yohandi - homework week 9

Exercise 15.1

3. $\int_{-1}^{0}\int_{-1}^{1}(x+y+1)\,dx\,dy = \int_{-1}^{0}\left(\frac{1}{2}x^2+xy+x\right)\Big|_{x=-1}^{1} dy = \int_{-1}^{0}(2y+2)\,dy = (y^2+2y)\Big|_{y=-1}^{0} = 1$

11. $\int_{-1}^{2}\int_{0}^{\pi/2} y\sin x\,dx\,dy = \int_{-1}^{2}(-y\cos x)\Big|_{x=0}^{\pi/2}dy = \int_{-1}^{2} y\,dy = \frac{1}{2}y^2\Big|_{y=-1}^{2} = \frac{3}{2}$

17. $\iint_R xy\cos y\,dA = \int_0^{\pi}\int_{-1}^{1} xy\cos y\,dx\,dy = \int_0^{\pi}\left(\frac{y\cos y\cdot x^2}{2}\right)\Big|_{x=-1}^{1}dy = \int_0^{\pi} 0\,dy = 0$

21. $\iint_R \frac{xy^3}{x^2+1}\,dA = \int_0^{2}\int_0^{1}\frac{xy^3}{x^2+1}\,dx\,dy = \int_0^{2}\left(y^3\frac{\ln(x^2+1)}{2}\right)_{x=0}^{1}dy = \frac{\ln(2)}{8}y^4\Big|_{y=0}^{2} = 2\ln(2)$

25. $\int_{-1}^{1}\int_{-1}^{1}(x^2+y^2)\,dx\,dy = \int_{-1}^{1}\left(y^2x+\frac{1}{3}x^3\right)\Big|_{x=-1}^{1}dy = \int_{-1}^{1}\left(2y^2+\frac{2}{3}\right)dy = \left(\frac{2}{3}y^3+\frac{2}{3}y\right)\Big|_{y=-1}^{1} = \frac{8}{3}$

Exercise 15.2

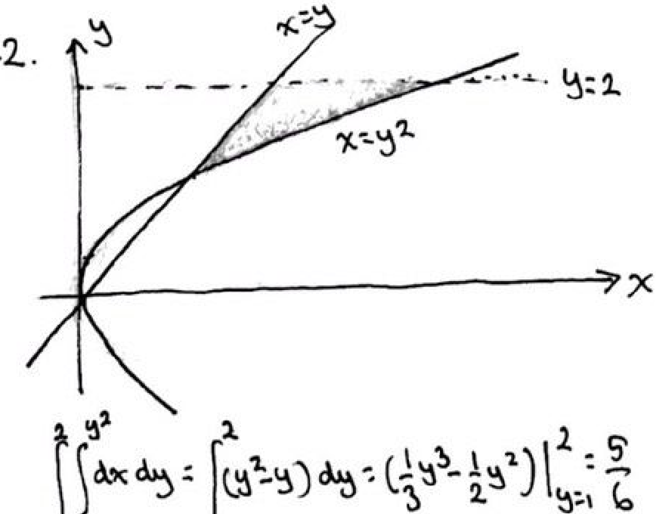
9a. $\int_0^{2}\int_{x^3}^{8} dy\,dx$

b. $\int_0^{8}\int_0^{y^{1/3}} dx\,dy$

17a. $\int_0^{1}\int_{x}^{3-2x} dy\,dx$

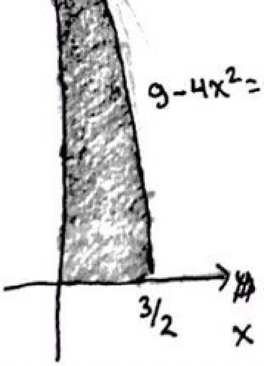
b. $\int_0^{1}\int_0^{y} dx\,dy + \int_1^{3}\int_1^{\frac{3-y}{2}} dx\,dy$

22.



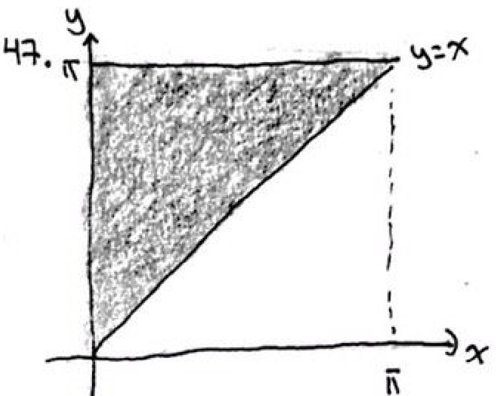
$\int_1^{2}\int_y^{y^2} dx\,dy = \int_1^{2}(y^2-y)\,dy = \left(\frac{1}{3}y^3-\frac{1}{2}y^2\right)\Big|_{y=1}^{2} = \frac{5}{6}$

39.



$\int_0^{3/2}\int_0^{9-4x^2} 16x\,dy\,dx = \int_0^{9}\int_0^{\frac{\sqrt{9-y}}{2}} 16x\,dx\,dy$

47.

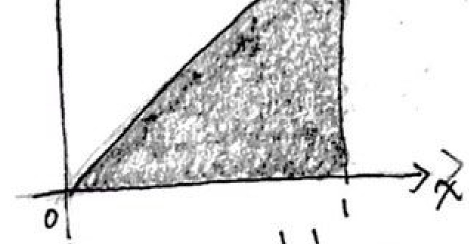


$\int_0^{\pi}\int_x^{\pi}\frac{\sin y}{y}\,dy\,dx = \int_0^{\pi}\int_0^{y}\frac{\sin y}{y}\,dx\,dy$

$= -\cos y\Big|_{y=0}^{\pi}$

$= 2$

49.



$\int_0^{1}\int_y^{1} x^2e^{xy}\,dx\,dy = \int_0^{1}\int_0^{x} x^2e^{xy}\,dy\,dx$

$= \int_0^{1} xe^{x^2} - x\,dx$

$= \frac{e-2}{2}$

59. $V = \int_{-4}^{1}\int_{3x}^{4-x^2} x+4 \, dy \, dx$

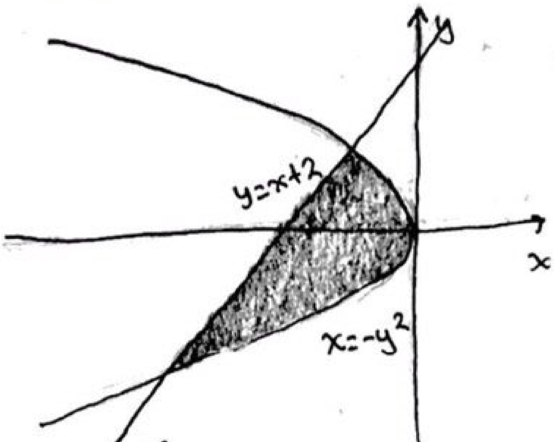
$= \int_{-4}^{1} -x^3 - 7x^2 - 8x + 16 \, dx$

$= \frac{625}{12}$

71. $\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \frac{dx\,dy}{(x^2+1)(y^2+1)} = \int_{-\infty}^{\infty} \frac{1}{y^2+1} (\arctan x)\Big|_{x=-\infty}^{\infty} dy = \pi \arctan y \Big|_{y=-\infty}^{\infty} = \pi^2$

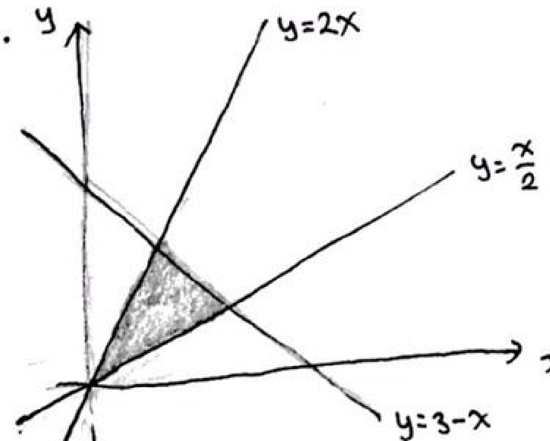
Exercise 15.3

3.



Area $= \int_{-2}^{1}\int_{y-2}^{-y^2} dx\,dy = \int_{-2}^{1} -y^2 - y + 2 \, dy = \frac{9}{2}$

11.

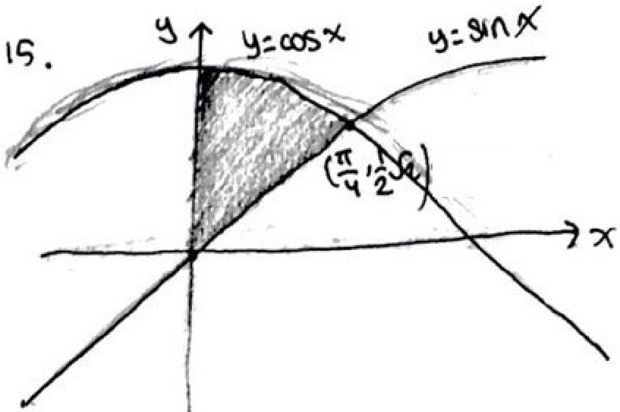


Area $= \int_{0}^{1}\int_{\frac{x}{2}}^{2x} dy\,dx + \int_{1}^{2}\int_{\frac{x}{2}}^{3-x} dy\,dx$

$= \int_{0}^{1} (2x - \frac{x}{2}) dx + \int_{1}^{2} (3 - \frac{3x}{2}) dx$

$= \frac{3}{2}$

15.



Area $= \int_{0}^{\pi/4}\int_{\sin x}^{\cos x} dy\,dx = \int_{0}^{\pi/4} \cos x - \sin x \, dx = \sqrt{2} - 1$

21. Average $= \frac{1}{2 \cdot 2} \int_{0}^{2}\int_{0}^{2} (x^2+y^2)\, dx\, dy$

$= \frac{1}{4} \int_{0}^{2} (2y^2 + \frac{8}{3}) dy$

$= \frac{8}{3}$

Exercise 15.4

1. $0 \le r \le 9$

$\frac{\pi}{2} \le \theta \le 2\pi$

3. $0 \le r \le \frac{1}{\sin\theta}$

$\frac{\pi}{4} \le \theta \le \frac{3\pi}{4}$

11. $\int_0^2 \int_0^{\sqrt{4-y^2}} (x^2+y^2)\,dx\,dy = \int_0^{\frac{\pi}{2}} \int_0^2 r^2 . r . dr\,d\theta$

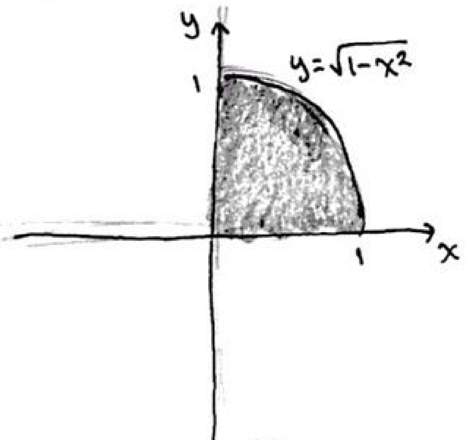
$= 2\pi$

13. $\int_0^6 \int_0^y x\,dx\,dy = \int_{\pi/4}^{\pi/2} \int_0^{6/\sin\theta} r\cos\theta . r\,dr\,d\theta$

$= 36$

23. $\int_0^{\pi/2} \int_0^1 (r\sin\theta)(r\cos\theta)\, r\,dr\,d\theta$

$= \int_0^1 \int_0^{\sqrt{1-x^2}} xy\,dy\,dx$



28. $\int_0^{\frac{\pi}{2}} \int_1^{1+\cos\theta} 2r\,dr\,d\theta = \int_0^{\pi/2} \cos^2\theta + 2\cos\theta\,d\theta$

$= \frac{\pi}{4} + 2$

34. $\int_0^{2\pi} \int_0^a \sqrt{(r\sin\theta)^2 + (r\cos\theta)^2}\, r\,dr\,d\theta$

$= \int_0^{2\pi} \int_0^a r^2\,dr\,d\theta$

$= \frac{2a^3\pi}{3}$

41a. $I^2 = \int_0^\infty \int_0^{\frac{\pi}{2}} e^{-((r\cos\theta)^2 + (r\sin\theta)^2)}\, r\,d\theta\,dr$

$= \int_0^\infty \frac{\pi}{2} . re^{-r^2}\,dr$

$= \frac{\pi}{4}$

$\Rightarrow I = \frac{1}{2}\sqrt{\pi}$ or $I = -\frac{1}{2}\sqrt{\pi}$ (invalid)

b. $\lim_{x\to\infty} \operatorname{erf}(x) = \lim_{x\to\infty} \int_0^x \frac{2e^{-t^2}}{\sqrt{\pi}}\,dt$

$= \frac{2}{\sqrt{\pi}} \lim_{x\to\infty} \int_0^x e^{-t^2}\,dt$

$= \frac{2}{\sqrt{\pi}} . \left(\frac{1}{2}\sqrt{\pi}\right)$

$= 1$

Exercise 15.5

11. $\int_0^{\pi/6} \int_0^1 \int_{-2}^3 y\sin z\,dx\,dy\,dz = \int_0^{\pi/6} \sin z . 5 \int_0^1 y\,dy$

$= -\frac{5}{2}\cos z \Big|_{z=0}^{\pi/6}$

$= \frac{5}{2} - \frac{5\sqrt{3}}{4}$

15. $\int_0^1 \int_0^{2-x} \int_0^{2-x-y} dz\,dy\,dx = \int_0^1 \int_0^{2-x} (2-x-y)\,dy\,dx$

$= \int_0^1 \left(2y - xy - \frac{1}{2}y^2\right)\Big|_{y=0}^{y=2-x} dx$

$= \int_0^1 \left(\frac{x^2}{2} - 2x + 2\right) dx$

$= \frac{7}{6}$

21a. $\int_{-1}^{1}\int_{0}^{1-x^2}\int_{x^2}^{1-z} dy\, dz\, dx$ b. $\int_{0}^{1}\int_{-\sqrt{1-z}}^{\sqrt{1-z}}\int_{x^2}^{1-z} dy\, dx\, dz$

c. $\int_{0}^{1}\int_{0}^{1-z}\int_{-\sqrt{y}}^{\sqrt{y}} dx\, dy\, dz$ d. $\int_{0}^{1}\int_{0}^{1-y}\int_{-\sqrt{y}}^{\sqrt{y}} dx\, dz\, dy$

e. $\int_{0}^{1}\int_{-\sqrt{y}}^{\sqrt{y}}\int_{0}^{1-y} dz\, dx\, dy$

27. line $(0,0,3)$ to $(1,0,0)$:

$3x+z=3$

line $(0,0,3)$ to $(0,2,0)$:

$3y+2z=6$

line $(1,0,0)$ to $(0,2,0)$:

$2x+y=2$

$$\int_{0}^{1}\int_{0}^{2-2x}\int_{0}^{\frac{6-6x-3y}{2}} dz\, dy\, dx = 1$$

29. Volume $= \int_{-1}^{1}\int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}}\int_{\sqrt{1-x^2}}^{\sqrt{1-x^2}} dz\, dy\, dx = \frac{16}{3}$

41. $z \in [0, 4]$

$y \in [0, 1]$ $\Rightarrow$ $x \in [0, 2]$

$x \in [2y, 2]$ $\quad$ $y \in [0, \frac{x}{2}]$

$$\int_{0}^{4}\int_{0}^{2}\int_{0}^{\frac{x}{2}} \frac{4\cos(x^2)}{2\sqrt{z}}\, dy\, dx\, dz$$

$$= \int_{0}^{4}\int_{0}^{2} \frac{4x}{2}\, \frac{\cos(x^2)}{2\sqrt{z}}\, dx\, dz$$

$$= \int_{0}^{4} \frac{1}{\sqrt{z}} \left(\frac{1}{2}\sin(x^2)\right)\Big|_{x=0}^{2} dz$$

$$= \frac{1}{2}\sin(4) \int_{0}^{4} 2 \cdot \frac{1}{2\sqrt{z}}\, dz$$

$$= 2\sin(4)$$