120040025
Yohandi

yohandi - assignment 11

1a. $P(2<X<9) = P(X\leq 8) - P(X\leq 2)$

≈ 0.8550

b. $Y \sim N(25\cdot 0.2,\ 25\cdot 0.2\cdot(1-0.2))$

$P(2<X<9) \approx P(2.5<Y<8.5)$

≈ 0.8543

2. $Y \sim N(100\cdot 0.1,\ 100\cdot 0.1\cdot(1-0.1))$

a. $P(12\leq X\leq 14) \approx P(11.5<Y<14.5)$

≈ 0.2417

$Z \sim Poi(100\cdot 0.1)$

b. $P(12\leq X\leq 14) \approx \sum_{z=12}^{14} \frac{10^z \cdot e^{-10}}{z!}$

≈ 0.2198

c. $P(12\leq X\leq 14) = \sum_{x=12}^{14} \binom{100}{x} 0.1^x (1-0.1)^{100-x}$

≈ 0.2244

3. $p' = 1 - P(-0.98\leq X\leq 0.98) \approx 0.0500$

$Y \sim Poi(100\cdot 0.0500)$

a. $P(Y\geq 7) = 1 - \sum_{y=0}^{6} \frac{5^y \cdot e^{-5}}{y!}$

≈ 0.2378

$Z \sim N(100\cdot 0.0500,\ 100\cdot 0.0500\cdot(1-0.0500))$

b. $P(Z>6.5) = 1 - P(Z\leq 6.5)$

≈ 0.2451

c. $1 - \sum_{i=0}^{6} \binom{100}{i} 0.0500^i (1-0.0500)^{100-i}$

≈ 0.2340

4. $P(|X-3.5|<2.5) = P(|X-\mu|<2.5)$

$= 1 - P(|X-\mu|\geq 2.5)$

$\geq 1 - \frac{\sigma^2}{2.5^2}$

≈ 0.5333

5. $P(|\frac{Y}{n} - 0.5| < 0.08) = 1 - P(|Y - \underbrace{0.5n}_{\mu}| \geq 0.08n)$

$\geq 1 - \frac{n(0.5)(1-0.5)}{(0.08n)^2}$

$= 1 - \frac{625}{16n}$

a. $n=100 \Rightarrow P(|\frac{Y}{n}-0.5|<0.08) \geq 0.6094$

b. $n=500 \Rightarrow P(|\frac{Y}{n}-0.5|<0.08) \geq 0.9219$

c. $n=1000 \Rightarrow P(|\frac{Y}{n}-0.5|<0.08) \geq 0.9609$

6. $P(75<\bar{X}<85) = P(|\bar{X}-80|<5) = 1 - P(|\bar{X}-80|\geq 5)$

$\geq 1 - \frac{4}{5^2} = 0.84$

7a $F(w) - F(-w) = 1 - \frac{1}{w^2}$

$F(w) + F(-w) = 1$ +

$2F(w) = 2 - \frac{1}{w^2} \Rightarrow w^2 \geq 1$

$2\cdot\frac{d}{dw}F(w) = \frac{2}{w^3}$

$f(w) = \frac{1}{|w^3|}$ $(f(w) = f(-w))$

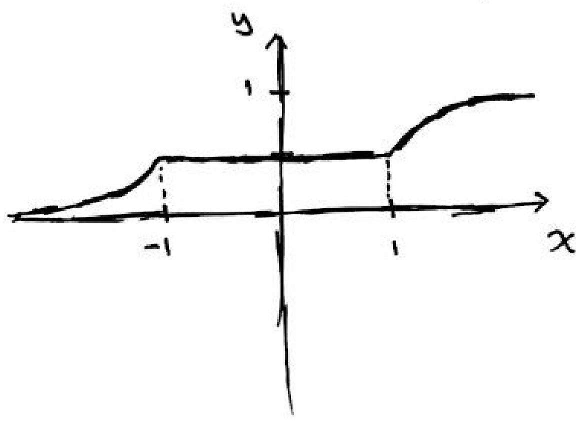
b. $E(X) = \int_{-\infty}^{\infty} w f(w)\,dw$ $\quad E(X^2) = \int_{-\infty}^{\infty} w^2 f(w)\,dw$

$= \int_{-\infty}^{-1} \frac{w}{|w^3|}\,dw + \int_{1}^{\infty} \frac{w}{|w|^3}\,dw$ $\quad = \int_{-\infty}^{-1} \frac{w^2}{|w^3|}\,dw + \int_{1}^{\infty} \frac{w^2}{|w^3|}\,dw$

$= 0$ $\quad = \infty$

$\Rightarrow Var(X) = \infty$

c.



8a. $P(0 \le Y \le 3) = \sum_{y=0}^{3} \binom{50}{y} (0.01)^y (1-0.01)^{50-y}$

≈ 0.9984

$Z \sim Poi(50 \cdot 0.01)$

b. $P(0 \le Y \le 3) \approx \sum_{z=0}^{3} \frac{0.5^z e^{-0.5}}{z!}$

≈ 0.9982

9. let $Y = \sum_{i=1}^{n} X_i$, $X_i \sim \chi^2(1)$

$\Rightarrow \mu_{X_1} = 1$, $\sigma_{X_1}^2 = 2$

by CLT,

$Y \sim N(n \cdot \mu, n \cdot \sigma^2) = N(n, 2n)$

$\Rightarrow W = \frac{Y-n}{\sqrt{2n}} \sim N(0, 1)$

10. let $Y = \sum_{i=1}^{n} X_i$, $X_i \sim Poi(3)$

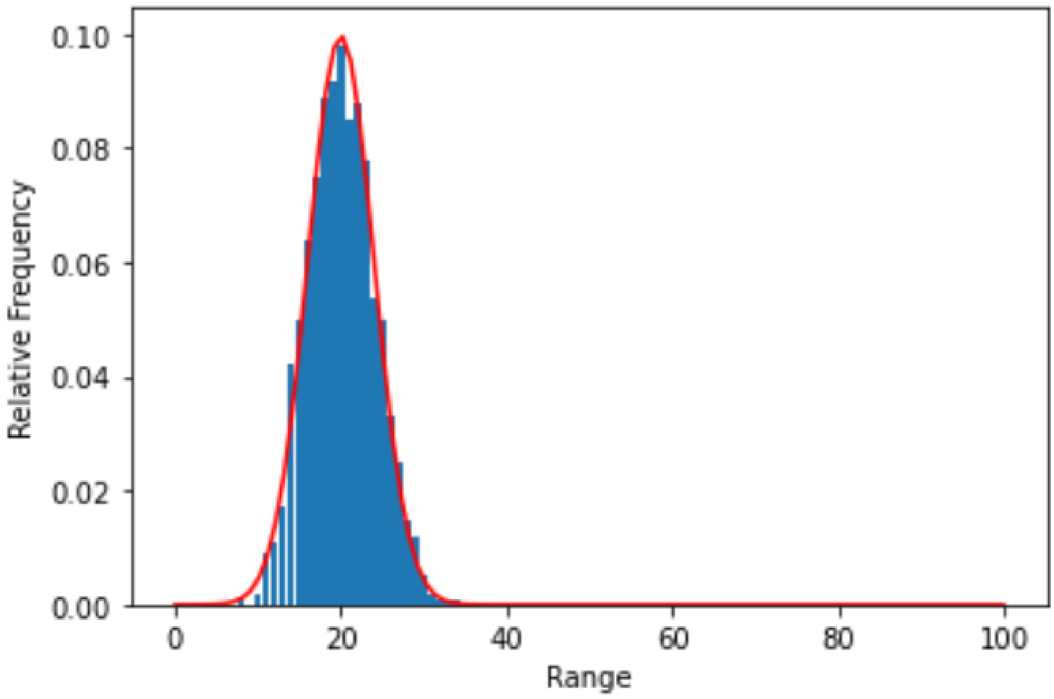
$\Rightarrow \mu_{X_1} = 3$, $\sigma_{X_i}^2 = 3$

by CLT,

$Y \sim N(n \cdot \mu, n \cdot \sigma^2) = N(3n, 3n)$

$\Rightarrow W = \frac{Y-3n}{\sqrt{3n}} \sim N(0, 1)$

Yohandi - Assignment 11 (Computer-based)



the approximation of normal distribution obtained from the Central Limit Theorem is highly accurate (as it represents most of the bars obtained) with only 1000 simulations.

this shows how important and useful the Central Limit Theorem especially in doing the statistical stuffs.

the code is shown below:

```
import math
import matplotlib.pyplot as plt
import numpy as np
import random
import scipy.stats as stats

#global constant
n = 100
p = 0.2
mean = n * p
variance = n * p * (1 - p)
m = 1000

def Y(n, p):
    return len(["?" for i in range(n) if random.uniform(0, 1) <= p])

freq_Y = {i : 0 for i in range(101)}

for i in range(m):
    global n, p
    freq_Y[Y(n, p)] += 1

plt.bar(tuple([i for i in range(101)]), tuple([value / m for (keys, value) in freq_Y.items()]), align = "center")
plt.xlabel("Range")
plt.ylabel("Relative Frequency")

range_x = np.linspace(0, 100, 100)
range_y = [i for i in stats.norm.pdf(range_x, mean, math.sqrt(variance))]
plt.plot(range_x, range_y, color = "r")
```