MAT1001 – T01-04

Student ID Name:

Quiz 3

(15 minutes on Tuesday, 29 Sep 2020)

Show your work for the questions below:

1. [8 points] In the motion along a coordinate line, the position of an object at time t is given by $s(t) = t^2 + \sin \pi t$. Find the following quantities at time $t = 1$:

(a) the velocity and speed of the object;

(b) the acceleration of the object.

In Questions 2 and 3 below, remember $(e^x)' = e^x$ and $(\ln x)' = 1/x$.

2. [15 points] Find the derivatives of the following functions $y = f(x)$:

(a) $y = \tan\left(x^3 e^{-x}\right)$

(b) $y = \sin^3\left(e^{\tan x}\right)$

(c) $y = f(x)$ determined by equation $y^5 \ln y = e^{xy} + \sec^2 x$

3. [7 points] Let $y = (1 - \cos x)^{\sqrt{x}}$. Find y' by differentiating $\ln y = \sqrt{x} \ln(1 - \cos x)$.

4. [10 points] Define a function $f(x)$ on $\mathbb{R}$ by

$$f(x) = \begin{cases} x^3 \sin\dfrac{1}{x}, & x \neq 0 \\ 0, & x = 0 \end{cases}$$

Show that $f(x)$ is differentiable and $f'(x)$ is continuous at $x = 0$.
You may use the limits:

$$\lim_{x \to 0} x \sin\frac{1}{x} = 0 \quad \text{and} \quad \lim_{x \to 0} x \cos\frac{1}{x} = 0$$