Error Analysis Assignment

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1 Significant figures

1.1 Determine the significant figures In this part, we are asked to determine the number of significant figures for the following numbers

numbers	number of significant figures
1.00101	**6**
1.0110×10^{-3}	**5**
0.0010011	**5**
3.140	**4**
1670.	**4**
1.68×10^{4}	**3**

1.2 Significant figures in calculations In this part, we are asked to express the results of the following calculations with the correct significant figures

calculations	results	results with correct significant figures
$3.1416 \times 0.28\ /\ 2.34$	$0.375917949\ldots$	**0.38**
$123.62 + 7.1 - 5.33$	125.39	**125.4**

2 Propagation of Uncertainty (Error)

In this part, we are asked to find the standard error σ_x in $x = f(u, v)$ as a function of the errors in σ_u and σ_v for the following functions:

- $x = u + v \Rightarrow \frac{\partial f}{\partial u} = 1, \frac{\partial f}{\partial v} = 1$

$$\sigma_x = \sqrt{(\frac{\partial f}{\partial u})^2\sigma_u^2 + (\frac{\partial f}{\partial v})^2\sigma_v^2} = \sqrt{1^2\sigma_u^2 + 1^2\sigma_v^2} = \sqrt{\sigma_u^2 + \sigma_v^2}$$

- $x = uv \Rightarrow \frac{\partial f}{\partial u} = v, \frac{\partial f}{\partial v} = u$

$$\sigma_x = \sqrt{(\frac{\partial f}{\partial u})^2\sigma_u^2 + (\frac{\partial f}{\partial v})^2\sigma_v^2} = \sqrt{v^2\sigma_u^2 + u^2\sigma_v^2} = x\sqrt{(\frac{\sigma_u}{u})^2 + (\frac{\sigma_v}{v})^2}$$

- $x = \frac{u}{v} \Rightarrow \frac{\partial f}{\partial u} = \frac{1}{v}, \frac{\partial f}{\partial v} = -\frac{u}{v^2}$

$$\sigma_x = \sqrt{(\frac{\partial f}{\partial u})^2\sigma_u^2 + (\frac{\partial f}{\partial v})^2\sigma_v^2} = \sqrt{(\frac{1}{v})^2\sigma_u^2 + (\frac{-u}{v^2})^2\sigma_v^2} = x\sqrt{(\frac{\sigma_u}{u})^2 + (\frac{\sigma_v}{v})^2}$$

- $x = uv^2 \Rightarrow \frac{\partial f}{\partial u} = v^2, \frac{\partial f}{\partial v} = 2uv$

$$\sigma_x = \sqrt{(\frac{\partial f}{\partial u})^2\sigma_u^2 + (\frac{\partial f}{\partial v})^2\sigma_v^2} = \sqrt{(v^2)^2\sigma_u^2 + (2uv)^2\sigma_v^2} = x\sqrt{(\frac{\sigma_u}{u})^2 + (\frac{2\sigma_v}{v^2})^2}$$

- $x = ue^{cv} \Rightarrow \frac{\partial f}{\partial u} = e^{cv}, \frac{\partial f}{\partial v} = cue^{cv}$

$\sigma_x = \sqrt{(\frac{\partial f}{\partial u})^2\sigma_u^2 + (\frac{\partial f}{\partial v})^2\sigma_v^2} = \sqrt{(e^{cv})^2\sigma_u^2 + (cue^{cv})^2\sigma_v^2} = x\sqrt{(\frac{\sigma_u}{u})^2 + (c\sigma_v)^2}$

- $x = \frac{1}{u} \Rightarrow \frac{\partial f}{\partial u} = -\frac{1}{u^2}, \frac{\partial f}{\partial v} = 0$

$\sigma_x = \sqrt{(\frac{\partial f}{\partial u})^2\sigma_u^2 + (\frac{\partial f}{\partial v})^2\sigma_v^2} = \sqrt{(-\frac{1}{u^2})^2\sigma_u^2} = x^2\sigma_u$

3 Snell's Law

According to Snell's law, the incident angle θ_1 of a ray traveling in a medium of index n_1 to the refraction angle θ_2 of the same light ray in the medium of refraction index n_2 relates with equation $n_1 \sin\theta_1 = n_2 \sin\theta_2$.

In this part, we are asked to find n_2 and its uncertainty from the following measurements:

$\theta_1 = 22.0^\circ \pm 0.2^\circ$

$\theta_2 = 16.3^\circ \pm 0.2^\circ$

$n_1 = 1.000$

$n_2 = \frac{n_1 \sin\theta_1}{\sin\theta_2} = 1.3347 ... = 1.33$

$\delta n_2 = \sqrt{(\frac{\partial \frac{\sin\theta_1}{\sin\theta_2}}{\partial\theta_1})^2\delta\theta_1^2 + (\frac{\partial \frac{\sin\theta_1}{\sin\theta_2}}{\partial\theta_2})^2\delta\theta_2^2} = \sqrt{(\frac{\cos\theta_1}{\sin\theta_2})^2\delta\theta_1^2 + (-\frac{\sin\theta_1\cos\theta_2}{\sin^2\theta_2})^2\delta\theta_2^2} = \frac{n_2}{n_1}\sqrt{(\frac{\cos\theta_1}{\sin\theta_1})^2\delta\theta_1^2 + (\frac{\cos\theta_2}{\sin\theta_2})^2\delta\theta_2^2} = 0.019598 ... = 0.02$

$\Rightarrow n_2 = 1.33 \pm 0.02$

4 Simple Pendulum

To determine the acceleration of gravity, we can measure the oscillation period T of a pendulum with length L and use equation $g = \frac{4\pi^2 L}{T^2}$.

In this part, we are asked to determine the value and uncertainty of g from the following measurements:

$T = 2.01 \pm 0.02\ s$

$L = 1.000 \pm 0.002\ m$

$g = \frac{4\pi^2 L}{T^2} = 9.7716 ... \frac{m}{s^2} = 9.77 \frac{m}{s^2}$

$\delta g = \sqrt{(\frac{\partial \frac{4\pi^2 L}{T^2}}{\partial T})^2\delta T^2 + (\frac{\frac{4\pi^2 L}{T^2}}{\partial L})^2\delta L^2} = \sqrt{(-\frac{8\pi^2 L}{T^3})^2\delta T^2 + (\frac{4\pi^2}{T^2})^2\delta L^2} = g\sqrt{(\frac{2}{T})^2\delta T^2 + (\frac{1}{L})^2\delta L^2} = 0.19541 ... \frac{m}{s^2} = 0.2 \frac{m}{s^2}$

$\Rightarrow g = (9.77 \pm 0.2)\ \frac{m}{s^2}$

From the derived formula, the uncertainty of the gravity acceleration is dominated by the uncertainty measure of T (at least by its coefficient compared to the uncertainty measure of L). In order to improve the measurement, we have to reduce the uncertainty of T (δT).