yohandi - assignment 10

10) $I = \frac{2}{3}MR^2$

$V_{com} = \omega R$

a. $K_T + K_R = K$

$\frac{1}{2}mv_{com}^2 + \frac{1}{2}I\omega^2 = 20\ J$

$\frac{1}{2}(M\omega^2R^2 + \frac{2}{3}M\omega^2R^2) = 20\ J$

$M\omega^2R^2 = \frac{20 \cdot 6}{5}\ J$

$M\omega^2R^2 = 24\ J$

$K_R = \frac{1}{2}I\omega^2 = \frac{1}{3}M\omega^2R^2 = \boxed{8\ J}$

b. $K_T = \frac{1}{2}mv_{com}^2 = 20\ J - 8\ J$

$mV_{com}^2 = 24\ J$

$V_{com} = \sqrt{\frac{24}{M}}\ m/s$

$= \sqrt{24 \cdot \frac{2R^2}{3I}}\ m/s$

$= \boxed{2.74\ m/s}$

c. $\Delta K = -\Delta U = -Mgh = -Mgx \sin\theta$

$= -\frac{2R^2}{3I} \cdot 9.8 \cdot 1 \cdot \frac{1}{2}\ J$

$= -15.68\ J$

$K_f = K_i + \Delta K = (20 - 15.68\ J)$

$= \boxed{4.32\ J}$

d $K = K_T + K_R = \frac{5}{6}M\omega^2R^2 = 4.32\ J$

$\omega^2R^2 = 1.62\ m^2/s^2$

$\boxed{V_{com} = \omega R = 1.27\ m/s}$

12) at the top of the loop, the ball loses its energy potential (converted to kinetics energy)

$\rightarrow U_i = U_f + K_f$

$mgh = mg2R' + \frac{1}{2}I\omega^2 + \frac{1}{2}mv^2$

$2\cancel{m}g(h-2R') = \frac{2}{5}\cancel{m}\omega^2R^2 + \cancel{m}\omega^2R^2$

$\frac{10g}{7}(h-2R') = \omega^2R^2 = v^2$ ....(1)

note that the normal force is $= 0$ when the ball is almost "fall"

$\Sigma F = W$

$\frac{mv^2}{R} - 0 = mg$

$v^2 = gR'$ ...(2)

(2)→(1):

$\frac{10\cancel{g}}{7}(h-2R') = \cancel{g}R'$

$h = \frac{27}{10}R'$

a) $\boxed{h = 32.4\ cm = 0.324\ m}$

b) $U_i = U_f + K_f$

$mgh = mgR' + \frac{1}{2}I\omega^2 + \frac{1}{2}mv^2$

$2g(h-R') = \frac{2}{5}\omega^2R^2 + \omega^2R^2$

$2g(h-R') = \frac{7}{5}\omega^2R^2 = \frac{7}{5}v^2$

$v^2 = \frac{10}{7}g(h-R')$

$v = \sqrt{\frac{10g}{7}(h-R')} = 2.898\ m/s$

Force component of the horizontal:

$F_c = \frac{mv^2}{R'} = \frac{0.320 \cdot 10^{-3}(2.898)^2}{12 \cdot 10^{-2}}\ N$

$\boxed{F_c = 0.0224\ N}$

c) $\boxed{\leftarrow}$ (to the left)

15) a) $V_{com} = \omega R = \boxed{0.11\omega}$

b) $F_{net} = m \cdot a_{com}$

$-\mu mg = m \cdot a_{com}$

$a_{com} = \mu \cdot g = 0.21 \cdot 9.8\ m/s^2 = \boxed{2.058\ m/s^2}$

c) $I\alpha = \tau_{net}$

$\frac{2}{5}MR^2\alpha = -\mu mg R$

$\alpha = -\frac{5\mu g}{2R} = \boxed{-46.77\ rad/s^2}$

d) $\omega = \omega_0 + \alpha t$ ...(1)

$V = V_0 + at$ ...(2)

(2)

$\omega R = 8.5 - 2.058t$

$\omega = \frac{8.5 - 2.058t}{0.11}\ rad/s$ ...(3)

(3)→(1)

$\frac{8.5 - 2.058t}{0.11} = 0 + 46.77t$

$8.5 = (0.11 \cdot 46.77 + 2.058)t$

$\boxed{t = 1.18\ s}$

e) $x = V_0 . t + \frac{1}{2} a t^2$

$x = 8.5 \cdot 1.18 + \frac{1}{2}(-2.058)\, 1.18^2$ m

$\boxed{x = 8.60 \text{ m}}$

f) $V = \omega R$

$V = (\omega_0 + \alpha t) R$

$V = \alpha t R$

$V = (46.77 \cdot 1.18)(0.11)$ m/s

$\boxed{V = 6.07 \text{ m/s}}$

22) $\bar{r} \times \bar{F} = \begin{vmatrix} i & j & k \\ 2 & -3 & 2 \\ F_x & 7 & -6 \end{vmatrix}$ Nm

$= 4\bar{i} + (-12 - 2F_x)\hat{j} + (14 + 3F_x)\hat{k}$ Nm

$\boxed{F_x = -5 \text{ N}}$

31) a) $\boxed{L = 0}$ (at the top $v_{ball} = 0$)

b) $L = m(\vec{r} \times \vec{v})$

$L = 0.4\left((2, \frac{v^2}{4g}) \times (0, -\frac{v}{\sqrt{2}})\right)$

$\boxed{L = -16\sqrt{2} \text{ kg m}^2/\text{s}}$

c) $\bar{\tau} = \bar{r} \times \bar{F}$

$\bar{\tau} = (2, \frac{v^2}{2g}) \times (0, -mg)$

$\boxed{\bar{\tau} = -7.84 \text{ Nm}}$.

d) $\bar{\tau} = \bar{r} \times \bar{F}$

$\bar{\tau} = (2, \frac{v^2}{4g}) \times (0, -mg)$

$\boxed{\bar{\tau} = -7.84 \text{ Nm}}$

39) a) $\bar{\tau}_{AVG} = \frac{\Delta L}{\Delta t} = \frac{0.8 - 3}{1.5}$ Nm

$= \boxed{-1.467 \text{ Nm}}$

b) $\theta = \omega_0 . t + \frac{1}{2}\alpha t^2$

$= \frac{L_i}{I} t + \frac{1}{2}\frac{\bar{\tau}}{I} t^2$

$= \boxed{20.36 \text{ rad}}$

c) Work $= \bar{\tau} . \theta = (-1.467)(20.36)$ J

$= \boxed{-29.86 \text{ J}}$

d) $P = \frac{W}{\Delta t} = + \frac{29.86}{1.5}$ J/s $\boxed{+19.91 \text{ watt}}$

52) a) $L_i = L_f$

$I_i\, \omega_i = I_f . \omega_f$

$\omega_f = \frac{I_i}{I_f} . \omega_i$

$= \left(\frac{\frac{1}{2} \cdot 4m\, R^2 + mR^2}{\frac{1}{2} 4mR^2 + \frac{1}{4} mR^2}\right) . 0.320$ rad/s

$= \boxed{0.4267 \text{ rad/s}}$

b) $\frac{K_f}{K_0} = \frac{\frac{1}{2} I_f \omega_f^2}{\frac{1}{2} I_i \omega_i^2} = \frac{I_i}{I_f}$ $\boxed{\frac{4}{3} = 1.333}$

c) the energy required for the cockroach to move from the edge to half the center of disc

61) $L_i = L_f$

$I_i\, \omega_i = I_f . \omega_f$

$\omega_f = \frac{I_i}{I_f} . \omega_i = \frac{(0.12)}{(0.12 + 0.2 \cdot 0.6^2)}$ 2.4 rad/s

$\boxed{\omega_f = 1.5 \text{ rad/s}}$