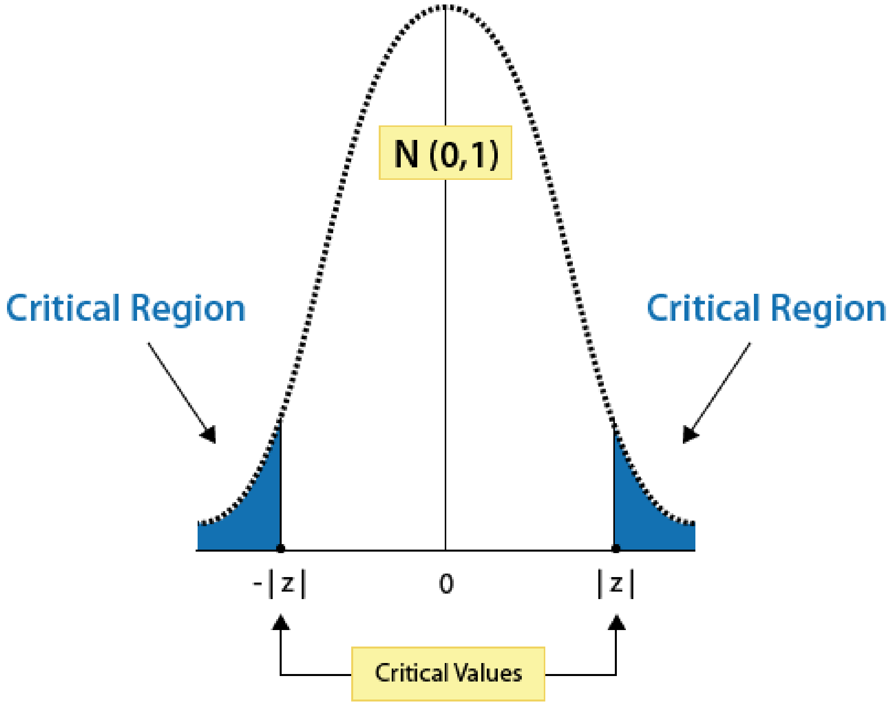
Hypothesis Testing for the Population Proportion p: The Critical Value Method

As noted earlier, before the widespread use of statistical software, it was common to use 'critical values' instead of p-values to assess the evidence provided by the data. Even though the critical values approach is not used in this course, students might find it insightful. Thus, the interested students are encouraged to review the critical value method on this page. If your instructor clearly states that you are required to have knowledge of the critical value method, you should definitely review the information.

We will be emphasizing the use of statistical software in obtaining the exact p-value. The critical value method provides the ability to get an understanding of whether or not a null hypothesis will be rejected at a given probability level (ex. 0.05 or 0.01). In addition, for the z test, the Normal Table can be used to determine the exact probability level, without the use of statistical software.

Concepts of the Critical Value Method

There are several concepts that are important to understand in the critical value method. They are the: 1) critical value and 2) the critical region. As shown in the graph below, the **critical value** is the value, which cuts off an area referred to as the **critical region (or area of rejection),** as applied to the z test.

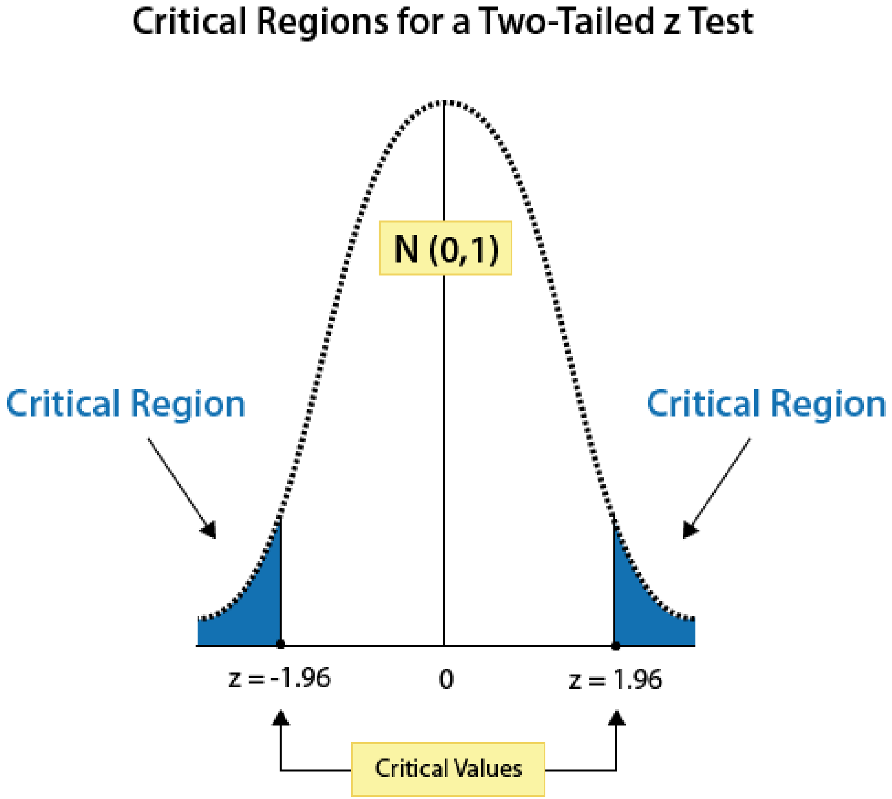


When z test statistics fall in the critical region (the **blue** shaded areas in the above graph), they are far enough from the mean that they are significantly different from the mean; therefore, in these instances, the null hypothesis would be rejected. The critical region is determined by a critical value that is based on two things: 1) the significance level of the test (either 0.05 or 0.01) **AND** the direction of the test (ex. left-tailed, right-tailed, or two-tailed).

Not Equal To

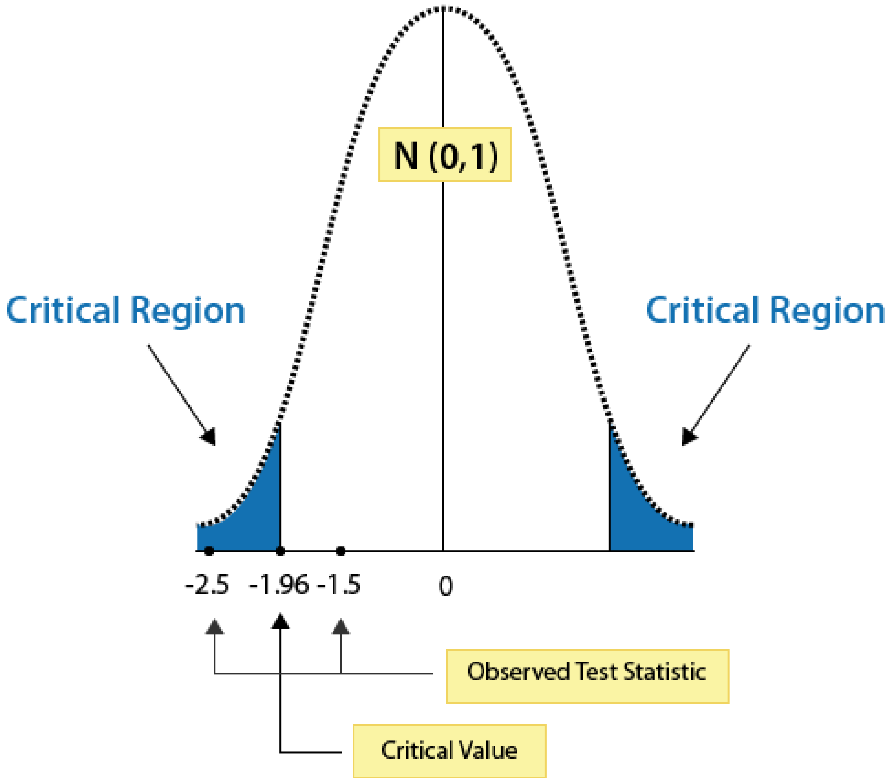
For a two-tailed z test, there will be critical regions on both sides of the distribution. For a two-tailed test using a 0.05 level of significance, we need to determine a value that would put 0.025 or 2.5%, in each tail. We can determine this value by using the Normal Table.

First, we need to look in the body of the normal table (click here), where we will see that the exact value 0.0250 is associated with the z score of -1.96; thus, -1.96 is the critical value that puts 0.025 (or 2.5%) in the left tail of the distribution. Since the standard normal distribution is symmetrical, +1.96 is the critical value that puts 0.025 (or 2.5%) in the right tail. Thus, the critical values of -1.96 and +1.96 would define the critical regions for a two-tailed z-test using a 0.05 significance level.



In order to test the null hypothesis, we need to look at where the z-test statistic falls in relation to the critical regions formed by the critical values. In a two-tailed z-test, a z-test statistic of -1.5 would not fall in a critical region Therefore, we know that the p-value would be more than 0.05. Thus, we would not reject the null hypothesis, since the p-value is greater than 0.05 (or, stated another way, p-value > 0.05).

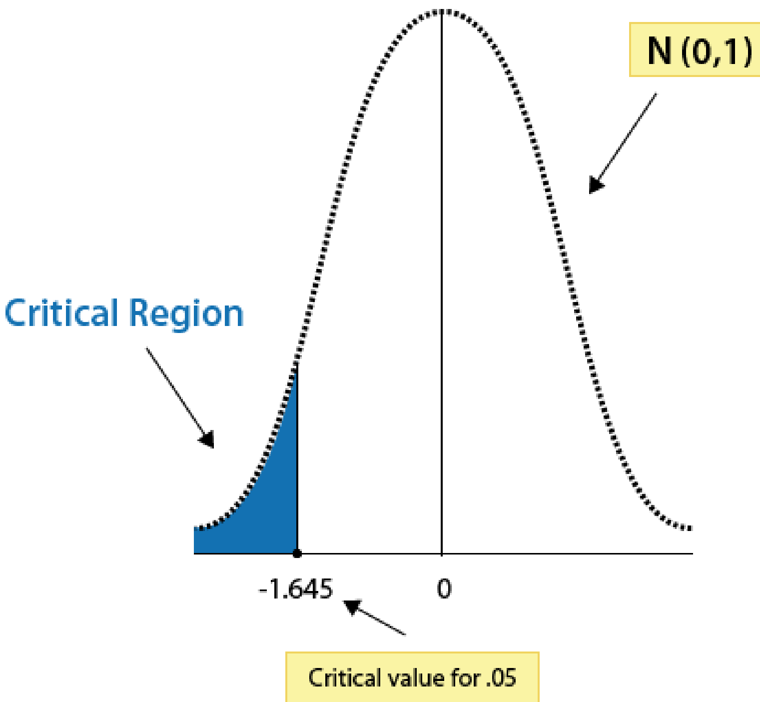
On the other hand, a z-test statistic of -2.5 would fall within the critical region on the left hand side of the distribution; therefore, we know that the p-value would be less than 0.05. In this instance, we would reject the null hypothesis at a less than 0.05 level (or p-value < 0.05). Furthermore, it is possible to figure out the exact p-value for the z-test statistic of -2.5, by using the Normal Table, which is 0.0062.



Less Than

The same logic applies to one-tailed z-tests. For the one-tailed "less than" z-test, the critical value for a 0.05 significance level is -1.645 (note: since the p-value for -1.64 is 0.0505 and the p-value for -1.65 is 0.0495, the critical value for 0.0500 would be between the two z scores or -1.645).

Critical Region for a "Less Than" One-Tailed Test

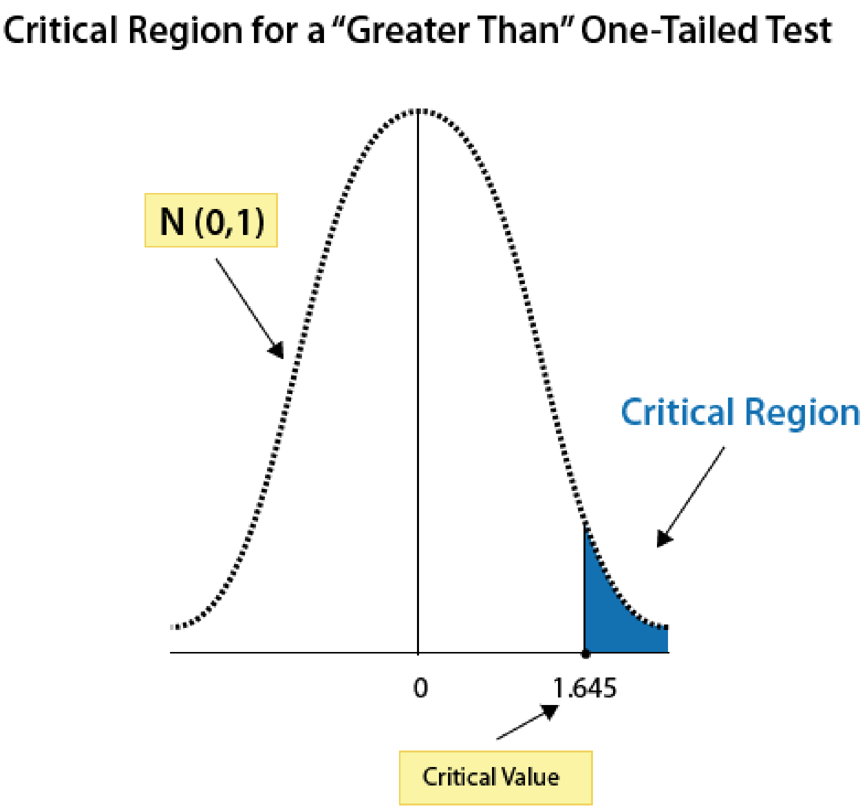


With a “less than” one-tailed z test, any z-test statistic that is **less** than -1.645, would fall in the critical region and therefore, would have a p-value less than 0.05. For instance, -2.5 would be less than -1.645 and would fall in the critical region. Thus, it would have a p-value less than 0.05 and the null hypothesis would be rejected.

Any z-test statistic that is **larger** than -1.645 would have a probability level of greater than 0.05 (or p-value > 0.05). For instance, -1.5 would be greater than -1.645 and, therefore, would not fall in the critical region. Thus, it would have a p-value greater than 0.05 and the null hypothesis would not be rejected.

Greater Than

For the one-tailed “greater than” z-test, the critical value for a 0.05 significance level is 1.645.



Thus, with a “greater than” one-tailed z test, any z-test statistic **larger** than 1.645, would fall in the critical region and therefore, would have a p-value less than 0.05. For instance, 2.5 would be greater than 1.645 and would fall in the critical region. Thus, it would have a p-value less than 0.05 (or p-value < 0.05) and the null hypothesis would be rejected .

Any z-test statistic that **less** than 1.645 would have a probability level of greater than 0.05 (or p-value > 0.05). For instance, 1.5 would be less than 1.645 and, therefore, would not fall in the critical region. Thus, it would have a p-value greater than 0.05 and the null hypothesis would not be rejected.

Wrap-up:

The critical value method uses two concepts: 1) the critical value and 2) the critical region. The critical value is used to determine the critical region and is based on two things: 1) the significance level of the test (either 0.05 or 0.01) **AND** the direction of the test (ex, left-tailed, right-tailed, or two-tailed).

When z-test statistic falls in the critical region, it is far enough from the mean that it is significantly different from the mean. Therefore, in this instance, the null hypothesis would be rejected at the significance level used to determine the critical region (either 0.05 or 0.01). Furthermore, the actual p-value can be determined by using the Normal Table.

When the z-test statistic does not fall in the critical region, it indicates that it is not far enough from the mean to be significantly different from the mean. In this instance, the null hypothesis would not be rejected.

Comment:

The critical value method has been traditionally used for hypothesis testing (note: there are different critical values and tables for t-tests, ANOVAs, and Chi Square tests). The emphasis now, however, is on the use of exact p-values, which are obtained through the use of statistical software packages.

Scenario: Medical Students

An Association of American Medical Colleges report stated that, in 2011-2012, 47.0% of all matriculated medical school students in the United States were female. It is thought that, perhaps, the number of matriculated female students may be lower in rural areas. Researchers collect a random sample of 1000 2011-2012 matriculated medical students from rural areas and find that 435 of the students were female.

Were there proportionately fewer females matriculated medical students in rural areas as compared to the national proportion of female matriculated medical students?

Did I Get It

1/1 point (graded)

What are the null and alternative hypotheses?

- ○ H_0: p = 0.435; H_a: p ≠ 0.435
- ○ H_0: p = 0.435; H_a: p > 0.435
- ○ H_0: p = 0.47; H_a: p ≠ 0.47
- ● H_0: p = 0.47; H_a: p < 0.47 ✔

Answer

Correct:
The population proportion of 0.47 is being assessed and the assessment is being used to determine whether there were proportionately fewer female matriculated students in rural areas as compared to the national proportion of female matriculated medical students.

Submit

Did I Get It

1/1 point (graded)
Using a 0.05 significance level, what critical value(s) should be used to determine whether there are proportionately fewer matriculated female medical students in rural areas as compared to the national proportion of matriculated female students.

- -1.96
- +1.96
- +1.645
- -1.645 ✔

Answer
Correct:
The alternative hypothesis would be "less than," so it would be a one-tailed test, with a negative critical value; thus, -1.645 would put 0.05 (or 5%) in the left side of the distribution.

Submit

Did I Get It

1/1 point (graded)
What is the z-test statistic?

- -2.218 ✔
- +2.218
- -140.51

- -2.232

Answer
Correct:
The z-test statistic is calculated by using the formula: (value-mean)/(standard deviation) or (.435-.47)/(.1578) = -2.218.

Submit

Did I Get It

1/1 point (graded)
Is the proportion of female students in the sample from the rural areas significantly lower than the population proportion at a 0.05 significance level?

- Yes, because the z-test statistic is more than the critical value for a .05 significance level.
- Yes, because the z-test statistic is less than the critical value for a .05 significance level. ✔
- No, because the z-test statistic is more than the critical value for a 0.05 significance level.
- No, because the z-test statistic is less than the critical value for a 0.05 significance level.

Answer
Correct:
Since the z-test statistic (-2.218) is less than the critical value for a one-tailed test at the .05 significance level (-1.645), it falls in the critical region. Thus, we can say that the probability of this z-test statistic is less than 0.05. Thus, the null hypothesis is rejected, and we can say that there are proportionately fewer female matriculated female medical students in the rural areas as compared to the national proportion.

Submit

Scenario: Pet Ownership

A Gallop poll in conducted in December 2006 found that 60% of Americans owned pets. A 2011 survey of 1,500 adults found that 945 had pets.

Do the results this survey show that there has been a change in the proportion of Americans, who own pets, between 2006 and 2011?

Did I Get It

1/1 point (graded)

What are the null and alternative hypotheses?

- ○ H_0: $p = 0.63$; H_a: $p \neq 0.63$
- ○ H_0: $p = 0.63$; H_a: $p > 0.63$
- ● H_0: $p = 0.60$; H_a: $p \neq 0.60$ ✓
- ○ H_0: $p = 0.60$; H_a: $p < 0.60$

Answer

Correct:

The population proportion of 0.60 is being assessed and the assessment is being used to determine whether there was a change, in either direction, in proportion of Americans, who own pets, between 2006 and 2011.

Submit

Did I Get It

1/1 point (graded)

Using a 0.05 significance level, what critical value(s) should be used to determine whether there was a change, in either direction, in proportion of Americans, who own pets, between 2006 and 2011?

- ○ -1.645
- ○ +1.645
- ● -1.96 and +1.96 ✓
- ○ Unable to determine from information provided

Answer

Correct:
The alternative hypothesis would be “not equal to” so it would be a two-tailed test, with both a negative and positive critical value; thus, -1.96 and +1.96 would put 0.025 (or 2.5%) in both the left and right sides of the distribution.

Submit

Did I Get It

1/1 point (graded)
What is the z-test statistic?

- -2.37
- 2.37 ✔
- 125
- 2.41

Answer
Correct:
The z-test statistic is calculated by using the formula: (value-mean)/(standard deviation) or (0.63 - 0.60)/(0.012649) = 2.37.

Submit

Did I Get It

1/1 point (graded)
Do the results this survey show that there has been a change in the proportion of Americans, who own pets, between 2006 and 2011 at a 0.05 significance level?

- Yes, because the z-test statistic is more than the critical value of 1.96 ✔
- Yes, because the z-test statistic is less than the critical value of 1.96
- No, because the z-test statistic is more than the critical value of 1.96.
- No, because the z-test statistic is less than the critical value for 1.96.

Answer

Correct:

Since the z-test statistic (2.37) is more than the critical value of +1.96 for a two-tailed test at the 0.05 significance level (1.96), it falls in the critical region. Thus, we can say that the probability of this z-test statistic is less than 0.05. The null hypothesis is rejected, and we can say that there are proportionately more Americans, who own pets in 2011, than 2006.

Submit