South-Eastern Finland
University of Applied Sciences
Jari Kortelainen

Matrices and Graphs
Exercises 1

Problems

1. Let $V = \mathbb{R}^2$ be a vectorspace and $S_1 = \{\mathbf{u}, \mathbf{v}\} \subseteq V$ such that $\mathbf{u} = (4,2)$ and $\mathbf{v} = (-2,3)$. Determine an example vector $\mathbf{x}$ such that it is a linear combination ($\neq \mathbf{0}$) of the vectors in S_1. Does S_1 form a linearly independent set?

2. Let $V = \mathbb{R}^2$ be a vectorspace and $S_1 = \{\mathbf{u}, \mathbf{v}\} \subseteq V$ such that $\mathbf{u} = (4,2)$ and $\mathbf{v} = (-2,3)$. Find scalars $s, k \in \mathbb{R}$ (if they exist) such that $\mathbf{x} = (6,7)$ is a linear combination of $\mathbf{u}$ and $\mathbf{v}$.

3. Let $\mathbf{u} = (4,-1,5,1)$ and $\mathbf{v} = (3,7,0,8)$ be vectors in $\mathbb{R}^4$. Give an example vector $\mathbf{x}$ such that it is a linear combination ($\neq \mathbf{0}$) of $\mathbf{u}$ and $\mathbf{v}$. Does $S_2 = \{\mathbf{u}, \mathbf{v}\}$ form a linearly independent set?

4. Let $\mathbf{u} = (4,-1,5,1)$ and $\mathbf{v} = (3,7,0,8)$ be vectors in $\mathbb{R}^4$. Find scalars $s, k \in \mathbb{R}$ (if they exist) such that the vector $\mathbf{x} = (6,-3,10,9)$ is a linear combination of $\mathbf{u}$ and $\mathbf{v}$.

5. Show that vectors $\mathbf{u}, \mathbf{v} \in \mathbb{R}^5$ does not form a linearly independent set $\{\mathbf{u}, \mathbf{v}\}$ ("are not linearly independent") if $\mathbf{u} = (-1,0,8,5,2)$, and $\mathbf{v} = (-6,0,48,30,12)$.

6. Let V be a vectorspace over a field $\mathscr{F}$. A subset $B \subseteq V$ is *a base for* V if B is a linearly independent set and for all $\mathbf{x} \in V$, $\mathbf{x}$ is a linear combination of vectors in B. Let S_1 and S_2 be given in the previous exercises. Does S_1 form a base for $\mathbb{R}^2$? Does S_2 form a base for $\mathbb{R}^4$?