MAT3007 - Assignment 5

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Problem 1.

(a) The dual problem of the linear program is obtained as follows:

$$\begin{aligned}
\texttt{min} \quad & 12y_1 + 8y_2 + 10y_3 \\
\texttt{s.t.} \quad & 2y_1 + y_2 \geq 3 \\
& y_1 + y_2 - y_3 \geq 4 \\
& -y_1 + y_2 + 2y_3 \geq 3 \\
& y_1 + y_2 + y_3 \geq 6 \\
& y_1 \leq 0, y_3 \geq 0
\end{aligned}$$

From the tableau, we notice that the optimal solution for x is $[4, 0, 0, 4]$. Along with the complementary conditions, we obtain that:

- $2y_1 + y_2 - 3 = 0$,
- $y_1 + y_2 + y_3 - 6 = 0$, and
- $y_3 = 0$ (since $-x_2 + 2x_3 + x_4 \neq 0$).

This implies that $y = [-3, 9, 0]$. As x is unique and y is derived accordingly from x, we conclude that the dual solution is unique.

(b) Since x_3 is not in B, we check $r_N^T + \lambda e_2 = [2\ 9\ 3]^T + \lambda[0\ -1\ 0]^T \geq 0$, which implies $\lambda \leq 9$. This means, as long as we change the c_3 to be somewhere around $(-\infty, 12]$, the basis that construct the optimal solution will not change.

- The above implies that the optimal solution for setting c_3 to 0 will not change. Moreover the objective value also stays the same as $y^{*T}\Delta b = 0$.
- Again, by the computed λ above, setting c_3 to 9 will not change the optimal solution and optimal value.

This time x_4 is in B, we check $r_N^T - \lambda e_2 A_B^{-1} A_N = [2\ 9\ 3]^T - \lambda([0\ -1\ 0][[0\ -2\ -1]\ [1\ 3\ 1]\ [-2\ -1\ -1]])^T = [2\ 9\ 3]^T + \lambda[1\ 3\ 1]^T \geq 0$, which implies $\lambda \geq -2$. As long as c_4 is changed to be somewhere around $[4, \infty)$, the basis that construct the optimal solution will not change.

- The above implies that changing the c_4 to 5 result in the same optimal solution. However, the optimal value indeed changes accordingly. As $y^{*T}\Delta b = 4$, the objective value becomes $36 - 4 = 32$.

As x_1 is in B, we check $r_N^T - \lambda e_1 A_B^{-1} A_N = [2\ 9\ 3]^T - \lambda([-1\ 0\ 0][[0\ -2\ -1]\ [1\ 3\ 1]\ [-2\ -1\ -1]])^T = [2\ 9\ 3]^T + \lambda[0\ -2\ -1]^T \geq 0$, which implies $\lambda \leq 3$. As long as c_1 is changed to be somewhere around $(-\infty, 6]$, the basis that construct the optimal solution will not change.

- The above implies that changing the c_1 to 7 result in different optimal solution, so does the optimal value.

(c) We need to check the range for λ that satisfies $x_B + \lambda A_B^{-1} e_2$.

$x_B + \lambda A_B^{-1} e_2 = [4\ 4\ 6] + \lambda[[1\ -1\ 0]\ [-1\ 2\ 0]\ [1\ -2\ 1]][0\ 1\ 0] = [4\ 4\ 6] + \lambda[-1\ 2\ -2] \geq 0$, which implies $-2 \leq \lambda \leq 3$. Then, the possible range is $[6, 11]$.

Problem 2.

(a) $y^* = A_B^{-T} c_B = [[1\ 0\ 0]\ [0\ 0\ \frac{1}{4}]\ [0\ 1\ \frac{1}{4}]]^T [1\ 8\ 0] = [1\ 0\ 2]$.

Since c_2 is not in B, we check $r_N^T + \lambda e_1 = [1\ 4\ 1\ 2]^T + \lambda[-1\ 0\ 0\ 0]^T \geq 0$, which implies that $\lambda \leq 1$. As long as c_2 is changed to be somewhere around $(\infty, 3]$, the basis that construct the optimal solution won't change.

(b) The above derivation directly answers the question, i.e., no basis construction change occurs. Moreover, as $x_2 = 0$, the change will not affect the optimal solution for which $x = [2\ 0\ 0\ 2]$ with optimal value 18.

(c) We need to ensure that:

$x_B + \lambda A_B^{-1} e_3 = [2\ 2\ 3]^T + \lambda[[1\ 0\ 0]\ [0\ 0\ \frac{1}{4}]\ [0\ 1\ \frac{1}{4}]][0\ 0\ 1]^T = [2\ 2\ 3]^T + \lambda[0\ \frac{1}{4}\ \frac{1}{4}]^T \geq 0$, which implies $\lambda \geq -8$. As long as b is changed to be somewhere around $[0, \infty)$, the basis that construct the optimal solution will not change.

(d) As 4 lies in the local sensitivity area, the basis that construct the optimal solution will not change. We notice that only x_4 is affected by the change; hence, x_4 becomes 1. The new optimal solution is $x = [2\ 0\ 0\ 1]$ that leads 10 objective value.