120040025

Yohandi

yohandi - assignment 6 (computer-based)

1. Theoretically,

$$\text{mean } (\mu) = E(X) = np = 40\left(\tfrac{1}{2}\right) = 20$$

$$\text{variance } (\sigma^2) = Var(X) = npq = 40\left(\tfrac{1}{2}\right)\left(\tfrac{1}{2}\right) = 10$$

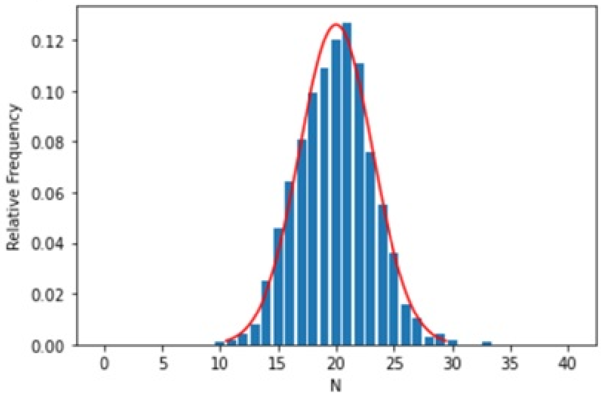
X follows the normal distribution with mean $\mu = 20$ and variance $\sigma^2 = 10$

$\Rightarrow X \sim N(20, 10)$

2&3.

```
import math
import matplotlib.pyplot as plt
import numpy as np
import pandas as pd
import random
import scipy.stats as stats

def experiment():
    ret = 0
    for i in range(40):
        if random.randint(1, 2) == 1:
            ret += 1
    return ret

simulation = [0 for i in range(41)]
relativeFrequency = []
for i in range(1000):
    simulation[experiment()] += 1
for i in range(41):
    relativeFrequency.append(simulation[i] / 1000)

mu = 20
variance = 10
sigma = math.sqrt(variance)
x = np.linspace(mu - 3 * sigma, mu + 3 * sigma, 100)
plt.plot(x, stats.norm.pdf(x, mu, sigma), color = 'r')

plt.bar(tuple([i for i in range(41)]), tuple(relativeFrequency), align = 'center')
plt.xlabel('N')
plt.ylabel('Relative Frequency')

plt.show()
```

4.



5. $P(19.5 < X < 20.5) = \phi\left(\frac{20.5-20}{\sqrt{10}}\right) - \phi\left(\frac{19.5-20}{\sqrt{10}}\right)$

$\approx 0.56282 - 0.43718$

≈ 0.12564

let Y be the number of heads that occurs,

$Y \sim B(40, \tfrac{1}{2})$

$P(Y=20) = \binom{40}{20}\left(\tfrac{1}{2}\right)^{20}\left(\tfrac{1}{2}\right)^{40-20}$

≈ 0.12537

from the result,

$$\varepsilon = |P(19.5<X<20.5) - P(Y=20)| \approx 0.00027 < 10^{-3}$$

the error is way less than 10^{-3}; therefore, the approximation using the normal distribution in this case is accurate to 3 decimal places.