第 2 周课后作业

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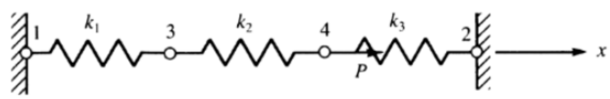
1.



For the spring system shown above, find

(a) the global stiffness matrix;

(b) the displacements at nodes 3 and 4;

(c) The reaction forces at nodes 1 and 2.

(a)

For $element1$ between nodes 1 and 3, we have

$$\mathbf{k}_1 = \begin{bmatrix} k1 & -k1 \\ -k1 & k1 \end{bmatrix}$$

For $element2$ between nodes 3 and e4, we have

$$\mathbf{k}_2 = \begin{bmatrix} k_2 & -k_2 \\ -k_2 & k_2 \end{bmatrix}$$

For *element*3 between nodes 2 and 4, we have

$$\mathbf{k}_3 = \begin{bmatrix} k_3 & -k_3 \\ -k_3 & k_3 \end{bmatrix}$$

Add them up, then we get the global stiffness matrix

$$\mathbf{K} = \begin{bmatrix} k_1 & 0 & -k_1 & 0 \\ 0 & k_3 & 0 & -k_3 \\ -k_1 & 0 & k_1 + k_3 & -k_2 \\ 0 & -k_3 & -k_2 & k_2 - k_3 \end{bmatrix}$$

(b)

The global stiffness equation $\mathbf{KU} = \mathbf{F}$ is shown as below

$$\begin{bmatrix} k_1 & 0 & -k_1 & 0 \\ 0 & k_3 & 0 & -k_3 \\ -k_1 & 0 & k_1 + k_3 & -k_2 \\ 0 & -k_3 & -k_2 & k_2 - k_3 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{bmatrix} = \begin{bmatrix} F_1 \\ F_2 \\ F_3 \\ F_4 \end{bmatrix}$$

$$u_1 = u_2 = 0, \qquad F_3 = 0, \qquad F_4 = P$$

Apply the BCs of nodes 1 and 2, or deleting the 1^{st} and 2^{nd} rows and columns, we have

$$\begin{bmatrix} k_1 + k_3 & -k_2 \\ -k_2 & k_2 - k_3 \end{bmatrix} \begin{bmatrix} u_3 \\ u_4 \end{bmatrix} = \begin{bmatrix} 0 \\ P \end{bmatrix}$$

Now we can easily get the displacements at nodes 3 and 4

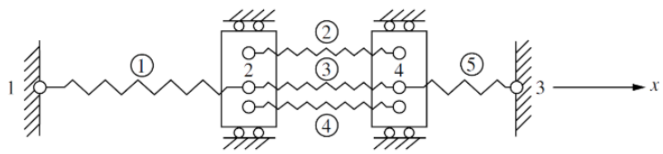
$$\begin{bmatrix} u_3 \\ u_4 \end{bmatrix} = \begin{bmatrix} \frac{Pk_2}{k_2^2+k_3^2+k_1k_2+k_2k_3+k_1k_3} \\ \frac{P(k_1+k_3)}{k_2^2+k_3^2+k_1k_2+k_2k_3+k_1k_3} \end{bmatrix}$$

(c)

From the 1^{st} and 2^{nd} equations, we get the reaction forces

$$F_1 = -k_1 u_3 = -\frac{Pk_1 k_2}{k_2^2 + k_3^2 + k_1 k_2 + k_2 k_3 + k_1 k_3}$$
$$F_2 = -k_3 u_4 = -\frac{Pk_3 (k_1 + k_3)}{k_2^2 + k_3^2 + k_1 k_2 + k_2 k_3 + k_1 k_3}$$

2.



For the spring system shown above, find the global stiffness matrix. All springs have the same stiffness k.

Surely we can solve this problem using the same method as before, but there is something interesting meriting our attemtion.

*element*2,*element*3 and *element*4, we can consider them as a whole. Now they become a single spring which has the stiffness $3k$

Then we can easily write the global stiffness matrix

$$\mathbf{K} = \begin{bmatrix} k & -k & 0 & 0 \\ -k & k & 0 & -3k \\ 0 & 0 & k & -k \\ 0 & -3k & -k & 4k \end{bmatrix}$$