Yohandi - Math homework week 7

Exercises 5.4

1. $\int_0^2 x(x-3)\,dx = \int_0^2 x^2-3x = \left[\frac{1}{3}x^3-\frac{3}{2}x^2\right]_0^2 = -\frac{10}{3}$

3. $\int_{-2}^2 \frac{3}{(x+3)^4}\,dx = \left[-\frac{1}{(x+3)^3}\right]_{-2}^2 = \frac{124}{125}$

4. $\int_{-1}^1 x^{299}\,dx = \left[\frac{1}{300}x^{300}\right]_{-1}^1 = 0$

8. $\int_1^{32} x^{-6/5}\,dx = \left[-5x^{-1/5}\right]_1^{32} = \frac{5}{2}$

9. $\int_0^{\pi/3} 2\sec^2 x\,dx = \left[2\tan x\right]_0^{\pi/3} = 2\sqrt{3}$

10. $\int_0^{\pi} (1+\cos x)\,dx = \left[x+\sin x\right]_0^{\pi} = \pi$

12. $\int_0^{\pi/3} 4\cdot\frac{\sin u}{\cos^2 u}\,du = \int_0^{\frac{\pi}{3}} 4\cdot\frac{\sin u}{\cos^2 u}\,\frac{d(\cos u)}{-\sin u}$

$= \left[\frac{4}{\cos u}\right]_0^{\frac{\pi}{3}}$

$= 4$

17. $\int_0^{\pi/8} \sin 2x\,dx = \left[-\frac{1}{2}\cos 2x\right]_0^{\pi/8}$

$= \frac{1}{4}(2-\sqrt{2})$

22. $\int_{-3}^1 y^2-\frac{2}{y^2}\,dy = \left[\frac{1}{3}y^3+\frac{2}{y}\right]_{-3}^{-1}$

$= \frac{22}{3}$

24. $\int_1^8 (1+x^{-1/3})(2-x^{2/3})\,dx$

$= \int_1^8 2-x^{2/3}+2x^{-1/3}-x^{1/3}$

$= \left[2x-\frac{3}{5}x^{5/3}+3x^{2/3}-\frac{3}{4}x^{4/3}\right]_1^8 = -\frac{137}{20}$

27. $\int_{-4}^4 |x|\,dx = 2\int_0^4 x\,dx = 2\left[\frac{1}{2}x^2\right]_0^4 = 16$

28. $\int_0^{\pi} \frac{1}{2}(\cos x+|\cos x|)\,dx$

$= \int_0^{\pi/2} \frac{1}{2}\cdot 2\cos x + \int_{\pi/2}^{\pi} 0\cdot dx$

$= \left[\sin x\right]_0^{\pi/2}$

$= 1$

30. $\int_1^{\pi^2} \frac{\sin\sqrt{x}}{\sqrt{x}}\cdot\frac{d(\sqrt{x})}{\frac{1}{2\sqrt{x}}} = 2\int_1^{\pi^2}\sin\sqrt{x}\cdot d(\sqrt{x})$

$= 2\left[-\cos\sqrt{x}\right]_1^{\pi^2}$

$= 2(1+\cos(1))$

40. $y = \int_1^x \frac{1}{t}\,dt = \left[\ln|t|\right]_1^x = \ln(x)$

$\frac{dy}{dx} = \frac{d(\ln(x))}{dx} = \frac{1}{x}$

44. $y = \left(\int_0^x (t^3+1)^{10}\,dt\right)^3$

$\frac{dy}{dx} = 3\,\frac{d}{dx}\left(\int_0^x (t^3+1)^{10}\,dt\right)\left(\int_0^x (t^3+1)^{10}\,dt\right)^2$

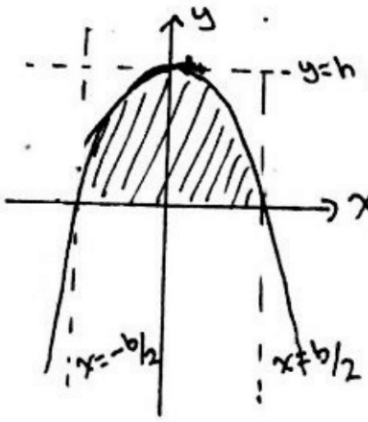
$= 3(x^3+1)^{10}\left(\int_0^x (t^3+1)^{10}\,dt\right)^2$

46. $y = \int_{\tan x}^0 \frac{dt}{1+t^2} = \left[\arctan(t)\right]_{\tan x}^0$

$= -x$

$\frac{dy}{dx} = -1$

61.



$A = \int_{-b/2}^{b/2} y\,dx$

$= \left[hx-\frac{4h}{3b^2}\cdot x^3\right]_{-b/2}^{b/2}$

$= 2\left(\frac{hb}{2}-\frac{4hb}{24}\right)$

$= hb-\frac{hb}{3}$

$= \frac{2}{3}hb$

63. $\frac{dc}{dx} = \frac{1}{2\sqrt{x}}$

$c(100) - c(1) = \int_1^{100} \frac{dc}{dx} \cdot dx$

$= [\sqrt{x}]_1^{100}$

$= 9$

68. $\int_0^x f(t)\,dt = x\cos \pi x$

$= x \cos \pi x - 0.\cos \pi.0$

$= [t \cos \pi t]_0^x$

$f(t) = \frac{d(t\cos \pi t)}{dt}$

$f(t) = \cos(\pi t) - \pi t \sin(\pi t)$

$\Rightarrow f(4) = \cos(4\pi) - 4\pi \sin(4\pi) = 1$

71a true, for $f(x)$ that is defined for all values of x

b. true, since $g(x)$ is a differentiable function, $g(x)$ is also a continuous function

c. true, $m(1) = g'(1) = f(1) = 0$ (horizontal)

d. false

e. true

since $g'(1) = 0$ and $g''(1) = f'(1) > 0$ $\Rightarrow (1, g(1))$ is a local minimum point

f. false, $g''(1) = f'(1) \neq 0$

g. true, $\frac{dg}{dx}(1) = f(1) = 0$ (crosses the x-axis at $x=1$)

73. $v = \frac{ds}{dt} = \frac{d(\int_0^t f(x)dx)}{dt} = f(t)$

a. $v(5) = f(5) = 2$ m/s

b. $a(5) = f'(5) < 0$ (decreasing) $\Rightarrow a(5)$ is negative

c. $s = \int_0^3 f(x)\,dx = \frac{1}{2}.3.3 = \frac{9}{2}$ m

d. $t = 6s$ since the area bounded by $f(x)$ and x is positive from $t=0$ to $t=6s$

e. $f'(t) = 0$ when $t \approx 4s$ and $t \approx 7s$

f. toward the origin when $6<t<9$ ($v>0$ & $s>0$) away from origin when $0<t<6$ ($v<0$ & $s>0$)

g. positive side (since $\int_0^9 f(x)\,dx$ is larger than 0)

Exercises 5.5

18. $\int \frac{1}{\sqrt{5s+4}}\, ds = 2 \cdot \frac{1}{5} \cdot \sqrt{5s+4} + C$

$= \frac{2\sqrt{5s+4}}{5} + C$

20. $\int 3y\sqrt{7-3y^2}\, dy = \int 3y\sqrt{7-3y^2}\, \frac{d(y^2)}{2y}$

$= \frac{3}{2} \cdot \frac{1}{(-3)} \cdot \frac{2}{3} \cdot (7-3y^2)^{3/2} + C$

$= -\frac{1}{3}(7-3y^2)^{3/2} + C$

21. $\int \frac{1}{\sqrt{x}(1+\sqrt{x})^2}\, dx = \int \frac{1}{\sqrt{x}(1+\sqrt{x})^2} \cdot \frac{d(\sqrt{x})}{\frac{1}{2\sqrt{x}}}$

$= 2\left(-\frac{1}{1+\sqrt{x}}\right) + C$

$= -\frac{2}{1+\sqrt{x}} + C$

22. $\int \sqrt{\sin x}\, \cos^3 x\, \frac{d(\sin x)}{\cos x} = \int \sqrt{\sin x} - \sin^2 x \sqrt{\sin x}\, .d(\sin x)$

$= \frac{2}{3}\sin x\sqrt{\sin x} - \frac{2}{7}\sin^3 x\sqrt{\sin x} + C$

26. $\int \tan^7 \frac{x}{2} \cdot \sec^2 \frac{x}{2} \cdot \frac{d(\tan\frac{x}{2})}{\frac{1}{2}\cdot\sec^2\frac{x}{2}} = 2 \cdot \frac{1}{8}\tan^8 \frac{x}{2} + C$

$= \frac{1}{4}\tan^8 \frac{x}{2} + C$

31. $\int \frac{\sin(2t+1)}{\cos^2(2t+1)}\, \frac{d(\cos(2t+1))}{-2\sin(2t+1)} = -\frac{1}{2} \cdot \left(-\frac{1}{\cos(2t+1)}\right) + C$

$= \frac{1}{2\cos(2t+1)} + C$

38. $\int \sqrt{\frac{x-1}{x^5}}\, dx$

let $x = \sec^2\theta$

$dx = 2\sec^2\theta\tan\theta\, d\theta$

$\int \frac{\sqrt{\sec^2\theta - 1}}{\sqrt{\sec^{10}\theta}} \cdot 2\sec^2\theta\tan\theta\, d\theta$

$= \int \frac{2(\sec^2\theta - 1)}{\sec^3\theta}\, d\theta$

$= \int 2\cos\theta - 2\cos^3\theta\, d\theta$

$= \int 2\cos\theta \cdot \sin^2\theta \cdot \frac{d(\sin\theta)}{\cos\theta}.$

$= \frac{2}{3}\sin^3\theta + C = \frac{2}{3}\left(\frac{x-1}{x}\right)^{3/2} + C$

42. $\int \sqrt{\frac{x^4}{x^3-1}}\, \frac{d(x^3-1)}{3x^2}$

$= \frac{1}{3} \cdot 2 \cdot (x^3-1)^{1/2} + C$

$= \frac{2}{3}\sqrt{x^3-1} + C$

43. $\int x(x-1)^{10}\, dx = \int (x-1)^{11}\, dx + \int (x-1)^{10}\, dx$

$= \frac{1}{12}(x-1)^{12} + \frac{1}{11}(x-1)^{11} + C$

46. $\int (x-5+10)(x-5)^{1/3}\, dx$

$= \int (x-5)^{4/3}\, dx + \int 10(x-5)^{1/3}\, dx$

$= \frac{3}{7}(x-5)^{7/3} + \frac{15}{2}(x-5)^{4/3} + C$

54. $\displaystyle\int \frac{\sin\sqrt{\theta}}{\sqrt{\theta\cos^3\sqrt{\theta}}}\,d\theta = \int \frac{\sin\sqrt{\theta}}{\sqrt{\theta\cos^3\sqrt{\theta}}}\;\frac{d(\cos\sqrt{\theta})}{\frac{-\sin\sqrt{\theta}}{2\sqrt{\theta}}}$

$\displaystyle = -2\int \frac{d(\cos\sqrt{\theta})}{\cos^{3/2}\sqrt{\theta}}$

$= -2(-2)\cos^{-1/2}\sqrt{\theta} + C$

$\displaystyle = \frac{4}{\sqrt{\cos\sqrt{\theta}}} + C$

56. $\displaystyle\frac{dy}{dx} = 4x(x^2+8)^{-1/3}$

$\Rightarrow dy = 4x(x^2+8)^{-1/3}\,dx$

$\displaystyle y = \int 4x(x^2+8)^{-1/3}\,\frac{d(x^2+8)}{2x}$

$\displaystyle = 2\cdot\frac{3}{2}(x^2+8)^{2/3} + C$

$= 3(x^2+8)^{2/3} + C$

when $x=0, y=0$

$0 = 3(0+8)^{2/3} + C$

$C = -12$

$y = 3(x^2+8)^{2/3} - 12$

62. $\displaystyle a = \frac{d^2s}{dt^2} = \pi^2\cos\pi t$

$\displaystyle\Rightarrow \frac{d^2s}{dt} = \pi^2\cos\pi t\cdot dt$

$\displaystyle\frac{ds}{dt} = \int \pi^2\cos\pi t\cdot dt$

$\displaystyle = \pi^2\cdot\frac{1}{\pi}\sin\pi t + C$

$= \pi\sin\pi t + C = V(t)$

$V(0) = 0 + C = 8$

$C = 8$

$\displaystyle V(t) = \pi\sin\pi t + 8 = \frac{ds}{dt}$

$\Rightarrow ds = \pi\sin\pi t + 8\;dt$

$\displaystyle S = \int \pi\sin\pi t + 8\;dt$

$= -\cos\pi t + 8t + C$

$S(0) = -1 + 0 + C = 0$

$C = 1$

$S(1) = -\cos\pi + 8 + 1 = 10$ m

Exercises 5.6

8a. $\int_0^1 \frac{10\sqrt{v}}{(1+v^{3/2})^2} \frac{d(1+v^{3/2})}{\frac{3}{2}\sqrt{v}}$

$= \frac{20}{3}\left[-\frac{1}{1+v^{3/2}}\right]_0^1$

$= \frac{10}{3}$

b. $\int_1^4 \frac{10\sqrt{v}}{(1+v^{3/2})^2}\, dv$

$= \frac{20}{3}\left[-\frac{1}{1+v^{3/2}}\right]_1^4$

$= \frac{70}{27}$

11a. $\int_0^1 t\sqrt{4+5t}\, \frac{d(5t+4)}{5}$

$= \frac{1}{5}\int_0^1 \frac{(5t+4)-4}{5}\sqrt{4+5t}\, d(5t+4)$

$= \frac{1}{25}\left[\frac{2}{5}(5t+4)^{5/2} - \frac{8}{3}(5t+4)^{3/2}\right]_0^1$

$= \frac{506}{375}$

b. $\int_1^9 t\sqrt{4+5t}\, dt$

$= \frac{1}{25}\left[\frac{2}{5}(5t+4)^{5/2} - \frac{8}{3}(5t+4)^{3/2}\right]_1^9$

$= \frac{86744}{375}$

12a. $\int_0^{\pi/6} (1-\cos 3t)\sin 3t\, \frac{d(1-\cos 3t)}{-3(-\sin 3t)}$

$= \frac{1}{3}\left[\frac{1}{2}(1-\cos 3t)^2\right]_0^{\pi/6}$

$= \frac{1}{6}$

b. $\int_{\pi/6}^{\pi/3} (1-\cos 3t)\sin 3t\, dt$

$= \frac{1}{3}\left[\frac{1}{2}(1-\cos 3t)\right]_{\pi/6}^{\pi/3}$

$= \frac{1}{2}$

21. $\int_0^1 (4y-y^2+4y^3+1)^{-2/3}(12y^2-2y+4)\, dy$

$= \int_0^1 (4y-y^2+4y^3+1)^{-2/3}\, d(4y-y^2+4y^3+1)$

$= 3\left[(4y-y^2+4y^3+1)^{1/3}\right]_0^1$

$= 3$

23. $\int_0^{\pi^{2/3}} \sqrt{\theta}\cos^2(\theta^{3/2})\, \frac{d(\theta^{3/2})}{\frac{3}{2}\theta^{1/2}}$

$= \frac{2}{3}\int_0^{\pi^{2/3}} \frac{1+\cos(2\theta^{3/2})}{2}$

$= \frac{1}{3}\left[\theta^{3/2} + \frac{1}{2}\sin(2\theta^{3/2})\right]_0^{\pi^{2/3}}$

$= \frac{1}{3}\pi$

25. $A = \left|\int_{-2}^{0} x\sqrt{4-x^2}\, \frac{d(4-x^2)}{-2x}\right| + \left|\int_0^2 x\sqrt{4-x^2}\, \frac{d(4-x^2)}{-2x}\right|$

$= -\frac{1}{2}\cdot\frac{2}{3}\cdot 2\left[(4-x^2)^{3/2}\right]_0^2$

$= \frac{16}{3}$

59. $4x^2+y=4 \ldots (1)$ $\quad x^4-y=1 \ldots (2)$

(1) + (2)

$4-4x^2 = x^4-1$

$\Rightarrow x=-1$ and $x=1$

when $-1 \le x \le 1$, $4-4x^2 \ge x^4-1$

$A = \int_{-1}^{1} (4-4x^2)-(x^4-1)\, dx$

$= \left[5x - \frac{4}{3}x^3 - \frac{1}{5}x^5\right]_{-1}^{1}$

$= \frac{104}{15}$

60. $x^3-y=0 \ldots (1)$ $\quad 3x^2-y=4 \ldots (2)$

(1) − (2)

$x^3 = 3x^2-4$

$\Rightarrow x=-1$ and $x=2$

when $-1 \le x \le 2$, $x^3 \ge 3x^2-4$

$A = \int_{-1}^{2} (x^3)-(3x^2-4)\, dx$

$= \left[\frac{1}{4}x^4 - x^3 + 4x\right]_{-1}^{2}$

$= \frac{27}{4}$

63. $A = \left|\int_0^{\pi} (2\sin x - \sin 2x)\, dx\right|$

$= \left|\left[-2\cos x + \frac{1}{2}\cos(2x)\right]_0^{\pi}\right|$

$= |4|$

$= 4$

70. $A = \left|\int_{-1}^{1} \left(\sec^2\left(\frac{\pi x}{3}\right) - x^{1/3}\right) dx\right|$

$= \left|\left[\frac{3}{\pi}\tan\left(\frac{\pi x}{3}\right) - \frac{3}{4}x^{4/3}\right]\right|_{-1}^{1}$

$= \frac{6\sqrt{3}}{\pi}$

71. $y=\sqrt[3]{x}$ ...(1) $\quad y=x$ ...(2)

$\underline{(1)\&(2)}$

$x^3=x$

$\Rightarrow x=\{-1,0,1\}$

when $-1\le x\le 0$, $x\ge\sqrt[3]{x}$

$0\le x\le 1$, $\sqrt[3]{x}\ge x$

$$A=\int_{-1}^{0} x-\sqrt[3]{x}\,dx+\int_0^1 \sqrt[3]{x}-x\,dx$$

$$=2\int_{-1}^{0}(x-\sqrt[3]{x})\,dx$$

$$=2\left[\tfrac{1}{2}x^2-\tfrac{3}{4}x^{4/3}\right]_{-1}^{0}$$

$$=\tfrac{1}{2}$$

72. $y=x^3$...(1) $\quad y=x^5$...(2)

$\underline{(1)\&(2)}$

$x^3=x^5$

$\Rightarrow x=\{-1,0,1\}$

when $-1\le x\le 0$, $x^5\ge x^3$

$0\le x\le 1$, $x^3\ge x^5$

$$A=\int_{-1}^{0} x^5-x^3\,dx+\int_0^1 x^3-x^5\,dx$$

$$=2\int_{-1}^{0} x^5-x^3\,dx$$

$$=2\left[\tfrac{1}{6}x^6-\tfrac{1}{4}x^4\right]_{-1}^{0}$$

$$=\tfrac{1}{6}$$

74. $y=\sin x$ ...(1) $\quad y=\cos(x)$ ...(2)

$\underline{(1)\&(2)}$

$\sin x=\cos x$

$x=k\pi+\frac{\pi}{4}$ (since $x>0$, $x_{min}=\frac{\pi}{4}$)

$$A=\int_0^{\pi/4}\cos x-\sin x\,dx$$

$$=[\sin x+\cos x]_0^{\pi/4}$$

$$=-1+\sqrt{2}$$

78. $$A=\int_0^1 2\sqrt{y}\,dy+\int_1^2 (3-y)-(y-1)^2\,dy$$

$$=\left[\tfrac{4}{3}y^{3/2}\right]_0^1+\left[2y+\tfrac{1}{2}y^2-\tfrac{1}{3}y^3\right]_1^2$$

$$=\tfrac{4}{3}-0+\tfrac{10}{3}-\tfrac{13}{6}=\tfrac{5}{2}$$

82. sometimes true, if we have $f(x)<g(x)$ in the interval bounded by $x=a$ and $x=b$, the value of:

$A=\int_a^b f(x)-g(x)\,dx<0$ (which is impossible)

83. $$\int_1^3 \frac{\sin 2x}{x}\,\frac{d(2x)}{2}=[F(2x)]_1^3=F(6)-F(2)$$

84. RHS: $$\int_0^1 f(1-x)\,\frac{d(1-x)}{-1}=-\int_0^1 f(1-x)\,d(1-x)$$

$$=-\int_1^0 f(x)\,dx$$

$$=\int_0^1 f(x)\,dx$$

85a $$\int_{-1}^{0} f(x)\,dx=\int_{-1}^{1} f(x)\,dx-\int_0^1 f(x)\,dx=0-3=-3$$

b. $$\int_{-1}^{0} f(x)\,dx=\int_{-1}^{1} f(x)\,dx-\int_0^1 f(x)\,dx=2\cdot 3-3=3$$

86a. $$\int_{-a}^{a} f(x)\,dx=\int_{-a}^{0} f(x)\,dx+\int_0^a f(x)\,dx$$

$$=\int_{-a}^{0} f(x)\,dx-\int_0^{-a} f(-x)\,dx$$ note that: $\int_0^{-a} f(-x)\,dx=-\int_0^{a} f(x)\,dx$

$$=0$$

b. $$\int_{-\pi/2}^{\pi/2}\sin x\,dx=[-\cos x]_{-\pi/2}^{\pi/2}=0$$ (it is true)

87. $$I=\int_0^a \frac{f(x)\,dx}{f(x)+f(a-x)}=\int_a^0 \frac{f(a-x)}{f(a-x)+f(x)}\,\frac{dx}{-1}$$

$$I=\int_0^a \frac{f(a-x)}{f(x)+f(a-x)}\,dx$$

$$I+I=\int_0^a \frac{f(x)}{f(x)+f(a-x)}+\frac{f(a-x)}{f(x)+f(a-x)}\,dx$$

$$2I=\int_0^a 1\,dx$$

$$2I=a$$

$$I=\tfrac{1}{2}a$$