

Yohandi - quiz 4

- 1a. ~~F~~
b. T
c. T
d. T

2. we know that if $x > y$, then $\sqrt{x} > \sqrt{y}$
(since both x and $y \geq 1$) and vice versa.

→ case $x > y$:

$$|\sqrt{x} - \sqrt{y}| \leq \frac{1}{2}|x - y|$$

$$\sqrt{x} - \sqrt{y} \leq \frac{1}{2}x - \frac{1}{2}y$$

$$\frac{2(\sqrt{x} - \sqrt{y})}{x - y} \leq 1$$

$$\frac{2}{\sqrt{x} + \sqrt{y}} \leq 1$$

since $\sqrt{x} + \sqrt{y} \geq \sqrt{1} + \sqrt{1} = 2$
(it is true)

→ case $x < y$:

similar as case $x > y$ with
swapping x and y

3. Let $f(x) = \tan x + e^{-x}$

$$\frac{d(f(x))}{dx} = f'(x) = \sec^2 x - e^{-x}$$

critical points when $f'(x) = 0$ or undefined

$$\rightarrow f'(x) = 0$$

$$\sec^2 x = e^{-x}$$

no solution for

x in $(0, \pi/3)$

→ $f'(x)$ undefined

e^{-x} undefined
(no solution)

$\sec^2 x$ undefined

no solution

Since there's no critical points
the graph is monotonically increasing
or decreasing resulting at most 1 solution

by IVT theorem,

$$f(0^+) = 1 \text{ and } f(\frac{\pi}{3}^-) > \sqrt{3}$$

there's no solution for
 $\tan x + e^{-x} = \ln(2, \frac{\pi}{3})$

It is proved that the solution
exist

$$4. y = f(x) \begin{cases} 1-x^2, & x < 0 \\ x^3 - 3x^2 + 1, & x \geq 0 \end{cases}$$

$$y' = f'(x) \begin{cases} -2x, & x < 0 \\ 3x^2 - 6x, & x \geq 0 \end{cases}$$

critical points when $y' = 0$

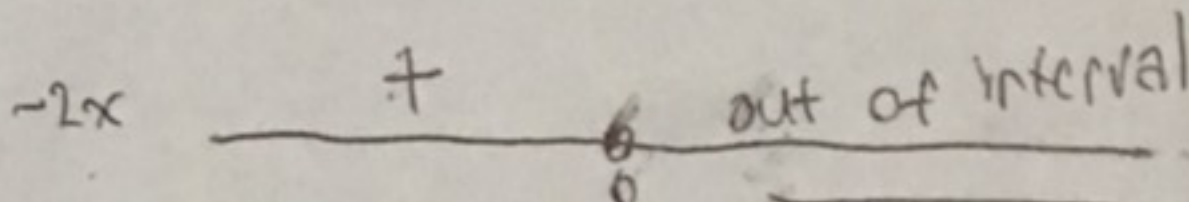
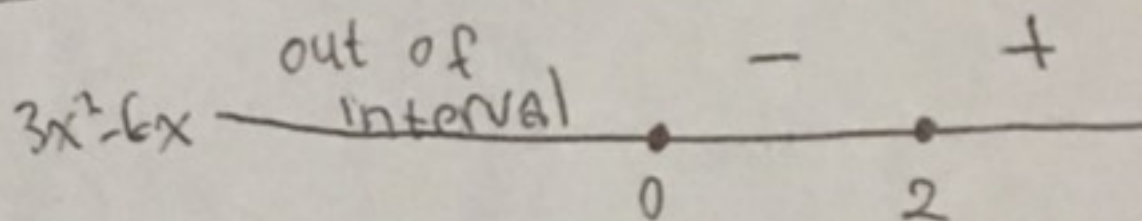
$$\rightarrow -2x = 0$$

$x = 0$ (since 0 is not < 0 , no
critical points for $1-x^2$)

$$\rightarrow 3x^2 - 6x = 0$$

$$3x(x-2) = 0$$

$$x = 0 \text{ or } x = 2$$



2. increasing on: $(-\infty, 0)$ and $[2, \infty)$

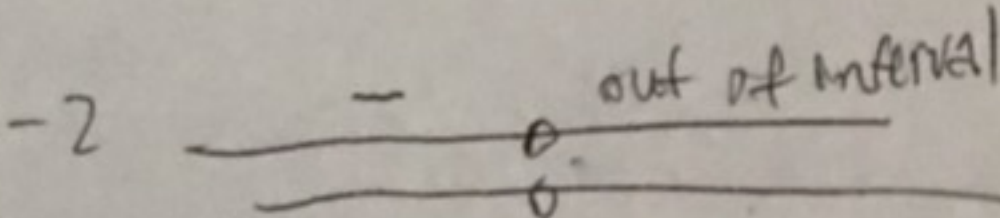
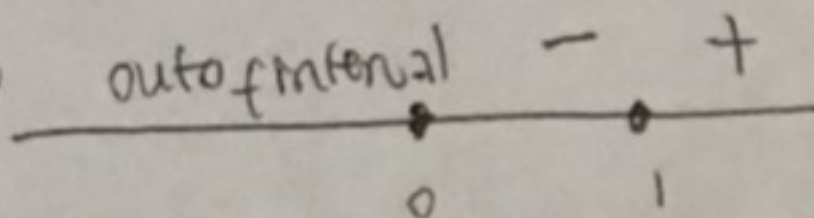
$$b. y'' = f''(x) \begin{cases} -2, & x < 0 \\ 6x - 6, & x \geq 0 \end{cases}$$

inflection points when $y'' = 0$

$$\rightarrow -2 \neq 0$$

$$\rightarrow 6x - 6 = 0$$

$$x = 1$$



concave up on: $(1, \infty)$

concave down on: $(-\infty, 1)$

c. extreme points: $x = 0, y = 1$

$x = 2, y = -3$

inflection point: $x = 1, y = -1$