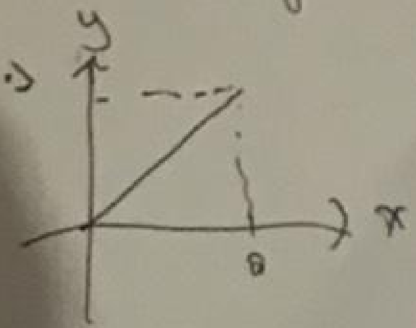
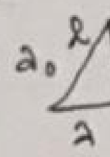
yohandi - quiz 8

1. length : $\int_0^a \sqrt{1+(f'(x))^2}\,dx = \sqrt{2}\,a$

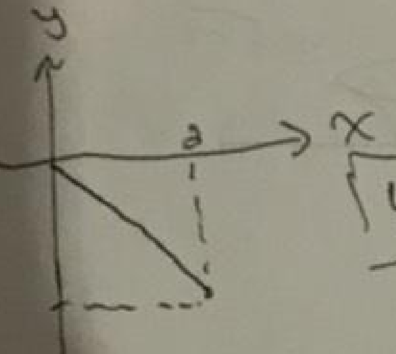


$\rightarrow$ if ~~we~~ we have $\boxed{y=f(x)=x}$.
we have length $= \sqrt{2}\,a$.



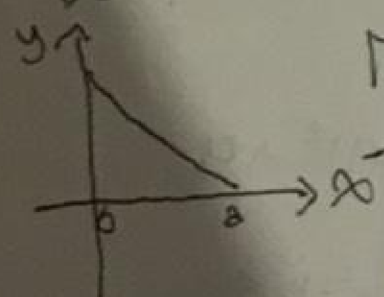
$l = \sqrt{a^2+a^2}$
$= 2\sqrt{a}$

$\rightarrow$ if we flip it to the x-axis, the length will still be the same



$\boxed{y=f(x)=-x}$

$\rightarrow$ it is also possible to flip it to the ~~y~~ $x=a/2$, the length will still be the same



$\boxed{y=f(x)=a-x}$

proof :

$\rightarrow \int_0^a \sqrt{1+((x)')^2}\,dx = \int_0^a \sqrt{1+(1)^2}\,dx = \sqrt{2}\,a$

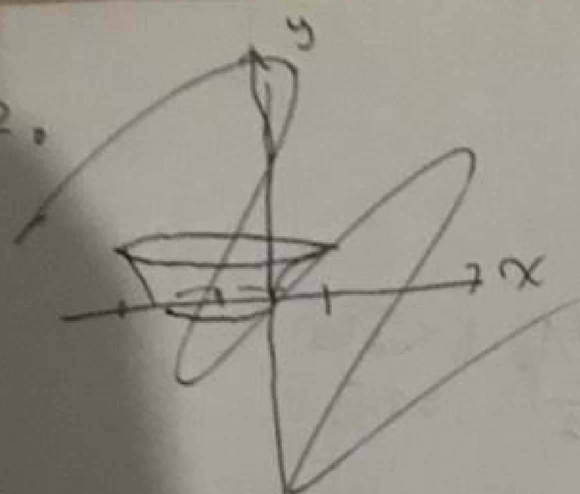
$\rightarrow \int_0^a \sqrt{1+((-x)')^2}\,dx = \int_0^a \sqrt{1+(-1)^2}\,dx = \sqrt{2}\,a$

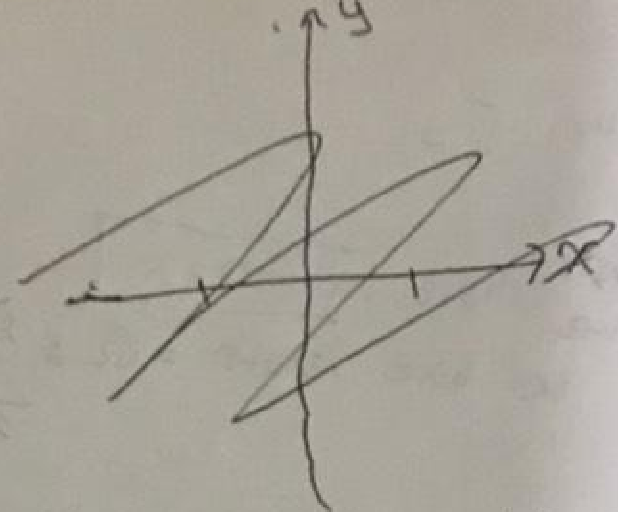
$\rightarrow \int_0^a \sqrt{1+((a-x)')^2}\,dx = \int_0^a \sqrt{1+(-1)^2}\,dx = \sqrt{2}\,a$.

note that it is possible to make infinite solutions such as ~~f(x)=...~~
$y = \boxed{f(x)=c\pm x}$

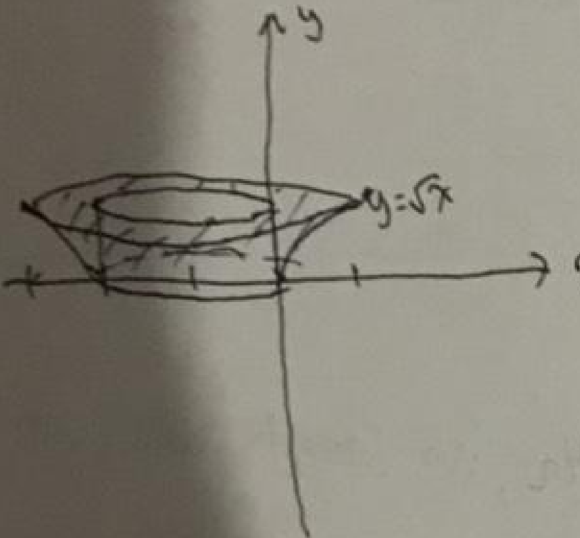
it doesn't have the other solution beside because if we ~~...~~ don't make a straight line, intuitively the length must be $> \sqrt{2}\,a$.

2.



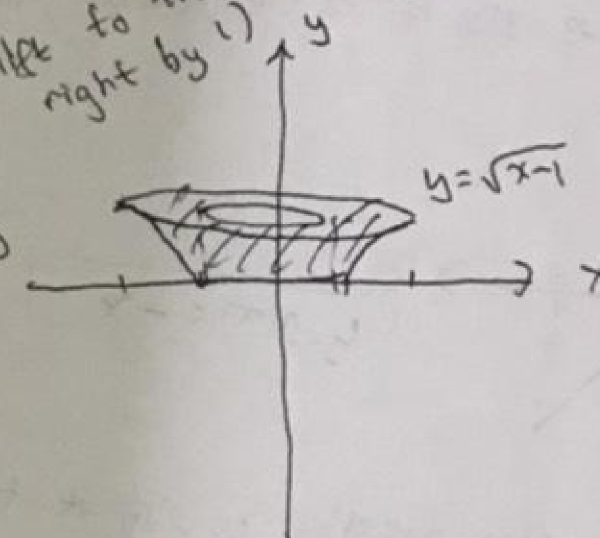


2.



have the volume as →

(shift to the right by 1)



$y=\sqrt{x-1}$

$\Rightarrow y^2+1=x$

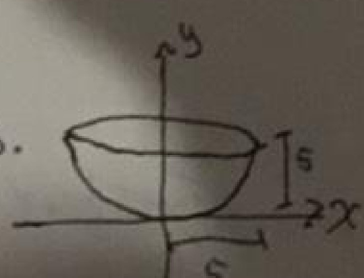
$$\text{volume} = \int_0^1 \pi (y^2+1)^2 \, dy$$

$$= \pi \int_0^1 y^4+2y^2+1 \, dy$$

$$= \pi \left[\frac{1}{5}y^5+\frac{2}{3}y^3+y\right]_0^1$$

$$= \boxed{\frac{28\pi}{15}}$$

3.



$x = \pm\sqrt{25-(y-5)^2}$

$= \pm\sqrt{10y-y^2}$

$$\text{volume}(y) = \int_0^y \pi\left(\sqrt{10y-y^2}\right)^2 dy$$

$(\pi(r(x))^2)$

given: $\frac{dV}{dt} = 0.2 \; m^3/s$

$\frac{dh}{dt} = \frac{dV}{dt}\cdot\frac{dh}{dV} = 0.2 \cdot \frac{1}{\pi(10y-y^2)} \; m/s$

when $y=1$ (4 m deep)

$\frac{dh}{dt}(1) = 0.2 \cdot \frac{1}{\pi(10\cdot 1-1^2)} \; m/s$

$= 0.2 \frac{1}{\pi 9} \; m/s$

$= \boxed{\cancel{\frac{1}{30}}}$ $\boxed{\frac{1}{45\pi} \; m/s}$