STA2001: Probability and Statistics I

Computer-based Exercise 9

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The goal of this exercise is to verify Theorem 5.1-1 [Random Number Generator] using Example 3 in Lecture 19.

Problem.

- Generate 100 realizations of the random variable $Y \sim U(0,1)$, namely $y_1, y_2, \ldots, y_{100}$.

  Define another random variable $X = F^{-1}(Y) = -\log(1-Y)$. Compute realizations $x_1 = -\log(1-y_1), x_2 = -\log(1-y_2), \ldots, x_{100} = -\log(1-y_{100})$.

  Plot

  $$G(x) = \frac{N(x)}{100},\ 0 \le x \le 10$$

  where $N(x)$ is the number of x_i's that are smaller than x.

  Compare it with the plot of

  $$F(x) = 1 - e^{-x},\ 0 \le x \le 10.$$

- Generate 1000 realizations of $Y \sim U(0,1)$ and repeat the above process. What do you find?