The Chinese University of Hong Kong, Shenzhen
School of Data Science

MAT3007 · Homework 3

Due: 11:59pm, Oct. 20 (Friday), 2023

Instructions:

- Homework problems must be carefully and clearly answered to receive full credit. Complete sentences that establish a clear logical progression are highly recommended.
- Please submit your assignment on Blackboard.
- The homework must be written in English.
- Late submission will not be graded.
- Each student must not copy homework solutions from another student or from any other source.

Problem 1 (20pts).
Consider the following linear program:

$$\begin{array}{rll} \text{maximize} & x_1 + 2x_2 + 3x_3 + 8x_4 & \\ \text{subject to} & x_1 - x_2 + x_3 & \le 2 \\ & x_3 - x_4 & \le 1 \\ & 2x_2 + 3x_3 + 4x_4 & \le 8 \\ & x_1, x_2, x_3, x_4 & \ge 0. \end{array}$$

Use simplex tableau to completely solve it. For each step, draw the simplex tableau. Clearly mark what is the current basis, the current basic solution, and the corresponding objective function value.

Problem 2 (30pts).
Apply the two-phase simplex method (implemented by simplex tableau) to solve the following linear program. For each step, draw the simplex tableau. Clearly mark what is the current basis, the current basic solution, and the corresponding objective function value.

$$\begin{array}{rll} \text{minimize} & x_1 - x_2 + 2x_3 & \\ \text{subject to} & 2x_1 - x_2 + 2x_3 & \le -1 \\ & x_1 - x_2 - x_3 & \le 4 \\ & x_2 - x_4 & = 0 \\ & x_1, x_2, x_3, x_4 & \ge 0. \end{array}$$

Problem 3 (30pts).
Use the two-phase simplex method (implemented by simplex tableau) to completely solve the linear optimization problem. For each step, draw the simplex tableau. Clearly mark what is the current basis, the current basic solution, and the corresponding objective function value.

$$\begin{aligned}
\text{minimize} \quad & x_1 + 3x_2 + x_4 - 2x_5 \\
\text{subject to} \quad & x_1 + 2x_2 + 4x_4 + x_5 &&= 2 \\
& x_1 + 2x_2 - 2x_4 + x_5 &&= 2 \\
& -x_1 - 4x_2 + 3x_3 &&= 1 \\
& x_1, x_2, x_3, x_4, x_5 &&\geq 0.
\end{aligned}$$

Problem 4 (20pts).
Consider a linear optimization problem in the standard form, described in terms of the following initial tableau (Table 2):

B	0	0	0	δ	3	γ	ξ	0
2	0	1	0	α	1	0	3	β
3	0	0	1	-2	2	η	-1	2
1	1	0	0	0	-1	2	1	3

Table 1

The entries α, β, γ, δ, η and ξ in the tableau are unknown parameters, and $B = \{2, 3, 1\}$. For each of the following statements, find (sufficient) conditions of the parameter values that will make the statement true.

1. This is an acceptable initial tableau (i.e., the basic variables are feasible for the problem).
2. The first row (in the constraint) indicates that the problem is infeasible.
3. The basic solution is feasible but we have not reached an optimal basic set B.
4. The basic solution is feasible and the first simplex iteration indicates that the problem is unbounded.
5. The basic solution is feasible, x_6 is a candidate for entering B, and when we choose x_6 as the entering basis, x_3 leaves B.