yohandi - homework for week 4

Exercises 12.1

1. A line that parallel to the z-axis through (2,3,0)

5. An xy- circle with radius 2 centered at (0,0,0)

11. $x^2+y^2+(0+3)^2=25$, $z=0$

$x^2+y^2=16$, $z=0$

An xy- circle with radius 4 centered at (0,0,0)

17a. first quadrant of the xy-plane

b. fourth quadrant of the xy-plane

20a. An xy- circle and its inferior with radius 1 centered at (0,0,0)

b. An xy- circle and its inferior with radius 1 centered at (0,0,3)

c. A tube where "an xy- circle and its inferior with radius 1" as its base without any height boundary

21a. The closed region bounded by the spheres of radius 1 and 4 centered at (0,0,0)

b. The closed region bounded by a half sphere of radius 1 centered at (0,0,0) and x-y plane (non-negative dimension of z)

25a. $x=3$ b. $y=-1$ c. $z=-2$

33. $x^2+y^2+3^2=5^2$, $z=3$

$\Rightarrow x^2+y^2=4^2$, $z=3$

35. $0 \le z \le 1$

40. $1 \le x^2+y^2+z^2 \le 4$

41. $|P_1P_2| = \sqrt{(1-3)^2+(1-3)^2+(1-0)^2} = 3$

44. $|P_1P_2| = \sqrt{(3-2)^2+(4-3)^2+(5-4)^2} = \sqrt{3}$

55. $x^2+y^2+z^2+4x-4z=0$

$(x+2)^2-4+y^2+(z-2)^2-4=0$

$(x+2)^2+y^2+(z-2)^2=8$

radius $= 2\sqrt{2}$, center $= (-2,0,2)$

58. $3x^2+3y^2+3z^2+2y-2z=9$

$x^2+y^2+z^2+(\frac{2}{3})y-(\frac{2}{3})z=3$

$x^2+(y+\frac{1}{3})^2-\frac{1}{9}+(z-\frac{1}{3})^2-\frac{1}{9}=3$

$x^2+(y+\frac{1}{3})^2+(z-\frac{1}{3})^2=\frac{29}{9}$

radius $= \frac{1}{3}\sqrt{29}$, center $= (0,-\frac{1}{3},\frac{1}{3})$

Exercises 12.2

7. $\frac{3}{5}u+\frac{4}{5}v=\langle \frac{3}{5}\cdot 3+\frac{4}{5}\cdot(-2), \frac{3}{5}(-2)+\frac{4}{5}\cdot(5)\rangle$

$=\langle \frac{1}{5}, \frac{14}{5}\rangle$

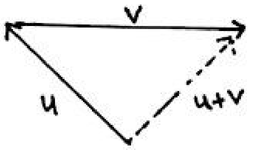
$\left|\frac{3}{5}u+\frac{4}{5}v\right| = \sqrt{\left(\frac{1}{5}\right)^2+\left(\frac{14}{5}\right)^2} = \frac{1}{5}\sqrt{197}$

12. $\vec{AB}+\vec{CD} = \langle (2-1)+(-2-(-1)), (0-(-1)) + (2-3)\rangle$

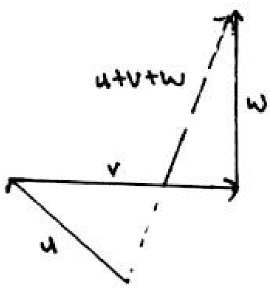
$=\langle 0,0\rangle$

14. $\langle \cos(-\frac{3\pi}{4}), \sin(-\frac{3\pi}{4})\rangle = \langle -\frac{1}{2}\sqrt{2}, -\frac{1}{2}\sqrt{2}\rangle$

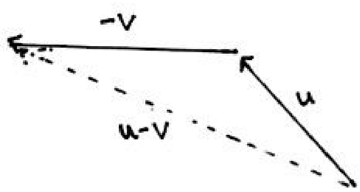
23a.



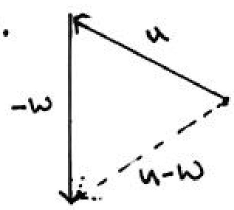
b.



c.



d.



26. $\vec{v}=\langle 9,-2,6\rangle$

$|\vec{v}|=\sqrt{9^2+(-2)^2+6^2}=11$

$\frac{\vec{v}}{|\vec{v}|}=\frac{9}{11}i+\frac{(-2)}{11}j+\frac{6}{11}k$

$\therefore \vec{v}=11\left(\frac{9}{11}i-\frac{2}{11}j+\frac{6}{11}k\right)$

31a. $2i$

b. $-\sqrt{3}k$

c. $\frac{3}{10}i+\frac{4}{10}j$

d. $6i-2j+3k$

35. $\vec{P_1P_2} = (2-(-1))i+(5-1)j+(0-5)k$

$= 3i+4j-5k$

$|\vec{P_1P_2}| = \sqrt{3^2+4^2+(-5)^2} = 5\sqrt{2}$

$\therefore \frac{\vec{P_1P_2}}{|\vec{P_1P_2}|} = \frac{3}{5\sqrt{2}}i+\frac{4}{5\sqrt{2}}j-\frac{1}{\sqrt{2}}k$

$M=\left(\frac{(-1)+2}{2}, \frac{1+5}{2}, \frac{5+0}{2}\right)$

$=\left(\frac{1}{2}, 3, \frac{5}{2}\right)$

39. $\vec{AB}=\vec{OB}-\vec{OA}$

$\Rightarrow \vec{OA}=\vec{OB}-\vec{AB}$

$=(5-1)i+(1-4)j+(3-(-2))k$

$=4i-3j+5k$

$\therefore A=(4,-3,5)$

43. $\vec{v}=\langle 800\sin 25°, 800\cos 25°\rangle$ km/h

$=\langle 338.0946, 725.0462\rangle$ km/h

48. as the weight is in an equilibrium position,

$\sum\vec{F_x}=0$ and $\sum\vec{F_y}=0$

$\Rightarrow \sum\vec{F_y}=0$

$F_1\cdot\sin\alpha+F_2\cdot\sin\beta-W=0$

$75\cdot\sin\alpha+75\cdot\sin\alpha-25=0$

$\sin\alpha=\frac{1}{6}$

$\alpha\approx 9.594°$

Exercises 12.3

12. $v \cdot u = (2 \cdot (-2)) + (-4 \cdot 4) + (\sqrt{5} \cdot -\sqrt{5})$

$= -25$

b. $|v| = \sqrt{2^2 + (-4)^2 + (\sqrt{5})^2} = 5$

$|u| = \sqrt{(-2)^2 + 4^2 + (-\sqrt{5})^2} = 5$

$\cos(\theta) = \dfrac{v \cdot u}{|v||u|} = -\dfrac{25}{5 \cdot 5} = -1$

c. $|u| \cos\theta = 5 \cdot -1 = -5$

d. $\text{proj}_v u = \left(\dfrac{u \cdot v}{v \cdot v}\right) v$

$= \left(\dfrac{v \cdot u}{|v|^2}\right) \cdot v$

$= \dfrac{-25}{5^2} \cdot \langle 2, -4, \sqrt{5} \rangle$

$= \langle -2, 4, -\sqrt{5} \rangle$

8 a. $v \cdot u = \left(\frac{1}{\sqrt{2}} \cdot \frac{1}{\sqrt{2}}\right) + \left(\frac{1}{\sqrt{3}} \cdot -\frac{1}{\sqrt{3}}\right)$

$= \frac{1}{6}$

$|v| = \sqrt{\left(\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{1}{\sqrt{3}}\right)^2} = \frac{1}{6}\sqrt{30}$

$|u| = \sqrt{\left(\frac{1}{\sqrt{2}}\right)^2 + \left(-\frac{1}{\sqrt{3}}\right)^2} = \frac{1}{6}\sqrt{30}$

b. $\cos(\theta) = \dfrac{v \cdot u}{|v||u|} = \dfrac{\frac{1}{6}}{\frac{1}{6}\sqrt{30} \cdot \frac{1}{6}\sqrt{30}} = \dfrac{1}{5}$

c. $|u| \cdot \cos\theta = \frac{1}{30}\sqrt{30}$

d. $\text{proj}_v u = \left(\dfrac{u \cdot v}{v \cdot v}\right) v$

$= \left(\dfrac{v \cdot u}{|v|^2}\right) \cdot v$

$= \left(\dfrac{\frac{1}{6}}{\frac{30}{36}}\right) \langle \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{3}} \rangle$

$= \langle \frac{1}{5\sqrt{2}}, \frac{1}{5\sqrt{3}} \rangle$

14. $\vec{AC} = \langle 3-1, 4-0 \rangle = \langle 2, 4 \rangle$

$\vec{BD} = \langle 4-0, 1-3 \rangle = \langle 4, -2 \rangle$

$\theta = \arccos\left(\dfrac{\vec{AC} \cdot \vec{BD}}{|\vec{AC}||\vec{BD}|}\right) = \arccos(0)$

$= \dfrac{\pi}{2}$

18. $\vec{CA} = -v - u$

$\vec{CB} = -v + u$

$\vec{CA} \cdot \vec{CB} = (-v-u) \cdot (-v+u)$

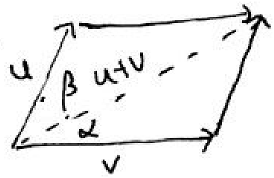
$= |v|^2 - |u|^2$

Since $|v| = |u|$

$\Rightarrow \vec{CA} \cdot \vec{CB} = 0$

$\Rightarrow \vec{CA}$ and $\vec{CB}$ are orthogonal

22.



$\cos(\alpha) = \dfrac{(u+v) \cdot v}{|u+v||v|} = \dfrac{u \cdot v + |v|^2}{|u+v||v|}$

$\cos(\beta) = \dfrac{(u+v) \cdot u}{|u+v||u|} = \dfrac{|u|^2 + u \cdot v}{|u+v||u|}$

since $|u| = |v|$, those 2 expressions are equal $\Rightarrow \cos(\alpha) = \cos(\beta)$

$\alpha = \beta$ (since $0 < \alpha, \beta < \frac{\pi}{2}$)

25a. note that $|\cos\theta| \le 1$,

$|u \cdot v| = \big||u||v|\cos\theta\big| \le |u||v|$ (it is true)

b. $|u \cdot v| = |u||v|$

$\big||u||v|\cos\theta\big| = |u||v|$

$\Rightarrow |\cos\theta| = 1$ or $\Rightarrow |u|$ or $|v| = 0$

$\theta = 0$ or π

(both vectors are parallel or antiparallel) (both vectors are zero)

27. $v \cdot u_1 = (a u_1 + b u_2) \cdot u_1$

$= a|u_1|^2 + b\underbrace{(u_2 \cdot u_1)}_{0}$

$= a|u_1|^2$

$= a$

29. $(u - proj_v u) \cdot proj_v u = \left(u - \left(\frac{u \cdot v}{|v|^2}\right)v\right) \cdot \left(\frac{u \cdot v}{|v|^2}\right)v$

$= \left(\frac{u \cdot v}{|v|^2}\right)u \cdot v - \left(\frac{u \cdot v}{|v|^2}\right)^2 |v|^2$

$= 0$

44. $F \cdot d = |F||d| \cos\theta$

$= 1000 \cdot 1000 \cdot \cos 60$

$= 500\,000$ Joule

Exercises 12.4

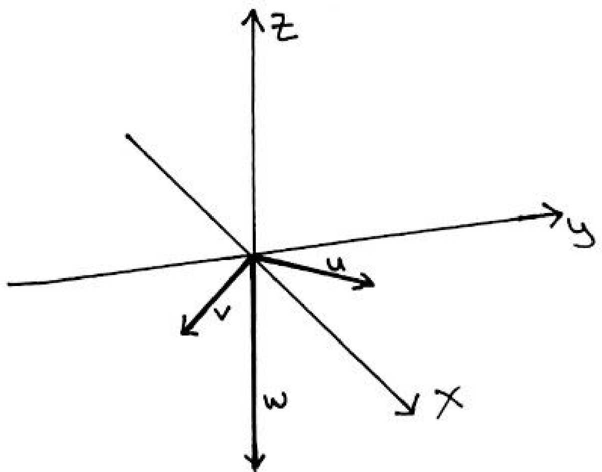
3. $u \times v = \begin{vmatrix} i & j & k \\ 2 & -2 & 4 \\ -1 & 1 & -2 \end{vmatrix}$

$= (4-4)i - (2-2)j + (2-2)k$

$= 0$

$v \times u = -u \times v = 0$

13. $w = u \times v = \begin{vmatrix} i & j & k \\ 1 & 1 & 0 \\ 1 & -1 & 0 \end{vmatrix} = -2k$



17a. Area $= \frac{1}{2}|\vec{PQ} \times \vec{PR}|$

$= \frac{1}{2}|\vec{PQ}||\vec{PR}| \sin\left(\arccos\left(\frac{\vec{PQ}\cdot\vec{PR}}{|\vec{PQ}\cdot\vec{PR}|}\right)\right)$

$= \frac{1}{2}\cdot\sqrt{3}\cdot\sqrt{2}\cdot\frac{1}{\sqrt{3}}$

$= \frac{1}{2}\sqrt{2}$

b. $\vec{v} = |\vec{PQ} \times \vec{PR}|$

$= \begin{vmatrix} i & j & k \\ 1 & 1 & 1 \\ 1 & 1 & 0 \end{vmatrix}$

$= -i + j$

$\frac{\vec{v}}{|v|} = -\frac{1}{2}\sqrt{2}\, i + \frac{1}{2}\sqrt{2}\, j$

21. $(u \times v)\cdot w = \begin{vmatrix} 2 & 1 & 0 \\ 2 & -1 & 1 \\ 1 & 0 & 2 \end{vmatrix} = -7$

$(v \times w)\cdot u = \begin{vmatrix} 2 & -1 & 1 \\ 1 & 0 & 2 \\ 2 & 1 & 0 \end{vmatrix} = -7$

$(w \times u)\cdot v = \begin{vmatrix} 1 & 0 & 2 \\ 2 & 1 & 0 \\ 2 & -1 & 1 \end{vmatrix} = -7$

Volume $= |(u \times v)\cdot w| = |-7| = 7.$

26. $|\vec{\tau}| = |\vec{r} \times \vec{F}|$

$= ||\vec{r}||\vec{F}| \sin\theta|$

$= |0.2 \cdot 15 \cdot \sin 45°|$ Nm

$= \frac{3}{2}\sqrt{2}$ Nm

27a. $\sqrt{u\cdot u} = \sqrt{|u||u|\cos 0°} = \sqrt{|u|^2} = |u|$

it is true for all vector u

b. $u \cdot u = |u||u|\cos° = |u|^2$

it is not always true as :

$|u|^2 = |u|$

when $|u| = 0$ or $|u| = 1$

c. $u \times 0 = \begin{vmatrix} i & j & k \\ u_1 & u_2 & u_3 \\ 0 & 0 & 0 \end{vmatrix} = 0$

$0 \times u = \begin{vmatrix} i & j & k \\ 0 & 0 & 0 \\ u_1 & u_2 & u_3 \end{vmatrix} = 0$

it is true for all vector u

d. $u \times -u = \begin{vmatrix} i & j & k \\ u_1 & u_2 & u_3 \\ -u_1 & -u_2 & -u_3 \end{vmatrix} = 0$

it is true for all vector u

e. $u \times v = \begin{vmatrix} i & j & k \\ u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \end{vmatrix}$

$= (u_2 v_3 - v_2 u_3)i - (u_1 v_3 - v_1 u_3)j + (u_1 \cdot v_2 - v_1 \cdot u_2)k$

$v \times u = \begin{vmatrix} i & j & k \\ v_1 & v_2 & v_3 \\ u_1 & u_2 & u_3 \end{vmatrix}$

$= (v_2 \cdot u_3 - u_2 v_3)i - (v_1 \cdot u_3 - u_1 \cdot v_3)j + (v_1 \cdot u_2 - u_1 \cdot v_2)k$

$= -(u \times v)$

it is only true when $(u \times v) = 0$

$\therefore$ is not always true for all vector u and v

f. $u\times(v+w)=(u\times v+u\times w)$

$\therefore$ it is always true for all vector u, v, and w

g. $(u\times v)\cdot v=\begin{vmatrix} u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \\ v_1 & v_2 & v_3 \end{vmatrix}=0$

it is true for all vector u and v

h. $(u\times v)\cdot w=\begin{vmatrix} u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \\ w_1 & w_2 & w_3 \end{vmatrix}$

$=-\begin{vmatrix} v_1 & v_2 & v_3 \\ u_1 & u_2 & u_3 \\ w_1 & w_2 & w_3 \end{vmatrix}$

$=\begin{vmatrix} v_1 & v_2 & v_3 \\ w_1 & w_2 & w_3 \\ u_1 & u_2 & u_3 \end{vmatrix}$

$=(v\times w)\cdot u$

$=u\cdot(v\times w)$

it is true for all vector u, v and w

28a. $u\cdot v=\langle u_1,u_2,u_3\rangle\cdot\langle v_1,v_2,v_3\rangle$

$=u_1.v_1+u_2v_2+u_3v_3$

$=v_1.u_1+v_2u_2+v_3u_3$

$=\langle v_1,v_2,v_3\rangle\cdot\langle u_1,u_2,u_3\rangle$

$=v\cdot u$

it is true for all vector u and v

b. as proved in 27e,

$-(u\times v)=v\times u$

$u\times v=-(v\times u)$

is true for all vector u and v

c. $(-u)\times v=\begin{vmatrix} i & j & k \\ -u_1 & -u_2 & -u_3 \\ v_1 & v_2 & v_3 \end{vmatrix}$

$=(-u_2v_3+u_3v_2)i-(-u_1v_3+u_3v_1)j+(-u_1v_2+u_2v_1)k$

$=-\Big((u_2v_3-u_3v_2)i-(u_1v_3-u_3v_1)j+(u_1v_2-u_2v_1)k\Big)$

$=-(u\times v)$

it is true for all vector u and v

d. $(cu)\cdot v=c.u_1.v_1+c.u_2v_2+c.u_3v_3$

$=u_1\cdot(c.v_1)+u_2(c.v_2)+u_3(c.v_3)$

$=u\cdot(cv)$

$=c(u_1.v_1+u_2v_2+u_3.v_3)$

$=c(u.v)$

it is true for all vector u and v

e. $c(u\times v)=c\begin{vmatrix} i & j & k \\ u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \end{vmatrix}$

$=\begin{vmatrix} i & j & k \\ c.u_1 & c.u_2 & c.u_3 \\ v_1 & v_2 & v_3 \end{vmatrix}$

$=(c.u)\times v$

$=\begin{vmatrix} i & j & k \\ u_1 & u_2 & u_3 \\ c.v_1 & c.v_2 & c.v_3 \end{vmatrix}$

$=u\times(cv)$

it is true for all vector u and v

f. $u\cdot u=|u||u|\cos 0^\circ=|u|^2$

it is true for all vector u

g. $(u\times u)\cdot u=\begin{vmatrix} u_1 & u_2 & u_3 \\ u_1 & u_2 & u_3 \\ u_1 & u_2 & u_3 \end{vmatrix}=0$

It is true for all vector u

h. $(u\times v)\cdot u=\begin{vmatrix} u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \\ u_1 & u_2 & u_3 \end{vmatrix}$

$=-\begin{vmatrix} v_1 & v_2 & v_3 \\ u_1 & u_2 & u_3 \\ u_1 & u_2 & u_3 \end{vmatrix}$

$=0$

$=\begin{vmatrix} u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \\ v_1 & v_2 & v_3 \end{vmatrix}$

$=(u\times v)\cdot v$

$=v\cdot(u\times v)$

it is true for all vector u and v

31a. $\underbrace{\underbrace{(u\times v)}_{\text{vector}}\cdot\underbrace{w}_{\text{vector}}}_{\text{scalar}}$ (make sense!)

b. $\underbrace{u}_{\text{vector}}\times\underbrace{(v\cdot w)}_{\text{scalar}}$ (undefined)

c. $\underbrace{\underbrace{u}_{\text{vector}}\times\underbrace{(v\times w)}_{\text{vector}}}_{\text{vector}}$ (make sense)

d. $\underbrace{u}_{\text{vector}}\cdot\underbrace{(v\cdot w)}_{\text{scalar}}$ (undefined)

32. note that:

- $(u\times v)\times w = \underbrace{(u\cdot w)}_{A}v - \underbrace{(v\cdot w)}_{B}u$

$= Av - Bu$

$\therefore (u\times v)\times w$ lies in the plane of u and v

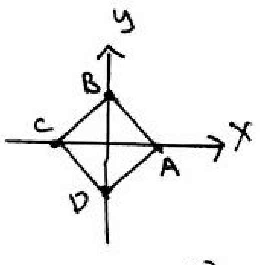
- $u\times(v\times w) = -(v\times w)\times u$

$= -\left(\underbrace{(v\cdot u)}_{A}w - \underbrace{(w\cdot u)}_{B}.v\right)$

$= A'w + B'v$

$\therefore u\times(v\times w)$ lies in the plane of v and w

35.



Area $= \sqrt{2}\cdot\sqrt{2}$

$= 2$

41. Area $= \frac{1}{2}|\vec{AB}\times\vec{AC}|$

$= \frac{1}{2}|\langle -2,3\rangle \times \langle 3,1\rangle|$

$= \frac{11}{2}$

Exercises 12.5

3. $\begin{pmatrix} P_x \\ P_y \\ P_z \end{pmatrix} + \lambda \begin{pmatrix} Q_x - P_x \\ Q_y - P_y \\ Q_z - P_z \end{pmatrix}$

$\begin{pmatrix} -2 \\ 0 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} 5 \\ 5 \\ -5 \end{pmatrix}$

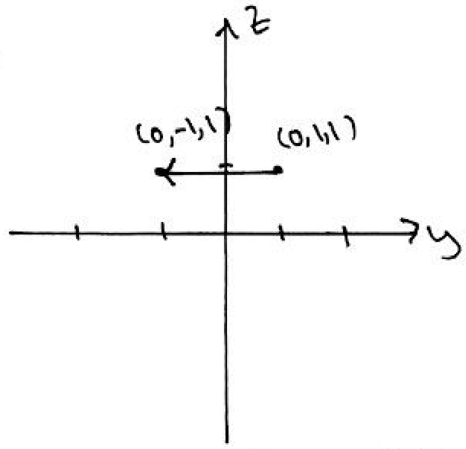
$\begin{pmatrix} -2 \\ 0 \\ 3 \end{pmatrix} + \lambda' \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}$

$x = -2 + t$
$y = t$
$z = 3 - t$

7. $\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$

$x = 1$
$y = 1$
$z = 1 + t$

17.



$\begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} 0 \\ -1 \\ 0 \end{pmatrix} \geq \begin{pmatrix} 0 \\ -1 \\ 1 \end{pmatrix}$

$\Rightarrow 1 - \lambda \geq -1$

$\lambda \leq 2$

$x = 0$
$y = 1 - t \quad , 0 \leq t \leq 2$
$z = 1$

23. $n = \vec{PQ} \times \vec{PR} = \begin{vmatrix} i & j & k \\ 4 & -1 & 3 \\ -1 & -3 & 2 \end{vmatrix}$

$= \langle 7, -5, -4 \rangle$

$7(x-1) - 5(y-1) - 4(z+1) = 0$

$7x - 5y - 4z = 6$

25. $V_{dir} = \begin{pmatrix} 1 \\ 3 \\ 4 \end{pmatrix}$

$(x-2) + 3(y-4) + 4(z-5) = 0$

$x + 3y + 4z = 34$

27. line 1: $\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 3 \\ 4 \end{pmatrix}$

line 2: $\begin{pmatrix} 2 \\ 4 \\ -1 \end{pmatrix} + \mu \begin{pmatrix} 1 \\ 2 \\ -4 \end{pmatrix}$

line 1 & line 2:

$1 + 2\lambda = 2 + \mu$
$\to 2\lambda - \mu = 1$
$2 + 3\lambda = 4 + 2\mu$
$\to 3\lambda - 2\mu = 2$

$\lambda = 0$
$\mu = -1$

intersection point at $(1, 2, 3)$

$n = u \times v = \begin{vmatrix} i & j & k \\ 2 & 3 & 4 \\ 1 & 2 & -4 \end{vmatrix}$

$= \langle -20, 12, 1 \rangle$

$-20(x-1) + 12(y-2) + (z-3) = 0$

$-20x + 12y + z = 7$

35. since $\begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix}$ is in the line $\begin{pmatrix} 2+2\lambda \\ 1+6\lambda \\ 3 \end{pmatrix}$

$\Rightarrow$ distance $= 0$

43. $n = \langle 2, 1, 2 \rangle$

$u = \langle 0, 4, 0 \rangle - \langle 0, -1, 0 \rangle = \langle 0, 5, 0 \rangle$

distance $= \dfrac{|u \cdot n|}{|n|} = \dfrac{5}{3}$

45. distance $= \dfrac{|D_1 - D_2|}{\sqrt{A^2 + B^2 + C^2}} = \dfrac{|10 - 1|}{\sqrt{1^2 + 2^2 + 6^2}} = \dfrac{9}{\sqrt{41}}$

47. $\theta = \arccos\left(\dfrac{n_1 \cdot n_2}{|n_1||n_2|}\right)$

$= \arccos\left(\dfrac{1 \cdot 2 + 1 \cdot 1 + 0}{\sqrt{2} \cdot \sqrt{9}}\right)$

$= \dfrac{\pi}{4}$

55. $(1+2t)+(1+5t)+(3t)=2$

$2+10t=2$

$\Rightarrow t=0$

$(x,y,z)=(1,1,0)$

59. let $z=0$,

$\left.\begin{array}{l} x-2y=2 \\ x+y=5 \end{array}\right\} \begin{array}{l} x=4 \\ y=1 \end{array}$

v dir $= n_1 \times n_2$

$= \begin{vmatrix} i & j & k \\ 1 & -2 & 4 \\ 1 & 1 & -2 \end{vmatrix}$

$=\langle 0,6,3\rangle$

line: $\begin{pmatrix} 4 \\ 1 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 0 \\ 6 \\ 3 \end{pmatrix}$

$x=4$
$y=1+6t$
$z=3t$

69. find 2 random vectors (plane's nomal)

for which $n_1 \times n_2 = \langle 1,-1,2\rangle$

$n_1 \times n_2 = \begin{vmatrix} i & j & k \\ A & B & C \\ D & E & F \end{vmatrix}$

$BF-CE=1$
$-(AF-CD)=-1$
$AE-BD=2$

$n_1=\langle A,B,C\rangle = \langle 5,3,-1\rangle$
$n_2=\langle D,E,F\rangle = \langle -4,-2,1\rangle$

for $t=0$, $x=1$, $y=2$, $z=3$

plane 1:
$5x+3y-z=8$

plane 2:
$-4x-2y+z=-5$

73a. $\vec{EP}=\lambda \vec{EP_1}$

$\begin{pmatrix} -x_0 \\ y \\ z \end{pmatrix} = \lambda \begin{pmatrix} x_1-x_0 \\ y_1 \\ z_1 \end{pmatrix}$

$\begin{pmatrix} 0 \\ y \\ z \end{pmatrix} = \begin{pmatrix} x_0 \\ 0 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} x_1-x_0 \\ y_1 \\ z_1 \end{pmatrix}$

$\Rightarrow \begin{array}{l} 0=x_0+\lambda(x_1-x_0) \\ y=\lambda y_1 \\ z=\lambda z_1 \end{array}$

b. for $x_1=0$,

$0=x_0+\lambda(0-x_0)$

$\Rightarrow \lambda=1$

$\Rightarrow y=y_1$
$z=z_1$

for $x_1=x_0$,

$0=x_0+\lambda(x_0-x_0)$

$\Rightarrow x_0=0$

$\Rightarrow \lambda$ is undefined

$\lim\limits_{x_0\to\infty} \lambda = \lim\limits_{x_0\to\infty} \frac{x_0}{x_0-x_1} = 1$

$\therefore$ for $x_0\to\infty$, $y=y_1$ and $z=z_1$

74. line:

$\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} -1 \\ 2 \\ 2 \end{pmatrix}$

plane:

$n=\langle 0,1,-1\rangle \times \langle -3,2,1\rangle$

$=\langle 3,3,3\rangle$

$3(x-1)+3y+3z(x-1)=0$

$x+y+z=2$

intersection:

$(1-\lambda)+(2\lambda)+(2\lambda)=2$

$\lambda=\frac{1}{3}$

at $\left(\frac{2}{3},\frac{2}{3},\frac{2}{3}\right)$

hidden ratio $= \dfrac{\text{distance intersection point to } (1,0,0)}{\text{distance } (2,0,0) \text{ to } (1,1,0)}$

$=\frac{1}{3}$.