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Confidence Intervals for the Population Mean: Understanding the General Structure

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Confidence Intervals for the Population Mean: Understanding the General Structure

Learning Objective: Explain what a confidence interval represents and determine how changes in sample size and confidence level affect the precision of the confidence interval.

Learning Objective: Find confidence intervals for the population mean and the population proportion (when certain conditions are met), and perform sample size calculations.

So far, we've developed the confidence interval for the population mean from scratch, based on results from probability, and discussed the trade-off between the level of confidence and the precision of the interval. The price you pay for a higher level of confidence is a lower level of precision of the interval (i.e., a wider interval).

Is there a way to bypass this trade-off? In other words, is there a way to increase the precision of the interval (i.e., make it narrower) *without* compromising on the level of confidence? We will answer this question shortly, but first we need to get a deeper understanding of the different components of the confidence interval and its structure.

Understanding the General Structure of the Confidence Intervals

We explored the confidence interval for μ for different levels of confidence and found that, in general, it has the following form:

$$\bar{x} \pm z^* \cdot \frac{\sigma}{\sqrt{n}},$$

where z* is a general notation for the multiplier that depends on the level of confidence. As we discussed before:

For a 90% level of confidence, z* = 1.645

For a 95% level of confidence, z* = 2 (or 1.96 if you want to be really precise)

For a 99% level of confidence, z* = 2.576

To start our discussion about the structure of the confidence interval, let's denote the $z^* \cdot \frac{\sigma}{\sqrt{n}}$ formula by m.

The confidence interval, then, has the form: $\bar{x} \pm m$:

$$\boxed{\bar{X}} \pm \underbrace{\boxed{z^* \cdot \frac{\sigma}{\sqrt{n}}}}_{m}$$

$\bar{x}$ is the sample mean, the point estimator for the unknown population mean (μ).

**m** is called the *margin of error*, since it represents the maximum estimation error for a given level of confidence.

For example, for a 95% confidence interval, we are 95% sure that our estimate will not depart from the true population mean by more than m, the margin of error.

m is further made up of the product of two components:

z*, the confidence multiplier, and

$\frac{\sigma}{\sqrt{n}}$, which is the standard deviation of $\bar{X}$, the point estimator of μ.

Here is a summary of the different components of the confidence interval and its structure:
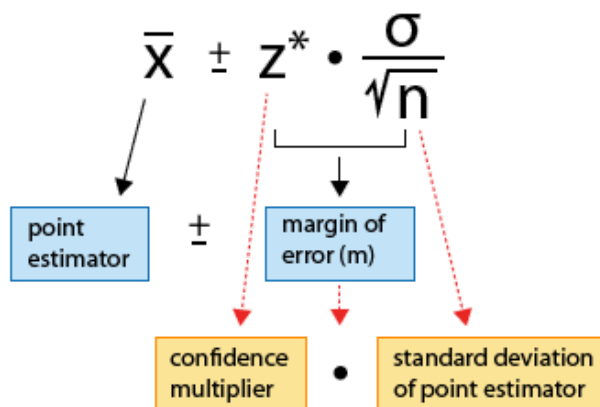
This structure:

$$estimate \pm margin\ of\ error,$$

where the margin of error is further composed of the product of a confidence multiplier and the standard deviation (or, as we'll see, the standard error) is the general structure of all confidence intervals that we will encounter in this course.

Obviously, even though each confidence interval has the same components, what these components actually are is different from confidence interval to confidence interval, depending on what unknown parameter the confidence interval aims to estimate.

Since the structure of the confidence interval is such that it has a margin of error on either side of the estimate, it is centered at the estimate (in our case, $\bar{x}$), and its width (or length) is exactly twice the margin of error:
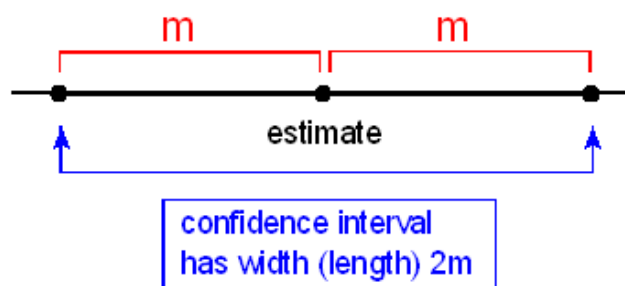
The margin of error, m, is therefore "in charge" of the width (or precision) of the confidence interval, and the estimate is in charge of its location (and has no effect on the width).

Did I Get This

1/1 point (graded)

Suppose the mean hours slept (per day) among a sample of college students was 7.1, and a 95% confidence interval for the population mean hours slept was constructed. Which one of the following could it be?

○ (7.1, 8.1)

◉ (6.9, 7.3) ✔

○ (7.0, 7.3)

Answer
Correct:
The confidence interval must be centered at the sample mean (7.1 in our case), and this is the only interval of the three that satisfies this requirement.

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Did I Get This

1/1 point (graded)
Suppose that based on a random sample, a 95% confidence interval for the mean hours slept (per day) among graduate students was found to be (6.5, 6.9). What is the margin of error of this confidence interval?

○ 6.7

○ 0.4

◉ 0.2 ✔

Answer
Correct:
Because the width of the confidence interval is 0.4 (6.9 - 6.5), the margin of error must be 0.4 / 2 = 0.2.

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Did I Get This

1/1 point (graded)
Suppose that based on a random sample, a 99% confidence interval for the mean hours slept (per day) among college freshmen was found to be (6.5, 7.3). What is the sample mean $\bar{x}$?

○ 0.8

○ 0.4

◉ 6.9 ✔

Answer

Correct: 6.9 is the center of the confidence interval [(6.5 + 7.3)/2] and is therefore the sample mean.

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Did I Get This

1/1 point (graded)

In a recent study, a random sample of "Intro Stats" students taking an introductory statistics course were asked about the number of hours they spend each week studying for their statistics class. Using the collected data, a 95% confidence interval and a 99% confidence interval for the population mean were constructed.

Which of the following could represent the 95% and 99% confidence intervals?

○ 95%: (5.4, 6.8); 99%: (5.5, 6.7)

○ 95%: (5.4, 6.8); 99%: (5.3, 7.0)

◉ 95%: (5.4, 6.8); 99%: (5.3, 6.9) ✔

○ 95%: (5.4, 6.8); 99%: (5.5, 6.6)

Answer

Correct:

The 99% confidence interval is wider than the 95% confidence interval, and since both confidence intervals were constructed using the same data, they are both centered at the same sample mean (6.1 in this case).

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