Introduction to Scientific Computing, Homework #1

Problem 1 (10 pts)

Consider an array of random numbers constructed in MATLAB using a for loop. Type the following code into MATLAB at the command line or in a script. Observe the form of the output randArr.

```
for jj = 1:20
    randArr(jj) = rand;
end
```

Note that **rand** outputs a random number.

Produce an array of random numbers, **myRandArr**, with the same dimensions as in the example above but using a single line of code and without the use of a ***for*** loop. Helpful hint: check the MATLAB help document for **rand**.

Problem 1 Solution

```
% Generate a random vector of numbers between 0 and 1 with
% dimensions of 1 x 20
MyRandArr = rand(1,20)
```

The output is:

```
>> MyRandArr = rand(1, 20)

MyRandArr =

  1 至 11 列

    0.8147    0.9058    0.1270    0.9134    0.6324    0.0975    0.2785    0.5469    0.9575    0.9649    0.1576

  12 至 20 列

    0.9706    0.9572    0.4854    0.8003    0.1419    0.4218    0.9157    0.7922    0.9595
```

Problem 2 (10 pts)

Write a MATLAB script that uses the law of cosines to compute the length of a third side of triangle ***c***, given the lengths of the other two sides, ***a*** and ***b***, and the opposing angle, θ . Your function should compute ***c*** based on the following expression:

$$c^2 = a^2 + b^2 - 2ab\cos\theta$$

Be wary of the trigonometric units used when computing cosine in MATLAB. Compute and report c for the following values of $a = 5, b = 3$, and $\theta = 45^\circ$.

Problem 2 Solution

```
a=5;
b=3;
theta=45;
c=sqrt(a**2+b**2-2*a*b*cos(theta*pi/180))
```

The output is: c = 3.5759

Problem 3 (20 pts)

Consider the following system of linear equations

$$
\begin{aligned}
-23x_1 - 18x_2 + 2x_3 - 19x_4 - 15x_5 &= 22 \\
-x_1 - 16x_2 + 10x_3 + x_4 + 3x_5 &= -21 \\
-15x_1 - 23x_2 + x_3 + 8x_4 + 7x_5 &= -20 \\
-19x_1 + 7x_2 + 2x_3 + 19x_4 - 4x_5 &= -18 \\
-15x_1 - 11x_2 - 3x_3 - 11x_4 - 15x_5 &= -17
\end{aligned}
$$

Solve this system of equations using three syntactically distinct methods in a single script file (m file). The arrays corresponding to the solutions of x should be named *xSol1*, *xSol2*, and *xSol3* for solution methods 1, 2, and 3, respectively. What are the values of x1 through x5 to three significant figures?

Problem 3 Solution

```
% Define the coefficient matrix A
A = [-23,-18,2,-19,-15;
     -1,-16,10,1,3;
     -15,-23,1,8,7;
     -19,7,2,19,-4;
     -15,-11,-3,-11,-15];
% Define a column vector b
b = [22;-21;-20;-18;-17];

% Solve the system of equations using 9 syntactically different
% methods
xsol1 = A\b;

xsol2 = inv(A)*b;

xsol3 = linsolve(A,b);
```

The output is:

```
xsol1 =        xsol2 =        xsol3 =

   -2.7530        -2.7530        -2.7530
    2.7370         2.7370         2.7370
    1.0380         1.0380         1.0380
   -3.8680        -3.8680        -3.8680
    4.5090         4.5090         4.5090
```

The solution to the system to three significant digits was found to be:

$$x_1 = -2.75\ x_2 = 2.74\ x_3 = 1.04\ x_4 = -3.87\ x_5 = 4.51$$

Problem 4 The world's oldest matrix equation.

The earliest recorded analysis of a simultaneous linear equation is found in the ancient Chinese book Chiu-chang Suan-shu (Nine Chapters on Arithmetic), estimated to have been written about 200 B.C. (they didn't have MATLAB then so they probably never figured out the answer).

> Three sheafs of a good crop, two sheafs of a mediocre crop and one sheaf of a bad crop are sold for 39 dou. Two sheafs of good, three mediocre and one bad are sold for 34 dou; and one good, two mediocre and three bad are sold for 26 dou. What is the price received for each sheaf of a good crop, each sheaf of a mediocre crop and each sheaf of a bad crop?

Write a MATLAB script to solve this problem.

Problem 4 Solution

```
%% problem4
A = [3,2,1;
    2,3,1;
    1,2,3];
b = [39;34;26];
x=A\b;
[good, midcore, bad] = deal(x(1), x(2), x(3))
```

The output is:

```
good =

    9.2500


midcore =

    4.2500


bad =

    2.7500
```

Problem 5

Hypocycloids, epicycloids, epitrochoids and hypotrochoids are all fancy names for curves generated when a circle rotates about another circle (a.k.a. roulettes). You can find interesting descriptions and .gifs for all of these roulettes on Wikipedia. You will write a short MATLAB function with the following function declaration:

```
function spirograph(R,r,d)
```

You function will generate curves of the following form

$$x(\theta) = (R+r)\cos\theta + d\cos\left(\left(\frac{R+r}{r}\right)\theta\right)$$

$$y(\theta) = (R+r)\sin\theta - d\sin\left(\left(\frac{R+r}{r}\right)\theta\right)$$

You should begin by creating an array called **theta** that has values between 0 and 10π in steps of no more than 0.001 radians. You can do this using MATLAB's colon operator or the **linspace** command. You will then compute values for x and y using array operations, addition, subtraction, the cosine function, etc. This should all be achieved without loops. Once you have the two arrays, x and y, your function should plot the curve using the command **plot(x,y)**. Look at the help pages for the **plot** command to determine the visualization options this command offers and how you can go about changing the color or style of the plotted line.

Include pretty plots for the following values of *R*, *r*, and *d* in your assignment submission.

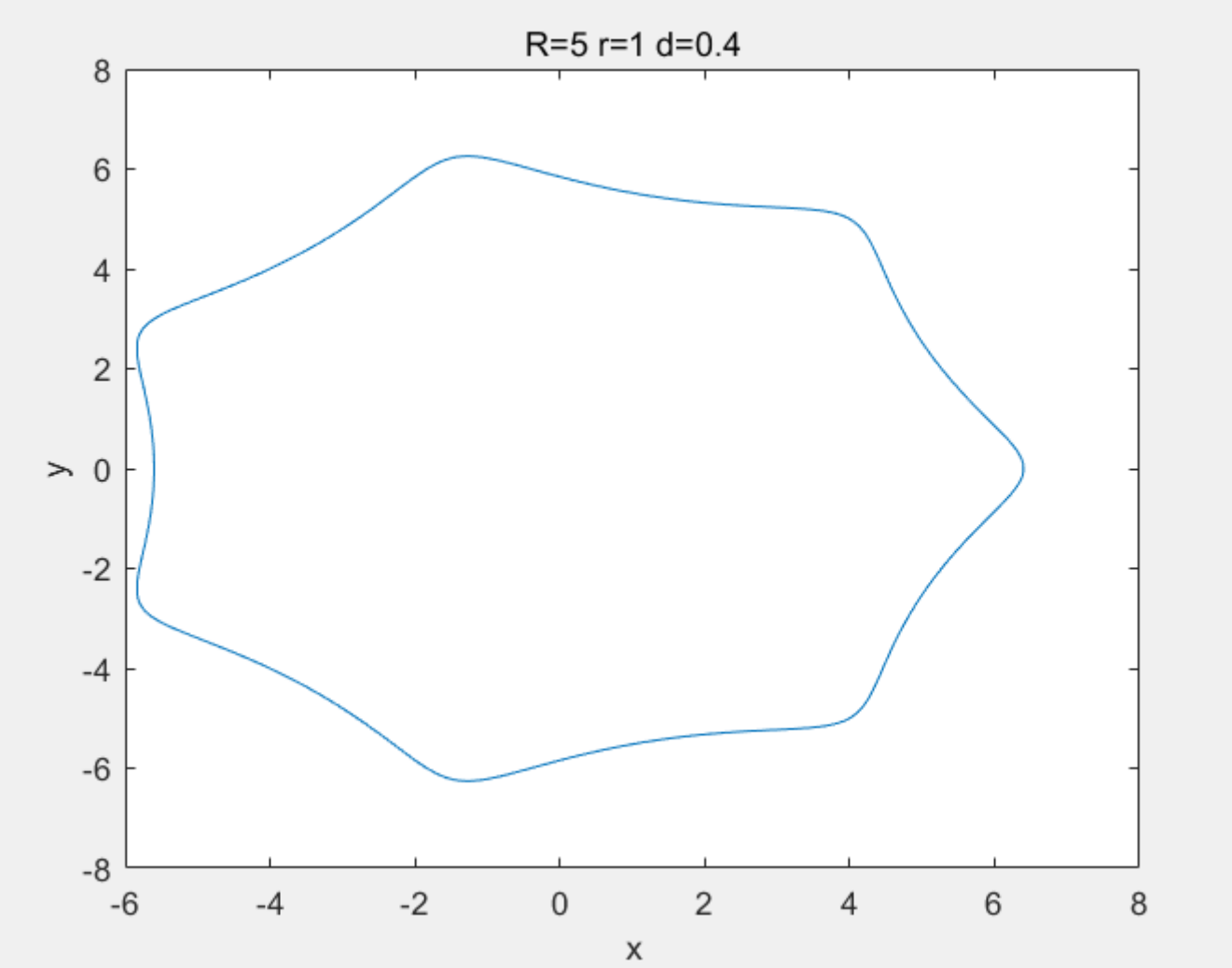
Plot number	***R***	***r***	***d***
1	5	1	0.4
2	12	-1	1.5

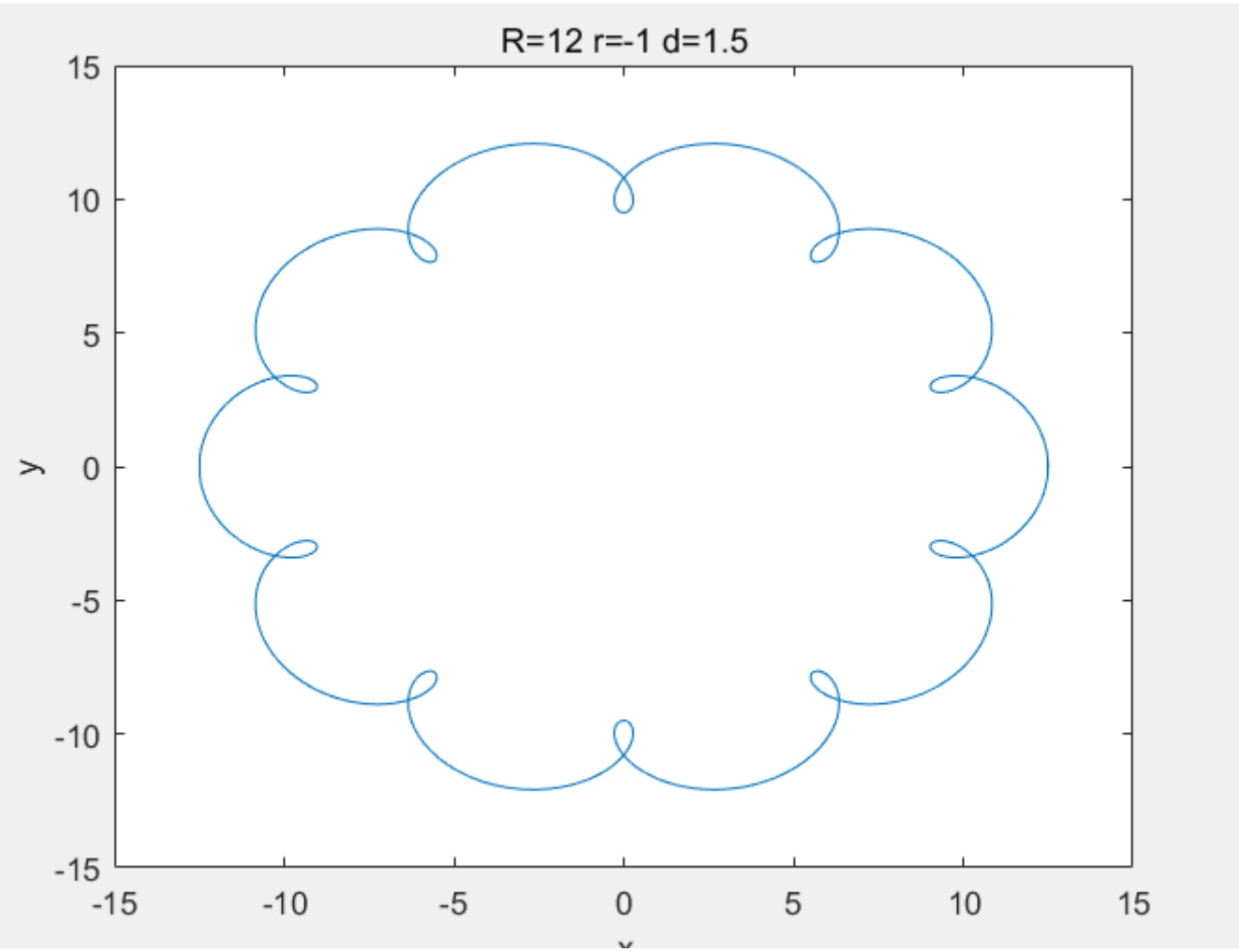
Plot number	*R*	*r*	*d*
3	7	-1	1

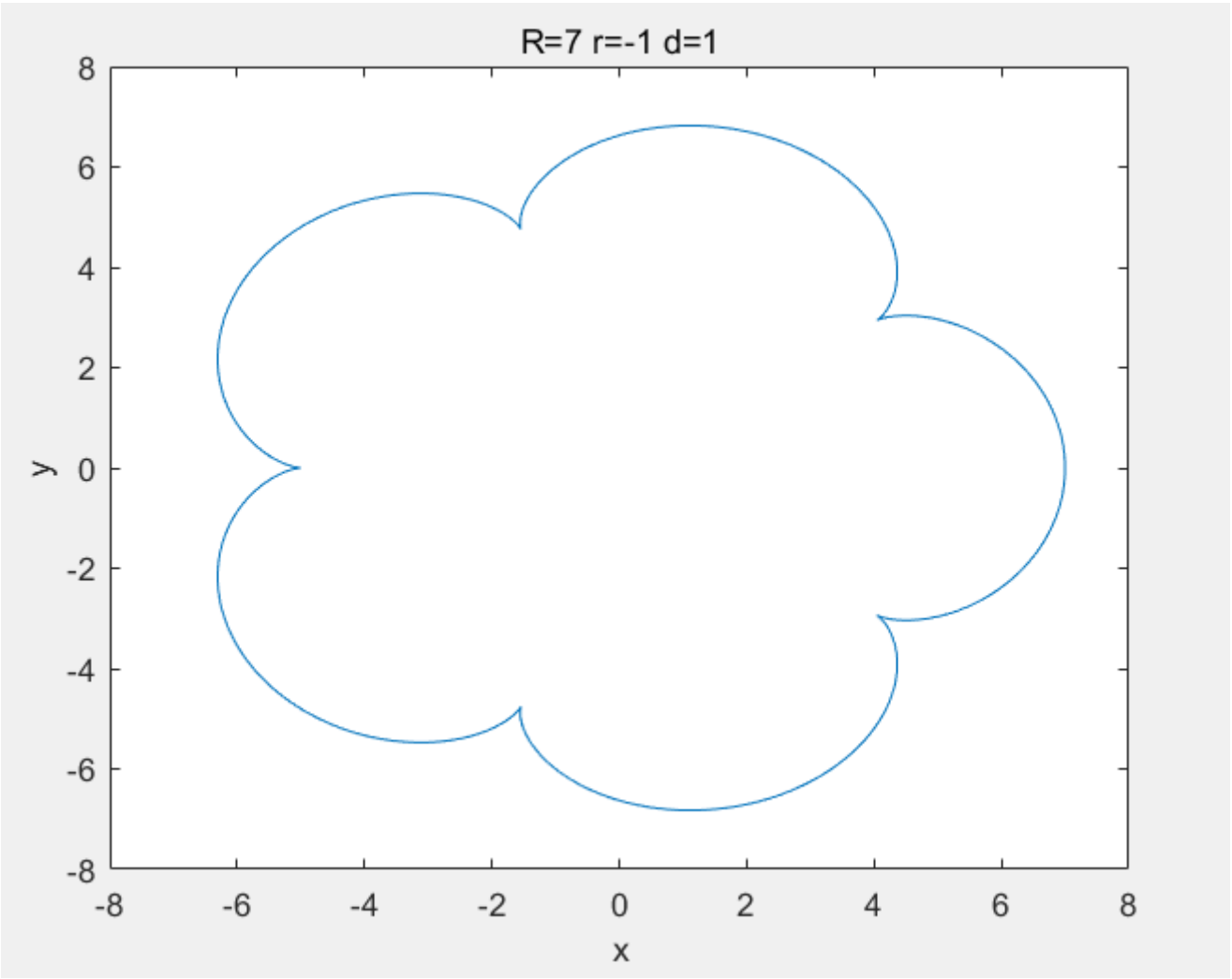
Problem 5 Solution

```
function spirograph(R,r,d)
theta = 0:0.001:10*pi;
x=(R+r)*cos(theta)+d*cos((R+r)/r*theta);
y=(R+r)*sin(theta)-d*sin((R+r)/r*theta);
plot(x,y);
xlabel('x');
ylabel('y');
title(['R=',num2str(R),' r=',num2str(r), ' d=',num2str(d)])
end
```

Plots corresponding to the three cases above are shown below.


R=5 r=1 d=0.4
8
6
4
2
0
-2
-4
-6
-8
y
-6
-4
-2
0
2
4
6
8
x


R=12 r=-1 d=1.5
15
10
5
0
-5
-10
-15
y
-15
-10
-5
0
5
10
15



Problem 6

Fibonacci sequences often appear in nature. See the following link for a description of Fibonacci numbers: Fibonacci Wikipedia

The Fibonacci sequence is described by the relationship

$$F_n = F_{n-1} + F_{n-2}$$

where $2 \leq n < N$ with N being some integer. The initial values of the Fibonnaci sequence are

$$F_0 = 0, F_1 = 1$$

The Fibonnaci sequence for $N = 6$ is: 0, 1, 1, 2, 3, 5, 8.

You are to write a program that returns a vector of the first N Fibonacci numbers with the following function declaration:

```
function nums = fib(N)
% fib(N) returns a list of the first N Fibonacci Numbers.
% N must be an integer.
```

What is the Fibonacci sequence for $N = 20$?

Problem 6 Solution

```
function nums=fib(N)
    if(N==0) nums=[0];
    else nums=[0,1];
    for i = 2:N
        nums(i+1)=nums(i)+nums(i-1);
    end
end
```

The Fibonacci sequence for N=20 is

```
>> fib(20)

ans =

  1 至 9 列

     0     1     1     2     3     5     8    13    21

  10 至 18 列

    34    55    89   144   233   377   610   987  1597

  19 至 21 列

  2584  4181  6765

>>
```