Course > Inference: Relationships C→C > Case C→C > Case C→C: Conditions and Chi-Square Test

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Case C→C: Conditions and Chi-Square Test

Learning Objective: In a given context, carry out the appropriate inferential method for comparing relationships and draw the appropriate conclusions.

Step 2: Checking the Conditions and Calculating the Test Statistic

Given our discussion on the previous page, it would be natural to present the test statistic, and then come back to the conditions that allow us to safely use the chi-square test, although in practice this is done the other way around.

The single number that summarizes the overall difference between observed and expected counts is the chi-square statistic χ^2 , which tells us in a standardized way how far what we observed (data) is from what would be expected if H_o were true.

Here it is:

$$\chi^2 = \sum_{all \quad cells} \frac{(Observed \ \ Count - ExpectedCount)^2}{Expected \quad Count}$$

Comment

As we expected, χ^2 is based on each of the differences: observed count - expected count (one such difference for each cell), but why is it squared? Why do we divide each square difference by the expected count? The reason we do that is so that the null distribution of χ^2 will have a known null distribution (under which p-values can be easily calculated). The details are really beyond the scope of this course, but we will just say that the null distribution of χ^2 is called chi-square (which is not very surprising given that the test is called the chi-square test), and like the t-distributions there are many chi-square distributions distinguished by the number of degrees of freedom associated with them.

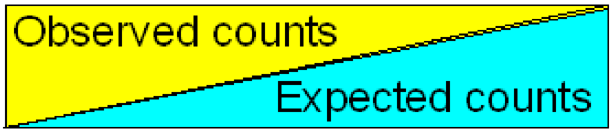
Conditions under Which the Chi-Square Test Can Safely Be Used

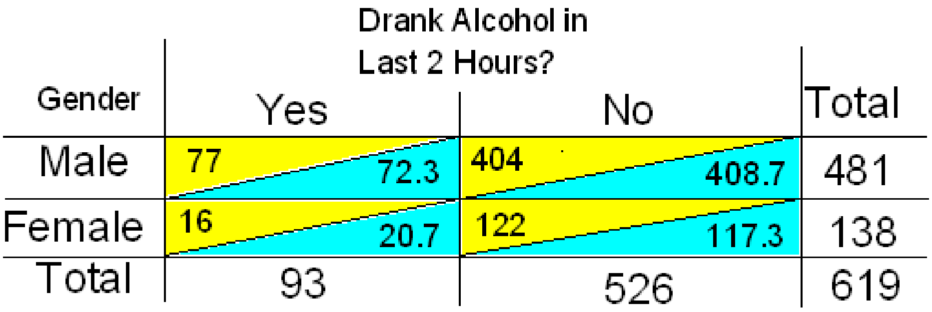
1. The sample should be random.

2. In general, the larger the sample, the more accurate and reliable the test results are. There are different versions of what the conditions are that will ensure reliable use of the test, all of which involve the expected counts. One version of the conditions says that all expected counts need to be greater than 1, and at least 80% of expected counts need to be greater than 5. A more conservative version requires that all expected counts are larger than 5.

Example

Here, again, are the observed and expected counts.





Gender	Drank Alcohol in Last 2 Hours? Yes	No	Total
Male	77 72.3	404 408.7	481
Female	16 20.7	122 117.3	138
Total	93	526	619

Checking the conditions:

1. The roadside survey is known to have been random.

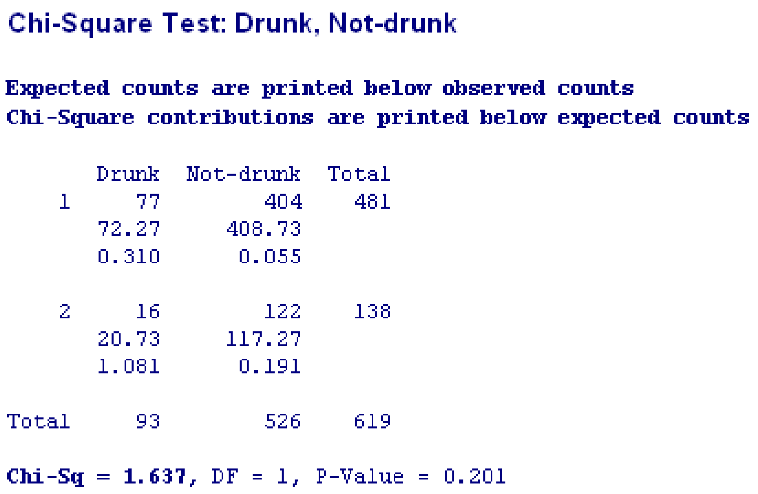
2. All the expected counts are above 5.

   We can therefore safely proceed with the chi-square test, and the chi-square test statistic is:

$$\frac{(77-72.3)^2}{72.3} + \frac{(404-408.7)^2}{408.7} + \frac{(16-20.7)^2}{20.7} + \frac{(122-117.3)^2}{117.3} = .306 + .054 + 1.067 + .188 = 1.62$$

Here is the Minitab output of the chi-square test (note: 1 = M, 2 = F). Look at it carefully. Can you identify the three numbers in each cell?

Actually, the output explains the content of each cell (explanation is highlighted).

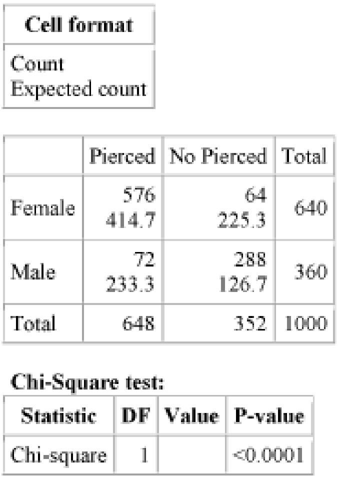


Chi-Square Test: Drunk, Not-drunk

```
Expected counts are printed below observed counts
Chi-Square contributions are printed below expected counts

       Drunk  Not-drunk  Total
    1     77        404    481
       72.27     408.73
       0.310      0.055

    2     16        122    138
       20.73     117.27
       1.081      0.191

Total     93        526    619

Chi-Sq = 1.637, DF = 1, P-Value = 0.201
```

Scenario: Ear Piercing and Gender

A study was done on the relationship between gender and piercing among high-school students. A sample of 1,000 students was chosen, and then classified according to both gender and whether or not they had either of their ears pierced. The following (edited) output is available:



Cell format
Count Expected count

	Pierced	No Pierced	Total
Female	576 414.7	64 225.3	640
Male	72 233.3	288 126.7	360
Total	648	352	1000

Chi-Square test:

Statistic	DF	Value	P-value
Chi-square	1		<0.0001

Did I Get This

1/1 point (graded)

What is the contribution of the first cell (pierced females) to the chi-square test statistic?

○ 45.16

● 62.72 ✓

○ 26.01

○ 40.14

○ 40.64

Answer
Correct:
Indeed, the contribution of the first cell is (observed count - expected count)^2 / expected count = (576 - 414.72)^2 / 414.72 = 62.72.

Submit

Did I Get This

1/1 point (graded)
What is the contribution of the fourth cell (no pierced males) to the chi-squared test statistic?

○ 1.27

○ 90.32

○ 26.01

○ 72.25

○ 73.9

● 205.27 ✓

Answer
Correct:
The contribution of the fourth cell (no-pierced males) to the chi-square statistic is: (Observed - Expected)^2/Expected = (288-126.72)^2 / 126.72 = 205.27

Submit

Did I Get This

1/1 point (graded)

What is the chi-square statistic in this case?

● 494.949 ✓

○ 1,000

○ 0.000

Answer

Correct:

Indeed, to find the test statistic, we need to add the contributions of each of the 4 cells, which in this case is: 62.72 + 115.462 + 111.502 + 205.265 = 494.949

Submit

Comment

Once the chi-square statistic has been calculated, we can get a feel for its size: is there a relatively large difference between what we observed and what the null hypothesis claims, or a relatively small one? It turns out that for a 2-by-2 case like ours, we are inclined to call the chi-square statistic "large" if it is larger than 3.84. Therefore, our test statistic is not large, indicating that the data are not different enough from the null hypothesis for us to reject it (we will also see that in the p-value not being small). For other cases (other than 2-by-2) there are different cut-offs for what is considered large, which are determined by the null distribution in that case. We are therefore going to rely only on the p-value to draw our conclusions. Even though we cannot really use the chi-square statistic, it was important to learn about it, since it encompasses the idea behind the test.

Scenario: Alcohol Problems Among 9/11 First Responders

The purpose of this activity is to continue to explore whether the risk of alcohol problems among New York firefighters and first responders is related to participation in the 911 rescue. In particular, in this activity, we will state the hypotheses that are being tested, learn how to carry out the chi-square test for independence, and check whether the conditions under which this test can be safely used are met.

Observed Data

	No risk for alchohol problems	Moderate to Servere risk for alcohol problems	Total
Participated in 911 rescue	793	309	**1102**
Did Not Participate in 911 rescue	441	110	**551**
Total	**1234**	**419**	**1653**

Learn By Doing

1/1 point (graded)

We are now ready to carry out the significance test, which will assess the significance of the relationship between the risk of alcohol problems and participation in the 9/11 rescue. Which of the following are the appropriate hypotheses for the chi-square test for independence in this case?

○ H_0: There is no relationship between the risk of alcohol problems and participation in the 9/11 rescue.

H_a: There is a significant relationship between the risk of alcohol problems and participation in the 9/11 rescue.

○ H_0: The risk of alcohol problems and participation in the 9/11 rescue are independent.

H_a: The risk of alcohol problems and participation in the 9/11 rescue are not independent.

● Both sets of hypotheses are correct. ✓

Answer

Correct:

Either format can be used to describe the null and alternative hypotheses for the chi-square test for independence. One way is to state that the null hypothesis can be no relationship between the two variables and the alternative hypothesis is that there is a significant relationship between the two variables. A second way is to state that the null hypothesis is that the two variables are independent and the alternative hypothesis is the two variables are not independent.

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The Work of First Responders in a Post 9/11 World

In this activity, we check whether the conditions under which the chi-square test can be safely used are met. In order to do this, you'll need to first launch the actual research report and read the last paragraph on page iii of the introduction (starting with the "The study was fully funded....").

Next we used statistics software to calculate the expected counts, since we do not have the raw data to construct a table.

	No risk for alchohol problems	**Moderate to Servere risk for alcohol problems**
Participated in 911 rescue	822.6667	279.3333
Did Not Participate in 911 rescue	411.3333	139.6667

Learn By Doing (1/1 point)

Are the conditions met under which the chi-square test can be safely used?

Your Answer:

Yes (because [i assumed it was] randomly sampled, and the expected values were all large).

Our Answer:

Both conditions are met. • According to the report, the sample was stratified using randomness. • As the output shows, all the expected counts are large.

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