MAT3007 - Assignment 3

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Problem 1.

Canonical form:

$$
\begin{aligned}
\texttt{min} \quad & -x_1 - 2x_2 - 3x_3 - 8x_4 \\
\texttt{s.t.} \quad & x_1 - x_2 + x_3 + s_1 = 2 \\
& x_3 - x_4 + s_2 = 1 \\
& 2x_2 + 3x_3 + 4x_4 + s_3 = 8 \\
& x_1, x_2, x_3, x_4 \geq 0
\end{aligned}
$$

Initial tableau:

B	x_1	x_2	x_3	x_4	s_1	s_2	s_3	RHS
	-1	-2	-3	-8	0	0	0	
s_1	1	-1	1	0	1	0	0	2
s_2	0	0	1	-1	0	1	0	1
s_3	0	2	3	4	0	0	1	8

We found that the maximum in the objective row is 8, so the entering variable is x_4. The test gives for s_3 with minimum ratio 2 (for s_2, it is -1 but we are going to ignore this as it is negative). So, the leaving variable is s_3.

Pivot on the element in the x_4 column and s_3 row.

B	x_1	x_2	x_3	x_4	s_1	s_2	s_3	RHS
	-1	2	3	0	0	0	2	
s_1	1	-1	1	0	1	0	0	2
s_2	0	$\frac{1}{2}$	$\frac{7}{4}$	0	0	1	0	3
x_4	0	$\frac{1}{2}$	$\frac{3}{4}$	1	0	0	$\frac{1}{4}$	2

We found that the maximum in the objective row is 1, so the entering variable is x_1. The test gives for s_1 with minimum ratio 2. So, the leaving variable is s_1.

Pivot on the element in the x_1 column and s_1 row.

B	x_1	x_2	x_3	x_4	s_1	s_2	s_3	RHS
	0	1	4	0	1	0	2	
x_1	1	-1	1	0	1	0	0	2
s_2	0	$\frac{1}{2}$	$\frac{7}{4}$	0	0	1	0	3
x_4	0	$\frac{1}{2}$	$\frac{3}{4}$	1	0	0	$\frac{1}{4}$	2

Now, all coefficients are non-negative; hence, we are done.

Problem 2.

$$
\begin{aligned}
\texttt{min} \quad & x_1 - x_2 + 2x_3 \\
\texttt{s.t.} \quad & 2x_1 - x_2 + 2x_3 \leq -1 \\
& x_1 - x_2 - x_3 \leq 4 \\
& x_2 - x_4 = 0 \\
& x_1, x_2, x_3, x_4 \geq 0
\end{aligned}
$$

As the first step for the Phase 1, we construct the auxiliary problem such that $b \geq 0$.

$$
\begin{aligned}
\texttt{min} \quad & y_1 + y_2 \\
\texttt{s.t.} \quad & -2x_1 + x_2 - 2x_3 - s_1 + y_1 = 1 \\
& x_1 - x_2 - x_3 + s_2 = 4 \\
& x_2 - x_4 + y_2 = 0 \\
& x_1, x_2, x_3, x_4, s_1, s_2, y_1, y_2 \geq 0
\end{aligned}
$$

Solve the auxiliary problem using the simplex method.

B	x_1	x_2	x_3	x_4	s_1	s_2	y_1	y_2	RHS
	-2	2	-2	-1	-1	0	0	0	
y_1	-2	1	-2	0	-1	0	1	0	1
s_2	1	-1	-1	0	0	1	0	0	4
y_2	0	1	0	-1	0	0	0	1	0

We found that the maximum in the objective row is 2, so the entering variable is x_2. The test gives for y_2 with minimum ratio 0. So, the leaving variable is y_2.

Pivot on the element in the x_2 column and y_2 row.

B	x_1	x_2	x_3	x_4	s_1	s_2	y_1	RHS
	-2	0	-2	1	-1	0	0	
y_1	-2	0	-2	1	-1	0	1	1
s_2	1	0	-1	-1	0	1	0	4
x_2	0	1	0	-1	0	0	0	0

We found that the maximum in the objective row is 1, so the entering variable is x_4. The test gives for y_1 with minimum ratio 1. So, the leaving variable is y_1.

Pivot on the element in the x_4 column and y_1 row.

B	x_1	x_2	x_3	x_4	s_1	s_2	RHS
	0	0	0	0	0	0	
x_4	-2	0	-2	1	-1	0	1
s_2	-1	0	-3	0	-1	1	5
x_2	-2	1	-2	0	-1	0	1

Then, we obtain the new tableu accordingly and BFS of the original problem as $x = (0, 1, 0, 1)$.

Phase 2 follows.

B	x_1	x_2	x_3	x_4	s_1	s_2	RHS
	1	0	0	0	1	0	
x_4	-2	0	-2	1	-1	0	1
s_2	-1	0	-3	0	-1	1	5
x_2	-2	1	-2	0	-1	0	1

Since all coefficients are non-negative; hence, we are done. The optimal solution is obtained with $x = (0, 1, 0, 1)$ and the objective value is -1.

Problem 3.

$$
\begin{aligned}
\texttt{min} \quad & x_1 + 3x_2 + x_4 - 2x_5 \\
\texttt{s.t.} \quad & x_1 + 2x_2 + 4x_4 + x_5 = 2 \\
& x_1 + 2x_2 - 2x_4 + x_5 = 2 \\
& -x_1 - 4x_2 + 3x_3 = 1 \\
& x_1, x_2, x_3, x_4, x_5 \geq 0
\end{aligned}
$$

As the first step for the Phase 1, we construct the auxiliary problem such that $b \geq 0$.

$$
\begin{aligned}
\texttt{min} \quad & y_1 + y_2 + y_3 \\
\texttt{s.t.} \quad & x_1 + 2x_2 + 4x_4 + x_5 + y_1 = 2 \\
& x_1 + 2x_2 - 2x_4 + x_5 + y_2 = 2 \\
& -x_1 - 4x_2 + 3x_3 + y_3 = 1 \\
& x_1, x_2, x_3, x_4, x_5, y_1, y_2, y_3 \geq 0
\end{aligned}
$$

Solve the auxiliary problem using the simplex method.

B	x_1	x_2	x_3	x_4	x_5	y_1	y_2	y_3	RHS
	1	0	3	2	2	0	0	0	
y_1	1	2	0	4	1	1	0	0	2
y_2	1	2	0	-2	1	0	1	0	2
y_3	-1	-4	3	0	0	0	0	1	1

We found that the maximum in the objective row is 3, so the entering variable is x_3. The test gives for y_3 with minimum ratio $\frac{1}{3}$. So, the leaving variable is y_3.

Pivot on the element in the x_3 column and y_3 row.

B	x_1	x_2	x_3	x_4	x_5	y_1	y_2	RHS
	2	4	0	2	2	0	0	
y_1	1	2	0	4	1	1	0	2
y_2	1	2	0	-2	1	0	1	2
x_3	$-\frac{1}{3}$	$-\frac{4}{3}$	1	0	0	0	0	$\frac{1}{3}$

We found that the maximum in the objective row is 4, so the entering variable is x_2. The test gives for y_1 with minimum ratio $\frac{1}{2}$. So, the leaving variable is y_1.

Pivot on the element in the x_2 column and y_1 row.

B	x_1	x_2	x_3	x_4	x_5	y_2	RHS
	0	0	0	-6	0	0	
x_2	$\frac{1}{2}$	1	0	2	$\frac{1}{2}$	0	1
y_2	0	0	0	-6	0	1	0
x_3	$\frac{1}{3}$	0	1	$\frac{8}{3}$	$\frac{2}{3}$	0	$\frac{5}{3}$

Then, we obtain the new tableu accordingly and BFS of the original problem as $x = (0, 1, \frac{5}{3}, 0, 0)$.

Phase 2 follows.

B	x_1	x_2	x_3	x_4	x_5	RHS
	$\frac{1}{2}$	0	0	5	$\frac{7}{2}$	
x_2	$\frac{1}{2}$	1	0	2	$\frac{1}{2}$	1
x_3	$\frac{1}{3}$	0	1	$\frac{8}{3}$	$\frac{2}{3}$	$\frac{5}{3}$

We found that the maximum in the objective row is 5, so the entering variable is x_4. The test gives for x_2 with minimum ratio $\frac{1}{2}$. So, the leaving variable is x_2.

Pivot on the element in the x_4 column and x_2 row.

B	x_1	x_2	x_3	x_4	x_5	RHS
	$-\frac{3}{4}$	$-\frac{5}{2}$	0	0	$\frac{9}{4}$	
x_4	$\frac{1}{4}$	$\frac{1}{2}$	0	1	$\frac{1}{4}$	$\frac{1}{2}$
x_3	$-\frac{1}{3}$	$-\frac{4}{3}$	1	0	0	$\frac{1}{3}$

We found that the maximum in the objective row is $\frac{9}{4}$, so the entering variable is x_5. The test gives for x_4 with minimum ratio 2. So, the leaving variable is x_4.

Pivot on the element in the x_5 column and x_4 row.

B	x_1	x_2	x_3	x_4	x_5	RHS
	-3	-7	0	-9	0	
x_5	1	2	0	4	1	2
x_3	$-\frac{1}{3}$	$-\frac{4}{3}$	1	0	0	$\frac{1}{3}$

Since all coefficients are non-negative; hence, we are done. The optimal solution is obtained with $x = (0, 0, \frac{1}{3}, 0, 2)$ and the objective value is -4.

Problem 4.

1. $\beta \geq 0$. For the tableau to be an acceptable initial tableau, the values in the rightmost column (under 0) for the basic variables should be non-negative.

2. Achieved when $\alpha \geq 0$ and $\beta < 0$. This is infeasible as all coefficients and variables are positive; consequently, β cannot be be negative.

3. One of the $\delta < 0$, $\gamma < 0$, or $\xi < 0$ must be true. $\beta \geq 0$ to make the tableau acceptable.

4. ($\alpha < 0$ and $\delta < 0$) or ($\alpha = 0$ and $\delta < 0$), which is equivalent to $\alpha \leq 0$ and $\delta < 0$ (to make all elements in the 4-th column becomes less than or equal to 0). $\beta \geq 0$ to make the tableau acceptable.

5. $\frac{3}{2} < \frac{2}{\eta}$ (to make x_3 become the minimum ratio) and $\eta > 0$ (to make the ratio positive). $\beta \geq 0$ to make the tableau acceptable. And lastly, $\gamma < 0$ (will be $\gamma < \min(0, \delta, \xi)$ if x_6 is a variable that for sure enter B, not only a candidate).