STA2001 Home Assignment 10

1. 5.4-22. Let X_1 and X_2 be two independent random variables. Let X_1 and $Y = X_1 + X_2$ be $\chi^2(r_1)$ and $\chi^2(r)$, respectively, where $r_1 < r$.
   (a) Find the mgf of X_2.
   (b) What is its distribution?

2. 5.4-23. Let X be N(0, 1). Use the mgf technique to show that $Y = X^2$ is $\chi^2(1)$. Hint: Evaluate the integral representing $E(e^{tX^2})$ by writing $w = x\sqrt{1-2t}$.

3. 5.5-2. Let X be N(50, 36). Using the same set of axes, sketch the graphs of the probability density functions of
   (a) X.
   (b) $\bar{X}$, the mean of a random sample of size 9 from this distribution.
   (c) $\bar{X}$, the mean of a random sample of size 36 from this distribution.

4. 5.5-4. Let X equal the weight of the soap in a 6-pound box. Assume that the distribution of X is N(6.05, 0.0004).
   (a) Find P(X<6.0171).
   (b) If nine boxes of soap are selected at random from the production line, find the probability that at most two boxes weigh less than 6.0171 pounds each. Hint: Let Y equal the number of boxes that weigh less than 6.0171 pounds.
   (c) Let $\bar{X}$ be the sample mean of the nine boxes. Find $P(\bar{X} \leq 6.035)$.

5. 5.5-13. Let Z_1,Z_2,and Z_3 have independent standard normal distributions, N(0,1).
(a) Find the distribution of

$$W = \frac{Z_1}{\sqrt{((Z_2)^2 + (Z_3)^2)/2}}$$

(b)Show that

$$V = \frac{Z_1}{\sqrt{((Z_1)^2 + (Z_2)^2)/2}}$$

has pdf

$$f(v) = \frac{1}{(\pi\sqrt{2 - v^2})}, -\sqrt{2}<v<\sqrt{2}$$

(c)Find the mean of V.
(d)Find the standard deviation of V.
(e)Why are the distribution of W and V so different?

6. 5.5-14. Let T have a t distribution with r degrees of freedom. Show that E(T) = 0 provided that $r \geq 2$, and Var(T) = r/(r-2) provided that $r \geq 3$, by first finding E(Z), $E(1/\sqrt{U})$, $E(Z^2)$, and E(1/U).

7. 5.5-16. Let n = 9 in the T statistic defined in Equation 5.5-2.
   (a) Find $t_{0.025}$ so that $P(-t_{0.025} \leq T \leq t_{0.025}) = 0.95$.
   (b) Solve the inequality $[-t_{0.025} \leq T \leq t_{0.025}]$ so that μ is in the middle.

8. 5.6-5. Let $X_1, X_2, ..., X_{18}$ be a random sample of size 18 from a chi-square distribution with r = 1. Recall that $\mu = 1$ and $\sigma^2 = 2$.
(a) How is $Y = \sum_{i=1}^{18} X_i$ distributed?
(b) Using the result of part (a), we see from Table IV in Appendix B that

$$P(Y \leq 9.390) = 0.05$$

and

$$P(Y \leq 34.80) = 0.99.$$

Compare these two probabilities with the approximations found with the use of the central limit theorem

9. 5.6-8. Let X equal the weight in grams of a miniature candy bar. Assume that $\mu = \mathrm{E(X)} = 24.43$ and $\sigma^2 = \mathrm{Var(X)} = 2.20$. Let $\bar{X}$ be the sample mean of a random sample of n = 30 candy bars. Find
(a) $E(\bar{X})$.
(b) $Var(\bar{X})$.
(c) $P(24.17 \leq \bar{X} \leq 24.82)$,approximately.

10. 5.6-14. Suppose that the sick leave taken by the typical worker per year has $\mu = 10$, $\sigma = 2$, measured in days. A firm has n = 20 employees. Assuming independence, how many sick days should the firm budget if the financial officer wants the probability of exceeding the number of days budgeted to be less than 20%?