yohandi - assignment 2

1a. $P(A|B) = \frac{P(A \cap B)}{P(B)}$ (given that $P(B) > 0$)

Since $P(A \cap B)$ is a probability function, $P(A \cap B) \geq 0$. Therefore, $P(A|B) \geq 0$

since $\{(A \cap B)\} \subseteq \{(B)\}$, $P(A \cap B) \leq P(B)$.

Therefore, $\frac{P(A \cap B)}{P(B)} = P(A|B) \leq 1$

b. $P(B|B) = \frac{P(B \cap B)}{P(B)} = \frac{P(B)}{P(B)} = 1$

c. $P\left(\bigcup_{i=1}^{k} A_i | B\right) = \frac{P\left(\bigcup_{i=1}^{k} (A_i \cap B)\right)}{P(B)} = \sum_{i=1}^{k} \frac{P(A_i \cap B)}{P(B)}$

$= \sum_{i=1}^{k} P(A_i|B)$

d. $P(A'|B) = \frac{P(B) - P(A \cap B)}{P(B)} = 1 - \frac{P(A \cap B)}{P(B)} = 1 - P(A|B)$

e. $P(A \cup C|B) = \frac{P((A \cup C) \cap B)}{P(B)}$

since $P(A \cup C) = P(A) + P(C) - P(A \cap C)$ and all $P(A), P(C), P(A \cap C)$ are mutually exclusive,

$\frac{P((A \cup C) \cap B)}{P(B)} = \frac{P(A \cap B) + P(C \cap B) - P(A \cap C \cap B)}{P(B)}$

$= P(A|B) + P(C|B) - P(A \cap C|B)$

2a. $\frac{\binom{3}{3}}{\binom{20}{3}} = \frac{1}{1140} = 8{,}7719 \cdot 10^{-4}$

b. $\frac{\binom{3}{2}\binom{17}{1}}{\binom{20}{3}} \cdot \frac{1}{17} = 2{,}6316 \cdot 10^{-3}$

c. to win on the i+1-th draw, 2 win balls must've been taken and 2i-2 lose balls must've also been taken given that player 1 succeeded in $\frac{1}{(20-2i)}$

$\sum_{i=1}^{9} \frac{\binom{3}{2}\binom{17}{2i-2}}{\binom{20}{2i}} \cdot \frac{1}{(20-2i)} = 0{,}46053$

d. draw second, since $0{,}46053 < 0{,}5$

3a. $P(A_1) = \frac{1}{4}$

$P(A_2) = \frac{3}{4} \cdot \frac{1}{3} = \frac{1}{4}$

$P(A_3) = \frac{3}{4} \cdot \frac{2}{3} \cdot \frac{1}{2} = \frac{1}{4}$

$P(A_4) = \frac{3}{4} \cdot \frac{2}{3} \cdot \frac{1}{2} \cdot \frac{1}{1} = \frac{1}{4}$

$P(A_i) = \frac{1}{4} = \frac{3!}{4!}$ for each i

b. $P(A_1 \cap A_2) = \frac{1}{4} \cdot \frac{1}{3} = \frac{1}{12}$

$P(A_1 \cap A_3) = \frac{1}{4} \cdot \frac{2}{3} \cdot \frac{1}{2} = \frac{1}{12}$

$P(A_1 \cap A_4) = \frac{1}{4} \cdot \frac{2}{3} \cdot \frac{1}{2} \cdot \frac{1}{1} = \frac{1}{12}$

$P(A_2 \cap A_3) = \frac{2}{4} \cdot \frac{1}{3} \cdot \frac{1}{2} = \frac{1}{12}$

$P(A_2 \cap A_4) = \frac{2}{4} \cdot \frac{1}{3} \cdot \frac{1}{2} \cdot \frac{1}{1} = \frac{1}{12}$

$P(A_3 \cap A_4) = \frac{2}{4} \cdot \frac{1}{3} \cdot \frac{1}{2} \cdot \frac{1}{1} = \frac{1}{12}$

$P(A_i \cap A_j) = \frac{1}{12} = \frac{2!}{4!}$ for $i \neq j$

c. $P(A_1 \cap A_2 \cap A_3) = \frac{1}{4} \cdot \frac{1}{3} \cdot \frac{1}{2} = \frac{1}{24}$

$P(A_1 \cap A_2 \cap A_4) = \frac{1}{4} \cdot \frac{1}{3} \cdot \frac{1}{2} \cdot \frac{1}{1} = \frac{1}{24}$

$P(A_1 \cap A_3 \cap A_4) = \frac{1}{4} \cdot \frac{1}{3} \cdot \frac{1}{2} \cdot \frac{1}{1} = \frac{1}{24}$

$P(A_2 \cap A_3 \cap A_4) = \frac{1}{4} \cdot \frac{1}{3} \cdot \frac{1}{2} \cdot \frac{1}{1} = \frac{1}{24}$

$P(A_i \cap A_j \cap A_k) = \frac{1}{24} = \frac{1}{4!}$ for $i \neq j$, $j \neq k$, $i \neq k$

d. $P(A_1 \cup A_2 \cup A_3 \cup A_4) = P(A_1) + P(A_2) + P(A_3) + P(A_4)$

$- \sum_{i \neq j} P(A_i \cap A_j)$

$+ \sum_{i \neq j, i \neq k, j \neq k} P(A_i \cap A_j \cap A_k)$

$- P(A_1 \cap A_2 \cap A_3 \cap A_4)$

$= 4 \cdot \frac{3!}{4!} - 6 \cdot \frac{2!}{4!} + 4 \cdot \frac{1}{4!} - \frac{1}{4!}$

$= 1 - \frac{1}{2!} + \frac{1}{3!} - \frac{1}{4!}$

e. $P\left(\bigcup_{i=1}^{n} A_i\right) = 1 - P\left(\bigcap_{i=1}^{n} A_i'\right)$

$= 1 - \frac{\text{no. of possible outcomes}}{\text{total}}$

by derangement formula in permutation and combination :

no. of possible outcome $= !n$

where $!n = n!\left(1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + \dots + \frac{(-1)^{n+1}}{n!}\right)$

$\rightarrow P\left(\bigcup_{i=1}^{n} A_i\right) = 1 - \frac{!n}{n!}$

$= 1 - \frac{\cancel{n!}\left(1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + \dots + \frac{(-1)^{n+1}}{n!}\right)}{\cancel{n!}}$

$= 1 - \left(1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + \dots + \frac{(-1)^{n}}{n!}\right)$

$= 1 - \frac{1}{2!} + \frac{1}{3!} - \frac{1}{4!} + \dots + \frac{(-1)^{n+1}}{n!}$

f. note that:

$!n = n!\sum_{i=0}^{n} \frac{(-1)^i}{i!}$ and $e^x = \sum_{i=0}^{\infty} \frac{x^i}{i!}$

for $x = -1$ and $n \to \infty$:

$\lim_{n\to\infty} P\left(\bigcup_{i=1}^{n} A_i\right) = \lim_{n\to\infty} 1 - \frac{!n}{n!}$

$= \lim_{n\to\infty} 1 - \sum_{i=0}^{\infty} \frac{(-1)^i}{i!}$

$= 1 - e^{-1}$ (converge)

4a. sample sum S:

	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

b. $P(S=7 \text{ or } S=11) = P(S=7) + P(S=11)$

$= \frac{6}{36} + \frac{2}{36}$

$= \frac{2}{9}$

c. $P(S=8 \mid (S=7 \text{ or } S=8)) = \frac{P(S=8)}{P(S=7 \text{ or } S=8)} = \frac{5/36}{11/36}$

$= 5/11$

d. $P(S=8)\,P(S=8 \mid (S=7 \text{ or } S=8)) = \frac{5}{36} \cdot \frac{5}{11} = \frac{25}{396}$

e. $A = \{4, 5, 6, 8, 9, 10\}$

$P_t = P(S=7) + P(S=11) + \sum_{i=1}^{6} P(S=A_i)\,P(S=A_i \text{ or } S=7)$

$= \frac{6}{36} + \frac{2}{36} + \frac{3}{36}\cdot\frac{3}{9} + \frac{4}{36}\cdot\frac{4}{10} + \frac{5}{36}\cdot\frac{5}{11} + \frac{5}{36}\cdot\frac{5}{11} + \frac{4}{36}\cdot\frac{4}{10} + \frac{3}{36}\cdot\frac{3}{9}$

≈ 0.49293

5a. $P(A \cap (B \cap C)) = P(A \cap B \cap C) = P(A)P(B \cap C)$

b. $P(A \cap (B \cup C)) = P((A \cap B) \cup (A \cap C))$

$= P(A \cap B) + P(A \cap C) - P((A \cap B) \cap (A \cap C))$

$= P(A)P(B) + P(A)P(C) - P(A)P(B)P(C)$

$= P(A)\left[P(B) + P(C) - P(B \cap C)\right]$

$= P(A)P(B \cup C)$

c. $P(A' \cap (B \cap C')) = P(B \cap C') - P(A \cap B \cap C')$

$= P(B)P(C') - P(A)P(B)P(C')$

$= P(A')P(B)P(C')$

$= P(A')P(B \cap C')$

d. $P(A' \cap B' \cap C') = P(A' \cap C') - P(A' \cap B \cap C')$

$= P(A')P(C') - P(A')P(B \cap C')$

$= P(A')P(C')(1 - P(B))$

$= P(A')P(B')P(C')$

6a. $P(A \cup B) = P(A) + P(B)$

$= P(A \cup B) + P(A \cap B)$

$\Rightarrow P(A \cap B) = 0$

independent when:

$P(A \cap B) = P(A)P(B)$

$= 0$

$\Rightarrow P(A) = 0$ or $P(B) = 0$

b. Since $A \subset B$,

$P(A \cap B) = P(A)$

independent when:

$P(A \cap B) = P(A)P(B)$

$= P(A)$

$\Rightarrow P(A) = 0$ or $P(B) = 1$

7. a. $\left(\frac{1}{2}\right)^8 = \frac{1}{256}$

b. $\left(\frac{1}{2}\right)^8 = \frac{1}{256}$

c. $\left(\frac{1}{2}\right)^8 = \frac{1}{256}$

d. $\frac{{}^8C_4}{2^8} = \frac{35}{128}$

8. a. # $4+2+1=7$

b. $\left(\frac{1}{2}\right)^7$

c. $32+16+8+4+2+1=63$

d. $\left(\frac{1}{2}\right)^{63} = \frac{1}{9,\ldots 10^{18}} \neq \frac{1}{7.5 \cdot 10^6}$

9. $P(D) = P(A_1) + P(A_2) + P(A_3)$

$= 60\% \cdot 0{,}01 + 30\% \cdot 0{,}008 + 10\% \cdot 0{,}007$

$= 0{,}0091$

a. $P(A_1 | D) = \frac{P(A_1 \cap D)}{P(D)} = \frac{P(A_1)}{P(D)} \approx 0{,}65934$

b. $P(A_2 | D) = \frac{P(A_2 \cap D)}{P(D)} = \frac{P(A_2)}{P(D)} \approx 0{,}26374$

c. $P(A_3 | D) = \frac{P(A_3 \cap D)}{P(D)} = \frac{P(A_3)}{P(D)} \approx 0{,}07692$

10. a. $P(D^+) = P(A^+) P(D^+ | A^+) + P(A^-) P(D^+ | A^-)$

$= 0.02 \quad 0.92 + 0.98 \quad 0.05$

$= 0.0674$

b. $P(A^- | D^+) = \frac{P(A^- \cap D^+)}{P(D^+)} = \frac{P(D^+ | A^-) P(A^-)}{P(D^+)}$

$= \frac{0.05 \quad 0.98}{0.0674}$

≈ 0.72700

$P(A^+ | D^+) = \frac{P(A^+ \cap D^+)}{P(D^+)} = \frac{P(D^+ | A^+) P(A^+)}{P(D^+)}$

$= \frac{0.92 \quad 0.02}{0.0674}$

≈ 0.27300

c. $P(D^-) = 1 - P(D^+) = 0.9326$

$P(A^- | D^-) = \frac{P(A^- \cap D^-)}{P(D^-)} = \frac{P(D^- | A^-) P(A^-)}{P(D^-)}$

$= \frac{0.95 \cdot 0.98}{0.9326}$

≈ 0.99828

$P(A^+ | D^-) = \frac{P(A^+ \cap D^-)}{P(D^-)} = \frac{P(D^- | A^+) P(A^+)}{P(D^-)}$

$= \frac{0.08 \cdot 0.02}{0.9326}$

≈ 0.00172

d. Yes, the calculatron shows us that $P(A^- | D^+) \approx 72.70\%$. It is alarming as 72.70% of those classified as abused were not abused (72.70% is not low) jupyter Statistic Assignment 2 (Computer-based) - Yohandi

Logout

File Edit View Insert Cell Kernel Widgets Help Trusted Kernel

Run Code

In [9]:

```
import pandas as pd
import matplotlib.pyplot as plt
import random

def probabilitySuccess(a,b):
    if random.randint(1,b)<=a:
        return True
    return False

r=[0]
b1=[0]
b2=[0]
b3=[0]
currentProbability1=[0]
currentProbability2=[0]
currentProbability3=[0]

for i in range(100000):
    notFound=True
    r.append(i+1)
    b1.append(b1[-1])
    b2.append(b2[-1])
    b3.append(b3[-1])
    while notFound:
        b=random.randint(1,6)
        if b<=2:
            if probabilitySuccess(2,6):
                b1[-1]+=1
                notFound=False
        elif b<=3:
            if probabilitySuccess(1,3):
                b2[-1]+=1
                notFound=False
        else:
            if probabilitySuccess(5,9):
                b3[-1]+=1
                notFound=False
    currentProbability1.append(float(b1[-1]/r[-1]))
    currentProbability2.append(float(b2[-1]/r[-1]))
    currentProbability3.append(float(b3[-1]/r[-1]))

data = {'R':r,'Probability1':currentProbability1}
df = pd.DataFrame(data,columns=['R','Probability1'])
df.plot(x ='R', y='Probability1', kind = 'scatter')
plt.show()
data = {'R':r,'Probability2':currentProbability2}
df = pd.DataFrame(data,columns=['R','Probability2'])
df.plot(x ='R', y='Probability2', kind = 'scatter')
plt.show()
data = {'R':r,'Probability3':currentProbability3}
df = pd.DataFrame(data,columns=['R','Probability3'])
df.plot(x ='R', y='Probability3', kind = 'scatter')
plt.show()
```

jupyter Statistic Assignment 2 (Computer-based) - Yohandi

Logout

File Edit View Insert Cell Kernel Widgets Help

Trusted

Kernel

Run Code

```
plt.show()
```