

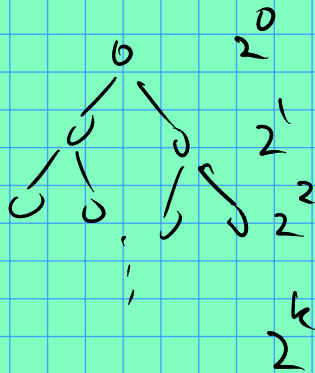
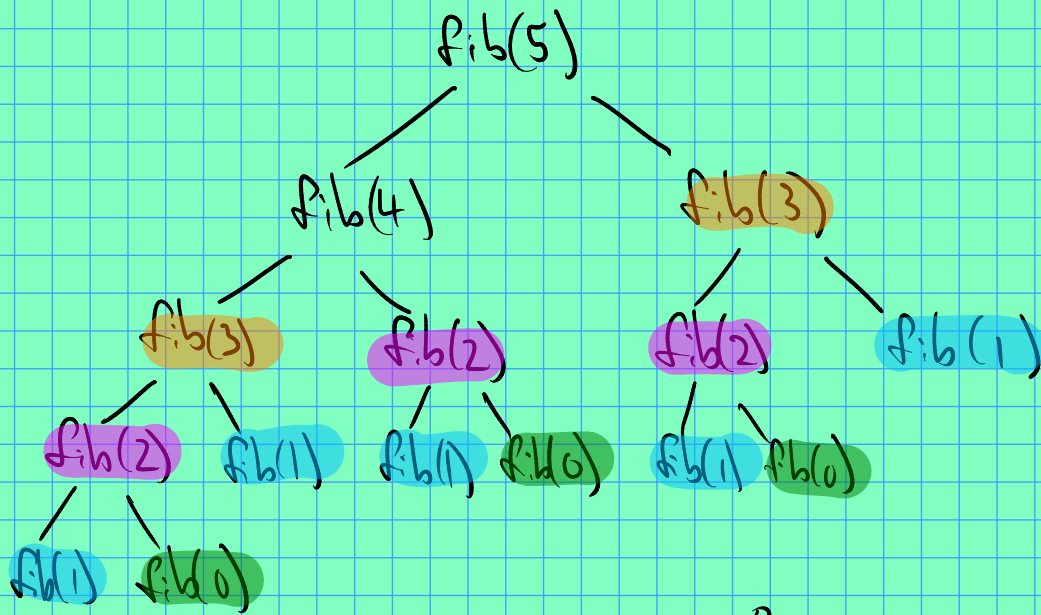
Let's Revisit Fibonacci:

$$a_n = \begin{cases} 1 & \text{if } n < 2 \\ a_{n-1} + a_{n-2} & \text{else} \end{cases}$$

1 1 2 3 5 8 ...

```
int fib(int n)
{
    if(n < 2) return 1;
    return fib(n-1) + fib(n-2);
}
```

Let's trace this (draw recursion tree)



$$\sum_{i=0}^n 2^i = \frac{(2^0 + 2^1 + 2^2 + \dots + 2^n)(1-2)}{(1-2)}$$

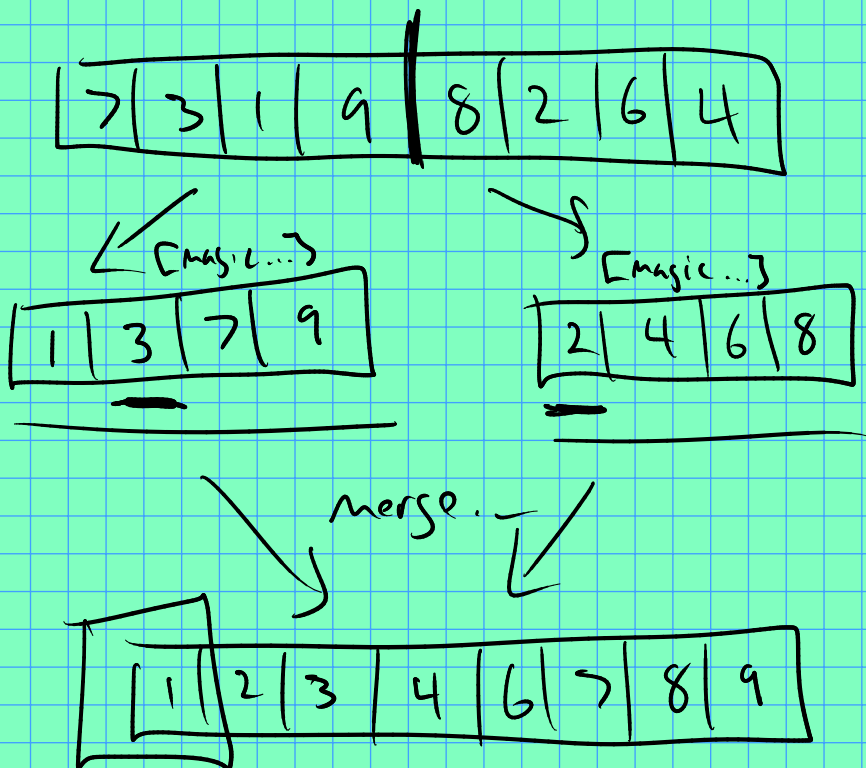
$$= \frac{(2^0 - \cancel{2^1}) + (\cancel{2^1} - \cancel{2^2}) + (\cancel{2^2} - \cancel{2^3}) + \dots + (\cancel{2^n} - \cancel{2^{n+1}})}{1-2}$$

$$= \frac{1 - 2^{n+1}}{1-2} = 2^{n+1} - 1$$

$$\sum_{i=0}^n r^i = \frac{1 - r^{n+1}}{1-r}$$

So size of recursion tree for fib
 $\approx 2^n$. (T-T)

Example: Sorting (merge sort)



```
void merge(int* L, int nL,  
           int* R, int nR,  
           int* S)
```

```
{
```

// we know L, R are sorted.

// nL = size of L, nR = size of R

// S is at least nL+nR elements.

// Then we will fill S with contents
of L, R in sorted order.

```
int iL = 0; iR = 0; iS = 0;
```

```
while (iL < nL && iR < nR) { // both still  
                             have elements
```

```
    if (L[iL] < R[iR])
```

```
        S[iS++] = L[iL++];
```

```
    else
```

```
        S[iS++] = R[iR++];
```

```
}
```

// one ran out..

```
while (iL < nL) S[iS++] = L[iL++];
```

```
while (iR < nR) S[iS++] = R[iR++];
```

```
}
```

```
void mergeSort(int* A, int n, int* Aux) {
```

// base case : $n < 2$

