yohandi - homework for week 5

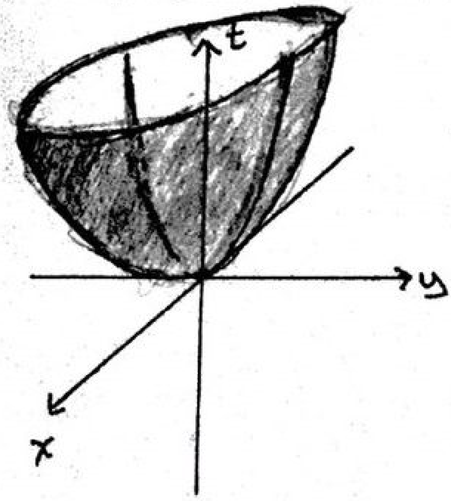
Exercises 12.6

6. (e)

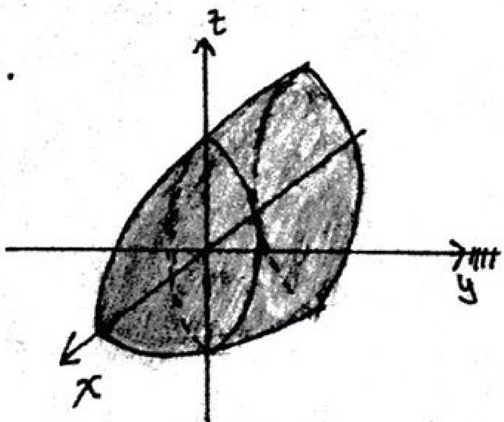
7. (b)

9. ~~[illegible]~~ (k)

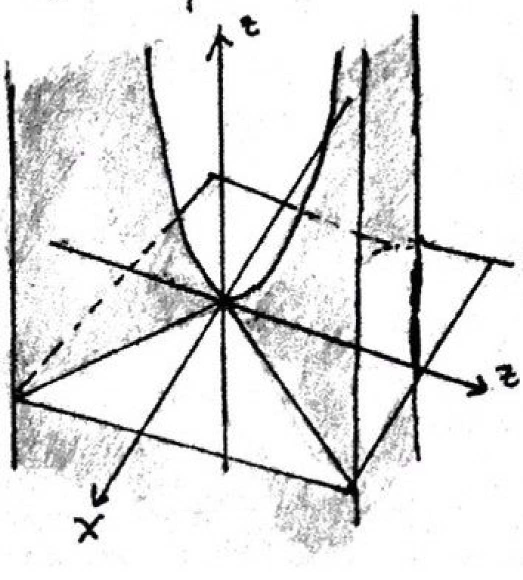
21.



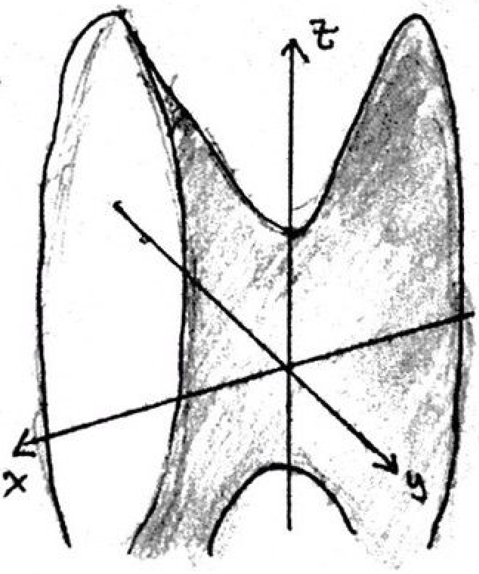
23.



31.



43.



Exercises 13.1

3. $v(t) = \frac{dr(t)}{dt} = e^t i + \frac{2}{9} \cdot 2e^{2t} j$
$= e^t i + \frac{4}{9} \cdot e^{2t} j$

$v(\ln(3)) = 3i + 4j$

$a(t) = \frac{dv(t)}{dt} = e^t i + \frac{4}{9} \cdot 2e^{2t} j$
$= e^t i + \frac{8}{9} e^{2t} j$

$a(\ln(3)) = 3i + 8j$

9. $v(t) = \frac{dr(t)}{dt} = i + 2t\, j + 2k$

$v(1) = i + 2j + 2k$

$a(t) = \frac{dv(t)}{dt} = 2j$

$a(1) = 2j$

$\text{speed}(1) = \sqrt{1^2 + 2^2 + 2^2} = 3$

$\text{direction}(1) = \frac{v(1)}{\text{speed}(1)} = \frac{1}{3} i + \frac{2}{3} j + \frac{2}{3} k$

$\Rightarrow i + 2j + 2k = 3\left(\frac{1}{3} i + \frac{2}{3} j + \frac{2}{3} k\right)$

11. $v(t) = \frac{dr(t)}{dt} = (-2\sin t)i + (3\cos t)j + 4k.$

$v(\frac{\pi}{2}) = -2i + 4k$

$a(t) = \frac{dv(t)}{dt} = (-2\cos t)i + (-3\sin t)j$

$a(\frac{\pi}{2}) = -3j$

$\text{speed}(\frac{\pi}{2}) = \sqrt{(-2)^2 + 4^2} = 2\sqrt{5}$

$\text{direction}(\frac{\pi}{2}) = \frac{v(\frac{\pi}{2})}{\text{speed}(\frac{\pi}{2})} = -\frac{1}{\sqrt{5}} i + \frac{2}{\sqrt{5}} k$

$\Rightarrow -2i + 4k = 2\sqrt{5}\left(-\frac{1}{\sqrt{5}} i + \frac{2}{\sqrt{5}} k\right)$

19. $v(t) = \frac{dr(t)}{dt} = (\cos t)i + (2t + \sin t)j + (e^t)k$

$\text{direction}(0) = \lambda . v(0) = \lambda i + \lambda k$

$r(0) = -j + k$

line ℓ:

$$\begin{pmatrix} 0 \\ -1 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} \Rightarrow \begin{matrix} x = t \\ y = -1 \\ z = 1 + t \end{matrix}$$

21. $v(t) = \frac{dr(t)}{dt} = \frac{1}{t} i + \frac{3}{(t+2)^2} j + (\ln t + 1)k$

$\text{direction}(1) = \lambda . v(1) = \lambda i + \frac{\lambda}{3} j + \lambda k$

$r(1) = 0$

line ℓ:

$$\lambda \begin{pmatrix} 1 \\ \frac{1}{3} \\ 1 \end{pmatrix} = \mu \begin{pmatrix} 3 \\ 1 \\ 3 \end{pmatrix} \Rightarrow \begin{matrix} x = 3t \\ y = t \\ z = 3t \end{matrix}$$

23a. i) $v(t) = (-\sin t)i + (\cos t)j$

$\text{speed}(t) = \sqrt{(-\sin t)^2 + (\cos t)^2} = 1$

the particle moves at a constant speed 1

ii) $a(t) = (-\cos t)i + (-\sin t)j$

$a(t) \cdot v(t) = \sin t \cdot \cos t - \sin t \cdot \cos t = 0$

the particle's acceleration vector always orthogonal to its velocity vector

iii) counter clockwise

iv) $r(0) = i$

the particle begins at $(1,0)$

b. i) $v(t) = (-2\sin 2t)i + (2\cos 2t)j$

$\text{speed}(t) = \sqrt{(-2\sin 2t)^2 + (2\cos 2t)^2} = 2$

the particle moves at a constant speed 2

ii) $a(t) = (-4\cos 2t)i + (-4\sin 2t)j$

$a(t) \cdot v(t) = 8\sin 2t \cos 2t - 8\sin 2t \cos 2t = 0$

the particle's acceleration vector always orthogonal to its velocity vector

iii) counter clockwise

iv) $r(0) = i$

the particle begins at $(1,0)$

c. i) let $\theta = t - \frac{\pi}{2} \Rightarrow \frac{d\theta}{dt} = 1$

$v(t) = (-\sin\theta)i + (\cos\theta)j$

$\text{speed}(t) = \sqrt{(-\sin\theta)^2 + (\cos\theta)^2} = 1$

the particle moves at a constant speed 1

ii) $a(t) = (-\cos\theta)i + (-\sin\theta)j$

$a(t) \cdot v(t) = \sin\theta\cos\theta - \sin\theta\cos\theta = 0$

the particle's acceleration vector always orthogonal to its velocity vector

iii) counter clock wise

iv) $r(0) = -j$

the particle doesn't begin at (1,0)

27. $\frac{d}{dt}(|r(t)|^2) = \frac{d}{dt}(r(t) \cdot r(t)) = \underbrace{r(t) \cdot \frac{d(r(t))}{dt}}_{0} \cdot 2$

$= 0$

$\Rightarrow \frac{d}{dt}(|r(t)|^2) = 2 \cdot |r(t)| \frac{d}{dt}(|r(t)|) = 0$

case $|r(t)| = 0$:

$r(t) = 0$ (constant)

case $\frac{d}{dt}(|r(t)|) = 0$:

$\Rightarrow |r(t)|$ is constant

$\therefore r(t) \cdot \frac{d(r(t))}{dt} = 0 \Rightarrow |r(t)|$ is constant

Exercises 13.2

3. $\int_{-\pi/4}^{\pi/4} [(\sin t)i + (1+\cos t)j + (\sec^2 t)k]\,dt$

$= -\cos t\, i + (t+\sin t)j + (\tan t)k \Big|_{t=-\pi/4}^{\pi/4}$

$= (\frac{\pi}{2} + \sqrt{2})j + 2k$

13. $r(t) = \int [\frac{3}{2}(t+1)^{1/2} i + e^{-t} j + \frac{1}{t+1} k]\,dt$

$= ((t+1)^{3/2} + C_1)i + (-e^{-t} + C_2)j + (\ln|t+1| + C_3)k$

$r(0) = (C_1+1)i + (C_2-1)j + (C_3)k$

$= k$

$\Rightarrow C_1 = -1,\ C_2 = 1,\ C_3 = 1$

$\Rightarrow r(t) = ((t+1)^{3/2} - 1)i + (1 - e^{-t})j + (\ln|t+1| + 1)k$

19. $x(t) = v_x(t) \cdot t$

$21000 = 840\cos(60°) \cdot t$

$\Rightarrow t = 50$ s

25. $\rightarrow t = \frac{2v_y(t)}{g} = \frac{2 \cdot 400\sin(\theta)}{9.8} \approx 81.633\sin\theta$

$\rightarrow t = \frac{x(t)}{v_x(t)} = \frac{16000}{400\cos\theta} = \frac{40}{\cos\theta}$

$\Rightarrow \frac{40}{\cos(\theta)} = 81.633\sin\theta$

$\sin(2\theta) = \frac{80}{81.633}$

$2\theta \approx 78.520°$ $\quad 2\theta \approx 101.480°$

$\theta \approx 39.260°$ $\quad \theta \approx 50.740°$

33a. $r(t) = v_x(t) \cdot t\ i + v_y(t) \cdot t \cdot j$

$= (12\cos 27° \cdot t \cdot i + (1.3 + 12\sin 27° t - \frac{1}{2} \cdot 9.8t^2)j)$ m

$\approx (10.692t\ i + (1.3 + 5.448t - 4.9t^2)j)$ m

b. $\frac{dv_y(t)}{dt} \approx 5.448 - 9.8t = 0 \Rightarrow t \approx 0.556$ s

$r_y(0.556) = (1.3 + 5.448 \cdot 0.556 - 4.9 \cdot 0.556^2)$ m

$= 2.814$ m

c. $r_y(t) = 1.3 + 5.448t - 4.9t^2$

$0 = 1.3 + 5.448t - 4.9t^2$

$\Rightarrow t \approx -0.202$ s (rejected) $\quad t \approx 1.314$ s

$x(1.314) \approx 10.692 \cdot 1.314$ m

≈ 14.049 m

d. $r_y(t) = 1.3 + 5.448t - 4.9t^2$

$2.3 = 1.3 + 5.448t - 4.9t^2$

$\Rightarrow t \approx 0.232$ s $\quad t \approx 0.880$ s

$\Rightarrow x(1.314 - 0.232)$ $\quad x(1.314 - 0.880)$

≈ 11.569 m $\quad \approx 4.640$ m

e. $r_x(t) \approx 10.692t$

$4 \approx 10.692t$

$t \approx 0.374$ s

$r_y(0.374) \approx 2.652$ m > 2.5 m

$\Rightarrow$ the ball will pass the net

41a. $R_1'(t) = R_2'(t)$

$\int R_1'(t)\,dt = \int R_2'(t)\,dt$

$R_1(t) + C_1 = R_2(t) + C_2$

$R_1(t) - R_2(t) = C_2 - C_1 = C$

since $R_1(t)$ and $R_2(t)$ are vectors,

C is also a vector.

$\Rightarrow$ both functions differ by a constant vector value

b. $R_1'(t) = R_2'(t) = r(t)$

$\Rightarrow R_1(t) - R_2(t) = C$

$\Rightarrow R_2(t) = R_1(t) + C'$

$\Rightarrow R_2(t) = R(t) + C'$ $(R_1(t) = R(t))$

42. Fundamental Theorem of Calculus:

$$\frac{d}{dx}\int_a^x f(t)\,dt = f(x)$$

$$\Rightarrow \frac{d}{dt}\int_a^t r(\tau)\,d\tau = \frac{d}{dt}\int_a^t f(\tau)d\tau + \frac{d}{dt}\int_a^t g(\tau)d\tau + \frac{d}{dt}\int_a^t h(\tau)d\tau$$

$$= f(t) + g(t) + h(t)$$

$$= r(t)$$

$$\Rightarrow \int_a^b r(t)\,dt = \int_a^x r(t)\,dt + \int_x^b r(t)\,dt$$

let $R(t) + C$ denotes the antiderative of vector $r(t)$,

$$= (R(t)+C)\Big|_{t=a}^{x} - (R(t)+C)\Big|_{t=b}^{x}$$

$$= (R(x) - R(a)) - (R(x) - R(b))$$

$$= R(b) - R(a)$$

44. $r_y(t) = V_y(0).t - \frac{1}{2}gt^2$

$r_y(t)_{max}$ when $V_y(t) = 0$,

$$0 = V_y(0) - gt$$

$$t = \frac{V_y(0)}{g}$$

$$\Rightarrow r_y(t)_{max} = \frac{V_y(0)^2}{g} - \frac{V_y(0)^2}{2g} = \frac{V_y(0)^2}{2g}$$

when $t = \frac{V_y(0)}{2g}$,

$$r_y(t) = \frac{V_y(0)^2}{2g} - \frac{1}{2}\cdot g\left(\frac{V_y(0)}{2g}\right)^2$$

$$= \frac{3V_y(0)^2}{8g}$$

$$= \frac{3}{4} r_y(t)_{max}$$

$\therefore$ a projectile attains $\frac{3}{4}$ of its maximum height in half the time it takes to reach the maximum height

Exercises 13.3

5. $v(t) = \frac{dr(t)}{dt} = (-3\sin t\cos^2 t)j + (3\cos t\sin^2 t)k$

speed(t) $= |v(t)| = |3\sin t\cos t\sqrt{\sin^2 t+\cos^2 t}|$

$= 3\sin t\cdot\cos t \quad , \quad t\in[0,\frac{\pi}{2}]$

$T = \frac{v(t)}{\text{speed}(t)} = (-\cos t)j + (\sin t)k$

$L = \int_0^{\pi/2} 3\sin t\cos t\, dt$

$= \frac{3}{2}\sin^2 t\Big|_{t=0}^{\pi/2}$

$= \frac{3}{2}$

9. $v(t) = \frac{dr(t)}{dt} = (5\cos t)i + (-5\sin t)j + (12)k$

speed(t) $= |v(t)| = \sqrt{(5\cos t)^2 + (-5\sin t)^2 + 12^2} = 13$

$L = \int_0^t 13\, dt = 26\pi$

$\Rightarrow t = 2\pi$

$r(2\pi) = \langle 0, 5, 24\pi\rangle$

11. $v(t) = \frac{dr(t)}{dt} = (-4\sin t)i + (4\cos t)j + 3k.$

speed(t) $= |v(t)| = \sqrt{(-4\sin t)^2 + (4\cos t)^2 + 3^2} = 5$

$L = \int_0^{\pi/2} 5\, dt = \frac{5\pi}{2}$

13. $v(t) = \frac{dr(t)}{dt} = (e^t\cos t - e^t\sin t)i + (e^t\sin t + e^t\cos t)j + e^t k$

speed(t) $= |v(t)| = \sqrt{(e^t\cos t - e^t\sin t)^2 + (e^t\sin t + e^t\cos t)^2 + (e^t)^2} = \sqrt{3}e^t$

$L = \int_{-\ln(4)}^{0} \sqrt{3}e^t\, dt = \sqrt{3} - \frac{\sqrt{3}}{4} = \frac{3\sqrt{3}}{4}$

17a. $t=0 \Rightarrow \langle 1,0,0\rangle$

$t=\frac{\pi}{2} \Rightarrow \langle 0,1,1\rangle$

$t=\pi \Rightarrow \langle -1,0,2\rangle$

$n = (r(\frac{\pi}{2}) - r(0)) \times (r(\pi) - r(0))$

$= \langle 2,0,2\rangle$

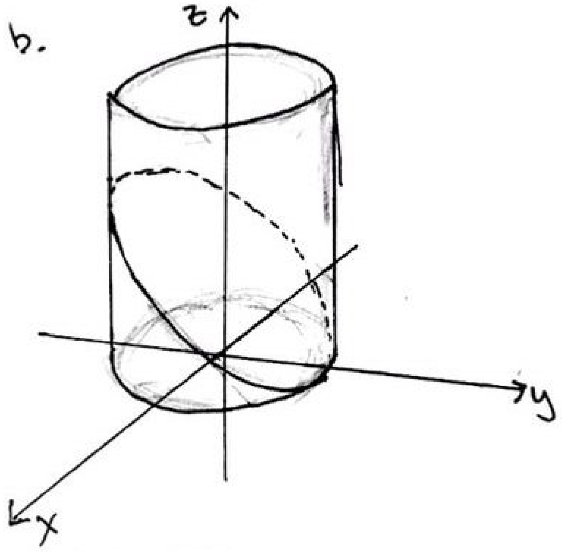
$2x + 2z = D$ & $(1,0,0)$:

$\Rightarrow x + z = 1$ (plane)

$x^2 + y^2 = C$ & $(1,0,0)$:

$\Rightarrow x^2 + y^2 = 1$ (cylinder)

b.



c. $\frac{d}{dt}\left(\frac{dr(t)}{dt}\right) = (-\cos t)i + (-\sin t)j + (\cos t)k$

$\langle -\cos t, -\sin t, \cos t\rangle \cdot \langle 2,0,2\rangle = 0$

$\Rightarrow$ the acceleration vector always lies parallel to the plane

d. $\int_0^{2\pi} \sqrt{(-\sin t)^2 + (\cos t)^2 + (\sin t)^2}\, dt$

≈ 7.6404

19. $v(t) = \frac{dr(t)}{dt} = (-\sin t)i + (\cos t)j$

$\text{speed}(t) = |v(t)| = 1$

$\Rightarrow T = v(t)$

$L = \int_0^t dt = t$

$r(t) - T \cdot L(t) = ((\cos t)i + (\sin t)j) - ((-t\sin t)i + (t\cos t)j)$

$= (\underbrace{\cos t + t\sin t}_{x})i + (\underbrace{\sin t - t\cos t}_{y})j$