MAT1001 – T01-04

Student ID Name:

Quiz 5

(25 minutes on Tuesday, 20 Oct 2020)

1. [12 points] Determine if the following statements are True or False (no need to show your work):

(a) If f has a continuous derivative an open interval I with $f'(a)>0$ for some $a \in I$, and f has no critical point in I, then $f'(x)>0$ for all $x \in I$.

(b) If f is a continuous at $x = x_0$, $f''(x)<0$ for $x \in (x_0-\delta, x_0)$ and $f''(x)>0$ for $x \in (x_0, x_0+\delta)$ with $\delta > 0$, then x_0 is an inflection point of f.

(c) $\lim\limits_{x\to 0^+} x^{1/x}$ is a limit of indeterminate form 0^∞.

(d) If $f'(x) = \sin x$ and $f(0)=1$, then $f(x) = 2-\cos x$.

Show your work for the questions below:

2. [12 points] Find the following limits by L'Hôpital's rule and/or other methods:

(a) $\lim\limits_{x\to 0} \dfrac{2\sin x - \sin 2x}{x\tan^2 x}$

(b) $\lim\limits_{x\to\infty} \left(x^2+2e^x\right)\ln\left(1+e^{-x}\right)$

Reminder: The equivalent replacement is valid for product:

$$\lim_{x\to x_0} \frac{f(x)}{g(x)} = 1 \;\Rightarrow\; \lim_{x\to x_0} f(x)h(x) = \lim_{x\to x_0} g(x)h(x) \quad (\text{including } x_0 = \pm\infty)$$

3. [8 points] Given that the surface area of a right circular cylinder is $A = 2\,\text{m}^2$, find the maximum volume of the cylinder.

4. [8 points] Let

$$f(x) = \frac{1}{2}\ln\frac{1+\sin x}{1-\sin x} \quad \text{and} \quad g(x) = \ln\left|\sec x + \tan x\right|$$

(a) Show that $f'(x) = g'(x)$.

(b) Use the result of part (a) to prove $f(x) = g(x)$.