Yohandi - Homework for Week 1

Exercises 2.2

1a. $\lim_{x\to1} g(x)$ does not exist because even both $\lim_{x\to1^-} g(x)$ and $\lim_{x\to1^+} g(x)$ exist, but they are not the same, which is a requirement for the (two-sided) limit to exist.

b. $\lim_{x\to2} g(x) = 1$, since $\lim_{x\to2^-} g(x) = \lim_{x\to2^+} g(x) = 1$

c. $\lim_{x\to3} g(x) = 0$, since $\lim_{x\to3^-} g(x) = \lim_{x\to3^+} g(x) = 0$

d. $\lim_{x\to2,5} g(x) = 0,5$, since $\lim_{x\to2,5^-} g(x) = \lim_{x\to2,5^+} g(x) = 0,5$

4a. false

b. false

c. true

d. true

e. true

9. the definition of limit itself is the approach from the $x \to x_0$ (from left and right), f does not have to be defined.
we can't conclude anything, $f(1)$ can be anything, because the requirement of $\lim_{x\to1} f(x)$ is only to be the same as $\lim_{x\to1^-} f(x)$ and $\lim_{x\to1^+} f(x)$ ($f(1)$ has nothing to do with it)

10 with the same idea with the previous number, $f(x_0)$ has nothing to do with $\lim_{x\to x_0} f(x)$, they are independent, therefore if $f(1)$ does exist $\lim_{x\to1} f(x)$ does not have to exist. because they are independent, we can't conclude anything.

18. $\lim_{y\to2} \frac{y+2}{y^2+5y+6} = \lim_{y\to2} \frac{1}{y+3} = \frac{1}{5}$

19. $\lim_{y\to-3} (5-y)^{4/3} = (5-(-3))^{4/3} = 16$

23. $\lim_{x\to5} \frac{x-5}{x^2-25} = \lim_{x\to5} \frac{1}{x+5} = \frac{1}{10}$

28. $\lim_{t\to-1} \frac{t^2+3t+2}{t^2-t-2} = \lim_{t\to-2} \frac{t+2}{t-2} = -\frac{1}{3}$

32. $\lim_{x\to0} \frac{\frac{1}{x-1}+\frac{1}{x+1}}{x} = \lim_{x\to0} \frac{2x}{x^3-x} = \lim_{x\to0} \frac{2}{x^2-1} = -2$

40. $\lim_{x\to-2} \frac{x+2}{\sqrt{x^2+5}-3} = \lim_{x\to-2} \frac{(x+2)(\sqrt{x^2+5}+3)}{x^2-4}$
$= \lim_{x\to-2} \frac{\sqrt{x^2+5}+3}{x-2} = -\frac{3}{2}$

45. $\lim_{x\to0} \sec x = 1$

46. $\lim_{x\to\pi/3} \tan x = \sqrt{3}$

54. $\lim_{x\to4} f(x) = 0 \qquad \lim_{x\to4} g(x) = -3$

a. $\lim_{x\to4} (g(x)+3) = \lim_{x\to4} g(x) + \lim_{x\to4} 3 = 0$

b. $\lim_{x\to4} xf(x) = \lim_{x\to4} x \cdot \lim_{x\to4} f(x) = 0$

c. $\lim_{x\to4} (g(x))^2 = \lim_{x\to4} g(x) \cdot \lim_{x\to4} g(x) = 9$

d. $\lim_{x\to4} \frac{g(x)}{f(x)-1} = \frac{\lim_{x\to4} g(x)}{\lim_{x\to4} f(x) - \lim_{x\to4} 1} = \frac{-3}{0-1} = 3$

57. $\lim_{x\to1} \lim_{h\to0} \frac{f(x+h)-f(x)}{h} = \lim_{x\to1} \lim_{h\to0} \frac{x^2+h^2-x^2+2xh}{h}$
$= \lim_{x\to1} \lim_{h\to0} h+2x$
$= 2$

65a. $\lim_{x\to0} 1-\frac{x^2}{6} = L_1, \; x\to0 \;\; L_1\to1$

$\lim_{x\to0} 1 = L_2, \; x\to0 \;\; L_2 = 1$

by using the Squeeze Theorem,

$\lim_{x\to0} \frac{x\sin x}{2-2\cos x}$ must be $\underline{1}$

75. we will know for sure for the value of $f(x)$ whereas,

$x^4 \le f(x) \le x^2$

when $x^4 = x^2 \Rightarrow x_0 = \{-1, 0, 1\}$,

since the function is defined for all real numbers and is continuous, the $\lim_{x\to x_0} f(x)$ does exist.

77. $\lim_{x\to4} \frac{f(x)-5}{x-2} = \frac{\lim_{x\to4} f(x) - \lim_{x\to4} 5}{\lim_{x\to4} x - \lim_{x\to4} 2} = 1$

$\lim_{x\to4} f(x) = 5+(4-2) = 7$

80. $\lim_{x\to0} \frac{f(x)}{x^2} = 1 \Rightarrow \lim_{x\to0} \frac{x^2 g(x)}{x^2} = 1 \Rightarrow \lim_{x\to0} g(x) = 1$

a. $\lim_{x\to0} f(x) = \lim_{x\to0} x^2 g(x) = 0$

b. $\lim_{x\to0} \frac{f(x)}{x} = \lim_{x\to0} \frac{x^2 g(x)}{x} = \lim_{x\to0} x g(x) = 0$

Exercises 2.4

1a. true
b. true
c. false
d. true
e. true
f. true
g. false
h. false
i. false
j. false
k. true
l. false

5a. the limit does not exist, as x approaches zero, its reciprocal, $1/x$, grows without bound and the values of $\sin(1/x)$ cycle repeatedly. there is no single number L that the function's values stay increasingly close to as x approaches zero

b. the limit exists, $\lim_{x\to 0^-} f(x) = 0$

c. the limit does not exist, it is required for two-sided limit to exist if $\lim_{x\to 0^-} f(x) = \lim_{x\to 0^+} f(x)$

6a. $\lim_{x\to 0^+} \sqrt{x}\sin(\frac{1}{x}) = \lim_{x\to 0^+}\sqrt{x} \cdot \lim_{x\to 0^+}\sin(\frac{1}{x}) = 0$, it is possible because $|\lim_{x\to 0^+}\sin(\frac{1}{x})| \le 1$ while $\lim_{x\to 0^+}\sqrt{x} = 0$

b. $\lim_{x\to 0^-} g(x)$ does not exist, it is because the function have sqrt in it which indicates $D: x\in R \cap x \ge 0$, therefore $\lim_{x\to 0^-} g(x)$ does not exist

c. the limit does not exist, it is required for two-sided limit to exist if $\lim_{x\to 0^-} g(x) = \lim_{x\to 0^+} g(x)$

17a. $\lim_{x\to -2^+} (x+3)\frac{|x+2|}{(x+2)} = \lim_{x\to -2^+}(x+3) = 1$

b. $\lim_{x\to -2^-} (x+3)\frac{|x+2|}{(x+2)} = \lim_{x\to -2^-}(x+3)(-1) = -1$

20a. $\lim_{t\to 4^+}(t-\lfloor t\rfloor) = \lim_{t\to 4^+} t - \lim_{t\to 4^+}\lfloor t\rfloor = 4-4=0$

b. $\lim_{t\to 4^-}(t-\lfloor t\rfloor) = \lim_{t\to 4^-} t - \lim_{t\to 4^-}\lfloor t\rfloor = 4-3=1$

28. $\lim_{x\to 0} 6x^2 \cdot \cot x \cdot \csc 2x = \lim_{x\to 0} \frac{6x^2 \cdot \cos x}{2\sin^2 x\cos x} = 3$

32. $\lim_{x\to 0}\frac{x - x\cos x}{\sin^2 3x} = \lim_{x\to 0}\frac{x(1-\cos x)}{\sin^2 3x} = \lim_{x\to 0}\frac{x \cdot 2\sin^2\frac{1}{2}x}{\sin^2 3x} = 0$

34. $\lim_{h\to 0}\frac{\sin(\sinh)}{\sinh} = \lim_{\sinh\to 0}\frac{\sin(\sinh)}{\sinh} = 1$

44. the reason that $\lim_{x\to c} f(x)$ exists is that it is equal to $\lim_{x\to c^-} f(x)$ and $\lim_{x\to c}$. it is possible to find $\lim_{x\to c} f(x)$ by calculating $\lim_{x\to c^+} f(x)$

45. suppose $x_0 = 0$
$\lim_{x\to x_0^-} f(x) = \lim_{x\to x_0^+} -f(x) = -\lim_{x\to x_0^+} f(x) = -3$
since the function is odd, $f(x) = -f(-x)$

46. since the function is even, $f(x_0) = f(-x_0)$
when $x_0 = 2^-$, $-x_0 = -2^+$ resulting
$\lim_{x\to 2^-} f(x) = \lim_{x\to -2^+} f(x) = 7$

Exercises 2.5

7a. f is not defined at $x=2$

b. f is not continuous at $x=2$

8. $[-1,0)\cup(0,1)\cup(1,2)\cup(2,3)$

9. $\lim_{x\to 2} f(x)=0$

10. $\lim_{x\to 1} f(x)=2$

26. $[\frac{1}{3},\infty)$

27. $(-\infty,\infty)$

32. $\lim_{t\to 0}\sin\left(\frac{\pi}{2}\cdot\cos(\tan t)\right)=\lim_{t\to 0}\sin\left(\frac{\pi}{2}\cdot 1\right)=1$

$\sin\left(\frac{\pi}{2}\cdot\cos(\tan 0)\right)=\sin\left(\frac{\pi}{2}\cdot 1\right)=1$

we can see that when $x\to 0$, the limit is approaching 1 and $f(0)=1$, therefore the function is continuous at the point

39. $\lim_{x\to 0}\sec\left(\frac{\pi(\sin 2x-\sin x)}{3x}\right)=\lim_{x\to 0}\sec\left(\frac{\pi\cdot 2\cos\frac{3}{2}x\cdot\sin\frac{1}{2}x}{3x}\right)$

$=2$

$f(0)$ is undefined.

since $f(0)$ is undefined, the limit is not continuous at the point

43. $\lim_{x\to 3^-} f(x)$ must be the same as $\lim_{x\to 3^+} f(x)$

and $f(3)$ must also be the same as $\lim_{x\to 3} f(x)$.

$\lim_{x\to 3^-} f(x)=\lim_{x\to 3^+} f(x)$

$(3^2-1)=2a(3)$

$a=\frac{4}{3}$

$f(3)=2\cdot\frac{4}{3}(3)=8$ (it is true), therefore the value of a is $\frac{4}{3}$

47. $\lim_{x\to -1^-} -2=\lim_{x\to -1^+} ax-b$

$-2=-a-b$ ...(1)

$\lim_{x\to 1^-} ax-b=\lim_{x\to 1^+} 3$

$a-b=3$ ...(2)

from (1) & (2): $a=2\frac{1}{2}$

$b=-\frac{1}{2}$

56. $f(x)=x^3-15x+1$

x	$f(x)$
-4	-3
-3	19
-2	23
-1	15
0	1
1	-13
2	-21
3	-17
4	5

there's a changing sign between -4 and -3

there's a changing sign between 0 and 1

there's a changing sign between 3 and 4

those changing sign mean that there exist x_0 such that $f(x_0)$ in the interval (since the function is continuous)

65. since the function is continuous, let's assume that there exists a and b in the interval such that $\max(f(a),f(b))>0$ and $\min(f(a),f(b))<0$. this means that there exists x_0 in $[a,b]$ such that $f(x_0)=0$. It contradicts with the statement "function that is never zero"

67. let $g(x)=f(x)-x$, we know that both $f(x)$ and x are continuous functions

$\Rightarrow g(x)$ is also a continuous function.

since $f(x)\in[0,1]$ for each $x\in[0,1]$,

$g(0)=f(0)-0=f(0)\geq 0$, $g(1)=f(1)-1\leq 0$.

note that $g(x)$ is a continuous function, and by IVT Theorem, there exist c such that $g(c)=0$ or $g(c)=f(c)-c=0$

$\Rightarrow f(c)=c$

Exercises 2.6

2a. 2

b. -3

c. 1

d. does not exist

e. $+\infty$

f. $+\infty$

g. $+\infty$

h. $+\infty$

i. $-\infty$

j. does not exist

k. 0

l. -1

10. $\lim_{\theta \to -\infty} \frac{\cos\theta}{3\theta} = 0$, this is because the value of $\cos\theta \in [-1,1]$ while the denominator $\to -\infty$

11. $\lim_{t\to-\infty} \frac{2-t+\sin t}{t+\cos t} = \lim_{t\to-\infty} \frac{2}{t+\cos t} + \frac{-t}{t+\cos t} + \frac{\sin t}{t+\cos t}$

$\lim_{t\to-\infty} \frac{2}{t+\cos t} = 0$

$\lim_{t\to-\infty} \frac{-t}{t+\cos t} = -1$

$\lim_{t\to-\infty} \frac{\sin t}{t+\cos t} = 0$

$\overline{}\,+$

-1

16a. $\lim_{x\to\infty} \frac{3x+7}{x^2-2} = \lim_{x\to\infty} \frac{3+\frac{7}{x}}{x-\frac{2}{x}} = 0$

b. $\lim_{x\to-\infty} \frac{3x+7}{x^2-2} = \lim_{x\to-\infty} \frac{3+\frac{7}{x}}{x-\frac{2}{x}} = 0$

21a. $\lim_{x\to\infty} \frac{3x^7+5x^2-1}{6x^3-7x+3} = \lim_{x\to\infty} \frac{3x^4+\frac{5}{x}-\frac{1}{x^3}}{6-\frac{7}{x^2}+\frac{3}{x^3}} = \infty$

b. $\lim_{x\to-\infty} \frac{3x^7+5x^2-1}{6x^3-7x+3} = \lim_{x\to-\infty} \frac{3x^4+\frac{5}{x}-\frac{1}{x^3}}{6-\frac{7}{x^2}+\frac{3}{x^3}} = \infty$

23. $\lim_{x\to\infty} \sqrt{\frac{8x^2-3}{2x^2+x}} = \lim_{x\to\infty} \sqrt{\frac{8-\frac{3}{x^2}}{2+\frac{1}{x}}} = \sqrt{4} = 2$

31. $\lim_{x\to\infty} \frac{2x^{5/3}-x^{1/3}+7}{x^{8/5}+3x+\sqrt{x}} = \lim_{x\to\infty} \frac{2-x^{-4/3}+7.x^{-5/3}}{x^{-1/15}+3x^{-2/3}+x^{-7/6}}$

$= \infty$

36. $\lim_{x\to-\infty} \frac{4-3x^3}{\sqrt{x^6+9}} = \lim_{x\to-\infty} \frac{-\frac{4}{x^3}+3}{\sqrt{1+\frac{9}{x^6}}} = 3$

39. $\lim_{x\to 2^-} \frac{3}{x-2} = -\infty$

43. $\lim_{x\to 7} \frac{4}{(x-7)^2} = +\infty$

63. $y = \frac{1}{x-1}$ will have an asymptote line $Ax+B$

where:

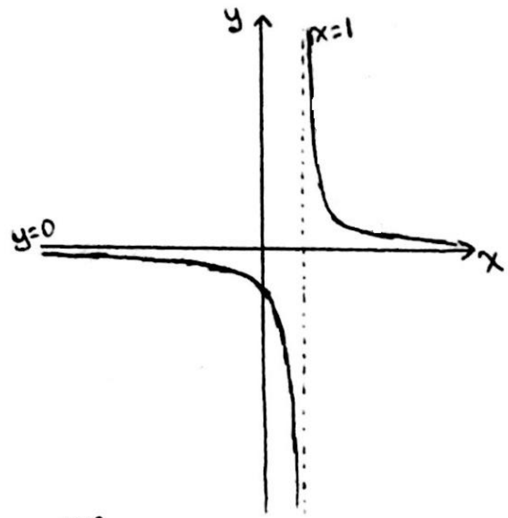
$A = \lim_{x\to\pm\infty} \frac{\frac{1}{x-1}}{x} = 0$

$B = \lim_{x\to\pm\infty} \frac{1}{x-1} - 0.x = 0$

asymptote line: $y = Ax+B = 0$

note that:

$\lim_{x\to 1^-} \frac{1}{x-1} = -\infty \quad \lim_{x\to 1^+} \frac{1}{x-1} = \infty$



67. $y = \frac{x+3}{x+2}$ will have an asymptote line $Ax+B$

where:

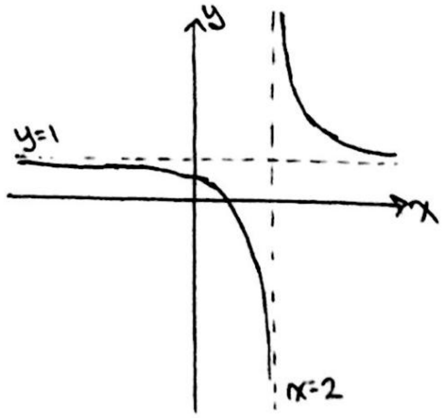
$A = \lim_{x\to\pm\infty} \frac{(x+3)}{x(x+2)} = 0$

$B = \lim_{x\to\pm\infty} \frac{x+3}{x+2} - 0.x = 1$

asymptote line: $y = Ax+B = 1$

note that:

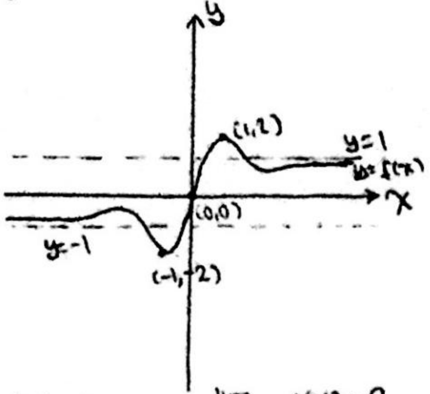
$\lim_{x\to 2^-} \frac{x+3}{x-2} = -\infty \quad \lim_{x\to 2^+} \frac{x+3}{x-2} = \infty$



69. $f(0)=0$ $\quad \lim_{x\to-\infty} f(x)=-1$

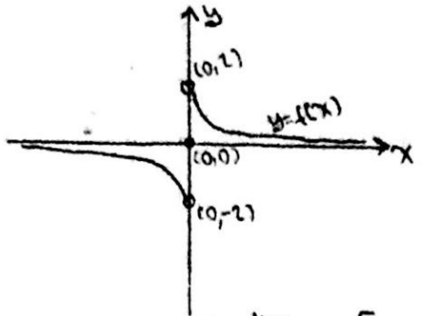
$f(1)=2$ $\quad \lim_{x\to\infty} f(x)=1$

$f(-1)=-2$



70. $f(0)=0$ $\quad \lim_{x\to0^+} f(x)=2$

$\lim_{x\to\pm\infty} f(x)=0$ $\quad \lim_{x\to0^-} f(x)=-2$



80. $\lim_{x\to\infty}(\sqrt{x+9}-\sqrt{x+4}) = \lim_{x\to\infty} \dfrac{5}{\sqrt{x+9}+\sqrt{x+4}} = 0$

82. $\lim_{x\to-\infty}(\sqrt{x^2+9}+x) = \lim_{x\to-\infty} \dfrac{x^2+9-x^2}{(\sqrt{x^2+9}-x)}$

$= \lim_{x\to-\infty} \dfrac{9}{\sqrt{x^2+9}-x}$

$= 0$

99. $y=\dfrac{x^2}{x-1}$ will have an asymptote line $Ax+B$

Where:

$A = \lim_{x\to\pm\infty} \dfrac{x^2}{x(x-1)} = \lim_{x\to\pm\infty} \dfrac{x}{x-1} = 1$

$B = \lim_{x\to\pm\infty} \dfrac{x^2}{x-1} - 1.x = \lim_{x\to\pm\infty} \dfrac{x}{x-1} = 1$

asymptote line: $y = Ax+B = x+1$

note that:

$\lim_{x\to1^-} \dfrac{x^2}{x-1} = -\infty \quad \lim_{x\to1^+} \dfrac{x^2}{x-1} = \infty$