Yohandi - quiz 6

1a. True

b. False

c. True

2. a. $\int 2x^5 - \sqrt[3]{6x+1}\, dx$

$\int 2x^5\, dx - \int \sqrt[3]{6x+1}\, dx$

let $u = 6x+1$

$du = 6\, dx$

$\frac{1}{3}x^6 + C_1 - \int \sqrt[3]{u}\, \frac{du}{6}$

$\frac{1}{3}x^6 + C_1 - \frac{1}{8}\sqrt[3]{(6x+1)^4} - C_2$

$\boxed{\frac{1}{3}x^6 - \frac{1}{8}(6x+1)^{4/3} + C}$

b. $\frac{d(x\cos x)}{dx} = \cos x - x\sin x$

$-\left(\frac{d(x\cos x)}{dx} - \cos x\right) = x\sin x$

$\int \ldots dx$.

$-\left[x\cos x - \sin x\right] = \int x\sin x\, dx$

$\boxed{\int x\sin x\, dx = \sin x - x\cos x + C}$

$\int \sec x = \ln|\tan x + \sec x| + C$

$x = 0 \to y = 1$

$1 = \ln|0 + 1| + C$

$C = 1$

$\ln|\tan x + \sec x| + \ln(e)$

$\ln(e(\tan x + \sec x))$

3. since x^2 is strictly increasing from $0 \le x \le 1$ (as $f(1^-) < f(1)$)

a. $U_n = \sum_{k=1}^{n} f(c_k).\Delta x_k$.

$= \Delta x_k \left(\sum_{k=1}^{n-1} f(c_k) + f(c_n)\right)$

$= \frac{1}{n}\left(\sum_{k=1}^{n-1} \frac{k^2}{n^2} + f(1)\right)$

$= \frac{1}{n^3} \cdot \frac{(n-1)(n)(2n-1)}{6} + \frac{1}{n} f(1)$

$= \frac{n(n-1)(2n-1)}{6n^3} + \frac{f(1)}{n}$

we can see that value of Un depends on f(1).

→ the smaller n, the bigger error

b. $\lim_{n\to\infty} U_n$

$= \lim_{n\to\infty} \frac{n(n-1)(2n-1)}{6n^3} + \lim_{n\to\infty} \frac{f(1)}{n}$

$= \frac{1}{3} + f(1) \lim_{n\to\infty} \frac{1}{n}$

$= \frac{1}{3} + f(1) . 0$

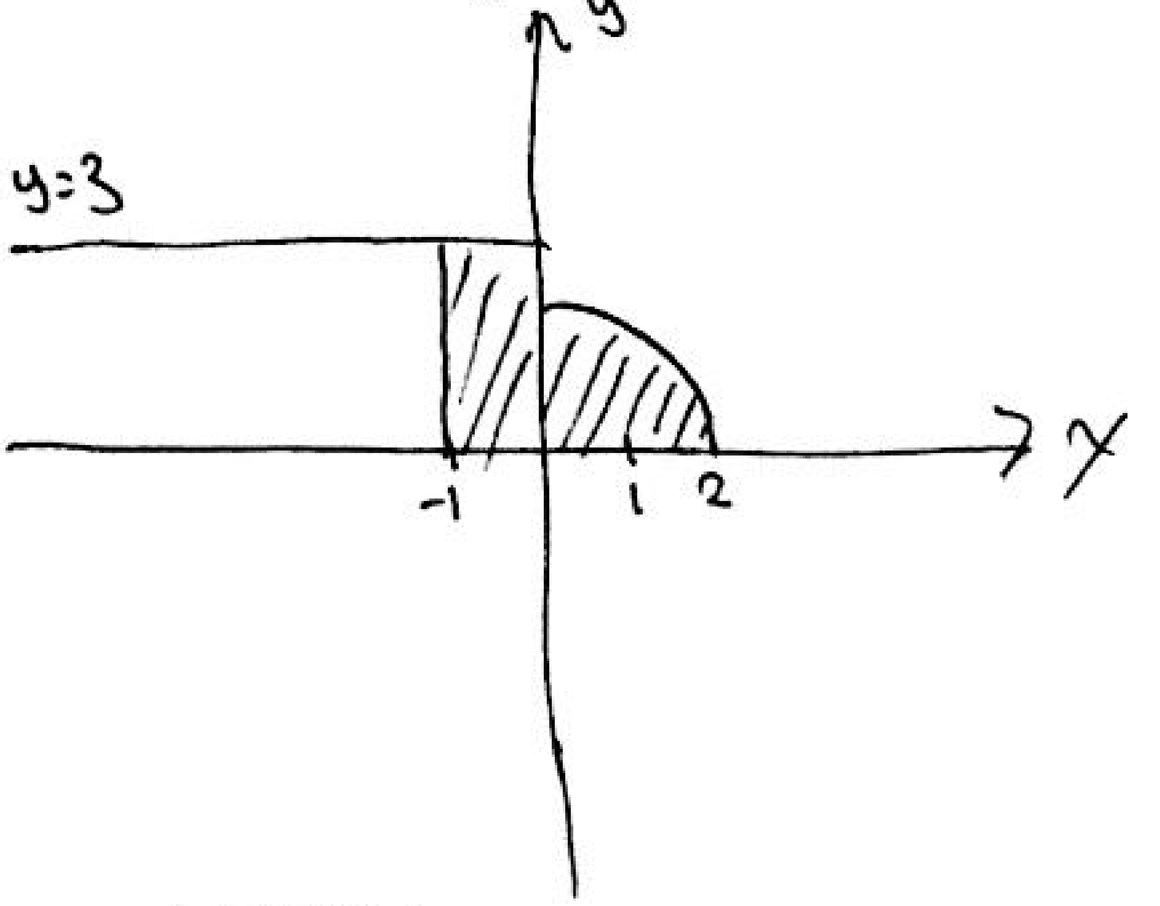
$= \frac{1}{3}$

we can see that as $n \to \infty$, the value of $\frac{f(1)}{n}$ will tend to 0 resulting $\lim_{n\to\infty} U_n$ doesn't depend on the value of f(1)

4. $f(x) = \begin{cases} 3, & -1 \le x < 0 \\ \sqrt{4-x^2}, & 0 \le x \le 2 \end{cases}$

$$\int_{-1}^{2} f(x)\,dx = \int_{-1}^{0} f(x)\,dx + \int_{0}^{2} f(x)\,dx$$

$$= \int_{-1}^{0} 3\,dx + \int_{0}^{2} \sqrt{4-x^2}\,dx.$$



for $-1 \le x < 0$:

Area $= b.h = (0-(-1)).3 = 3$

for $0 \le x \le 2$:

Area $= \frac{1}{4}\pi R^2 = \frac{1}{4}\pi(2)^2 = \pi$

Total Area $= \boxed{\pi+3} = \int_{-1}^{2} f(x)\,dx$