STA2001 Assignment 6

1. 3.2-7. Find the moment-generating function for the gamma distribution with parameters α and θ.
   Hint: In the integral representing $E(e^{tX})$, change variables by letting $y = (1 - \theta t)x/\theta$, where $1 - \theta t > 0$.

2. 3.2-9. If the moment-generating function of a random variable W is
   $$M(t) = (1 - 7t)^{-20},$$
   find the pdf mean and the variance of W.

3. 3.2-11 If X is $\chi^2(17)$, find
   (a) $P(X < 7.564)$.
   (b) $P(X > 27.59)$.
   (c) $P(6.408 < X < 27.59)$.
   (d) $\chi^2_{0.95}(17)$.
   (e) $\chi^2_{0.025}17$

4. 3.2-22 Let X have a logistic distribution with pdf
   $$f(x) = \frac{e^{-x}}{(1 + e^{-x})^2}, \quad -\infty < x < \infty.$$
   Show that
   $$Y = \frac{1}{1 + e^{-X}}$$
   has a $U(0, 1)$ distribution.
   Hint: Find $G(y) = P(Y \leq y) = P(\frac{1}{1+e^{-X}} \leq y)$, where $0 < y < 1$.

5. 3.3-2 If Z is $N(0, 1)$, find
   (a) $P(0 \leq Z \leq 0.87)$.
   (b) $P(-2.64 \leq Z \leq 0)$.
   (c) $P(-2.13 \leq Z \leq -0.56)$.

(d) $P(|Z| > 1.39)$.

(e) $P(Z < -1.62)$.

(f) $P(|Z| > 1)$.

(g) $P(|Z| > 2)$.

(h) $P(|Z| > 3)$.

6. 3.3-3 If Z is $N(0,1)$, find values of c such that

   (a) $P(Z \geq c) = 0.025$

   (b) $P(|Z| \leq c) = 0.95$

   (c) $P(Z > c) = 0.05$

   (d) $P(|Z| \leq c) = 0.90$.

7. 3.3-5. If X is normally distributed with a mean of 6 and a variance of 25, find

   (a) $P(6 \leq X \leq 12)$

   (b) $P(0 \leq X \leq 8)$

   (c) $P(-2 < X \leq 0)$

   (d) $P(X > 21)$

   (e) $P(|X - 6| < 5)$

   (f) $P(|X - 6| < 10)$

   (g) $P(|X - 6| < 15)$

   (h) $P(|X - 6| < 12.41)$

8. 3.3-6. If the moment-generating function of X is $M(t) = \exp(166t + 200t^2)$, find

   (a) The mean of X

   (b) The variance of X

   (c) $P(170 < X < 200)$

   (d) $P(148 \leq X \leq 172)$

9. 3.3-10 If X is $N(\mu, \sigma^2)$, show that the distribution of $Y = aX + b$ is $N(a\mu + b, a^2\sigma^2)$, $a \neq 0$. Hint: Find the cdf $P(Y \leq y)$ of Y, and in the resulting integral, let $w = ax + b$ or, equivalently, $x = (w - b)/a$.

10. 3.3-14. The strength X of a certain material is such that its distribution is found by $X = e^Y$, where Y is $N(10, 1)$. Find the cdf and pdf of X , and compute $P(10,000 < X < 20,000)$.Note:$F(x) = P(X \leq x) = P(e^Y \leq x) = P(Y \leq \ln x)$ so that the random variable X is said to have a lognormal distribution.