

$$3. f = \frac{1}{2\pi} \sqrt{\frac{k}{m}}$$

$$k = (2\pi f)^2 \cdot m$$

$$k_T = k_L + k_R$$

$$\Rightarrow f_T = \frac{1}{2\pi} \sqrt{\frac{k_L + k_R}{m}}$$

$$= \sqrt{f_L^2 + f_R^2}$$

$$= \sqrt{30^2 + 50^2} \text{ Hz}$$

$$= 10\sqrt{34} \text{ Hz}$$

$$7a. I = \sum k_i \cdot m_i \cdot R_i^2$$

$$= \frac{1}{2} m r^2 + \frac{1}{12} M L^2 + M \left(\frac{L}{2}\right)^2 + m(L+r)^2$$

$$= 0,203 \text{ kg m}^2$$

$$b. y_{\text{com}} = \frac{m_1 \cdot y_1 + m_2 \cdot y_2}{m_1 + m_2}$$

$$= \frac{0,5(-0,6) + 0,25(-0,25)}{0,5 + 0,25} \text{ m}$$

$$= -0,483 \text{ m}$$

$$d = |y_{\text{com}}| = 0,483 \text{ m}$$

$$c. T = 2\pi \sqrt{\frac{I}{(\sum m_i) g \cdot d}}$$

$$= 1,5 \text{ s}$$

$$12a. k = \frac{F}{\Delta x} = \frac{mg}{\Delta x} = 58800 \text{ N/m}$$

$$b. \text{damping ratio} = \frac{b}{2\sqrt{mk}}$$

$$\frac{1}{2} = \frac{b}{2\sqrt{600 \cdot 58800}}$$

$$b = 5939,7 \text{ kg/s}$$

$$13a. m_1 v_1 + m_2 v_2 = (m_1 + m_2) v'$$

$$v' = \frac{m_1 v_1}{m_1 + m_2} = 1,19 \text{ s}$$

$$b. \frac{1}{2} (m_1 + m_2) v'^2 = \frac{1}{2} k x_m^2$$

$$x_m = \sqrt{\frac{(m_1 + m_2) v'^2}{k}}$$

$$= v' \sqrt{\frac{m_1 + m_2}{k}}$$

$$= 0,036 \text{ m}$$

$$14. f = \frac{1}{2\pi} \sqrt{\frac{k}{m}}$$

$$f_T = \frac{1}{2\pi} \sqrt{\frac{2500}{0,490}} \text{ Hz}$$

$$= 11,368 \text{ Hz}$$

$$29. d = |\Delta x| = |x_f - x_i|$$

$$= \left| \frac{Mg}{k} - \frac{(M+4m)g}{k} \right|$$

$$= \frac{4mg}{k}$$

$$= \frac{4mg}{(M+4m)\omega^2}$$

$$= \frac{4mg}{M+4m} \cdot \left(\frac{d}{2\pi v}\right)^2$$

$$= 0,0776 \text{ m}$$

$$53a. f = \frac{1}{2\pi} \sqrt{\frac{A}{X}} = 5,982 \text{ Hz}$$

$$b. \left| \frac{A}{X} \right| = \frac{k}{m}$$

$$m = k \cdot \left| \frac{X}{A} \right| = 0,3455 \text{ kg}$$

$$c. E_1 = E_2$$

$$\frac{1}{2} k x_m^2 = \frac{1}{2} k x^2 + \frac{1}{2} m v^2$$

$$x_m = \sqrt{x^2 + \frac{m}{k} \cdot v^2} = 0,400 \text{ m}$$

$$59a. \Omega_{\text{max}} = \omega \cdot \theta_m = \left(\frac{2\pi}{T}\right) \pi = 32,90 \text{ rad/s}$$

$$b. \Omega = \left| \frac{d(\theta(t))}{dt} \right|$$

$$= |-\omega \theta_m \sin(\omega t + \varphi)|$$

$$\text{when } \cos(\omega t + \varphi) = \frac{x}{x_m} = \frac{\pi/2}{\pi} = \frac{1}{2}$$

$$\Rightarrow \sin(\omega t + \varphi) = \sqrt{1 - \left(\frac{1}{2}\right)^2} = \frac{\sqrt{3}}{2}$$

$$\Omega = |-\omega \cdot \theta_m \cdot \frac{\sqrt{3}}{2}| = 28,49 \text{ rad/s}$$

$$c. \alpha = \frac{d(\Omega)}{dt}$$

$$= -\omega^2 \theta_m \cos(\omega t + \varphi)$$

$$= -\omega^2 \cdot \theta$$

$$\text{when } \theta = \frac{\pi}{4}$$

$$\Rightarrow \alpha = -\omega^2 \cdot \frac{\pi}{4} = -86,13 \text{ rad/s}^2$$