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Hypothesis Testing for the Population Proportion p: Using a Test Statistic

Learning Objective: Carry out hypothesis testing for the population proportion and mean (when appropriate), and draw conclusions in context.

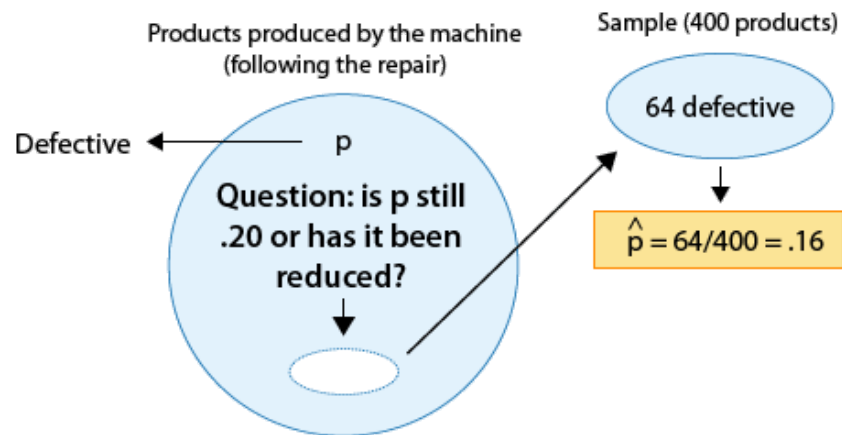## 2. Collecting and Summarizing the Data (Using a Test Statistic)

After the hypotheses have been stated, the next step is to obtain a **sample** (on which the inference will be based), **collect relevant data**, and **summarize** them.

It is extremely important that our sample is representative of the population about which we want to draw conclusions. This is ensured when the sample is chosen at **random.** Beyond the practical issue of ensuring representativeness, choosing a random sample has theoretical importance that we will mention later.
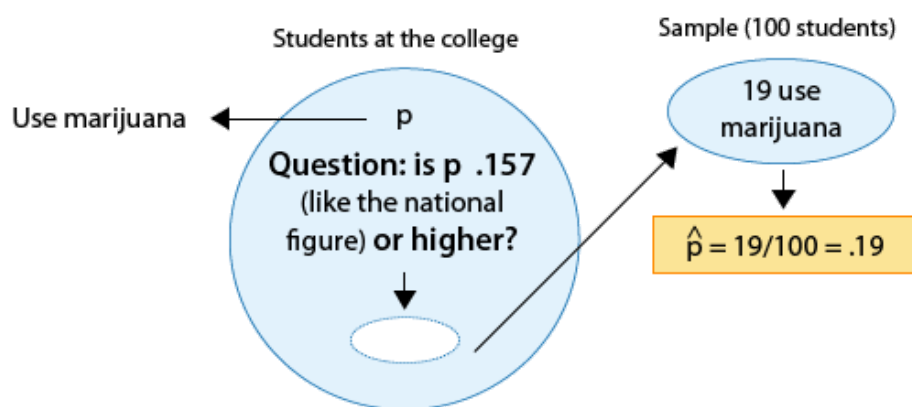
In the case of hypothesis testing for the population proportion (p), we will collect data on the relevant categorical variable from the individuals in the sample and start by calculating the sample proportion, $\hat{p}$ (the natural quantity to calculate when the parameter of interest is p).

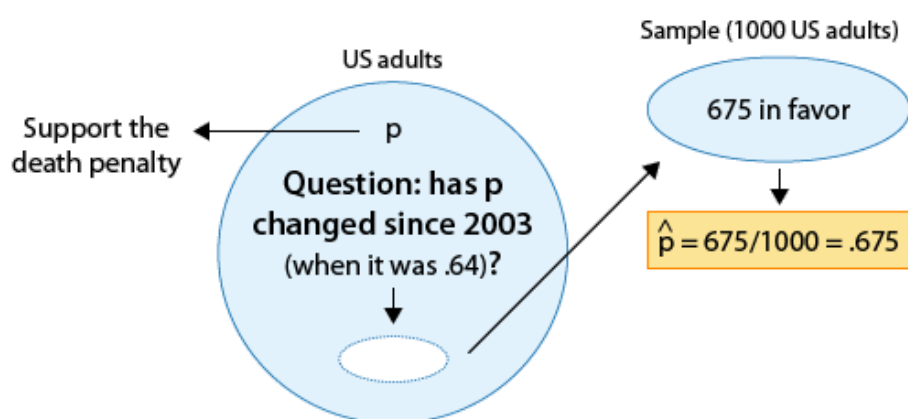Let's go back to our three examples and add this step to our figures.

Example: 1

Products produced by the machine
(following the repair)

Sample (400 products)

Defective

p

Question: is p still
.20 or has it been
reduced?

64 defective

$\hat{p} = 64/400 = .16$

Example: 2

Students at the college

Sample (100 students)

Use marijuana

p

Question: is p .157
(like the national
figure) or higher?

19 use
marijuana

$\hat{p} = 19/100 = .19$

Example: 3

US adults

Sample (1000 US adults)

Support the
death penalty

p

Question: has p
changed since 2003
(when it was .64)?

675 in favor

$\hat{p} = 675/1000 = .675$

As we mentioned earlier without going into details, when we summarize the data in hypothesis testing, we go a step beyond calculating the sample statistic and summarize the data with a **test statistic**. Every test has a test statistic, which to some degree captures the essence of the test. In fact, the p-

value, which so far we have looked upon as "the king" (in the sense that everything is determined by it), is actually determined by (or derived from) the test statistic. We will now gradually introduce the test statistic.

The test statistic is **a measure** of how far the sample proportion $\hat{p}$ is from the null value p_0, the value that the null hypothesis claims is the value of p. In other words, since $\hat{p}$ is what the data estimates p to be, the test statistic can be viewed as a measure of the "distance" between what the data tells us about p and what the null hypothesis claims p to be.

Let's use our examples to understand this:

Example: 1

The parameter of interest is p, the proportion of defective products following the repair.

The data estimate p to be $\hat{p} = 0.16$

The null hypothesis claims that p = 0.20

The data are therefore 0.04 (or 4 percentage points) below the null hypothesis with respect to what they each tell us about p.

It is hard to evaluate whether this difference of 4% in defective products is enough evidence to say that the repair was effective, but clearly, the larger the difference, the more evidence it is against the null hypothesis. So if, for example, our sample proportion of defective products had been, say, 0.10 instead of 0.16, then I think you would all agree that cutting the proportion of defective products in half (from 20% to 10%) would be extremely strong evidence that the repair was effective.

Example: 2

The parameter of interest is p, the proportion of students in a college who use marijuana.

The data estimate p to be $\hat{p} = 0.19$.

The null hypothesis claims that p = 0.157

The data are therefore 0.033 (or 3.3 percentage points) above the null hypothesis with respect to what they each tell us about p.

Example: 3

The parameter of interest is p, the proportion of U.S. adults who support the death penalty for convicted murderers.

The data estimate p to be $\hat{p} = 0.675$

The null hypothesis claims that p = 0.64.

There is a difference of 0.035 (3.5 percentage points) between the data and the null hypothesis with respect to what they each tell us about p.

There is a problem with just looking at the difference between the sample proportion $\hat{p}$ and the null value p_o.

Examples 2 and 3 illustrate this problem very well.

In example 2 we have a difference of 3.3 percentage points between the data and the null hypothesis, which is approximately the same as the difference in example 3 of 3.5 percentage points. However, the difference in example 3 of 3.5 percentage points is based on a **sample of size of 1,000** and therefore it is much **more impressive** than the difference of 3.3 percentage points in example 2, which was obtained from a sample of size of only 100.

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The parameter of interest is p, the proportion of community college students who work full-time. The null hypothesis claims that p = 0.45. Which of the following give the most convincing evidence against H_o?

○   A sample of 1,000 community college students in which 605 report working full-time.

○   A sample of 1,000 community college students in which 650 report working full-time.

◉   A sample of 1,000 community college students in which 700 report working full-time. ✔

Answer
Correct:
For a given sample size, sample proportions further from p_o are stronger evidence against H_o. Here the sample proportion is 0.70 which is the furthest from 0.45.

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1/1 point (graded)

The parameter of interest is p, the proportion of college students who report that computer gaming is an obstacle to their success in college. The null hypothesis claims that p = 0.30. Which of the following give the most convincing evidence against H_o?

- ○   A sample of 10 students in which 4 report that computer gaming an obstacle to their success in college.

- ○   A sample of 100 students in which 40 report that computer gaming an obstacle to their success in college.

- ●   A sample of 1,000 students in which 400 report that computer gaming an obstacle to their success in college. ✔

- ○   All of these are equally strong evidence against Ho since $\hat{p}$ = 0.40 for each sample.

Answer

Correct:

While the sample proportion is 0.40 for each situation, this scenario gives the most accurate picture of the true population proportion because it has the largest sample size.

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