Student ID Name:

Quiz 6

(15 minutes on Tuesday, 27 Oct 2020)

1. [12 points] Determine if the following statements are True or False (no need to show your work):

(a) If $y' = \sec x$ and $y = 1$ at $x = 0$, then $y = \ln\left(e|\sec x + \tan x|\right)$.

(b) If $f(x) = 1 - |x|$, then $\int_{-1}^{1} f(x)dx = 2$.

(c) For any $\varepsilon > 0$, if $\delta = \varepsilon$, then $|x - y| < \delta \Rightarrow |\sin x - \sin y| < \varepsilon$ for all $x, y \in \mathbb{R}$.

Show your work for the questions below:

2. [10 points] Calculate the following indefinite integrals:

(a) $\int \left(2x^5 - \sqrt[3]{6x+1}\right)dx$

(b) $\int x \sin x dx$ (hint: get the derivative of $x\cos x$)

3. [12 points] Define $f(x) = x^2$ if $0 \le x < 1$ and $f(1) > 1$. Partition interval [0,1] by $P_n : 0 = x_0 < x_1 < \cdots < x_n = 1$ with $x_k = k/n$, $k = 1, 2, \ldots, n$.

(a) Determine the maximum Riemann sum $U_n = f(c_1)\Delta x_1 + \cdots + f(c_n)\Delta x_n$ under P_n based on n and $f(1)$, where $c_k \in [x_{k-1}, x_k]$ and $\Delta x_k = x_k - x_{k-1}$, $k = 1, 2, \ldots, n$. Does U_n depend on the value of $f(1)$?

(b) Find the limit of U_n as $n \to \infty$. Does this limit depend on the value of $f(1)$?

Reminder: $1^2 + 2^2 \cdots + n^2 = n(n+1)(2n+1)/6$.

4. [6 points] A function $f(x)$ is defined by

$$f(x) = \begin{cases} 3, & -1 \le x < 0 \\ \sqrt{4 - x^2}, & 0 \le x \le 2 \end{cases}$$

Use known formulae of areas to calculate $\int_{-1}^{2} f(x)dx$.