Rules for Means and Variances of Random Variables: Sum of Two Variables

Learning Objective: Apply the rules of means and variances to find the mean and variance of a linear transformation of a random variable and the sum of two independent random variables.

Besides taking a linear transformation of a random variable, another way to form a new random variable is to combine two or more existing random variables in some way, such as finding the sum of two random variables. Again, we'll start with a motivating example.

Example: Xavier's and Yves' Production Lines

Recall previous examples for the number X of defective parts coming out of Xavier's production line, and Y from Yves' line. Consider the total number of defective parts coming out of both production lines together. This is the **new random variable X + Y**. Since we know the means and standard deviations of X and of Y, is there a simple, quick way to figure out the mean and standard deviation of X + Y?

In general, the mean of the sum of random variables is the sum of the means. As long as the random variables are independent, the variance of the sum equals the sum of the variances. The formal rules are as follows:

Rules for X + Y (Sum of Two Random Variables)

Let X and Y be random variables with means μ_X and μ_Y and with variances σ^2_X and σ^2_Y. Then the new random variable X + Y has a mean of

$$\mu_{X+Y} = \mu_X + \mu_Y$$

and as long as the random variables are independent, the variance of X + Y is:

$$\sigma^2_{X+Y} = \sigma^2_X + \sigma^2_Y$$

What these two rules tell us is that if we take two random variables and add them, then the new mean is the sum of the original two means, and the new variance—not standard deviation—is the sum of the original two variances, as long as those variables are independent.

Comment

We've talked a lot about independent and dependent **events**, but not about what it means for two **random variables** to be independent or dependent. Basically, the same reasoning extends from events to random variables. Two random variables will be independent if knowing that one random variable takes any of its possible values has no effect on the probability that the other random variable takes a certain value.

While in the case of **events** we formalized the definition of independence using conditional probability, doing the same for random variables is beyond the scope of this course. Therefore, whenever we want to use the rule:

$\sigma^2_{X+Y} = \sigma^2_X + \sigma^2_Y$, we will assume that X and Y are independent, or it will be clear from the context of the problem.

Example: Combined Mean and Standard Deviation

We want to find the mean and standard deviation of X + Y, the total number of defective products coming out of both production lines in an hour. In the previous section, we found that:

$$\mu_X = 1.8$$

$$\sigma_X = 1.21$$

$$\mu_Y = 2.7$$

$$\sigma_Y = 0.85$$

Using the rules for X + Y, we find that:

$$\mu_{X+Y} = \mu_X + \mu_Y = 1.8 + 2.7 = 4.5$$

$$\sigma^2_{X+Y} = \sigma^2_X + \sigma^2_Y = 1.21^2 + 0.85^2 = 2.19$$

$$\sigma_{X+Y} = \sqrt{2.19} = 1.48$$

We can conclude, then, that the total number of defective parts coming out of both production lines in an hour is on average 4.5, and typically the combined number of defective products is about 1.48 away from that average.

Comment

It is important to remember that while variances are additive, standard deviations are not:

$$\sigma_X + \sigma_Y = 1.21 + 0.85 = 2.06 \neq \sigma_{X+Y} = 1.48$$

Scenario: Department Store Doors

A big department store has two entrance doors, one on 5th Ave. and the other on 6th Ave. The number of shoppers who enter the store through the 5th Ave. door in an hour, X, has mean of $\mu_X = 25$ and standard deviation $\sigma_X = 5$, and the number of shoppers who enter the store through the 6th Ave. door in an hour, Y, has a mean of $\mu_Y = 35$, and standard deviation of $\sigma_Y = 8$. Let the random variable T be the total number of shoppers who enter the store in an hour.

Did I Get This

1/1 point (graded)

What is μ_T, the mean of T?

- 30
- 43
- 10
- 60 ✔

Answer

Correct: Indeed, since T = X + Y, $\mu_T = \mu_X + \mu_Y = 25 + 35 = 60$.

Submit

Did I Get This

1/1 point (graded)

Assuming that shoppers enter the store through the two doors independently, what is σ_T, the standard deviation of T?

- 13
- 89
- sqrt(13), or the square root of 13
- sqrt(89), or the square root of 89 ✔

Answer

Correct:

Indeed, since X and Y are independent: $\sigma^2_T = \sigma^2_{X+Y} = \sigma^2_X + \sigma^2_Y = 25 + 64 = 89$. Therefore σ_T is the square root of 89.

Submit

Did I Get This

1/1 point (graded)

The number of computer monitors manufactured per day by CompScreens Inc. is a random variable X, with a mean $\mu_x = 120$ and a variance of $\sigma^2_x = 36$. The cost of manufacturing the monitors is $500 base cost plus $15 per monitor.

What is the mean total cost per day for CompScreens Inc. to manufacture computer monitors?

- $515
- $1040
- $1800
- $2300 ✔

Answer

Correct:
Let T be the total cost of manufacturing monitors for a day. T is composed of two parts: the fixed cost of \$500 (regardless of the number of monitors) and \$15 for every monitor made. The total cost can be written as T = 500 + 15X. To find the mean of T you use the rule of means: $\mu_{(a+bX)} = a + b\ \mu_X = 500 + 15(120) = \$2300$.

Submit

Did I Get This

1/1 point (graded)

A national park has 2 entrance gates, one at the north end and a second gate at the south end of the park. Let X be the number of cars entering the North gate per hour and let Y be the number of cars entering the South gate per hour. Assume that X is random variable with mean $\mu_X = 23$ and standard deviation $\sigma_X = 3$. Assume Y is a random variable with mean $\mu_Y = 18$ and standard deviation $\sigma_Y = 4$. Assume that the two gates operate independently.

What is the mean of Z, the total number of cars entering the park each hour?

- 18
- 23
- 41 ✔
- 48

Answer

Correct: Since Z = X + Y, $\mu_Z = \mu_X + \mu_Y = 23 + 18 = 41$.

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