120040025
Yohandi

yohandi - assignment 10

1a. $M_{X_2}(t) = E(e^{tX_2})$

$= E(e^{t(Y-X_1)})$

$= E(e^{tY}) E(e^{-tX_1})$

$= (1-2t)^{-r/2} (1-2t)^{r_1/2}$

$= (1-2t)^{(r_1-r)/2}$

b. $X_2 \sim \chi^2(r-r_1)$

2. $M_Y(t) = E(e^{tX^2})$

$= \int_{-\infty}^{\infty} e^{tx^2} \cdot \frac{1}{\sqrt{2\pi}} \cdot e^{-\frac{x^2}{2}} dx$

$= \int_{-\infty}^{\infty} e^{x^2(t-\frac{1}{2})} \cdot \frac{1}{\sqrt{2\pi}} dx$

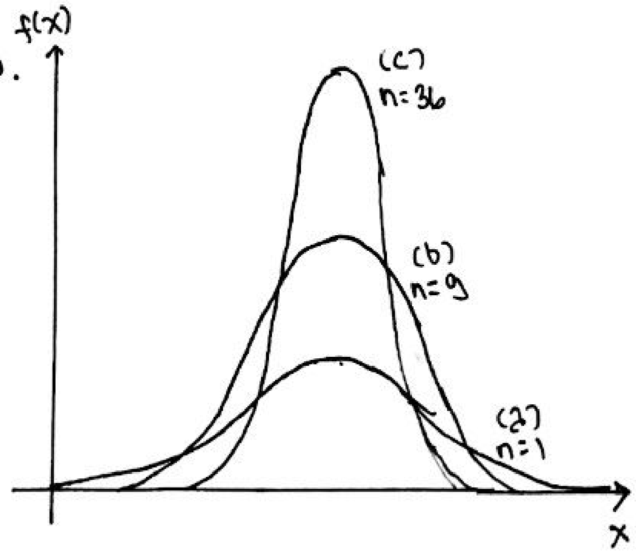
let $w = x\sqrt{1-2t}$

$\frac{dw}{dx} = \sqrt{1-2t}$

$\Rightarrow M_Y(t) = \int_{-\infty}^{\infty} e^{-\frac{w^2}{2}} \cdot \frac{1}{\sqrt{2\pi}} \frac{dw}{\sqrt{1-2t}}$

$= \frac{1}{\sqrt{1-2t}}$

this implies that $Y \sim \chi^2(1)$

3. 

4. $X \sim N(6.05, 0.0004)$

a. $P(X < 6.0171) = \Phi\left(\frac{6.0171 - 6.05}{\sqrt{0.0004}}\right) \approx 0.050$

$Y \sim B(9, 0.050)$

b. $P(Y \leq 2) = \sum_{i=0}^{2} \binom{9}{i} (0.050)^i (1-0.050)^{9-i} \approx 0.992$

$\bar{X} \sim N(6.05, \frac{0.0004}{9})$

c. $P(\bar{X} \leq 6.035) = \Phi\left(\frac{6.035 - 6.05}{\sqrt{\frac{0.0004}{9}}}\right) \approx 0.012$

6a. $W = \frac{Z_1}{\sqrt{\frac{Z_2^2 + Z_3^2}{2}}} = \frac{Z}{\sqrt{\frac{U}{r}}}$ where $Z \sim N(0,1)$, $U \sim \chi^2(2)$, $r = 2$

this implies that $W \sim t(2)$

b. $V = \frac{Z_1}{\sqrt{\frac{Z_1^2 + Z_2^2}{2}}}$

$F(v) = P(V \leq v)$

$= P\left(\frac{Z_1}{\sqrt{\frac{Z_1^2+Z_2^2}{2}}} \leq v\right)$

$= P\left(-\frac{v}{\sqrt{2-v^2}} \sqrt{Z_2^2} \leq Z_1 \leq \frac{v}{\sqrt{2-v^2}} \sqrt{Z_2^2}\right)$

$= \int_0^{\infty} \int_{-\infty}^{\frac{v|z_2|}{\sqrt{2-v^2}}} \frac{1}{2\pi} e^{-\frac{(z_1^2+z_2^2)}{2}} dz_1 dz_2 - \int_0^{\infty} \int_{-\infty}^{\frac{-v|z_2|}{\sqrt{2-v^2}}} \frac{1}{2\pi} e^{-\frac{(z_1^2+z_2^2)}{2}} dz_1 dz_2$

$f(v) = \frac{d}{dv} F(v)$

$= \frac{1}{\pi(2-v^2)^{3/2}} \left(\int_0^{\infty} z_2 \cdot e^{\left(\frac{2}{v^2-2}\right) z_2^2 \left(\frac{1}{2}\right)} dz_2 \right)$

$= \frac{2}{\pi(2-v^2)^{3/2}\left(\frac{4}{v^2-2}\right)} \int_0^{\infty} e^{-w/2} dw$

$= \frac{1}{\pi\sqrt{2-v^2}}$

c. $E(V) = \int_{-\infty}^{\infty} v f(v)$

$= \underbrace{\int_{-\sqrt{2}}^{\sqrt{2}} v \cdot \frac{1}{\pi\sqrt{2-v^2}} dv}_{\text{odd function}}$

$= 0$

d. $Var(v) = E(v^2) - E(v)^2$

$= \int_{-\infty}^{\infty} v^2 f(v)$

$= \int_{-\sqrt{2}}^{\sqrt{2}} \frac{v^2}{\pi\sqrt{2-v^2}} dv$

$= \frac{\arcsin(\frac{v}{\sqrt{2}})}{\pi} - \frac{v\sqrt{2-v^2}}{2\pi} \Big|_{v=-\sqrt{2}}^{\sqrt{2}}$

$= 1$

$\sigma = \sqrt{\sigma_v^2} = \sqrt{Var(v)} = 1$

e. In succinct way, both (a) & (b) are mainly different on their dependencies. Since Z_1, Z_2, and Z_3 have independent distribution,

Z_1 is independent with $\sqrt{\frac{Z_2^2+Z_3^2}{2}}$.

however,

Z_1 might not be independent with $\sqrt{\frac{Z_1^2+Z_2^2}{2}}$

6. $T = \frac{Z}{\sqrt{\frac{U}{r}}}$ (here, $r=2$)

$\Rightarrow E(T) = \underbrace{E(Z)}_{0 \text{ (since } Z \sim N(0,1))} \cdot E\left(\frac{\sqrt{r}}{\sqrt{U}}\right)$

$= 0$

$\Rightarrow Var(T) = E\left(\frac{rZ^2}{U}\right) - E(T)^2$

$= r \cdot E(Z^2) E(\frac{1}{U})$

$= r(E(Z)^2 + Var(Z)) E(\frac{1}{U})$

$= r \cdot (0^2+1) \cdot \int_0^{\infty} \frac{1}{u}\left(\frac{u^{r/2-1}e^{-u/2}}{2^{r/2}\Gamma(\frac{r}{2})}\right) du$

$= r \cdot \frac{1}{r-2}$

$= \frac{r}{r-2}$

7a. $t_{0.025}(9-1) = 2.306$

b. $-t_{0.025} \le T \le t_{0.025}$

$= -t_{0.025} \le \frac{\bar{x}-\mu}{\frac{s}{\sqrt{n}}} \le t_{0.025}$

$= -2.306 \le \frac{\bar{x}-\mu}{\frac{s}{\sqrt{9}}} \le 2.306$

$= \bar{x} - \frac{2.306}{3}s \le \mu \le \bar{x} + \frac{2.306}{3}s$

8a. $Y \sim \chi^2(18)$

b. $Y \sim N(18, 36)$

$P(Y \le 9.390) \approx 0.07565$ (CLT) & $P(Y \le 9.390) \approx 0.05$ (table)

$P(Y \le 34.80) \approx 0.9974$ (CLT) & $P(Y \le 34.80) \approx 0.99$ (table)

9a. $E(\bar{x}) = E(x) = 24.43$

b. $Var(\bar{x}) = \frac{Var(x)}{n} = 0.07\bar{3}$

c. $\phi\left(\frac{24.82-24.43}{\sqrt{0.07\bar{3}}}\right) - \phi\left(\frac{24.17-24.43}{\sqrt{0.07\bar{3}}}\right) = 0.7566$

10. $Y \sim N(10 \cdot 20, 4 \cdot 20)$

$P(Z \ge \frac{Y-200}{\sqrt{80}}) = 1 - P(Z \le \frac{Y-200}{\sqrt{80}}) \approx 20\%$

$\Rightarrow \phi\left(\frac{Y-200}{\sqrt{80}}\right) = \phi(0.84)$

$\Rightarrow Y \approx 207.513$

$\Rightarrow \lceil Y \rceil = 208$ days for budget yohandi - assignment 10 (computer-based)

S	-1	1
f(S)	0.5	0.5

$$E(X) = E\left(\frac{1}{N}\sum_{i=1}^{N} S_i\right) = \frac{1}{N}\underbrace{\sum_{i=1}^{N} E(S_i)}_{0} = 0$$

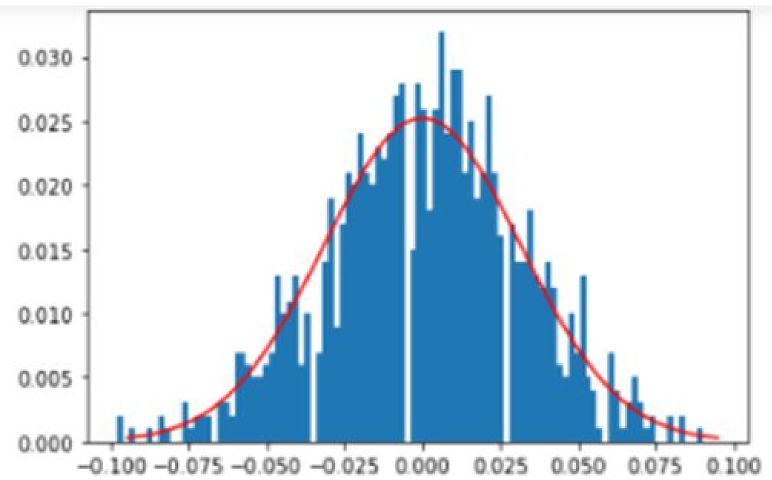
$$Var(X) = Var\left(\frac{1}{N}\sum_{i=1}^{N} S_i\right) = \frac{1}{N^2}\underbrace{\sum_{i=1}^{N} Var(S_i)}_{N} = \frac{1}{N}$$

from the obtained Var(X),

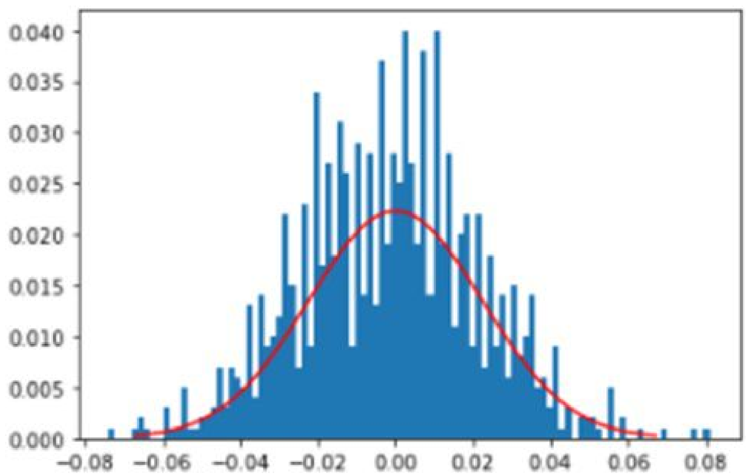
the bigger N, the smaller Var(X)

$\Rightarrow$ the graph tends to be more compact

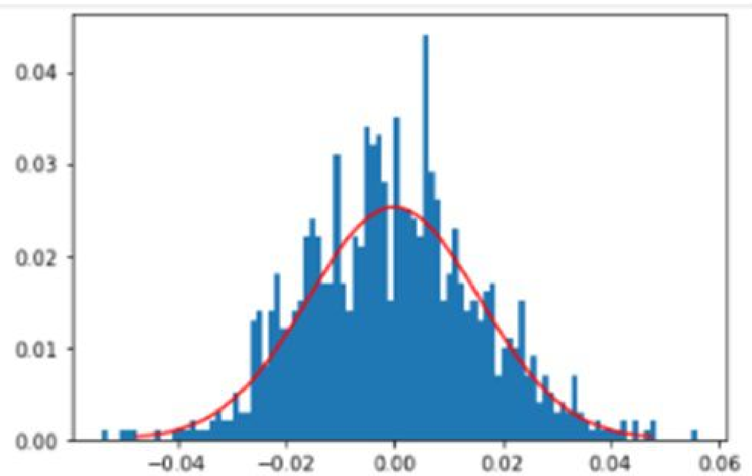
N=1000



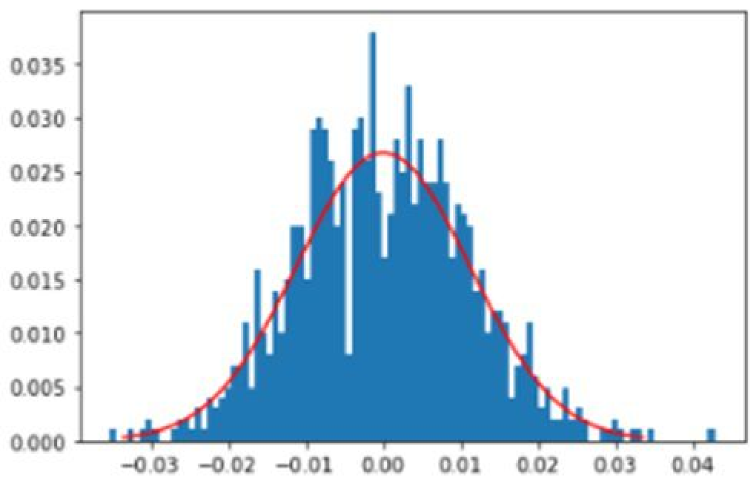
N=2000



N=4000



N=8000



```python
import math
import matplotlib.pyplot as plt
import numpy as np
import random
import scipy.stats

total_experiment = 4
N_list = [(1000 * 2 ** i) for i in range(total_experiment)]

def random_walk(N):
    return sum([(random.randint(0, 1) * 2 - 1) for i in range(N)]) / N

def experiment(ith, N, M = 1000, K = 100):
    ith_list = [0.002, 0.00125, 0.001, 0.00075]
    sample_X = np.array([random_walk(N) for i in range(M)])
    ax = plt.figure().add_subplot(111)
    n, bins, patches = ax.hist(sample_X, bins = K, cumulative = 0, weights = np.zeros_like(sample_X) + 1. / len(sample_X))
    ax.legend
    mean = 0
    variance = 1 / N
    sigma = math.sqrt(variance)
    X = np.linspace(mean - 3 * sigma, mean + 3 * sigma, 100)
    Y = [i * ith_list[ith] for i in scipy.stats.norm.pdf(X, mean, sigma)]
    plt.plot(X, Y, color = "r")
    plt.show()

for i in range(total_experiment):
    experiment(i, N_list[i])
```