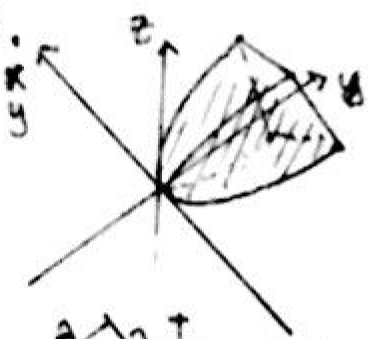
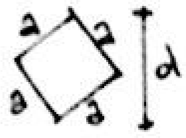
yohandi - math homework for week 8

Exercises 6.1

1.





$d(x) = \sqrt{x} - (-\sqrt{x})$

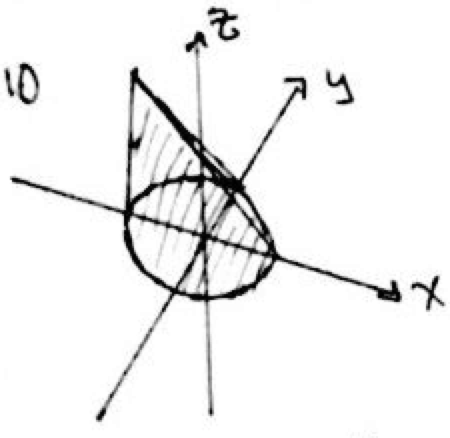
$= 2\sqrt{x}$

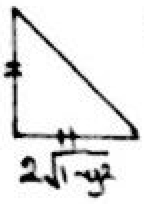
$a(x) = \frac{d(x)}{\sqrt{2}}$

$= \sqrt{2x}$

Volume $= \int_0^4 A(x)\,dx = \int_0^4 2x\,dx = 16$

10





$A(y) = \frac{1}{2} b^2$

$= \frac{1}{2} 4(1-y^2)$

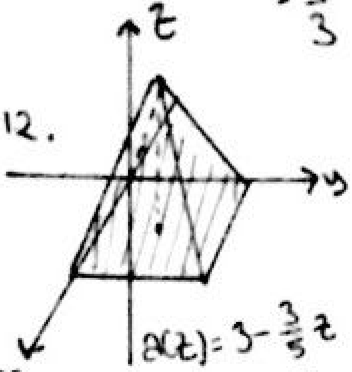
$= 2(1-y^2)$

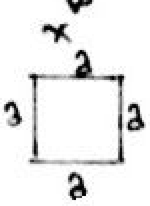
Volume $= \int_{-1}^{1} A(y)\,dy = \int_{-1}^{1} 2(1-y^2)\,dy$

$= \left[2y - \frac{2}{3}y^3\right]_{-1}^{1}$

$= \frac{8}{3}$

12.





$a(z) = 3 - \frac{3}{5}z$

$A(z) = a(z)^2$

$= (3 - \frac{3}{5}z)^2$

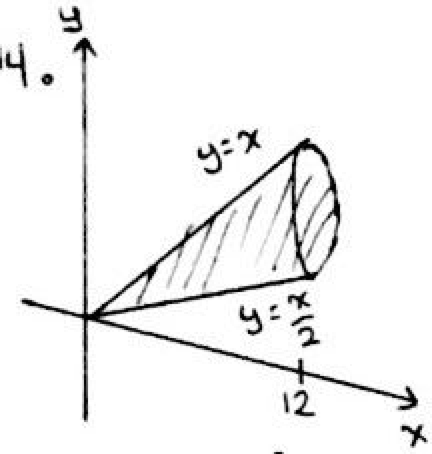
Volume $= \int_0^5 A(z)\,dz$

$= \int_0^5 (3 - \frac{3}{5}z)^2\,dz$

$= \left[-\frac{5}{9}(3 - \frac{3}{5}z)^3\right]_0^5$

$= 15$

14.



$A(x) = \pi r^2$

$= \pi\left(\frac{x}{4}\right)^2$

Volume $= \int_0^{12} A(x)\,dx$

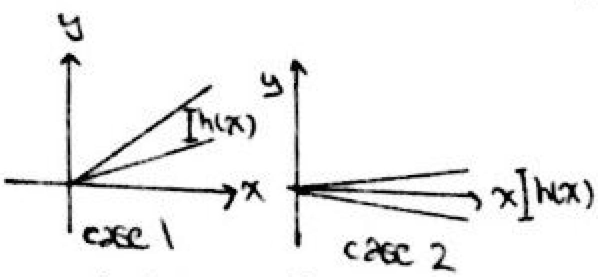
$= \frac{1}{16}\pi\left[\frac{1}{3}x^3\right]_0^{12}$

$= 36\pi$

Volume cone $= \frac{1}{3}\pi r^2 h$

$= \frac{1}{3}\pi \cdot 3^2\ 12$

$= 36\pi$

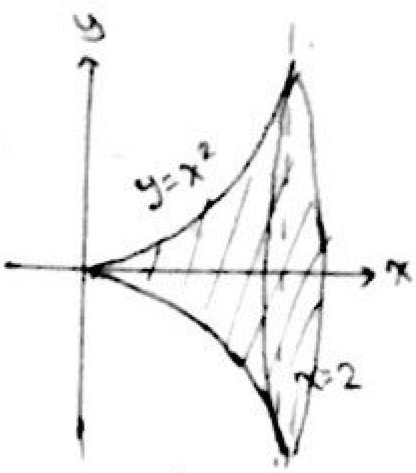


$h_1(x) = x - \frac{x}{2} = \frac{x}{2}$

$h_2(x) = \frac{x}{4} - \left(-\frac{x}{4}\right) = \frac{x}{2}$

Since $h_1(x) = h_2(x)$, both V_1 and V_2 have the same volume

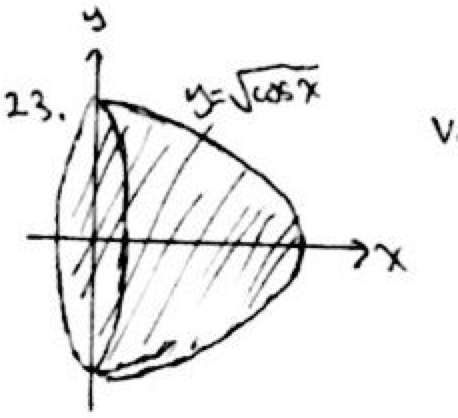
19.



Volume $= \int_0^2 \pi (x^2)^2\,dx$

$= \frac{1}{5}\left[x^5\right]_0^2 \cdot \pi = \frac{32\pi}{5}$

23.

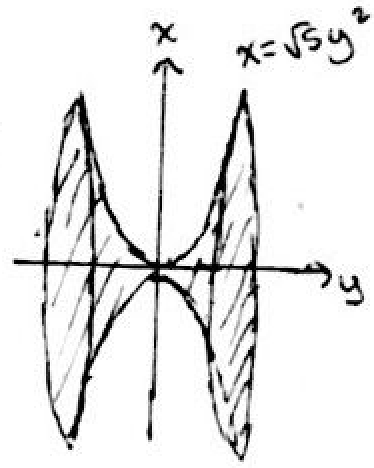


Volume $= \int_0^{\pi/2} \pi(\sqrt{\cos x})^2\,dx$

$= \pi\left[\sin x\right]_0^{\pi/2}$

$= \pi$

27.

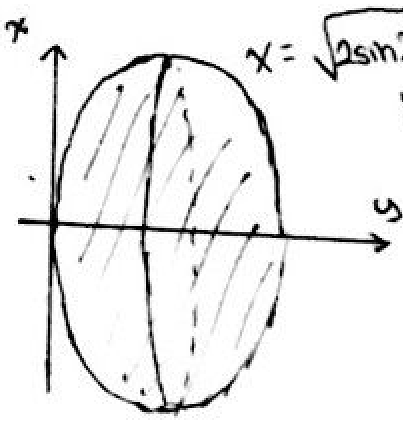


Volume $= \int_{-1}^{1} (\sqrt{5}y^2)^2 \pi\,dy$

$= \pi\left[y^5\right]_{-1}^{1}$

$= 2\pi$

29.



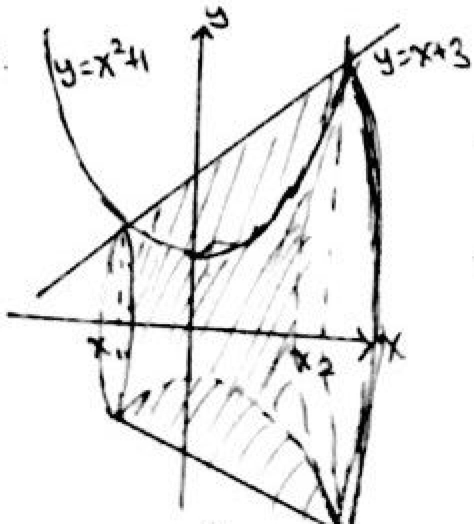
Volume $= \int_0^{\pi/2} \pi(\sqrt{2\sin 2y})^2\,dy$

$= \pi \int_0^{\pi/2} 2\sin 2y\,dy$

$= \pi\left[-\cos 2y\right]_0^{\pi/2}$

$= 2\pi$

37.



$y_1 = y_2$

$x^2 + 1 = x + 3$

$x^2 - x - 2 = 0$

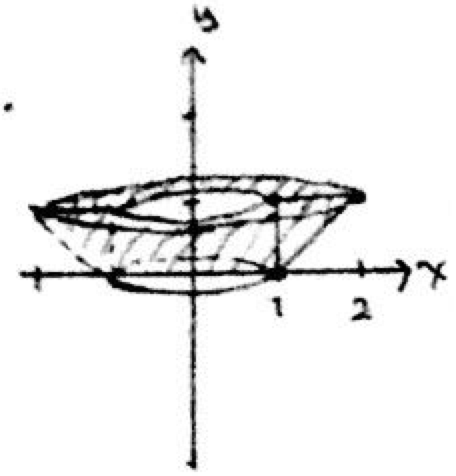
$\Rightarrow x_1 = -1,\ x_2 = 2$

Volume $= \int_{-1}^{2} \pi\left((x+3) - (x^2+1)\right)^2\,dx$

$= \pi\left[-\frac{x^5}{5} - \frac{x^3}{3} + 3x^2 + 8x\right]_{-1}^{2}$

$= \frac{117\pi}{5}$

41.



outer: $y = 1-(x-1)$ $\Rightarrow x = 2-y$

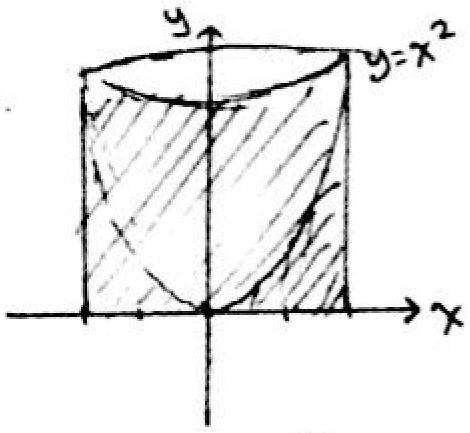
inner: $x = 1$

$$\text{Volume} = \int_0^1 \pi\left((2-y)^2-(1)^2\right) dy$$

$$= \int_0^1 \pi(y^2-4y+3)\, dy$$

$$= \left[\tfrac{1}{3}y^3-2y^2+3y\right]_0^1 \cdot \pi = \frac{4\pi}{3}$$

43.



outer: $x=2$

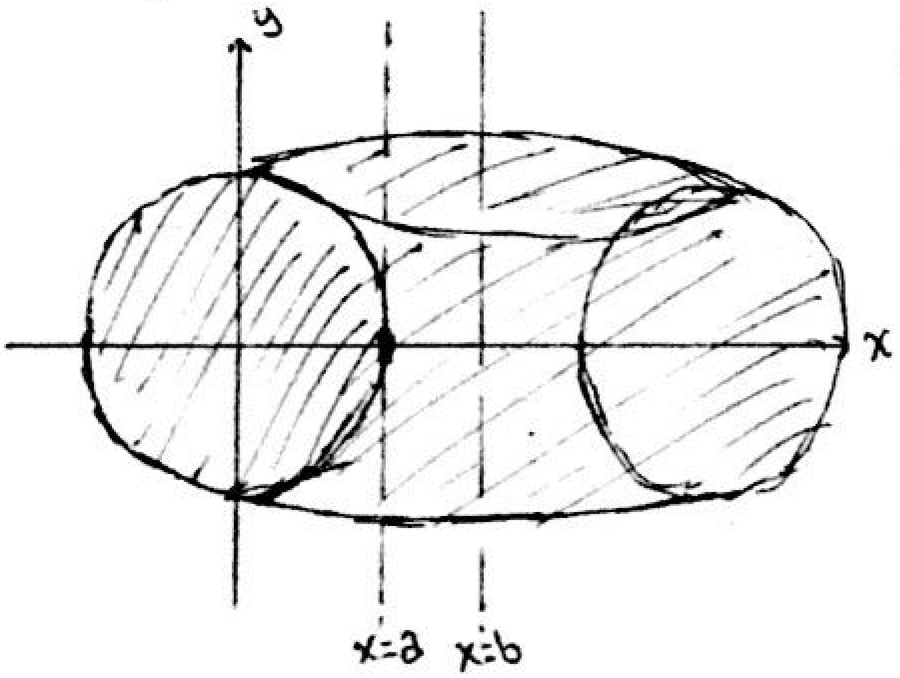
inner: $y=x^2$

$\Rightarrow x = \pm\sqrt{y}$

$= \sqrt{y}$ (considering first quadrant $\Rightarrow x \geq 0$ and $y \geq 0$)

$$\text{Volume} = \int_0^4 \pi\left((2)^2-(\sqrt{y})^2\right) dy = \pi\left[4y-\tfrac{1}{2}y^2\right]_0^4 = 8\pi$$

51.



consider $x=b$ as "y-axis":

$(x-b)^2+y^2=a^2$

$\Rightarrow x = b \pm \sqrt{a^2-y^2}$

outer: $x = b+\sqrt{a^2-y^2}$

inner: $x = b-\sqrt{a^2-y^2}$

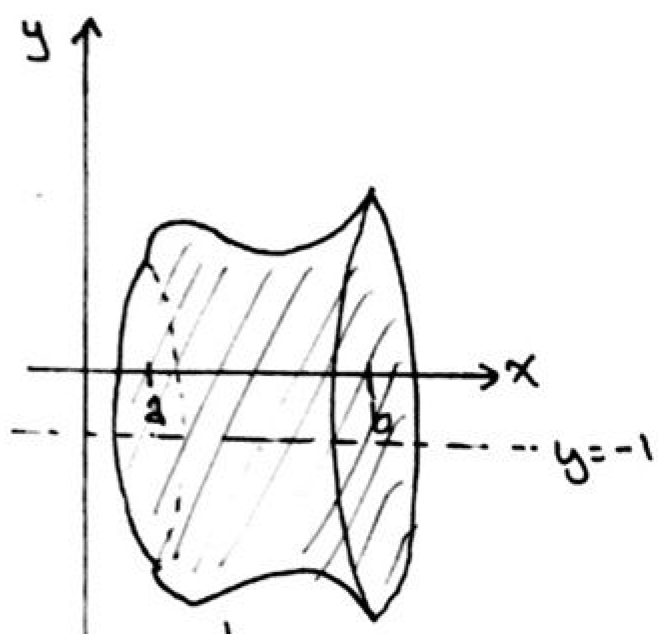
$$\text{Volume} = \int_{-a}^{a} \pi\left((b+\sqrt{a^2-y^2})^2-(b-\sqrt{a^2-y^2})^2\right) dy$$

$$= \int_{-a}^{a} \pi\left(2b\sqrt{a^2-y^2}\right) dy$$

$$= 4\pi b \int_{-a}^{a} \sqrt{a^2-y^2}\, dy$$

$$= 4\pi b \cdot \frac{\pi a^2}{2} = 2\pi^2 a^2 b$$

59.



given: $\int_a^b \pi f(x)^2 dx = 4\pi \ldots (1)$

$$\text{Volume} = \int_a^b \pi(f(x)+1)^2\, dx$$

$$8\pi = \pi \int_a^b f(x)^2+2f(x)+1\, dx \quad \ldots (2)$$

(1) $\mapsto$ (2)

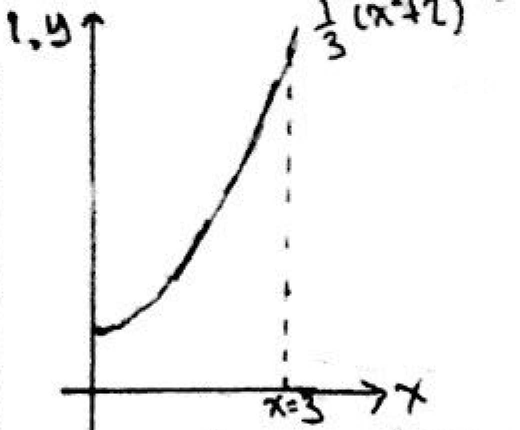
$$8\pi = 4\pi + \pi\int_a^b 2f(x)+1\, dx$$

$$4 = 2\int_a^b f(x)\, dx + [x]_a^b$$

$$\Rightarrow \int_a^b f(x)\, dx = 2 + \frac{(a-b)}{2}$$

Exercises 6.3

1.



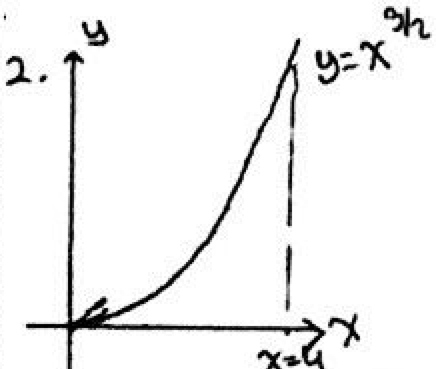
$$\text{length} = \int_0^3 \sqrt{1+\left(\frac{d\left(\frac{1}{3}(x^2+2)^{3/2}\right)}{dx}\right)^2}\,dx$$

$$= \int_0^3 \sqrt{1+(x\sqrt{x^2+2})^2}\,dx$$

$$= \int_0^3 (x^2+1)\,dx = \left[\frac{1}{3}x^3+x\right]_0^3$$

$$= 12$$

2.



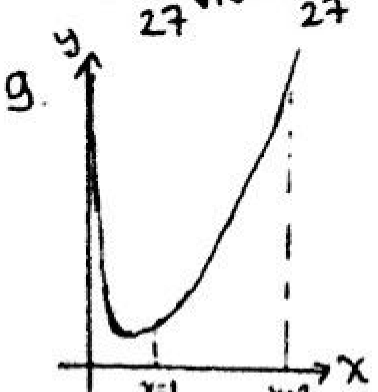
$$\text{length} = \int_0^4 \sqrt{1+\left(\frac{d(x^{3/2})}{dx}\right)^2}\,dx$$

$$= \int_0^4 \sqrt{1+\frac{9}{4}x}\,dx$$

$$= \frac{4}{9}\cdot\frac{2}{3}\left[\left(1+\frac{9}{4}x\right)^{3/2}\right]_0^4$$

$$= \frac{80}{27}\sqrt{10} - \frac{8}{27}$$

9.



$$\text{length} = \int_1^3 \sqrt{1+\left(\frac{d\left(\frac{x^3}{3}+\frac{1}{4x}\right)}{dx}\right)^2}\,dx$$

$$= \int_1^3 \sqrt{1+\left(x^2-\frac{1}{4x^2}\right)^2}\,dx$$

$$= \int_1^3 x^2+\frac{1}{4x^2}\,dx$$

$$= \left[\frac{1}{3}x^3-\frac{1}{4x}\right]_1^3 = \frac{53}{6}$$

23. $y = \int_0^x \sqrt{\cos 2t}\,dt$

$$\frac{dy}{dx} = \sqrt{\cos 2x}$$

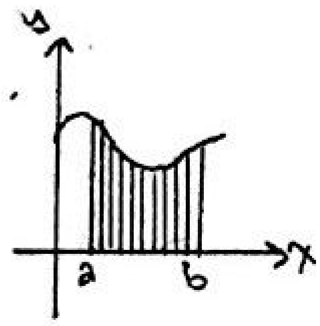
$$\text{length} = \int_0^{\pi/4} \sqrt{1+(\sqrt{\cos 2x})^2}\,dx$$

$$= \int_0^{\pi/4} \sqrt{2}\cos x\,dx$$

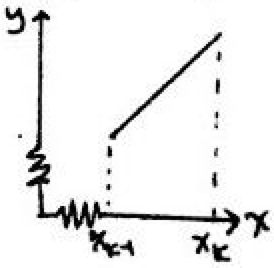
$$= \sqrt{2}\left[\sin x\right]_0^{\pi/4}$$

$$= 1$$

30.



a. for a very small partition (i.e. $\Delta x_k \to 0$)



$$\text{length} = \sqrt{\Delta y_k^2 + \Delta x_k^2}$$

$$= \sqrt{\Delta x_k^2 + (y_k - y_1)^2}$$

$$= \sqrt{\Delta x_k^2 + (f'(x_k-1)\Delta x_k)^2}$$

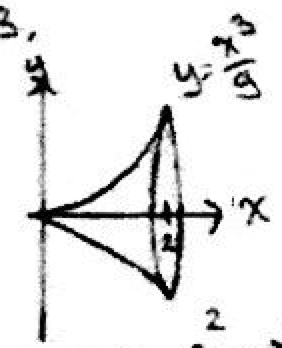
b. $$\lim_{n\to\infty} \sqrt{\Delta x_k^2 + (f'(x_k-1)\,\Delta x_k^2}$$

$$= \lim_{n\to\infty} \sqrt{\Delta x_k^2 (1+f'(x_{k-1})^2)}$$

$$= \int_a^b \sqrt{1+f'(x)^2}\,dx$$

Exercises 6.4

13.



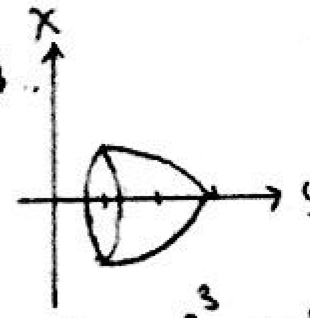
$$\text{surface} = \int_0^2 2\pi \frac{x^3}{9}\sqrt{1+\left(\frac{x^2}{3}\right)^2}\,dx$$

$$= 2\pi\int_0^2 \frac{1}{9}\sqrt{1+\frac{1}{9}x^4}\ \frac{d(1+\frac{1}{9}x^4)}{\frac{4}{9}}$$

$$= \frac{\pi}{2}\left[\frac{2}{3}\left(1+\frac{1}{9}x^4\right)^{3/2}\right]_0^2$$

$$= \frac{98\pi}{81}$$

18.



$$\text{surface} = \int_1^3 2\pi x\sqrt{1+\left(\frac{dx}{dy}\right)^2}\,dy$$

$$= 2\pi\int_1^3\left(\frac{y^{3/2}}{3}-y^{1/2}\right)\sqrt{1+\left(\frac{y^{1/2}}{2}-\frac{y^{-1/2}}{2}\right)^2}\,dy$$

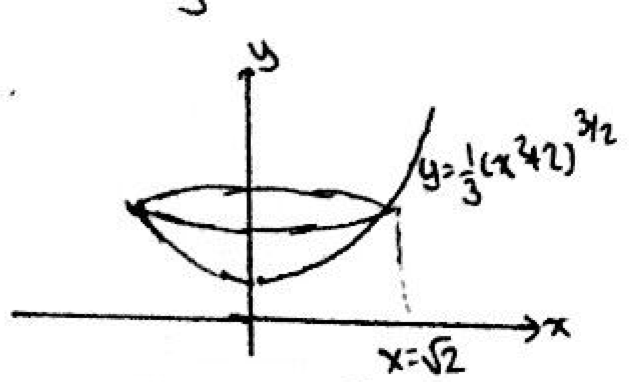
$$= 2\pi\int_1^3\left(\frac{y^{3/2}}{3}-y^{1/2}\right)\left(\frac{y^{1/2}}{2}+\frac{y^{-1/2}}{2}\right)dy$$

$$= \pi\int_1^3 \frac{1}{3}y^2-\frac{2}{3}y-1\ dy$$

$$= -\pi\left[\frac{1}{9}y^3-\frac{1}{3}y^2-y\right]_1^3$$

$$= \frac{16\pi}{9}$$

22.



$\Rightarrow \frac{dy}{dx} = \frac{1}{2}\,2x(x^2+2)^{1/2}$

$\Rightarrow dy = x\sqrt{x^2+2}\,dx$

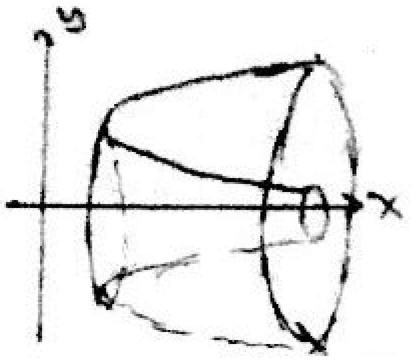
$\Rightarrow ds = \sqrt{dx^2+dy^2}$

$\Rightarrow ds = \sqrt{1+x^4+2x^2}\,dx$

$= (x^2+1)\,dx$

$$\text{Surface} = \int_0^{\sqrt{2}} 2\pi x(x^2+1)\,dx = 2\pi\left[\frac{1}{4}x^4+\frac{1}{2}x^2\right]_0^{\sqrt{2}} = 4\pi$$

23



$\Rightarrow \frac{dx}{dy} = y^3 - \frac{1}{4y^3}$

$\Rightarrow dx = \left(y^3-\frac{1}{4y^3}\right)dy$

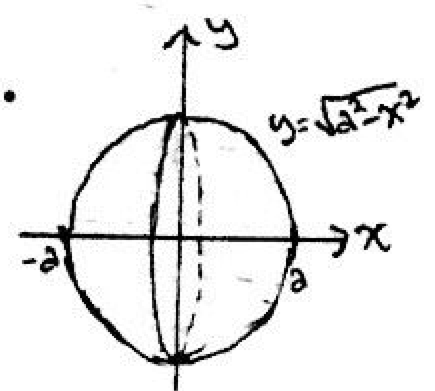
$\Rightarrow ds = \sqrt{dx^2+dy^2}$

$\Rightarrow ds = \sqrt{y^6+\frac{1}{2}+\frac{1}{16y^6}}\ dy$

$= \left(y^3+\frac{1}{4y^3}\right)dy$

$$\text{surface} = \int_1^2 2\pi y\left(y^3+\frac{1}{4y^3}\right)dy = 2\pi\left[\frac{1}{5}y^5-\frac{1}{4y}\right]_1^2 = \frac{253\pi}{20}$$

25.



$$\text{surface} = \int_{-a}^{a} 2\pi\sqrt{a^2-x^2}\sqrt{1+\left(-\frac{x}{\sqrt{a^2-x^2}}\right)^2}\,dx$$

$$= \int_{-a}^{a} 2\pi\sqrt{a^2-x^2+x^2}\,dx$$

$$= 2\pi a\int_{-a}^{a} dx$$

$$= 2\pi a[x]_{-a}^{a}$$

$$= 4\pi a^2$$

28. for all $h>0$:

suppose we have a point "a" such as $a+h \le r$ and $a \ge -r$,

$$\text{surface} = \int_a^{a+h} 2\pi y\sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx \quad \text{where } y=\sqrt{a^2-x^2}$$

$$= \int_a^{a+h} 2\pi\sqrt{r^2-x^2}\sqrt{1+\left(\frac{-x}{\sqrt{r^2-x^2}}\right)^2}\,dx$$

$$= 2\pi r\int_a^{a+h} dx$$

$$= 2\pi r[x]_a^{a+h}$$

$$= 2\pi r h$$

this shows us that the value of surface is independent with the value of a ($S(a,h)=S(h)$).

$\therefore$ the area swept out by AB doesn't depend on the location of interval

Exercises 6.5

22. a. $k \Delta x = F$

$$k = \frac{F}{\Delta x}$$

$$= \frac{800\ N}{(14cm - 10cm)}$$

$$= 200\ N/cm$$

b. $F(x) = k|10-x|$

$$W = \int_{10}^{12} F(x)\, dx$$

$$= \int_{0}^{2} kx\, dx$$

$$= \left[\frac{1}{2} kx^2\right]_0^2$$

$$= 400\ Ncm = 4\ \text{Joule}$$

c. $F(x) = k|10-x|$

"stretch" $\Rightarrow x > 10$

$$F(x) = k(x-10)$$

$$1600 = 200x - 2000$$

$$x = 18\ cm$$

$$\Rightarrow \Delta x = 18cm - 10cm$$

$$= 8cm$$