ychandi - quiz 5

1 a. true

b. true

c. false

d. true

2 a. $$\lim_{x\to 0} \frac{2\sin x - \sin 2x}{x\tan^2 x} = \lim_{x\to 0} \frac{2\sin x}{x} \cdot \lim_{x\to 0} \frac{(1-\cos x)}{\tan^2 x} = 2 \cdot \lim_{x\to 0} \frac{\frac{1}{2}(x)^2}{\tan^2 x} = \boxed{1}$$

b. $$\lim_{x\to\infty} (x^2+2e^x)\cdot e^{-x} = \lim_{x\to\infty} \frac{x^2}{e^x} + 2 = \cancel{\infty} \text{ (indeterminate form)}$$

$$= \lim_{x\to\infty} \frac{\frac{d(x^2+2)}{dx}}{\frac{d(e^x)}{dx}} + 2 = \lim_{x\to\infty} \frac{2x}{e^x} + 2 = \lim_{x\to\infty} \frac{\frac{d(2x)}{dx}}{\frac{d(e^x)}{dx}} + 2$$

$$= \lim_{x\to\infty} \frac{2}{e^x} + 2$$

$$= 0 + 2$$

$$= \boxed{2}$$

3. $$\text{Area} = 2\pi r^2 + 2\pi r h = 2\pi(r^2 + rh) = 2$$

$$2\pi r(r+h) = 2$$

$$(r+h) = \frac{1}{\pi r}$$

$$h = \frac{1}{\pi r} - r = \frac{1-\pi r^2}{\pi r}$$

$$\text{volume} = \pi r^2 h = \pi(r^2)\left(\frac{1-\pi r^2}{\pi r}\right) = r - \pi r^3$$

$$\frac{d(\text{volume})}{dr} = 1 - 3\pi r^2 = 0$$

$$r = \sqrt{\frac{1}{3\pi}}$$

$$\text{volume}_{max} = \pi\left(\frac{1}{3\pi}\right) \cdot h = \frac{1}{3}h = \frac{1}{3}\left(\frac{1-\pi r^2}{\pi r}\right) = \frac{2\sqrt{3\pi}}{9\pi} = \boxed{\frac{2}{\sqrt{3\pi}}}$$

4. $f(x) = \frac{1}{2} \ln\left(\frac{1+\sin x}{1-\sin x}\right)$ $\qquad g(x) = \ln|\sec x + \tan x|$

$$\cancel{f'(x) = \frac{1}{2} \cdot \frac{1}{\left(\frac{1+\sin x}{1-\sin x}\right)} \cdot \left(1 + \frac{2}{1}\right)}$$

$$f(x) = \frac{1}{2}\left[\ln(1+\sin x) - \ln(1-\sin x)\right]$$

$$f'(x) = \frac{1}{2}\left[\frac{\cos x}{1+\sin x} - \frac{-\cos x}{1-\sin x}\right]$$

$$= \frac{1}{2}\left[\frac{\cos x(1-\sin x) + \cos x(1+\sin x)}{1-\sin^2 x}\right]$$

$$= \frac{1}{\cos x}$$

$$g(x) = \ln|\sec x + \tan x|$$

$$\cancel{g'(x) = \frac{1}{\sec x\tan x + \sec^2 x}}$$

$$g'(x) = \frac{1}{\sec x + \tan x}\left(\sec x \tan x + \sec^2 x\right)$$

$$= \sec x \; \frac{(\tan x + \sec x)}{(\sec x + \tan x)}$$

since $f'(x) = \frac{1}{\cos x} = \sec x$

a.) $\therefore f'(x) = g'(x)$

b.) $\int f'(x)\,dx = \int g'(x)\,dx + C$

$$f(x) = g(x) + C$$

$$f(0) = g(0) + C$$

$$\frac{1}{2}\ln\frac{1+0}{1-0} = \ln(\sec 0 + \tan 0) + C$$

$$0 = 0 + C$$

$$c = 0$$

$\therefore f(x) = g(x)$