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Matrices and Graphs

Exercises 2

Problems

1. Let $\mathbf{x} \in \mathbb{R}^4$ such that $\mathbf{x} = (-2,5,3,6)$. Calculate $\|\mathbf{x}\|$, $\widehat{\mathbf{x}}$ and $\mathbf{x} \bullet 3\mathbf{x}$.

2. Let V be any inner product space over $\mathbb{R}$ and $\mathbf{u}, \mathbf{v} \in V$. We determine vectors $\mathbf{a} = 7\mathbf{u} - 3\mathbf{v}$ and $\mathbf{b} = 2\mathbf{u} + 3\mathbf{v}$, thus, $\mathbf{a}$ and $\mathbf{b}$ are linear combinations of $\mathbf{u}$ and $\mathbf{v}$.

   a) If the angle α between $\mathbf{u}$ and $\mathbf{v}$ is $\alpha = \frac{2\pi}{3}$ and $\|\mathbf{u}\| = 3$ and $\|\mathbf{v}\| = 4$ then calculate $\mathbf{a} \bullet \mathbf{b}$.

   b) If $\mathbf{u}$ and $\mathbf{v}$ are *orthogonal*, i.e. perpendicular, and $\|\mathbf{u}\| = \|\mathbf{v}\| = 1$ then calculate $\mathbf{a} \bullet \mathbf{b}$.

   Recall that in an inner product space V we have for any $\mathbf{u}, \mathbf{v} \in V$, $\mathbf{u} \bullet \mathbf{v} = \|\mathbf{u}\|\|\mathbf{v}\| \cos(\alpha)$, where α is the angle between $\mathbf{u}$ and $\mathbf{v}$.

3. Calculate $\widehat{\mathbf{a}}$, when $\mathbf{a}$ is given in exercise 2 a).

4. Let $P(-1,3,4)$ and $Q(5,2,8)$ be two *points* on the 3D real space. If $\overline{P}$ and $\overline{Q}$ are the corresponding vectors then determine the angle between $\overline{P}$ and $\overline{Q}$ and the distance $d(\overline{P}, \overline{Q})$

5. Consider an object living in the 3D real space. Its current position is $P(2,5,3)$. The object is heading towards the point $Q(4,10,5)$, and moves to that direction exactly 3 length units. Calculate the position of this object after movement. Hint! Calculate $\widehat{\mathbf{x}}$, where $\mathbf{x} = \overline{Q} - \overline{P}$.