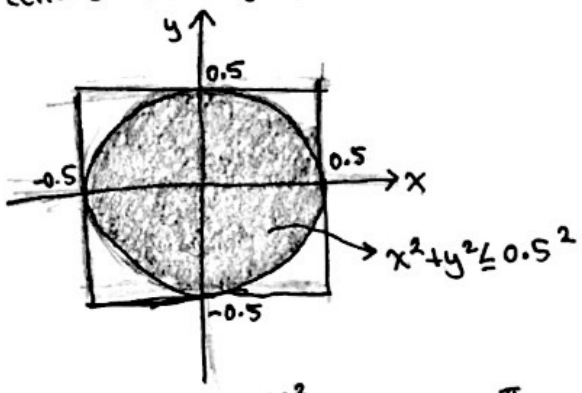
yohandi - assignment 7 (computer-based)

$X \sim U(-0.5, 0.5)$ and $Y \sim U(-0.5, 0.5)$,

consider $A \sim X^2+Y^2$

$P(A \leq (0.5)^2) = P(x^2+y^2 \leq (0.5)^2)$

-> to compute the given probability let's consider the circular region with radius 0.5 centered at origin,

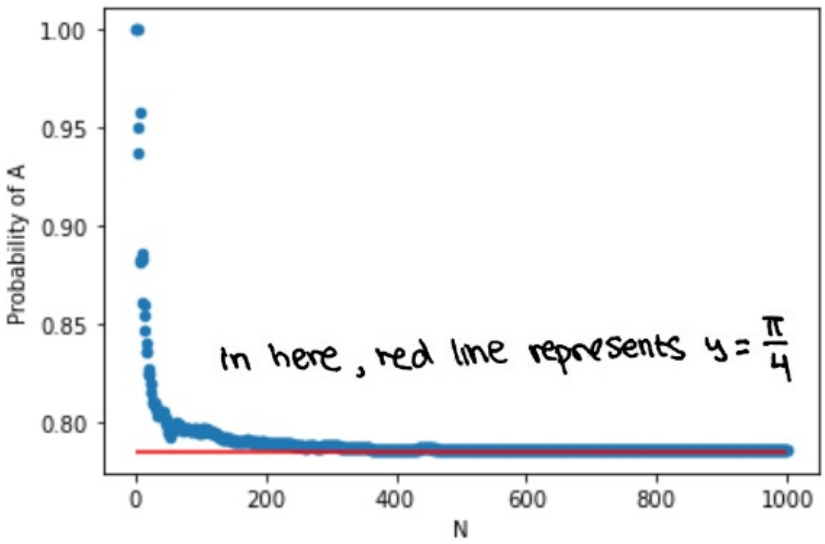


$$\Rightarrow P(A \leq (0.5)^2) = \frac{\pi(0.5)^2}{(0.5-(-0.5))^2} = 0.25\pi = \frac{\pi}{4}$$

the probability can also be estimated using the relative frequency interpretation of $P(A)$. for every values of N, we count the number of pairs (x_i, y_i) $i=1,\dots,N$ where $x_i^2 + y_i^2 \leq (0.5)^2$.

$$\Rightarrow P(A) = \lim_{N \to \infty} \frac{N(A)}{N}$$

below is the graph obtained from computing:



we can see that the bigger N the more accurate the probability

```python
import math
import matplotlib.pyplot as plt
import numpy as np
import pandas as pd

def randomExperiment(N):
    ret = 0
    x = np.random.uniform(-0.5, 0.5, N)
    y = np.random.uniform(-0.5, 0.5, N)
    for i in range(N):
        if x[i] ** 2 + y[i] ** 2 <= (0.5) ** 2:
            ret += 1
    return ret

N = []
totalP = [0]
P = []
for i in range(1, 1001):
    N.append(i)
    totalP.append(totalP[-1])
    totalP[-1] += randomExperiment(i) / i
    P.append(totalP[-1] / N[-1])

data = {'N' : N, 'Probability of A' : P}
df = pd.DataFrame(data, columns = ['N', 'Probability of A'])
df.plot(x = 'N', y = 'Probability of A', kind = 'scatter')
plt.hlines(math.pi / 4, 0, 1000, colors = 'red')
plt.show()
```