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Assignment 1

Q1. i. $p \wedge q \rightarrow r$

ii. $r \leftrightarrow p \vee q$

Q2.

p	q	$\neg q$	$p \oplus \neg q$
F	F	T	T
F	T	F	F
T	F	T	F
T	T	F	T

Q3. a.

p	q	$p \rightarrow q$	$p \wedge (p \rightarrow q)$	$[p \wedge (p \rightarrow q)] \rightarrow q$
F	F	T	F	T
F	T	T	F	T
T	F	F	F	T
T	T	T	T	T

(tautology)

b. $[p \wedge (p \rightarrow q)] \rightarrow q$

$= [p \wedge (\neg p \vee q)] \rightarrow q$

$= [(p \wedge \neg p) \vee (p \wedge q)] \rightarrow q$

$= (p \wedge q) \rightarrow q$

$= \neg (p \wedge q) \vee q$

$= (\neg p \vee \neg q) \vee q$

$= \neg p \vee (\neg q \vee q)$

$= \neg p \vee T$

$= T$ (tautology)

Q4. assume that:

$(p \wedge q) \rightarrow r \equiv (p \rightarrow r) \wedge (q \rightarrow r)$

consider a case where $p = F$, $q = T$, and $r = F$, we have:

LHS:

$(F \wedge T) \rightarrow F = T$

RHS:

$(F \rightarrow F) \rightarrow (T \rightarrow F) = F$

both sides have a different result. (contradicting)

hence proved

Q5. i. $\exists x (C(x) \wedge D(x) \wedge F(x))$

ii. $\neg \exists x (C(x) \wedge D(x) \wedge F(x))$

iii. $(\exists x\, C(x)) \wedge (\exists x\, D(x)) \wedge (\exists x\, F(x))$

Q6. i. $\neg (\exists x \exists y (Q(x,y) \leftrightarrow Q(y,x)))$

$= \forall x \forall y \neg ((Q(x,y) \rightarrow Q(y,x)) \wedge (Q(y,x) \rightarrow Q(x,y)))$

$= \forall x \forall y (\neg (Q(x,y) \rightarrow Q(y,x)) \vee \neg (Q(y,x) \rightarrow Q(x,y)))$

$= \forall x \forall y ((Q(x,y) \wedge \neg Q(y,x)) \vee (Q(y,x) \wedge \neg Q(x,y)))$

ii. $\neg (\forall y \exists x \exists z (T(x,y,z) \vee Q(x,y)))$

$= \exists y \forall x \forall z \neg (T(x,y,z) \vee Q(x,y))$

$= \exists y \forall x \forall z (\neg T(x,y,z) \wedge \neg Q(x,y))$

Q7. Let $A = \{x\}$ where x is an element & $B = \{x, \{x\}\}$.

these sets are true as $\{x\} \in B$ and $x \in B$.

Q8 i. $A \times B \times C = \{(a,x,0), (a,x,1), (a,y,0), (a,y,1), (b,x,0), (b,x,1), (b,y,0), (b,y,1), (c,x,0), (c,x,1), (c,y,0), (c,y,1)\}$

ii. $C \times A \times B = \{(0,a,x), (0,a,y), (0,b,x), (0,b,y), (0,c,x), (0,c,y), (1,a,x), (1,a,y), (1,b,x), (1,b,y), (1,c,x), (1,c,y)\}$

iii. $D = A \times B = \{(a,x), (a,y), (b,x), (b,y), (c,x), (c,y)\}$

$D \times C = \{((a,x),0), ((a,x),1), ((a,y),0), ((a,y),1), ((b,x),0), ((b,x),1), ((b,y),0), ((b,y),1), ((c,x),0), ((c,x),1), ((c,y),0), ((c,y),1)\}$

Q9. consider $A \cup (A \cap B)$ and A:

- $x \in A \cup (A \cap B)$ implies $x \in A$ or $x \in (A \cap B)$
  implies $x \in A$

therefore, $A \cup (A \cap B) \subseteq A$

- $x \in A$ implies $x \in A$ or $x \in (A \cap B)$
  implies $x \in A \cup (A \cap B)$

therefore, $A \subseteq A \cup (A \cap B)$

hence $A \cup (A \cap B) = A$ (proved)

Q10. i. $x \in A \cap B \cap C$ implies $x \in A \cap B$ and $x \in C$
implies $x \in A \cap B$

therefore, $A \cap B \cap C \subseteq A \cap B$

ii. $x \in (A - B) - C$ implies $x \in A - B$ and $x \notin C$
implies $x \in A$ and $x \notin B$ and $x \notin C$
implies $x \in A$ and $x \notin C$
implies $x \in A - C$

therefore, $(A - B) - C \subseteq A - C$

iii. $x \in (B - A) \cup (C - A)$ implies $x \in (B - A)$ or $x \in (C - A)$

case $x \in B - A$:
$x \in B$ and $x \notin A$ implies $x \in B \cup C$ and $x \notin A$
implies $x \in (B \cup C) - A$

case $x \in C - A$:
$x \in C$ and $x \notin A$ implies $x \in B \cup C$ and $x \notin A$
implies $x \in (B \cup C) - A$

therefore, $(B - A) \cup (C - A) \subseteq (B \cup C) - A$

$x \in (B \cup C) - A$ implies $x \in B \cup C$ and $x \notin A$
implies $(x \in B$ and $x \notin A)$ or $(x \in C$ and $x \notin A)$
implies $x \in (B - A)$ or $x \in (C - A)$
implies $x \in (B - A) \cup (C - A)$

therefore, $(B \cup C) - A \subseteq (B - A) \cup (C - A)$

hence $(B - A) \cup (C - A) = (B \cup C) - A$ (proved)