The Chinese University of Hong Kong, Shenzhen

PHY1002

Physics Laboratory

Lab Report for Conservation of Angular Momentum

Author:
Yohandi

Student Number:
120040025

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1 Introduction

1.1. Objective

The preservation of angular momentum, a core concept in physics, describes the way a spinning system retains its angular momentum in the absence of external torques. This principle states that a system's total angular momentum will stay constant, regardless of changes in rotation or shape. In this experiment, a spinning disk is hit by a stationary ring. Angular momentum is measured before the ring is dropped and after it has ceased moving across the disk. The final section will delve into the significance and implications of the angular momentum conservation principle, as well as its connection to rotational kinetic energy.

1.2. Literature Review

1.2.1. Angular Momentum

The angular momentum of an object quantifies its rotational motion concerning a specific rotational axis. It represents the product of the object's moment of inertia and angular velocity, essentially capturing the rate and orientation of its rotation. The angular momentum equation is given by:

$$L = I_i \cdot \omega_i = I_f \cdot \omega_f \tag{1}$$

where I_i denotes the initial moment of inertia, ω_i the initial angular velocity, I_f the final moment of inertia, and ω_f the final angular velocity. The moment of inertia depends on the object's mass and its distribution relative to the rotational axis, reflecting the difficulty in changing the object's rotational motion. Angular velocity describes the speed at which an object rotates around an axis. The rotational axis and the object's angular velocity are both parallel to the direction of the angular momentum. If either the moment of inertia or angular velocity varies, the direction of the angular momentum will change accordingly. Since angular momentum is a conserved property, it remains constant in the absence of external torques acting on the system.

1.2.2. Rotational Inertia

In physics, rotational inertia refers to an object's opposition to changes in its rotational motion. It is determined by the object's mass and the distribution of that mass around the rotational axis. Objects with a more widespread mass distribution and further from the rotational axis exhibit greater rotational inertia due to the increased "lever arm" length, which requires more torque to rotate. The initial rotational inertia of a disk, with an axis perpendicular to the disk and passing through its center of mass, is given by the following formula:

$$I_i = I_d = \frac{1}{2} \cdot M \cdot R^2 \tag{2}$$

where M represents the object's mass and R denotes the radius. The rotational inertia of a ring around an axis through its center of mass and parallel to the symmetry axis is calculated using this formula:

$$I_{rcm} = \frac{1}{2} \cdot M \cdot ({R_1}^2 + {R_2}^2) \tag{3}$$

where R_1 is the inner radius and R_2 is the outer radius. If the rotation axis is shifted by a distance (x) from the center of mass, the rotational inertia can be computed with this formula:

$$I_r = \frac{1}{2} \cdot M \cdot ({R_1}^2 + {R_2}^2) + (Mx^2) \tag{4}$$

1.2.3. Rotational Kinetic Energy

Rotational kinetic energy pertains to the energy associated with an object's rotating motion. An object's resistance to rotational motion, known as the moment of inertia, depends on its mass and how that mass is distributed around the rotational axis. The rotational kinetic energy of a rotating object can be determined using this formula:

$$KE = \frac{1}{2} \cdot I \cdot \omega^2 \tag{5}$$

2 Method

2.1. Setup

First, connect the Rotary Motion Sensor to the support rod attached to the 550 Universal Interface. Next, attach the pulley and disk to the rotary motion sensor as depicted in figure 1. Use a level to make sure the disk is level, adjusting the feet as necessary to ensure accurate results. The full experimental setup can be seen in figure 2.

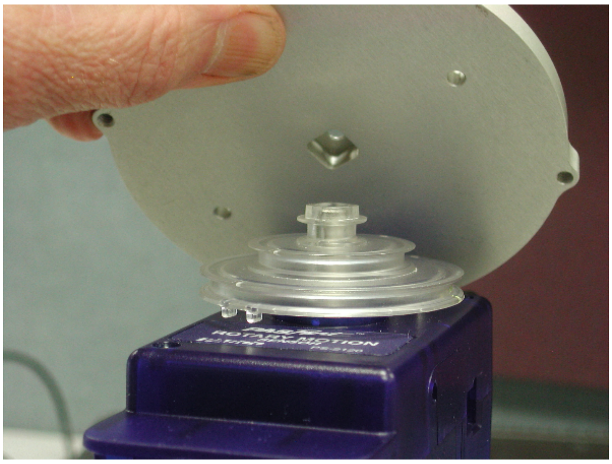

Figure 1: Attaching the Disk

Figure 2: Setup

2.2. Procedure

The experiment procedure is split into four parts: physical data measurement, setup completion, leveling, and experiment analysis. Measurements are taken using a digital scale and calipers, including the plastic pulley, disk, ring, and borrowed disk. This data will be recorded and presented later in section four. The setup is completed as described in the previous section.

2.2.1. Leveling

The leveling step verifies that the disk is perfectly level. Conduct the level check (angular speed vs. time) as illustrated in figure 3.

2.2.2. Experiment Analysis

Complete the following three steps: First, rotate the disk clockwise at around 20 to 30 radians per second and start recording data. Allow the disk to spin for about two seconds, then drop the ring onto the disk as centrally as possible from a height of 2 to 3 millimeters. Avoid dropping the ring from too high, as this may decrease its angular momentum due to a stronger downward force. After another two seconds, stop collecting data. Second, measure the minimum distance between the ring and the disk's edge by marking the tangent disk to the ring with a piece of paper. This provides an accurate measurement of the distance. Lastly, calculate the distance from the ring's center of mass using the formula below:

$$x = 0.95 - (\text{minimum distance}) \tag{6}$$

If the ring is perfectly centered on the disk, the minimum distance from the edge will be approximately 9.5 mm, which is the difference between the two outer radii. Repeat all steps from the beginning three times: twice with the same ring and once with a different disk (borrowed disk).

3 Raw Data

The data gathered in this experiment consists of graphs depicting Angular Velocity (rad/s) as a function of Time (t). The respective graph for each trial can be found in the subsequent figures provided below.

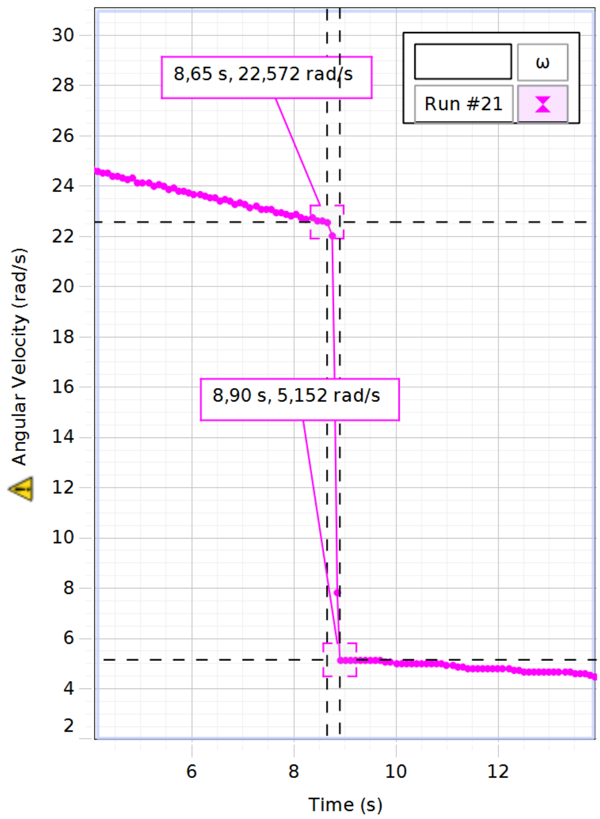

Figure 3: ω-t graph of Ring 1

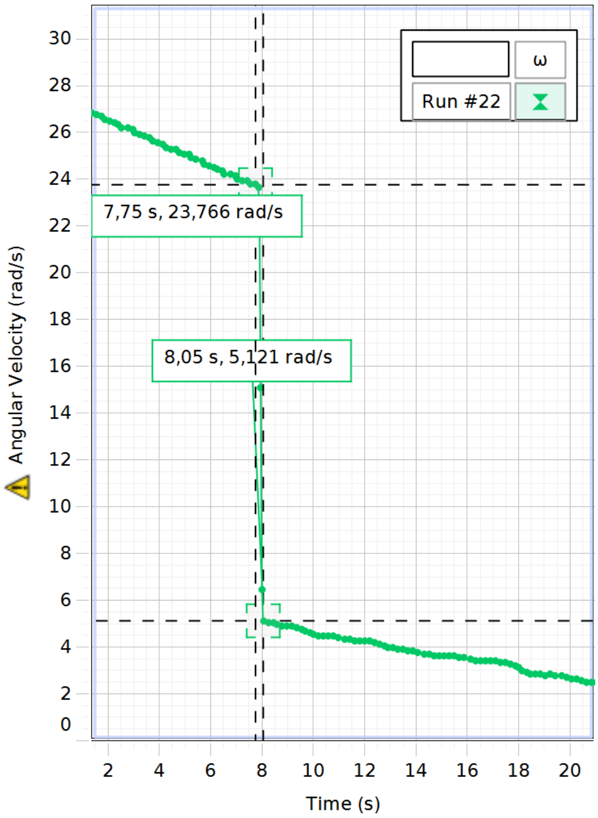

Figure 4: ω-t graph of Ring 2

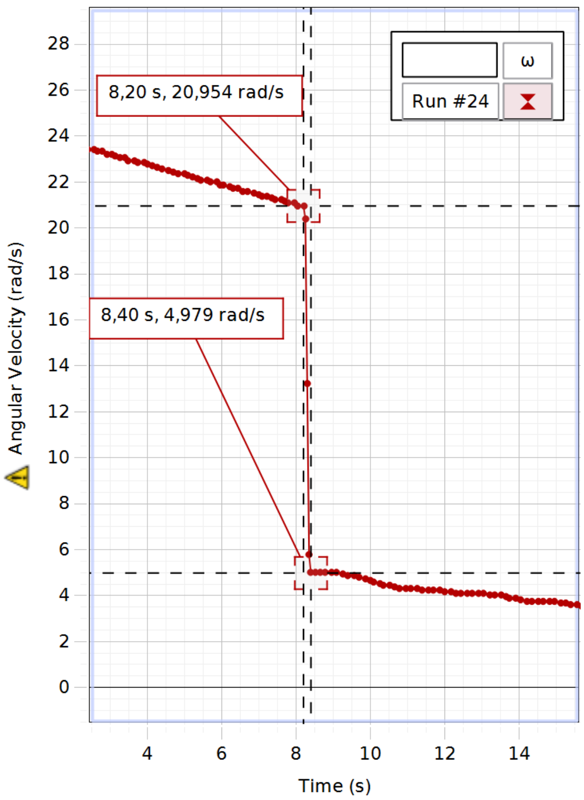

Figure 5: ω-t graph of Ring 3

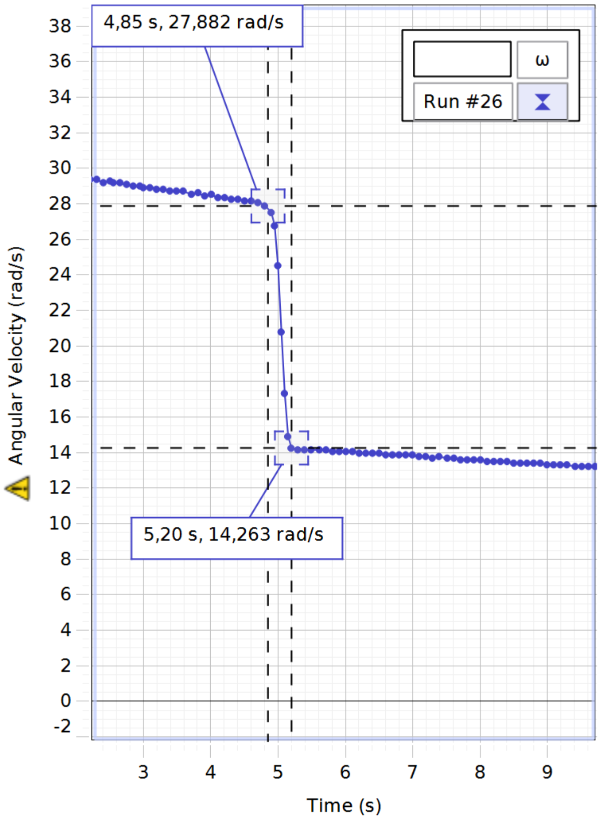

Figure 6: ω-t graph of Disk

4 Data and Error Analysis

4.1. Measurement Data Analysis

The experiment requires information about all measured attributes related to the equation. The information is as follows:

Pulley Mass (g)	Pulley Radius (cm)	Obj	Mass (g)	R out (cm)	R in (cm)
6,97 $\pm 0,01$	2,395 $\pm 0,001$	Ring run 1	469,57 $\pm 0,06$	3,8800 $\pm 0,001$	2,6868 $\pm 0,001$
6,97 $\pm 0,01$	2,395 $\pm 0,001$	Ring run 2	469,57 $\pm 0,06$	3,8800 $\pm 0,001$	2,6868 $\pm 0,001$
6,97 $\pm 0,01$	2,395 $\pm 0,001$	Ring run 3	469,57 $\pm 0,06$	3,8800 $\pm 0,001$	2,6868 $\pm 0,001$
6,97 $\pm 0,01$	2,395 $\pm 0,001$	Disk 1	120,49 $\pm 0,02$	4,7675 $\pm 0,001$	0,0000
6,97 $\pm 0,01$	2,395 $\pm 0,001$	Disk 2	120,49 $\pm 0,02$	4,7675 $\pm 0,001$	0,0000

Table 1: Measurements of attributes used in experiment

When two objects collide, the shortest distance between their respective center of masses (COMs) is denoted as x. Assuming the ring is placed perfectly in the center of the disk, the distance between the inner and outer radii is 0.95 cm. Using this information, we can determine the formula for x:

$$x = 0.95 - (\texttt{minimum distance})(cm). \tag{7}$$

We can use the formula mentioned above to obtain the value of x.

Obj Set	minimum distance (cm)	x (cm)
Ring run 1	0,87 $\pm 0,01$	0,12 $\pm 0,01$
Ring run 2	0,51 $\pm 0,01$	0,44 $\pm 0,01$
Ring run 3	0,13 $\pm 0,01$	0,82 $\pm 0,01$

Table 2: The distance that is the least amount of space between the disk and the object.

4.2. Rotational Inertia and Angular Momentum Analysis

The rotational inertia of an object is its resistance to rotational motion and is dependent on the object's mass, shape, and distribution of mass. In this lab, we measured the rotational inertia of various objects by attaching them to a rotating platform and measuring the angular acceleration produced by a known torque.

Based on the theory, one can compute the values of rotational inertia and angular momentum using the following methods:

$$I_r = \frac{1}{2}M(R_1 + R_2)^2 + Mx^2 \tag{8}$$

$$L = I_i\omega_i = I_f\omega_f \tag{9}$$

By using these methods, we have the tables as followings:

System	IRI (g cm^2)	FRI (g cm^2)	IAV (rad/s)	FAV (rad/s)
ring on disk (1)	1389 ±3	6645 ±19	22,572 ±0, 001	5,152 ±0, 001
ring on disk (2)	1389 ±3	6729 ±15	23,766 ±0, 001	5,121 ±0, 001
ring on disk (3)	1389 ±3	6950 ±22	20,954 ±0, 001	4,979 ±0, 001
disk 2 on disk	1389 ±3	2778 ±4	27,882 ±0, 001	14,263 ±0, 001

Table 3: Rotational Inertia and Angular Velocity

System	IAM (g cm^2/s)	FAM (g cm^2/s)
ring on disk (1)	31350 ±70	34240 ±80
ring on disk (2)	33010 ±70	34460 ±80
ring on disk (3)	29110 ±70	34600 ±110
disk 2 on disk	38730 ±60	39620 ±60

Table 4: Angular Momentum

4.3 Rotational Kinetic Energy Analysis

Using the calculated values of the moment of inertia (I) and angular velocity (ω), compute the rotational kinetic energy (K) with the following formula:

$$K = \frac{1}{2}I\omega^2 \tag{10}$$

By using these formula, we have the table as follows:

System	Initial K (g cm^2/s^2)	Final K (g cm^2/s^2)
ring on disk (1)	353800 ±800	88190 ±200
ring on disk (2)	392300 ±800	88230 ±200
ring on disk (3)	304900 ±700	86150 ±300
disk 2 on disk	539900 ±900	282600 ±400

Table 5: Rotational Kinetic Energy

By comparing the `Initial K` and `Final K`, we obtain the `Energy Difference %` as follows:

System	Energy Difference %
ring on disk (1)	75
ring on disk (2)	78
ring on disk (3)	72
disk 2 on disk	48

4.4. Answer to Lab Questions

1. What effect should each of the following have on the value you calculate for the final angular momentum? State whether each would cause the final value to be low, high, or unchanged, and explain why.

   (a) If the axis of the Rotary Motion Sensor has small rotational inertia (in addition to the pulley)?
   The final value of angular momentum would be higher than expected. If the axis of the Rotary Motion Sensor has a small, but non-zero, rotational inertia, this additional inertia must be taken into account when calculating the total angular momentum of the system. As angular momentum is the product of moment of inertia and angular velocity, the increase in moment of inertia will lead to an increase in the total angular momentum, resulting in a higher final value.

   (b) If the frictional drag on the bearings cannot be ignored?
   The final value of angular momentum would be lower than expected. Frictional drag on the bearings acts as an external force, introducing an external torque on the system. This torque opposes the rotational motion of the system, causing a decrease in the angular velocity. As the angular momentum is the product of moment of inertia and angular velocity, a decrease in angular velocity results in a decrease in the total angular momentum, leading to a lower final value.

2. Does the experimental result support the Law of Conservation of Angular Momentum? Explain fully.

   To assess if the experimental results support the Law of Conservation of Angular Momentum, we will need to compare the initial and final angular momentum values of the system. According to the law, the total angular momentum of a closed system should remain constant if there are no external torques acting on it.

   To determine if our results align with this principle, we need to calculate the initial angular momentum and the final angular momentum. If the initial and final angular momentum values are approximately equal (considering experimental uncertainties and errors), then our results support the Law of Conservation of Angular Momentum. This would indicate that angular momentum was conserved during the experiment, as predicted by the law. From the experiment, it is seen that both initial and final angular momentum values differ by at most 18%, which is quite a good standard for values comparison. This implies that the experiment was conducted successfully.

   However, if there is a significant discrepancy between the initial and final angular momentum values, further investigation is needed to identify potential sources of error, such as friction or other external forces, that might have affected the outcome of the experiment. In this case, the experimental results may not fully support the Law of Conservation of Angular Momentum.

3. Was Kinetic Energy conserved in the collision? Explain how you know.

   To determine whether kinetic energy was conserved in the collision, we need to compare the initial and final kinetic energies of the system. The conservation of kinetic energy implies that the total kinetic energy before the collision is equal to the total kinetic energy after the collision.

   To assess if our experimental results support the conservation of kinetic energy, we need to calculate the initial rotational kinetic energy and the final rotational kinetic energy.

   If the initial and final kinetic energy values are approximately equal (considering experimental uncertainties and errors), then our results suggest that kinetic energy was conserved during the collision.

   However, from our experiment, the energy difference was quite huge. It reaches 78% of `Energy Difference`. It is important to note that in real-life situations, collisions are rarely perfectly elastic, meaning that some energy is typically lost in the form of heat, sound, or vibrations. As a result, the conservation of kinetic energy may not be strictly observed in practice. If there is a significant discrepancy between the initial and final kinetic energy values, this could indicate that some energy was lost during the collision, suggesting that kinetic energy was not conserved.

4. Typically, you should see a loss of angular moment for the ring of $5\% - 15\%$. If you did the disk drop, it should have shown a few percent drops.

   (a) Why should the disk drop work better?
   The disk drop typically works better than the ring drop for a couple of reasons:

   1. Similar shapes: When a disk is dropped onto another disk, their shapes are more similar than when a ring is dropped onto a disk. This similarity allows for better contact and more effective transfer of angular momentum between the two objects.

   2. Lower losses due to air resistance: A disk has a more compact shape than a ring, which can reduce the effect of air resistance during the drop. As a result, a disk may experience less loss of angular momentum due to air resistance compared to a ring.

   (b) What causes the small percentage of angular momentum loss after the drop?

   There are several factors that can contribute to the small percentage of angular momentum loss after the drop:

   1. Friction: Friction between the two objects in contact during the collision can dissipate some energy in the form of heat. This energy loss can cause a reduction in the system's total angular momentum.

   2. Air resistance: As the objects move through the air, they experience air resistance, which can slow them down and cause a loss of angular momentum.

   3. Imperfect alignment: If the ring or disk is not dropped precisely at the center of the spinning disk, the impact force may not be perfectly aligned with the axis of rotation, causing a loss of angular momentum.

   4. Inelastic collision: In real-world scenarios, collisions are often inelastic, meaning that some of the kinetic energy is converted into other forms of energy, such as heat or sound, during the collision. This energy conversion can result in a loss of angular momentum.

5. In the ideal case, how can angular momentum be conserved, but energy not be conserved?
   In an ideal case, angular momentum is conserved when no external torques act on a closed system. However, the conservation of energy and the conservation of angular momentum are separate principles, and energy conservation depends on the nature of the interaction between objects in the system.

   If a system experiences an inelastic collision, some of the kinetic energy may be converted into other forms of energy, such as heat, sound, or internal energy due to deformation. In this scenario, the total mechanical energy of the system is not conserved, even though angular momentum may still be conserved. This occurs because energy conservation depends on the type of collision, whereas angular momentum conservation depends solely on the absence of external torques.

   For example, imagine two objects with different masses rotating on a frictionless axis. If they collide inelastically and stick together, their combined moment of inertia will change, and their combined angular velocity will adjust to conserve angular momentum. However, some of the initial kinetic energy may be transformed into heat or sound during the collision. In this case, angular momentum is conserved, but the total mechanical energy is not conserved due to the inelastic nature of the collision.

5 Conclusion

In summary, the conservation of angular momentum experiment serves as a valuable and powerful tool for exploring rotational motion and understanding the underlying physical phenomena. This experiment demonstrates the fundamental principle of angular momentum conservation, stating that a rotating system's total angular momentum remains constant in the absence of external torques. Through conducting this experiment, we successfully showcased that angular momentum is a conserved quantity, while its kinetic energy is not. During an impact, energy may be converted into other forms, such as heat or sound.