MAT1001 – T01-04

Student ID   Name:

Quiz 4

(15 minutes on Tuesday, 13 Oct 2020)

1. [12 points] Determine if the following statements are True or False (no need to show your work):

(a) To find all maximum and minimum points, it suffices to check every critical point and the boundary points of the domain (if there is any).

(b) If x_0 is an extreme point of a differential function f on $\mathbb{R}$, then the derivative f' of f is monotone in $(x_0, x_0+\delta)$ for some $\delta > 0$.

(c) If $f'(x) \neq 0$ for all $x \in (a,b)$, then $f(x) = 0$ has at most one solution in (a,b).

(d) If a function f is twice differentiable and concave up on an open interval I, then its second derivative $f''(x) > 0$ for all $x \in I$.

Show your work for the questions below:

2. [4 points] Explain how the Lagrange Mean Value Theorem implies

$$\left|\sqrt{x} - \sqrt{y}\right| \leq \frac{1}{2}|x-y| \quad \text{for all } x \geq 1 \text{ and } y \geq 1 \text{ with } x \neq y$$

3. [6 points] Justify that $\tan x + e^{-x} = \sqrt{3}$ has exactly one solution in $(0, \pi/3)$.

4. [18 points] Given the following function defined on $\mathbb{R}$:

$$y = f(x) = \begin{cases} 1 - x^2, & x < 0 \\ x^3 - 3x^2 + 1, & x \geq 0 \end{cases},$$

find:

(a) the intervals on which f is increasing or decreasing;

(b) the intervals on which f is concave up or down;

(c) all extreme points and values, and any inflection point of f.