Rules for Means and Variances of Random Variables: Linear Transformation

Learning Objective: Apply the rules of means and variances to find the mean and variance of a linear transformation of a random variable and the sum of two independent random variables.

The four observations we made on the previous page help illustrate a general rule for how random variables transform if we add, subtract and/or multiply by a constant.

Rules for a + bX (Linear Transformation of One Random Variable)

If X is a random variable with the mean μ_X and a variance of σ_X^2, then the new random variable **a + bX** has a mean and variance (respectively) of:

$$\mu_{a+bX} = a + b\mu_X$$

$$\sigma^2_{a+bX} = b^2\sigma^2_X$$

Comment

If we take a random variable's distribution and shift it over "a" units, and stretch or shrink its spread by "b" (stretch if b is greater than 1, shrink if b is less than 1), then the mean is shifted and the distribution is stretched or shrunk accordingly. For instance, if we multiply a random variable by 6 and add 3, then the mean is also transformed. The mean is also multiplied by 6 and 3 is added. Shifting by "a," however, has no effect on the variance (or standard deviation) of a random variable, because the spread would not be changed. On the other hand, stretching or shrinking the distribution of a random variable entails stretching or shrinking its spread accordingly. Doubling a random variable's values produces a new random variable whose variance is four times the original variance, but the standard deviation is just double the original standard deviation, as we might expect.

Example: Shifting and Stretching

Recall that X is the number of defective parts per hour in Xavier's production line, and in the previous section we calculated that:

$\mu_x = 1.8$ and that $\sigma_X = 1.21$.

We are interested in a new random variable, "50 + 5X," which represents the hourly cost of operation for Xavier's production line. Note that 50 + 5X is of the form "a + bX" (where a = 50 and b = 5), so in order to find the mean and standard deviation of this new random variable, we can use the rules above:

$\mu_{50+5X} = 50 + 5\mu_x = 50 + 5(1.8) = 59$

$\sigma^2_{50+5X} = 5^2\sigma^2_X = 25(1.46) = 36.5$

and therefore: $\sigma_{50+5X} = \sqrt{36.5} = 6.04$

So, we can conclude that the hourly costs for Xavier's production line average \$59, and typically the cost is about \$6 away from that average.

Scenario: Bridge Toll

The number of people in a car that crosses a certain bridge is a random variable X having a mean value of 2.7 and variance of 1.2. The toll on the bridge is \$3 per car plus \$0.50 (50 cents) per person in the car.

Did I Get This

1/1 point (graded)

An expression for the amount of money that is collected from a car that crosses the bridge is:

- ○ 0.50 + 3X
- ◉ 3 + 0.50X ✔

- 3.50X
- 3.50

Answer
Correct:
There is a $3 charge per car (regardless of the number of people in the car), plus $.50 for each person in the car.

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Did I Get This

1/1 point (graded)
Given your answer to the previous question, and using the first set of rules for means and variances, what is the mean and standard deviation of the amount of money that is collected from a car that crosses the bridge?

- $mean = \$4.35, standard\ deviation = \$\sqrt{3.60}$
- $mean = \$1.35, standard\ deviation = \$\sqrt{.30}$
- $mean = \$9.45, standard\ deviation = \$\sqrt{.30}$
- $mean = \$4.35, standard\ deviation = \$\sqrt{.30}$ ✔
- $mean = \$4.35, standard\ deviation = \$\sqrt{.60}$

Answer
Correct:
Indeed, $\mu_{3+50X} = 3 + 0.50 * \mu_X = 3 + 0.50 * 2.7 = 4.35$, and $\sigma^2_{3+0.50X} = 0.50^2 * \sigma^2_X = 0.25 * 1.2 = 0.30$. Therefore, the standard deviation is sqrt(0.30).

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