The Chinese University of Hong Kong, Shenzhen
School of Data Science

MAT3007 · Homework 1

Due: 11:59pm, September 22 (Friday), 2023

Instructions:

- Homework problems must be carefully and clearly answered to receive full credit. Complete sentences that establish a clear logical progression are highly recommended.
- Please submit your assignment on Blackboard.
- The homework must be written in English.
- Late submission will not be graded.
- Each student must not copy homework solutions from another student or from any other source.
- For those questions that ask you to write MATLAB/Python codes to solve the problem. Please attach the code to the homework. You also need to clearly state (write or type) the optimal solution and the optimal value you obtained. However, you do not need to attach the outputs in the command window of MATLAB/Python.

Problem 1 (25pts). Modeling

A company produces two kinds of products. A product of the first type requires 1/8 hours of assembly labor, 1/2 hours of testing, and \$1.2 worth of raw materials. A product of the second type requires 1/4 hours of assembly, 1/6 hours of testing, and \$0.9 worth of raw materials. Given the current personnel of the company, there can be at most 90 hours of assembly labor and 80 hours of testing each day. Products of the first and second type have a market value of \$9 and \$8 respectively.

(a) Formulate a linear optimization that maximizes the daily profit of the company.

(b) Write the standard form of the LP you formulated in part (a).

(c) Consider the following modification to the original problem: Suppose that up to 40 hours of overtime assembly labor can be scheduled, at a cost of \$8 per hour. Can it be easily incorporated into the linear optimization formulation and how?

(d) Solve the LP using software (for the optimization problem formulated in part (a)).

Problem 2 (25pts). Reformulate NLP as LP

Reformulate the following problems as linear programming:

$$\begin{aligned} \text{minimize} \quad & 2x_2 + |x_1 - x_3| \\ \text{subject to} \quad & |x_1 + 2| + |x_2| \le 5 \\ & x_3^2 \le 1 \end{aligned}$$

Please also write down its standard form.

Problem 3 (25pts). The China Railroad Ministry is in the process of planning relocations of freight cars among 5 regions of the country to get ready for the fall harvest. Table1 shows the cost of moving a car between each pair of regions. Table2 shows the current number of cars in each region and the number needed for harvest shipping.
Write down a linear optimization to compute the least costly way to move the cars such that the need is met.

(a) Formulate an optimization problem to achieve this task.

(b) Solve the formulated optimization problem using software. If you have integer constraints in your formulation, you can first ignore the integer constraints and solve the relaxed problem. What are the optimal solution of the relaxed problem? What is the optimal value of the true problem and why?

From/To	1	2	3	4	5
1	–	20	13	11	28
2	20	–	18	8	46
3	13	18	–	9	27
4	11	8	9	–	20
5	28	46	27	20	–

Table 1: Cost of moving a car

	1	2	3	4	5
Present	110	335	400	420	610
Need	150	200	600	200	390

Table 2: Number of current and needed cars

Problem 4 (25pts). Write a software code to use linear optimization to solve the shortest path problem. Suppose the input of the problem will be an $n \times n$ matrix of W, where w_{ij} is the length of the path from i to j. In our implementation, we always use 1 to denote the source node (the S node in the lecture slides), and n to denote the terminal node (the T node in the lecture slides). In addition, we assume for any i and j, there is a path, i.e., the set of E is all pairs of nodes. This is without loss of generality since one can set w_{ij} and w_{ji} to be an extremely large number if there is no edge between i and j, effectively eliminating it from consideration.
You are asked to solve the optimization problem formulated in the lecture slides using software, with the given labeling shown in Figure 1. Basically, you need to input the W matrix for this case, and then solve it. To solve this problem, you can relax the binary integer constraint $x_{ij} \in \{0, 1\}$ to $0 \leq x_{ij} \leq 1$. What is the optimal solution, i.e., the optimal path, for the relaxed problem? Is it optimal to the original problem? If yes, justify it.

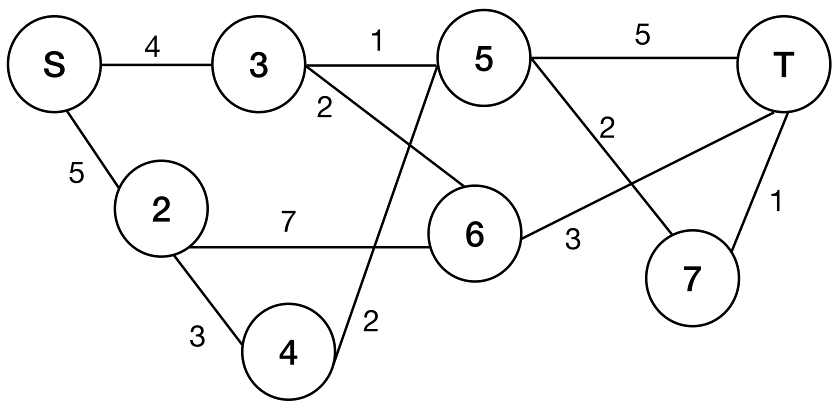

Figure 1: The graph of the shortest path problem