Yohandi - quiz 9

1.a. True

b. False, ~~example:~~

for $g(x) = c$

$g \circ f(x)$ is also equals to c

(not one-to-one function)

2. $f(x) = x^3 - 3x^2 - 1$

$f'(x) = 3x^2 - 6x$

~~critical points~~ stationary points when $f'(x) = 0$

$3x^2 - 6x = 0$

$3x(x-2) = 0$

$x = \{0, 2\}$

$f'(x)$ + − | +

0 2

a) we can see for $x \geq 0$, $f'(x) > 0 \Rightarrow f(x)$ is increasing, thus, $f(x)$ is a one-to-one function $\Rightarrow$ $\boxed{\text{has an inverse}}$

b) $(f^{-1})'(-1) = \dfrac{1}{f'(f^{-1}(-1))}$

~~$f^{-1}(2) = f(2)$~~

Let $f^{-1}(-1) = a$

$f(a) = -1$

$a^3 - 3a^2 - 1 = -1$

$a^3 - 3a^2 = 0$

$a = \{0, 3\}$

not to the domain

$(f^{-1})'(-1) = \dfrac{1}{f'(a)} = \dfrac{1}{f'(3)} = \dfrac{1}{3(3^2) - 6(3)}$

$= \boxed{\dfrac{1}{9}}$

3a. $\int \frac{3\sec^2 t}{6+3\tan t}\, dt$

$= \int \frac{3\cancel{\sec^2 t}}{6+3\tan t} \frac{d(6+3\tan t)}{3\cancel{\sec^2 t}}$

$= \int \frac{d(6+3\tan t)}{6+3\tan t}$

$= \boxed{\ln|6+3\tan t| + C_1} = \boxed{\ln|2+\tan t| + C_2^*}$

b. $y = \sqrt{\frac{(x+1)^{10}}{(2x+1)^5}}$

$\ln(y) = \frac{1}{2}\left[\ln(x+1)^{10} - \ln(2x+1)^5\right]$

$\ln(y) = 5\ln(x+1) - \frac{5}{2}\ln(2x+1)$

$\xrightarrow{\frac{d(\sim)}{dx}}$

$\frac{\frac{dy}{dx}}{y} = \frac{5}{x+1} - \frac{5 \cdot \cancel{2}}{\cancel{2}(2x+1)}$

$\boxed{\frac{dy}{dx} = \sqrt{\frac{(x+1)^{10}}{(2x+1)^5}}\left(\frac{5}{x+1} - \frac{5}{2x+1}\right)}$