Homework 2

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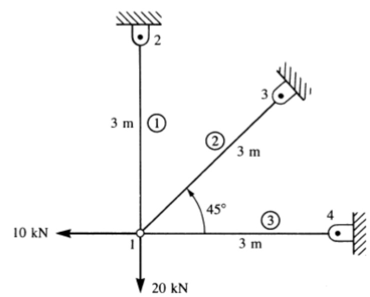
1.

For the plane truss shown below, determine the horizontal and vertical displacements of node 1 and the stresses in each element. All elements have $E = 200$ GPa and $A = 4.0\times10^{-4}$ m^2.



For $element1$, we have

$$\theta = 90^\circ, \qquad C = 0, \qquad S = 1$$

$$\mathbf{k_1} = \frac{EA}{L}\begin{bmatrix} C^2 & S & -C^2 & -S \\ CS & S^2 & -CS & -S^2 \\ -C^2 & -S & C^2 & S \\ -CS & -S^2 & CS & S^2 \end{bmatrix} = \frac{EA}{L}\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 1 \end{bmatrix}$$

For $element2$, we have

$$\theta = 45^\circ, \qquad C = S = \frac{\sqrt{2}}{2}$$

$$\mathbf{k_2} = \frac{EA}{L}\begin{bmatrix} C^2 & S & -C^2 & -S \\ CS & S^2 & -CS & -S^2 \\ -C^2 & -S & C^2 & S \\ -CS & -S^2 & CS & S^2 \end{bmatrix} = \frac{EA}{2L}\begin{bmatrix} 1 & 1 & -1 & -1 \\ 1 & 1 & -1 & -1 \\ -1 & -1 & 1 & 1 \\ -1 & -1 & 1 & 1 \end{bmatrix}$$

For $element3$, we have

$$\theta = 0^\circ, \qquad C = 1, \qquad S = 0$$

$$\mathbf{k_3} = \frac{EA}{L}\begin{bmatrix} C^2 & S & -C^2 & -S \\ CS & S^2 & -CS & -S^2 \\ -C^2 & -S & C^2 & S \\ -CS & -S^2 & CS & S^2 \end{bmatrix} = \frac{EA}{L}\begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Add them up, and we get

$$\mathbf{K} = \frac{EA}{2L}\begin{bmatrix} 3 & 1 & 0 & 0 & -1 & -1 & -2 & 0 \\ 1 & 3 & 0 & -2 & -1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -2 & 0 & 2 & 0 & 0 & 0 & 0 \\ -1 & -1 & 0 & 0 & 1 & 1 & 0 & 0 \\ -1 & -1 & 0 & 0 & 1 & 1 & 0 & 0 \\ -2 & 0 & 0 & 0 & 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

so the equation $\mathbf{F} = \mathbf{KU}$ can be write as below

$$\frac{4}{3}\times 10^7\begin{bmatrix}3 & 1 & 0 & 0 & -1 & -1 & -2 & 0\\ 1 & 3 & 0 & -2 & -1 & -1 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & -2 & 0 & 2 & 0 & 0 & 0 & 0\\ -1 & -1 & 0 & 0 & 1 & 1 & 0 & 0\\ -1 & -1 & 0 & 0 & 1 & 1 & 0 & 0\\ -2 & 0 & 0 & 0 & 0 & 0 & 2 & 0\\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\end{bmatrix}\begin{bmatrix}u_1\\ v_1\\ u_2\\ v_2\\ u_3\\ v_3\\ u_4\\ v_4\end{bmatrix}=\begin{bmatrix}F_{1x}\\ F_{1y}\\ F_{2x}\\ F_{2y}\\ F_{3x}\\ F_{3y}\\ F_{4x}\\ F_{4y}\end{bmatrix}$$

Then apply the BCs

$$u_2 = v_2 = u_3 = v_3 = u_4 = v_4 = 0$$
$$F_{1x} = -10kN, \qquad F_{1y} = -20kN$$

So now we have

$$\frac{4}{3}\times 10^7\begin{bmatrix}3 & 1\\ 1 & 3\end{bmatrix}\begin{bmatrix}u1\\ v1\end{bmatrix}=\begin{bmatrix}-10000\\ -20000\end{bmatrix}$$

Solve the equation and we get

$$u_1 = -9.375\times 10^{-5}m$$
$$v_1 = -4.6875\times 10^{-4}m$$

Now we can calculate the stresses in each element using the fomular as below

$$\sigma = \frac{E}{L}\begin{bmatrix}-C & -S & C & S\end{bmatrix}\begin{bmatrix}u_i\\ v_i\\ u_j\\ v_j\end{bmatrix}$$

So the result is

$$\sigma_1 = 31.25MPa$$

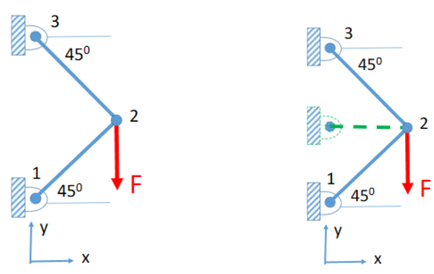
$$\sigma_2 = 26.52MPa$$

$$\sigma_3 = 6.25MPa$$

2.

For the plane bar structure below in Figure (a). Each bar has a length l, cross-sectional area A, and Young's modulus E. An externa load F is applied at node 2 ($F_{2y} = -F$). Please

1) find the displacements at node 2 (u_2 and v_2).
2) find the reaction force at node 1 (F_{1x}, F_{1y}) and node 3(F_{3x}, F_{3y}). Check if the external forces of all nodes are balanced in x and y direction.
3) Someone wants to improve the structure by adding another horizontal bar as shown in Figure (b). Do you think it will work or not? Briefly explain your reason.



(a) (b)

1)

for $element1$ between nodes 1 and 2, we have

$$\theta = 45°, \qquad C = S = \frac{\sqrt{2}}{2}$$

$$\mathbf{k_1} = \frac{EA}{L}\begin{bmatrix} C^2 & S & -C^2 & -S \\ CS & S^2 & -CS & -S^2 \\ -C^2 & -S & C^2 & S \\ -CS & -S^2 & CS & S^2 \end{bmatrix} = \frac{EA}{2L}\begin{bmatrix} 1 & 1 & -1 & -1 \\ 1 & 1 & -1 & -1 \\ -1 & -1 & 1 & 1 \\ -1 & -1 & 1 & 1 \end{bmatrix}$$

for $element2$ between nodes 2 and 3, we have

$$\theta = 135^\circ, \qquad C = -\frac{\sqrt{2}}{2}, \qquad S = \frac{\sqrt{2}}{2}$$

$$\mathbf{k_1} = \frac{EA}{L}\begin{bmatrix} C^2 & S & -C^2 & -S \\ CS & S^2 & -CS & -S^2 \\ -C^2 & -S & C^2 & S \\ -CS & -S^2 & CS & S^2 \end{bmatrix} = \frac{EA}{2L}\begin{bmatrix} 1 & -1 & -1 & 1 \\ -1 & 1 & 1 & -1 \\ -1 & 1 & 1 & -1 \\ 1 & -1 & -1 & 1 \end{bmatrix}$$

Add them up, and we can write $\mathbf{F} = \mathbf{KU}$ as below

$$\frac{EA}{2L}\begin{bmatrix} 1 & 1 & -1 & -1 & 0 & 0 \\ 1 & 1 & -1 & -1 & 0 & 0 \\ -1 & -1 & 2 & 0 & -1 & 1 \\ -1 & -1 & 0 & 2 & 1 & -1 \\ 0 & 0 & -1 & 1 & 1 & -1 \\ 0 & 0 & 1 & -1 & -1 & 1 \end{bmatrix}\begin{bmatrix} u_1 \\ v_1 \\ u_2 \\ v_2 \\ u_3 \\ v_3 \end{bmatrix} = \begin{bmatrix} F_{1x} \\ F_{1y} \\ F_{2x} \\ F_{2y} \\ F_{3x} \\ F_{3y} \end{bmatrix}$$

Then apply the BCs.

$$u_1 = v_1 = u_3 = v_3 = 0$$
$$F_{2x} = 0, \quad F_{2y} = -F$$

So now we have

$$\frac{EA}{2L}\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}\begin{bmatrix} u2 \\ v2 \end{bmatrix} = \begin{bmatrix} 0 \\ -F \end{bmatrix}$$

Solve the equation and we get

$$u_2 = 0$$
$$v_2 = -\frac{FL}{EA}$$

2)

Now we have all displacements, so it is easy to calculate the reaction force.

$$F_{1x} = \frac{F}{2}$$
$$F_{1y} = \frac{F}{2}$$
$$F_{3x} = -\frac{F}{2}$$
$$F_{3y} = \frac{F}{2}$$

Check the external forcesof all nodes

$$F_x = F_{1x} + F_{2x} + F_{3x} = 0$$
$$F_y = F_{1y} + F_{2y} + F_{3y} = 0$$

They are balanced in x and y direction.

3)

It will not work. Because the horizontal bar can only apply horizontal force. However, there is no horizontal displacement at node 2. In fact, the new bar apply no force to node 2, so it will not work.