STA2001 Home assignment 11

1. 5.7-2. Suppose that among gifted seventh-graders who score very high on a mathematics exam, approximately 20% are left-handed or ambidextrous. Let X equal the number of left-handed or ambidextrous students among a random sample of n = 25 gifted seventh-graders. Find $P(2 < X < 9)$.
   (a) Using Table II in Appendix B.
   (b) Approximately, using the central limit theorem

2. 5.7-12. If X is b(100, 0.1), find the approximate value of $P(12 \leq X \leq 14)$, using
   (a) The normal approximation.
   (b) The Poisson approximation.
   (c) The binomial.

3. 5.7-18. Assume that the background noise X of a digital signal has a normal distribution with $\mu = 0$ volts and $\sigma = 0.5$ volt. If we observe n = 100 independent measurements of this noise, what is the probability that at least 7 of them exceed 0.98 in absolute value?
   (a) Use the Poisson distribution to approximate this probability.
   (b) Use the normal distribution to approximate this probability.
   (c) Use the binomial distribution to approximate this probability.

4. 5.8-3. Let X denote the outcome when a fair die is rolled. Then $\mu = 7/2$ and $\sigma^2 = 35/12$. Note that the maximum deviation of X from μ equals 5/2. Express this deviation in terms of the number of standard deviations; that is, find k, where $k\sigma = 5/2$. Determine a lower bound for $P(|X - 3.5| \ < 2.5)$.

5. 5.8-4. If the distribution of Y is b(n, 0.5), give a lower bound for$P(|Y/n - 0.5\ | < 0.08)$ when
   (a) n = 100.
   (b) n = 500.
   (c) n = 1000.

6. 5.8-6. Let $\bar{X}$ be the mean of a random sample of size n = 15 from a distribution with mean $\mu = 80$ and variance $\sigma^2 = 60$. Use Chebyshevs inequality to find a lower bound for $P(75 < \bar{X} < 85)$.

7. 5.8-7. Suppose that W is a continuous random variable with mean 0 and a symmetric pdf f (w) and cdf F(w), but for which the variance is not specified (and may not exist). Suppose further that W is such that

$$P(|W - 0|<k) = 1 - \frac{1}{k^2}$$

for $k \geq 1$. (Note that this equality would be equivalent to the equality in Chebyshevs inequality if the variance of W were equal to 1.) Then the cdf satisfies

$$F(w) - F(-w) = 1 - \frac{1}{w^2}, \quad w \geq 1.$$

Also, the symmetry assumption implies that

$$F(-w) = 1 - F(w).$$

(a) Show that the pdf of W is

$$f(w) = \begin{cases} \frac{1}{|w^3|} & |w| > 1, \\ 0 & |w| \leq 1. \end{cases}$$

(b) Find the mean and the variance of W and interpret your results.
(c) Graph the cdf of W.

8. 5.9-1. Let Y be the number of defectives in a box of 50 articles taken from the output of a machine. Each article is defective with probability 0.01. Find the probability that $Y = 0$, 1, 2, or 3
(a) By using the binomial distribution.
(b) By using the Poisson approximation.

9. 5.9-4. Let Y be $\chi^2(n)$. Use the central limit theorem to demonstrate that $W = (Y - n)/\sqrt{2n}$ has a limiting cdf that is N(0, 1). Hint: Think of Y as being the sum of a random sample from a certain distribution.

10. 5.9-5. Let Y have a Poisson distribution with mean 3n. Use the central limit theorem to show that the limiting distribution of $W = (Y - 3n)/\sqrt{3n}$ is N(0, 1).