

$$\begin{aligned}
 (a) \overline{A\bar{B}(C+D)} &= \bar{A} + \overline{\bar{B}(C+D)} \\
 &= \bar{A} + B + \overline{(C+D)} \\
 &= \bar{A} + B + \bar{C} \cdot \bar{D} \\
 (b) \overline{AB(CD+EF)} &= \bar{A}\bar{B} + \overline{(CD+EF)} \\
 &= \bar{A}\bar{B} + \bar{C}\bar{D} \cdot \bar{E}\bar{F} \\
 &= \bar{A}\bar{B} + (\bar{C}+\bar{D})(\bar{E}+\bar{F}) \\
 &= \bar{A}\bar{B} + \bar{C}\bar{E} + \bar{C}\bar{F} + \bar{D}\bar{E} + \bar{D}\bar{F} \\
 (c) \overline{(A+\bar{B}+\bar{C}+\bar{D}) + ABC\bar{D}} &= \bar{A}\bar{B}\bar{C}\bar{D} + \bar{A} + \bar{B} + \bar{C} + \bar{D} \\
 &= \bar{A} + \bar{B} + \bar{C} + D(\bar{A}\bar{B}\bar{C} + 1) \\
 &= \bar{A} + \bar{B} + \bar{C} + D
 \end{aligned}$$

$$\begin{aligned}
 (d) \overline{(\bar{A}+B+C+D)(A\bar{B}\bar{C}\bar{D})} &= \bar{A} + B + C + D + A\bar{B}\bar{C}\bar{D} \\
 &= \bar{A} + B + C + D(1 + A\bar{B}\bar{C}) \\
 &= \bar{A} + B + C + D \\
 (e) \overline{\bar{A}\bar{B}(CD+\bar{E}\bar{F})(\bar{A}\bar{B}+\bar{C}\bar{D})} &= AB + \overline{(CD+\bar{E}\bar{F})} + \overline{(\bar{A}\bar{B}+\bar{C}\bar{D})} \\
 &= AB + \bar{C}\bar{D} \cdot \bar{E}\bar{F} + ABCD \\
 &= AB(1 + \bar{C}\bar{D}) + (\bar{C}+\bar{D})(\bar{E}+\bar{F}) \\
 &= AB + \bar{C}\bar{E} + \bar{C}\bar{F} + \bar{D}\bar{E} + \bar{D}\bar{F}
 \end{aligned}$$

2.(a)

A	B	C	A+B+C
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	1
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	1

(b)

A	B	C	ABC
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	0
1	0	0	0
1	0	1	0
1	1	0	0
1	1	1	1

(c)

A	B	C	AB	BC	CA	AB+BC+CA
0	0	0	0	0	0	0
0	0	1	0	0	0	0
0	1	0	0	0	0	0
0	1	1	0	1	0	1
1	0	0	0	0	0	0
1	0	1	0	0	1	1
1	1	0	1	0	0	1
1	1	1	1	1	1	1

(d)

A	B	C	A+B	B+C	C+A	(A+B)(B+C)(C+A)
0	0	0	0	0	0	0
0	0	1	0	1	1	0
0	1	0	1	1	0	0
0	1	1	1	1	1	1
1	0	0	1	0	1	0
1	0	1	1	1	1	1
1	1	0	1	1	1	1
1	1	1	1	1	1	1

(e)

A	B	C	A \bar{B}	B \bar{C}	C \bar{A}	A \bar{B} +B \bar{C} +C \bar{A}
0	0	0	0	0	0	0
0	0	1	0	0	1	1
0	1	0	0	1	0	1
0	1	1	0	0	1	1
1	0	0	1	0	0	1
1	0	1	1	0	0	1
1	1	0	0	1	0	1
1	1	1	0	0	0	0

$$\begin{aligned}
 3.(a) x &= A \cdot (\bar{B} + \bar{C}\bar{D}) + \bar{A}\bar{B}C = A\bar{B} + AC\bar{D} + \bar{A}\bar{B}C \\
 (b) x &= A\bar{B} + AC\bar{D} + \bar{A}\bar{B}C = A\bar{B}(1+C) + AC\bar{D} = A\bar{B} + AC\bar{D} \\
 (c) x &= \bar{B}\bar{C}\bar{D} + A\bar{B} \\
 (d) x &= A\bar{B} + AC\bar{D}
 \end{aligned}$$

Figure (b) & (d) are equivalent

$$4.(a) f(A, B, C, D) = A\bar{B}\bar{C}\bar{D} + AB\bar{C}\bar{D} + \bar{A}\bar{B}C\bar{D} + \bar{A}\bar{B}\bar{C}D$$

A	B	C	D	f(A, B, C, D)
0	0	0	0	1
0	0	0	1	0
0	0	1	0	0
0	0	1	1	1
0	1	0	0	0
0	1	0	1	0
0	1	1	0	0
0	1	1	1	0

A	B	C	D	f(A, B, C, D)
1	0	0	0	0
1	0	0	1	0
1	0	1	0	1
1	0	1	1	0
1	1	0	0	1
1	1	0	1	0
1	1	1	0	0
1	1	1	1	0

$$(b) f(w, x, y, z) = wxyz + \bar{w}x\bar{y}z + w\bar{x}y\bar{z} + \bar{w}\bar{x}yz + wxy\bar{z}$$

w	x	y	z	f(w, x, y, z)
0	0	0	0	0
0	0	0	1	0
0	0	1	0	0
0	0	1	1	1
0	1	0	0	0
0	1	0	1	1
0	1	1	0	0
0	1	1	1	0

w	x	y	z	f(w, x, y, z)
1	0	0	0	0
1	0	0	1	0
1	0	1	0	1
1	0	1	1	0
1	1	0	0	1
1	1	0	1	0
1	1	1	0	0
1	1	1	1	1

$$(b). f(A, B, C, D) = (A + \bar{B} + C + \bar{D})(\bar{A} + B + \bar{C} + D)(A + B + \bar{C} + D)(\bar{A} + \bar{B} + C + D)$$

$$5(a) f(A, B, C) = (\bar{A} + \bar{B} + \bar{C})(A + B + C)(A + B + \bar{C})$$

A	B	C	f(A, B, C)
0	0	0	0
0	0	1	0
0	1	0	1
0	1	1	1
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	0

A	B	C	D	f(A, B, C, D)
0	0	0	0	1
0	0	0	1	1
0	0	1	0	1
0	0	1	1	0
0	1	0	0	1
0	1	0	1	0
0	1	1	0	1
0	1	1	1	1

A	B	C	D	f(A, B, C, D)
1	0	0	0	1
1	0	0	1	1
1	0	1	0	0
1	0	1	1	1
1	1	0	0	0
1	1	0	1	1
1	1	1	0	1
1	1	1	1	1

$$6(a). \bar{A}\bar{B}\bar{C} + \bar{A}\bar{B}C + \bar{A}B\bar{C}$$

$$\downarrow \quad \downarrow \quad \downarrow$$

\therefore minterms of given function are $m(0, 1, 5)$

K-Map,

	$\bar{B}\bar{C}$	$\bar{B}C$	BC	$B\bar{C}$
\bar{A}	1	1	0	0
A	0	1	0	0

$$\therefore \text{minimum SOP} = \bar{A}\bar{B} + \bar{B}\bar{C}$$

$$(d) \bar{A}\bar{B}\bar{C} + \bar{A}\bar{B}C + \bar{A}B\bar{C} + \bar{A}BC$$

$$\downarrow \quad \downarrow \quad \downarrow \quad \downarrow$$

$$0 \quad 4 \quad 2 \quad 6$$

K-Map,

	$\bar{B}\bar{C}$	$\bar{B}C$	BC	$B\bar{C}$
\bar{A}	1	0	0	1
A	1	0	0	1

$$\therefore \text{minimum SOP} = \bar{C}$$

$$(b) AC(\bar{B} + C) = AC\bar{B} + AC$$

$$= AC\bar{B} + ACB + AC\bar{B}$$

$$= AC\bar{B} + ACB$$

$$\downarrow \quad \downarrow$$

$$5 \quad 7$$

\therefore minterms of given function are $m(5, 7)$

K-Map,

	$\bar{B}\bar{C}$	$\bar{B}C$	BC	$B\bar{C}$
\bar{A}	0	0	0	0
A	0	1	1	0

$$\therefore \text{minimum SOP} = AC$$

$$(c) \bar{A}(BC + \bar{B}\bar{C}) + A(B\bar{C} + \bar{B}C)$$

$$= \bar{A}BC + \bar{A}\bar{B}\bar{C} + AB\bar{C} + A\bar{B}C$$

$$\downarrow \quad \downarrow \quad \downarrow \quad \downarrow$$

$$3 \quad 2 \quad 7 \quad 6$$

\therefore minterms of given function are $m(2, 3, 6, 7)$

K-Map,

	$\bar{B}\bar{C}$	$\bar{B}C$	BC	$B\bar{C}$
\bar{A}	0	0	1	1
A	0	0	1	1

$$\therefore \text{minimum SOP} = B$$

7. Given $X = m(1, 2, 6, 7, 8, 10, 12, 13, 15)$

	$\bar{C}\bar{D}$	$\bar{C}D$	CD	$C\bar{D}$
$\bar{A}\bar{B}$	0	1	0	1
$\bar{A}B$	0	0	1	1
AB	1	1	1	0
$A\bar{B}$	1	0	0	1

$$X = A\bar{C}\bar{D} + ABD + \bar{A}BC + \bar{B}C\bar{D} + \bar{A}\bar{B}C\bar{D}$$

8(a) $X = (A+B+C)(\bar{A}+\bar{B}+\bar{C})(A+B+C) = \pi(0, 2, 7)$

	$\bar{B}\bar{C}$	$\bar{B}C$	BC	$B\bar{C}$
\bar{A}	0	1	1	0
A	1	1	0	1

$$X = \overline{ABC} + (\bar{A}\bar{C})$$

$$= (\bar{A} + \bar{B} + \bar{C}) \cdot (A + C)$$

(b) $X = (X + \bar{Y})(\bar{X} + Z)(X + \bar{Y} + \bar{Z})(\bar{X} + \bar{Y} + Z)$

	$\bar{Y}\bar{Z}$	$\bar{Y}Z$	YZ	$Y\bar{Z}$
\bar{X}	1	1	0	0
X	0	1	1	0

$$X = (\bar{X}\bar{Y}) + (X\bar{Z})$$

$$= (X + \bar{Y}) \cdot (\bar{X} + Z)$$

(c) $X = A(B + \bar{C})(\bar{A} + C)(A + \bar{B} + C)(\bar{A} + B + C)$

	$\bar{B}\bar{C}$	$\bar{B}C$	BC	$B\bar{C}$
\bar{A}	0	0	0	0
A	0	0	1	0

$$X = (\bar{A} + \bar{B} + \bar{C}) = ABC$$

10 (a) $X = \bar{A}\bar{B}\bar{C}\bar{D} + \bar{A}\bar{B}\bar{C}D + \bar{A}\bar{B}C\bar{D} + \bar{A}B\bar{C}\bar{D} + \bar{A}B\bar{C}D + \bar{A}BC\bar{D} + \bar{A}BCD$

\downarrow \downarrow \downarrow \downarrow \downarrow \downarrow \downarrow
 $0000_{(2)} \quad 0001_{(2)} \quad 0101_{(2)} \quad 1100_{(2)} \quad 1010_{(2)} \quad 0110_{(2)} \quad 1001_{(2)}$
 \downarrow \downarrow \downarrow \downarrow \downarrow \downarrow \downarrow
 $0_{(10)} \quad 1_{(10)} \quad 5_{(10)} \quad 12_{(10)} \quad 10_{(10)} \quad 6_{(10)} \quad 9_{(10)}$

$$= m(0, 1, 5, 6, 9, 10, 12)$$

(b) number of 1s in minterm	Minterm	ABCD	First level
0	m_0	0000	$(m_0, m_1) \quad 000x$
1	m_1	0001	$(m_1, m_5) \quad 0x01$
2	m_5	0101	$(m_1, m_5) \quad x001$
	m_6	0110	
	m_9	1001	
	m_{10}	1010	
	m_{12}	1100	

(c) number of 1s in First level	Second level
0	(m_0, m_1)
1	(m_1, m_5)
	(m_1, m_9)

note that second level minterms are those minterms which are listed in the first level by excluding the essential prime implicants

9.

	$\bar{B}\bar{C}$	$\bar{B}C$	BC	$B\bar{C}$
\bar{A}	1	1	1	0
A	1	1	1	0

$$X = (\bar{B}\bar{C}) = \bar{B} + C$$

(d)

Prime implicants	m_0	m_1	m_5	m_6	m_9	m_{10}	m_{12}
$\bar{A}\bar{B}\bar{C} (m_0, m_1)$	✓	✓					
$\bar{A}\bar{C}\bar{D} (m_1, m_5)$		✓	✓				
$\bar{A}BC\bar{D} (m_6)$				✓			
$A\bar{B}\bar{C}\bar{D} (m_9)$					✓		
$A\bar{B}C\bar{D} (m_{10})$						✓	
$ABC\bar{D} (m_{12})$							✓
$\bar{B}\bar{C}\bar{D} (m_1, m_9)$		✓			✓		

(e) the table shows that all prime implicants are essential and they will be included in the final expression as they have a single check mark (except for m_9)

therefore,

$$X = \bar{A}\bar{B}\bar{C} + \bar{A}\bar{C}\bar{D} + \bar{A}BC\bar{D} + A\bar{B}\bar{C}\bar{D} + A\bar{B}C\bar{D} + \bar{B}\bar{C}\bar{D}$$