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CSC3001 Assignment 5

1) G is bipartite ⇒ can be partitioned into V_1 & V_2

let $|V_1| = n \Rightarrow |V_2| = v-n$

for V_1 & V_2, maximum number of edges are $f(n) = n(v-n)$. here, $0 \le n \le v$.

we aim to find n that maximizes $f(n)$,

$$\frac{df(n)}{dn} = \frac{d(n(v-n))}{dn} = v-2n$$

when $\frac{df(n)}{dn} = 0$, $n = \frac{v}{2}$

$\Rightarrow n = \frac{v}{2}$ maximizes $f(n) = f\left(\frac{v}{2}\right) = \frac{v^2}{4}$

$$\therefore e = f(n) \le \frac{v^2}{4}$$

2) $|K| = C(|V|, 2) = \frac{|V|(|V|-1)}{2} = 15+13 = 28$

$\Rightarrow |V| = 8$

3a) V_1 = {Ping, Quiggley, Ruiz, Sitea}

V_2 = {Hardware, Software, Networking, Wireless}.

Ping Quiggley Ruiz Sitea

Hardware Software Networking Wireless

b) compute all possibilities:

let P, Q, R, and S denote Ping, Quiggley, Ruiz, and Sitea respectively and A, B, C, and D denote Hardware, Software, Networking, and Wireless.

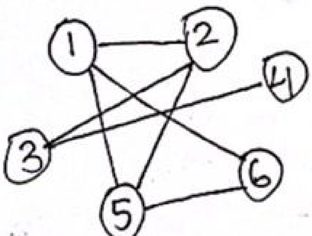
subsets V_1	neighborhood	|subsets|	|neighborhood|
∅	∅	0	0
P	A, C, D	1	3
Q	B, C	1	2
R	C, D	1	2
S	A, B	1	2
P, Q	A, B, C, D	2	4
P, R	A, C, D	2	3
P, S	A, B, C, D	2	4
Q, R	B, C, D	2	3
Q, S	A, B, C	2	3
R, S	A, B, C, D	2	4
P, Q, R	A, B, C, D	3	4
P, Q, S	A, B, C, D	3	4
P, R, S	A, B, C, D	3	4
Q, R, S	A, B, C, D	3	4
P, Q, R, S	A, B, C, D	4	4

Since |subsets| ≥ |neighborhood|, there exists at least one assignment

c) Ping - Hardware, Quiggley - Networking, Ruiz - Wireless, Sitea - Software

4.

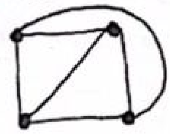
	1	2	3	4	5	6
1	-	85	175	200	50	100
2	85	-	125	175	100	160
3	175	125	-	100	200	250
4	200	175	100	-	210	220
5	50	100	200	210	-	100
6	100	160	250	220	100	-



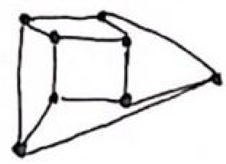
channel 1: 2, 4, 6
channel 2: 1, 3
channel 3: 5

} 3 channels are needed

5a) $v = 4$
$e = 6$
$f = 5$



b) $v = 8$
$e = 12$
$f = 8$



c) $v = 6$
$e = 9$
$f = 13$

we know that $v \ge 3$ and there are not cycles w/ of length 3

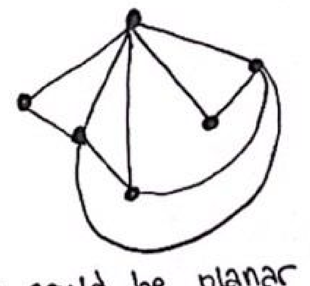
If the graph is planar,

$$e \le 2v - 4$$

however, $2v - 4 = 8 \ngeq 9$; therefore, the graph is non-planar

6. $e = \frac{5+4+4+3+2+2}{2} = 10$

$v - e + f = 2 \Rightarrow f = 6$



G could be planar

7. LHS:

How many subsets of size r from a set ~~of~~ of size m+n ? $\binom{m+n}{r}$

RHS:

We split the set into two subsets of size m and n.

let $|S_1| = m$ & $|S_2| = n$.

if we take 0 element from S_2, total combinations are $\binom{m}{r}\binom{n}{0}$

if we take 1 element from S_2, total combinations are $\binom{m}{r-1}\binom{n}{1}$

$\vdots$

if we take r elements from S_2, total combinations are $\binom{m}{0}\binom{n}{r}$

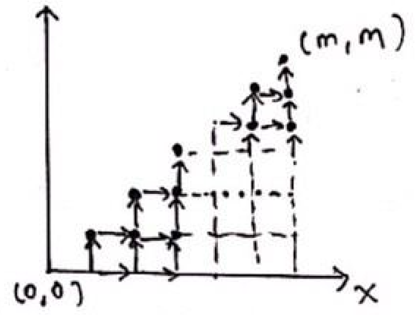
$\Rightarrow$ there are

$$\binom{m}{r}\binom{n}{0} + \binom{m}{r-1}\binom{n}{1} + \dots + \binom{m}{0}\binom{n}{r}$$

$$= \sum_{k=0}^{r} \binom{m}{r-k}\binom{n}{k}$$

combinations

8. Suppose we have a cartesian diagram, let (x, y) be a position where x denotes the total ~~num~~ of zeroes in the partitioned sequence from 1 to x+y and y denotes the total of ones. we aim to find the total combinations - of path from (0,0) to (m, m) without having x < y, even for a single time.



suppose we don't have the constraint of $x \geq y$, then we can ~~...~~ count the possible combinations by arranging

$$\underbrace{RRR\_\_\_RR}_{m}\underbrace{UUU\_\_\_UU}_{m}$$

where R denotes right, and U denotes up.

$\Rightarrow \binom{2m}{m}$ possible combinations without the constraint.

to calculate the "bad" path, we consider the (m+1) ~~steps~~ up steps and (m-1) right steps where the monotonic path in the $(m-1)\times(m+1)$ grid meets the higher diagonal

$$\Rightarrow \binom{m-1+m+1}{m-1} = \binom{2m}{m-1} = \binom{2m}{m+1}$$

$\therefore$ the number of abnormal 0-1 sequences is $\binom{2m}{m+1}$

$\therefore$ the number of normal 0-1 sequences is $\binom{2m}{m} - \binom{2m}{m+1}$

a) LHS:

$$\binom{2m}{m+1} = \binom{(m-1)+(m+1)}{m+1}$$

= number of sequences $\{a_n\}$ with (m+1) terms and (m-1) terms

(shown)

b) m=4

$\Rightarrow \binom{8}{4} - \binom{8}{5} = 14$ combinations