yohandi - quiz 4

1a. F
b. T
c. T
d. T

2. we know that if $x>y$, then $\sqrt{x}>\sqrt{y}$ (since both x and $y \geq 1$) and vice versa.

→ case $x>y$:

$|\sqrt{x}-\sqrt{y}| \leq \frac{1}{2}|x-y|$

$\sqrt{x}-\sqrt{y} \leq \frac{1}{2}x-\frac{1}{2}y$

$\frac{2(\sqrt{x}-\sqrt{y})}{x-y} \leq 1$

$\boxed{\frac{2}{\sqrt{x}+\sqrt{y}} \leq 1}$

since $\sqrt{x}+\sqrt{y} \geq \sqrt{1}+\sqrt{1}=2$ (it is true)

→ case $x=y$:

$|\sqrt{x}-\sqrt{x}| \leq \frac{1}{2}|x-x|$

$\boxed{0 \leq 0}$ (true)

→ case $x<y$:

similar as case $x>y$ with swapping x and y

3. let $f(x)=\tan x+e^{-x}$

$\frac{d(f(x))}{dx}=f'(x)=\sec^2 x-e^{-x}$

critical points when $f'(x)=0$ or undefined

→ $f'(x)=0$

$\sec^2 x=e^{-x}$

no solution for x in $(0,\pi/3)$

→ $f'(x)$ undefined

e^{-x} undefined (no solution)

$\sec^2 x$ undefined

(no solution)

since there's no critical points the graph is monotonically increasing or decreasing resulting at most 1 solution

by IVT theorem,

$f(0^+)=1$ and $f(\frac{\pi}{3}^-)>\sqrt{3}$

~~there's no solution for $\tan x+e^{-x}$ in $(0,\frac{\pi}{3})$~~

It is proved that the solution exist

4. $y=f(x)\begin{cases}1-x^2, & x<0\\ x^3-3x^2+1, & x\geq 0\end{cases}$

$y'=f'(x)\begin{cases}-2x, & x<0\\ 3x^2-6x, & x\geq 0\end{cases}$

critical points when $y'=0$,

→ $-2x=0$

$x=0$ (since 0 is not <0, no critical points for $1-x^2$)

→ $3x^2-6x=0$

$3x(x-2)=0$

$x=0$ or $x=2$

$3x^2-6x$: out of interval | 0 | $-$ | 2 | $+$

$-2x$: $+$ | 0 | out of interval

a. increasing on: $(-\infty,0)$ and $[2,\infty)$

b. $y''=f''(x)\begin{cases}-2, & x<0\\ 6x-6, & x\geq 0\end{cases}$

inflection points when $y''=0$

→ $-2\neq 0$

→ $6x-6=0$

$x=1$

$6x-6$: out of interval | 0 | $-$ | 1 | $+$

-2: $-$ | 0 | out of interval

concave up on: $(1,\infty)$

concave down on: $(-\infty,1)$

c. extreme points: $x=0$, $y=1$

$x=2$, $y=-3$

inflection point: $x=1$, $y=-1$