# Assignment 8

**Question 1.** Let $u_1$, $u_2$ be an orthonormal basis for $\mathbb{C}^2$, and let $z = (4+2i)u_1 + (6-5i)u_2$.

(a) What are the values of $u_1^H z, z^H u_1, u_2^H z$ *and* $z^H u_2$?

(b) Determin the value of $||z||$

**Question 2.** Find all othogonal matrices that diagonalize

$$B = \begin{bmatrix} 9 & 12 \\ 12 & 16 \end{bmatrix}$$

**Question 3.** Let A= $\begin{bmatrix} \frac{1}{2} & -\frac{1}{2} \\ \frac{1}{2} & -\frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{bmatrix}$,

(a) Find the orthonormal basis for $N(A^T)$.

(b) Determine the projection matrix Q that projects vectors in $\mathbb{R}^4$ onto $N(A^T)$.

**Question 4.** Find the matrix associated with each of the following quadratic forms:

(a) $3x^2 - 5xy + y^2$

(b) $2x^2 + 3y^2 + z^2 + xy - 2xz + 3yz$

(c) $3x^2 + 2y^2 + 5z^2 + 2xy - 4xz - 7yz$

**Question 5.** Show that if $A$ is a symmetric positive definite matrix, then $A$ is nonsingular and $A^{-1}$ also positive definite.

**Question 6.** If two real symmetric matrices $A$ and $B$ are similar, then show that there exists an orthogonal matrix $M$ such that $B = MAM^{-1}$.

**Question 7.** Suppose that $u$ is a unit vector in $\mathbb{R}^n$, so $u^T u = 1$. This problem is about the $n$ by $n$ symmetric matrix $H = I - 2uu^T$

(a) Show that $H$ is an orthogonal matrix.

(b) One eigenvector of $H$ is $u$ itself. Find the corresponding eigenvalue.

**Question 8.** Given $A$ is $m$ by $n$ real matrices and $n < m$, show that $A^T A$ is positive definite matrices if and only if rank$(A) = n$

**Question 9.** Let $A$, $B$ are $n$ by $n$ positive definite matrix, show that $D = \begin{bmatrix} A & 0 \\ 0 & B \end{bmatrix}$ is also positive definite matrix.

**Question 10.** Given quadratic form $f(x_1, x_2, x_3) = 5x_1^2 + x_2^2 + 5x_3^2 + 4x_1x_2 - 8x_1x_3 - 4x_2x_3$ determine it is positive definite or not.