yohandi - quiz 3
(12004025)

1a. velocity $= \frac{d(s(t))}{dt} = v(t) = 2t + \pi \cos(\pi t)$

speed $= |v(t)|$
when $t=1$:
velocity $= v(1) = 2 - \pi$
speed $= |2-\pi| = \pi - 2$

b. acceleration $= \frac{d(v(t))}{dt} = a(t) = 2 - \pi^2 \sin(\pi t)$

when $t=1$:
acceleration $= a(1) = 2$

2a. $y = \tan(x^3 . e^{-x})$ $\quad \frac{d(\tan x)}{dx} = \sec^2 x$

$\frac{dy}{dx} = \sec^2(x^3 . e^{-x}) . [3x^2 e^{-x} + x^3 . -e^{-x}]$

$= \sec^2(x^3 . e^{-x}) . [3x^2 . e^{-x} - x^3 . e^{-x}]$

$= \frac{x^2}{e^x} . \sec^2(x^3 . e^{-x}) (3 - x)$

b. $y = \sin^3(e^{\tan x})$ $\quad \frac{d(\sin x)}{dx} = \cos x$

$\frac{dy}{dx} = 3\sin^2(e^{\tan x}) . \cos(e^{\tan x}) . \sec^2 x . e^{\tan x} .$

c. $y^5 . \ln y = e^{xy} + \sec^2 x$ $\quad \frac{d(\ln x)}{dx} = \frac{1}{x}$

$5y^4 \ln y \frac{dy}{dx} + y^5 . \frac{1}{y} . \frac{dy}{dx} = (y + x . \frac{dy}{dx}) . e^{xy} + 2 . \sec^2 x . \tan x$

$\frac{dy}{dx}[5y^4 \ln y + y^4 - x . e^{xy}] = y . e^{xy} + 2\sec^2 x \tan x$

$\frac{dy}{dx} = \frac{y . e^{xy} + 2\sec^2 x \tan x}{5y^4 \ln y + y^4 - x e^{xy}}$

3. $y = (1 - \cos x)^{\sqrt{x}}$

$\ln(y) = \sqrt{x} . \ln(1 - \cos x)$ $\quad \frac{d(\ln y)}{dx} = \frac{1}{y} \frac{dy}{dx}$

$\frac{1}{y} . \frac{dy}{dx} = \frac{1}{2\sqrt{x}} \ln(1 - \cos x) + \sqrt{x} . \frac{1}{1 - \cos x} . \sin x$

$\frac{dy}{dx} = y \left(\frac{1}{2\sqrt{x}} \ln(1 - \cos x) + \sqrt{x} \frac{\sin x}{1 - \cos x} \right)$

$= \frac{(1 - \cos x)^{\sqrt{x}}}{2\sqrt{x}} \ln(1 - \cos x) + (1 - \cos x)^{\sqrt{x} - 1} \sqrt{x} \sin x$

4. $\lim_{x \to 0^-} x^3 \sin \frac{1}{x} = \lim_{x \to 0^+} x^3 \sin \frac{1}{x} = 0$

$\Rightarrow \lim_{x \to 0} x^3 \sin \frac{1}{x} = 0$

now since $\lim_{x \to 0} f(x) = f(0)$, $f(x)$ is continuous
which satisfy the first requirement to be differentiable

$$f'(x) \begin{cases} \frac{d(x^3 \sin \frac{1}{x})}{dx} = 3x^2 \sin \frac{1}{x} - x \cos \frac{1}{x}, & x \neq 0 \\ 0, & x = 0 \end{cases}$$

$\lim_{x \to 0^-} f'(x) = \lim_{x \to 0^-} 3x . \lim_{x \to 0^-} \sin \frac{1}{x} . x - \lim_{x \to 0^-} x \cos \frac{1}{x}$

$= 0 . 0 - 0 = 0$

$\lim_{x \to 0^+} f'(x) = \lim_{x \to 0^+} 3x . \lim_{x \to 0^+} \sin \frac{1}{x} . x - \lim_{x \to 0^+} x \cos \frac{1}{x}$

$= 0 . 0 - 0 = 0$

$\Rightarrow \lim_{x \to 0} f(x) = 0$

since $\lim_{x \to 0} f'(x) = f'(0^-) = f'(0^+) = f(0)$
$= 0$,

$f(x)$ is differentiable at $x = 0$

$f'(x)$ is continuous at $x = 0$.

thus, $f(x)$ is differentiable at $x = 0$