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Matrices and Graphs

Linear Algebra Methods 2

Problems

1. Let

$$\mathbf{A} = \begin{bmatrix} 7 & 5 \\ 6 & 14 \end{bmatrix}$$

   Calculate the eigenvalues and eigenvectors of $\mathbf{A}$.

2. Let

$$\mathbf{M} = \begin{bmatrix} 1 & 4 & 8 \\ -6 & 14 & 7 \\ 0 & 9 & -2 \end{bmatrix}$$

   Calculate the eigenvalues and eigenvectors of $\mathbf{M}$ using `SAGE`.

3. Suppose we want to draw a best fitting line through three points $(7,9)$, $(6,12)$ and $(4,4)$, so a linear function $y = c_0 + c_1 x$ should be given. Determine this function by means of the Method of Least Squares (LSM). Perform calculations by `SAGE` and study by yourself how to plot points and the line in the same coordinate system.

4. Suppose we want to draw a best 4th order polynomial line through 6 points $(7,9)$, $(6,12)$ and $(4,4)$, $(2,-1)$, $(-1,0)$, $(6,16)$. Determine this function by means of the Method of Least Squares (LSM). Perform calculations by `SAGE` and plot the points and the line in the same coordinate system.

5. Suppose points $(0,4),(1,8),(1,10),(2,5),(2,6)$, $(8,10)$ and $(6,10)$ are given. Approximate the points by a *real trigonometric polynomial of degree* 2. Plot the points and the graph of this function in the same coordinate system. Hint! The function must be periodic, and the data points represent the values of the function in one period.

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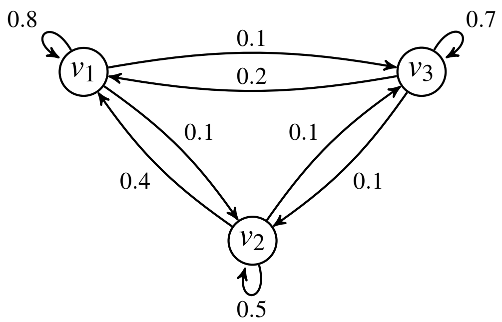
6. Consider there are three Web-pages, namely v_1, v_2 and v_3. The transitions between Web-pages are given as a diagram below. For example, if a person is at the page v_3 then he exits Web by probability 70%, transfers to page v_1 by probability 20%, and transfers to page v_2 by 10% probability. Create the adjacency matrix $\mathbf{A}$ and the stochastic matrix (transition matrix) $\mathbf{S}$. Which page is the most important? Sort the pages in the (decreasing) importance order.



Use `SAGE` in your calculations.

7. Consider the stochastic matrix $\mathbf{S}$ in the problem 6. Suppose $\mathbf{x}^{(0)} = (0.3, 0.1, 0.6)$. Show by calculations (with `SAGE`) that there is $k \in \mathbb{N}$ such that $\|\mathbf{x}^{(k+1)} - \mathbf{x}^{(k)}\| < \varepsilon$ as $\varepsilon = 0.001$ and $\mathbf{x}^{(k)} \approx \mathbf{x}^{\star}$, where $\mathbf{x}^{\star}$ denotes the steady state of the system (giving the importance of the Web-pages).