yohandi - quiz 2

1a. T

b. T

c. T

2. $(\sec x)' = \lim_{h\to 0} \frac{\sec(x+h) - \sec x}{h}$

$= \lim_{h\to 0} \frac{\frac{1}{\cos(x+h)} - \frac{1}{\cos x}}{h}$

$= \lim_{h\to 0} \frac{\frac{1}{\cos x\cosh - \sin x \sinh} - \frac{1}{\cos x}}{h}$

$= \lim_{h\to 0} \frac{\cos x - \cos x\cosh + \sin x \sinh}{h \cdot \cos x (\cos(x+h))}$

$= \lim_{h\to 0} \left[\frac{\cos x(1-\cosh)}{h \cdot \cos x \cdot \cos(x+h)} + \frac{\sin x \sinh}{h \cdot \cos x \cdot \cos(x+h)}\right]$

note that when $h\to 0$, $1-\cosh \sim 2\sin^2\frac{1}{2}h$

$= \lim_{h\to 0} \left[\frac{2\cos x \cdot \sin\frac{1}{2}h}{\cos x \cdot \cos(x+h)} \cdot \left(\frac{\sin\frac{1}{2}h}{h}\right) + \frac{\sin x}{\cos x \cdot \cos(x+h)}\left(\frac{\sinh}{h}\right)\right]$

$= \lim_{h\to 0} \left[\frac{2\cos x \cdot \sin\frac{1}{2}h \cdot \frac{1}{2}}{\cos x \cdot \cos(x+h)} + \frac{\sin x}{\cos x \cdot \cos(x+h)}\right]$

$= \lim_{h\to 0} \left[0 + \frac{\tan x}{\cos(x+h)}\right]$

$= \frac{\tan x}{\cos x}$

$= \boxed{\sec x \tan x}$

3a. $y = 3\cot x \ln x + x^8 4^x$

$\frac{dy}{dx} = 3\left[\frac{d(\cot x)}{dx} \cdot \ln x + \frac{d(\ln x)}{dx} \cdot \cot x\right] + \left[\frac{d(x^8)}{dx} \cdot 4^x + \frac{d(4^x)}{dx} \cdot x^8\right]$

$= 3\left[-\csc^2 x \cdot \ln x + \frac{\cot x}{x}\right] + \left[8x^7 \cdot 4^x + 4^x \cdot x^8 \ln(4)\right]$

$= \boxed{-3\csc^2 x \ln x + 3\frac{\cot x}{x} + 4^x(8x^7 + x^8 \cdot \ln(4))}$

b. $y = \frac{x^5 - 3x^3 + 1}{x + \sqrt{x}}$

$\frac{dy}{dx} = \frac{\frac{d(x^5-3x^3+1)}{dx}(x+\sqrt{x}) - (x^5-3x^3+1)\cdot\frac{d(x+\sqrt{x})}{dx}}{(x+\sqrt{x})^2}$

$\frac{dy}{dx} = \frac{(5x^4 - 9x^2)(x+\sqrt{x}) - (x^5-3x^3+1)(1+\frac{1}{2\sqrt{x}})}{(x+\sqrt{x})^2}$

$\frac{dy}{dx} = \frac{5x^4 - 9x^2}{x+\sqrt{x}} - \frac{\left[x^5 + \frac{1}{2}x^4 \cdot \sqrt{x} - 3x^3 - \frac{3}{2}x^2 \cdot \sqrt{x} + 1 + \frac{1}{2\sqrt{x}}\right]}{(x+\sqrt{x})^2}$

or

$\frac{dy}{dx} = \frac{5x^4 - 9x^2}{x+\sqrt{x}} - \frac{(1+\frac{1}{2\sqrt{x}})(x^5 - 3x^3 + 1)}{(x+\sqrt{x})^2}$

the most simplified version

3b. $\frac{dy}{dx} = \frac{4x^5 + \frac{9}{2}x^4\sqrt{x} - 6x^3 - \frac{15}{2}x^2\sqrt{x} - 1 - \frac{1}{2\sqrt{x}}}{(x+\sqrt{x})^2}$

3c. $y = h^{-1}(x)$. $h(x) = e^x + 2x \rightarrow h'(x) = e^x + 2$

$x = h(y)$

$\frac{d \dots}{dx}$

$1 = \frac{dy}{dx} h'(y)$

$\boxed{\frac{dy}{dx} = \frac{1}{h'(y)} = \frac{1}{e^y + 2}}$