STA2001 Probability and Statistics I
Computer-based Exercise 7

Pooi Yuen KAM

Problem 1. A Probabilistic Approach for Determining the Value of π

The constant π is a well-known scientific constant, and many methods have been devised to measure it accurately. Here, we consider a probabilistic approach.

Let X and Y be two independent random variables, both of which are uniformly distributed over the interval $[-0.5, 0.5]$ and take on values in the xy-plane. Consider the circular region A of radius 0.5 centered at the origin, i.e., $A = \{(x, y) : x^2 + y^2 \leq (0.5)^2\}$. The probability $P(A)$ that the pair $(X, Y) \in A$ is given by

$$P(A) = 0.25\pi$$

You can also estimate this probability accurately using the relative frequency interpretation of $P(A)$. Using the random number generator (e.g., *rand* in MATLAB, *numpy.random.uniform* in Python), generate N pairs of values of the random vector (X, Y). Count the number $N(A)$ of pairs that fall inside the circular region A. You can then estimate $P(A)$ as

$$P(A) = \lim_{N \to \infty} \frac{N(A)}{N}$$

Comment on the accuracy of this method of determining the value of π as a function of the number of trials N. You can also try this out experimentally in practice by using a dartboard.