STA2001 Assignment 7

1. 4.1-3. Let the joint pmf of X and Y be defined by

$$f(x, y) = \frac{x + y}{32}$$

   x = 1, 2, y = 1, 2, 3, 4.

   (a) Find $f_X(x)$, the marginal pmf of X.

   (b) Find $f_Y(y)$, the marginal pmf of Y.

   (c) Find $P(X > Y)$.

   (d) Find $P(Y = 2X)$.

   (e) Find $P(X + Y = 3)$.

   (f) Find $P(X \leq 3 - Y)$.

   (g) Are X and Y independent or dependent?Why or why not?

   (h) Find the means and the variances of X and Y.

2. 4.1-4. Select an (even) integer randomly from the set $\{12, 14, 16, 18, 20, 22\}$. Then select an integer randomly from the set $\{12, 13, 14, 15, 16, 17\}$. Let X equal the integer that is selected from the first set and let Y equal the sum of the two integers.

   (a) Show the joint pmf of X and Y on the space of X and Y.

   (b) Compute the marginal pmfs.

   (c) Are X and Y independent?Why or why not?

3. 4.1-5. Roll a pair of four-sided dice, one red and one black. Let X equal the outcome on the red die and let Y equal the sum of the two dice.

   (a) On graph paper, describe the space of X and Y.

   (b) Define the joint pmf on the space (similar to Figure 4.1-1).

   (c) Give the marginal pmf of X in the margin.

   (d) Give the marginal pmf of Y in the margin.

   (e) Are X and Y dependent or independent?Why or why not?

4. 4.1-8. In a smoking survey among men between the ages of 25 and 30. 63% prefer to date nonsmokers, 13% prefer to date smokers, and 24% dont care. Suppose nine such men are selected randomly. Let X equal the number who prefer to date nonsmokers and Y equal the number who prefer to date smokers.

   (a) Determine the joint pmf of X and Y. Be sure to include the support of the pmf.

   (b) Find the marginal pmf of X. Again include the support.

5. 4.1-9. A manufactured item is classified as good, a second, or defective with probabilities 6/10, 3/10, and 1/10, respectively. Fifteen such items are selected at random from the production line. Let X denote the number of good items, Y the number of seconds, and 15-X-Y the number of defective items.

(a) Give the joint pmf of X and Y, f (x, y).

(b) Sketch the set of integers (x, y) for which f(x, y)>0. From the shape of this region, can X and Y be independent?Why or why not?

(c) Find $P(X = 10, Y = 4)$.

(d) Give the marginal pmf of X.

(e) Find $P(X \leq 11)$.

6. 4.2-2. Let X and Y have the joint pmf defined by f (0, 0) = f (1, 2) = 0.2, f (0, 1) = f (1, 1) = 0.3.

(a) Depict the points and corresponding probabilities on a graph.

(b) Give the marginal pmfs in the 'margins.'

(c) Compute μ_X, μ_Y, σ_x^2 , σ_Y^2 , Cov(X,Y), and ρ.

(d) Find the equation of the least squares regression line and draw it on your graph. Does the line make sense to you intuitively?

7. 4.2-3. Roll a fair four-sided die twice. Let X equal the outcome on the first roll, and let Y equal the sum of the two rolls.

   (a) Determine μ_X, μ_Y, σ_x^2 , σ_Y^2 , Cov(X,Y), and ρ

   (b) Find the equation of the least squares regression line and draw it on your graph. Does the line make sense to you intuitively?

8. 4.2-5. Let X and Y be random variables with respective means μ_X and μ_Y, respective variances $(\sigma_x)^2$,$(\sigma_Y)^2$ and correlation coefficient ρ. Fit the line y = a + bx by the method of least squares to the probability distribution by minimizing the expectation

$$K(a, b) = E[(Y - a - bX)^2]$$

with respect to a and b. Hint: Consider $\partial k/\partial a = 0$ and $\partial k/\partial b = 0$, and solve simultaneously.

9. 4.2-9. A car dealer sells X cars each day and always tries to sell an extended warranty on each of these cars. (In our opinion, most of these warranties are not good deals.) Let Y be the number of extended warranties sold; then $Y \leq X$. The joint pmf of X and Y is given by

$$f(x, y) = c(x+1)(4-x)(y+1)(3-y),$$

x=0,1,2,3, y=0,1,2, with $y \leq x$

(a) Find the value of c.

(b) Sketch the support of X and Y.

(c) Record the marginal pmfs $f_X(x)$ and $f_Y(y)$ in the margins.

(d) Are X and Y independent?

(e) Compute μ_X and σ_X^2.

(f) Compute μ_Y and σ_Y^2.

(g) Compute Cov(X,Y).

(h) Determine ρ, the correlation coefficient.

(i) Find the best-fitting line and draw it on your figure.

10. 4.2-10. If the correlation coefficient ρ exists, show that ρ satisfies the inequality $1 \leq \rho \leq 1$. Hint: Consider the discriminant of the nonnegative quadratic function that is given by $h(v) = E\{[(X - \mu_X) + v(Y - \mu_Y)]^2\}$.