yohandi - quiz 7

1a. True

b. True

c. False

2a. $\int x^3 \sqrt{x^2+1}\ \frac{d(x^2+1)}{2x} = \frac{1}{2}\int (x^2+1-1)\sqrt{x^2+1}\ d(x^2+1) = \frac{1}{2}\int (x^2+1)^{3/2} - (x^2+1)^{1/2}\ d(x^2+1)$

$= \frac{1}{2}\left[\frac{2}{5}(x^2+1)^{5/2} - \frac{2}{3}(x^2+1)^{3/2}\right] + C$

$= \boxed{\frac{(x^2+1)^{5/2}}{5} - \frac{(x^2+1)^{3/2}}{3} + C}$

b. $\int \frac{1}{x}\cos^2(\ln x)\ \frac{d(\ln x)}{\frac{1}{x}} = \int \cos^2(\ln x)\ d(\ln x) = \int \frac{\cos(2\ln x)+1}{2}\ d(\ln x) = \frac{1}{2}\left[\frac{1}{2}\sin(2\ln x) + \ln x\right] + C$

$= \boxed{\frac{1}{4}\sin(2\ln x) + \frac{1}{2}\ln x + C}$

3a. let $u = \sin x$ when $x = 0 \Rightarrow u = 0$

$du = \cos x\ dx$ $x = \frac{\pi}{2} \Rightarrow u = 1$

$\int_0^1 \frac{du}{\cos x}\cdot\frac{1}{\sec x + \tan x} = \int_0^1 \frac{du}{1+u} = \left[\ln(u+1)\right]_0^1 = \boxed{\ln(2)}$

b. $\int_{-2}^{2} \underbrace{\frac{x^4}{|x|^5+1}}_{\text{even}} + \underbrace{\frac{-3x^3+6x}{|x|^5+1}}_{\text{odd} \Rightarrow 0}\ dx = 2\int_0^2 \frac{x^4}{x^5+1}\ dx = 2\int_0^2 \frac{x^4}{x^5+1}\ \frac{d(x^5+1)}{5x^4} = \frac{2}{5}\left[\ln(x^5+1)\right]_0^2$

$= \boxed{\frac{2}{5}\ln(33)}$

4. $y = 4\sqrt{x}$ ...(1) $y = x^{5/2}$ ...(2)

(1) & (2)

$4\sqrt{x} = x^{5/2}$ $(x \geq 0)$

$x^{5/2} - 4\sqrt{x} = 0$

$\sqrt{x}(x+2)(x-2) = 0$

$x = \{0, 2\}$

when $0 \leq x \leq 2$:

$4\sqrt{x} \geq x^{5/2}$

$\therefore$ Area $= \int_0^2 4\sqrt{x} - x^{5/2}\ dx = \left[\frac{8}{3}x^{3/2} - \frac{2}{7}x^{7/2}\right]_0^2 = \frac{8}{3}\cdot 2\sqrt{2} - \frac{2}{7}\cdot 8\sqrt{2} = 16\sqrt{2}\cdot\frac{4}{21}$

$= \boxed{\frac{64\sqrt{2}}{21}}$