STA2001 Assignment 8: Bivariate Distribution: Conditional, Continuous, Normal

1. 4.3-1. Let X and Y have the joint pmf

$$f(x,y) = \frac{x+y}{32}, \quad x = 1,2 \quad y = 1,2,3,4.$$

(a) Display the joint pmf and the marginal pmfs on a graph like Figure 4.3-1(a).

(b) Find $g(x|y)$ and draw a figure like Figure 4.3-1(b), depicting the conditional pmfs for y = 1, 2, 3, and 4.

(c) Find $h(y|x)$ and draw a figure like Figure 4.3-1(c), depicting the conditional pmfs for x = 1 and 2.

(d) Find $P(1 \leq Y \leq 3|X = 1)$, $P(Y \leq 2|X = 2)$, and $P(X = 2|Y = 3)$.

(e) Find $E(Y|X = 1)$ and $\text{Var}(Y|X = 1)$.

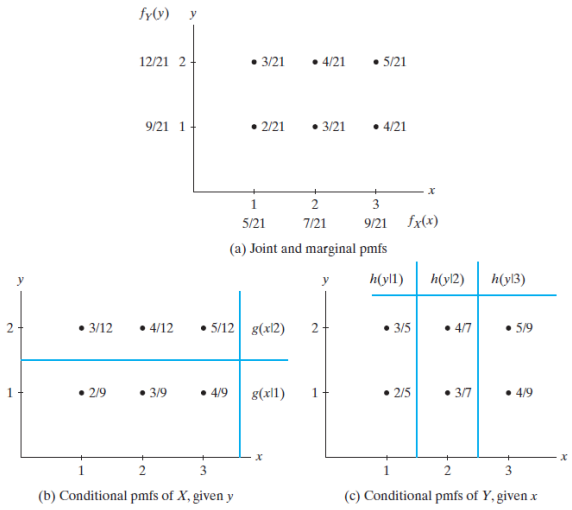

Figure 4.3-1 Joint, marginal, and conditional pmfs

2. 4.3-4. The alleles for eye color in a certain male fruit fly are (R, W). The alleles for eye color in the mating female fruit fly are (R, W). Their offspring receive one allele for eye color from each parent. If an offspring ends up with either (W, W), (R, W), or (W, R), its eyes will look white. Let X equal the number of offspring having white eyes. Let Y equal the number of white-eyed offspring having (R, W) or (W, R) alleles.

   (a) If the total number of offspring is n = 400, how is X distributed?

   (b) Give the values of E(X) and Var(X).

   (c) Given that X = 300, how is Y distributed?

   (d) Give the value of $E(Y|X = 300)$ and the value of $\text{Var}(Y|X = 300)$.

3. 4.3-8. A fair six-sided die is rolled 30 independent times. Let X be the number of ones and Y the number of twos.

   (a) What is the joint pmf of X and Y?

   (b) Find the conditional pmf of X, given Y = y.

   (c) Compute $E(X^2 - 4XY + 3Y^2)$.

4. 4.4-7. Let f (x, y) = 4/3, $0 < x < 1$, $x^3 < y < 1$, zero elsewhere.

   (a) Sketch the region where $f(x, y) > 0$.

   (b) Find $P(X > Y)$.

5. 4.4-9. Two construction companies make bids of X and Y (in \$100,000's) on a remodeling project. The joint pdf of X and Y is uniform on the space $2 < x < 2.5$, $2 < y < 2.3$. If X and Y are within 0.1 of each other, the companies will be asked to rebid; otherwise, the low bidder will be awarded the contract. What is the probability that they will be asked to rebid?

6. 4.4-15. An automobile repair shop makes an initial estimate X (in thousands of dollars) of the amount of money needed to fix a car after an accident. Say X has the pdf

$$f(x) = 2e^{-2(x-0.2)}, \quad 0.2 < x < \infty$$

Given that X = x, the final payment Y has a uniform distribution between $x - 0.1$ and $x + 0.1$.What is the expected value of Y?

7. 4.4-18. Let f (x, y) = 1/8, $0 \leq y \leq 4$, $y \leq x \leq y + 2$, be the joint pdf of X and Y.

   (a) Sketch the region for which $f(x, y) > 0$.

   (b) Find $f_X(x)$, the marginal pdf of X.

   (c) Find $f_Y(y)$, the marginal pdf of Y.

   (d) Determine $h(y|x)$, the conditional pdf of Y, given that X = x.

   (e) Determine $g(x|y)$, the conditional pdf of X, given that Y = y.

   (f) Compute $E(Y|x)$, the conditional mean of Y, given that X = x.

   (g) Compute $E(X|y)$, the conditional mean of X, given that Y = y.

   (h) Graph $y = E(Y|x)$ on your sketch in part (a). Is $y = E(Y|x)$ linear?

   (i) Graph $x = E(X|y)$ on your sketch in part (a). Is $x = E(X|y)$ linear?

8. 4.5-1. Let X and Y have a bivariate normal distribution with parameters $\mu_X = -3$, $\mu_Y = 10$, $\sigma_X^2 = 25$,$\sigma_Y^2 = 9$, and $\rho = 3/5$. Compute

   (a) $P(-5 < X < 5)$.

   (b) $P(-5 < X < 5|Y = 13)$.

   (c) $P(7 < Y < 16)$.

   (d) $P(7 < Y < 16|X = 2)$.

9. 4.5-6. For a freshman taking introductory statistics and majoring in psychology, let X equal the student's ACT mathematics score and Y the students ACT verbal score. Assume that X and Y have a bivariate normal distribution with $_X = 22.7$, $\sigma_X^2 = 17.64$, $_Y = 22.7$, $\sigma_Y^2 = 12.25$, and $\rho = 0.78$.

   (a) Find $P(18.5 < Y < 25.5)$.

   (b) Find $E(Y|x)$.

   (c) Find $\text{Var}(Y|x)$.

   (d) Find $P(18.5 < Y < 25.5|X = 23)$.

   (e) Find $P(18.5 < Y < 25.5|X = 25)$.

   (f) For x = 21, 23, and 25, draw a graph of $z = h(y|x)$ similar to Figure 4.5-1.

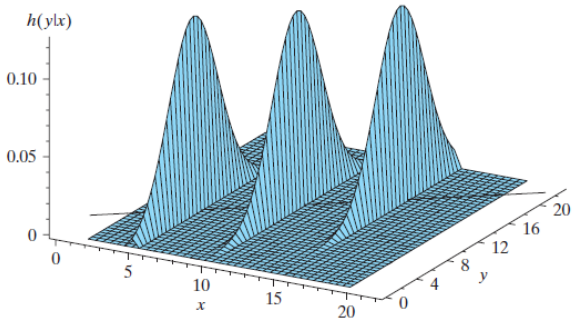

Figure 4.5-1 Conditional pdf of Y, given that $x = 5, 10, 15$

10. Let

$$f(x,y) = (\frac{1}{2\pi})e^{-(x^2+y^2)/2}[1 + xye^{-(x^2+y^2-2)/2}], \quad -\infty < x < \infty, -\infty < y < \infty.$$

Show that $f(x,y)$ is a joint pdf and the two marginal pdfs are each normal. Note that X and Y can each be normal, but their joint pdf is not bivariate normal.