yohandi - assignment 1

1. $P(A) = 0.4$

$P(B) = 0.5$

$P(A \cap B) = 0.3$

a) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
$= 0.6$

b) $P(A \cap B') = P(A) - P(A \cap B)$
$= 0.1$

c) $P(A' \cup B') = 1 - P(A \cap B)$
$= 0.7$

2. $A_1 = \{1 \text{ or } 2 \text{ on the first roll}\}$

$A_2 = \{3 \text{ or } 4 \text{ on the second roll}\}$

$A_3 = \{5 \text{ or } 6 \text{ on the third roll}\}$

a) $$P(A_1 \cup A_2 \cup A_3) = P(A_1) + P(A_2) + P(A_3) - P(A_1 \cap A_2) - P(A_1 \cap A_3) - P(A_2 \cap A_3) + P(A_1 \cap A_2 \cap A_3)$$
$$= \frac{19}{27}$$

b) $$P(A_1 \cup A_2 \cup A_3) = \frac{19}{27} = 1 - \frac{8}{27} = 1 - \left(\frac{2}{3}\right)^3$$
$$= 1 - \left(1 - \frac{1}{3}\right)^3$$

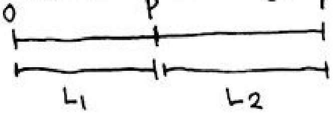
3. Note that:

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$.

LHS:

$$P((A \cup B) \cup C) = P(A \cup B) + P(C) - P((A \cup B) \cap C)$$
$$= P(A) + P(B) - P(A \cap B) + P(C) - P((A \cap C) \cup (B \cap C))$$
$$= P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C) \quad \text{(proved)}$$

4. let L_1 denotes the length of the first segment and L_2 denotes the length of the second segment,



$L_1 = P - 0 = P$

$L_2 = 1 - P$

case $L_1 \geq 2L_2$:

$P \geq 2(1 - P)$

$P \geq \frac{2}{3}$

case $2L_1 \leq L_2$:

$2P \leq 1 - P$

$P \leq \frac{1}{3}$

The probability of getting $P \leq \frac{1}{3}$ or $P \geq \frac{2}{3} = \frac{2}{3}$

5. a) $9! = 362880$

b) $\binom{9}{3} = 84$

c) $2^9 = 512$

6. note that:

$$(a+b)^n = \sum_{r=0}^{n} \binom{n}{r} a^r b^{n-r}$$

proof 1:

$$\sum_{r=0}^{n} (-1)^r \binom{n}{r} = \sum_{r=0}^{n} \binom{n}{r} (-1)^r (1)^{n-r}$$
$$= (-1+1)^n$$
$$= 0$$

proof 2:

$$\sum_{r=0}^{n} \binom{n}{r} = \sum_{r=0}^{n} \binom{n}{r} (1)^r (1)^{n-r}$$
$$= (1+1)^n$$
$$= 2^n$$

7 a) $\dfrac{{}^{13}C_5 \cdot {}^{13}C_4 \cdot {}^{13}C_3 \cdot {}^{13}C_1}{{}^{52}C_{13}} = 0{,}0053878.$

b) $\dfrac{{}^{13}C_5 \cdot {}^{13}C_4 \cdot {}^{13}C_2 \cdot {}^{13}C_2}{{}^{52}C_{13}} = 0{,}0088164$

c) $\dfrac{{}^{13}C_5 \cdot {}^{13}C_4 \cdot {}^{13}C_1 \cdot {}^{13}C_3}{{}^{52}C_{13}} = 0{,}0053878$

d) since $\binom{13}{3}\binom{13}{1} \cdot 3 \cdot 2 > \binom{13}{2}\binom{13}{2} \cdot 3$,
the probability of having split 3 and 1 is greater than split 2 and 2

10 a) $\dfrac{13 \cdot 48}{{}^{52}C_5} = 0{,}00024001$

b) $\dfrac{{}^4C_2 \cdot {}^4C_3 \cdot 2! \cdot {}^{13}C_2}{{}^{52}C_5} = 0{,}0014406$

c) $\dfrac{{}^4C_3 \cdot 4 \cdot 4 \cdot {}^3C_1 \cdot {}^{13}C_3}{{}^{52}C_5} = 0{,}021128$

d) $\dfrac{{}^4C_2 \cdot {}^4C_2 \cdot 4 \cdot {}^3C_2 \; {}^{13}C_3}{{}^{52}C_5} = 0{,}047539$

e) $\dfrac{{}^4C_2 \cdot 4 \cdot 4 \cdot 4 \cdot {}^4C_1 \cdot {}^{13}C_4}{{}^{52}C_5} = 0{,}42257$

8. let A_i denotes the i-th flavor of suckers,

$A_1 + A_2 + A_3 + \dots + A_{10} = 36 \quad (0 \leq A_i \leq 36)$

by Stars and Bars approach,

$$\text{total combinations} = \binom{36+10-1}{10-1} = \binom{45}{9} = 886163135$$

(there are 36 stars and 9 bars)

9. to select n_1 position $\binom{n}{n_1}$ ways

to select n_2 position $\binom{n-n_1}{n_2}$ ways

$\vdots$

to select n_s position $\binom{n-\sum_{i=1}^{s-1} n_i}{n_s}$ ways

by multiplication rule,

$$\text{total combinations} = \frac{n!}{\cancel{(n-n_1)!}\, n_1!} \cdot \frac{\cancel{(n-n_1)!}}{\cancel{(n-n_1-n_2)!}\, n_2!} \cdots \frac{\cancel{(n-n_1-\dots n_{s-1})!}}{n_s!}$$

$$= \frac{n!}{n_1!\, n_2! \dots n_s!}$$

jupyter Statistic Assignment 1 (Computer-based) - Yohandi

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In [37]:

```
import pandas as pd
import matplotlib.pyplot as plt
import random

n=[0]
sample=[0]
eventCount=[0]
currentProbability=[0]

for i in range(5000):
    n.append(i+1)
    sample.append(random.randint(1,6))
    eventCount.append(eventCount[-1])
    if(sample[i]==1 or sample[i]==2):
        eventCount[-1]+=1
    currentProbability.append(float(eventCount[-1]/n[-1]))

data = {'N':n,'Probability':currentProbability}
df = pd.DataFrame(data,columns=['N','Probability'])
df.plot(x ='N', y='Probability', kind = 'scatter')
plt.show()
```