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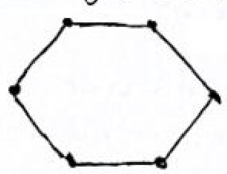
CSC3001 Assignment 4

1. -> An undirected graph is said to be a simple graph if it has no multiple edges and loops
-> In an undirected graph, the degree of a vertex is the number of edges incident on it
-> The degree sequence of a graph is the sequence of the degrees of the vertices of the graph in non decreasing order
-> A sequence $\{d_1, d_2, d_3, \dots, d_n\}$ is called graphic if it is the degree sequence of a simple graph

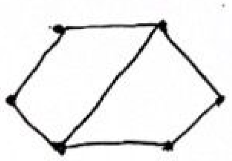
(a) since the sum is $5+4+3+2+1+0=15$ is an odd number, the sequence is not graphic
since the sum is $2+2+2+2+2+2=12$

(b) since the sum is $6+5+4+3+2+1=21$ is an odd number, the sequence is not graphic.

(c) is an even number and contains all even edges the sequence is graphic.



(d) since the sum is $3+3+2+2+2+2=14$ is an even number and the number of odd degrees is even, the sequence is graphic



2. suppose $d(x)$ denotes the degree sequence that is valid by constructing x edges. in other words, $d(x) = \{a, b, c, d, e\}$. notice that each edge that is to be added will increase two values by 1.

$d(0) = \{0,0,0,0,0\}$

$\underbrace{d(0) \to \{1,1,0,0,0\}}_{d(1)}$ — $\underbrace{\{2,2,0,0,0\},\ \{2,1,1,0,0\},\ \{1,1,1,1,0\}}_{d(2)}$

$\{2,2,0,0,0\}$ — $\{3,3,0,0,0\}$ (not simple), $\{3,2,1,0,0\}$ (not simple), $\{2,2,1,1,0\}$

$\{2,1,1,0,0\}$ — $\{3,2,1,0,0\}$ (isomorphic), $\{3,1,1,1,0\}$, $\{2,2,2,0,0\}$, $\{2,2,1,1,0\}$ (isomorphic)

$\{1,1,1,1,0\}$ — $\{2,2,1,1,0\}$ (isomorphic), $\{2,1,1,1,1\}$

hence, there are 4 non-isomorphic simple graphs with 5 vertices & 3 edges

3. Suppose the preferences are:
A: C D
B: D C
C: A B
D: B A
we notice A-C, B-D is stable as the favorite of the boys. However, the match A-D, B-C is also stable which shows more than one stable matching.
note: A & B are boys, C & D are girls

4(i) current_free = {Adam, Bill, Carl, Dan, Eric}
perform algorithm:

{(Amy, Dan), (Beth, Carl), (Diane, Bill)}

current - free = {Adam, Eric}

{(Amy, Adam), (Beth, Carl), (Diane, Eric)}

current_free : {Dan, Bill}

{(Amy, Adam), (Beth, Carl) (Diane, Eric)}

current - free = {Bill, Dan}

{(Amy, Adam), (Beth, Carl), (Cara, Dan), (Diane, Eric)}

current - free = {Bill}

{(Amy, Adam), (Beth, Carl), (Cara, Dan), (Diane, Eric)}

current - free = {Bill}

{(Amy, Adam), (Beth, Carl), (Cara, Bill), (Diane, Eric)}

current - free = {Dan}

{(Amy, Adam), (Beth, Carl), (Cara, Bill), (Diane, Eric)}

current - free = {Dan}

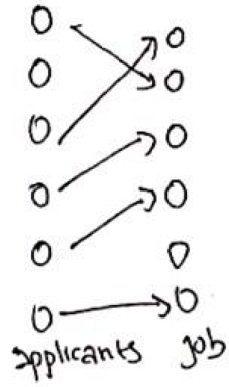
{(Amy, Adam), (Beth, Carl), (Cara, Bill), (Diane, Eric), (Ellen, Dan)}

(ii) similarly we can perform the algorithm and obtain:

{(Adam, Diane), (Bill, Cara), (Carl, Beth), (Dan, Ellen), (Eric, Amy)}

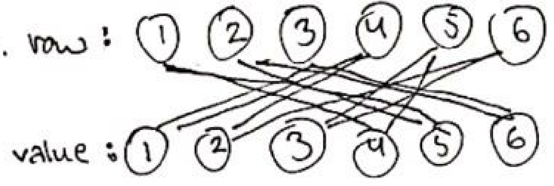
5. assume a bipartite graph where one side corresponds to the jobs with r vertices and another side corresponds to the s applicants, corresponding to

each job in the graph, create an edge from the vertex to all the applicants in the graph that are qualified for the job. maximum matching corresponds to the maximum number of jobs vertex ~~being~~ assigned to any vertex corresponding to the applicants



maximum filled jobs is 5

6. let X and Y denote the left and right side of the graph respectively.
we denote d|X| as the number of edges adjacent to X. similarly for Y.
suppose we have S as a subset of X, the edges, which are adjacent to S, are adjaent to vertices in N(S) in Y.
if |N(S)| < |S|
some vertex must have degree more than d, which is a contradiction
=> |N(S)| ≥ |S| ~~forever~~ for every possible subsets of S.
by Hall's theorem, there is left-saturated matching.
=> there is a perfect matching.
remove the perfect matching, we have a d-1 regular graph.
repeat the argument until the graph is empty. hence, proved

7. 

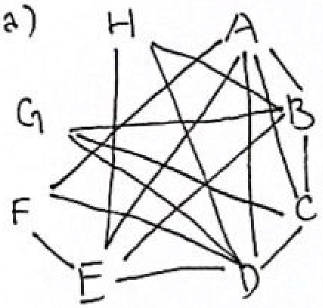
hence, there exists a solution (or more solutions):

...

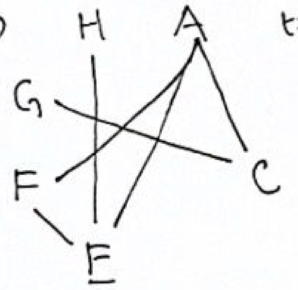
4	5
5	6
6	1
1	2
3	4
2	3

8. let A denotes Corn Feed Committee, B denotes Dorm Policy Committee, ..., H denotes Student Fees Committee respectively. We connect two committees if they share at least one student. each edge, which connect two committees, implies only either one vertice in the edge can be done at a time. we greedily do the most degrees that a vertice has.

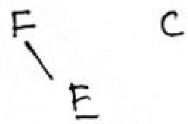
(a)



take B&D



take A, G, & H



take C & E

then take F

(b) 4