yohandi - homework for week 3

Exercises 10.10

1. $(1+x)^{1/2} = 1 + \sum_{k=1}^{\infty} \binom{1/2}{k} x^k$

$= 1 + \frac{1}{2}x - \frac{1}{8}x^2 + \frac{1}{16}x^3 + \ldots$

7. $(1+x^3)^{-1/2} = 1 + \sum_{k=1}^{\infty} \binom{-1/2}{k} x^{3k}$

$= 1 - \frac{1}{2}x^3 + \frac{3}{8}x^6 - \frac{5}{16}x^9 + \ldots$

16. $\int_0^{0.4} \frac{e^{-x}-1}{x} dx = \int_0^{0.4} \sum_{n=1}^{\infty} (-1)^n \frac{x^{n-1}}{n!} dx$

as $\frac{(0.4)^6}{6\ 6!} < 10^{-5}$, the integral value is accurate to the 5 decimal places

$= \int_0^{0.4} \sum_{n=1}^{5} (-1)^n \frac{x^{n-1}}{n!} dx$

$= \sum_{n=1}^{5} (-1)^n \cdot \frac{0.4^n}{n.n!}$

$= -0.3633060$

17. $\int_0^{0.5} \frac{1}{\sqrt{1+x^4}} dx = \int_0^{0.5} \sum_{n=1}^{\infty} \frac{x^{4(n-1)}}{(n-1)!} \binom{-\frac{1}{2}}{n-1} dx$

$= \int_0^{0.5} \sum_{n=0}^{\infty} \frac{x^{4n}}{n!} \binom{-\frac{1}{2}}{n} dx.$

as $\frac{5(0.5)^{13}}{16 \cdot 13} < 10^{-5}$, the integral value is accurate to the 5 decimal places,

$= \int_0^{0.5} \sum_{n=0}^{3} \frac{x^{4n}}{n!} \binom{-\frac{1}{2}}{n} dx$

$= 0.4969564$

24. $|E| = \left| \int_0^1 \cos\sqrt{t}\, dt - \sum_{n=0}^{3} \frac{t^n}{(2n)!} (-1)^n \right|$

$< \frac{t^5}{5.8!} \Big|_{t=0}^{1} = 4.960 . 10^{-6}$

25. $F(x) = \int_0^x \sin t^2\, dt$

$= \int_0^x \sum_{n=0}^{\infty} \frac{t^{2+4n}}{(1+2n)!} (-1)^n dt.$

as $\frac{1}{11.5!} < 10^{-3}$, the integral value is accurate to the 3 decimal places,

$= \int_0^{x} \sum_{n=0}^{1} \frac{t^{2+4n}}{(1+2n)!} (-1)^n dt$

$= \frac{x^3}{3} - \frac{x^7}{7 \cdot 3!}$

29. $\lim_{x \to 0} \frac{e^x - (x+1)}{x^2}$

$= \lim_{x \to 0} \frac{\left(\sum_{n=0}^{\infty} \frac{x^n}{n!}\right) - (x+1)}{x^2}$

$= \lim_{x \to 0} \frac{\sum_{n=2}^{\infty} \frac{x^n}{n!}}{x^2}$

$= \lim_{x \to 0} \left(\sum_{n=2}^{2} \frac{x^{n-2}}{n!} + \sum_{n=3}^{\infty} \frac{x^{n-2}}{n!} \right)$

$= \frac{1}{2}$

33. $\lim_{y \to 0} \frac{y - \tan^{-1} y}{y^3}$

$= \lim_{y \to 0} \frac{y - \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)} y^{2n+1}}{y^3}$

$= \lim_{y \to 0} \frac{1 - \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)} y^{2n}}{y^2}$

$= \lim_{y \to 0} \frac{\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{(2n+1)} \cdot y^{2n}}{y^2}$

$= \lim_{y \to 0} \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+3)} y^{2n} = \frac{1}{3}$

35. $$\lim_{x \to \infty} x^2 (e^{-1/x^2} - 1)$$

let $y = \frac{1}{x}$,

$$= \lim_{y \to 0} \frac{1}{y^2} (e^{-y^2} - 1)$$

$$= \lim_{y \to 0} \frac{1}{y^2} \left(\sum_{n=0}^{\infty} \frac{(-y^2)^n}{n!} - 1 \right)$$

$$= \lim_{y \to 0} \sum_{n=1}^{\infty} \frac{(-1)^n \cdot y^{2n-2}}{n!}$$

$$= -1$$

38. $$\lim_{x \to 2} \frac{x^2 - 4}{\ln(x-1)}$$

$$= \lim_{x \to 2} \frac{(x+2)(x-2)}{\ln(1+(x-2))}$$

$$= \lim_{x \to 2} \frac{(x+2)(x-2)}{\sum_{n=1}^{\infty} \frac{(-1)^{n+1} (x-2)^n}{n}}$$

$$= \lim_{x \to 2} \frac{(x+2)}{\sum_{n=1}^{\infty} \frac{(-1)^{n+1} (x-2)^{n-1}}{n}}$$

$$= 4$$

39. $$\lim_{x \to 0} \frac{\sin 3x^2}{1 - \cos 2x}$$

$$= \lim_{x \to 0} \frac{\sum_{n=0}^{\infty} \frac{(-1)^n (3x^2)^{2n+1}}{(2n+1)!}}{\sum_{n=1}^{\infty} \frac{(-1)^{n+1} (2x)^{2n}}{(2n)!}}$$

$$= \lim_{x \to 0} \frac{\sum_{n=0}^{\infty} \frac{(-1)^n 3 \cdot 9^n x^{4n}}{(2n+1)!}}{\sum_{n=1}^{\infty} \frac{(-1)^{n+1} \cdot 4^n x^{2n-2}}{(2n)!}}$$

$$= \frac{3}{2}$$

40. $$\lim_{x \to 0} \frac{\ln(1+x^3)}{x \cdot \sin x^2}$$

$$= \lim_{x \to 0} \frac{\sum_{n=1}^{\infty} \frac{(-1)^{n+1} (x^3)^n}{n}}{x \sum_{n=0}^{\infty} \frac{(-1)^n (x^2)^{2n+1}}{(2n+1)!}}$$

$$= \lim_{x \to 0} \frac{\sum_{n=1}^{\infty} \frac{(-1)^{n+1} x^{3n-3}}{n}}{\sum_{n=0}^{\infty} \frac{(-1)^n (x)^{4n}}{(2n+1)!}}$$

$$= 1$$

45. $$\sum_{n=0}^{\infty} \frac{\pi^{2n+1} (-1)^n}{3^{2n+1} (2n+1)!}$$

$$= \sin\left(\frac{\pi}{3}\right)$$

$$= \frac{1}{2}\sqrt{3}$$

50. $$\sum_{n=0}^{\infty} \frac{x^{2+n} (-1)^n \cdot 2^n}{n!}$$

$$= x^2 \cdot \sum_{n=0}^{\infty} \frac{(-2x)^n}{n!}$$

$$= x^2 \cdot e^{-2x}$$

55. $$\frac{\pi}{4} = \arctan(1)$$

$$= \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)}$$

$$|E| = \left| \frac{\pi}{4} - \sum_{n=0}^{k-1} \frac{(-1)^n}{2n+1} \right| < \left| \frac{(-1)^k}{2k+1} \right|$$

$$\frac{1}{2k+1} < 10^{-3}$$

$$\Rightarrow k \geq 500$$

$(500-1)-(0)+1 = 500$ terms needed to approximate the value of $\frac{\pi}{4}$ with $|E| < 10^{-3}$

69. $e^{i\theta} = \sum_{n=0}^{\infty} \frac{(i\theta)^n}{n!}$

$e^{-i\theta} = \sum_{n=0}^{\infty} \frac{(-i\theta)^n}{n!}$

$$\frac{e^{i\theta}+e^{-i\theta}}{2} = \frac{2\sum_{n=0}^{\infty}\frac{(i\theta)^{2n}}{(2n)!}}{2}$$

$$= \sum_{n=0}^{\infty} \frac{i^{2n}\theta^{2n}}{(2n)!}$$

$$= \sum_{n=0}^{\infty} \frac{(-1)^n \theta^{2n}}{(2n)!}$$

$$= \cos\theta$$

$$\frac{e^{i\theta}-e^{-i\theta}}{2i} = \frac{2\sum_{n=0}^{\infty}\frac{(i\theta)^{2n+1}}{(2n+1)!}}{2i}$$

$$= \sum_{n=0}^{\infty} \frac{(i)^{2n}(\theta)^{2n+1}}{(2n+1)!}$$

$$= \sum_{n=0}^{\infty} \frac{(-1)^n \theta^{2n+1}}{(2n+1)!}$$

$$= \sin\theta$$

72. $\frac{d(e^{(a+ib)x})}{dx} = \frac{d}{dx}(e^{ax}(\cos bx + i\sin bx))$

$$= ae^{ax}(\cos bx + i\sin bx) + e^{ax}(-b\sin bx + ib\cos bx)$$

$$= a\cdot e^{(a+ib)x} + be^{ax}(i\cos bx - \sin bx)$$

$$= ae^{(a+ib)x} + ibe^{ax}(\cos bx + i\sin bx)$$

$$= (a+ib)e^{(a+ib)x}$$

74. $\int e^{(a+ib)x}\,dx = \underbrace{\int e^{ax}\cos bx\,dx}_{\text{real}} + i\underbrace{\int e^{ax}\sin bx\,dx}_{\text{complex}}$.

$$\frac{(a-ib)}{a^2+b^2}e^{(a+ib)x} = \frac{(a-ib)}{a^2+b^2}e^{ax}(\cos bx + i\sin bx)$$

$$= \underbrace{\frac{e^{ax}}{a^2+b^2}(a\cos bx + b\sin bx)}_{\text{real}} +$$

$$\underbrace{i\frac{e^{ax}}{a^2+b^2}(a\sin bx - b\cos bx)}_{\text{complex}}.$$

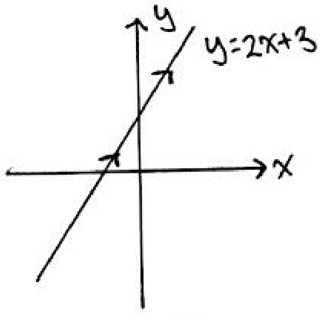
therefore,

$$\int e^{ax}\cos bx\,dx = \frac{e^{ax}}{a^2+b^2}(a\cos bx + b\sin bx) + C_1$$

$$\int e^{ax}\sin bx\,dx = \frac{e^{ax}}{a^2+b^2}(a\sin bx - b\cos bx) + C_2$$

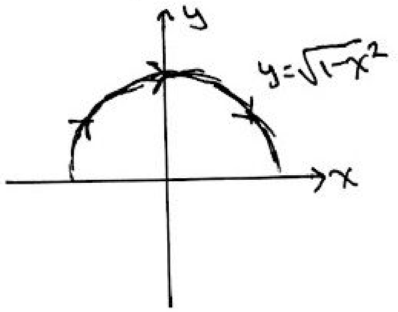
Exercises 11.1

3. $x=2t-5$
$y=4t-7$



$y=2(2t)-7$
$=2(x+5)-7$
$=2x+3$

6. $x=\cos(\pi-t)$
$y=\sin(\pi-t)$



$y=\sqrt{\sin^2(\pi-t)}$
$=\sqrt{1-\cos^2(\pi-t)}$
$=\sqrt{1-x^2}$

19. $\sin^2\theta+\cos^2\theta=1$
$a^2\sin^2\theta+a^2\cos^2\theta=a^2$
let $x=a\cos t$
$y=a\sin t$
both x and y satisfy the starting position of the particle as when $t=0 \Rightarrow (x(0),y(0))=(a,0)$

a. note that as y is increasing when $t\in(0,\frac{\pi}{2})$, the particle is moving counter clock wise. Hence,
$x=a\cos t$ and $y=-a\sin t$ $(0\le t\le 2\pi)$

b. $x=a\cos t$ and $y=a\sin t$ $(0\le t\le 2\pi)$

c. $x=a\cos t$ and $y=-a\sin t$ $(0\le t\le 4\pi)$

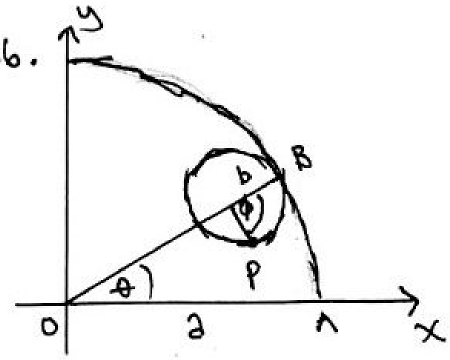
d. $x=a\cos t$ and $y=a\sin t$ $(0\le t\le 4\pi)$

27. $\sin^2\theta+\cos^2\theta=1$
$4\sin^2\theta+4\cos^2\theta=4$
as the particle is only moving in the top half's trace $\Rightarrow$ y' is negative when $t\in(\pi,2\pi)$ or $t\in(3\pi,4\pi)$
$\Rightarrow y=|y'|=|2\sin t| \Rightarrow x=2\cos t$
$x=2\cos t$
$y=|2\sin t|$

32. from the curve, we know that:
$\tan\theta=\frac{y}{x}$
$\Rightarrow \tan\theta=\frac{\sqrt{x}}{x}$
$=\frac{1}{\sqrt{x}}$
$\Rightarrow \sqrt{x}=\cot\theta=y$
$\Rightarrow x=\cot^2\theta$
$\Big\}\ 0<\theta\le\frac{\pi}{2}$

35. $x=AQ$
$=2\cot t$
$y=2-AB\sin t$
$=2-\frac{AQ^2}{OA}\sin t$
$=2-AQ\sin t\cdot\cos t$
$=2-2\cos^2 t$
$=2\sin^2 t$
$P(x,y)=P(2\cot t, 2\sin^2 t)$ $t\in[0,\pi]$

36.



by definition of sine and cosine, $P(x,y)$ is defined by:
$x=(a-b)\cos\theta+b\cos(\phi-\theta)$
$y=(a-b)\sin\theta-b\sin(\phi-\theta)$
the arc of smaller circle between P and B is the same as the arc of larger circle between A and B, by arc length:

$a\theta=b\phi$
$\Rightarrow \phi-\theta=\frac{a}{b}\theta-\theta=(\frac{a-b}{b})\theta$
$\Rightarrow x=(a-b)\cos\theta+b\cos((\frac{a-b}{b})\theta)$
$\Rightarrow y=(a-b)\sin\theta-b\sin((\frac{a-b}{b})\theta)$

given $b = \frac{a}{4}$

$\Rightarrow x = \frac{3}{4}a\cos\theta + \frac{1}{4}a\cos(3\theta)$

$\Rightarrow y = \frac{3}{4}a\sin\theta - \frac{1}{4}a\sin(3\theta)$

note that:

$\cos(3\theta) = 4\cos^3\theta - 3\cos\theta$

$\sin(3\theta) = 3\sin\theta - 4\sin^3\theta$

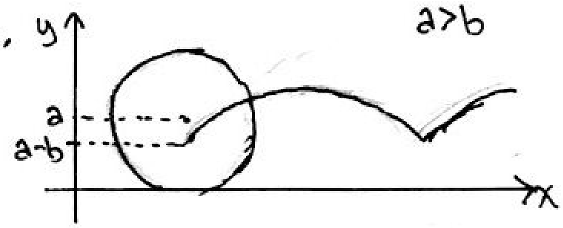
$\Rightarrow x = a\left(\frac{3}{4}\cos\theta + \cos^3\theta - \frac{3}{4}\cos\theta\right)$

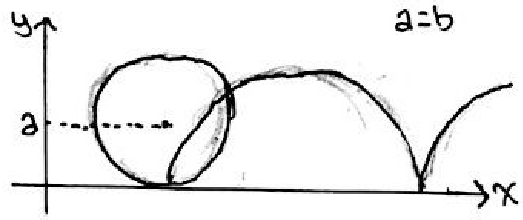
$= a\cos^3\theta$

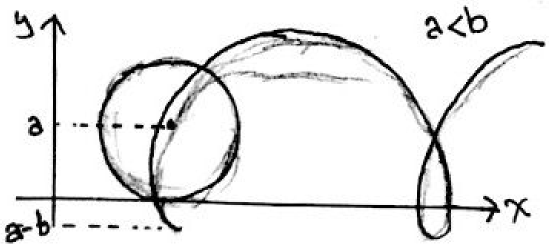
$\Rightarrow y = a\left(\frac{3}{4}\sin\theta - \frac{3}{4}\sin\theta + \sin^3\theta\right)$

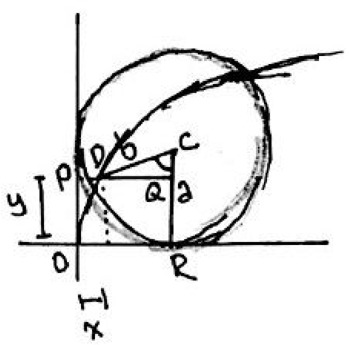
$= a\sin^3\theta$.

38.









$CD = b$

$CQ = b\cos\theta \quad DQ = b\sin\theta$

$OR = a\theta$

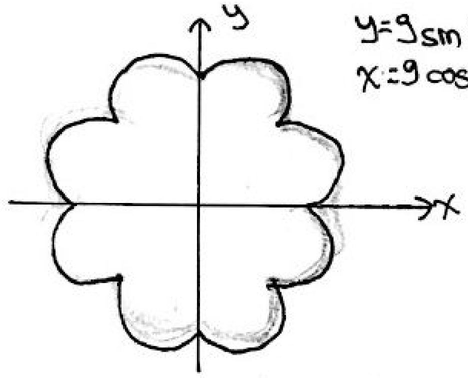
$\Rightarrow x = OR - DQ$

$= a\theta - b\sin\theta$

$\Rightarrow y = CR - QR$

$= a - b\cos\theta$

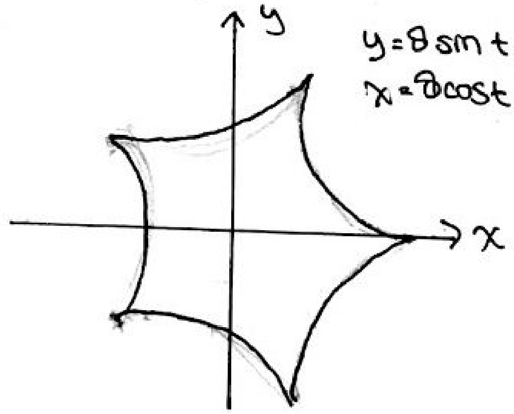
47a



$y = 9\sin t - \sin 9t$

$x = 9\cos t - \cos 9t$

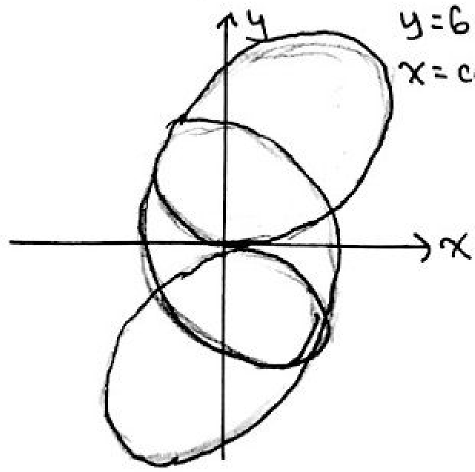
b.



$y = 8\sin t - 2\sin 4t$

$x = 8\cos t + 2\cos 4t$

c.



$y = 6\cos t - 5\sin 3t$

$x = \cos t + 5\cos 3t$

Exercises 11.2

3. $\frac{dy}{dt} = -2\sin t$

$\frac{dx}{dt} = 4\cos t$

$y' = \frac{dy/dt}{dx/dt} = -\frac{\tan t}{2}$

$\frac{dy'}{dt} = \frac{d}{dt}\left(-\frac{\sqrt{3}}{4}\tan t\right) = -\frac{\sec^2 t}{2}$

$\frac{d^2y}{dx^2} = \frac{dy'/dt}{dx/dt} = -\frac{\sec^3 t}{8}$

when $t = \frac{\pi}{4} \Rightarrow x = 2\sqrt{2},\ y = \sqrt{2}$

$\Rightarrow \frac{dy}{dx} = -\frac{1}{2}$

tangent line ℓ:

$y - \sqrt{2} = -\frac{1}{2}(x - 2\sqrt{2})$

$y = -\frac{1}{2}x + 2\sqrt{2}$

$\frac{d^2y}{dx^2}\left(\frac{\pi}{4}\right) = -\frac{1}{4}\sqrt{2}$

11. $\frac{dy}{dt} = \sin t$

$\frac{dx}{dt} = 1 - \cos t$

$y' = \frac{dy/dt}{dx/dt} = \frac{\sin t}{1-\cos t}$

$\frac{dy'}{dt} = \frac{d}{dt}\left(\frac{\sin t}{1-\cos t}\right) = \frac{d}{dt}\left(\frac{1+\cos t}{\sin t}\right)$

$= -\csc t(\cot t + \csc t)$

$\frac{d^2y}{dx^2} = \frac{dy'/dt}{dx/dt} = \frac{-(\cot t + \csc t)}{\sin t(1-\cos t)}$

when $t = \frac{\pi}{3} \Rightarrow x = \frac{\pi}{3} - \frac{1}{2}\sqrt{3},\ y = \frac{1}{2}$

$\Rightarrow \frac{dy}{dx} = \sqrt{3}$

tangent line ℓ:

$y - \frac{1}{2} = \sqrt{3}\left(x - \left(\frac{\pi}{3} - \frac{1}{2}\sqrt{3}\right)\right)$

$y = \sqrt{3}x - \frac{\pi\sqrt{3}}{3} + 2$

$\frac{d^2y}{dx^2}\left(\frac{\pi}{3}\right) = -4$

16. $y(t-1) = \sqrt{t}$

$\frac{dy}{dt}(t-1) + y = \frac{1}{2\sqrt{t}}$

$\Rightarrow \frac{dy}{dt} = \frac{\frac{1}{2\sqrt{t}} - y}{t-1}$

$x = \sqrt{5 - \sqrt{t}}$

$\Rightarrow \frac{dx}{dt} = -\frac{1}{4\sqrt{5t - t\sqrt{t}}}$

when $t = 4$,

$\frac{dy}{dx}(4) = \frac{\frac{dy}{dt}(4)}{\frac{dx}{dt}(4)}$

$= \frac{-\frac{5}{36}}{-\frac{1}{8\sqrt{3}}}$

$= \frac{10\sqrt{3}}{9}$

23. $\text{Area} = 2\int_{-a}^{a} y\,dx$

$= 2\int_{\pi}^{0} b\sin t\,(-a\sin t)\,dt$

$= -2ab\int_{\pi}^{0}\left(\frac{1-\cos 2t}{2}\right)dt$

$= ab\pi$

25. $\text{length} = \int_0^{\pi}\sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\,dt$

$= \int_0^{\pi}\sqrt{\sin^2 t + (1+\cos t)^2}\,dt$

$= 4\sin\left(\frac{t}{2}\right)\Big|_{t=0}^{\pi}$

$= 4$

29. $\text{length} = \int_0^{\pi/2}\sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\,dt$

$= \int_0^{\pi/2}\sqrt{(8t\cos t)^2 + (8t\sin t)^2}\,dt$

$= 4t^2\Big|_{t=0}^{\pi/2}$

$= \pi^2$

34. $ds = \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\,dt$

$= \sqrt{(\sec(t) - \cos(t))^2 + (-\sin(t))^2}\,dt$

$= \tan(t)\,dt$

surface area $= \int_0^{\pi/3} 2\pi\cos(t)\tan(t)\,dt$

$= -2\pi\cos(t)\Big|_{t=0}^{\pi/3}$

$= \pi$

42a assuming y as a function t where the function's slope is constant (we take slope=1),

$$\frac{dy}{dt} = 1$$

$$y(t) = t + c'$$

$$\Rightarrow \quad x = g(y)$$

$$x = g(t+c')$$

$$\Rightarrow \frac{dx}{dt} = g'(t+c')$$

proof:

$$L = \int_c^d \sqrt{1+\left(\frac{dx}{dy}\right)^2}\, dy$$

$$= \int_a^b \sqrt{1+\left(\frac{dx}{dt}\right)^2}\, dt$$

$$= \int_a^b \sqrt{\left(\frac{dy}{dt}\right)^2+\left(\frac{dx}{dt}\right)^2}\, dt.$$

b. $L = \int_0^{4/3} \sqrt{1+\frac{9}{4}y}\, dy$

$$= \frac{56}{27}$$

c. $L = \int_0^1 \sqrt{1+y^{-2/3}}\, dy$

$$= 2\sqrt{2}-1$$

48. Volume $= \int_0^{2\pi} \pi y^2\, dx$

$$= \int_0^{2\pi} \pi(1-\cos t)^2 (1-\cos t)\, dt$$

$$= 5\pi^2$$

Exercises 11.3

2a $x=r\cos\theta=-2\cdot\frac{1}{2}=-1$

$y=r\sin\theta=-2\cdot\frac{1}{2}\sqrt{3}=-\sqrt{3}$

b. $x=r\cos\theta=2\cdot\frac{1}{2}=1$

$y=r\sin\theta=2\cdot-\frac{1}{2}\sqrt{3}=-\sqrt{3}$

c. $x=r\cos\theta$

$y=r\sin\theta$

d. $x=r\cos(\theta+\pi)=-r\cos\theta$

$y=r\sin(\theta+\pi)=-r\sin\theta$

e. $x=-r\cos\theta$

$y=-r\sin\theta$

f. $x=r\cos\theta=2\cdot-\frac{1}{2}=-1$

$y=r\sin\theta=2\cdot-\frac{1}{2}\sqrt{3}=-\sqrt{3}$

g. $x=-r\cos(\theta+\pi)=r\cos\theta$

$y=-r\sin(\theta+\pi)=r\sin\theta$

h. $x=r\cos\theta=-2\cdot-\frac{1}{2}=1$

$y=r\sin\theta=-2\cdot\frac{1}{2}\sqrt{3}=-\sqrt{3}$

(a) & (f)

(b) & (h)

(c) & (g)

(d) & (e)

6. a. $(\sqrt{2}\cos(\frac{\pi}{4}),\sqrt{2}\sin(\frac{\pi}{4}))=(1,1)$

b. $(1\cos(0),\ 1\sin(0))=(1,0)$

c. $(0\cos(\frac{\pi}{2}),\ 0\sin(\frac{\pi}{2}))=(0,0)$

d. $(-\sqrt{2}\cos(\frac{\pi}{4}),-\sqrt{2}\sin(\frac{\pi}{4}))=(-1,-1)$

e. $(-3\cos(\frac{5\pi}{6}),-3\sin(\frac{5\pi}{6}))=(\frac{3\sqrt{3}}{2},-\frac{3}{2})$

f. $(5\cos(\tan^{-1}(\frac{4}{3})),5\sin(\tan^{-1}(\frac{4}{3})))=(3,4)$

g. $(-1\cos(7\pi),-1\sin(7\pi))=(1,0)$

h. $(2\sqrt{3}\cos(\frac{2\pi}{3}),2\sqrt{3}\sin(\frac{2\pi}{3}))=(-\sqrt{3},3)$

10a. $x=r\cos\theta=-2$

$y=r\sin\theta=0$

$\tan\theta=\frac{y}{x}=0 \Rightarrow \theta=\{-\pi,0,\pi\}$

$r=\frac{-2}{\cos\theta} \Rightarrow r=-2$

$\Rightarrow (-2,0)$

b. $x=r\cos\theta=1$

$y=r\sin\theta=0$

$\tan\theta=\frac{y}{x}=0 \Rightarrow \theta=\{-\pi,0,\pi\}$

$r=\frac{1}{\cos\theta} \Rightarrow r=-1$

$\Rightarrow (-1,-\pi)$ and $(-1,\pi)$

c. $x=r\cos\theta=0$

$y=r\sin\theta=-3$

$\cot\theta=\frac{x}{y}=0 \Rightarrow \theta=\{-\frac{\pi}{2},\frac{\pi}{2},\frac{3\pi}{2}\}$

$r=\frac{-3}{\sin\theta}=\{-3\}$

$\Rightarrow (-3,\frac{\pi}{2})$

d. $x=r\cos\theta=\frac{1}{2}\sqrt{3}$

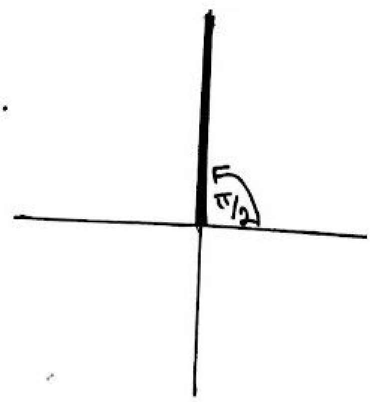
$y=r\sin\theta=\frac{1}{2}$

$\tan\theta=\frac{1}{3}\sqrt{3} \Rightarrow \theta=\{-\frac{5\pi}{6},\frac{\pi}{6},\frac{7\pi}{6}\}$

$r=\frac{\frac{1}{2}}{\sin\theta} \Rightarrow r=-1$

$\Rightarrow (-1,-\frac{5\pi}{6})$ and $(-1,\frac{7\pi}{6})$

19.



37. $r = \frac{5}{\sin\theta - 2\cos\theta}$

$r\sin\theta - 2r\cos\theta = 5$

∘> $x = r\cos\theta$
∘> $y = r\sin\theta$

$y - 2x = 5$

$y = 2x + 5$

=> a line graph with slope = 2
and y-intersect = 5

42. $r\sin\theta = \ln r + \ln \cos\theta$

$r\sin\theta = \ln(r\cos\theta)$

∘> $x = r\cos\theta$
∘> $y = r\sin\theta$

$y = \ln(x)$

=> a natural logarithmic function

61. $y^2 = 4x$

∘> $r\cos\theta = x$
∘> $r\sin\theta = y$

$r^2\sin^2\theta = 4r\cos\theta$

$r\sin^2\theta - 4\cos\theta = 0$

64. $(x-5)^2 + y^2 = 25$

∘> $r\cos\theta = x$
∘> $r\sin\theta = y$

$(r\cos\theta - 5)^2 + (r\sin\theta)^2 = 25$

$r^2\cos^2\theta - 10r\cos\theta + 25 + r^2\sin^2\theta = 25$

$r^2 - 10r\cos\theta = 0$

$r - 10\cos\theta = 0$

Exercises 11.4

5. $r_0 = 2+\sin\theta_0$

for $\theta_0 = -\theta$:

$r = 2+\sin(-\theta)$

$= 2-\sin\theta$

(is not symmetric to the x-axis)

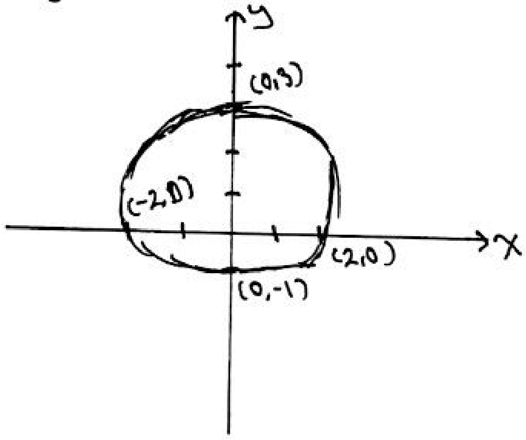
for $\theta_0 = \pi-\theta$:

$r = 2+\sin(\pi-\theta)$

$= 2+\sin\theta$

(is symmetric to the y-axis)

$\therefore$ the curve is symmetric to the y-axis



12. $r_0^2 = -\cos\theta$

for $\theta_0 = -\theta$:

$r^2 = -\cos(-\theta)$

$= -\cos\theta$

(is symmetric to the x-axis)

for $r_0 = -r$ & $\theta_0 = -\theta$:

$(-r)^2 = -\cos(-\theta)$

$r^2 = -\cos\theta$

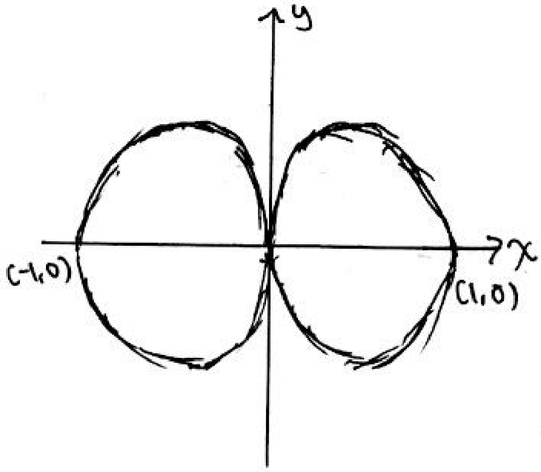
(is symmetric to the y-axis)

for $r_0 = -r$:

$(-r)^2 = -\cos(\theta)$

$r^2 = -\cos\theta$

(is symmetric to the origin)



16. $r_0^2 = -\cos(2\theta_0)$

for $\theta_0 = -\theta$:

$r_0^2 = -\cos(-2\theta)$

$= -\cos(2\theta)$

(is symmetric to the x-axis

for $r_0 = -r$ & $\theta_0 = -\theta$:

$(-r)^2 = -\cos(-2\theta)$

$r^2 = -\cos(2\theta)$

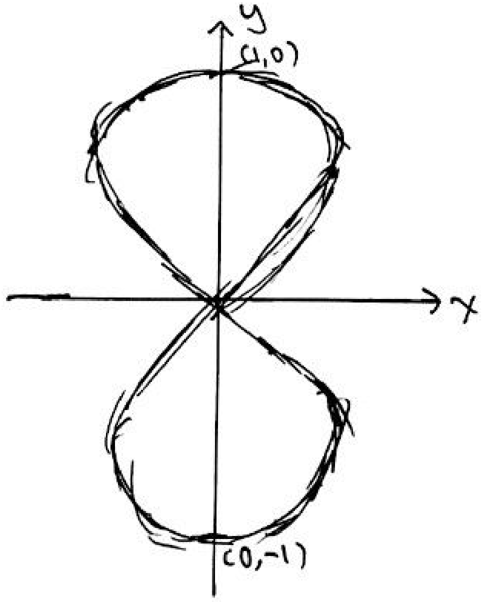
(is symmetric to the y-axis)

for $r_0 = -r$:

$(-r)^2 = -\cos(2\theta)$

$r^2 = -\cos(2\theta)$

(is symmetric to the origin)



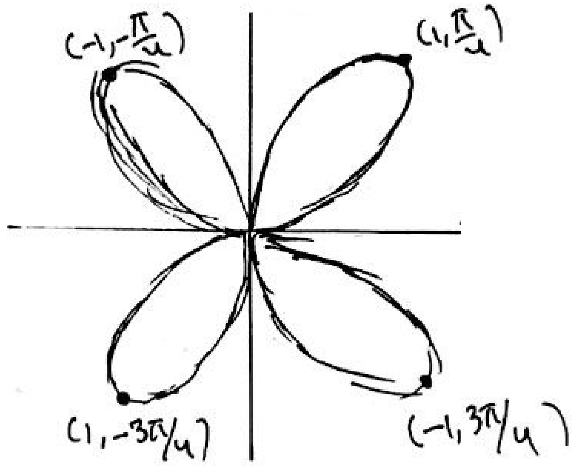
19. $\left.\frac{dy}{dx}\right|_{(r,\theta)} = \frac{2\cos(2\theta)\sin(\theta) + \sin(2\theta)\cos\theta}{2\cos(2\theta)\cos(\theta) - \sin(2\theta)\sin\theta}$

when $\theta = \frac{\pi}{4} \Rightarrow r = 1 \Rightarrow \frac{dy}{dx} = -1$

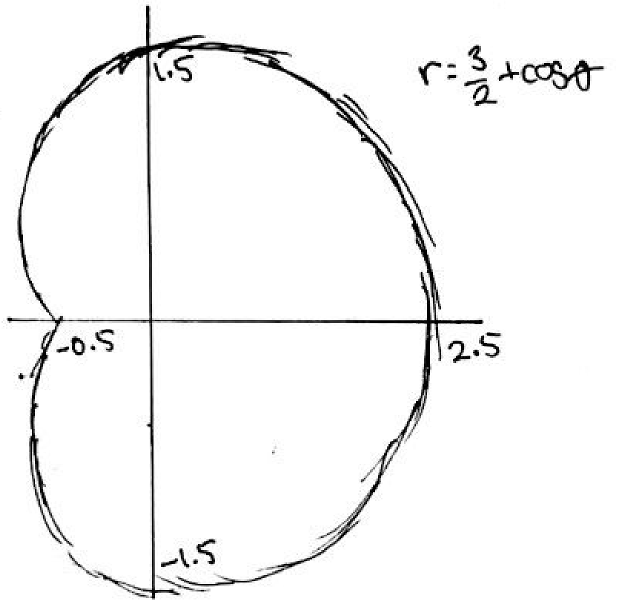
when $\theta = -\frac{\pi}{4} \Rightarrow r = -1 \Rightarrow \frac{dy}{dx} = 1$

when $\theta = \frac{3\pi}{4} \Rightarrow r = -1 \Rightarrow \frac{dy}{dx} = 1$

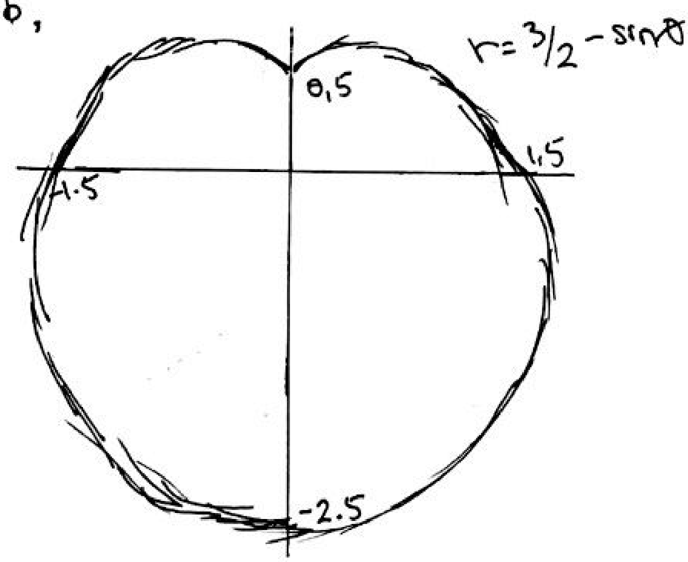
when $\theta = -\frac{3\pi}{4} \Rightarrow r = 1 \Rightarrow \frac{dy}{dx} = -1$



23a.



b,



Exercises 11.5

2. $\text{Area} = \int_{\pi/4}^{\pi/2} \frac{1}{2}(2\sin\theta)^2\, d\theta$

$= \int_{\pi/4}^{\pi/2} 1 - \cos 2\theta \, d\theta$

$= \left[\theta - \frac{1}{2}\sin 2\theta\right]\Big|_{\theta=\pi/4}^{\pi/2}$

$= \frac{\pi}{4} + \frac{1}{2}$

8. $\text{Area} = 6 \cdot \int_0^{\pi/3} \frac{1}{2}(2\sin 3\theta)\, d\theta$

$= \left[-2\cos 3\theta\right]\Big|_{\theta=0}^{\pi/3}$

$= 4$

18. $\left.\begin{array}{l} r_1 = 3\csc\theta \\ r_2 = 4\sin\theta. \end{array}\right\}$ $3\csc\theta = 4\sin\theta.$

$\frac{3}{4} = \sin^2\theta$

$\pm\frac{1}{2}\sqrt{3} = \sin\theta \quad (\theta \in (0,\pi))$

$\theta = \frac{\pi}{3}$ or $\theta = \frac{2\pi}{3}$

$\text{Area} = \int_0^{\pi/3} \frac{1}{2}(4\sin\theta)^2\, d\theta + \int_{\pi/3}^{2\pi/3} \frac{1}{2}(3\csc\theta)^2\, d\theta + \int_{2\pi/3}^{\pi} \frac{1}{2}(4\sin\theta)^2\, d\theta$

$= \left(\frac{4\pi}{3} - \sqrt{3}\right) + (3\sqrt{3}) + \left(\frac{4\pi}{3} - \sqrt{3}\right)$

$= \frac{8\pi}{3} + \sqrt{3}$

22. $L = \int_0^{\pi} \sqrt{\frac{e^{2\theta}}{2} + \frac{e^{2\theta}}{2}}\, d\theta$

$= e^{\theta}\Big|_{\theta=0}^{\pi}$

$= e^{\pi} - 1$

29. $L = \int_\alpha^\beta \sqrt{\left(\frac{d(f(\theta)\cos\theta)}{d\theta}\right)^2 + \left(\frac{d(f(\theta)\sin\theta)}{d\theta}\right)^2}\, d\theta$

$= \int_\alpha^\beta \sqrt{(f'(\theta)\cos\theta + f(\theta)(-\sin\theta))^2 + (f'(\theta)\sin\theta + f(\theta)\cos\theta)^2}\, d\theta$

$= \int_\alpha^\beta \sqrt{f'(\theta)^2[\cos^2\theta + \sin^2\theta] + f(\theta)^2[\sin^2\theta + \cos^2\theta]}\, d\theta$

$= \int_\alpha^\beta \sqrt{f'(\theta)^2 + f(\theta)^2}\, d\theta$

$= \int_\alpha^\beta \sqrt{\left(\frac{dr}{d\theta}\right)^2 + r^2}\, d\theta$

32. when $r_1 = f(\theta)$,

$$L_1 = \int_{\alpha}^{\beta} \sqrt{r_1^2 + \left(\frac{dr_1}{d\theta}\right)^2}\, d\theta$$

$$= \int_{\alpha}^{\beta} \sqrt{f(\theta)^2 + f'(\theta)^2}\, d\theta.$$

when $r_2 = 2f(\theta)$

$$L_2 = \int_{\alpha}^{\beta} \sqrt{r_2^2 + \left(\frac{dr_2}{d\theta}\right)^2}\, d\theta$$

$$= \int_{\alpha}^{\beta} \sqrt{4f(\theta)^2 + 4f'(\theta)^2}\, d\theta.$$

$$= 2\int_{\alpha}^{\beta} \sqrt{f(\theta)^2 + f'(\theta)^2}\, d\theta$$

$$= 2L_1$$

since $L_2 = 2L_1$, the length of r_2 is twice of the length of r_1