can have, and their corresponding probabilities. (Hint: There are three possibilities here: no fire, total loss due to fire, 50% damage due to fire).

Your Answer:

x = total loss, 50% damage, no loss
x = -98650, -48650, +1350
P(X=x) = 0.002, 0.008, and 0.99

Our Answer:
There are three possibilities that correspond to the three possible values of X in this example. In every case, the insurance company is gaining the amount of the policy cost, which is $1,350. • If there is total loss due to fire (which happens with probability .002), the insurance company has to pay the customer $100,000, and therefore the company's gain is: $1,350 - $100,000. • If there is 50% damage due to fire (which happens with probability .008), the insurance company has to pay the customer $50,000, and therefore the company's gain is: $1,350 - $50,000. • If there is no fire (which must happen with the "remaining probability" of 1 - .002 - .008 = .99), the insurance company has to pay nothing, and so is left with the premium gain of $1,350. To summarize, the probability distribution of X is:

Resubmit Reset

Learn By Doing (1/1 point)

What is the mean (expected) annual gain for a policy of this type? In other words, what is the mean of X?

Your Answer:

750

Our Answer:
Using the definition of the mean of a discrete random variable, we will average the possible values weighted by their corresponding probabilities. Mean of X = (1,350 - 100,000)(.002) + (1,350 - 50,000)(.008) + 1,350(.99) = 750. The expected gain of such a policy is $750. Since the insurance company probably sells a lot of policies of this type, in the majority of them (99%) the company will gain money ($1,350), and in the remaining 1% of them, it will lose (either 1,350 - 100,000 or 1,350 - 5,0000). Averaging over all such policies, the company is going to make about $750 from each such fire insurance policy.

Resubmit Reset

Learn By Doing (1/1 point)

The insurance company gets information about gas leakage in several houses that use the same gas provider that your customer does. In light of this new information, the probabilities of total loss and 50% damage (that were originally 0.002 and 0.008, respectively) are tripled (to 0.006 for total loss and 0.024 for 50% damage). Obviously, this change in the probabilities should be reflected in the annual premium, to account for the added risk that the insurance company is taking. What should be the new annual premium (instead of $1,350), if the company wants to keep its expected gain of $750? Guidance: Let the new premium (instead of 1,350) be denoted by N, for new. Set up the new probability distribution of X using the updated probabilities, and using N instead of 1,350. (The answer to question 1 will help.) The question now is: What should the value of N (the new premium) be, if we want the mean of X to remain 750? Set up an equation with N as unknown, and solve for N.

Your Answer:

(N-100k)*0.006 + (N-50k)*0.024 + 0.97*N = 750
0.006N - 600 + 0.024 - 1200 + 0.97N = 750
N = 2550

Our Answer:

Here is the new probability distribution of X: Note that we updated the probabilities of total loss and 50% damage by tripling them, and the probability of no fire has changed to 1 - 0.006 - 0.024 = 0.97. The company wants to keep the same annual gain from the policy ($750), and the question is, what should the new premium (N) be that will satisfy this? In other words, we need to solve the following equation for N: 750 = (N - 100,000)(0.006) + (N - 50,000)(0.024) + N(0.97) Thus, 750 = N - 600 - 1,200, or N - 1,800. And therefore, N = 750 + 1,800 = 2,550. In order to account for the added risk that the insurance company is taking by continuing to insure the customer, the premium changes from $1,350 to $2,550.

Resubmit Reset

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