STA2001 Assignment 1

Question 6 and 10 and Computer exercise are optional questions. Please submit your hardcopy of solultion in assignment box which is next to TC414 by 17:00, **22th,Jan. 2021.** For the students who cannot return to campus, please submit your solution in PDF form on BB on time.

1. 1.1-6. If $P(A) = 0.4$, $P(B) = 0.5$, and $P(A \cap B) = 0.3$, find
   (a) $P(A \cup B)$
   (b) $P(A \cap B')$
   (c) $P(A' \cup B')$

2. 1.1-9. Roll a fair six-sided die three times. Let A_1={1 or 2 on the first roll}, A_2={3 or 4 on the second roll}, and A_3={5 or 6 on the third roll}. It is given that $P(A_i) = \frac{1}{3}$, i=1,2,3; $P\left(A_i \cap A_j\right) = (1/3)^2, i \neq j$; and $P(A_1 \cap A_2 \cap A_3) = (1/3)^3$.
   (a) Use Theorem 1.1-6 to find $P(A_1 \cup A_2 \cup A_3)$.
   (b) Show that $P(A_1 \cup A_2 \cup A_3) = 1 - (1 - 1/3)^3$.

3. 1.1-10. Prove Theorem 1.1-6.

   Theorem 1.1-6 If A, B and C are any three events, then
   $$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C)$$
   $$-P(B \cap C) + P(A \cap B \cap C)$$

4. 1.1-13. Divide a line segment into two parts by selecting a point at random. Use your intuition to assign a probability to the event that the longer segment is at least two times longer than the shorter segment.

5. 1.2-11. Three students (S) and six faculty members (F) are on a panel discussing a new college policy.

   (a) In how many different ways can the nine participants be lined up at a table in the front of the auditorium?
   (b) How many lineups are possible, considering only the labels S and F?
   (c) For each of the nine participants, you are to decide whether the participant did a good job or a poor job stating his or her opinion of the new policy; that is, give each of the nine participants a grade of G or P. How many different scorecards are possible?

6. 1.2-12 Prove

   $$\sum_{r=0}^{n}(-1)^r\binom{n}{r} = 0 \qquad \text{and} \qquad \sum_{r=0}^{n}\binom{n}{r} = 2^n.$$

   HINT: Consider $(1-1)^n$ and $(1+1)^n$, or use Pascal's equation and proof by induction.

7. 1.2-13 A bridge hand is found by taking 13 cards at random and without replacement from a deck of 52 playing cards. Find the probability of drawing each of the following hands.
   (a) One in which there are 5 spades, 4 hearts, 3 diamonds, and 1 club.
   (b) One in which there are 5 spades, 4 hearts, 2 diamonds, and 2 clubs.
   (c) One in which there are 5 spades, 4 hearts, 1 diamond, and 3 clubs.
   (d) Suppose you are dealt 5 cards of one suit, 4 cards of another. Would the probability of having the other suits split 3 and 1 be greater than the probability of having them split 2 and 2?

8. 1.2-14 A bag of 36 dum-dum pops (suckers) contains up to 10 flavors. That is, there are from 0 to 36 suckers of each of 10 flavors in the bag. How many different flavor combinations are possible?

9. Prove Equation 1.2-2.

The foregoing results can be extended. Suppose that in a set of n objects, n_1 are similar, n_2 are similar, … , n_s are similar, where $n_1 + n_2 + \cdots + n_s = n$. Then the number of distinguishable permutations of the n objects is (see Exercise 1.2-15)

$$\binom{n}{n_1, n_2, \ldots, n_s} = \frac{n!}{n_1! n_2! \ldots n_s!}. \qquad (1.2\text{-}2)$$

Hint: First select n_1 position in $\binom{n}{n_1}$ ways. Then select $n_2$from the remaining $n - n_1$ position in $\binom{n-n_1}{n_2}$ ways, and so on. Finally, use the multiplication rule.

10. 1.2-17. A poker hand is defined as drawing 5 cards at random without replacement from a deck of 52 playing cards. Find the probability of each of the following poker hands:
   (a) Four of a kind (four cards of equal face value and one card of a different value).
   (b) Full house (one pair and one triple of cards with equal face value).
   (c) Three of a kind (three equal face values plus two cards of different values).
   (d) Two pairs (two pairs of equal face value plus one card of a different value).
   (e) One pair (one pair of equal face value plus three cards of different values).