quiz 11 - yohandi (120040025)

1a. $\int x^3 e^x \, dx$

let $u = x^3$ $\quad \frac{dv}{dx} = e^x$

$\frac{du}{dx} = 3x^2$ $\quad v = e^x$

$= x^3 e^x - 3\int x^2 e^x \, dx$

let $u = x^2$. $\quad \frac{dv}{dx} = e^x$

$\frac{du}{dx} = 2x$ $\quad v = e^x$

$= x^3 e^x - 3x^2 e^x + 3 \cdot 2 \int x e^x \, dx$

let $u = x$ $\quad \frac{dv}{dx} = e^x$

$\frac{du}{dx} = 1$ $\quad v = e^x$

$= x^3 e^x - 3x^2 e^x + 6x e^x - \int 6e^x dx$

$= \boxed{x^3 e^x - 3x^2 e^x + 6x e^x - 6e^x + C}$

b. $\int \sec^3 x \tan^3 x \, dx$

let $u = \sec x$

$\frac{du}{dx} = \sec x \tan x$

$= \int u^2 (u^2 - 1) \, du$

$= \frac{1}{5}u^5 - \frac{1}{3}u^3 + C$

$= \boxed{\frac{1}{5}\sec^5 x - \frac{1}{3}\sec^3 x + C}$

c. $\int_1^e \frac{dy}{y\sqrt{1+(\ln y)^2}}$

let $\ln(y) = \tan\theta$

$\frac{1}{y} = \sec^2\theta \cdot \frac{d\theta}{dy}$

$= \int_0^{\pi/4} \frac{\sec^2\theta \, d\theta}{\sqrt{\sec^2\theta}}$

$= \ln|\sec\theta + \tan\theta| \Big|_{\theta=0}^{\pi/4}$

$= \boxed{\ln(\sqrt{2}+1)}$

d). $\lim_{x\to 0} \frac{(1-\cos(3x))\sin(9x)}{\ln(1+2x)}$

$= \lim_{x\to 0} \frac{\sin 9x - [\sin 9x \cos 3x]}{\ln(1+2x)}$

$= \lim_{x\to 0} \frac{\sin 9x - \frac{1}{2}\sin 6x - \frac{1}{2}\sin 12x}{\ln(1+2x)}$.

"$\frac{0}{0}$" indeterminate form
Apply L'Hôpital

$= \lim_{x\to 0} \frac{9\cos 9x - \frac{1}{2}6\cos 6x - \frac{12}{2}\cos 12x}{\frac{2}{2x+1}}$

$= \lim_{x\to 0} \left(\frac{2x+1}{2}\right)\left[9\cos 9x - 3\cos 6x - 6\cos 12x\right]$

$= \boxed{0}$