yohandi - Homework for Week 3

Exercises 3.4

3. $s = -t^3 + 3t^2 - 3t$, $0 \le t \le 3$

a. displacement $= \Delta s = s(3) - s(0)$
$= (-3^3 + 3.3^2 - 3.3) - (0)$
$= -9 m$

average velocity $= \frac{\Delta s}{\Delta t} = \frac{-9}{(3-0)} = -3\ m/s$

b. speed $= \left|\frac{ds}{dt}(0)\right| = |(-3t^2+6t-3)|(0) = 3\ m/s$

acceleration $= \frac{d^2s}{dt^2}(0) = (-6t+6)(0) = 6\ m/s^2$

speed $= \left|\frac{ds}{dt}(3)\right| = |(-3t^2+6t-3)|(3) = |-3.3^2+6.3-3|$
$= 12\ m/s$

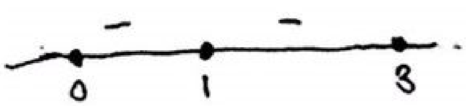
acceleration $= \frac{d^2s}{dt^2}(3) = (-6t+6)(3) = -12\ m/s^2$

c. direction changes when velocity's sign also changes

velocity $= v = \frac{ds}{dt} = -3t^2 + 6t - 3$

key value when $v = 0$

$\hookrightarrow -3t^2 + 6t - 3 = 0$
$t^2 - 2t + 1 = 0$
$(t-1)^2 = 0$
$t = 1$



we can see that there's no changing sign in $[0,3]$. this implies that there's no change in direction.

7. $s = t^3 - 6t^2 + 9t$ m.

$\frac{ds}{dt} = 3t^2 - 12t + 9$

$\frac{d^2s}{dt^2} = 6t - 12$

a. velocity is 0 when $3t^2 - 12t + 9 = 0$
$t^2 - 4t + 3 = 0$
$t = 1s$ or $t = 3s$

at $t = 1s$, acceleration $= 6(1) - 12 = -6\ m/s^2$

at $t = 3s$, acceleration $= 6(3) - 12 = 6\ m/s^2$

b. acceleration is 0 when $6t - 12 = 0$
$t = 2s$

at $t = 2s$, velocity $= 3(2)^2 - 12(2) + 9 = -3\ m/s$, speed $= 3\ m/s$

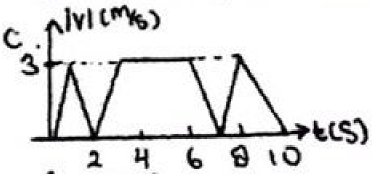
c. note that there's a changing direction at $t = 1s$

distance $= |\Delta s_{01}| + |\Delta s_{12}|$
$= |(1^3 - 6.1^2 + 9.1) - (0)| + |(2^3 - 6.2^2 + 9.2) - (4)|\ m$
$= |4| + |-2|\ m$
$= 6\ m$

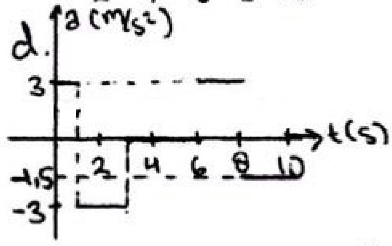
15 a. at $t = 2s$ and $t = 7s$

b. at $t = 3s$ until $t = 6s$

c.



d.



21. we can see that graph A is a parabolic equation while graph B is a linear equation..
since the derivative of parabolic function is linear.
velocity must be either graph C or A.
assume that $\frac{ds}{dt}$ is C), s must be a function that is increasing with a changing rate over time and there's no graph that satisfying that statement.
therefore, velocity must be A), acceleration must be B and s must be C.

23 a. $c(100) = \frac{(2000 + 10000 - 1000)}{100}$ dollars/unit
$= 110$ dollars/unit

b. $c'(100) = 100 - 0,2(100) = 80$ dollars

c. $c(101) - c(100) = (2000 + 100.101 - 0,1(101)^2) - (11000)$
$= 79,9$ dollars
$\approx c'(100)$

25. $b = 10^6 + 10^4 t - 10^3 t^2$

$\frac{db}{dt} = 10^4 - 2.10^3 t$

a. $\frac{db}{dt}(0) = 10^4$ bacteria/hour

b. $\frac{db}{dt}(5) = 0$ bacteria/hour

c. $\frac{db}{dt}(10) = -10^4$ bacteria/hour

26. $S = \frac{1}{60}\sqrt{w.180}$

$= \frac{1}{20}\sqrt{20}.\sqrt{w}$

$\frac{ds}{dw} = \frac{1}{40}\sqrt{20}.\frac{1}{\sqrt{w}}$.

we can see from the equation obtaned that

$\frac{ds}{dw}$ is going larger as the weight getting lower ($\frac{1}{\sqrt{w_0}} > \frac{1}{\sqrt{w_1}}$ when $w_0 < w_1$)

Exercises 3.6

5. $y=\sqrt{u}=\sqrt{\sin x}$

$$\frac{dy}{dx}=\frac{1}{2}(\sin^{1/2}x).\cos x$$
$$=\frac{\cos x}{2\sqrt{\sin x}}$$

7. $y=\tan u=\tan(\pi x^2)$

$$\frac{dy}{dx}=\sec^2(\pi x^2).\pi.2x$$
$$=2\pi x.\sec^2(\pi x^2)$$

10. $y=(4-3x)^9=(u)^9$, $u=4-3x$

$$\frac{dy}{dx}=9(4-3x)^8.-3$$
$$=-27(4-3x)^8$$

15. $y=\sec(\tan x)=\sec(u)$, $u=\tan x$

$$\frac{dy}{dx}=\sec(\tan x).\tan(\tan x).\sec^2 x$$

29. $y=(4x+3)^4(x+1)^{-3}$

$$\frac{dy}{dx}=4(4x+3)^3.4.(x+1)^{-3}+(4x+3)^4.(-3)(x+1)^{-4}$$
$$=(4x+3)^3(x+1)^{-3}\left[16+\frac{(4x+3).-3}{x+1}\right]$$
$$=(4x+3)^3(x+1)^{-3}(4x+7).(x+1)^{-1}$$
$$=\frac{(4x+3)^3(4x+7)}{(x+1)^4}$$

34. $g(x)=\tan(3x).(x+7)^{-4}$

$$\frac{d(g(x))}{dx}=\sec^2(3x).3(x+7)^{-4}+\tan(3x).(x+7)^{-5}.(-4)$$
$$=\frac{3\sec^2(3x)}{(x+7)^4}-\frac{4\tan(3x)}{(x+7)^5}$$

41. $y=\sin^2(\pi t-2)$

$$\frac{dy}{dt}=2.\sin(\pi t-2).\cos(\pi t-2).\pi$$
$$=\pi.\sin(2\pi t-4)$$

49. $y=\sin(\cos(2t-5))$

$$\frac{dy}{dt}=\cos(\cos(2t-5)).(-\sin(2t-5)).2$$
$$=-2\sin(2t-5).\cos(\cos(2t-5))$$

54. $y=4\sin(\sqrt{1+\sqrt{t}})$

$$\frac{dy}{dt}=4.\cos(\sqrt{1+\sqrt{t}}).\frac{1}{2\sqrt{1+\sqrt{t}}}.\frac{1}{2\sqrt{t}}$$
$$=\frac{\cos(\sqrt{1+\sqrt{t}})}{\sqrt{t}\sqrt{1+\sqrt{t}}}$$

65. $f(u)=u^5+1$; $u=g(x)=\sqrt{x}$, $x=1$

$$\frac{d(f(g(x)))}{dx}=\frac{d(x^{5/2}+1)}{dx}$$
$$\frac{d(f(g(x)))}{dx}=\frac{5}{2}x^{3/2}\quad , x=1 , (f\circ g)'(x)=\frac{5}{2}$$

70. $f(x)=\left(\frac{x-1}{x+1}\right)^2$, $u=g(x)=\frac{1}{x^2}-1$, $x=-1$

$$f'(x)=2\left(\frac{x-1}{x+1}\right)\left(\frac{(x+1)-(x-1)}{(x+1)^2}\right)$$
$$=4\frac{(x-1)}{(x+1)^3}$$
$$f(g(x))=f(u)$$
$$\frac{d(f(g(x)))}{dx}=\frac{d(f(u))}{dx}$$
$$=f'(u).g'(x)$$
$$=4\frac{(u-1)}{(u+1)^3}.-2x^{-3}$$
$$x=-1,\ u=0,\ \frac{d(f(g(x)))}{dx}(-1)=4.\frac{-1}{1}.-2.(-1)^{-3}$$
$$=-8$$

73a. $\frac{d(2f(x))}{dx}(2) = 2f'(2) = \frac{2}{3}$

b. $\frac{d(f(x)+g(x))}{dx}(3) = f'(3)+g'(3) = 2\pi+5$

c. $\frac{d(f(x)g(x))}{dx}(3) = f'(3)\cdot g(3)+f(3)\cdot g'(3) = 15-8\pi$

d. $\frac{d(f(x)/g(x))}{dx}(2) = \frac{f'(2)\cdot g(2) - f(2)\cdot g'(2)}{g^2(2)} = \frac{37}{6}$

e. $\frac{d(f(g(x)))}{dx}(2) = f'(g(2))\cdot g'(2) = -1$

f. $\frac{d(\sqrt{f(x)})}{dx}(2) = \frac{1}{2\sqrt{f(2)}} f'(2) = \frac{1}{24}\sqrt{2}$

g. $\frac{d(1/g^2(x))}{dx}(3) = -2\frac{1}{g^3(3)} g'(3) = \frac{5}{32}$

h. $\frac{d(\sqrt{f^2(x)+g^2(x)})}{dx}(2) = \frac{1}{2\sqrt{f^2(2)+g^2(2)}}\cdot(2 f(2) f'(2)+2 g(2) g'(2)) = \frac{-5}{3\sqrt{17}}$

74a. $\frac{d(5f(x)-g(x))}{dx}(1) = 5 f'(1)-g'(1) = 1$

b. $\frac{d(f(x)\cdot g^3(x))}{dx}(0) = f'(0)\cdot g^3(0)+f(0)\cdot 3g^2(0)\cdot g'(0) = 6$

c. $\frac{d(f(x)/(g(x)+1))}{dx}(1) = \frac{f'(1)(g(1)+1)-f(1)\cdot g'(1)}{(g(1)+1)^2} = 1$

d. $\frac{d(f(g(x)))}{dx}(0) = f'(g(0))\cdot g'(0) = -\frac{1}{9}$

e. $\frac{d(g(f(x)))}{dx}(0) = g'(f(0))\, f'(0) = -\frac{40}{3}$

f. $\frac{d((x^{11}+f(x))^{-2})}{dx}(1) = -2\,((1)^{11}+f(1))^{-3}(11(1)^{10}+f'(1)) = -\frac{1}{3}$

g. $\frac{d(f(x+g(x)))}{dx}(0) = f'(0+g(0))\,(1+g'(0)) = -\frac{4}{9}$

82a. $y_1 = \sin 2x$ $\quad y_2 = -\sin(\frac{x}{2})$

$m_1 = \frac{dy}{dx}(0) = 2\cos(2\cdot 0) = 2$ $\quad m_2 = \frac{dy}{dx}(0) = -\frac{1}{2}\cos(\frac{0}{2}) = -\frac{1}{2}$

$y = 2x$ $\quad y = -\frac{1}{2}x$.

since both share the same points at $(0,0)$ and $m_1 \cdot m_2 = -1$, both tangent lines are perpendicular to each other

b. $y_1 = \sin mx$ $\quad y_2 = -\sin(\frac{x}{m})$

$\frac{dy}{dx} = m\cos(mx)$ $\quad \frac{dy}{dx} = -\frac{1}{m}\cdot\cos(\frac{x}{m})$

we can see that at origin, no matter what is the value of m (given that $m \neq 0$), the tangent line of both curves will always perpendicular to each other.

c. since the value of $|\cos(mx)| \le 1$, the largest values are $|m|$ and $|-\frac{1}{m}|$ respectively. (note that there exist x such that $mx \equiv 0 \pmod{2\pi}$ and $-\frac{x}{m} \equiv 0 \pmod{2\pi}$)

d. $y = \sin mx$ completes $|m|$ periods in the $[0, 2\pi]$.

and the slope value at origin will always be equal to m ($m\cos(0) = m$).

therefore, the total periods completed is |slope at origin|

84a. $y = 20\sin(\frac{2\pi}{365}(x-101)) - 4$

$\frac{dy}{dx} = 20\cos(\frac{2\pi}{365}(x-101))\cdot\frac{2\pi}{365}$

fastest when $\cos(\frac{2\pi}{365}(x-101)) = 1$

$\Rightarrow \frac{2\pi}{365}(x-101) = 0 \Rightarrow x = 101$ (April 11th)

b. $\frac{dy}{dx}(101) = 20\cdot 1\cdot\frac{2\pi}{365} = \frac{8\pi}{73}$

Exercises 3.7

1. $x^2y+xy^2=6$

$$\frac{d(\ldots)}{dx}$$

$$2x.y+x^2.\frac{dy}{dx}+y^2+2x.y.\frac{dy}{dx}=0$$

$$\frac{dy}{dx}(x^2+2xy)=-y^2-2xy$$

$$\Rightarrow \frac{dy}{dx}=\frac{-y^2-2xy}{x^2+2xy}$$

12. $x^4+\sin y=x^3.y^2$

$$\frac{d(\ldots)}{dx}$$

$$4x^3+\cos y.\frac{dy}{dx}=3x^2.y^2+x^3.2y.\frac{dy}{dx}$$

$$\frac{dy}{dx}(\cos y-2y.x^3)=3x^2y^2-4x^3$$

$$\Rightarrow \frac{dy}{dx}=\frac{3x^2y^2-4x^3}{\cos y-2y.x^3}$$

19. $x^2+y^2=1$

$$\frac{d(\ldots)}{dx}$$

$$2x+2y.\frac{dy}{dx}=0$$

$$\Rightarrow \frac{dy}{dx}=\frac{-2x}{2y}=-\frac{x}{y}$$

$$x+y.\frac{dy}{dx}=0$$

$$\frac{d(\ldots)}{dx}$$

$$1+\left[\left(\frac{dy}{dx}\right)^2+y.\frac{d^2y}{dx^2}\right]=0$$

$$\Rightarrow \frac{d^2y}{dx^2}=\frac{-1-\frac{x^2}{y^2}}{y}=-\frac{1}{y}-\frac{x^2}{y^3}$$

29. $x^2+xy-y^2=1$

$$\frac{d(\ldots)}{dx}$$

$$2x+y+x.\frac{dy}{dx}-2y.\frac{dy}{dx}=0$$

$$\frac{dy}{dx}(x-2y)=-2x-y$$

$$\frac{dy}{dx}=\frac{-2x-y}{x-2y}$$

$$\frac{dy}{dx}(2,3)=\frac{-2.2-3}{2-2.3}=\frac{-7}{-4}=\frac{7}{4}$$

tangent line: $y-3=\frac{7}{4}(x-2)$

$$y=\frac{7}{4}x-\frac{1}{2}$$

normal line: $y-3=-\frac{4}{7}(x-2)$

$$y=-\frac{4}{7}x+\frac{29}{7}$$

41. $y^4=y^2-x^2$

$$\frac{d(\ldots)}{dx}$$

$$4y^3.\frac{dy}{dx}=2y.\frac{dy}{dx}-2x$$

$$\frac{dy}{dx}=\frac{-2x}{4y^3-2y}$$

$$\frac{dy}{dx}\left(\frac{\sqrt3}{4},\frac{\sqrt3}{2}\right)=\frac{-\frac{1}{2}\sqrt3}{\left(\frac{3\sqrt3}{2}-\sqrt3\right)}$$

$$=-1$$

$$\frac{dy}{dx}\left(\frac{\sqrt3}{4},\frac{1}{2}\right)=\frac{-\frac{1}{2}\sqrt3}{\frac{1}{2}-1}=\sqrt3$$

Exercise. 38

7. $x^2+y^2=25$ $\quad \frac{d(\dots)}{dx}$

$$2x+2y\frac{dy}{dx}=0$$

$$\frac{dy}{dx}=-\frac{x}{y}$$

$$\frac{dy}{dt}=\frac{dy}{dx}(3,-4)\cdot\frac{dx}{dt}$$

$$=\frac{3}{4}\cdot -2=-\frac{3}{2}$$

23. $l=3.9$ m

when $x=3.6$ m $\frac{dx}{dt}=1.5$ m/s

a. note that: $l^2=x^2+y^2$

$$0=2x\cdot\frac{dx}{dt}+2y\cdot\frac{dy}{dt}$$

$$\frac{dy}{dt}=-\frac{x}{y}\cdot\frac{dx}{dt}=\frac{-3.6}{\sqrt{3.9^2-3.6^2}}(1.5)=-3.6 \text{ m/s}$$

b. note that: Area $=\frac{1}{2}\cdot x\cdot y$

$$\frac{d(\text{Area})}{dt}=\frac{1}{2}\left(\frac{dx}{dt}\cdot y+x\cdot\frac{dy}{dt}\right)$$

$$=\frac{1}{2}\left(1.5\cdot\sqrt{3.9^2-3.6^2}+3.6\cdot(-3.6)\right)$$

$$=-5.355 \text{ m/s}$$

c. note that when $x=3.6$, $\cos\theta=\frac{x}{l}=\frac{12}{13}$, $\sin\theta=\frac{5}{13}$

$$\rightarrow \cos\theta=\frac{x}{l}$$

$$-\sin\theta\cdot\frac{d\theta}{dt}=\frac{1}{3.9}\cdot\frac{dx}{dt}$$

$$\frac{d\theta}{dt}=-\frac{1}{1.5}\cdot 1.5 \text{ rad/s}$$

$$=-1 \text{ rad/s}$$

27. $\frac{dV}{dt}=10$ m³/min

$h=\frac{3}{8}d=\frac{3}{4}r \qquad V=\frac{1}{3}\pi r^2 h$

when $h=4$m; $r=\frac{16}{3}$m

a. $V=\frac{1}{3}\pi r^2 h=\frac{1}{3}\pi\cdot\frac{16}{9}\cdot h^3$

$$\frac{dV}{dt}=\frac{16}{27}\pi\cdot 3\cdot 4^2\cdot\frac{dh}{dt}=\frac{256}{9}\pi\cdot\frac{dh}{dt}$$

$$\frac{dh}{dt}=\frac{45}{128\pi} \text{ m/min}=\frac{1125}{32\pi} \text{ cm/min}$$

b. $V=\frac{1}{3}\pi r^2 h=\frac{1}{4}\pi r^3$

$$\frac{dV}{dt}=\frac{3}{4}\pi\cdot\left(\frac{16}{3}\right)^2\cdot\frac{dr}{dt}=\frac{64\pi}{3}\cdot\frac{dr}{dt}$$

$$\frac{dr}{dt}=\frac{15}{32\pi} \text{ m/min}=\frac{375}{8\pi} \text{ cm/min}$$

32. $\frac{dl}{dt}=0.5$ m/s

$h=2$

a. note that: $l^2=h^2+x^2$

$$2l\cdot\frac{dl}{dt}=0+2x\cdot\frac{dx}{dt}$$

when $l=3$m, $\frac{dx}{dt}=\frac{3}{\sqrt{3^2-2^2}}\cdot 0.5$

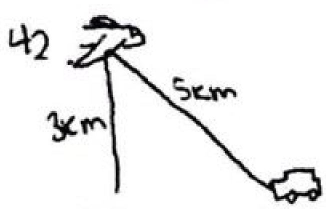
$$=\frac{3\sqrt{5}}{10} \text{ m/s}$$

b $\cos\theta=\frac{2}{l} \qquad \sin\theta=\frac{x}{l}$

$$-\sin\theta\cdot\frac{d\theta}{dt}=-\frac{2}{l^2}\cdot\frac{dl}{dt}$$

$$\frac{d\theta}{dt}=-\frac{2}{3^2}\cdot\frac{1}{2}\cdot-\frac{3}{\sqrt{5}}=\frac{\sqrt{5}}{15} \text{ rad/s}$$

42



$\frac{dl}{dt}=160$ km/h

$l^2=x^2+h^2$

$$2l\cdot\frac{dl}{dt}=2x\cdot\frac{dx}{dt}+0$$

$$\frac{dx}{dt}=\frac{l}{x}\cdot\frac{dl}{dt}=\frac{5}{4}\cdot 160=200 \text{ km/h}$$

since 200 km/h (the speed) is relative to the sight of the plane

speed of car $=200$ km/h -120 km/h

$=80$ km/h

Exercises 3.9

1. $f(x) = x^3 - 2x + 3$, $a = 2$

$f'(x) = 3x^2 - 2$

$L(x) = f(a) + f'(a)(x-a)$

$= 7 + 10(x-2)$

$= 10x - 13$

6a. $f(x) = \sin x$, $a = 0$

$f'(x) = \cos x$

$L(x) = f(a) + f'(a)(x-a)$

$= 0 + 1.(x-0)$

$= x$

b. $f(x) = \cos x$, $a = 0$

$f'(x) = -\sin x$

$L(x) = f(a) + f'(a)(x-a)$

$= 1 + 0(x-0)$

$= 1$

c. $f(x) = \tan x$, $a = 0$

$f'(x) = \sec^2 x$

$L(x) = f(a) + f'(a)(x-a)$

$= 0 + 1(x-0)$

$= x$

17. $y = x^3 - 3\sqrt{x}$

$\frac{dy}{dx} = 3x^2 - \frac{3}{2\sqrt{x}}$

$dy = (3x^2 - \frac{3}{2\sqrt{x}})dx$

18. $y = x\sqrt{1-x^2}$

$\frac{dy}{dx} = \sqrt{1-x^2} + x.\frac{-2x}{2\sqrt{1-x^2}}$

$dy = (\frac{1-2x^2}{\sqrt{1-x^2}})dx$

29 a. $f(1.1) - f(1) = 0{,}41$

b. $f'(x) = 2x + 2$

$df = f'(1).0.1 = 0{,}4$

c. $|\Delta f - df| = |0.41 - 0.4|$

$= 0.01$

37. $S = 6x^2$

$\frac{dS}{dx} = 12x$

$dS = 12x.dx$, when $x = x_0 \rightarrow dS = 12x_0.dx$

42. $\text{perimeter}_0 = \pi.d = 25\pi$ cm

$\text{Perimeter}_1 = 25\pi + 5$ cm

a. $d_1 = \frac{\text{Perimeter}_1}{\pi} = \frac{25\pi+5}{\pi} = 25 + \frac{5}{\pi}$ cm , $\Delta d = \frac{5}{\pi}$ cm

b. $\Delta A = A_1 - A_0 = \frac{1}{4}\pi(d_1^2 - d_0^2) = \frac{\pi}{4}(\frac{250}{\pi} + \frac{25}{\pi^2}) = 62{,}5 + \frac{25}{4\pi}$ cm^2

51. $W = a + \frac{b}{g}$

$\frac{dW}{dg} = -\frac{b}{g^2}$

$\frac{dW_{moon}}{dW_{earth}} = \frac{g_{earth}^2}{g_{moon}^2} = \frac{9.8^2}{1.6^2} = 37{,}515625$

about 38 times

55. $Q(x) = b_0 + b_1(x-a) + b_2(x-a)^2$

$Q'(x) = b_1 + 2b_2(x-a)$

$Q''(x) = 2b_2$

a. i. $Q(a) = f(a) = b_0$

ii. $Q'(a) = f'(a) = b_1$

iii. $Q''(a) = f''(a) = 2b_2 \Rightarrow \frac{f''(a)}{2} = b_2$

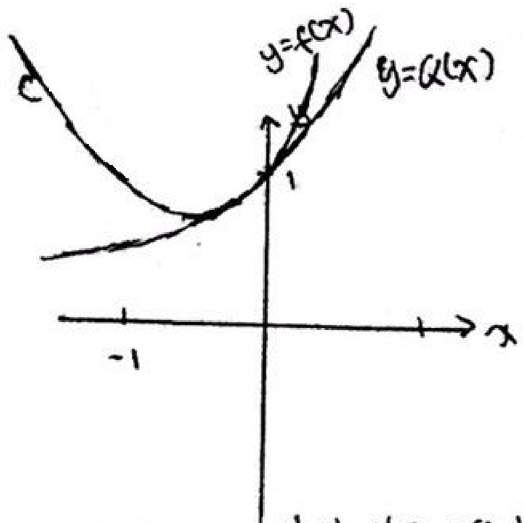
b. $f(x) = (1-x)^{-1}$ $\quad b_0 = f(0) = 1$

$f'(x) = (1-x)^{-2}$ $\quad b_1 = f'(0) = 1$

$f''(x) = 2(1-x)^{-3}$ $\quad b_2 = \frac{f''(0)}{2} = 1$

$Q(x) = 1 + x + x^2$

c.



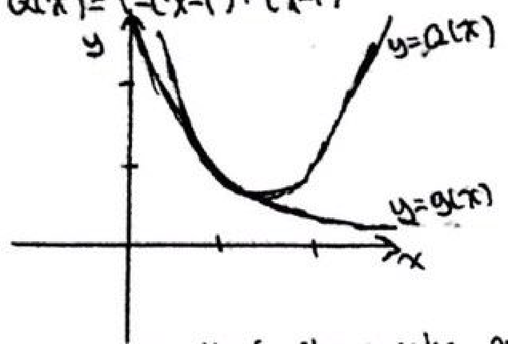
we can see that the graphs are similar at (0,1) since Q(x) is an approximation of f(x) when x=0

d. $g(x) = x^{-1} \Rightarrow b_0 = g(1) = 1$

$g'(x) = -x^{-2} \Rightarrow b_1 = g'(1) = -1$

$g''(x) = 2x^{-3} \Rightarrow b_2 = \frac{g''(1)}{2} = \frac{2}{2} = 1$

$Q(x) = 1-(x-1)+(x-1)^2$



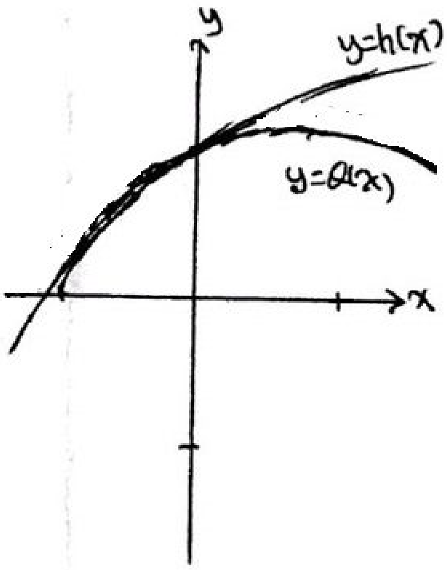
we can see that the graphs are similar at (1,1) since $Q(x)$ is approximation of $g(x)$ when $x=1$

e. $h(x) = \sqrt{1+x} \Rightarrow b_0 = h(0) = 1$

$h'(x) = \frac{1}{2\sqrt{1+x}} \Rightarrow b_1 = h'(0) = \frac{1}{2}$

$h''(x) = -\frac{1}{4(1+x)^{3/2}} \Rightarrow b_2 = \frac{h''(0)}{2} = -\frac{1}{8}$

$Q(x) = 1 + \frac{1}{2}x - \frac{1}{8}x^2$



we can see that the graphs are similar since $Q(x)$ is an approximation of $h(x)$ when $x=0$

f. $L_f(x) = f(0) + f'(0)(x)$
$= 1+x$

$L_g(x) = g(1) + g'(1)(x-1)$
$= 1-1.(x-1)$
$= 2-x$

$L_h(x) = h(0) + h'(0)(x)$
$= 1+\frac{1}{2}x$

56. i. $E(a) = 0 = f(a) - g(a)$

$0 = f(a) - c$

$c = f(a)$

2. $f'(a) = \lim_{x\to a} \frac{f(x)-f(a)}{x-a}$

$= \lim_{x\to a} \frac{f(x)-g(x)}{x-a} + \frac{g(x)-c}{x-a}$

$= \lim_{x\to a} \frac{E(x)}{x-a} + m = m$

now, since $g = f'(a)(x-a) + f(a)$
$= m(x-a)+c$ (proved)