STA2001 Assignment 4: Discrete Distribution

1. 2.3-16. Let X equal the number of flips of a fair coin that are required to observe the same face on consecutive flips.
(a) Find the pmf of X. Hint: Draw a tree diagram.
(b) Find the moment-generating function of X.
(c) Use the mgf to find the values of (i) the mean and (ii) the variance of X.
(d) Find the values of (i) $P(X \leq 3)$, (ii) $P(X \geq 5)$, and (iii) $P(X = 3)$.

2. 2.3-19. Given a random permutation of the integers in the set $\{1, 2, 3, 4, 5\}$, let X equal the number of integers that are in their natural position. The moment-generating function of X is

$$M(t) = \frac{44}{120} + \frac{45}{120}e^{t} + \frac{20}{120}e^{2t} + \frac{10}{120}e^{3t} + \frac{1}{120}e^{5t}$$

(a) Find the mean and variance of X.
(b) Find the probability that at least one integer is in its natural position.
(c) Draw a graph of the probability histogram of the pmf of X.

3. 2.4-5. In a lab experiment involving inorganic syntheses of molecular precursors to organometallic ceramics, the final step of a five-step reaction involves the formation of a metalCmetal bond. The probability of such a bond forming is p = 0.20. Let X equal the number of successful reactions out of n = 25 such experiments.
(a) Find the probability that X is at most 4.
(b) Find the probability that X is at least 5.
(c) Find the probability that X is equal to 6.
(d) Give the mean, variance, and standard deviation of X.

4. 2.4-7. Suppose that 2000 points are selected independently and at random from the unit square $\{(x, y) : 0 \leq x < 1, 0 \leq y < 1\}$. Let W equal the number of points that fall into $A = \{(x, y) : x^2 + y^2 < 1\}$.
(a) How is W distributed?
(b) Give the mean, variance, and standard deviation of W.
(c) What is the expected value of $W/500$?

5. 2.4-20. (i) Give the name of the distribution of X (if it has a name), (ii) find the values of μ and σ^2, and (iii) calculate $P(1 \leq X \leq 2)$ when the moment-generating function of X is given by

$$
\begin{aligned}
&(a) \quad M(t) = (0.3 + 0.7e^t)^5 \\
&(b) \quad M(t) = \frac{0.3e^t}{1 - 0.7e^t} \quad , \quad t < -ln(0.7) \\
&(c) \quad M(t) = 0.45 + 0.55e^t \\
&(d) \quad M(t) = 0.3e^t + 0.4e^{2t} + 0.2e^{3t} + 0.1e^{4t} \\
&(e) \quad M(t) = \sum_{x=1}^{10} (0.1)e^{tx}
\end{aligned}
$$

6. 2.5-10. In 2012, Red Rose tea randomly began placing 1 of 12 English porcelain miniature figurines in a l00-bag box of the tea, selecting from 12 nautical figurines.
(a) On the average, how many boxes of tea must be purchased by a customer to obtain a complete collection consisting of the 12 nautical figurines?
(b) If the customer uses one tea bag per day, how long can a customer expect to take, on the average, to obtain a complete collection?

7. 2.6-1. Let X have a Poisson distribution with a mean of 10. Find
$(a)P(4 \leq X \leq 9)$.
$(b)P(X \geq 4)$.
$(c)P(X \leq 4)$.

8. 2.6-5. Flaws in a certain type of drapery material appear on the average of one in 150 square feet. If we assume a Poisson distribution, find the probability of at most one flaw appearing in 225 square feet.

9. 2.6-8. Suppose that the probability of suffering a side effect from a certain flu vaccine is 0.005. If 1000 persons are inoculated, find the approximate probability that
(a) At most 1 person suffers.
(b) 4, 5, or 6 persons suffer.

10. 2.6-11. An airline always overbooks if possible. A particular plane has 95 seats on a flight in which a ticket sells for \$300. The airline sells 100 such tickets for this flight.
(a) If the probability of an individual not showing up is 0.05, assuming independence, what is the probability that the airline can accommodate all the passengers who do show up?
(b) If the airline must return the \$300 price plus a penalty of \$400 to each passenger that cannot get on the flight, what is the expected payout (penalty plus ticket refund) that the airline will pay?