yohandi - Math Homework Week 6

Exercises set 5.1

1a $\left(\frac{1}{2}\right)\left[0+\frac{1}{4}\right]=\frac{1}{8}$

b $\left(\frac{1}{4}\right)\left[0+\frac{1}{16}+\frac{1}{4}+\frac{9}{16}\right]=\frac{7}{32}$

c. $\left(\frac{1}{2}\right)\left[\frac{1}{4}+1\right]=\frac{5}{8}$

d. $\left(\frac{1}{4}\right)\left[\frac{1}{16}+\frac{1}{4}+\frac{9}{16}+1\right]=\frac{15}{32}$

3a $(2)\left[\frac{1}{3}+\frac{1}{5}\right]=\frac{16}{15}$

b $(1)\left[\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\frac{1}{5}\right]=\frac{77}{60}$

c $(2)\left[1+\frac{1}{3}\right]=\frac{8}{3}$

d. $(1)\left[1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}\right]=\frac{25}{12}$

5 two rectangles:

$\left(\frac{1}{2}\right)\left[\frac{1}{16}+\frac{9}{16}\right]=\frac{5}{16}$

four rectangles:

$\left(\frac{1}{4}\right)\left[\frac{1}{64}+\frac{9}{64}+\frac{25}{64}+\frac{49}{64}\right]=\frac{21}{64}$

7 . two rectangles:

$(2)\left[\frac{1}{2}+\frac{1}{4}\right]=\frac{3}{2}$

four rectangles:

$(1)\left[\frac{2}{3}+\frac{2}{5}+\frac{2}{7}+\frac{2}{9}\right]=\frac{496}{315}$

9a $\sum_{i=0}^{9} v_i \cdot \Delta x = 1 \cdot \sum_{i=0}^{9} v_i$

$=(0+12+22+10+5+13+11+6+2+6)$ cm

$=87$ cm

b. $\sum_{i=1}^{10} v_i \cdot \Delta x = 1 \cdot \sum_{i=1}^{10} v_i$

$=(12+22+10+5+13+11+6+2+6+0)$ cm

$=87$ cm

19a. $U=\sum_{i=1}^{5} v_i \cdot \Delta t = 1 \cdot (70+97+136+190+265)$ L

$=758$ L

$L=\sum_{i=0}^{4} v_i \cdot \Delta t = 1 \cdot (50+70+97+136+190)$ L

$=543$ L

b. $U=\sum_{i=1}^{8} v_i \cdot \Delta t = 1 \cdot (70+97+136+190+265+369+516+720)$ L

$=2363$ L

$L=\sum_{i=0}^{7} v_i \cdot \Delta t = 1 \cdot (50+70+97+136+190+265+369+516)$ L

$=1693$ L

c. worst case:

$t=\frac{25000-2363}{720}$ h $=31.44$ h

best case:

$t=\frac{25000-1693}{720}$ h $=32.37$ h

Exercises Set 5.2

8a $\sum_{k=1}^{6}(-2)^{k-1}$

b $\sum_{k=0}^{5}(-1)^{k}(2^{k})$

12 $\sum_{k=1}^{4}k^{2}$

15 $\sum_{k=1}^{5}(-1)^{k+1}\left(\frac{1}{k}\right)$

30a $\sum_{k=9}^{36}k = \sum_{k=1}^{36}k - \sum_{k=1}^{8}k = \frac{36\cdot 37}{2} - \frac{8\cdot 9}{2} = 630$

b $\sum_{k=3}^{17}k^{2} = \sum_{k=1}^{17}k^{2} - \sum_{k=1}^{2}k^{2} = \frac{17(18)(35)}{6} - 5 = 1780$

c $\sum_{k=18}^{71}k(k-1) = \sum_{k=1}^{71}k^{2} - \sum_{k=1}^{71}k - \sum_{k=1}^{17}k^{2} + \sum_{k=1}^{17}k$

$= \frac{71(72)(143)}{6} - \frac{71\cdot 72}{2} - \frac{17(18)(35)}{6} + \frac{17\cdot 18}{2}$

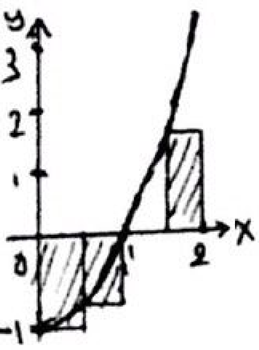
$= 117648$

32a $\sum_{k=1}^{n}\left(\frac{1}{n}+2n\right) = \left(\frac{1}{n}+2n\right)\sum_{k=1}^{n} = 2n^{2}+1$

b $\sum_{k=1}^{n}\frac{c}{n} = \frac{c}{n}\sum_{k=1}^{n} = c$

c $\sum_{k=1}^{n}\frac{k}{n^{2}} = \frac{1}{n^{2}}\sum_{k=1}^{n}k = \frac{n(n+1)}{2n^{2}} = \frac{n+1}{2n}$

33a

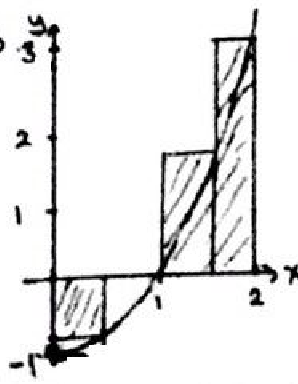


$\sum_{k=0}^{3} f\left(\frac{k}{2}\right)\Delta x = \frac{1}{2}\left[f(0)+f\left(\frac{1}{2}\right)+f(1)+f\left(\frac{3}{2}\right)\right]$

$= \frac{1}{2}(-1-0.75+0+1.25)$

$= -0.25$

b

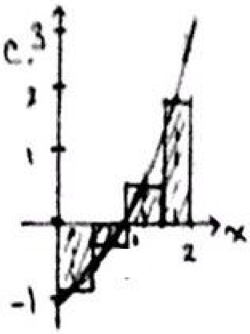


$\sum_{k=1}^{4} f\left(\frac{k}{2}\right)\Delta x = \frac{1}{2}\left[f\left(\frac{1}{2}\right)+f(1)+f\left(\frac{3}{2}\right)+f(2)\right]$

$= \frac{1}{2}(-0.75+0+1.25+3)$

$= 1.75$

c

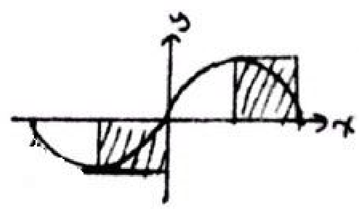


$\sum_{k=1}^{4} f\left(\frac{k}{2}-\frac{1}{4}\right)\Delta x$

$= \frac{1}{2}\left[f\left(\frac{1}{4}\right)+f\left(\frac{3}{4}\right)+f\left(\frac{5}{4}\right)+f\left(\frac{7}{4}\right)\right]$

$= 0.625$

35a

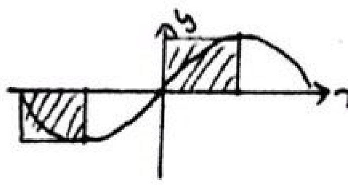


$\sum_{k=0}^{3} f\left(\pi\frac{(k-2)}{2}\right)\Delta x$

$= \frac{\pi}{2}\left[f(-\pi)+f\left(-\frac{\pi}{2}\right)+f(0)+f\left(\frac{\pi}{2}\right)\right]$

$= 0$

b

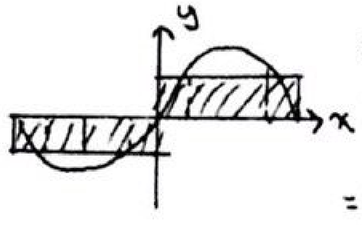


$\sum_{k=1}^{4} f\left(\pi\frac{(k-2)}{2}\right)\Delta x$

$= \frac{\pi}{2}\left[f\left(-\frac{\pi}{2}\right)+f(0)+f\left(\frac{\pi}{2}\right)+f(\pi)\right]$

$= 0$

c



$\sum_{k=1}^{4} f\left(\pi\frac{(2k-3)}{4}\right)\Delta x$

$= \frac{\pi}{2}\left[f\left(-\frac{3\pi}{4}\right)+f\left(-\frac{\pi}{4}\right)+f\left(\frac{\pi}{4}\right)+f\left(\frac{3\pi}{4}\right)\right]$

$= 0$

37 $\Delta x_1 = 1.2-0 = 1.2$

$\Delta x_2 = 1.5-1.2 = 0.3$

$\Delta x_3 = 2.3-1.5 = 0.8$

$\Delta x_4 = 2.6-2.3 = 0.3$

$\Delta x_5 = 3-2.6 = 0.4$

$\Delta x_{max} = \text{norm} = \max(\Delta x_1, \ldots, \Delta x_5) = 1.2$

39 $\Delta x_1 = -1.6-(-2) = 0.4$

$\Delta x_2 = -0.5-(-1.6) = 1.1$

$\Delta x_3 = 0-(-0.5)\ 0.5$

$\Delta x_4 = 0.8-0 = 0.8$

$\Delta x_5 = 1-0.8 = 0.2$

$\Delta x_{max} = \text{norm} = \max(\Delta x_1, \ldots, \Delta x_5) = 1.1$

39. $f(x) = 1 - x^2$

$$\sum_{i=1}^{n} f\left(\frac{i}{n}\right)\frac{1}{n} = \frac{1}{n}\sum_{i=1}^{n} 1 - \frac{i^2}{n^2}$$

$$= 1 - \frac{1}{n^3}\sum_{i=1}^{n} i^2$$

$$= 1 - \frac{n(n+1)(2n+1)}{6n^3}$$

when $n \to \infty \Rightarrow \sum_{i=1}^{n} f\left(\frac{i}{n}\right)\frac{1}{n} = 1 - \frac{2}{6} = \frac{2}{3}$

40. $f(x) = 2x$

$$\sum_{i=1}^{n} f\left(\frac{3i}{n}\right)\cdot\frac{3}{n} = \frac{3}{n}\cdot\sum_{i=1}^{n}\frac{6i}{n}$$

$$= \frac{18}{n^2}\sum_{i=1}^{n} i$$

$$= \frac{18n(n+1)}{2n^2}$$

when $n \to \infty \Rightarrow \sum_{i=1}^{n} f\left(\frac{3i}{n}\right)\frac{3}{n} = \frac{18}{2} = 9$

Exercises Set 5.3

1. $\lim_{\|P\|\to 0} \sum_{k=1}^{n} c_k^2 \Delta x_k$, where P is a partition $[0,2]$

$\Rightarrow \int_0^2 x^2\,dx$

3. $\lim_{\|P\|\to 0} \sum_{k=1}^{n} (c_k^2 - 3c_k)\,\Delta x_k$, where P is a partition $[-7,5]$

$\Rightarrow \int_{-7}^{5} (x^2-3x)\,dx$

6. $\lim_{\|P\|\to 0} \sum_{k=1}^{n} \sqrt{4-c_k^2}\,\Delta x_k$, where P is a partition $[0,1]$

$\Rightarrow \int_0^1 \sqrt{4-x^2}\,dx$

8. $\lim_{\|P\|\to 0} \sum_{k=1}^{n} \tan(c_k)\,\Delta x_k$, where P is a partition $[0,\pi/4]$

$\Rightarrow \int_0^{\pi/4} \tan(x)\,dx$

9a. 0 (zero width interval)

b. -8 (order of integration)

c. $3(-4)=-12$ (constant multiple)

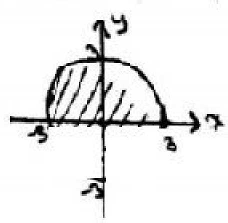
d. $6-(-4)=10$ (additivity)

e. $6-8=-2$ (sum and difference)

f. $4(6)-8=16$ (constant multiple, sum and difference)

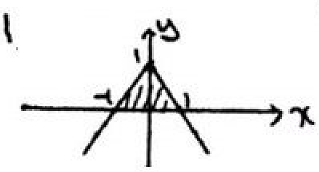
17. we recognize that $f(x)=\sqrt{9-x^2}$ as a function whose graph is the upper semi circle of radius 3 centered at the origin

$$\int_{-3}^{3}\sqrt{9-x^2}\,dx = \frac{1}{2}\pi R^2 = \frac{9}{2}\pi$$



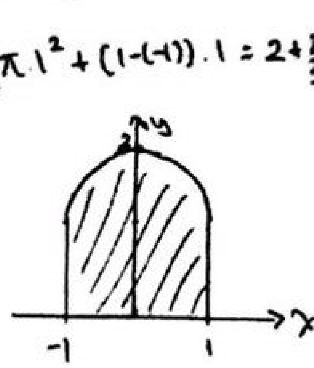
20. we recognize that $f(x)=1-|x|$ as a function whose graph is a triangle

$$\int_{-1}^{1}(1-|x|)\,dx = \frac{1}{2}b.h = \frac{1}{2}(1-(-1))\cdot 1 = 1$$

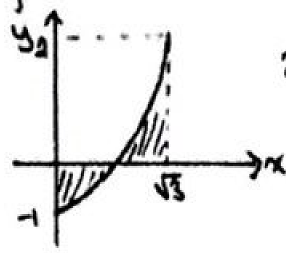


22. we recognize that $f(x)=1+\sqrt{1-x^2}$ as a function whose graph is the union of upper semi circle of radius 1 centered at $(0,1)$ with rectangle

$$\int_{-1}^{1} 1+\sqrt{1-x^2}\,dx = \frac{1}{2}\pi R^2 + b.h = \frac{1}{2}\pi\, 1^2 + (1-(-1)).1 = 2+\frac{\pi}{2}$$



55. $f(x)=x^2-1$ on $[0,\sqrt{3}]$



$$\text{average} = \frac{1}{\sqrt{3}-0}\int_0^{\sqrt{3}} x^2-1$$

$$= \frac{1}{\sqrt{3}}\left[\frac{1}{3}x^3 - x\right]_0^{\sqrt{3}}$$

$$= 0$$

71. $\int_a^b (x-x^2)\,dx = \left[\frac{1}{2}x^2 - \frac{1}{3}x^3\right]_a^b$

the integrand is positive when:

$x - x^2 > 0$

$x(1-x) > 0$

key points $= \{0, 1\}$

− + −

0 1

$0 < x < 1$

take $a = \min(0,1) = 0$

$b = \max(0,1) = 1$

73. $L = f_{min}.(b-a)$

$= \frac{1}{1+1^2}(1-0)$

$= \frac{1}{2}$

$U = f_{max}(b-a)$

$= \frac{1}{1+0^2}(1-0)$

$= 1$

$L \le \int_a^b f(x)\,dx \le U$

$\therefore \frac{1}{2} \le \int_0^1 \frac{1}{1+x^2}\,dx \le 1$

76. $L = f_{min}(b-a)$

$= \sqrt{0+8}\,(1-0)$

$= \sqrt{8}$

$= 2\sqrt{2}$

$U = f_{max}(b-a)$

$= \sqrt{1+8}\,(1-0)$

$= \sqrt{9}$

$= 3$

$L \le \int_a^b f(x)\,dx \le U$

$\therefore 2\sqrt{2} \le \int_0^1 \sqrt{x+8}\,dx \le 3$

77. $L = f_{min}(b-a)$

$= 0 \cdot (b-a)$

$= 0$

$L \le \int_a^b f(x)\,dx$

$\Rightarrow 0 \le \int_a^b f(x)\,dx$ (it is true)

78. $U = f_{max}(b-a)$

$= 0\,(b-a)$

$= 0$

$\int_a^b f(x)\,dx \le U$

$\Rightarrow \int_a^b f(x)\,dx \le 0$ (it is true)

81. since av(f) is a constant value,

$\int_a^b av(f)\,dx = av(f)\int_a^b dx$

$= \left[\frac{1}{b-a}\int_a^b f(x)\,dx\right](b-a)$

$= \int_a^b f(x)\,dx$

(it is true)

83a. $x = \left\{a, a+\frac{(b-a)}{n}, a+\frac{2(b-a)}{n}, \ldots, a+\frac{n(b-a)}{n} = b\right\}$

since f is an increasing function:

$L = \sum_{i=0}^{n-1} \min(f(x_i), f(x_{i+1}))\,.\,\Delta x$

$= \Delta x \sum_{i=0}^{n-1} f(x_i)$

$= \frac{(b-a)}{n}\sum_{i=0}^{n-1} f(x_i)$

$U = \sum_{i=1}^{n} \max(f(x_i)), f(x_{i-1}))\,.\,\Delta x$

$= \Delta x \cdot \sum_{i=1}^{n} f(x_i)$

$= \frac{(b-a)}{n}\sum_{i=1}^{n} f(x_i)$

$U-L = \frac{(b-a)}{n}\sum_{i=1}^{n} f(x_i) - \frac{(b-a)}{n}\sum_{i=0}^{n-1} f(x_i)$

$= \frac{(b-a)}{n}\left[\sum_{i=1}^{n} f(x_i) - \sum_{i=0}^{n-1} f(x_i)\right]$

$= \frac{(b-a)}{n}\left[f(x_n) - f(x_0)\right]$

$= \underbrace{\frac{(b-a)}{n}}_{\Delta x}(f(b)-f(a))$ (it is true)

b. case $\frac{b-a}{n} = (\Delta x)max$:

$\frac{(b-a)}{n}(f(b)-f(a)) = (\Delta x)max\,.\,|f(b)-f(a)|$

case $\frac{b-a}{n} > (\Delta x)max$:

this ~~is~~ case is impossible since the value of (Δx)max can't be lower than the average of the sum Δx

case $\frac{b-a}{n} < (\Delta x)max$:

$\frac{(b-a)}{n}(f(b)-f(a)) = (\Delta x)max\,|f(b)-f(a)|$

$\Rightarrow \frac{(b-a)}{n}(f(b)-f(a)) = U - L \le (\Delta x)max\,|f(b)-f(a)|$

for $(\Delta x)max \to 0$, $U-L \to 0$