The Chinese University of Hong Kong, Shenzhen
School of Data Science

MAT3007 · Homework 5

Due: 11:59am, Nov. 2nd (Thursday), 2023

Instructions:

- Homework problems must be carefully and clearly answered to receive full credit. Complete sentences that establish a clear logical progression are highly recommended.
- Please submit your assignment on Blackboard.
- The homework must be written in English.
- Late submission will not be graded.
- Each student must not copy homework solutions from another student or from any other source.

Problem 1 (50pts). Consider the following linear program:

$$\begin{array}{lll} \text{maximize} & 3x_1 + 4x_2 + 3x_3 + 6x_4 & \\ \text{subject to} & 2x_1 + x_2 - x_3 + x_4 & \geq 12 \\ & x_1 + x_2 + x_3 + x_4 & = 8 \\ & -x_2 + 2x_3 + x_4 & \leq 10 \\ & x_1, x_2, x_3, x_4 & \geq 0. \end{array} \tag{1}$$

After transforming the problem into standard form and apply Simplex method, we obtain the final tableau as follow:

B	0	2	9	0	3	0	36
1	1	0	-2	0	-1	0	4
4	0	1	3	1	1	0	4
6	0	-2	-1	0	-1	1	6

a) Derive the dual problem of the linear program (1) and calculate a dual solution based on complementarity conditions. Given that the optimal solution to the primal solution is unique, investigate whether the dual solution is unique.

b) Do the optimal solution and the objective function value change if we

- decrease the objective function coefficient for x_3 to 0?
- increase the objective function coefficient for x_3 to 9?
- decrease the objective function coefficient for x_4 to 5?
- increase the objective function coefficient for x_1 to 7?

c) Find the possible range for adjusting the coefficient 8 of the second constraint such that the current basis is kept optimal.

Problem 2 (50pts). Consider the following linear program:

$$
\begin{array}{lll}
\text{maximize} & x_1 + cx_2 + 3x_3 + 8x_4 & \\
\text{subject to} & x_1 - x_2 + x_3 & \leq 2 \\
 & x_3 - x_4 & \leq 1 \\
 & 2x_2 + 3x_3 + 4x_4 & \leq b \\
 & x_1, x_2, x_3, x_4 & \geq 0,
\end{array}
\tag{2}
$$

where $c = 2, b = 8$. Denote $x = (x_1; x_2; x_3; x_4; s_1; s_2; s_3)$ as the decision variable to the standard form of the above problem, where s_1, s_2, s_3 are the slack variables corresponding to the three constraints. The following table gives the final simplex tableau when solving the standard form of the above problem:

B								
B	0	1	4	0	1	0	2	18
1	1	-1	1	0	1	0	0	2
6	0	$\frac{1}{2}$	$\frac{7}{4}$	0	0	1	$\frac{1}{4}$	3
4	0	$\frac{1}{2}$	$\frac{3}{4}$	1	0	0	$\frac{1}{4}$	2

a) What is the current dual optimal solution? In what range can we change the coefficient c so that the current optimal basis still remains optimal?

b) If we change $c = 2$ to $c = -100$, what will be the new optimal solution to problem (2) and the optimal value of problem (2)?

c) In what range can we change the coefficient of the third constraint $b = 8$ so that the current optimal basis still remains optimal?

d) What will be the new optimal solution to problem (2) when we change $b = 8$ to $b = 4$ and what will be the optimal value?