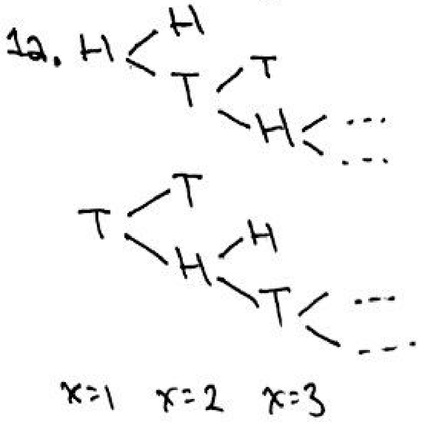
yohandi - assignment 4.

1a.



$P(X=1)=0$

$P(X=2)=\frac{1}{2}(1-P(X=1))$
$=\frac{1}{2}$

$P(X=3)=\frac{1}{2}(1-P(X=2))$
$=\frac{1}{4}$

$\vdots$

$P(X=x)=\frac{1}{2^{x-1}}$

b. $MGF_X(t)=E(e^{tX})=\sum_{x=2}^{\infty} e^{tx}\,p(X=x)$

$=\sum_{x=2}^{\infty} e^{tx}\left(\frac{1}{2}\right)^x(2)$

$=2\sum_{x=2}^{\infty}\left(\frac{e^t}{2}\right)^x$

$=2\cdot\frac{\frac{e^{2t}}{2^2}}{1-\frac{e^t}{2}}$

$=\frac{e^{2t}}{2-e^t}$

c. $\frac{d}{dt}MGF_X(t)=\frac{2e^{2t}(2-e^t)-e^{2t}(0-e^t)}{(2-e^t)^2}$

$=\frac{e^{2t}(4-e^t)}{(2-e^t)^2}$

$\frac{d}{dt}\left(\frac{d}{dt}MGF_X\right)(t)=\frac{(8e^{2t}-3e^{3t})(2-e^t)^2-(4e^{2t}-e^{3t})(2-e^t)(-e^t)}{(2-e^t)^3}$

$=\frac{e^{2t}(e^{2t}-6e^t+16)}{(2-e^t)^3}$

$\mu=E(X)=\frac{d}{dt}MGF_X(0)=3$

$\sigma^2=E(X^2)-E(X)^2$

$=\frac{d}{dt}\left(\frac{d}{dt}MGF_X\right)(0)-\left(\frac{d}{dt}MGF_X(0)\right)^2$

$=11-3^2$

$=2$

d. $P(X\le 3)=P(X=2)+P(X=3)$

$=\frac{1}{2}+\frac{1}{4}$

$=\frac{3}{4}$

$P(X\ge 5)=1-P(X\le 4)$

$=1-\left(\frac{3}{4}+\frac{1}{8}\right)$

$=\frac{1}{8}$

$P(X=3)=\frac{1}{2^2}=\frac{1}{4}$

2a. $\frac{d}{dt}M(t)=\frac{45}{120}e^t+\frac{40}{120}e^{2t}+\frac{30}{120}e^{3t}+\frac{5}{120}e^{5t}$

$\frac{d}{dt}\left(\frac{d}{dt}M\right)(t)=\frac{45}{120}e^t+\frac{80}{120}e^{2t}+\frac{90}{120}e^{3t}+\frac{25}{120}e^{5t}$

$\mu=\frac{d}{dt}M(0)=1$

$\sigma^2=\frac{d}{dt}\left(\frac{d}{dt}M\right)(0)-\left(\frac{d}{dt}M(0)\right)^2=2-1^2=1$

b. $M(t)=\sum_{x=0}^{5}e^{tx}p(X=x)=\frac{44}{120}+\frac{45}{120}e^t+\frac{20}{120}e^{2t}+\frac{10}{120}e^{3t}+\frac{1}{120}e^{5t}$

$P(X=0)=\frac{44}{120}$

$P(X=1)=\frac{45}{120}$

$P(X=2)=\frac{20}{120}$

$P(X=3)=\frac{10}{120}$

$P(X=5)=\frac{1}{120}$

$P(X\ge 1)=1-P(X=0)$

$=\frac{76}{120}$

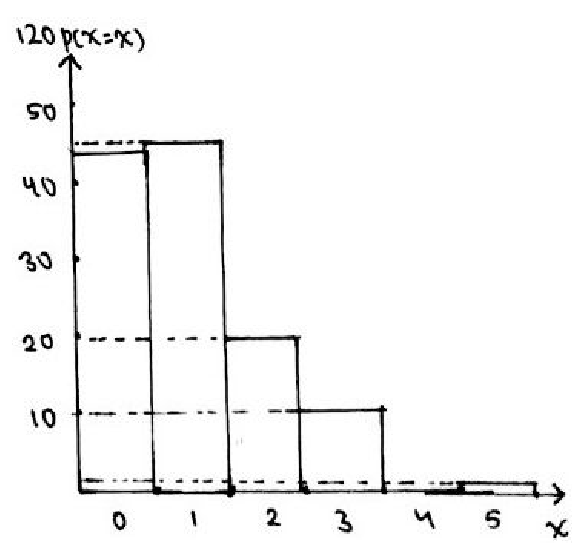
$P(X\ge 5)=1-P(X\le 4)$

$=1-\left(\frac{3}{4}+\frac{1}{8}\right)$

$=\frac{1}{8}$

$P(X=3)=\frac{1}{2^2}=\frac{1}{4}$

c.



3a. $P(X=0) = \binom{25}{0}(0.2)^0(0.8)^{25}$

$P(X=1) = \binom{25}{1}(0.2)^1(0.8)^{24}$

$P(X=2) = \binom{25}{2}(0.2)^2(0.8)^{23}$

$P(X=3) = \binom{25}{3}(0.2)^3(0.8)^{22}$

$P(X=4) = \binom{25}{4}(0.2)^4(0.8)^{21}$ $+$

$P(X \le 4) \approx 0.42067$

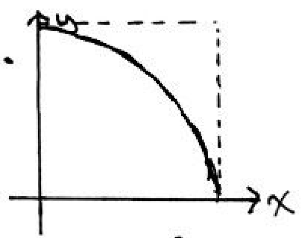
b. $P(X \ge 5) = 1 - P(X \le 4) \approx 0.57933$

c. $P(X=6) = \binom{25}{6}(0.2)^6(0.8)^{19} \approx 0.16335$

d. $\mu = 25(0.2) = 5$

$\sigma^2 = 25(0.2)(0.8) = 4$

$\sigma = \sqrt{4} = 2$

4a. 

$p = \dfrac{\frac{1}{4}\pi(1)^2}{(1)^2} = \frac{1}{4}\pi$

b. $\mu = 2000 \cdot (\frac{1}{4}\pi) = 500\pi$

$\sigma^2 = 2000(\frac{1}{4}\pi)(1 - \frac{1}{4}\pi) = 500\pi - 125\pi^2$

$\sigma = \sqrt{500\pi - 125\pi^2}$

c. $E(\frac{W}{500}) = \frac{1}{500}E(W) = \frac{1}{500} \cdot 500\pi = \pi$

5a. $M(t) = (0.3 + 0.7e^t)^5$

$= 0.3^5 + 5 \cdot 0.3^4(0.7e^t) + \dots + (0.7e^t)^5$

$= \sum_{x=0}^{5} \underbrace{\binom{5}{x}(0.3)^{(5-x)}(0.7)^x}_{P(X=x)} e^{tx}$

(i). $X \sim B(5, 0.7)$ (binomial dist.)

(ii). $\mu = 5(0.7) = 3.5$

$\sigma^2 = 5(0.7)(0.3) = 1.05$

(iii). $P(1 \le X \le 2) = P(X=1) + P(X=2)$

$= 0.16065$

b. $M(t) = \dfrac{0.3e^t}{1 - 0.7e^t}$

$= 0.3e^t(1 + 0.7e^t + (0.7e^t)^2 + \dots)$

$= \sum_{x=1}^{\infty} (0.3e^t)(0.7e^t)^{x-1}$

$= \sum_{x=1}^{\infty} \underbrace{0.3(0.7)^{x-1}}_{P(X=x)} e^{tx}$

(i) $X \sim G(0.3)$ (geometric dist.)

(ii) $\mu = \frac{1}{p} = \frac{10}{3}$

$\sigma^2 = \frac{1-p}{p^2} = \frac{70}{9}$

(iii) $P(1 \le X \le 2) = P(X=1) + P(X=2)$

$= 0.51$

c. $M(t) = 0.45 + 0.55e^t$

$= (1 - 0.55 + \underbrace{0.55}_{p}e^t)^1$

(i) $X \sim$ Bernoulli(0.55) (bernoulli dist.)

(ii) $\mu = p = 0.55$

$\sigma^2 = p(1-p) = 0.2475$

(iii) $P(1 \le X \le 2) = P(X=1) = p = 0.55$

d. $M(t) = \underbrace{0.3}_{P(X=1)}e^t + \underbrace{0.4}_{P(X=2)}e^{2t} + \underbrace{0.2}_{P(X=3)}e^{3t} + \underbrace{0.1}_{P(X=4)}e^{4t}$

(i) discrete dist.

(ii) $\mu = 0.3 + 2(0.4) + 3(0.2) + 4(0.1) = 2.1$

$\sigma^2 = -1.1^2 \cdot 0.3 + 0.1^2\, 0.4 + 0.9^2\, 0.2 + 1.9^2\, 0.1$

$= 0.89$

(iii) $P(1 \le X \le 2) = P(X=1) + P(X=2)$

$= 0.7$

e. $M(t) = \sum_{x=1}^{10} \underbrace{(0.1)}_{P(X=x)} e^{tx}$

(i). uniform dist.

(ii). $\mu = 0.1 \sum_{x=1}^{10} x = 5.5$

$\sigma^2 = (-4.5^2 + -3.5^2 + \dots + 4.5^2)0.1 = 8.25$

(iii) $P(1 \le X \le 2) = 2P(X=1) = 0.2$

6a. let x_i denotes the i-th item,

$$E(X_1) = 1$$

$$E(X_2) = \frac{11}{12} + \frac{1}{12}\cdot\frac{11}{12}\cdot 2 + \left(\frac{1}{12}\right)^2\cdot\frac{11}{12}\cdot 3 + \dots$$

$$= \frac{12}{11}$$

$$E(X_3) = \frac{10}{12} + \frac{2}{12}\cdot\frac{10}{12}\cdot 2 + \left(\frac{2}{12}\right)^2\cdot\frac{10}{12}\cdot 3 + \dots$$

$$= \frac{12}{10}$$

$\vdots$

$$E(X_i) = \frac{12}{13-i}$$

$\vdots$

$$E(X) = E\left(\sum_{i=1}^{12} X_i\right) = \sum_{i=1}^{12} E(X_i) \approx 37.23853$$

b. It is expected that to obtain a complete collection, 37.24 boxes = 3724 bags. Therefore 3724 days are expected on average.

7a. $P(4 \le X \le 9) = \sum_{x=4}^{9} P(X=x) = \sum_{x=4}^{9} \frac{10^x e^{-10}}{x!}$

≈ 0.44759

b. $P(X \ge 4) = 1 - \sum_{x=0}^{3} P(X=x) = 1 - \sum_{x=0}^{3} \frac{10^x \cdot e^{-10}}{x!}$

≈ 0.98966

c. $P(X \le 4) = \sum_{x=0}^{4} P(X=x) = \sum_{x=0}^{4} \frac{10^x \cdot e^{-10}}{x!}$

≈ 0.02925

8. $X \sim Poi\left(\frac{225}{150} = \frac{3}{2}\right)$

$$P(X \le 1) = \sum_{x=0}^{1} P(X=x) = \sum_{x=0}^{1} \frac{\left(\frac{3}{2}\right)^x \cdot e^{-\frac{3}{2}}}{x!}$$

≈ 0.55783

9. $X \sim Poi(0.005 \cdot 1000 = 5)$

a. $P(X \le 1) = \sum_{x=0}^{1} \frac{5^x e^{-5}}{x!} \approx 0.04043$

b. $P(4 \le X \le 6) = \sum_{x=4}^{6} \frac{5^x e^{-5}}{x!} \approx 0.49716$

10. let X be the number of people who don't show up,

$$X \sim B(100, 0.05)$$

a. $P(X \ge 5) = 1 - P(X < 5)$

$$= 1 - \sum_{x=0}^{4} \binom{100}{x}(0.05)^x(0.95)^{100-x}$$

≈ 0.56402

b. $\sum_{x=0}^{4} 700(5-x)\binom{100}{x}(0.05)^x(0.95)^{100-x}$

≈ 598.55928

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Trusted Python 3

Run Code

In [53]:

```
import pandas as pd
import matplotlib.pyplot as plt
import random

def C(n, r):
    ret = 1
    for i in range(n):
        ret *= (i + 1)
    for i in range(r):
        ret /= (i + 1)
    for i in range(n - r):
        ret /= (i + 1)
    return ret

def f(x):
    return C(50, x) * C(50, 5-x) / C(100, 5)

redObtained = [0, 0, 0, 0, 0, 0]
relativeFrequency = [0, 0, 0, 0, 0, 0]

for i in range(1000):
    red = 50
    black = 50
    redBall = 0
    for j in range(5):
        if random.randint(1,red+black) <= red:
            red -= 1
            redBall += 1
        else:
            black -= 1
    redObtained[redBall] += 1

for i in range(6):
    relativeFrequency[i] = float(redObtained[i]/1000)

x = tuple([i for i in range(6)])
y = tuple([relativeFrequency[i] for i in range(6)])
z = tuple([f(i) for i in range(6)])
plt.bar(x, y, align = 'center')
plt.xlabel('Red Balls Obtained')
plt.ylabel('Relative Frequency')
for i in range(3):
    plt.hlines(z[i+3], -0.6, x[i+3] + 0.4, colors = 'green')
for i in range(3):
    plt.hlines(z[i], -0.6, x[i] + 0.4, colors = 'red')
plt.show()
```