Homework 3

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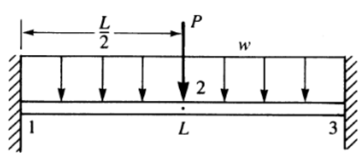
1.

For the beam shown below subjected to the concentrated load P and distributed load w, determine the midspan displacement and the reaction force and moment at the ends. Let EI be constant throughout the beam.

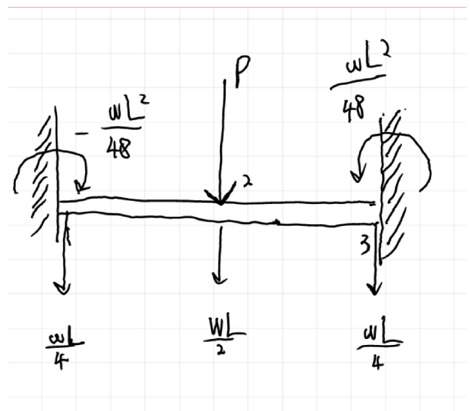


We can have the equivalent nodal loads as below.

Considering the part that between nodes 1 and 2 as $element1$, then we have

$$\mathbf{k_1} = \frac{8EI}{L^3}\begin{bmatrix} 12 & 3L & -12 & 3L \\ 3L & L^2 & -3L & \frac{L^2}{2} \\ -12 & -3L & 12 & -3L \\ 3L & \frac{L^2}{2} & -3L & L^2 \end{bmatrix}$$

Considering the part that between nodes 2 and 3 as $element2$, then we



have

$$\mathbf{k_2} = \frac{8EI}{L^3}\begin{bmatrix} 12 & 3L & -12 & 3L \\ 3L & L^2 & -3L & \frac{L^2}{2} \\ -12 & -3L & 12 & -3L \\ 3L & \frac{L^2}{2} & -3L & L^2 \end{bmatrix}$$

Now we can get the global FE equation.

$$\frac{8EI}{L^3}\begin{bmatrix} 12 & 3L & -12 & 3L & 0 & 0 \\ 3L & L^2 & -3L & \frac{L^2}{2} & 0 & 0 \\ -12 & -3L & 24 & 0 & -12 & 3L \\ 3L & \frac{L^2}{2} & 0 & 2L^2 & -3L & \frac{L^2}{2} \\ 0 & 0 & -12 & -3L & 12 & -3L \\ 0 & 0 & 3L & \frac{L^2}{2} & -3L & L^2 \end{bmatrix}\begin{bmatrix} v_1 \\ \theta_1 \\ v_2 \\ \theta_2 \\ v_3 \\ \theta_3 \end{bmatrix} = \begin{bmatrix} F_{1y} \\ M_1 \\ F_{2y} \\ M_2 \\ F_{3y} \\ M_3 \end{bmatrix}$$

Applying the BCs.

$$v_1 = \theta_1 = v_3 = \theta_3 = 0$$
$$F_{2y} = -P, M2 = 0$$

Solving the midspan displacement first.

$$\frac{8EI}{L^3}\begin{bmatrix} 24 & 0 \\ 0 & 8L^2 \end{bmatrix}\begin{bmatrix} v_2 \\ \theta_2 \end{bmatrix} = \begin{bmatrix} -P - \frac{wL}{2} \\ 0 \end{bmatrix}$$

Solvong the equations and we can get the displacement.

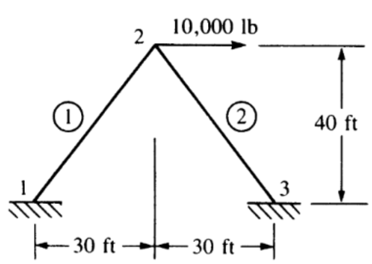
$$v_2 = -\frac{PL^3 + \frac{wL^4}{2}}{192EI}$$
$$\theta_2 = 0$$

Then we can easily calculate the reaction force and moment at ends.

$$F_{1y} = \frac{P + wL}{2}$$
$$M_1 = \frac{3P + 2wL^2}{24}$$
$$F_{2y} = \frac{P + wL}{2}$$
$$M_2 = -\frac{3P + 2wL^2}{24}$$

2.

For the rigid frame shown below. Determine (1) the displacement components and the rotation at node 2; (2) the support reactions.



(1)

We have known that

$$\frac{E}{L}\begin{bmatrix} AC^2+\frac{12I}{L^2}S^2 & \left(A-\frac{12I}{L^2}\right)CS & -\frac{6I}{L}S & -\left(AC^2+\frac{12I}{L^2}S^2\right) & -\left(A-\frac{12I}{L^2}\right)CS & -\frac{6I}{L}S \\ & AS^2+\frac{12I}{L^2}C^2 & \frac{6I}{L}C & -\left(A-\frac{12I}{L^2}\right)CS & -\left(AS^2+\frac{12I}{L^2}C^2\right) & \frac{6I}{L}C \\ & & 4I & \frac{6I}{L}S & -\frac{6I}{L}C & 2I \\ & & & AC^2+\frac{12I}{L^2}S^2 & \left(A-\frac{12I}{L^2}\right)CS & \frac{6I}{L}S \\ & & & & AS^2+\frac{12I}{L^2}C^2 & -\frac{6I}{L}C \\ \text{Symmetry} & & & & & 4I \end{bmatrix} \begin{Bmatrix} u_i \\ v_i \\ \theta_i \\ u_j \\ v_j \\ \theta_j \end{Bmatrix} = \begin{Bmatrix} F_{iX} \\ F_{iY} \\ M_i \\ F_{jX} \\ F_{jY} \\ M_j \end{Bmatrix}$$

For *element*1 between nodes 1 and 2

$$C = 0.6, \qquad S = 0.8$$

$$\mathbf{k_1} = 5 \times 10^4 \begin{bmatrix} 3.611 & 4.792 & -4.000 & -3.611 & -4.792 & -4.000 \\ 4.792 & 6.406 & 3.000 & -4.792 & -6.406 & 3.000 \\ -4.000 & 3.000 & 2000 & 4.000 & -3.000 & 1000 \\ -3.611 & -4.792 & 4.000 & 3.611 & 4.792 & 4.000 \\ -4.792 & -6.406 & -3.000 & 4.792 & 6.406 & -3.000 \\ -4.000 & 3.000 & 1000 & 4.000 & -3.000 & 2000 \end{bmatrix}$$

For *element*2 between nodes 2 and 3

$$C = 0.6, \qquad S = -0.8$$

$$\mathbf{k_2} = 5 \times 10^4 \begin{bmatrix} 3.611 & -4.792 & 4.000 & -3.611 & 4.792 & 4.000 \\ -4.792 & 6.406 & 3.000 & 4.792 & -6.406 & 3.000 \\ 4.000 & 3.000 & 2000 & -4.000 & -3.000 & 1000 \\ -3.611 & 4.792 & -4.000 & 3.611 & -4.792 & -4.000 \\ 4.792 & -6.406 & -3.000 & -4.792 & 6.406 & -3.000 \\ 4.000 & 3.000 & 1000 & -4.000 & -3.000 & 2000 \end{bmatrix}$$

Add them up, and we may have

$$5 \times 10^4 \begin{bmatrix} 3.611 & 4.792 & -4 & -3.611 & -4.792 & -4 & 0 & 0 & 0 \\ 4.792 & 6.406 & 3 & -4.792 & -6.406 & 3 & 0 & 0 & 0 \\ -4 & 3 & 2000 & 4 & -3 & 1000 & 0 & 0 & 0 \\ -3.611 & -4.792 & 4 & 7.222 & 0 & 8 & -3.611 & 4.792 & 4 \\ -4.792 & -6.406 & -3 & 0 & 12.812 & 0 & 4.792 & -6.406 & 3 \\ -4 & 3 & 1000 & 8 & 0 & 4000 & -4 & -3 & 1000 \\ 0 & 0 & 0 & -3.611 & 4.792 & -4 & 3.611 & -4.792 & -4 \\ 0 & 0 & 0 & 4.792 & -6.406 & -3 & -4.792 & 6.406 & -3 \\ 0 & 0 & 0 & 4 & 3 & 1000 & -4 & -3 & 2000 \end{bmatrix} \begin{bmatrix} u_1 \\ v_1 \\ \theta_1 \\ u_2 \\ v_2 \\ \theta_2 \\ u_3 \\ v_3 \\ \theta_3 \end{bmatrix} = \begin{bmatrix} F_{1x} \\ F_{1y} \\ M_1 \\ F_{2x} \\ F_{2y} \\ M_2 \\ F_{3x} \\ F_{3y} \\ M_3 \end{bmatrix}$$

Applying the BCs

$$u_1 = v_1 = \theta_1 = 0$$
$$u_3 = v_3 = \theta_3 = 0$$
$$F_{2x} = -P, F_{2y} = M_2 = 0$$

So we can get the equations of node 2.

$$5 \times 10^4 \begin{bmatrix} 7.222 & 0 & 8 \\ 0 & 12.812 & 0 \\ 8 & 0 & 4000 \end{bmatrix} \begin{bmatrix} u_2 \\ v_2 \\ \theta_2 \end{bmatrix} = \begin{bmatrix} 10000 \\ 0 \\ 0 \end{bmatrix}$$

Solving the equations, and we get

$$u_2 = 0.0277in$$
$$v_2 = 0$$
$$\theta_2 = -5.54 * 10^{-5} rad$$

(2)

With the solution in (1), we can easily calculate the support reactions.

$$F_{1x} = -4998lb$$
$$F_{1y} = -6645lb$$
$$M_1 = 2770lb - in$$
$$F_{3x} = -4990lb$$
$$F_{3y} = 6645lb$$
$$M_3 = 2270lb - in$$