yohandi - homework for week 5

Exercises 4.5

1. Area $= 16cm^2 = b.h$

perimeter $= 2(b+h)$

$= 2(b + \frac{16}{b})$ (cm)

$= 2b + \frac{32}{b}$ (cm)

$\frac{d(\text{perimeter})}{db} = 2 - \frac{32}{b^2} = 0$

$b^2 = 16cm^2$

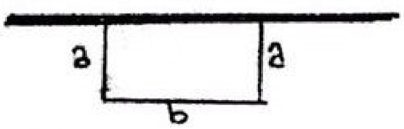
$b = 4cm$ (since $b>0$)

$h = \frac{16cm^2}{4cm} = 4cm$

a. perimeter $= 2(4cm + 4cm) = 16cm$

b. dimension $= b \times h = 4cm \times 4cm$

7.



$2a+b = 800m$

Area $= a.b = a(800-2a)$ (m)

$= 800a - 2a^2$ (m)

$\frac{d(\text{Area})}{da} = 800 - 4a = 0$

$a = 200$ m

$b = 400$ m

a. Area $= a.b = 80000$ m^2

$200m . 400m =$

b. ~~dess~~ dimension $= a \times b = 200m \times 400m$

12. $x^2+y^2 = 3^2 = 9$

volume $= \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi x^2(y+3) = \frac{1}{3}\pi(9-y^2)(y+3)$

$\frac{d(\text{volume})}{dy} =$

$= \frac{1}{3}\pi(-3(y^2+2y-3)) = 0$

$y=1$ $\quad y=-3$

(y must >0)

volume$_{max} = \frac{1}{3}\pi(9-y^2)(y+3)$

$= \frac{32\pi}{3}$

20. $[x+4y]_{max} = 276$ cm

volume $= xy^2 = (276-4y)y^2$ (cm^3)

$\frac{d(\text{volume})}{dy} = 552y - 12y^2 = 0$

$y = 0$ (y must be >0)

$y = 46$cm $\quad x = 92$cm

a. volume $= 92 . 46^2$ cm^3 $= 194672$ cm^3

dimension $= 92$cm $\times$ 46cm $\times$ 46cm

22. perimeter $= 2h + 2r + \pi r = p$

light $= 2h.r + \frac{1}{2}.\frac{1}{2}.\pi r^2 = \frac{r}{4}(8h + \pi r)$

$= \frac{r}{4}(4p - 4\pi r - 8r + \pi r)$

$= \frac{r}{4}(4p - 3\pi r - 8r)$

$\frac{d(\text{light})}{dr} = p - \frac{3\pi}{2}r - 4r = 0$

$r = \frac{2p}{3\pi+8}$

$h = \frac{p}{2}\left[1 - \frac{2(\pi+2)}{3\pi+8}\right]$

$= \frac{p(\pi+4)}{2(3\pi+8)}$

$\frac{2r}{h} = \frac{\frac{4p}{3\pi+8}}{\frac{p(\pi+4)}{2(3\pi+8)}} = \frac{8}{\pi+4}$

28. $\frac{x}{a} + \frac{y}{b} = 1$

$bx + ay = ab$

distance $= \sqrt{x^2+y^2}$

$= \sqrt{x^2 + (b - \frac{b}{a}x)^2}$

$= \sqrt{(1+\frac{b^2}{a^2})x^2 - \frac{2b^2}{a}x + b^2}$

$\frac{d(\text{distance})}{dx} = \frac{2(1+\frac{b^2}{a^2})x - \frac{2b^2}{a}}{2 \,.\, \text{distance}} = 0$

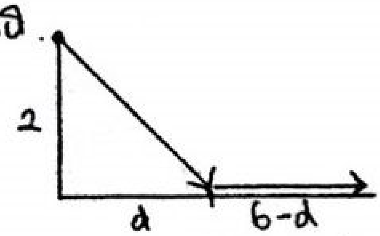
$x = \frac{\frac{b^2}{a}}{1+\frac{b^2}{a^2}}$

$x = \frac{ab^2}{a^2+b^2}$

$y = \frac{a^2b}{a^2+b^2}$

$\left(\frac{ab^2}{a^2+b^2}, \frac{a^2b}{a^2+b^2}\right)$

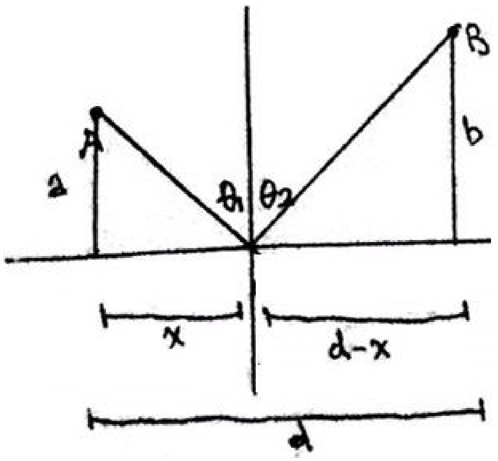
38.



time $= \frac{\sqrt{d^2+2^2}}{2} + \frac{6-d}{5}$

$$\frac{d(\text{time})}{d(d)} = \frac{d}{2\sqrt{d^2+4}} - \frac{1}{5} = 0$$
$$5d = 2\sqrt{d^2+4}$$
$$21d^2 = 16$$
$$d = \frac{4}{21}\sqrt{21} \text{ (since } d \geq 0) \text{ km}$$
(from the nearest point the boat)

48.



$$\text{length} = \sqrt{a^2+x^2} + \sqrt{(d-x)^2+b^2}$$
$$\frac{d(\text{length})}{dx} = \frac{2x}{2\sqrt{a^2+x^2}} + \frac{2x-2d}{2\sqrt{(d-x)^2+b^2}} = 0$$
$$\frac{x}{\sqrt{a^2+x^2}} = \frac{d-x}{\sqrt{(d-x)^2+b^2}}$$
$$\sin(\theta_1) = \sin(\theta_2)$$
since $0 \leq \theta_1, \theta_2 < \frac{\pi}{2}$,
$$\theta_1 = \theta_2$$

49. $v(x) = kax - kx^2$

$v'(x) = ka - 2kx = 0$

$x = \frac{a}{2}$ → critical points

$v''(x) = -2k$

since $v''(\frac{a}{2})$ is negative ⇒ $(\frac{a}{2}, v(\frac{a}{2}))$ is a global maximum point

a. $x = \frac{a}{2}$

b. $v(\frac{a}{2}) = \frac{1}{4}ka^2$

51. $n = \frac{a}{x-c} + b(100-x)$

$\text{profit} = cn - x.n$
$= -a + b(c-x)(100-x)$

$$\frac{d(\text{profit})}{dx} = -b(100+c) + 2bx = 0$$
$$x = \frac{c}{2} + 50$$

54. Let $c(x)$ denotes the cost function

average cost: $\frac{c(x)}{x}$ marginal cost: $c'(x)$

$$d\left(\frac{c(x)}{x}\right) = \frac{c'(x).x - c(x)}{x^2} = 0$$
$$c'(x) = \frac{c(x)}{x}$$

since $c'(x)$ is the marginal cost, this implies that the average cost is the smallest when $c'(x) = \frac{c(x)}{x}$

67a. $\text{distance} = \sqrt{(x-\frac{3}{2})^2+(y)^2} = \sqrt{(x-\frac{3}{2})^2+x}$

$$\frac{d(\text{distance})}{dx} = \frac{2x-2}{\text{distance}} = 0$$
$$x = 1$$
$$y = 1$$
$$\text{distance} = \sqrt{(1-\frac{3}{2})^2+(1)^2}$$
$$= \frac{1}{2}\sqrt{5}$$

Exercises 4.6

1. $f(x)=x^2+x-1$

$f'(x)=2x+1$

using Newton's method,

$$x_{n+1}=x_n-\frac{f(x_n)}{f'(x_n)}$$

a) $x_1=x_0-\frac{f(x_0)}{f'(x_0)}$

$=-1-\frac{(-1)^2+(-1)-1}{2(-1)+1}$

$=-2$

$x_2=x_1-\frac{f(x_1)}{f'(x_1)}$

$=-2-\frac{(-2)^2+(-2)-1}{2(-2)+1}$

$=-\frac{5}{3}$

b) $x_1=x_0-\frac{f(x_0)}{f'(x_0)}$

$=1-\frac{(1)^2+(1)-1}{2(1)+1}$

$=\frac{2}{3}$

$x_2=x_1-\frac{f(x_1)}{f'(x_1)}$

$=\frac{2}{3}-\frac{(\frac{2}{3})^2+(\frac{2}{3})-1}{2(\frac{2}{3})+1}$

$=\frac{13}{21}$

3. $f(x)=x^4+x-3$

$f'(x)=4x^3+1$

using Newton's method,

$$x_{n+1}=x_n-\frac{f(x_n)}{f'(x_n)}$$

a) $x_1=x_0-\frac{f(x_0)}{f'(x_0)}$

$=-1-\frac{(-1)^4+(-1)-3}{4(-1)^3+1}$

$=-2$

$x_2=x_1-\frac{f(x_1)}{f'(x_1)}$

$=-2-\frac{(-2)^4+(-2)-3}{4(-2)^3+1}$

$=-\frac{51}{31}$

b) $x_1=x_0-\frac{f(x_0)}{f'(x_0)}$

$=1-\frac{(1)^4+(1)-3}{4(1)^3+1}$

$=\frac{6}{5}$

$x_2=x_1-\frac{f(x_1)}{f'(x_1)}$

$=\frac{6}{5}-\frac{(\frac{6}{5})^4+(\frac{6}{5})-3}{4(\frac{6}{5})^3+1}$

$=\frac{5763}{4945}$

7. Since x_0 is the root of $f(x)$, $f(x_0)=0$.

by Newton's method,

$$x_{n+1}=x_n-\frac{f(x_n)}{f'(x_n)}$$

$$x_1=x_0-\frac{f(x_0)}{f'(x_0)}=x_0-0=x_0$$

then for $n\geq 1$ (integer) $x_n=x_0$

9. $f(x)=\begin{cases}\sqrt{x}, & x\geq 0\\ \sqrt{-x}, & x<0\end{cases}$

$f'(x)=\begin{cases}\frac{1}{2\sqrt{x}}, & x\geq 0\\ -\frac{1}{2\sqrt{-x}}, & x<0\end{cases}$

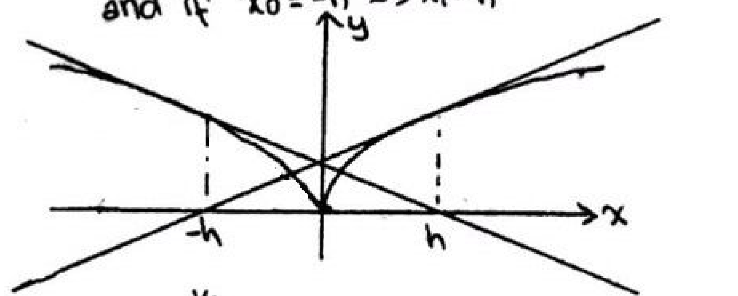
by Newton's method,

$$x_{n+1}=x_n-\frac{f(x_n)}{f'(x_n)}$$

$$x_{n+1}=\begin{cases}x_n-\frac{\sqrt{x_n}}{\frac{1}{2\sqrt{x_n}}}=-x_n, & x_n\geq 0\\ x_n-\frac{\sqrt{-x_n}}{-\frac{1}{2\sqrt{-x_n}}}=-x_n, & x_n<0\end{cases}$$

$=-x_n$

we can see that if $x_0=h \Rightarrow x_1=-h$ and if $x_0=-h \Rightarrow x_1=h$



10. $f(x)=x^{1/3}$

$f'(x)=\frac{1}{3}x^{-2/3}$

by Newton's method,

$$x_{n+1}=x_n-\frac{f(x_n)}{f'(x_n)}=g(x_n)$$

$x_0=1$

$x_1=g(x_0)=-2$

$x_2=g(x_1)=4$

$x_3=g(x_2)=-8$

$x_4=g(x_3)=16$

$g(x_n)=x_n-\frac{x_n^{1/3}}{\frac{1}{3}x_n^{-2/3}}$

$=x_n-3x_n$

$=-2x_n$

$\Rightarrow x_{n+1}=-2x_n$

$|x_{n+1}|=2|x_n|$

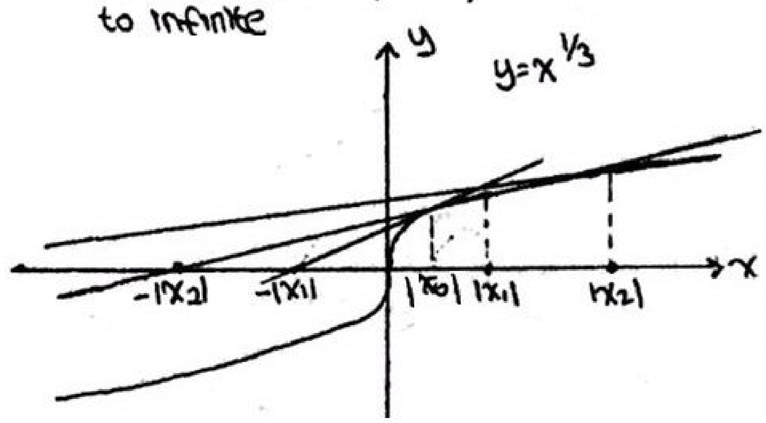
$=2\cdot 2\cdot|x_{n-1}|$

$=\underbrace{2\cdot 2\cdot\ldots\ldots}_{n+1}|x_{n-n}|$

$=2^{n+1}|x_0|$

$\lim_{n\to\infty}|x_n|=2^{\lim_{n\to\infty}n}\cdot|x_0|=+\infty$

as n tends to infinite, $|x_n|$ also tends to infinite



20. let $f(x) = x + \cos x$

$\Rightarrow f'(x) = 1 - \sin x$

by Newton's method,

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$= x_n - \frac{x_n + \cos(x_n)}{1 - \sin(x_n)}$$

let's take an arbitrary number for x_0,

$x_0 = \pi$

$x_1 = 1$

$x_2 \approx -8.716217$

$x_3 \approx -2.976061$

$x_4 \approx 0.425785$

$x_5 \approx -1.851184$

$x_6 \approx -0.766040$

$x_7 \approx -0.739241$

$x_8 \approx -0.739085$

$x_9 \approx -0.739085$

$\vdots$

$x_n \approx -0.739085 \ (n > 9)$

since the value of $f'(x) \geq 0$ (the value of $|\sin x| \leq 1$), this implies that $f(x)$ is an increasing function resulting $f(x)$ will only intersect $y=0$ once (only one solution for $\cos x = -x$. it is approximately -0.739085

Exercises 4.7

1 a $\int 2x\,dx = 2\ \frac{1}{2}x^2 + C = x^2 + C$

$\frac{d(x^2+C)}{dx} = 2x$ (true)

b. $\int x^2\,dx = \frac{1}{3}x^3 + C$

$\frac{d(\frac{1}{3}x^3+C)}{dx} = 3 \cdot \frac{1}{3} \cdot x^2 = x^2$ (true)

c. $\int x^2 - 2x + 1\,dx = \frac{1}{3}x^3 - x^2 + x + C$

$\frac{d(\frac{1}{3}x^3 - x^2 + x + C)}{dx} = x^2 - 2x + 1$ (true)

7. a. $\int \frac{3}{2}\sqrt{x}\,dx = \frac{3}{2} \cdot \frac{2}{3} x\sqrt{x} + C = x\sqrt{x} + C$

$\frac{d(x\sqrt{x}+C)}{dx} = \frac{3}{2} x^{\frac{1}{2}} = \frac{3}{2}\sqrt{x}$ (true)

b. $\int \frac{1}{2\sqrt{x}}\,dx = 2 \cdot \frac{1}{2}\sqrt{x} + C = \sqrt{x} + C$

$\frac{d(\sqrt{x}+C)}{dx} = \frac{1}{2} \cdot \frac{1}{\sqrt{x}} = \frac{1}{2\sqrt{x}}$ (true)

c. $\int \sqrt{x} + \frac{1}{\sqrt{x}}\,dx = \frac{2}{3}x\sqrt{x} + 2 \cdot \sqrt{x} + C$

$\frac{d(\frac{2}{3}x\sqrt{x} + 2\sqrt{x} + C)}{dx} = \sqrt{x} + \frac{1}{\sqrt{x}}$ (true)

12 a. $\int \pi \cos \pi x\,dx = \pi \cdot \frac{1}{\pi} \sin \pi x + C$

$= \sin \pi x + C$

$\frac{d(\sin \pi x + C)}{dx} = \pi \cdot 1 \cos(\pi x)$ (true)

b. $\int \frac{\pi}{2} \cos \frac{\pi}{2} x\,dx = \frac{\pi}{2} \cdot \frac{2}{\pi} \sin(\frac{\pi}{2}x) + C$

$= \sin(\frac{\pi}{2}x) + C$

$\frac{d(\sin \frac{\pi}{2}x + C)}{dx} = \frac{\pi}{2} \cdot \cos(\frac{\pi}{2}x)$ (true)

c. $\int \cos \frac{\pi}{2}x + \pi \cos x = \frac{2}{\pi} \sin \frac{\pi}{2}x + \pi \sin x + C$

$\frac{d(\frac{2}{\pi}\sin\frac{\pi}{2}x + \pi \sin x + C)}{dx} = \cos \frac{\pi}{2}x + \pi \cos x$ (true)

23. $\int \frac{1}{x^2} - x^2 - \frac{1}{3}\,dx = -\frac{1}{x} - \frac{1}{3}x^3 - \frac{1}{3}x + C$

$\frac{d(-\frac{1}{x} - \frac{1}{3}x^3 - \frac{1}{3}x + C)}{dx} = \frac{1}{x^2} - 3 \cdot \frac{1}{3}x^2 - \frac{1}{3}$

$= \frac{1}{x^2} - x^2 - \frac{1}{3}$ (true)

32. $\int x^{-3}(x+1)\,dx = \int x^{-2} + x^{-3}\,dx$

$= -\frac{1}{x} - \frac{1}{2x^2} + C$

$\frac{d(-\frac{1}{x} - \frac{1}{2x^2} + C)}{dx} = \frac{1}{x^2} + \frac{1}{2} \cdot 2 \cdot \frac{1}{x^3} = \frac{1}{x^3}(x+1)$ (true)

34. $\int \frac{4+\sqrt{t}}{t^3}\,dt = \int 4t^{-3} + t^{-5/2}\,dt$

$= -\frac{2}{t^2} - \frac{2}{3t\sqrt{t}} + C$

$\frac{d(-\frac{2}{t^2} - \frac{2}{3t\sqrt{t}} + C)}{dt} = \frac{4}{t^3} + \frac{2}{3} \cdot \frac{3}{2} \frac{1}{t^2\sqrt{t}} = \frac{4+\sqrt{t}}{t^3}$ (true)

52. $\int (2 + \tan^2\theta)\,d\theta = \int 1 + \sec^2\theta\,d\theta$

$= \theta + \tan\theta + C$

$\frac{d(\theta + \tan\theta + C)}{d\theta} = 1 + \sec^2\theta = 2 + \tan^2\theta$ (true)

59. $\frac{d(\frac{1}{5}\tan(5x-1) + C)}{dx} = \frac{1}{5} \cdot 5 \cdot \sec^2(5x-1)$

$= \sec^2(5x-1)$

Since the derivative of $\frac{1}{5}\tan(5x-1)+C$ is $\sec^2(5x-1)$, $\frac{1}{5}\tan(5x-1)+C$ is the anti-derivative of $\sec^2(5x-1)$

63a. $\frac{d(\frac{x^2}{2}\sin x + C)}{dx} = x \cdot \sin x + \frac{x^2}{2} \cdot \cos x \neq x \sin x$ (wrong)

b. $\frac{d(-x\cos x + C)}{dx} = -\cos x + x \sin x \neq x \sin x$ (wrong)

c. $\frac{d(-x\cos x + \sin x + C)}{dx} = -\cos x + x\sin x + \cos x$

$= x \sin x$ (right!)

75. $\frac{dy}{dx} = 3x^{-2/3}$

$y = \int \frac{dy}{dx}\,dx = \int 3x^{-2/3}\,dx = 3\ \frac{3}{1} \cdot x^{1/3} + C$

$= 9x^{1/3} + C$

$y(-1) = -5 = -9 + C$

$C = 4$

$y = 9x^{1/3} + 4$

85. $\frac{d^2r}{dt^2} = \frac{2}{t^3}$

$\int \frac{d^2r}{dt^2}\,dt = \int \frac{2}{t^3}\,dt$

$\frac{dr}{dt} = -\frac{1}{2} \cdot 2 \cdot \frac{1}{t^2} + C$

$= -\frac{1}{t^2} + C$

$\frac{dr}{dt}\Big|_{t=1} = 1 = -\frac{1}{1^2} + C$

$C = 2$

$\frac{dr}{dt} = -\frac{1}{t^2} + 2$

$\frac{dr}{dt} = -\frac{1}{t^2} + 2$

$\int \frac{dr}{dt}\,dt = \int -\frac{1}{t^2} + 2\,dt$

$r = \frac{1}{t} + 2t + C$

$r(1) = 1 = \frac{1}{1} + 2 + C$

$C = -2$

$r = \frac{1}{t} + 2t - 2$

95. $\frac{dy}{dx} = \sin x - \cos x$

$\int \frac{dy}{dx}\, dx = \int (\sin x - \cos x)\, dx$

$y = -\cos x - \sin x + C$

$y(-\pi) = -1 = -\cos(-\pi) + \sin(\pi) + C$

$C = -2$

$y = -(\sin x + \cos x) - 2$

99 (1) $\frac{d^2s}{dt^2} = -k$

$\int \frac{d^2s}{dt^2}\, dt = \int -k\, dt$

$\frac{ds}{dt} = -kt + C$

$\frac{ds}{dt}(0) = 30 = 0 + C$

$C = 30$

$\frac{ds}{dt} = -kt + 30$

$\int \frac{ds}{dt}\, dt = \int (-kt + 30)\, dt$

$S = -\frac{1}{2}kt^2 + 30t + C$

$S(0) = 0 = 0 + 0 + C$

$C = 0$

$S = -\frac{1}{2}kt^2 + 30t$

(2) $\frac{ds}{dt} = 0 = -kt + 30$

$t = \frac{30}{k}$

(3) $S(\frac{30}{k}) = 75 = -\frac{1}{2}k(\frac{30}{k})^2 + 30(\frac{30}{k})$

$75 = -\frac{900}{2k} + \frac{900}{k}$

$k = 6$

103. $\frac{d^2s}{dt^2} = a$

$\int \frac{d^2s}{dt^2}\, dt = \int a\, dt$

$\frac{ds}{dt} = at + C$

$\frac{ds}{dt}(0) = V_0 = 0 + C$

$C = V_0$

$\frac{ds}{dt} = at + V_0$

$\frac{ds}{dt} = at + V_0$

$\int \frac{ds}{dt}\, dt = \int (at + V_0)\, dt$

$S = \frac{1}{2}at^2 + V_0 . t + C$

$S(0) = S_0 = 0 + 0 + C$

$C = S_0$

$S = \frac{1}{2}at^2 + V_0 . t + S_0$