ychandi - quiz 10

1 a) false

b) false

2. LHS $= \log_5(48) \cdot \log_{48}(x)$

$= \frac{\ln(48)}{\ln(5)} \cdot \frac{\ln(x)}{\ln(48)}$

$= \frac{\ln(x)}{\ln(5)}$

$= \log_5(x)$. for $(x>0)$

(true)

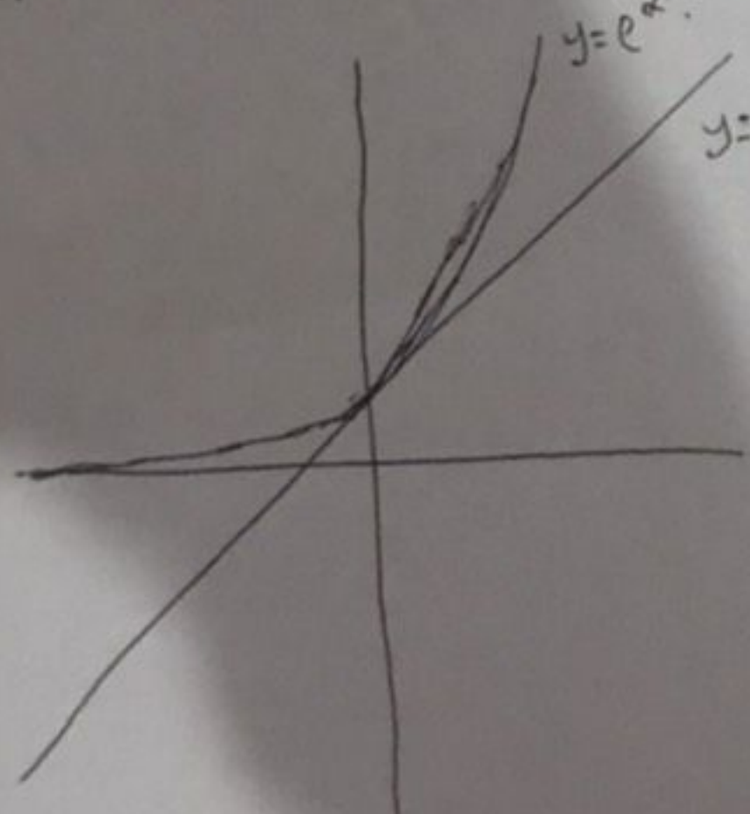
3 consider the case whereas $e^x = x+1$

$\Rightarrow x = 0$

gradient of $x+1$ at $x=0$ $= 1$

gradient of e^x at $x=0$ $= e^0 = 1$

this shows that $x+1$ is also a tangent line of e^x at $x=0$.



for $x>0$ the gradient of e^x $= e^x$ while gradient of $x+1 = 1$

since $e^x > 1$ for $x>0$,

equation holds true for $x>0$

for $x=0$,

$e^x = e^0 = 1 = x(0)+1$

equation holds true for $x=0$

since $e^x = x+1$ has only 1 solution

and e^x is a concave up $\Rightarrow$ equation holds true for $x<0$

$\therefore e^x \geq x+1$ for all x

4a) $\lim_{x\to 0} (1-5x)^{3/x}$

$= \lim_{x\to 0} e^{\ln((1-5x)^{3/x})}$

$= e^{\lim_{x\to 0} 3/x \ln(1-5x)}$

apply L'Hopital ↑ "$\frac{0}{0}$" indeterminate form

$= e^{\lim_{x\to 0} 3 \cdot \frac{-5}{1-5x}} = e^{-15}$

b) $= \lim_{x\to\infty} \sqrt{x}\, e^{-x/2}$ ↩ turn

let $y = \sqrt{x}$

$= \lim_{y\to\infty} y \cdot e^{-y^2/2} = \lim_{y\to\infty} \frac{y}{e^{y^2/2}}$

$\overset{\infty}{=}$ "$\frac{\infty}{\infty}$" indeterminate form

$= \lim_{y\to\infty} \frac{1}{\text{↑}}$

4b) $\lim_{x\to\infty} \sqrt{x}\, e^{-x/2}$

$= \lim_{x\to\infty} \frac{\sqrt{x}}{e^{x/2}}$ "$\frac{\infty}{\infty}$" indeterminate form

$= \lim_{x\to\infty} \frac{\frac{1}{2\sqrt{x}}}{\frac{1}{2}\cdot e^{x/2}}$

$= \lim_{x\to\infty} \frac{1}{\sqrt{x}\, e^{x/2}}$

$= 0.$