yohandi - homework week 7.

## Exercises 14.4

3. ~~$w = x^2 + y^2$~~ $w = \frac{x}{z} + \frac{y}{z}$, $x = \cos^2 t$, $y = \sin^2 t$, $z = \frac{1}{t}$

a. $\frac{dw}{dt} = \frac{\partial w}{\partial x}\frac{dx}{dt} + \frac{\partial w}{\partial y}\frac{dy}{dt} + \frac{\partial w}{\partial z}\cdot\frac{dz}{dt}$

$= \left(\frac{1}{z}\right)(-2\cos t \sin t) + \left(\frac{1}{z}\right)(2\sin t \cos t)$

$+ \left(-\frac{(x+y)}{z^2}\right)\left(-\frac{1}{t^2}\right)$

$= \frac{x+y}{z^2 t^2}$

$= 1$

b. $\frac{dw}{dt}(3) = 1$

7. $z = 4e^x \ln(y)$, $x = \ln(u\cos v)$, $y = u \sin v$

a. $\frac{\partial z}{\partial u} = \frac{\partial z}{\partial x}\cdot\frac{\partial x}{\partial u} + \frac{\partial z}{\partial y}\cdot\frac{\partial y}{\partial u}$

$= (4e^x \ln(y))\left(\frac{\cos v}{u\cos v}\right) + \left(\frac{4e^x}{y}\right)(\sin v)$

$= 4\cos v\,(\ln(u\sin v) + 1)$

$\frac{\partial z}{\partial v} = \frac{\partial z}{\partial x}\cdot\frac{\partial x}{\partial v} + \frac{\partial z}{\partial y}\cdot\frac{\partial y}{\partial v}$

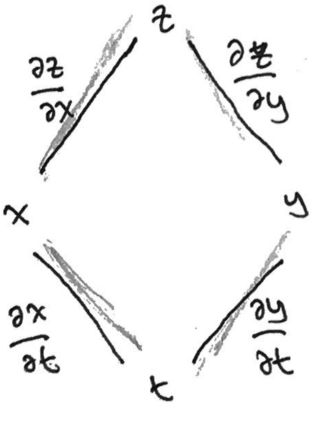
$= (4e^x \ln(y))\left(\frac{-u\sin v}{u\cos v}\right) + \left(\frac{4e^x}{y}\right)(u\cos v)$

$= -4u\sin v \ln(u\sin v) + \frac{4u\cos^2 u}{\sin v}$

b. at $(2, \frac{\pi}{4})$

$\frac{\partial z}{\partial u} = \sqrt{2}(\ln(2)+2)$, $\frac{\partial z}{\partial v} = -2\sqrt{2}(\ln(2)-2)$

13. $\frac{dz}{dt} = \frac{d(f(g(t), h(t)))}{dt}$

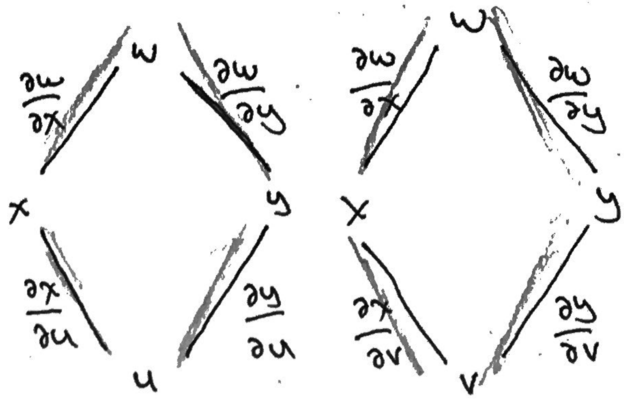



chain rule,

$\frac{dz}{dt} = \frac{\partial z}{\partial x}\cdot\frac{\partial x}{\partial t} + \frac{\partial z}{\partial y}\cdot\frac{\partial y}{\partial t}$

17. $\frac{\partial w}{\partial u} = \frac{\partial(g(h(u,v), k(u,v)))}{\partial u}$

$\frac{\partial w}{\partial v} = \frac{\partial(g(h(u,v), k(u,v)))}{\partial v}$

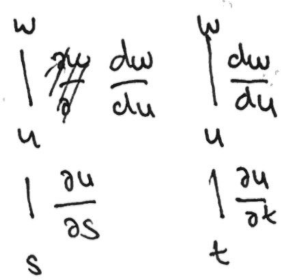



$\frac{\partial w}{\partial u} = \frac{\partial w}{\partial x}\cdot\frac{\partial x}{\partial u} + \frac{\partial w}{\partial y}\cdot\frac{\partial y}{\partial u}$

$\frac{\partial w}{\partial v} = \frac{\partial w}{\partial x}\cdot\frac{\partial x}{\partial v} + \frac{\partial w}{\partial y}\cdot\frac{\partial y}{\partial v}$

21. ~~$\frac{\partial(g(h(s,t)))}{\partial s}$~~ $\frac{\partial w}{\partial s} = \frac{\partial(g(h(s,t)))}{\partial s}$

$\frac{\partial w}{\partial t} = \frac{\partial(g(h(s,t)))}{\partial t}$




~~$\frac{\partial w}{\partial s} = \frac{\partial w}{\partial}$~~ $\frac{\partial w}{\partial s} = \frac{dw}{du}\cdot\frac{\partial u}{\partial s}$

$\frac{\partial w}{\partial t} = \frac{dw}{du}\cdot\frac{\partial u}{\partial t}$

27. $x^2 + xy + y^2 - 7 = 0$

$\frac{dy}{dx} = -\frac{F_x}{F_y} = -\frac{2x+y}{x+2y}$

at $(1,2)$,

$\frac{dy}{dx} = -\frac{2(1)+2}{(1)+2(2)} = -\frac{4}{5}$

32. $xe^y + ye^z + 2\ln x - 2 - 3\ln 2 = 0$

$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z} = -\frac{(e^y + \frac{2}{x})}{ye^z}$

$\frac{\partial z}{\partial y} = -\frac{F_y}{F_z} = -\frac{xe^y + e^z}{ye^z}$

at $(1, \ln 2, \ln 3)$

$\frac{\partial z}{\partial x}(1, \ln 2, \ln 3) = -\frac{4}{3\ln 2}$

$\frac{\partial z}{\partial y}(1, \ln 2, \ln 3) = -\frac{5}{3\ln 2}$

35. $u=0, v=0$ if $w = x^2 + \left(\frac{y}{x}\right), x = u-2v+1,$

$y = 2u+v-2$

$\frac{\partial w}{\partial v} = \frac{\partial w}{\partial x}\cdot\frac{\partial x}{\partial v} + \frac{\partial w}{\partial y}\cdot\frac{\partial y}{\partial v}$

$= \left(2x - \frac{y}{x^2}\right)(-2) + \left(\frac{1}{x}\right)(1)$

$\frac{\partial w}{\partial v}\Big|_{u=0, v=0} = -7$

44. $x = r\cos\theta,\ y = r\sin\theta,\ w = f(x,y)$

a. $\frac{\partial w}{\partial r} = \frac{\partial w}{\partial x}\cdot\frac{\partial x}{\partial r} + \frac{\partial w}{\partial y}\cdot\frac{\partial y}{\partial r}$

$= f_x\cos\theta + f_y\sin\theta.$

$\frac{\partial w}{\partial \theta} = \frac{\partial w}{\partial x}\cdot\frac{\partial x}{\partial \theta} + \frac{\partial w}{\partial y}\cdot\frac{\partial y}{\partial \theta}$

$= f_x\, r(-\sin\theta) + f_y\, r\cos\theta$

$\Rightarrow \frac{1}{r}\cdot\frac{\partial w}{\partial \theta} = -f_x\sin\theta + f_y\cos\theta$

b. note that

$x = r\cos\theta \Rightarrow \cos\theta = \frac{x}{r}$

$y = r\sin\theta \Rightarrow \sin\theta = \frac{y}{r}$

$x^2+y^2 = r^2$

$\cos^2\theta + \sin^2\theta = r^2$

$\Rightarrow$ from part (a),

$\frac{\partial w}{\partial \theta} = -f_x\cdot y + f_y\cdot x$

$\Rightarrow f_y = \frac{1}{x}\frac{\partial w}{\partial \theta} + f_x\frac{y}{x}\ \ldots(1)$

$(1) \to (2)$

$\frac{\partial w}{\partial r} = f_x\cdot\frac{x}{r} + \left(\frac{1}{x}\cdot\frac{\partial w}{\partial \theta} + f_x\cdot\frac{y}{x}\right)\frac{y}{r}$

$= f_x\cdot\frac{r^2}{xr} + \frac{y}{rx}\cdot\frac{\partial w}{\partial \theta}$

$f_x = -\frac{\sin\theta}{r}\cdot\frac{\partial w}{\partial r} + \cos\theta\cdot\frac{\partial w}{\partial r}$

$f_x \to (1)$

$f_y = \frac{1}{x}\frac{\partial w}{\partial \theta} + \left(-\frac{y}{r^2}\cdot\frac{\partial w}{\partial \theta} + \frac{x}{r}\cdot\frac{\partial w}{\partial r}\right)\frac{y}{x}$

$= \sin\theta\cdot\frac{\partial w}{\partial r} + \frac{\cos\theta}{r}\frac{\partial w}{\partial \theta}.$

c. $f_x^2 + f_y^2 = \left(-\frac{\sin\theta}{r}\cdot\frac{\partial w}{\partial \theta} + \cos\theta\cdot\frac{\partial w}{\partial r}\right)^2 +$

$\left(\sin\theta\cdot\frac{\partial w}{\partial r} + \frac{\cos\theta}{r}\cdot\frac{\partial w}{\partial \theta}\right)^2$

$= \left(\frac{\partial w}{\partial r}\right)^2 + \frac{1}{r^2}\left(\frac{\partial w}{\partial \theta}\right)^2$

49. $x = \cos t,\ y = \sin t,\ 0 \le t \le 2\pi \Rightarrow T = f(x,y)$

$\frac{\partial T}{\partial x} = 8x - 4y,\quad \frac{\partial T}{\partial y} = 8y - 4x$

a. $\frac{dT}{dt} = \frac{\partial T}{\partial x}\left(\frac{dx}{dt}\right) + \frac{\partial T}{\partial y}\left(\frac{dy}{dt}\right)$

$= (8x-4y)(-\sin t) + (8y-4x)(\cos t)$

$= -4\cos 2t$

$\frac{d}{dt}(-4\cos 2t) = 8\sin(2t)$

when $-4\cos 2t = 0$,

$t = \left\{\frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}\right\}$

$\therefore$ max $= \left\{\left(-\frac{\sqrt2}{2}, \frac{\sqrt2}{2}\right), \left(\frac{\sqrt2}{2}, -\frac{\sqrt2}{2}\right)\right\}$

min $= \left\{\left(\frac{\sqrt2}{2}, \frac{\sqrt2}{2}\right), \left(-\frac{\sqrt2}{2}, -\frac{\sqrt2}{2}\right)\right\}$

b. $T = 4x^2 - 4xy + 4y^2$

$f(x,y) = f\left(\frac{\sqrt2}{2}, \frac{\sqrt2}{2}\right) = 2$

$f(x,y) = f\left(-\frac{\sqrt2}{2}, \frac{\sqrt2}{2}\right) = 6$

$\therefore$ $T_{max} = 6$ and $T_{min} = 2$

51. let $u = x^2$ and $g(t,x) = \sqrt{t^4 + x^3}$

$F(x) = G(u,x) = \int_0^4 g(t,x)\,dt = \int_0^{x^2}\sqrt{t^4+x^3}\,dt$

by chain rule,

$F'(x) = G_u(u,x)\cdot u_x + G_x(u,x)\,x'$

$= 2x\ G_u(u,x) + G_x(u,x)$

$G_u(u,x) = \frac{d}{du}\int_0^u g(u,x)\,dt$ $\quad G_x(u,x) = \int_0^u g_x(t,x)\,dt$

$= \sqrt{x^8+x^3}$ $\quad = \int_0^{x^2} \frac{3x^2}{2\sqrt{t^4+x^3}}\,dt$

$\therefore F'(x) = 2x\sqrt{x^8+x^3} + \int_0^{x^2} \frac{3x^2}{2\sqrt{t^4+x^3}}\,dt$

52. let $u = x^2$ and $g(t,x) = \sqrt{t^3+x^2}$.

$F(x) = G(u,x) = \int_u^1 g(t,x)\,dt = -\int_1^{x^2} \sqrt{t^3+x^2}\,dt$

by chain rule,

$F'(x) = G_u(u,x).u_x + G_x(u,x)x'$

$= 2x\, G_u(u,x) + G_x(u,x)$

$G_u(u,x) = -\frac{d}{du}\int_1^u g(t,x)\,dt$

$= -\sqrt{x^6+x^2}$

$G_x(u,x) = -\int_1^u g_x(t,x)\,dt$

$= -\int_1^{x^2} \frac{x}{2\sqrt{t^3+x^2}}\,dt$

$\therefore F'(x) = -2x\sqrt{x^6+x^2} - \int_1^{x^2} \frac{x}{2\sqrt{t^3+x^2}}\,dt$

Exercises 14.5

3. $g(x,y) = xy^2 \quad (2,-1)$

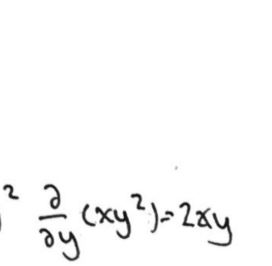
$\frac{\partial}{\partial x}(xy^2) = y^2 \quad \frac{\partial}{\partial y}(xy^2) = 2xy$

$\nabla g = \langle y^2, 2xy \rangle$

$\nabla g(2,-1) = \langle (-1)^2, 2(2)(-1) \rangle$

$= i - 4j$

$g(2,-1) = (2)(-1)^2 = 2$




$\therefore \nabla g = i - 4j$ starts at $(2,-1)$ on the curve $xy^2 = 2$ and ends at the point $(3,-5)$

9. $\frac{\partial f}{\partial x} = -\frac{1}{2}(x^2+y^2+z^2)^{-3/2}(2x) + \frac{1}{xyz}(yz)$

$= -x(x^2+y^2+z^2)^{-3/2} + \frac{1}{x}$

$\frac{\partial f}{\partial y} = -\frac{1}{2}(x^2+y^2+z^2)^{-3/2}(2y) + \frac{1}{xyz}(xz)$

$= -y(x^2+y^2+z^2)^{-3/2} + \frac{1}{y}$

$\frac{\partial f}{\partial z} = -\frac{1}{2}(x^2+y^2+z^2)^{-3/2}(2z) + \frac{1}{xyz}(xy)$

$= -z(x^2+y^2+z^2)^{-3/2} + \frac{1}{z}$

$\frac{\partial f(-1,2,-2)}{\partial x} = -\frac{26}{27}$

$\frac{\partial f(-1,2,-2)}{\partial y} = \frac{23}{54} \quad \frac{\partial f(-1,2,-2)}{\partial z} = -\frac{23}{54}$

$\nabla f(x,y,z) = \langle \frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z} \rangle$

$\nabla f(1,1,1) = \langle -\frac{26}{27}, \frac{23}{54}, \frac{-23}{54} \rangle$

13. $g(x,y) = \frac{x-y}{xy+2}$, $P_0(1,-1)$, $u = 12i + 5j$

$\frac{\partial g}{\partial x} = \frac{(xy+2) - (x-y)(y)}{(xy+2)^2} = \frac{y^2+2}{(xy+2)^2}$

$\frac{\partial g}{\partial y} = \frac{(xy+2)(-1) - (x-y)(x)}{(xy+2)^2} = -\frac{x^2+2}{(xy+2)^2}$

$\frac{\partial f(1,-1)}{\partial x} = 3 \quad \frac{\partial f}{\partial y}(1,-1) = -3$

$\nabla f(1,-1) = \langle 3,-3 \rangle$

$v = \frac{u}{|u|} = \frac{\langle 12,5 \rangle}{13} = \langle \frac{12}{13}, \frac{5}{13} \rangle$

$D_v f = \nabla f \cdot v = \langle 3,-3 \rangle \cdot \langle \frac{12}{13}, \frac{5}{13} \rangle = \frac{21}{13}$

17. $g(x,y,z) = 3e^x \cos(yz)$, $P_0(0,0,0)$, $u = 2i + j - 2k$

$\frac{\partial g}{\partial x} = 3e^x \cos yz. \quad \frac{\partial g}{\partial z} = -3ye^x \sin yz$

$\frac{\partial g}{\partial y} = -3ze^x \sin yz$

$\frac{\partial g}{\partial x}(0,0,0) = 3 \quad \frac{\partial g}{\partial z}(0,0,0) = 0$

$\frac{\partial g}{\partial y}(0,0,0) = 0$

$\nabla g(2,0,0) = \langle 3,0,0\rangle$

$v = \frac{u}{|u|} = \frac{\langle 2,1,-2\rangle}{\sqrt{2^2+1^2+(-2)^2}} = \langle \frac{2}{3}, \frac{1}{3}, -\frac{2}{3}\rangle$

$D_v g = \nabla g \cdot v = \langle 3,0,0\rangle \cdot \langle \frac{2}{3}, \frac{1}{3}, -\frac{2}{3}\rangle = 2$

19. $f(x,y)\cancel{z} = x^2+xy+y^2$, $P_0(-1,1)$

$\frac{\partial f}{\partial x} = 2x+y \qquad \frac{\partial f}{\partial y} = x+2y$

$\frac{\partial f}{\partial x}(-1,1) = -1 \qquad \frac{\partial f}{\partial y}(-1,1) = 1$

$\nabla f(-1,1) = \langle -1,1\rangle$

$u_{max} = \frac{\nabla f(-1,1)}{|\nabla f(-1,1)|} = \langle -\frac{1}{\sqrt2}, \frac{1}{\sqrt2}\rangle$

$u_{min} = -u_{max} = \langle \frac{1}{\sqrt2}, -\frac{1}{\sqrt2}\rangle$

$D_v f_{max} = \nabla f(-1,1) \cdot u_{max} = \sqrt2$

$D_u f_{min} = \nabla f(-1,1) \cdot u_{min} = -\sqrt2$

21. $f(x,y,z) = (\frac{x}{y}) - yz$, $P_0(4,1,1)$

$\frac{\partial f}{\partial x} = \frac{1}{y} \qquad \frac{\partial f}{\partial y} = -\frac{x}{y^2} - z \qquad \frac{\partial f}{\partial z} = -y$

$\frac{\partial f}{\partial x}(4,1,1) = 1 \qquad \frac{\partial f}{\partial y}(4,1,1) = -5 \qquad \frac{\partial f}{\partial z}(4,1,1) = -1$

$\nabla f(4,1,1) = \langle 1,-5,-1\rangle$

$u_{max} = \frac{\nabla f(4,1,1)}{|\nabla f(4,1,1)|} = \langle \frac{1}{3\sqrt3}, -\frac{5}{3\sqrt3}, -\frac{1}{3\sqrt3}\rangle$

$u_{min} = -u_{max} = \langle -\frac{1}{3\sqrt3}, \frac{5}{3\sqrt3}, \frac{1}{3\sqrt3}\rangle$

$D_u f_{max} = \nabla f(4,1,1) \cdot u_{max} = 3\sqrt3$

$D_u f_{min} = -D_u f_{max} = -3\sqrt3$

27. $xy = -4 \quad (2,-2)$

→ gradient:

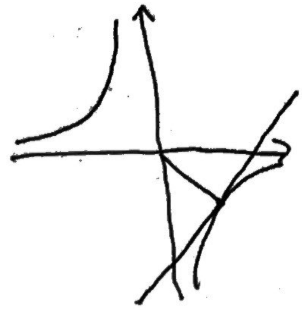
$\nabla f(x,y) = \frac{\partial f}{\partial x} + \frac{\partial f}{\partial y} = yi + xj$

$\nabla f(2,-2) = -2i + 2j$

→ at $(2,-2)$ is the line:

$-2(x-2) + 2(y+2) = 0$

$-2x + 2y = -8$



31. $f(x,y) = xy + y^2 \qquad P(3,2)$

$\frac{\partial f}{\partial x} = y = 2 \qquad \frac{\partial f}{\partial y} = x+2y = 7$

$\nabla f = 2i + 7j$

$D_u f = \nabla f \cdot u$

$\cancel{\partial = \frac{7}{\sqrt{35}} \quad +2} \quad 0 = \langle \frac{2}{\sqrt{53}}, \frac{7}{\sqrt{53}}\rangle > u$

$u = \frac{\pm 7}{\sqrt{35}} i + \frac{-2}{\sqrt{53}} j$

36. $v = i + j - k$.

$\nabla f = kv = k(i+j-k)$

$\Rightarrow |kv| = \sqrt3 k$

from the equation

$2\sqrt3 = \sqrt3 k$

$k = 2$

$\nabla f = 2i + 2j - 2k$

$D_u f = \nabla f \cdot u = \langle 2,2,-2\rangle \cdot \langle \frac{1}{\sqrt2}, \frac{1}{\sqrt2}\rangle = 2\sqrt2$

37. Directional derivative $D_u f$ is a scalar component. If the gradient vector $\nabla f$ is evaluated at point p in the direction of u.

$\Rightarrow D_u f = \nabla f \cdot u$

Exercises 14.6

3. $2z - x^2 = 0$, $P_0(2,0,2)$

a. $f_x = -2x \qquad f_x(2,0,2) = -4$

$f_y = 0 \qquad f_y(2,0,2) = 0$

$f_z = 2 \qquad f_z(2,0,2) = 2$

$\nabla f(2,0,2) = \langle -4, 0, 2 \rangle$

$-4(x-2) + 0(y-0) + 2(z-2) = 0$

$-4x + 8 + 2z - 4 = 0$

$-2x + z = -2$

b. $x = 2 - 4t$

$y = 0 + 0t = 0$

$z = 2 + 2t$

8. $x^2 + y^2 - 2xy - x + 3y - z = -4$, $P_0(2,-3,18)$

a. $f_x = 2x - 2y - 1 \qquad f_x(2,-3,18) = 9$

$f_y = 2y - 2x + 3 \qquad f_y(2,-3,18) = -7$

$f_z = -1 \qquad f_z(2,-3,18) = -1$

$\nabla f(2,-3,18) = \langle 9, -7, -1 \rangle$

$9(x-2) - 7(y+3) - 1(z-18) = 0$

$9x - 18 - 7y - 21 - z + 18 = 0$

$9x - 7y - z = 21$

b. $x = 2 + 9t$

$y = -3 - 7t$

$z = 18 - t$

11. given:

$z = \sqrt{y-x}$ , $(1,2,1)$

$f_x = -\frac{1}{2}(y-x)^{-1/2} \qquad f_y = \frac{1}{2}(y-x)^{-1/2} \qquad f_z = -1$

$f_x(1,2,1) = -\frac{1}{2} \qquad \nabla f(1,2,1) = \langle -\frac{1}{2}, \frac{1}{2}, -1 \rangle$

$f_y(1,2,1) = \frac{1}{2}$

$f_z(1,2,1) = -1$

$-\frac{1}{2}(x-1) + \frac{1}{2}(y-2) - (z-1) = 0$

$x - y + 2z = 1$

17. surfaces: $x^3 + 3x^2y^2 + y^3 + 4xy - z^2 = 0$

$x^2 + y^2 + z^2 = 11$

point: $(1,1,3)$

$\nabla f(1,1,3) = \left((3x^2 + 6xy^2 + 4y)\,i + (6x^2y + 3y^2 + 4x)\,j - (2z)\,k\right)_{(1,1,3)}$

$= 13i + 13j - 6k.$

$\nabla g(1,1,3) = \left((2x)i + (2y)j + (2z)k\right)_{(1,1,3)}$

$= 2i + 2j + 6k$

$$\begin{vmatrix} i & j & z \\ 13 & 13 & -6 \\ 2 & 2 & 6 \end{vmatrix} = 90i - 90j$$

$\Rightarrow$ tangent line in parametric equations:

$x = 1 + 90t$

$y = 1 - 90t$

$z = 3$

19. $f(x,y,z) = \ln(\sqrt{x^2+y^2+z^2}) = \frac{1}{2}\ln(x^2+y^2+z^2)$

$\frac{\partial f}{\partial x} = \frac{1}{2} \cdot \frac{2x}{x^2+y^2+z^2} = \frac{x}{x^2+y^2+z^2}$

$\frac{\partial f}{\partial y} = \frac{1}{2} \cdot \frac{2y}{x^2+y^2+z^2} = \frac{y}{x^2+y^2+z^2}$

$\frac{\partial f}{\partial z} = \frac{1}{2} \cdot \frac{2z}{x^2+y^2+z^2} = \frac{z}{x^2+y^2+z^2}$

$\Rightarrow \nabla f = \left\langle \frac{x}{x^2+y^2+z^2}, \frac{y}{x^2+y^2+z^2}, \frac{z}{x^2+y^2+z^2} \right\rangle$

$\nabla f(3,4,12) = \left\langle \frac{3}{169}, \frac{4}{169}, \frac{12}{169} \right\rangle$

$u = \frac{v}{|v|} = \frac{\langle 3, 6, -2 \rangle}{\sqrt{3^2 + 6^2 + (-2)^2}} = \left\langle \frac{3}{7}, \frac{6}{7}, -\frac{2}{7} \right\rangle$

$df = (\nabla f(3,4,12) \cdot u)\, ds$

$= \left( \left\langle \frac{3}{169}, \frac{4}{169}, \frac{12}{169} \right\rangle \cdot \left\langle \frac{3}{7}, \frac{6}{7}, -\frac{2}{7} \right\rangle \right) \left(\frac{1}{10}\right)$

$= \frac{9}{11830} \approx 0.00076$

23a. at $P(\frac{1}{2}, \frac{1}{2}\sqrt{3})$ in the direction of motion is

$u = \frac{1}{2}\sqrt{3}\, i - \frac{1}{2} j$

the gradient of T:

$$\nabla T\big|_{(\frac{1}{2},\frac{1}{2}\sqrt{3})} = \left((\sin 2y)i + (2x\cos 2y)j\right)\big|_{(\frac{1}{2},\frac{1}{2}\sqrt{3})}$$

$$= \sin\sqrt{3}\, i + \cos\sqrt{3}\, j$$

$$\Rightarrow D_uT\left(\tfrac{1}{2},\tfrac{\sqrt{3}}{2}\right) = \nabla T\cdot u = \left((\sin\sqrt{3})i + (\cos\sqrt{3})j\right)\cdot\left(\tfrac{\sqrt{3}}{2}i - \tfrac{1}{2}j\right)$$

$$= \left(\frac{\sqrt{3}}{2}\sin\sqrt{3} - \frac{1}{2}\cos\sqrt{3}\right)^{\circ} c/m$$

b. $r(t) = (\sin 2t)i + (\cos 2t)j$

$v(t) = (2\cos 2t)i - (2\sin 2t)j \Rightarrow |v| = 2$

$$\frac{dT}{dt} = \frac{dT}{dx}\cdot\frac{dx}{dt} + \frac{dT}{dy}\cdot\frac{dy}{dt} = (\nabla T, v) = \left(\nabla T, \frac{v}{|v|}\right)|v|$$

$$= (D_uT)|v| = \left(\frac{\sqrt{3}}{2}\sin\sqrt{3} - \frac{1}{2}\cos\sqrt{3}\right)\cdot 2$$

$$= \sqrt{3}\sin\sqrt{3} - \cos\sqrt{3} \approx 1.87^{\circ}C/sec$$

25. $f(x,y) = x^2+y^2+1$

$f_x(x,y) = 2x$

$f_y(x,y) = 2y$

a. let $(x_0,y_0) = (0,0)$

$L(x,y) = f(0,0) + f_x(0,0)(x-0) + f_y(0,0)(y-0)$

$= (0+0+1) + 2(0)x + 2(0)y$

$= 1$

b. let $(x_0,y_0) = (1,1)$

$L(x,y) = f(1,1) + f_x(1,1)(x-1) + f_y(1,1)(y-1)$

$= 3+2x-2+2y-2$

$= 2(x+y)-1$

29. $f(x,y) = e^x\cos y$

$f_x(x,y) = e^x\cos y$

$f_y(x,y) = -e^x\sin y$

a. let $(x_0,y_0) = (0,0)$

$L(x,y) = f(0,0) + f_x(0,0)(x-0) + f_y(0,0)(y-0)$

$= 1 + 1(x-0) + 0(y-0)$

$= 1+x$

b. let $(x_0,y_0) = (0,\frac{\pi}{2})$

$L(x,y) = f(0,0) + f_x(0,0)(x-0) + f_y(0,0)(y-0)$

$= 0 + 0(x-0) - 1(y-\frac{\pi}{2})$

$= y - \frac{\pi}{2}$

35. $f(x,y) = 1+y+x\cos y$ at $P_0(0,0)$

$R: |x| \le 0.2,\ |y| \le 0.2$

$f_x(x,y) = \cos y \Rightarrow f_x(0,0) = 1$

$f_y(x,y) = 1 - x\sin y \Rightarrow f_y(0,0) = 1$

$L(x,y) = f(0,0) + f_x(0,0)(x-0) + f_y(0,0)(y-0)$

$= 1 + 1(x-0) + 1(y-0)$

$= x+y+1$

since

$f_{xx} = 0,\ f_{yy} = -x\cos y,\ f_{xy} = \sin y$, and

$|-x\cos y| \le |x| \le 0.2$ and $|\sin y| \le 1$

$|E| \le \frac{1}{2}(1)(|x|+|y|)^2 \le \frac{1}{2}(0.2+0.2)^2 = 0.08$

$\therefore |E| \le 0.08$

41. $f(x,y,z) = \sqrt{x^2+y^2+z^2}$

$$f_x(x,y,z) = \frac{x}{\sqrt{x^2+y^2+z^2}}$$

$$f_y(x,y,z) = \frac{y}{\sqrt{x^2+y^2+z^2}}$$

$$f_z(x,y,z) = \frac{z}{\sqrt{x^2+y^2+z^2}}$$

a. at $P_0(1,0,0)$

$f(1,0,0) = 1$

$f_x(1,0,0) = 1$

$f_y(1,0,0) = 0$

$f_z(1,0,0) = 0$

$L(x,y,z) = f(P_0) + f_x(P_0)(x-x_0) + f_y(P_0)(y-y_0) + f_z(P_0)(z-z_0)$

$= x$

b. at $P_0(1,1,0)$

$f(1,1,0) = \sqrt{2}$

$f_x(1,1,0) = \frac{1}{\sqrt{2}}$

$f_y(1,1,0) = \frac{1}{\sqrt{2}}$

$f_z(1,1,0) = 0$

$$L(x,y,z) = f(P_0) + f_x(P_0)(x-x_0) + f_y(P_0)(y-y_0) + f_z(P_0)(z-z_0)$$
$$= \sqrt{2} + \frac{1}{\sqrt{2}}(x-1) + \frac{1}{\sqrt{2}}(y-1) + 0(z-0)$$
$$= \frac{1}{\sqrt{2}}x + \frac{1}{\sqrt{2}}y$$

c. at $P_0(1,2,2)$

$f(1,2,2) = 3$

$f_x(1,2,2) = \frac{1}{3}$

$f_y(1,2,2) = \frac{2}{3}$

$f_z(1,2,2) = \frac{2}{3}$

$$L(x,y,z) = f(P_0) + f_x(P_0)(x-x_0) + f_y(P_0)(y-y_0) + f_z(P_0)(z-z_0)$$
$$= 3 + \frac{1}{3}(x-1) + \frac{2}{3}(y-2) + \frac{2}{3}(z-2)$$
$$= \frac{1}{3}x + \frac{2}{3}y + \frac{2}{3}z$$

47. $f(x,y,z) = xy + 2yz - 3xz$ at $P_0(1,1,0)$

$R: |x-1| \le 0.01,\ |y-1| \le 0.01,\ |z| \le 0.01$

$f(1,1,0) = 1$

$f_x(x,y,z) = y - 3z \Rightarrow f_x(1,1,0) = 1$

$f_y(x,y,z) = x + 2z \Rightarrow f_y(1,1,0) = 1$

$f_z(x,y,z) = 2y - 3x \Rightarrow f_z(1,1,0) = -1$

$$L(x,y,z) = f(P_0) + f_x(P_0)(x-x_0) + f_y(P_0)(y-y_0) + f_z(P_0)(z-z_0)$$
$$= 1 + (x-1) + (y-1) + (z-0)$$
$$= x + y + z - 1$$

$|E| \le \frac{1}{2} \cdot 3(0.01 + 0.01 + 0.02)^2 = 0.00135$

51. let B be required area of a rectangle, x its length and y its width. We know from the task that $x > y$.

The area is calculate as

$$B = xy \Rightarrow dB = y\,dx + x\,dy$$

From the previous result from, we can notice that value of y will change more its area than dx and we already can see that dy will be greater than dx and we already know that $x > y_0$ ~~so they~~ therefore $x\,dy > y\,dx$

(Pay more attention to the smaller of the two dimensions)

55. at $(x_0, y_0)$

$L(x_0, y_0) = f(x_0, y_0)$

~~form~~

the plane contains $x = x_0$, $y = y_0$, and $z = f(x_0, y_0)$

$\Rightarrow$ it contains point $(x_0, y_0, f(x_0, y_0))$

- vector n is a vector perpendicular to the surface $z = f(x,y)$.
- given surface is the level surface $F(x,y,z) = 0$ of the function $F(x,y,z) = f(x,y) - z$.

$\Rightarrow n = \nabla F = \langle F_x, F_y, F_z \rangle = \langle f_x, f_y, -1 \rangle$

$\therefore$ the tangent plane is a plane passing through $P_0(x_0, y_0, f(x_0, y_0))$ with normal vector $n = \langle f_x(x_0,y_0), f_y(x_0,y_0), -1 \rangle$