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Matrices and Graphs

Exercises 4

Problems

1. Let us consider a 3D real point $P(3,1,-1)$. Rotate this point counterclockwise $\frac{\pi}{6}$ (radians) around a) x-axis, b) y-axis and c) z-axis.

2. Let $\mathbf{a} = (2,4)$ and $\mathbf{b} = (1,-3)$ be vectors in $\mathbb{R}^2$. If these vectors are considered as edges of a parallelogram then calculate its area A.

3. Let $\mathbf{a} = (2,4,0)$, $\mathbf{b} = (1,-3,2)$ and $\mathbf{c} = (-1,2,1)$ be vectors in $\mathbb{R}^3$. If these vectors are edges of a parallelepiped (a polyhedron where each face is a parallelogram) then calculate its volume V. How would you calculate the volume of a *tetrahedron* with $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{c}$ as its edges?

4. Let $\mathbf{a} = (2,4,0)$ and $\mathbf{b} = (1,-3,2)$ be vectors in $\mathbb{R}^3$. Calculate $\mathbf{a} \times \mathbf{b}$ and $\mathbf{b} \times \mathbf{a}$. Sketch a vector $\mathbf{c} = (2,3,2)$ in the standard real coordinate system. Calculate $\mathbf{c} \times \mathbf{k}$ (where is naturally $\mathbf{k} = (0,0,1)$) and position this vector in the same coordinate system with $\mathbf{c}$.

5. Let $\mathbf{v}$ and $\mathbf{u}$ be vectors in $\mathbb{R}^3$. Consider the vector $\mathbf{u}$ as a unit vector representing a *rotation axis*. We want to rotate $\mathbf{v}$ around $\mathbf{u}$ by an angle θ (using the right hand rule). Then, the rotated vector $\mathbf{v}_{\text{rot}}$ is given by *Rodrigues' rotation formula*

   $$\mathbf{v}_{\text{rot}} = (\cos\theta)\mathbf{v} + (\sin\theta)(\mathbf{u} \times \mathbf{v}) + (1 - \cos\theta)(\mathbf{u} \bullet \mathbf{v})\mathbf{u}.$$

   Calculate (using `SAGE`) $\mathbf{v}_{\text{rot}}$ if $\mathbf{v} = (2,3,4)$, $\mathbf{u} = (\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}})$ and $\theta = \frac{\pi}{4}$. Sketch in a standard real coordinate system.

6. a) Let

   $$\mathbf{A} = \begin{bmatrix} 7 & 5 \\ 6 & 14 \end{bmatrix}$$

   Calculate the eigenvalues and eigenvectors of $\mathbf{A}$.

   b) Study by yourself, how to calculate the eigenvalues and eigenvectors of matrices using `SAGE`. Then, calculate by `SAGE` the eigenvalues and eigenvectors of $\mathbf{A}$ and

   $$\mathbf{B} = \begin{bmatrix} 1 & 4 & 8 \\ -6 & 14 & 7 \\ 0 & 9 & -2 \end{bmatrix}$$