yohandi - homework week 12

Exercise 8.5

6. $\frac{z}{z^3-z^2-6z} = \frac{1}{z^2-z-6} = \frac{1}{(z-3)(z+2)}$

$= \frac{A}{z-3} + \frac{B}{z+2}$

$A(z+2) + B(z-3) = 1$

$z=-2 \Rightarrow B = -\frac{1}{5}$

$z=3 \Rightarrow A = \frac{1}{5}$

$= \frac{1}{5(z-3)} - \frac{1}{5(z+2)}$

7. $\frac{t^2+8}{t^2-5t+6} = 1 + \frac{5t+2}{(t-2)(t-3)} = 1 + \frac{A}{t-2} + \frac{B}{t-3}$

$(t-2)(t-3) + A(t-3) + B(t-2) = t^2+8$

$t=2 \Rightarrow A = -12$

$t=3 \Rightarrow B = 17$

$= 1 - \frac{12}{t-2} + \frac{17}{t-3}$

13. $\int_4^8 \frac{y}{y^2-2y-3} dy = \int_4^8 \left(\frac{A}{y-3} + \frac{B}{y+1} \right) dy$

$A(y+1) + B(y-3) = y$

$y=-1 \Rightarrow B = \frac{1}{4}$

$y=3 \Rightarrow A = \frac{3}{4}$

$= \int_4^8 \frac{3}{4(y-3)} + \frac{1}{4(y+1)} dy = \frac{3}{4}\ln|y-3| + \frac{1}{4}\ln|y+1| \Big|_{y=4}^{8}$

$= \frac{1}{2}\ln(3) + \frac{1}{2}\ln(5)$

16. $\int \frac{x+3}{2x^3-8x} dx = \int \left(\frac{A}{2x} + \frac{B}{x+2} + \frac{C}{x-2} \right) dx$

$A(x+2)(x-2) + B(2x)(x-2) + C(2x)(x+2) = x+3$

$x=2 \Rightarrow C = \frac{5}{16}$

$x=-2 \Rightarrow B = \frac{1}{16}$

$x=0 \Rightarrow A = \frac{3}{-4}$

$= \int -\frac{3}{8x} + \frac{1}{16(x+2)} + \frac{5}{16(x-2)} dx$

$= -\frac{3}{8}\ln|x| + \frac{1}{16}\ln|x+2| + \frac{5}{16}\ln|x-2| + C$

20. $\int \frac{x^2}{(x-1)(x^2+2x+1)} dx = \int \frac{A}{(x-1)} + \frac{B}{(x+1)} + \frac{C}{(x+1)^2} dx$

$A(x+1)^2 + B(x-1)(x+1) + C(x-1) = x^2$

$x=-1 \Rightarrow C = -\frac{1}{2}$

$x=1 \Rightarrow A = \frac{1}{4}$

$x=0 \Rightarrow A - B - C = 0 \Rightarrow B = \frac{3}{4}$

$= \int \frac{1}{4(x-1)} + \frac{3}{4(x+1)} - \frac{1}{2(x+1)^2} dx$

$= \frac{1}{4}\ln|x-1| + \frac{3}{4}\ln|x+1| + \frac{1}{2(x+1)} + C$

23. $\int \frac{y^2+2y+1}{(y^2+1)^2} dy = \int \frac{y^2+1}{(y^2+1)^2} + \frac{2y}{(y^2+1)^2} dy$

$= \int \frac{1}{y^2+1} dy + \int \frac{d(y^2+1)}{(y^2+1)^2}$

$= \arctan(y) - \frac{1}{(y^2+1)} + C$

34. $\int \frac{x^4}{x^2-1} dx = \int x^2 + 1 + \frac{1}{x^2-1} dx$

$= \int x^2 + 1 + \frac{1}{2(x-1)} - \frac{1}{2(x+1)} dx$

$= \frac{1}{3}x^3 + x + \frac{1}{2}\ln\left|\frac{x-1}{x+1}\right| + C$

35. $\int \frac{9x^3-3x+1}{x^3-x^2} dx = \int 9 + \frac{9x^2-3x+1}{x^2(x-1)} dx$

$= \int 9 + \frac{A}{x} + \frac{B}{x^2} + \frac{C}{(x-1)} dx$

$A(x)(x-1) + B(x-1) + C(x)^2 = 9x^2-3x+1$

$x=0 \Rightarrow B = -1$

$x=1 \Rightarrow C = 7$

$x=2 \Rightarrow 2A + B + 4C = 31 \Rightarrow A = 2$

$= \int 9 + \frac{2}{x} - \frac{1}{x^2} + \frac{7}{(x-1)} dx$

$= 9x + 2\ln|x| + \frac{1}{x} + 7\ln|x-1| + C$

41. $\int \frac{\cos y \, dy}{\sin^2 y + \sin y - 6} = \int \frac{d(\sin y)}{(\sin y+3)(\sin y-2)}$

$= \int \frac{A}{\sin y+3} + \frac{B}{\sin y-2} \, d(\sin y)$

$A(\sin(y)-2) + B(\sin(y)+3) = 1$

$\sin(y) = -1 \Rightarrow -3A+2B = 1$

$\sin(y) = 1 \Rightarrow -A + 4B = 1$

$\Rightarrow A = -\frac{1}{5}, \; B = \frac{1}{5}$

$= \int -\frac{1}{5(\sin(y)+3)} + \frac{1}{5(\sin(y)-2)} d(\sin(y))$

$= -\frac{1}{5}\ln|\sin(y)+3| + \frac{1}{5}\ln|\sin(y)-2| + C$

Exercise 8.7

8.I

a. $T = \sum_{i=1}^{4} \frac{\Delta x}{2}(y_{i-1} + y_i)$

$= \dfrac{\left(1 + 2\cdot\frac{1}{2{,}25} + 2\cdot\frac{1}{4} + 2\cdot\frac{1}{6{,}25} + \frac{1}{9}\right)}{4} = \frac{141}{200}$

$|E_T| \le U = \frac{M(b-a)^3}{12n^2}$

$f(s) = \frac{1}{(s-1)^2}$ $\quad f''(s) = \frac{6}{(s-1)^4}$

$f'(s) = -\frac{2}{(s-1)^3}$

since $f''(s)$ is decreasing from 2 to 4

$\therefore M = |f''_{max}| = 6$

$U = 6 \cdot \frac{(4-2)^3}{12 \cdot 4^2} = \frac{1}{4}$

b. $\int_2^4 \frac{1}{(s-1)^2}\, ds = -\frac{1}{s-1}\Big|_{s=2}^{4} = \frac{2}{3} = T - E_T$

$|E_T| = \left|T - \frac{2}{3}\right| = \frac{23}{600}$

c. $\frac{|E_T|}{\text{true value}} \times 100\,\% = 5{,}75\%$

II

a. $S = \sum_{i=1}^{2} \frac{\Delta x}{3}(y_{2i-2} + 4y_{2i-1} + y_{2i})$

$= \dfrac{\left(1 + 4\cdot\frac{1}{2{,}25} + 2\cdot\frac{1}{4} + 4\cdot\frac{1}{6{,}25} + \frac{1}{9}\right)}{6} = \frac{1813}{2700}$

$|E_S| \le U = \frac{M(b-a)^5}{180n^4}$

$f(s) = \frac{1}{(s-1)^2}$

$f'(s) = -\frac{2}{(s-1)^3}$

$\vdots$

$f^{(4)}(s) = \frac{120}{(s-1)^6}$

since $f^{(4)}(s)$ is decreasing from 2 to 4

$\therefore M = |f^{(4)}_{max}| = 120$

$U = \frac{120(4-2)^5}{180(4)^4} = \frac{1}{12}$

b. $|E_S| = \left|S - \frac{2}{3}\right| = \frac{13}{2700}$

c. $\frac{|E_S|}{\text{true value}} \times 100\,\% = 0{,}7\bar{2}\,\%$

24. $T = \frac{1}{2}\Big[(2,2)\left[0 + \frac{30}{3.6}\right] + (1,0)\left[\frac{30}{3.6} + \frac{40}{3.6}\right] + (1.3)\left[\frac{40}{3.6} + \frac{50}{3.6}\right]$

$+ (1,4)\left[\frac{50}{3.6} + \frac{60}{3.6}\right] + (1,9)\left[\frac{60}{3,6} + \frac{70}{3,6}\right] + (2,4)\left[\frac{70}{3,6} + \frac{80}{3,6}\right]$

$+ (2,5)\left[\frac{80}{3,6} + \frac{90}{3,6}\right] + (3,3)\left[\frac{90}{3,6} + \frac{100}{3,6}\right] + (4,6)\left[\frac{100}{3,6} + \frac{110}{3,6}\right]$

$+ (5,6)\left[\frac{110}{3,6} + \frac{120}{3,6}\right] + (10,9)\left[\frac{120}{3,6} + \frac{130}{3,6}\right]\Big]$ m

$= 978{,}47\bar{2}$ m

25. $V = A \cdot h$

$\frac{m}{\rho} = \sum_{i=1}^{3} \frac{\Delta x}{3}(y_{2i-2} + 4y_{2i-1} + y_{2i}) \cdot h$

$\frac{2000\text{ kg}}{673\text{ kg/m}^3} = 0{,}1(0{,}5 + 4\cdot 0{,}55 + 2\cdot 0{,}6 + 4\cdot 0{,}65 + 2\cdot 0{,}7 + 5\cdot 0{,}75)\text{m}^2 \cdot h$

$h \approx 2{,}551$ m

28 a. $S = \sum_{i=1}^{5} \frac{\Delta x}{3}(y_{2i-2} + 4y_{2i-1} + y_{2i})$

$= \frac{2}{\sqrt{\pi}} \cdot \frac{1}{30}\Big[e^{-0} + 4 e^{-(0.1)^2} + 2 e^{-(0.2)^2} + 4 e^{-(0.3)^2} + 2 e^{-(0.4)^2} + 4e^{-(0.5)^2} + 2e^{-(0.6)^2} + 4e^{-(0.7)^2} + 2e^{-(0.8)^2} + 4e^{-(0.9)^2} + e^{-1^2}\Big]$

$\approx 0{,}84270$

b. $|E_S| \le U = \frac{M(b-a)^5}{180n^4} = \frac{12\cdot(1)^5}{180\cdot 10^4} = \frac{2}{3}\cdot 10^{-5}$

29. $T = \frac{b-a}{2n}\sum_{i=1}^{n}(f(x_i) + f(x_{i-1}))$

$= \frac{b-a}{n}\sum_{i=1}^{n}\left[\frac{f(x_i) + f(x_{i-1})}{2}\right]$

since f is a continuous function there exist $c_i \in [x_{i-1}, x_i]$ such that $2f(c_i) = f(x_i) + f(x_{i-1})$

$T = \frac{b-a}{n}\sum_{i=1}^{n} f(c_i)$ (this is Riemann Sum)

31 a. length $\approx T = \frac{4(1)}{2} \cdot \frac{\left(\frac{\pi}{2} - 0\right)}{10}\Big[\sqrt{\frac{1-\cos^2(0)}{4}} + \sqrt{4-\cos^2\left(\frac{\pi}{20}\right)}$

$+ \sqrt{4-\cos^2\left(\frac{\pi}{10}\right)} + \sqrt{4-\cos^2\left(\frac{3\pi}{20}\right)} + \sqrt{4-\cos^2\left(\frac{\pi}{5}\right)}$

$+ \sqrt{4-\cos^2\left(\frac{\pi}{4}\right)} + \sqrt{4-\cos^2\left(\frac{3\pi}{10}\right)} + \sqrt{4-\cos^2\left(\frac{7\pi}{20}\right)}$

$+ \sqrt{4-\cos^2\left(\frac{2\pi}{5}\right)} + \sqrt{4-\cos^2\left(\frac{9\pi}{20}\right)} + \sqrt{\frac{1-\cos^2\left(\frac{\pi}{2}\right)}{4}}\Big]$

$\approx 5{,}86985$

b. $|E_T| \le \frac{M\left(\frac{\pi}{2} - 0\right)^3}{12(10)^2}$ $(M=1)$ $= \frac{\pi^3}{9600}$

$= 0.00323$

Exercise 8.8

4. $\int_0^4 \frac{dx}{\sqrt{4-x}} = \lim_{c\to 4^-} \int_0^c \frac{dx}{\sqrt{4-c}}$

$= \lim_{c\to 4^-} -2\sqrt{4-c} + 4 = 4$

10. $\int_{-\infty}^{2} \frac{2dx}{x^2+4} = \lim_{c\to -\infty} \int_c^2 \frac{2dx}{x^2+4}$

$= \arctan(1) - \lim_{c\to -\infty} \arctan(c) = \frac{3\pi}{4}$

14. $\int_{-\infty}^{\infty} \frac{x\,dx}{(x^2+4)^{3/2}} = \lim_{a\to -\infty,\, b\to \infty} \int_a^b \frac{\frac{1}{2} d(x^2+4)}{(x^2+4)^{3/2}}$

$= \lim_{b\to\infty} -\frac{1}{(b^2+4)^{1/2}} + \lim_{a\to -\infty} \frac{1}{(a^2+4)^{1/2}}$

$= 0$

20. $\int_0^{\infty} \frac{16\tan^{-1}(x)}{1+x^2} dx = \int_0^{\infty} 16\tan^{-1}(x)\, d(\tan^{-1}(x))$

$= \lim_{c\to\infty} \int_0^c 16\tan^{-1}(x)\, d(\tan^{-1}(x))$

$= \lim_{c\to\infty} 8\arctan(c)^2 - 0$

$= 2\pi^2$

23. $\int_{-\infty}^{0} e^{-|x|} dx = \lim_{c\to -\infty} \int_c^0 e^x dx$

$= e^0 - \lim_{c\to -\infty} e^c$

$= 1$

27. $\int_0^2 \frac{ds}{\sqrt{4-s^2}} = \lim_{c\to 2^-} \int_0^c \frac{ds}{\sqrt{4-s^2}}$

$= \lim_{c\to 2^-} \arcsin(\frac{c}{2}) - 0$

$= \frac{\pi}{2}$

32. $\int_0^2 \frac{dx}{\sqrt{|x-1|}}$

$= \int_0^1 \frac{dx}{\sqrt{|x-1|}} + \int_1^2 \frac{dx}{\sqrt{|x-1|}}$

$= \lim_{a\to 1^-} \int_0^a \frac{dx}{\sqrt{1-x}} + \lim_{b\to 1^+} \int_b^2 \frac{dx}{\sqrt{x-1}}$

$= \lim_{a\to 1^-} -2\sqrt{1-a} + 2 + 2 - \lim_{b\to 1^+} 2\sqrt{b-1}$

$= 4$