Probability Distribution: Using Conditional Probabilities

Learning Objective: Find the probability distribution of discrete random variables, and use it to find the probability of events of interest.

Here is another example in which we'll use a probability distribution that is associated with a random variable of interest to find probabilities. What will be new in this example is the use of conditional probabilities.

Example: Xavier's Production Line

The number of defective parts produced each hour by Xavier's production line is a random variable X with the following probability distribution:

X	0	1	2	3	4
P(X=x)	.15	.30	.25	.20	.10

Using the probability distribution of a random variable, we can answer some probability questions:

(a) What is the probability of at least 2 defects in a randomly chosen hour?

$$P(X \geq 2) = P(X = 2) + P(X = 3) + P(X = 4) = 0.25 + 0.20 + 0.10 = 0.55$$

(Note that the addition principle has been applied.)

(b) Suppose it is known that more than 2 defects were produced in a particular hour. What is the probability that the number of defects was fewer than 4?

We use the conditional probabilities definition $P\left(B \mid A\right) = \frac{P(AandB)}{P(A)}$ to solve:

$$P\left(X < 4 \mid X > 2\right) = \frac{P((X<4)and(X>2))}{P(X>2)} = \frac{P(X=3)}{P(X>2)} = \frac{0.2}{0.3} = 0.67$$

Note that we are substituting the event "X < 4" for event B, and the event "X > 2" for event A.

Also note that the only way that (X < 4) and (X > 2) can happen together is if X = 3.

The purpose of the next activity is to give you guided practice at using the probability distribution of a random variable to find probabilities of interest.

Scenario: Telemarketing Sales

Recall the following example:

The number of sales that a telemarketing salesperson makes in an hour is a random variable X having the following probability distribution:

x	0	1	2	3	4
P(X=x)	10/50	12/50	12/50	10/50	6/50

Learn By Doing

1/1 point (graded)

What is the probability that the salesperson makes at least one sale in an hour?

- 12/50
- 22/50
- 28/50
- 40/50 ✔
- 10/50

Answer

Correct:
The probability of at least one sale is $P(X \geq 1) = P(X = 1) + P(X = 2) + P(X = 3) + P(X = 4) = (12 + 12 + 10 + 6) / 50 = 40/50$, (using the addition principle). Alternatively (and more efficiently), you can use complements and the fact that the complementary event of $X \geq 1$ is $X = 0$. Therefore, $P(X \geq 1) = 1 - P(X = 0) = 1 - 10/50 = 40/50$.

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Learn By Doing (1/1 point)

Ten minutes after the salesperson has started working, he made a sale. What is the probability that this is the only sale that the salesperson will make within the first hour? This is a bit tricky. ... Let's rephrase the question in a way that will make it easier to translate to the language of probability. We are given that one sale has been made 10 minutes into the hour. This means that the number of sales that will be made within the hour, X, is at least one. In other words, we are given that X is greater than or equal to 1. Given that information, we are asked to find the probability that this will be the only sale, i.e., that $X = 1$. Putting this together, the question asks you to find: $P(X=1 \mid X \geq 1)$.

Your Answer:

0.3

Our Answer:
We use the conditional probabilities definition $P(B \mid A) = P(A \text{ and } B) / P(A)$ to solve: (Note that we are treating "$X = 1$" as event B and "$X \geq 1$" as event A.) Our answer indicates that there is a 30% chance that the sale that the salesperson has made is the only one he or she will make during the hour.

Resubmit Reset

Scenario: Changing Majors

Data were collected from a survey given to graduating college seniors on the number of times they had changed majors. From that data, a probability distribution was constructed. The random variable X is defined as the number of times a graduating senior changed majors. It is shown below:

x	0	1	2	3	4	5
P(X = x)	.28	.37	.23	.09	.02	.01

Did I Get This

1/1 point (graded)

What is the probability that a randomly selected student changed his or her major at least once?

- 0.12
- 0.35
- 0.37
- 0.65
- 0.72 ✔

Answer

Correct:

At least once means one or more times. So P(changed majors at least once) = $P(X \geq 1) = P(X = 1) + P(X = 2) + P(X = 3) + P(X = 4) + P(X = 5)$. This is .37 + .23 + .09 + .02 + .01 = 0.72. It is a little easier using complements. P(changed majors at least once) = $1 - P(X = 0)$. This is 1 - .28, which also is 0.72.

Submit

Did I Get This

1/1 point (graded)

What is the probability that a randomly selected student changed his or her major at most twice?

- 0.12
- 0.23
- 0.35
- 0.88 ✔

Answer

Correct:

We want P(changed majors at most twice). At most means that number or less. $P(X \leq 2) = P(X = 0) + P(X = 1) + P(X = 2)$ = .28 + .37 + .23 = 0.88. We also could have used complements. The complement of at most 2 is more than 2. So, $P(X \leq 2) = 1 - P(X > 2)$. $P(X \leq 2) = 1 - P(X > 2) = 1 - [\,P(X = 3) + P(X = 4) + P(X = 5)\,]$. The only advantage to using this method is that the numbers (.09 + .02 + .01) are easier to add.

Submit

Did I Get This

1/1 point (graded)

Given that a randomly selected person did change majors, what is the probability that he or she changed majors more than three times?

- 0.03
- 0.04 ✔
- 0.35
- 0.72

Answer

Correct: We want $P(X > 3 \mid X \geq 1) = .03 / .72 = 0.04$.

Submit