yohandi - homework week 10

Exercises 15.7

3. $\int_0^{2\pi}\int_0^{\theta/2\pi}\int_0^{3+24r^2} dz\, r\, dr\, d\theta$

$= \int_0^{2\pi}\int_0^{\theta/2\pi} (3r+24r^3)\, dr\, d\theta$

$= \int_0^{2\pi} \frac{3\theta^2}{8\pi^2} + \frac{3\theta^4}{8\pi^4}\, d\theta$

$= \frac{17\pi}{5}$

9. $\int_0^1\int_0^{\sqrt{z}}\int_0^{2\pi} (r^2\cos^2\theta + z^2) r\, d\theta\, dr\, dz$

$= 2\int_0^1\int_0^{\sqrt{z}} (\pi r^3 + 2\pi r z^2)\, dr\, dz$

$= \int_0^1 (\frac{\pi z^2}{4} + \pi z^3)\, dz$

$= \frac{\pi}{3}$

14. $\int_{-1}^1\int_0^{\sqrt{1-y^2}}\int_0^x (x^2+y^2)\, dz\, dx\, dy$

$= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\int_0^1\int_0^{r\cos\theta} r^2\, dz\, r\, dr\, d\theta$

$= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\int_0^1 r^4\cos\theta\, dr\, d\theta$

$= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{\cos\theta}{5}\, d\theta$

$= \frac{2}{5}$

17. $\theta \in [-\frac{\pi}{2}, \frac{\pi}{2}]$

$r \in [1, 1+\cos\theta]$

$z \in [0, 4]$

$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\int_1^{1+\cos\theta}\int_0^4 r\, f(r,\theta,z)\, dz\, dr\, d\theta$

21. $\int_0^{\pi}\int_0^{\pi}\int_0^{2\sin\phi} \rho^2\sin\phi\, d\rho\, d\phi\, d\theta$

$= \int_0^{\pi}\int_0^{\pi} \frac{8}{3}\sin^4\phi\, d\phi\, d\theta$

$= \int_0^{\pi} \pi\, d\theta$

$= \pi^2$

25. $\int_0^{2\pi}\int_0^{\pi/3}\int_{\sec\phi}^2 3\rho^2\sin\phi\, d\rho\, d\phi\, d\theta$

$= \int_0^{2\pi}\int_0^{\pi/3} (8\sin\phi - \frac{\sin\phi}{\cos^3\phi})\, d\phi\, d\theta$

$= \int_0^{2\pi} \frac{5}{2}\, d\theta$

$= 5\pi$

33a. $\theta \in [0, 2\pi]$

$\phi \in [0, \frac{\pi}{2}]$

$\rho \in [\cos\phi, 2]$

b. $\int_0^{2\pi}\int_0^{\frac{\pi}{2}}\int_{\cos\phi}^2 \rho^2\sin\phi\, d\rho\, d\phi\, d\theta$

$= \int_0^{2\pi}\int_0^{\pi/2} (\frac{8}{3}\sin\phi - \frac{\sin\phi\cos^3\phi}{3})\, d\phi\, d\theta$

$= \int_0^{2\pi} \frac{31}{12}\, d\theta$

$= \frac{31\pi}{6}$

47. $\int_0^{\pi/2}\int_0^{\sin\theta}\int_0^{\sqrt{1-r^2}} r\, dz\, dr\, d\theta$

$= \int_0^{\pi/2}\int_0^{\sin\theta} r\sqrt{1-r^2}\, dr\, d\theta$

$= \int_0^{\pi/2} (-\frac{\cos^3\theta}{3} + \frac{1}{3})\, d\theta$

$= \frac{\pi}{6} - \frac{2}{9}$

Exercises 15.8

12. $\frac{\partial(x,y)}{\partial(u,v)} = \frac{\partial x}{\partial u}\cdot\frac{\partial y}{\partial v} - \frac{\partial x}{\partial v}\cdot\frac{\partial y}{\partial u}$

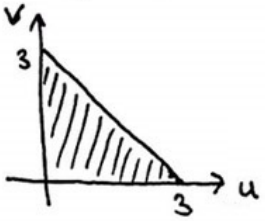
$= \frac{1}{3}\cdot\frac{1}{3} - \frac{1}{3}\cdot(-\frac{2}{3}) = \frac{1}{3}$

b. $x=0 \Rightarrow u=-v$

$x=1 \Rightarrow u=3-v$

$y=-2x \Rightarrow v=0$

$y=x \Rightarrow u=0$



6. $\iint_R (x-y)(2x+y)\, dx\, dy$ ($u = x-y$, $v = 2x+y$)

$\frac{\partial(x,y)}{\partial(u,v)} = \frac{\partial x}{\partial u}\cdot\frac{\partial y}{\partial v} - \frac{\partial x}{\partial v}\cdot\frac{\partial y}{\partial u} = \frac{1}{3}\cdot\frac{1}{3} - \frac{1}{3}(-\frac{2}{3}) = \frac{1}{3}$

$y=-2x+4 \Rightarrow v=4$

$y=-2x+7 \Rightarrow v=7$

$y=x-2 \Rightarrow u=2$

$y=x+1 \Rightarrow u=-1$

$\int_{-1}^2\int_4^7 \frac{1}{3}uv\, dv\, du = \int_{-1}^2 \frac{1}{3}\cdot\frac{33}{2}u\, du = \frac{33}{4}$

13. $\int_0^{2/3}\int_y^{2-2y} (x+2y)e^{(y-x)}\, dx\, dy$ ($u = x+2y$, $v = y-x$)

$\frac{\partial(x,y)}{\partial(u,v)} = \frac{\partial x}{\partial u}\cdot\frac{\partial y}{\partial v} - \frac{\partial x}{\partial v}\cdot\frac{\partial y}{\partial u} = \frac{1}{3}(-\frac{1}{3}) - \frac{1}{3}\cdot\frac{2}{3} = -\frac{1}{3}$

$y=0 \Rightarrow u=v$

$x=y \Rightarrow v=0$

$x=2-2y \Rightarrow u=2$

$\int_0^2\int_0^u u\cdot e^{-v}(\frac{1}{3})\, dv\, du$

$= \int_0^2 (-\frac{1}{3})(ue^{-u} - u)\, du$

$= e^{-2} + \frac{1}{3}$