yohandi - assignment 3

1a. (1) $f(x) > 0$

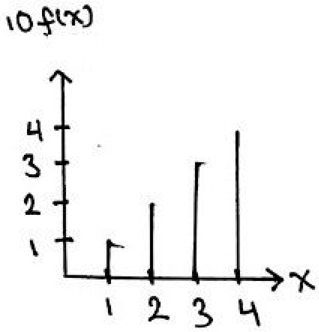
$\frac{x}{c} > 0$

as $x = 1, 2, 3, 4$ $(x > 0) \Rightarrow c > 0$

(2) $\sum_{x \in S} f(x) = 1$

$\frac{1}{c} + \frac{2}{c} + \frac{3}{c} + \frac{4}{c} = 1$

$c = 10$



b. (1) $f(x) > 0$

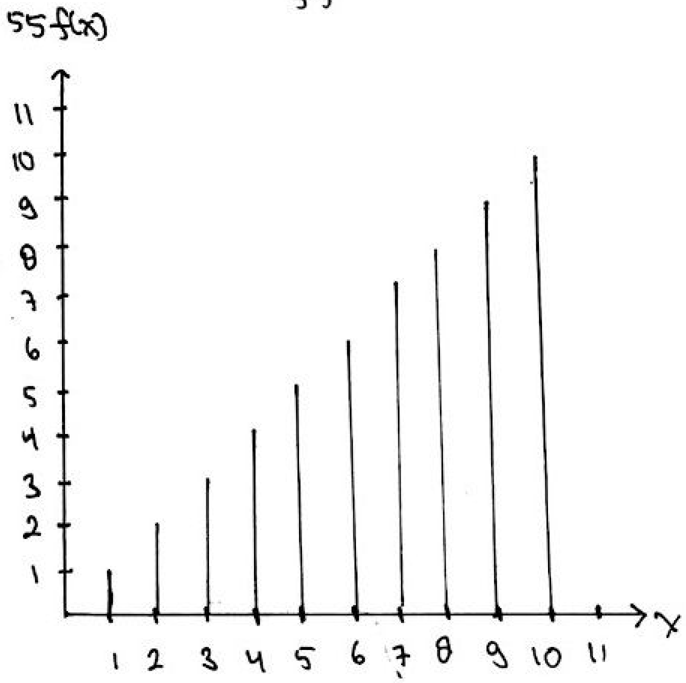
$xc > 0$

as $x = 1, 2, 3, \dots, 10$ $(x > 0) \Rightarrow c > 0$

(2) $\sum_{x \in S} f(x) = 1$

$c + 2c + 3c + \_ + 10c = 1$

$c = \frac{1}{55}$



c. (1) $f(x) > 0$

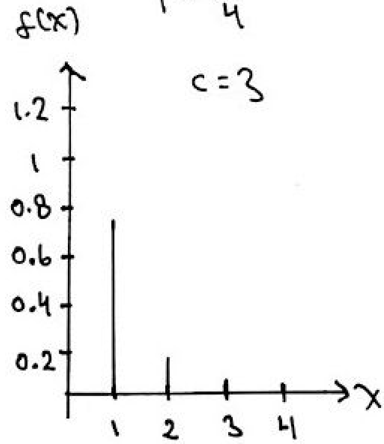
$c\left(\frac{1}{4}\right)^x > 0$

as $x = 1, 2, 3, \dots$ $\left(\left(\frac{1}{4}\right)^x > 0\right) \Rightarrow c > 0$

(2) $\sum_{x \in S} f(x) = 1$

$\frac{c\left(\frac{1}{4}\right)^1}{1 - \frac{1}{4}} = 1$

$c = 3$



d. (1) $f(x) > 0$

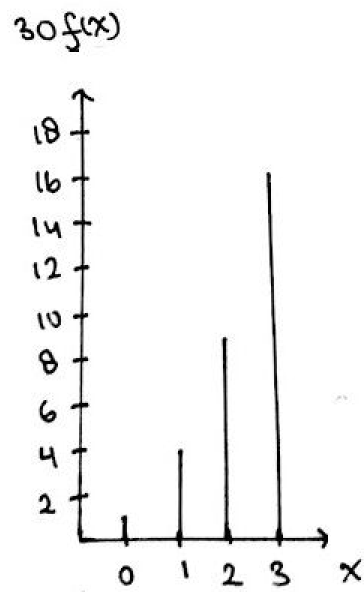
$c(x+1)^2 > 0$

$c > 0$

(2) $\sum_{x \in S} f(x) = 1$

$c + 4c + 9c + 16c = 1$

$c = \frac{1}{30}$



e. (1) $f(x) > 0$

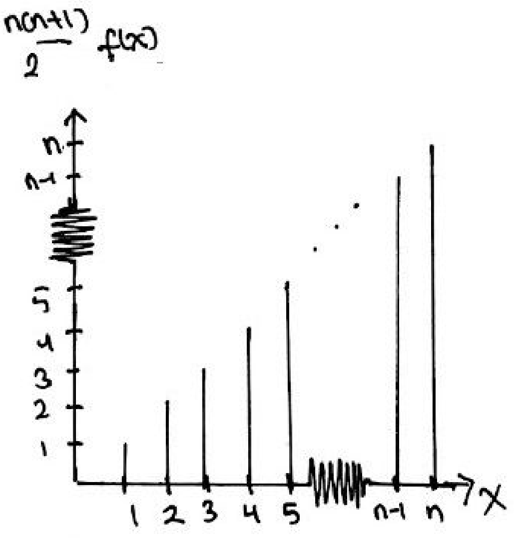
$\frac{x}{c} > 0$

as $x = 1, 2, \ldots, n$ $(x > 0) \Rightarrow c > 0$

(2) $\sum_{x \in S} f(x) = 1$

$\frac{1}{c} + \frac{2}{c} + \ldots + \frac{n}{c} = 1$

$c = \frac{n(n+1)}{2}$



f. (1) $f(x) \geq 0$

$\frac{c}{(x+1)(x+2)} > 0$

as $x = 0, 1, 2, \ldots$ $((x+1)(x+2) > 0) \Rightarrow c > 0$

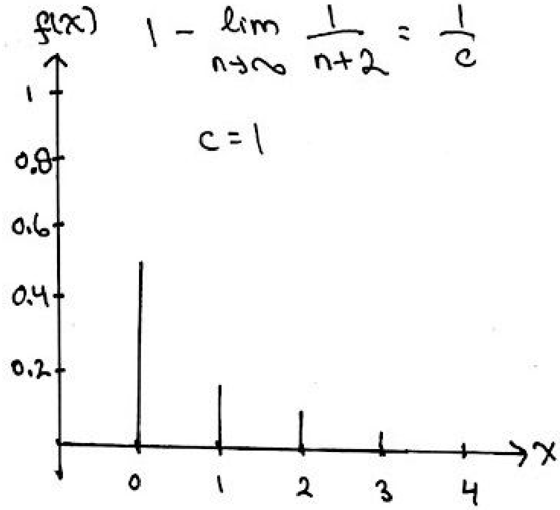
(2) $\sum_{x \in S} f(x) = 1$

$\sum_{x=0}^{\infty} \frac{c}{(x+1)(x+2)} = 1$

$\sum_{x=0}^{\infty} c\left(\frac{1}{x+1} - \frac{1}{x+2}\right) = 1$

$1 - \lim_{n \to \infty} \frac{1}{n+2} = \frac{1}{c}$

$c = 1$



2. $P(X \geq 4 \mid X \geq 1) = \frac{1 - \sum_{x=0}^{3} f(x)}{1 - \sum_{x=0}^{0} f(x)} = \frac{1 - \left(\frac{1}{0+1} - \frac{1}{3+2}\right)}{1 - \left(\frac{1}{0+1} - \frac{1}{0+2}\right)} = \frac{2}{5}$

3a. $\sum_{x=1}^{4} x f(x) = \sum_{x=1}^{4} \frac{x^2}{10} = \frac{1}{10} \cdot \frac{1}{6} \cdot 4 \cdot 5 \cdot 9 = 3$

b. $\sum_{x=1}^{10} x f(x) = \sum_{x=1}^{10} \frac{x^2}{55} = \frac{1}{55} \cdot \frac{1}{6} \cdot 10 \cdot 11 \cdot 21 = 7$

c. $\sum_{x=1}^{\infty} x f(x) = \sum_{x=1}^{\infty} 3x\left(\frac{1}{4}\right)^x = 3 \sum_{x=1}^{\infty} x\left(\frac{1}{4}\right)^x$

to compute $\sum_{x=1}^{\infty} x\left(\frac{1}{4}\right)^x$:

$$\frac{3}{4} \sum_{x=1}^{\infty} x\left(\frac{1}{4}\right)^x = \left(1 - \frac{1}{4}\right) \sum_{x=1}^{\infty} x\left(\frac{1}{4}\right)^x$$

$$= \left(\left(\frac{1}{4}\right) + 2\left(\frac{1}{4}\right)^2 + 3\left(\frac{1}{4}\right)^3 + \ldots\right)$$

$$- \left(\left(\frac{1}{4}\right)^2 + 2\left(\frac{1}{4}\right)^3 + 3\left(\frac{1}{4}\right)^4 + \ldots\right)$$

$$= \sum_{x=1}^{\infty} \left(\frac{1}{4}\right)^x$$

$$= \frac{\frac{1}{4}}{1-\frac{1}{4}}$$

$$= \frac{1}{3}$$

hence, $\sum_{x=1}^{\infty} x\left(\frac{1}{4}\right)^x = \frac{4}{3}\cdot\frac{1}{3} = \frac{4}{9}$

$\Rightarrow 3\sum_{x=1}^{\infty} x\left(\frac{1}{4}\right)^x = \frac{4}{3}$

d. $\sum_{x=0}^{3} xf(x) = \sum_{x=0}^{3} \frac{x(x+1)^2}{30} = \frac{4+18+48}{30}$

$$= \frac{7}{3}$$

e. $\sum_{x=0}^{n} xf(x) = \sum_{x=0}^{n} \frac{2}{n(n+1)}\cdot x^2 = \frac{2}{n(n+1)}\sum_{x=0}^{n} x^2$

$$= \frac{2}{n(n+1)}\cdot\frac{n(n+1)(2n+1)}{6}$$

$$= \frac{2n+1}{3}$$

f. $\sum_{x=0}^{\infty} xf(x) = \sum_{x=0}^{\infty}\left(\frac{x}{x+1}-\frac{x}{x+2}\right) = \sum_{x=2}^{\infty}\frac{1}{x}$

since $\sum \frac{1}{x}$ diverges, $E(x)$ doesn't exist

4. $\sum_{x=0}^{6} f(x) = 1$

$\Rightarrow \sum_{x=0}^{0} f(x) + \sum_{x=1}^{6} f(x) = 0.9 + \sum_{x=1}^{6}\frac{c}{x}$

$$= 0.9 + c\,\frac{49}{20}$$

$$= 1$$

$\Rightarrow c = \frac{2}{49}$

$E(x-1) = \sum_{x=0}^{6} \underbrace{\max(0, x-1)}\, f(x)$

"deductible of one unit"

$$= \frac{2}{49}\left(\frac{1}{2}+\frac{2}{3}+\frac{3}{4}+\frac{4}{5}+\frac{5}{6}\right) = \frac{71}{490}$$

5. $Z = u(X) = X^3$

a. $h(z) = f(x) = f(z^{1/3}) = \frac{4-z^{1/3}}{6}$ $(z = 1^3, 2^3, 3^3)$

b. $E(Z) = 1^3\cdot\frac{4-1}{6} + 2^3\cdot\frac{4-2}{6} + 3^3\cdot\frac{4-3}{6} = \frac{23}{3}$

c. $E(10) - E(Z) = 10 - \frac{23}{3} = \frac{7}{3}$

therefore, the average profit is $\frac{7}{3}$ dollars for each play

6. $E(X) = \sum_{x=1}^{\infty} xf(x) = \sum_{x=1}^{\infty}\frac{6}{\pi^2 x} = \frac{6}{\pi^2}\sum_{x=1}^{\infty}\frac{1}{x}$

note that $\sum_{x=1}^{\infty}\frac{1}{x} \geq \int_1^{\infty}\frac{1}{x}dx = \ln(x)\Big|_{x=1}^{\infty}$

$$= +\infty$$

$\Rightarrow \sum_{x=1}^{\infty}\frac{1}{x} = +\infty$ (diverge)

$\Rightarrow \sum_{x=1}^{\infty}\frac{6}{\pi^2 x} = +\infty$

7. $S = \{1,2,3,5,15,25,50\}$

$E(|x-c|) = \sum_{x\in S}|x-c|f(x) = \frac{1}{7}\sum_{x\in S}|x-c|$

case $c\in(-\infty,1]$:

$E(|x-c|) = \frac{1}{7}(101-7c) = \frac{101}{7} - c$

$\Rightarrow \min_{c\in(-\infty,1)} E(|x-c|) = \frac{94}{7}$ $(c=1)$

case $c\in(1,2]$:

$E(|x-c|) = \frac{1}{7}(99-5c) = \frac{99}{7} - \frac{5c}{7}$

$\Rightarrow \min_{c\in(1,2]} E(|x-c|) = \frac{89}{7}$ $(c=2)$

case $c\in(2,3]$:

$E(|x-c|) = \frac{1}{7}(95-3c) = \frac{95}{7} - \frac{3c}{7}$

$\Rightarrow \min_{c\in(2,3]} E(|x-c|) = \frac{86}{7}$ $(c=3)$

case $c\in(3,5]$:

$E(|x-c|) = \frac{1}{7}(89-c) = \frac{89}{7} - \frac{c}{7}$

$\Rightarrow \min_{c\in(3,5]} E(|x-c|) = 12$ $(c=5)$

case $c \in [5,15)$:

$$E(|x-c|) = \frac{1}{7}(79+c) = \frac{79}{7} + \frac{c}{7}$$

$$\Rightarrow \min_{c \in [5,15)} E(|x-c|) = 12 \quad (c=5)$$

case $c \in [15,25)$:

$$E(|x-c|) = \frac{1}{7}(49+3c) = 7 + \frac{3c}{7}$$

$$\Rightarrow \min_{c \in [15,25)} E(|x-c|) = \frac{94}{7} \quad (c=15)$$

case $c \in [25,50)$:

$$E(|x-c|) = \frac{1}{7}(5c-1) = \frac{5c}{7} - \frac{1}{7}$$

$$\Rightarrow \min_{c \in [25,50)} E(|x-c|) = \frac{124}{7} \quad (c=25)$$

case $c \in [50,\infty)$:

$$E(|x-c|) = \frac{1}{7}(7c-101) = c - \frac{101}{7}$$

$$\Rightarrow \min_{c \in [50,\infty)} E(|x-c|) = \frac{249}{7} \quad (c=50)$$

therefore,

$$\min_{c \in (-\infty,\infty)} E(|x-c|) = 12 \text{ with } c=5$$

$$E((x-b)^2) = E(x^2 - 2xb + b^2)$$

$$= E(x^2) - 2bE(x) + b^2$$

$$= \sum_{x \in S} \frac{x^2}{7} - 2b \sum_{x \in S} \frac{x}{7} + b^2$$

$$= \frac{3389}{7} - \frac{202}{7} b + b^2$$

$$\frac{d(E((x-b)^2))}{db} = -\frac{202}{7} + 2b = 0$$

$$\Rightarrow b = \frac{101}{7}$$

$\therefore$ the value of b is larger than c

8a. $\mu = E(x) = \sum_{x=0}^{3} x f(x)$

$$= \sum_{x=1}^{3} \frac{3!}{(x-1)!(3-x)!} \left(\frac{1}{4}\right)^x \left(\frac{3}{4}\right)^{3-x}$$

$$= \frac{27}{64} + \frac{9}{32} + \frac{3}{64}$$

$$= \frac{3}{4}$$

$$E(x(x-1)) = \sum_{x=0}^{3} x(x-1) f(x)$$

$$= \sum_{x=2}^{3} \frac{3!}{(x-2)!(3-x)!} \left(\frac{1}{4}\right)^x \left(\frac{3}{4}\right)^{3-x}$$

$$= \frac{9}{32} + \frac{3}{32}$$

$$= \frac{3}{8}$$

$$\sigma^2 = \left(\frac{3}{8}\right) + \left(\frac{3}{4}\right) - \left(\frac{3}{4}\right)^2 = \frac{9}{16}$$

b. $\mu = E(x) = \sum_{x=0}^{4} x f(x)$

$$= \sum_{x=1}^{4} \frac{4!}{(x-1)!(4-x)!} \left(\frac{1}{2}\right)^4$$

$$= \frac{1}{4} + \frac{3}{4} + \frac{3}{4} + \frac{1}{4}$$

$$= 2$$

$$E(x(x-1)) = \sum_{x=0}^{4} x(x-1) f(x)$$

$$= \sum_{x=2}^{4} \frac{4!}{(x-2)!(4-x)} \left(\frac{1}{2}\right)^4$$

$$= \frac{3}{4} + \frac{3}{2} + \frac{3}{4}$$

$$= 3$$

$$\sigma^2 = (3) + (2) - (2)^2 = 1$$

9. $E\left(\frac{(X-\mu)}{\sigma}\right) = \frac{1}{\sigma}\left(E(X) - E(\mu)\right)$

$= \frac{1}{\sigma}\left(E(X) - \mu\right)$

$= \frac{1}{\sigma}\left(E(X) - E(X)\right)$

$= 0$

$E\left(\frac{(X-\mu)^2}{\sigma^2}\right) = \frac{1}{\sigma^2}\left(E(X^2) - E(2\mu X) + E(\mu^2)\right)$

$= \frac{1}{\sigma^2}\left(E(X^2) - 2\mu E(X) + \mu^2\right)$

$= \frac{1}{\sigma^2}\left(E(X^2) - 2E(X)^2 + E(X)^2\right)$

$= \frac{1}{\sigma^2}\left(E(X^2) - E(X)^2\right)$

$= \frac{1}{\sigma^2} \cdot \sigma^2$

$= 1$

10. let $f(x|x \in S)$ denotes the probability of the value x chip being drawn randomly,

$$f(x) = \begin{cases} \frac{3}{8}, & x=1 \\ \frac{2}{8}, & x=2 \\ \frac{3}{8}, & x=3 \end{cases}$$

$\mu = E(X) = \sum_{x=1}^{3} x f(x)$

$= \frac{3}{8} + \frac{4}{8} + \frac{9}{8}$

$= 2$

$\sigma^2 = E(X^2) - E(X)^2 = \sum_{x=1}^{3} x^2 f(x) - \mu^2$

$= \left(\frac{3}{8} + 1 + \frac{27}{8}\right) - 2^2$

$= \frac{3}{4}$

jupyter Statistic Assignment 3 (Computer-based) - Yohandi

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Run Code

In [296]:

```
import pandas as pd
import matplotlib.pyplot as plt
import random

n=[0]
totalPayment=[0]
averagePayment=[0]

for i in range(10000):
    n.append(i+1)
    dice=random.randint(1,6)
    totalPayment.append(totalPayment[-1])
    if dice==1 or dice==2 or dice==3:
        totalPayment[-1]+=1
    elif dice==4 or dice==5:
        totalPayment[-1]+=2
    else:
        totalPayment[-1]+=3
    averagePayment.append(float(totalPayment[-1]/n[-1]))

data = {'N':n,'Average Payment':averagePayment}
df = pd.DataFrame(data,columns=['N','Average Payment'])
df.plot(x ='N', y='Average Payment', kind = 'scatter')
plt.show()
```