

yphandi - quiz 3  
(120040025)

1a. velocity =  $\frac{d(s(t))}{dt} = v(t) = 2t + \pi \cos(\pi t)$

speed =  $|v(t)|$

when  $t=1$ :

velocity =  $v(1) = 2 - \pi$

speed =  $|2 - \pi| = \pi - 2$

b. acceleration =  $\frac{d(v(t))}{dt} = a(t) = 2 - \pi^2 \sin(\pi t)$

when  $t=1$ :

acceleration =  $a(1) = 2$

2a.  $y = \tan(x^3 \cdot e^{-x})$   $\frac{d}{dx}$   
 $\frac{dy}{dx} = \sec^2(x^3 \cdot e^{-x}) \cdot [3x^2 \cdot e^{-x} + x^3 \cdot (-e^{-x})]$   
 $= \sec^2(x^3 \cdot e^{-x}) \cdot [3x^2 \cdot e^{-x} - x^3 \cdot e^{-x}]$   
 $= \frac{x^2}{e^x} \cdot \sec^2(x^3 \cdot e^{-x}) \cdot (3 - x)$

b.  $y = \sin^3(e^{\tan x})$   $\frac{d}{dx}$   
 $\frac{dy}{dx} = 3 \sin^2(e^{\tan x}) \cdot \cos(e^{\tan x}) \cdot \sec^2 x \cdot e^{\tan x}$

c.  $y^5 \cdot \ln y = e^{xy} + \sec^2 x$   $\frac{d}{dx}$   
 $5y^4 \ln y \cdot \frac{dy}{dx} + y^5 \cdot \frac{1}{y} \cdot \frac{dy}{dx} = (y + x \cdot \frac{dy}{dx}) \cdot e^{xy} + 2 \sec^2 x \tan x$   
 $\frac{dy}{dx} [5y^4 \ln y + y^4 - x \cdot e^{xy}] = y \cdot e^{xy} + 2 \sec^2 x \tan x$   
 $\frac{dy}{dx} = \frac{y \cdot e^{xy} + 2 \sec^2 x \tan x}{5y^4 \ln y + y^4 - x \cdot e^{xy}}$

3.  $y = (1 - \cos x)^{\sqrt{x}}$

$\ln(y) = \sqrt{x} \cdot \ln(1 - \cos x)$   $\frac{d}{dx}$

$\frac{1}{y} \cdot \frac{dy}{dx} = \frac{1}{2\sqrt{x}} \cdot \ln(1 - \cos x) + \sqrt{x} \cdot \frac{1}{1 - \cos x} \cdot \sin x$

$\frac{dy}{dx} = y \left( \frac{1}{2\sqrt{x}} \cdot \ln(1 - \cos x) + \frac{\sqrt{x} \cdot \sin x}{1 - \cos x} \right)$

$= \frac{(1 - \cos x)^{\sqrt{x}}}{2\sqrt{x}} \ln(1 - \cos x) + (1 - \cos x)^{\sqrt{x}-1} \cdot \sqrt{x} \sin x$

4.  $\lim_{x \rightarrow 0^-} x^3 \sin \frac{1}{x} = \lim_{x \rightarrow 0^+} x^3 \sin \frac{1}{x} = 0$

$\Rightarrow \lim_{x \rightarrow 0} x^3 \sin \frac{1}{x} = 0$

now since  $\lim_{x \rightarrow 0} f(x) = f(0)$ ,  $f(x)$  is continuous

which satisfy the first requirement to be differentiable

$f'(x) = \begin{cases} \frac{d}{dx}(x^3 \sin \frac{1}{x}) = 3x^2 \sin \frac{1}{x} - x \cos \frac{1}{x}, & x \neq 0 \\ 0, & x = 0 \end{cases}$

$\lim_{x \rightarrow 0^-} f'(x) = \lim_{x \rightarrow 0^-} 3x^2 \sin \frac{1}{x} - \lim_{x \rightarrow 0^-} x \cos \frac{1}{x}$   
 $= 0 \cdot 0 - 0 = 0$

$\lim_{x \rightarrow 0^+} f'(x) = \lim_{x \rightarrow 0^+} 3x^2 \sin \frac{1}{x} - \lim_{x \rightarrow 0^+} x \cos \frac{1}{x}$   
 $= 0 \cdot 0 - 0 = 0$

$\Rightarrow \lim_{x \rightarrow 0} f'(x) = 0$

since  $\lim_{x \rightarrow 0} f'(x) = f'(0^-) = f'(0^+) = f'(0)$   
 $= 0$ ,

~~$f(x)$  is differentiable at  $x=0$~~

$f'(x)$  is continuous at  $x=0$ .

thus,  $f(x)$  is differentiable at  $x=0$