Homework 2

Due Monday, September 28

Please either give the assignment to Loraine at the CDS or send it via email to the graders **before 1pm**.

1. *Spider on a wall* There's a spider living on a wall of your living room that has a painting under which the spider likes to hide. Figure 1 shows a diagram of the wall; it is 10 feet high and 10 feet wide.

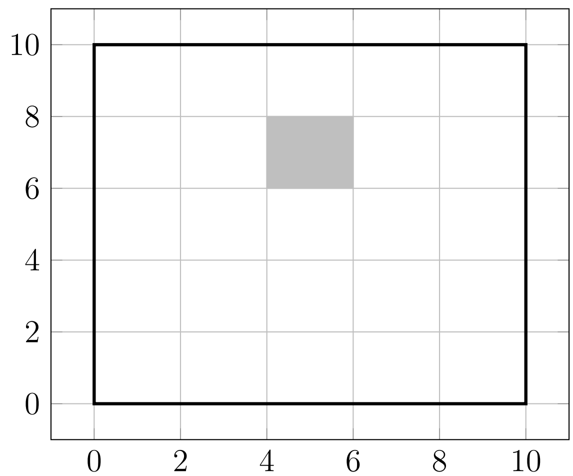

Figure 1: Wall and painting (in gray).

After observing the spider for a while you determine that (1) it spends twice the time under the painting than on the rest of the wall, (2) it never crawls on the painting or leaves the wall, (3) if it is not under the painting then it is equally likely to be anywhere on the wall. Since you cannot see it under the painting, you assume that when it is there it is also equally likely to be at any spot.

a. Model the position of the spider as a bivariate random variable and give its pdf.

b. Compute the pdf of the height at which the spider is located and sketch it.

c. Compute the conditional cdf of the height at which the spider is located, given that you can see it (i.e. it's not under the painting) and sketch it.

2. *Pizza delivery* Pat and Robbie work in competing pizzerias that are on the same street. Over time, they have realized that at any given point the time until the next call is distributed as an exponential random variable with parameter λ calls/day for both pizzerias.

a. Pat sees that Robbie is waiting for a call. Being an optimist, he wonders about the distribution of the time until one of them receives a call. Can you give help him out (i.e. compute the cdf of the waiting time)? If you make any reasonable assumptions please state them.

b. Robbie sees that Pat is also waiting for a call. Being a pessimist, he wonders about the distribution of the time until both of them have received a call. Can you give help him out (i.e. compute the cdf of the waiting time)?

c. Plot the cdfs you have obtained and also the cdf of the waiting time for either Pat or Robbie. Do the results make sense? Explain.

3. *Cars* The number of mechanical problems in a car of a certain model is distributed as a Poisson random variable with parameter λ problems/month. You and your sister have the same car. You are interested in the total number of mechanical problems that you will encounter each month in total.

   a. Prove that

$$\sum_{k=0}^{n} \binom{n}{k} = 2^n \tag{1}$$

   using the axioms of probability and the expression for the pmf of a binomial random variable.

   b. Using (1) determine the pmf of the number of mechanical problems that you will encounter per month.

4. *Race* Mary and Hannah are running a 10-mile race. You have no idea when the race started and how fast they run with respect to each other, so you decide to model their positions as independent and uniformly distributed over ten miles. You wonder about the pdf of the distance between your friends under this model.

   a. Draw a diagram of the joint pdf of the positions of Mary and Hannah.

   b. Shade the area that corresponds to the event *the distance between them is smaller than d.*

   c. Compute the pdf of the distance between them and sketch it.

5. *Cheating at coin flips* Your cousin Marvin wants to bet on the outcome of the flip of a coin. He chooses heads right away. You suspect that he is being sneaky and the coin is biased. You model the outcome of the coin flip as a Bernouilli random variable where heads is assigned to 1 and tails to 0 and the bias is *random*.

   a. Before the coin flip you decide to model the bias as being uniform between 1/2 and 1. Briefly justify the model and compute the probability that the result of coin flip is heads or tails under this model.

   b. After the coin flip you should update your model of the bias of the coin. Describe the updated model if the outcome is tails and if the outcome is heads. Sketch any distributions you compute and explain why the drawing makes sense.