STA2001 Assignment 3: Random Variables of The Discrete Type, Mathematical Expectation and Special Mathematical Expectations

1. (2.1-3) For each of the following determine the constant c so that f(x) satisfies the conditions of being a pmf for a random variable X, and then depict each pmf as a line graph:

   (a) $f(x) = x/c$, $\quad x = 1, 2, 3, 4.$

   (b) $f(x) = cx$, $\quad x = 1, 2, 3, ..., 10.$

   (c) $f(x) = c(1/4)^x$, $\quad x = 1, 2, 3, ...$

   (d) $f(x) = c(x+1)^2$, $\quad x = 0, 1, 2, 3.$

   (e) $f(x) = x/c$, $\quad x = 1, 2, 3, ..., n.$

   (f) $f(x) = \frac{c}{(x+1)(x+2)}$, $\quad x = 0, 1, 2, 3, ...$

   Hint:In part(f). write $f(x) = 1/(x+1) - 1/(x+2)$

2. (2.1-12) let X be the number of accidents per week in a factory. Let the pmf of X be

$$f(x) = \frac{1}{(x+1)(x+2)} = \frac{1}{x+1} - \frac{1}{x+2}, \qquad x = 0, 1, 2, ...$$

   Find the conditional probability of $X \geqslant 4$, given that $X \geqslant 1$.

3. (2.2-1) Find E(X) for each of the distributions given in Exercise 2.1-3.

4. (2.2-4) An insurance company sells an automobile policy with a deductible of one unit. Let X be the amount of the loss having pmf

$$f(x) = \begin{cases} 0.9, & x = 0, \\ \frac{c}{x}, & x = 1, 2, 3, 4, 5, 6. \end{cases} \tag{1}$$

where c is a constant. Determine c and the expected value of the amount the insurance company must pay.

5. (2.2-5) In Example 2.2-1 let $Z = u(X) = X^3$.

   (a) Find the pmf of Z, say h(z).

   (b) Find E(Z).

   (c) How much, on average, can the young man expect to win on each play if he charges 10 dollars per play?

6. (2.2-6) Let the pmf of X be defined by $f(x) = 6/(\pi^2 x^2), x = 1, 2, 3, \ldots$. Show that $E(X) = +\infty$ and thus, does not exist.

7. (2.2-8) Let X be a random variable with support $\{1, 2, 3, 5, 15, 25, 50\}$, each point of which has the same probability 1/7. Argue that c=5 is the value that minimizes $h(c) = E(|X - c|)$. Compare c with the value of b that minimizes $g(b) = E[(X - b)^2]$.

8. (2.3-2) For each of the following distributions, find $\mu = E(X)$, $E[X(X-1)]$,and $\sigma^2 = E[X(X-1)] + E(X) - \mu^2$:

   (a) $f(x) = \frac{3!}{x!(3-x)!}(\frac{1}{4})^x(\frac{3}{4})^{3-x}$, $\quad x = 0, 1, 2, 3.$

   (b) $f(x) = \frac{4!}{x!(4-x)!}(\frac{1}{2})^4$, $\quad x = 0, 1, 2, 3, 4.$

9. (2.3-4) Let μ and σ^2 denote the mean and variance of the random variable X. Determine $E[(X-\mu)/\sigma]$ and $E\{[(X-\mu)/\sigma]^2\}$.

10. (2.3-6) Place eight chips in a bowl: Three have the number 1 on them, two have the number 2, and three have the number 3. Say each chip has a probability of 1/8 of being drawn at random, let the random variable X equal the number on the chip that is selected, so that the space of X is S=$\{1, 2, 3\}$.Make reasonable probability assignments to each of these three outcomes, and compute the mean μ and the variance σ^2 of this probability distribution.