General Physics II

Homework #7

(Abridged)

P7-1. A special kind of lightbulb emits monochromatic light of wavelength 630 nm. Electrical energy is supplied to it at the rate of 60 W, and the bulb is 93% efficient at converting that energy to light energy. How many photons are emitted by the bulb during its lifetime of 730 h?

Solution: The total energy emitted by the bulb is $E = 0.93Pt$, where $P = 60$ W and

$$t = 730 \text{ h} = 2.628 \times 10^6 \text{ s}.$$

The energy of each photon emitted is $E_{\text{ph}} = hc/\lambda$. Therefore, the number of photons emitted is

$$\begin{aligned} N &= \frac{E}{E_{\text{ph}}} = \frac{0.93Pt}{hc/\lambda} \\ &= \frac{0.93 \cdot 60 \cdot 2.628 \times 10^6}{6.63 \times 10^{-34} \cdot 2.998 \times 10^8/(630 \times 10^{-9})} \\ &= 4.7 \times 10^{26}. \end{aligned}$$

P7-2. In a photoelectric experiment using a sodium surface, you find a stopping potential of 1.85 V for a wavelength of 300 nm and a stopping potential of 0.820 V for a wavelength of 400 nm. From these data find (a) a value for the Planck constant, (b) the work function Φ for sodium, and (c) the cutoff wavelength λ_0 for sodium.

Solution:
(a) For the first and second case (labeled 1 and 2) we have

$$eV_{01} = hc/\lambda_1 - \Phi, \qquad eV_{02} = hc/\lambda_2 - \Phi,$$

from which h and Φ can be determined. Thus,

$$\begin{aligned} h &= \frac{e(V_1 - V_2)}{c(\lambda_1^{-1} - \lambda_2^{-1})} \\ &= \frac{1.85\mathrm{eV} - 0.820\mathrm{eV}}{(3.00 \times 10^{17}\mathrm{nm/s})[(300\mathrm{nm})^{-1} - (400\mathrm{nm})^{-1}]} \\ &= 4.12 \times 10^{-15}\ \mathrm{eV \cdot s}. \end{aligned}$$

(b) The work function is

$$\Phi = \frac{3(V_2\lambda_2 - V_1\lambda_1)}{\lambda_1 - \lambda_2} = \frac{0.820 \cdot 400 - 1.85 \cdot 300}{300 - 400}$$
$$= 2.27 \text{ eV}.$$

(c) Let $\Phi = hc/\lambda_{\mathrm{max}}$ to obtain

$$\lambda_{\mathrm{max}} = \frac{hc}{\Phi} = \frac{1240}{2.27} = 545 \text{ nm}.$$

P7-3. Show that when a photon of energy E is scattered from a free electron at rest, the maximum kinetic energy of the recoiling electron is given by

$$K_{\mathrm{max}} = \frac{E^2}{E + mc^2/2}.$$

Solution: Referring to Sample Problem — "Compton scattering of light by electrons," we see that the fractional change in photon energy is

$$\frac{E - E_n}{E} = \frac{\Delta\lambda}{\lambda + \Delta\lambda} = \frac{(h/mc)(1 - \cos\phi)}{(hc/E) + (h/mc)(1 - \cos\phi)}.$$

Energy conservation demands that $E\breve{\ }E' = K$, the kinetic energy of the electron. In the maximal case, $\phi = 180^\circ$, and we find

$$\begin{aligned}\frac{K}{E} &= \frac{(h/mc)(1 - \cos 180^\circ)}{(hc/E) + (h/mc)(1 - \cos 180^\circ)} \\ &= \frac{2h/mc}{(hc/E) + (2h/mc)}.\end{aligned}$$

Multiplying both sides by E and simplifying the fraction on the right-hand side leads to

$$K = E\frac{2/mc}{c/E + 2/mc} = \frac{E^2}{mc^2/2 + E}.$$

P7-4. The highest achievable resolving power of a microscope is limited only by the wavelength used; that is, the smallest item that can be distinguished has dimensions about equal to the wavelength. Suppose one wishes to “see” inside an atom. Assuming the atom to have a diameter of 100 pm, this means that one must be able to resolve a width of, say, 10 pm. (a) If an electron microscope is used, what minimum electron energy is required? (b) If a light microscope is used, what minimum photon energy is required? (c) Which microscope seems more practical? Why?

Solution: (a) Setting $\lambda = h/p = h/\sqrt{E/c^2 - m_e^2 c^2}$, we solve for $K = E \breve{} m_e c^2$:

$$
\begin{aligned}
K &= \sqrt{\left(\frac{hc}{\lambda}\right)^2 + m_e^2 c^4} - m_e c^2 \\
&= \sqrt{\left(\frac{1240\mathrm{eV}\cdot\mathrm{nm}}{10\times 10^{-3}\mathrm{nm}}\right)^2 + (0.511\mathrm{MeV})^2} - 0.511\mathrm{MeV} \\
&= 15\ \mathrm{keV}.
\end{aligned}
$$

(b) Using the value $hc = 1240\ \mathrm{eV}\cdot\mathrm{nm}$,

$$
E = \frac{hc}{\lambda} = \frac{1240\mathrm{eV}\cdot\mathrm{nm}}{10\times 10^{-3}\mathrm{nm}} = 1.2\times 10^5\mathrm{eV} = 120\ \mathrm{keV}.
$$

(c) The electron microscope is more suitable, as the required energy of the electrons is much less than that of the photons.