STA2001 Probability and Statistics I

Computer-based Exercise 5

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The goal of this exercise is to understand the relationship between the exponential and gamma random variables and their applications.

Problem

Consider a computer system consisting of 3 processors, where the arrival of tasks for processing conforms to a Poisson process with rate λ=2 per minute. Suppose the arrivals are routed to individual processors in a cyclic manner so that every *k*th arrival is routed to the *k*th processor. Let T be the inter-arrival time of tasks to the third processor, then

$$T = T_1 + T_2 + T_3$$

where T_1, T_2, and T_3 are each exponentially distributed with mean 1/2.

1. What is the theoretical pdf of T?
2. Obtain 1,000 samples of T from simulation by adding together 3 exponential variables each with mean 1/2
3. Produce a relative frequency histogram of the 1,000 samples of T using 20 classes of equal sizes
4. Compare the actual observed distribution of the values of T obtained from (3) with the theoretical pdf distribution of (1) by plotting the latter in the same figure
5. Note: when the gamma distribution is obtained as the sum of n exponential random variables, it is known as the Erlang Distribution (the n-Erlang distribution) and has been widely used in queueing theory and telephony.

You may use either Python or Matlab. In addition to the mechanisms for random number generation indicated in previous exercises, the plotting functions (such as plot.hist in Python, or plot in Matlab) may also be used.