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Matrices and Graphs

Exercises 3

Problems

1. Let $\mathbf{a} = (-1,3,4)$ and $\mathbf{b} = (5,2,8)$ be two vectors in the 3D real space. If $\overline{P}$ and $\overline{Q}$ are the corresponding vectors then determine the angle between $\overline{P}$ and $\overline{Q}$, the distance $d(\overline{P},\overline{Q})$ and orthogonal projection of $\mathbf{a}$ along $\mathbf{b}$, i.e. the vector $\mathbf{a_b}$.

2. Let $\mathbf{p}_0 \in \mathbb{R}^2$ be such that $\mathbf{p}_0 = (5,3)$ and the corresponding point in $\mathbb{R}^2$ is $P(5,3)$. We define a function $f\colon \mathbb{R}^2 \to \mathbb{R}^2$ such that $f(\mathbf{x}) = \mathbf{x} - \mathbf{p}_0$. Calculate$f((0,0))$, $f((-5,9))$ and $f(\mathbf{p}_0)$. Describe in your own words the action of this function.

3. A river flows from west to east by constant speed 1.5 km/h. You know that your swimming speed is 3 km/h in still water. To which direction you should head yourself in order to move exactly to the north?

4. Let $\mathbf{n} \neq \mathbf{0}$ be a vector in $\mathbb{R}^3$. Describe the set of those points x which satisfy the equation $\mathbf{n} \bullet \mathbf{x} = 0$, thus, describe the set $S = \{x \in \mathbb{R}^3 \mid \mathbf{n} \bullet \mathbf{x} = 0\}$, where $\mathbf{n}$ is some given non-zero vector.

5. Let us have two matrices
$$\mathbf{A} = \begin{bmatrix} 7 & 5 \\ 6 & 14 \end{bmatrix} \quad \textit{and} \quad \mathbf{B} = \begin{bmatrix} -2 & 6 \\ -7 & 9 \end{bmatrix}.$$
Calculate $\mathbf{A} + 2\mathbf{B}$ and $\mathbf{B} - 3\mathbf{A}$.

6. Suppose we have a cube in $\mathbb{R}^3$ such that the vertices are at the points $(0,0,0)$, $(0,0,1)$, $(0,1,0)$ $(0,1,1)$, $(1,0,0)$, $(1,0,1)$, $(1,1,0)$ and $(1,1,1)$. Write this knowledge as a matrix $\mathbf{C}$ such that the points are written as the columns ("vertical rows ") of $\mathbf{C}$. Study how to determine the *center of the cube*, thus one point, C_0. Determine a function $f\colon \mathbb{R}^{3\times 8} \to \mathbb{R}^{3\times 8}$ such that the cube is centered at the *origin of the coordinate system*, thus the center of the cube is mapped to $\mathbf{0}$.