Homework 7

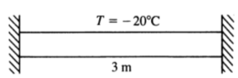
Huang Yuchong
3210105423

May 27, 2024

1

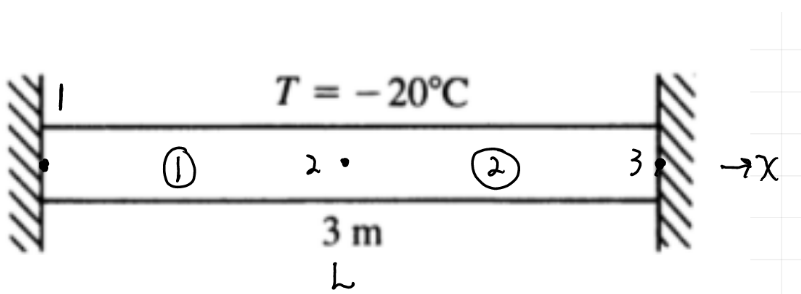
Problem

1. For the one-dimensional steel bar fixed at each end and subjected to a uniform temperature drop of 20 °C. Determine the reactions at the fixed ends and the stress in the bar. Let $E = 210$ GPa, $A = 10^{-2}$ m^2, and $\alpha = 11.7 \times 10^{-6}$ (mm/mm)/°C



Solution

First, we make some seperation as below.



The thermal force matrix for each element are as below

$$f_t^{(1)} = f_t^{(2)} = \begin{bmatrix} -E\alpha TA \\ E\alpha TA \end{bmatrix}$$

So, as a result

$$\{f_t\} = \begin{bmatrix} -E\alpha TA \\ 0 \\ E\alpha TA \end{bmatrix}$$

Then, with the fomular $\mathbf{Kd} = \mathbf{F}$, we have

$$\frac{AE}{L/2}\begin{bmatrix} 1 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{bmatrix}\begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} = \begin{bmatrix} F_{1x} + f_{1t} \\ F_{2x} + f_{2t} \\ F_{3x} + f_{3t} \end{bmatrix}$$

Applying the BCs, $u_1 = u_3 = F2x = 0$
We may find the reactions at the fixed ends.

$$\begin{aligned} F_{1x} &= E\alpha TA = -491400N \qquad (1) \\ F_{3x} &= -E\alpha TA = 491400N \end{aligned}$$

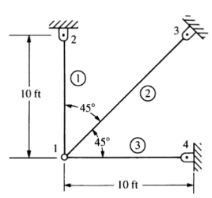
And then easily get the stress.

$$\sigma = \frac{F_{1x}}{A} = -49.14MPa(expansive)$$

2

Problem

2. For the plane truss shown below, bar element 2 is subjected to a uniform temperature of 50 ºF. Let $E = 30\times10^6$ psi, $A = 2$ in^2, and $\alpha = 7.0\times10^{-6}$ (in./in.)/°F. The lengths of the truss elements are shown in the figure. Determine the stresses in each bar.



Solution

First, we focus on the thermal force, which occurs in element2.

$$f_t^{(2)} = \begin{bmatrix} f_{1t} \\ f_{3t} \end{bmatrix} = \begin{bmatrix} -E\alpha TA \\ E\alpha TA \end{bmatrix} = \begin{bmatrix} -21000 \\ 21000 \end{bmatrix} lb$$

With calculation, we have

$$\begin{bmatrix} f_{1x} \\ f_{1y} \\ f_{2x} \\ f_{2y} \\ f_{3x} \\ f_{3y} \\ f_{4x} \\ f_{4y} \end{bmatrix} = 10500\sqrt{2} \begin{bmatrix} -1 \\ -1 \\ 0 \\ 0 \\ 1 \\ 1 \\ 0 \\ 0 \end{bmatrix} lb$$

Also, we can have the global stiffness matrix.

$$\mathbf{K} = 6 \times 10^6 \begin{bmatrix} 1+\sqrt{2} & 1 & 0 & 0 & -1 & -1 & -\sqrt{2} & 0 \\ 1 & 1+\sqrt{2} & 0 & -\sqrt{2} & -1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -\sqrt{2} & 0 & \sqrt{2} & 0 & 0 & 0 & 0 \\ -1 & -1 & 0 & 0 & 1 & 1 & 0 & 0 \\ -1 & -1 & 0 & 0 & 1 & 1 & 0 & 0 \\ -\sqrt{2} & 0 & 0 & 0 & 0 & 0 & \sqrt{2} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Applying the BCs, $u_2 = u_3 = u_4 = v_2 = v_3 = v_4 = F_1 = 0$
We got $u_1 = v_1 = -7.25 \times 10^{-4} ft$
So now we can calculate the stress with formular

$$\sigma = \frac{E}{L} \begin{bmatrix} -C & -S & C & S \end{bmatrix} \begin{bmatrix} u_i & v_i & u_j & v_j \end{bmatrix}^T - E\alpha T$$

For bar1

$$\sigma^{(1)} = 2175 \times \begin{bmatrix} 0 & -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} -1 & -1 & 0 & 0 \end{bmatrix}^T = 2175 psi$$

For bar2

$$\sigma^{(2)} = \frac{2175}{\sqrt{2}} \times \begin{bmatrix} -\sqrt{2} & -\sqrt{2} & \sqrt{2} & \sqrt{2} \end{bmatrix} \begin{bmatrix} -1 & -1 & 0 & 0 \end{bmatrix}^T - E\alpha T = 2250 psi$$

For bar3

$$\sigma^{(3)} = 2175 \times \begin{bmatrix} -1 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} -1 & -1 & 0 & 0 \end{bmatrix}^T = 2175 psi$$