

# Homework 8

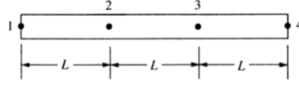
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## Problem

For the one-dimensional bar discretized into three elements, determine the lumped- and consistent mass matrices. Let the bar properties be  $E$ ,  $\rho$ , and  $A$  throughout the bar.



## Solution

First, it is easy to get the lumped mass matrix. Setting the parts as element 1 to element 3 from left to right.

Each element has its own element mass matrix, and with the formular  $m = \frac{\rho AL}{2}$ , we can know the matrices are all as below.

$$\mathbf{M}^{(i)} = \begin{bmatrix} \frac{\rho AL}{2} & 0 \\ 0 & \frac{\rho AL}{2} \end{bmatrix}$$

Add them up, and we can easily get the final result

$$\mathbf{M} = \begin{bmatrix} \frac{\rho AL}{2} & 0 & 0 & 0 \\ 0 & \rho AL & 0 & 0 \\ 0 & 0 & \rho AL & 0 \\ 0 & 0 & 0 & \frac{\rho AL}{2} \end{bmatrix}$$

As for consistent mass matrix, each element also has the same element mass matrix as below.

$$\mathbf{M}^{(i)} = \frac{\rho AL}{6} \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \quad (1)$$

Add them up, and we can easily get the final result

$$\mathbf{M} = \frac{\rho AL}{6} \begin{bmatrix} 2 & 1 & 0 & 0 \\ 1 & 4 & 1 & 0 \\ 0 & 1 & 4 & 1 \\ 0 & 0 & 1 & 2 \end{bmatrix}$$