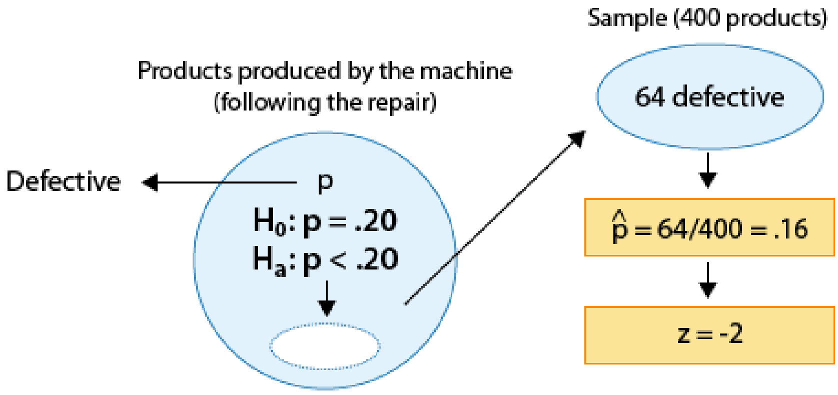
Hypothesis Testing for the Population Proportion p: Interpreting the p-value

Learning Objective: Carry out hypothesis testing for the population proportion and mean (when appropriate), and draw conclusions in context.

Example: 1



The p-value in this case is:

* The probability of observing a test statistic as small as -2 or smaller, assuming that H_o is true.

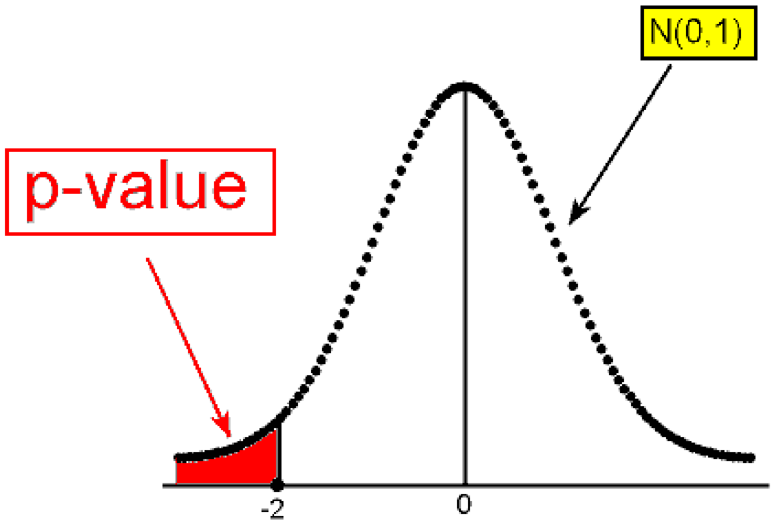
OR (recalling what the test statistic actually means in this case),

* The probability of observing a sample proportion that is 2 standard deviations or more below $p_0 = 0.20$, assuming that p_0 is the true population proportion.

OR, more specifically,

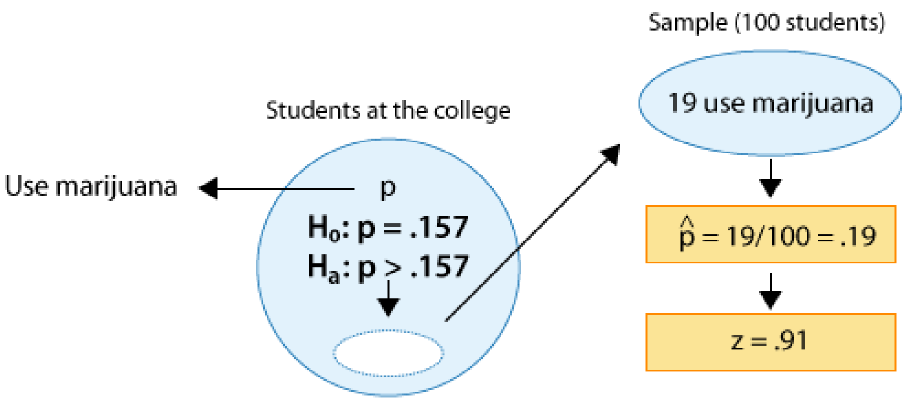
* The probability of observing a sample proportion of 9.16 or lower in a random sample of size 400, when the true population proportion is $p_0 = 0.20$.

In either case, the p-value is found as shown in the following figure:



To find $P(Z \le -2)$ we can either use a table or software. Eventually, after we understand the details, we will use software to run the test for us and the output will give us all the information we need. The p-value that the statistical software provides for this specific example is 0.023. The p-value tells me that it is pretty unlikely (probability of 0.023) to get data like those observed (test statistic of -2 or less) assuming that H_o is true.

Example: 2



The p-value in this case is:

* The probability of observing a test statistic as large as 0.91 or larger, assuming that H_o is true.

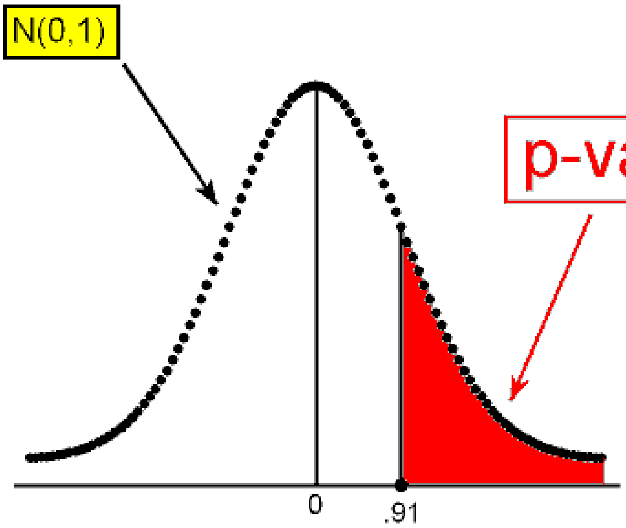
OR (recalling what the test statistic actually means in this case),

* The probability of observing a sample proportion that is 0.91 standard deviations or more above $p_0 = 0.157$, assuming that p_0 is the true population proportion.

OR, more specifically,

* The probability of observing a sample proportion of 0.19 or higher in a random sample of size 100, when the true population proportion is $p_0 = 0.157$.

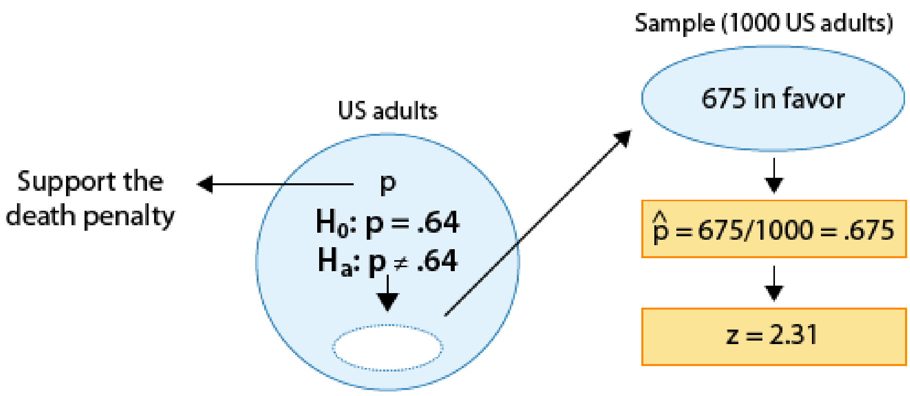
In either case, the p-value is found as shown in the following figure:



Again, at this point we can either use a table or software to find that the p-value is 0.182.

The p-value tells us that it is not very surprising (probability of 0.182) to get data like those observed (which yield a test statistic of 0.91 or higher) assuming that the null hypothesis is true.

Example: 3



The p-value in this case is:

* The probability of observing a test statistic as large as 2.31 (or larger) or as small as -2.31 (or smaller), assuming that H_o is true.

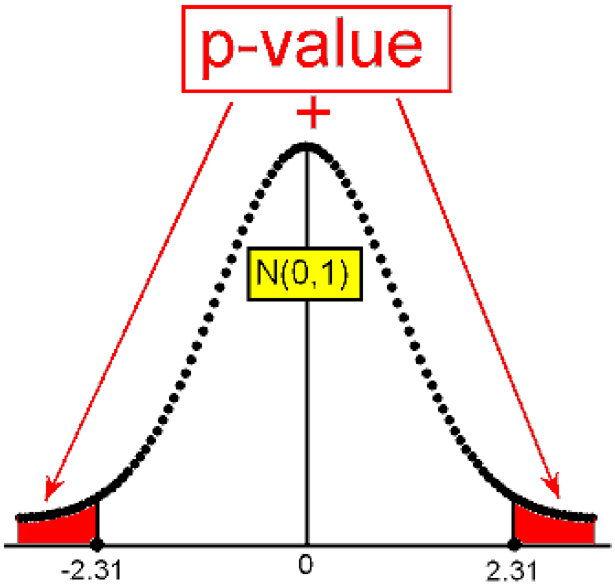
OR (recalling what the test statistic actually means in this case),

* The probability of observing a sample proportion that is 2.31 standard deviations or more away from $p_0 = 0.64$, assuming that p_0 is the true population proportion.

OR, more specifically,

* The probability of observing a sample proportion as different as 0.675 is from 0.64, or even more different (i.e. as high as 0.675 or higher or as low as 0.605 or lower) in a random sample of size 1,000, when the true population proportion is $p_0 = 0.64$.

In either case, the p-value is found as shown in the following figure:



Again, at this point we can either use a table or software to find that the p-value is 0.021.

The p-value tells us that it is pretty unlikely (probability of 0.021) to get data like those observed (test statistic as high as 2.31 or higher or as low as -2.31 or lower) assuming that H_o is true.

Comment

We've just seen that finding p-values involves probability calculations about the value of the test statistic assuming that H_o is true. In this case, when H_o is true, the values of the test statistic follow a standard normal distribution (i.e., the sampling distribution of the test statistic when the null hypothesis is true is N(0,1)). Therefore, p-values correspond to areas (probabilities) under the standard normal curve.

Similarly, in **any test**, p-values are found using the sampling distribution of the test statistic when the null hypothesis is true (also known as the "null distribution" of the test statistic). In this case, it was relatively easy to argue that the null distribution of our test statistic is N(0,1). As we'll see, in other tests, other distributions come up (like the t-distribution and the F-distribution), which we will just mention briefly, and rely heavily on the output of our statistical package for obtaining the p-values.

We've just completed our discussion about the p-value, and how it is calculated both in general and more specifically for the z-test for the population proportion. Let's go back to the four-step process of hypothesis testing and see what we've covered and what still needs to be discussed.

The Four Steps in Hypothesis Testing

1. State the appropriate null and alternative hypotheses, H_o and H_a.

2. Obtain a random sample, collect relevant data, and **check whether the data meet the conditions under which the test can be used.** If the conditions are met, summarize the data using a test statistic.

3. Find the p-value of the test.

4. Based on the p-value, decide whether or not the results are significant, and **draw your conclusions in context.**

With respect to the z-test the population proportion:

Step 1: Completed

Step 2: Completed

Step 3: Completed

Step 4: This is what we will work on next.

Scenario: Financial Situation Rating

In 2007, a Gallup poll estimated that 45% of U.S. adults rated their financial situation as “good.” We want to know if the proportion is smaller this year. We gather a random sample of 100 U.S. adults this year and find that 39 rate their financial situation as “good.” Use the output below to answer the following questions about the p-value.

Test and CI for One Proportion

Test of p = 0.45 vs p < 0.45

Sample	X	N	Sample p	95% Upper Bound	Z-Value	P-Value
1	39	100	0.390000	0.485600	-1.21	0.114

Learn By Doing

1/1 point (graded)

A p-value of 0.114 tells us that it is likely to get data like those observed assuming:

- (selected) Ho is true ✔
- Ha is true

Answer

Correct: Every hypothesis test is based on the assumption that H_o is true.

Submit

Learn By Doing

1/1 point (graded)

This p-value is the probability of observing a test statistic as small or smaller than __________ , assuming Ho is true.

-1.21 ✔

-1.21

Submit

Learn By Doing

1/1 point (graded)

The p-value is the probability of observing a sample proportion that is 1.21 standard deviations or more __________ $p_o = 0.45$, assuming that the true population proportion is 0.45.

above

below ✔

away from

Answer

Correct: The "<" in H_a tells us to look at sample proportions below the observed $\hat{p}$ from the data.

Submit

Learn By Doing

1/1 point (graded)

The p-value is the probability of observing a sample proportion of 0.39 or lower in a random sample of size 100, when the true population proportion is:

0.45 ✔

0.45

Answer

Correct: The true population proportion is the p_o in the null hypothesis.

Submit

Scenario: School Bond Measure

The trustees of a local school district commission a survey to determine voter opinions about a possible bond measure to fund school upgrades. In a poll of 293 of the district's 5,019 registered voters, 178 would support the bond measure. A hypothesis test was conducted using Minitab to determine if such a bond would pass with the required 55% of the vote.

Test and CI for One Proportion

Test of p = 0.55 vs p > 0.55

Sample	X	N	Sample p	95% Upper Bound	Z-Value	P-Value
1	178	293	0.607509	0.66342	1.98	0.024

Learn By Doing (1/1 point)

Interpret the meaning of the p-value as a probability statement that relates to this scenario.

Your Answer:

p-value is 0.024. That means there's a 2.4% probability of observing a test-statistic of 1.98 standard deviations above 0.55, assuming that the true population proportion is 0.55. That means it's unlikely to see 0.55 (Ho).

Our Answer:

Here are several ways you could describe the p-value: This p-value of 0.024 tells us that it is very unlikely to get data like those observed, assuming Ho is true. (This is the least precise description of the four given here.) The p-value is the probability of observing a test statistic as large as 1.98 or larger, assuming Ho is true. The p-value is the probability of observing a sample proportion that is 1.98 standard deviations or more above po = 0.55, assuming that the true population proportion is 0.55. The p-value is the probability of observing a sample proportion of 0.608 or higher in a random sample of size 293 when the true population proportion is 0.55. Check your response to make sure it includes the following elements: (a) The p-value is a probability. (b) The probability is based on the assumption that Ho is true. (c) A statement about the test statistic or sample proportion being higher than (since Ha contains "greater than") what we observed in the data .

Resubmit Hint Reset

Scenario: Zinc Supplements Reducing Colds

Do zinc supplements reduce a child's risk of catching a cold? A medical study reports a p-value of 0.03. Are the following interpretations of the p-value valid or invalid?

Learn By Doing

1/1 point (graded)

The p-value is the probability of getting results as extreme as or more extreme than the ones in this study if zinc is actually not effective.

- valid ✔
- not valid

Answer
Correct:
This has the elements of a good interpretation of p-value: 1) The p-value is a probability. 2) The probability is based on the assumption that H_o is true. 3) A statement about results (test statistics or sample proportions) being more extreme than observed in the data.

Submit

Learn By Doing

1/1 point (graded)
The p-value is the probability that the drug is not effective.

- valid
- not valid ✔

Answer
Correct:
We cannot make a probability statement about whether the drug is not effective. It either is or it isn't. We can only make probability statements about random events, such as the results from random samples.

Submit

Learn By Doing

1/1 point (graded)
The p-value is the probability that the drug is effective.

- valid
- not valid ✔

Answer

Correct:
We cannot make a probability statement about whether the drug is effective. It either is or it isn't. We can only make probability statements about random events, such as the results from random samples.

Submit