Sample Spaces

Learning Objective: Determine the sample space of a given random experiment.

As we saw in the previous section, probability questions arise when we are faced with a situation that involves uncertainty. Such a situation is called a **random experiment**, an experiment that produces an outcome that cannot be predicted in advance (hence the uncertainty).

Here are a few examples of random experiments:

1. Toss a coin once and record whether you get heads (H) or tails (T). The possible outcomes that this random experiment can produce are: {H, T}.

2. Toss a coin twice. The possible outcomes that this random experiment can produce are: {HH, HT, TH, TT}.

3. Toss a coin 3 times. The possible outcomes in this case are: {HHH, THH, HTH, HHT, HTT, THT, TTH, TTT}.

4. Toss a coin until you get the first tails (T). When we conduct this experiment, one possible outcome is that we get T in the first toss and we are done. Another possible outcome is that we get H in the first toss, toss a second time, get T and be done. We might need three tosses until we get the first T, etc. The possible outcomes of this random experiment are therefore: {T, HT, HHT, HHHT, ...}. (Note that in this example the list of possible outcomes is not finite as in examples 1-3. This is not an important distinction at this point, just a noteworthy observation.)

5. Choose a person at random and check his or her blood type. In this random experiment the possible outcomes are the four blood types: {A, B, AB, O}.

6. There are two job openings for a staff position at a certain college, and 4 equally qualified candidates for the job (Ann, Beth, Jim and Dan). For fairness, the human resources department decides to choose two of the four candidates at random. The possible outcomes of this random experiment are all possible pairs of candidates: { (Ann, Beth), (Ann, Jim), (Ann, Dan), (Beth, Jim), (Beth, Dan), (Jim, Dan) }.

Comment: Does Order Matter?

Note that when a coin is tossed twice, as in example 2, the possible outcome HT (indicating that the first toss was H and the second T) is NOT the same as the outcome TH (indicating that T occurred first and then H), and therefore both outcomes were listed separately. This is an example of a situation when order does matter. However, order does not always matter. Example 6 is a case in which order does not matter. The outcome (Ann, Beth) indicates that Ann and Beth are the two randomly chosen to get the jobs. Whether Ann appears first or Beth does is irrelevant in this case, and therefore (Beth, Ann) was **not** listed as a separate outcome.

There is really no rule that dictates when order matters and when it doesn't. It is sometimes clear from the way the random experiment is defined. For example, suppose I were to change example 6 slightly:

There are two job openings for similar staff positions at a certain college: one in the Registrar's Office, and one in the Office of Admissions. The Human Resources Department has identified four equally qualified candidates for the jobs (Ann, Beth, Jim and Dan), and for fairness decides to choose two of the four candidates at random. The first chosen will fill the position in the Registrar's Office, and the second will fill the position in the Office of Admissions.

Now order **is** relevant—the two outcomes (Ann, Beth) and (Beth, Ann) are not the same in this scenario. The first outcome indicates that Ann got the position at the Registrar's Office and Beth got the position at the Office of Admissions, while the second outcome indicates the reverse. In this case, therefore, all the possible outcomes are:

{ (Ann, Beth), (Beth, Ann), (Ann, Jim), (Jim, Ann), (Ann, Dan), (Dan, Ann),

(Beth, Jim), (Jim, Beth), (Beth, Dan), (Dan, Beth), (Jim, Dan), (Dan, Jim) }

Each random experiment has a set of possible outcomes, and there is **uncertainty** as to which of the outcomes we are actually going to get once the experiment is conducted. This list of possible outcomes is called **the sample space** of the random experiment, and is denoted by the (capital) letter **S.**

Going back to the 6 examples above, we can write:

Example 1: **S** = {H, T}

Example 2: **S** = {HH, HT, TH, TT}

Example 3: **S** = {HHH, THH, HTH, HHT, HTT, THT, TTH, TTT}

Example 4: **S** = {T, HT, HHT, HHHT, ...}

Example 5: **S** = {A, B, AB, O}

Example 6: **S** = { (Ann, Beth), (Ann, Jim), (Ann, Dan), (Beth, Jim), (Beth, Dan), (Jim, Dan) }.

Learn By Doing

1/1 point (graded)

In each of the following situations, choose the correct sample space (S) for the random experiment that is described.

A pair of dice is rolled, and the sum of the dots on the two faces that come up is recorded:

- S = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12}
- S = {2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12} ✔
- S = {2, 4, 6, 8, 10, 12}
- S = {1, 3, 5, 7, 9, 11}

Answer

Correct:

When rolling two dice, the possible outcomes are: (1,1) (1,2) (1,3) (1,4) (1,5) (1,6) (2,1) (2,2) ... (6,6). Thus, the sums can be 2-12.

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Learn By Doing

1/1 point (graded)

A coin is tossed three times, or until the first "head" appears. (whichever occurs first):

- S = {H, T, HT, TH, HH, TT, THH, HTH, HHT, HTT, THT, TTH, HHH, TTT}
- S = {THH, HTH, HHT, HTT, THT, TTH, HHH, TTT}
- S = {H, TH, TTH}

S = {H, TH, TTH, TTT} ✔

Answer
Correct:
We stop the experiment when we get the first "head" (H, TH, TTH), or after three tosses, even if we didn't get any "heads" (TTT).

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Did I Get This

1/1 point (graded)
A couple decides to have children until they have one boy and one girl, but they will not have more than three children. Choose the correct sample space for this random experiment.

- S = {GGG, GGB, GBG, GBB, BBB, BBG, BGB, BGG}
- S = {GGG, GGB, BBB, BGG}
- S = {GB, BG, BBG, GGB}
- S = {GB, BG, BBG, GGB, BBB, GGG} ✔

Answer
Correct:
Each outcome has one boy and one girl, except if the couple has three children of the same gender (in which case they stop having children because they originally decided to have no more than 3.)

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