Assignment 1

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Question 1

The following statement is true or false?

- An inconsistent system has more than one solution
- Elementary row operations on an augmented matrix never change the solution set of the linear system
- If $AB = O$, then $A = O$ or $B = O$
- $A^2 - I^2 = (A + I)(A - I)$
- $(A + B)^2 = A^2 + 2AB + B^2$

Question 2

Determine whether the following linear systems are equivalent or not.

$$1)\begin{cases} x - y + 2z = 1 \\ 2x + y + z = 8 \\ x + y = 5 \end{cases} \qquad 2)\begin{cases} x + y = 5 \\ x + z = 3 \\ y - z = 2 \end{cases}$$

Question 3

For the following system of the linear equations

$$\begin{cases} 4x_1 + 5x_2 + 3x_3 + 3x_4 + 4x_5 = -5 \\ 2x_1 + 3x_2 + x_3 + x_5 = -3 \\ 3x_1 + 4x_2 + 2x_3 + x_4 + x_5 = -1 \end{cases}$$

1) Write down the coefficient matrix and the augmented matrix

2) Solve the system by Gauss-Jordan elimination

Question 4

If following linear system has nonzero solutions, find the value of k.

$$\begin{cases} 3x + ky + z = 0 \\ 4y + z = 0 \\ kx - 5y - z = 0 \end{cases}$$

Question 5

Here is a linear system,

$$\begin{cases} x + 4y - 2z = 1 \\ x + 7y - 6z = 6 \\ 3y + pz = q \end{cases}$$

1) Find the value of p and q to make the system has no solution

2) Find the value of p and q to make the system has only one solution

3) Find the value of p and q to make the system has infinitely many solutions

Question 6

Find b such that the homogeneous system $Ax = 0$ has infinitely many solutions.

$$\mathbf{A} = \begin{bmatrix} 1 & 0 & -1 & 1 \\ 0 & 1 & 2 & 1 \\ 0 & 0 & 1 & -1 \\ 1 & 2 & 4 & b \end{bmatrix}$$

Question 7

How to choose the value of a and b to make the following system

$$\begin{cases} x_1 + x_2 + x_3 = 2 \\ x_1 + 2x_2 + ax_3 = -1 \\ 2x_1 + 3x_2 = b \end{cases}$$

1) has no solution

2) has only one solution

3) has infinitely many solutions

Question 8

Suppose the following linear system exists at least one solution. Try to find the solution and the value of a.

$$\begin{cases} x_1 + x_2 + x_3 = 0 \\ x_1 + 2x_2 + ax_3 = 0 \\ x_1 + 4x_2 + a^2x_3 = 0 \\ x_1 + 2x_2 + x_3 = a - 1 \end{cases}$$

Question 9

One may raise a square matrix to any non-negative integer power multiplying it by itself repeatedly in the same way as for ordinary numbers. That is,

$$A^0 = I$$
$$A^1 = A$$
$$A^k = \underbrace{AA \cdots A}_{\text{k times}}$$

Suppose

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

Calculate the value of A^k

Question 10

If $A = A^T$ and $B = B^T$, which of the following matrices are symmetric?
(a)$A^2 - B^2$
(b)$(A + B)(A - B)$
(c)ABA
(d)$ABAB$

Question 11

Let $a = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$, $b = \begin{bmatrix} 1 \\ \frac{1}{2} \\ 0 \end{bmatrix}$, $A = ab^T$, then what is the value of A^{11}?

Question 12

If $A = \begin{bmatrix} 1 & -2 & 2 \\ -2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$, find the value of $A^8 - 6400I$.

Question 13

Suppose $A^k = 0$, calculate $(I - A)(I + A + A^2 + ... + A^{k-1})$.

Question 14

Suppose the size of the the following matrices are 2×2, give a example to show the following results are false:

(a) $AB = O$, *then* $A = O$ or $B = O$.

(b) $AB - BA = 0$, *then* $A = B$.

Question 15

Suppose the size of the the following matrices are 2×2, give a example to show the following results are false:

(a) $A^2 = O$, then $A = O$.

(b) $A^2 = A$, then $A = O$ or $A = I$.

Question 16

Please give an exmaple to show following statement is false

$(A - B)^2 = O$ implies $A = B$.

Question 17

Prove that: if A is a real square matrix, and $A^T A = O$, then $A = O$

Question 18

Let A and B be $n \times n$ square matrices, $A^2 = A$ and $(A + B)^2 = A^2 + B^2$. Prove that $AB(A + I) = O$.

Question 19

Let

$$A = \begin{bmatrix} \frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} \\ -\frac{1}{2} & \frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} \\ -\frac{1}{2} & -\frac{1}{2} & \frac{1}{2} & -\frac{1}{2} \\ -\frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} & \frac{1}{2} \end{bmatrix}$$

Compute A^2 and A^3. What will A^{2n} and A^{2n+1} turn out to be?

Question 20

Let α is a 3 dimensional column vector, if $\alpha\alpha^T = \begin{bmatrix} 1 & -1 & 1 \\ -1 & 1 & -1 \\ 1 & -1 & 1 \end{bmatrix}$, find the value of $\alpha^T\alpha$