yohandi - quiz 1

1 a. false

b. ~~true~~ false

c. ~~fa~~ true

2 a. $\lim_{x \to 0} \frac{1-\cos 4x}{\ln^2(1+3x)}$

when $x \to 0$, $1-\cos x \sim 2\sin^2 \frac{1}{2}x$, $\ln(1+x) \sim x$

$$\lim_{x \to 0} \frac{1-\cos 4x}{\ln^2(1+3x)} = \lim_{x \to 0} \frac{2\sin^2 2x}{(3x)^2} = 2 \lim_{x \to 0} \left(\frac{\sin 2x}{2x} \cdot \frac{2x}{3x}\right)^2 = 2 \cdot \frac{4}{9} = \boxed{\frac{8}{9}}$$

b $$\lim_{x \to 2^+} \frac{x^2+x-6}{x^3-4x^2+4x} = \lim_{x \to 2^+} \frac{(x+3)(x-2)}{x(x-2)^2} = \lim_{x \to 2^+} \frac{(x+3)}{x(x-2)} = \boxed{+\infty}$$

c. $$\lim_{x \to \infty} \tan\left(\frac{3x^{3/2}+2}{2x^{5/3}+x-4}\right) = \tan\left(\lim_{x \to \infty} \frac{3+2x^{-3/2}}{2x^{1/6}+x^{-1/2}-4x^{-3/2}}\right) = \tan(0) = \boxed{0}$$

3. $f(x) = \frac{x-\pi}{|\sin x|}$

~~we know that the f(x)'s limit is not defined when $x \to 0^+$ resulting $-\infty$~~

~~when $x=\pi$, $\lim_{x \to \pi} f(x) = 0$~~,

$$\lim_{x \to \pi^+} \frac{x-\pi}{|\sin x|} = \lim_{x \to \pi} \frac{x-\pi}{-\sin x} = \lim_{x \to \pi} \frac{x-\pi}{-\sin(\pi-x)} = 1$$

$$\lim_{x \to \pi^-} \frac{x-\pi}{|\sin x|} = \lim_{x \to \pi} \frac{x-\pi}{\sin x} = \lim_{x \to \pi} \frac{x-\pi}{\sin(\pi-x)} = -1$$

Since $\lim_{x \to \pi^+} f(x) \neq \lim_{x \to \pi^-} f(x)$, the function is discontinuous at $x=\pi$.

we also know that for all $x \neq \pi$, and $x = k.\pi$, value of $|\sin x| = 0$

~~where~~, $\lim_{x \to k.\pi} f(x) = \pm\infty$ (vertical asymptote)

∴ discontinuous at $x=\pi$, vertical asymptote at $x = k.\pi$ $(k \neq 1)$, for all x that is not multiple of π, f(x) is continuous