

yohandi - quite 5

1 a. true

b. true

c. false

d. true

2 a. $\lim_{x \rightarrow 0} \frac{2 \sin x - \sin 2x}{x \tan^2 x} = \lim_{x \rightarrow 0} \frac{2 \sin x}{x} \cdot \lim_{x \rightarrow 0} \frac{(1 - \cos x)}{\tan^2 x} = 2 \cdot \lim_{x \rightarrow 0} \frac{\frac{1}{2} x^2}{x^2} = \boxed{1}$

b. $\lim_{x \rightarrow \infty} (x^2 + 2e^x) \ln(1 + e^{-x}) \sim \sim$ (Indeterminate form)

$\lim_{x \rightarrow \infty} (2x + 2e^x) \ln(1 + e^{-x}) + (x^2 + 2e^x) (-e^{-x})$

Let $y = \frac{1}{x}$, when $x \rightarrow \infty$, $y \rightarrow 0$

$\ln(1 + e^{-x}) = \ln(1 + e^{-\frac{1}{y}}) \sim e^{-\frac{1}{y}}$

$\lim_{y \rightarrow 0} \left(\frac{1}{y^2} + \frac{2}{e^{1/y}} \right) \ln(1 + e^{-1/y})$

$\lim_{x \rightarrow \infty} \frac{x^2 + 2e^x}{e^x} = \lim_{x \rightarrow \infty} x^2 (x^2 + 2e^x) = \boxed{2}$

$\lim_{x \rightarrow \infty} (x^2 + 2e^x) \cdot e^{-x} = \lim_{x \rightarrow \infty} \frac{x^2}{e^x} + 2 = \sim$ (Indeterminate form)

$= \lim_{x \rightarrow \infty} \frac{\frac{d(x^2 + 2)}{dx}}{\frac{d(e^x)}{dx}} = \lim_{x \rightarrow \infty} \frac{2x + 2}{e^x} = \lim_{x \rightarrow \infty} \frac{2}{e^x} = 0$

3. Area $= 2\pi r^2 + 2\pi r h = 2\pi(r^2 + rh) = 2$

$2\pi r(r+h) = 2$

$(r+h) = \frac{1}{\pi r}$

$h = \frac{1}{\pi r} - r = \frac{1 - \pi r^2}{\pi r}$

Volume $= \pi r^2 h = \pi(r^2) \left(\frac{1 - \pi r^2}{\pi r} \right) = r - \pi r^3$

$\frac{d(\text{Volume})}{dr} = 1 - 3\pi r^2 = 0$
 $r = \sqrt{\frac{1}{3\pi}}$

Volume $_{\max} = \pi \left(\frac{1}{3\pi} \right) \cdot h = \frac{1}{3} h = \frac{1}{3} \left(\frac{1 - \pi r^2}{\pi r} \right) = \frac{2\sqrt{3}\pi}{9\pi} = \boxed{\frac{2}{\sqrt{3\pi}}}$

$$4. \quad f(x) = \frac{1}{2} \ln \left(\frac{1+\sin x}{1-\sin x} \right) \quad g(x) = \ln |\sec x + \tan x|$$

~~$$f'(x) = \frac{1}{2} \left[\frac{1}{1+\sin x} - \frac{-1}{1-\sin x} \right] = \frac{1}{2} \left[\frac{1}{1+\sin x} + \frac{1}{1-\sin x} \right]$$~~

$$f(x) = \frac{1}{2} [\ln(1+\sin x) - \ln(1-\sin x)]$$

$$\begin{aligned} f'(x) &= \frac{1}{2} \left[\frac{\cos x}{1+\sin x} - \frac{-\cos x}{1-\sin x} \right] \\ &= \frac{1}{2} \left[\frac{\cos x(1-\sin x) + \cos x(1+\sin x)}{1-\sin^2 x} \right] \\ &= \frac{1}{\cos x} \end{aligned}$$

$$g(x) = \ln |\sec x + \tan x|$$

~~$$g'(x) = \frac{\sec x \tan x + \sec^2 x}{\sec x + \tan x}$$~~

$$\begin{aligned} g'(x) &= \frac{\sec x \tan x + \sec^2 x}{\sec x + \tan x} \quad (\sec x \tan x + \sec^2 x) \\ &= \sec x \frac{(\tan x + \sec x)}{(\sec x + \tan x)} \end{aligned}$$

$$\text{since } f'(x) = \frac{1}{\cos x} = \sec x$$

$$2.) \quad \therefore f'(x) = g'(x)$$

$$b.) \quad \int f'(x) dx = \int g'(x) dx + C$$

$$f(x) = g(x) + C$$

$$f(0) = g(0) + C$$

$$\frac{1}{2} \ln \frac{1+0}{1-0} = \ln(\sec 0 + \tan 0) + C$$

$$0 = 0 + C$$

$$C = 0$$

$$\therefore f(x) = g(x)$$