yohandi - math homework week 11

~~Exercises 16.1~~ Exercises 15.8

17. $\cancel{\int_0^3\int_0^4\int_{\frac{y}{2}}^{\frac{y}{2}+1}}$

17. $\int_0^3\int_0^4\int_{\frac{y}{2}}^{\frac{y}{2}+1}\left(\frac{2x-y}{2}+\frac{z}{3}\right)dx\,dy\,dz$

$=\int_0^3\int_0^4\left(\frac{1}{2}+\frac{z}{3}\right)dy\,dz$

$=\int_0^3\left(2+\frac{4z}{3}\right)dz$

$=12$

18. let $x=au$, $y=bv$, $z=cw$

$\Rightarrow u^2+v^2+w^2=1$ (sphere with radius 1)

$J(u,v,w)=\begin{vmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{vmatrix}=abc$

$V=abc\,V_{sphere}(1)$

$=abc\cdot\frac{4\pi}{3}(1)^3$

$=\frac{4}{3}\pi abc$

20. let $u=x$, $v=xy$, $w=3z$

$x\in[1,2]\Rightarrow u\in[1,2]$

$xy\in[0,2]\Rightarrow v\in[0,2]$

$z\in[0,1]\Rightarrow w\in[0,3]$

$J(u,v,w)=\begin{vmatrix} 1 & 0 & 0 \\ -\frac{v}{u^2} & \frac{1}{u} & 0 \\ 0 & 0 & \frac{1}{3} \end{vmatrix}=\frac{1}{3u}$

$\int_0^3\int_0^2\int_1^2 \frac{(uv+3vw)}{3u}\,du\,dv\,dw$ #

$=\int_0^3\int_0^2\left(\frac{1}{3}v+\cancel{3}\frac{1}{3}vw\ln(2)\right)dv\,dw$

$=\int_0^3\left(\frac{2}{3}+\frac{2}{3}w\ln(2)\right)dw$

$=2+3\ln(2)$

Exercises 16.1

11. $\int_C xy+y+z\,ds$

$=\int_0^1(2t^2+t+2-2t)\sqrt{2^2+1^2+(-2)^2}\,dt$

$=\frac{13}{2}$

15. $\int_{C_1}x+\sqrt{y}-z^2\,ds+\int_{C_2}x+\sqrt{y}-z^2\,ds$

$=\int_0^1 2t\sqrt{1+(2t)^2+0}\,dt+\int_0^1(2-t^2)\sqrt{0^2+0^2+1^2}\,dt$

$=\frac{5\sqrt{5}}{6}+\frac{3}{2}$

19. $\int_C x\,ds$

a. $\int_0^4 t\sqrt{(1)^2+\left(\frac{1}{2}\right)^2}\,dt=4\sqrt{5}$

b. $\int_0^2 t\sqrt{(1)^2+(2t)^2}\,dt=\frac{17\sqrt{17}}{12}-\frac{1}{12}$

23. $\int_C\frac{x^2}{y^{4/3}}\,ds$

$=\int_1^2\frac{t^4}{t^4}\sqrt{(2t)^2+(3t^2)^2}\,dt=\frac{\sqrt{40^3}-\sqrt{13^3}}{27}$

31. $\int_C x+\sqrt{y}\,ds$

$=\int_0^2(x+x)\sqrt{1+(2x)^2}\,dx=\frac{1}{6}\left(\sqrt{17^3}-1\right)$

33. $m=\int_0^1\frac{3}{2}t\sqrt{(2t)^2+2^2}\,dt=2\sqrt{2}-1$

35 a. $m=\int_0^1 3t\sqrt{2+2+(-2t)^2}\,dt=4\sqrt{2}-2$

b. $m=\int_0^1\sqrt{2+2+(-2t)^2}\,dt=\ln(\sqrt{2}+1)+\sqrt{2}$

Exercises 16.2

7a. $\int_0^1 (3t+2t+4t)\,dt = \frac{9}{2}$

b. $\int_0^1 (3t^2+4t^2+16t^4)\,dt = \frac{13}{3}$

c. $\int_0^1 (3t+2t+4t)\,dt = \frac{9}{2}$

13. $\int_0^3 (-t-1)\,dt = -\frac{15}{2}$

17a. $\int_0^1 (t-1-t^2)\,dt = -\frac{5}{6}$

b. $\int_0^1 (t-1-t^2)\cdot 0 \cdot dt = 0$

c. $\int_0^1 (t-1-t^2)\,dt\,dt = -\frac{5}{6}$

21. $W = \int_0^{2\pi} (t\cos t - \sin^2 t + \cos t)\,dt$

$= -\pi$

26. $x^2+y^2=1 \Rightarrow x=\cos t$ & $y=\sin t$

$\int_0^{\frac{\pi}{2}} -\sin^2 t - \cos^2 t\,dt = -\frac{\pi}{2}$

28. $x^2+y^2=4 \Rightarrow x=2\cos t$ & $y=2\sin t$

$\int_0^{2\pi} (2\cos t+2\sin t)(-2\sin t+2\cos t)\,dt$

$= 0$

29a. circulation$_1 = \int_C x\,dx + y\,dy$

$= \int_0^{2\pi} \cos t\,(-\sin t) + \sin t(\cos t)\,dt$

$= 0$

flux$_1 = \int_C -y\,dx + x\,dy$

$= \int_0^{2\pi} -\sin t(-\sin t) + \cos t(\cos t)\,dt$

$= 2\pi$

circulation$_2 = \int_C -y\,dx + x\,dy$

$= \int_0^{2\pi} -\sin t(-\sin t) + \cos t(\cos t)\,dt = 2\pi$

flux$_2 = \int_C -x\,dx - y\,dy$

$= \int_0^{2\pi} -\sin t(\cos t) - \cos t(-\sin t)\,dt$

$= 0$

b. circulation$_1 = \int_C x\,dx + y\,dy$

$= \int_0^{2\pi} \cos t(-\sin t) + 4\sin t(4\cos t)\,dt$

$= 0$

flux$_1 = \int_C -y\,dx + x\,dy$

$= \int_0^{2\pi} -4\sin t(-\sin t) + \cos t(4\cos t)\,dt$

$= 8\pi$

circulation$_2 = \int_C -y\,dx + x\,dy$

$= \int_0^{2\pi} -4\sin t(-\sin t) + 4\cos t\,(\cos t)\,dt$

$= 8\pi$

flux$_2 = \int_C -x\,dx - y\,dy$

$= \int_0^{2\pi} -4\sin t\,(4\cos t)\,dt - \cos t(-\sin t)\,dt$

$= 0$

30. ~~flux$_1$ = $\int_0^{2\pi} (2x\,dy) + C$~~

30. flux$_1 = \int_0^{2\pi} 2a\cos t(a\cos t) + 3a\sin t(-a\sin t)\,dt$

$= -a^2\pi$

flux$_2 = \int_0^{2\pi} 2a\cos t(a\cos t) - (a\cos t - a\sin t)(-a\sin t)\,dt$

$= a^2\pi$

37a. flow $= \int_0^2 (2x)^2 + 2x(2x)2 \cdot dx = 32$

b. flow $= \int_0^2 (x^2)^2 + 2x(x^2)2x\,dx = 32$

c. flow $= \int_0^2 (4\sin(\frac{\pi x}{4}))^2 + 2x \cdot 4\sin(\frac{\pi x}{4})\,dx$

$= 32$

Exercises 16.3

1. curl $= \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ yz & xz & xy \end{vmatrix} = (x-x)i + (y-y)j + (z-z)k = 0$
(conservative)

5. curl $= \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ (z+y) & z & (y+x) \end{vmatrix} = (1-1)i + (1-1)j + (0-1)k \neq 0$
(non-conservative)

7. $f(x,y,z) = \int \frac{\partial f}{\partial x}dx + \frac{\partial f}{\partial y}dy + \frac{\partial f}{\partial z}dz$

$= x^2 + \frac{3}{2}y^2 + 2z^2 + C$

9. $f(x,y,z) = \int \frac{\partial f}{\partial x}dx + \frac{\partial f}{\partial y}dy + \frac{\partial f}{\partial z}dz$

$= xe^{y+2z} + C$

19. $f(x,y,z) = \int \frac{\partial f}{\partial x}dx + \frac{\partial f}{\partial y}dy + \frac{\partial f}{\partial z}dz$

$= x^3 + z^2\ln(y)$

$f(1,2,3) - f(1,1,1) = 9\ln(2)$

29. $f(x,y,z) = \int \frac{\partial f}{\partial x}dx + \frac{\partial f}{\partial y}dy + \frac{\partial f}{\partial z}dz$

$= (\frac{x^3}{3} + xy) + (\frac{y^3}{3}) + (ze^z - e^z)$

$f(1,0,1) - f(1,0,0) = 1$

33a. $M = ay^2 + 2czx$

$N = y(bx + cz)\,dy$

$P = (ay^2 + cx^2)$

$\Rightarrow \frac{\partial P}{\partial y} = \frac{\partial N}{\partial z}$

$2ay = cy$

$2a = c$

$\Rightarrow \frac{\partial M}{\partial z} = \frac{\partial P}{\partial x}$

$2cx = 2cx$

$c = c$

$\Rightarrow \frac{\partial N}{\partial x} = \frac{\partial M}{\partial y}$

$by = 2ay$

$2a = b$

b. from (a), $a = 1$

$\Rightarrow b = 2$

$\Rightarrow c = 2$