Homework 4

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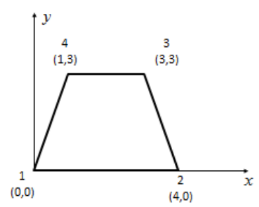
Problem

Consider a 4 node isoparametric element shown in the following Figure. The element has a trapezoidal shape. The shape functions are $N_i = (1 + ss_i)(1 + tt_i)/4$. The nodal coordinates are given in the figure.



1) Give the relation between the global coordinate (x, y) and the natural coordinate (s, t)

2) Determine the Jacobian matrix [J] at point (s, t) = (1, 1).

3) Determine $\dfrac{\partial N_3}{\partial x}$ at point (s, t)=(1, 1).

Solution

1)

First we have the horizontal and vertical coordinates of the four points.

$$x_1 = 0, \quad x_2 = 4, \quad x_3 = 3, \quad x_4 = 1$$

$$y_1 = 0, \quad y_2 = 0, \quad y_3 = 3, \quad y_4 = 3$$

Also the shape function.

$$N_1 = \frac{1}{4}(1-s)(1-t)$$
$$N_2 = \frac{1}{4}(1+s)(1-t)$$
$$N_3 = \frac{1}{4}(1+s)(1+t)$$
$$N_4 = \frac{1}{4}(1-s)(1+t)$$

With shape interpolation,

$$x = \sum_{i=1}^{4} N_i(s,t)\, x_i$$
$$y = \sum_{i=1}^{4} N_i(s,t)\, y_i$$

So we can easily get

$$x = 2 + \frac{3}{2}s - \frac{1}{2}st$$
$$y = \frac{3}{2}(1+t)$$

2)

We may calculate the Jacobian matrix directly.

$$\mathbf{J} = \begin{bmatrix} \frac{\partial x}{\partial s} & \frac{\partial y}{\partial s} \\ \frac{\partial x}{\partial t} & \frac{\partial y}{\partial t} \end{bmatrix} = \begin{bmatrix} \frac{3}{2} - \frac{1}{2}t & 0 \\ -\frac{1}{2}s & \frac{3}{2} \end{bmatrix}$$

Substituting $(s,t) = (1,1)$ into the matrix, we can get

$$\mathbf{J} = \begin{bmatrix} 1 & 0 \\ -\frac{1}{2} & \frac{3}{2} \end{bmatrix}$$

at point $(s,t) = (1,1)$.

3)

We have

$$\begin{bmatrix} \frac{\partial N_i}{\partial s} \\ \frac{\partial N_i}{\partial t} \end{bmatrix} = \mathbf{J} \begin{bmatrix} \frac{\partial N_i}{\partial x} \\ \frac{\partial N_i}{\partial y} \end{bmatrix}$$

So

$$\mathbf{J}^{-1} \begin{bmatrix} \frac{\partial N_i}{\partial s} \\ \frac{\partial N_i}{\partial t} \end{bmatrix} = \begin{bmatrix} \frac{\partial N_i}{\partial x} \\ \frac{\partial N_i}{\partial y} \end{bmatrix}$$

$$\mathbf{J}^{-1} = \frac{2}{9-3t} \begin{bmatrix} 3 & 0 \\ s & 3-t \end{bmatrix}$$

$$\frac{\partial N_3}{\partial s} = \frac{1}{4}(1+t)$$

$$\frac{\partial N_3}{\partial t} = \frac{1}{4}(1+s)$$

So

$$\frac{\partial N_3}{\partial x} = \frac{2}{3-t} \frac{\partial N_3}{\partial s}$$

Substituting $(s,t) = (1,1)$ into the matrix, we can get

$$\frac{\partial N_3}{\partial x} = \frac{1}{2}$$

at point $(s,t) = (1,1)$.