

Quiz 2

(15 minutes on Tuesday, 22 Sep 2020)

1. [12 points] Determine if the following statements are True or False (no need to show your work):

(a) $y = x - 1$ is a tangent line of the curve $y = f(x) = \ln x$ at some point $x > 0$.

(b) $f(x) = \begin{cases} x^2 & x \leq 1 \\ 4\sqrt{x} & x > 1 \end{cases} \Rightarrow f'(x) = \begin{cases} 2x & x \leq 1 \\ 2x^{-1/2} & x > 1 \end{cases} \Rightarrow f'(1-) = f'(1) = f'(1+) = 2$

(c) $\lim_{x \rightarrow 8} \frac{x^{2/3} - 4}{x - 8} = \frac{1}{3}$ can be obtained by the derivative formula $(x^\alpha)' = \alpha x^{\alpha-1}$.

Show your work for the questions below:

2. [10 points] Obtain the derivative of $y = \sec x$ by calculating the limit:

$$(\sec x)' = \lim_{h \rightarrow 0} \frac{\sec(x+h) - \sec x}{h}$$

You may use the trigonometric identity $\cos(a+b) = \cos a \cos b - \sin a \sin b$.

3. [18 points] Find the derivatives of the following functions $y = f(x)$ by the rules of differentiation:

(a) $y = 3 \cot x \ln x + x^8 4^x$

(b) $y = \frac{x^5 - 3x^3 + 1}{x + \sqrt{x}}$

(c) $y = h^{-1}(x)$ for $h(x) = e^x + 2x$ (you can keep y in the result)