Простой пример

ТЕХНОАТОМ

Дано:

$$f(x, y, z) = (x + y)z$$

$$x = -2, y = 5, z = -4$$

Найти:

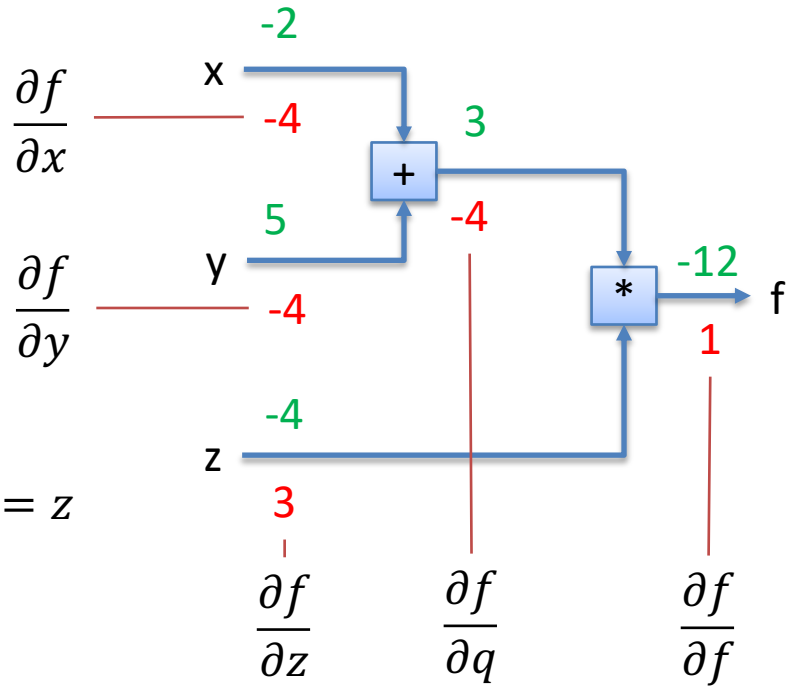
$$\frac{\partial f}{\partial x}, \quad \frac{\partial f}{\partial y}, \quad \frac{\partial f}{\partial z}$$

Решение:

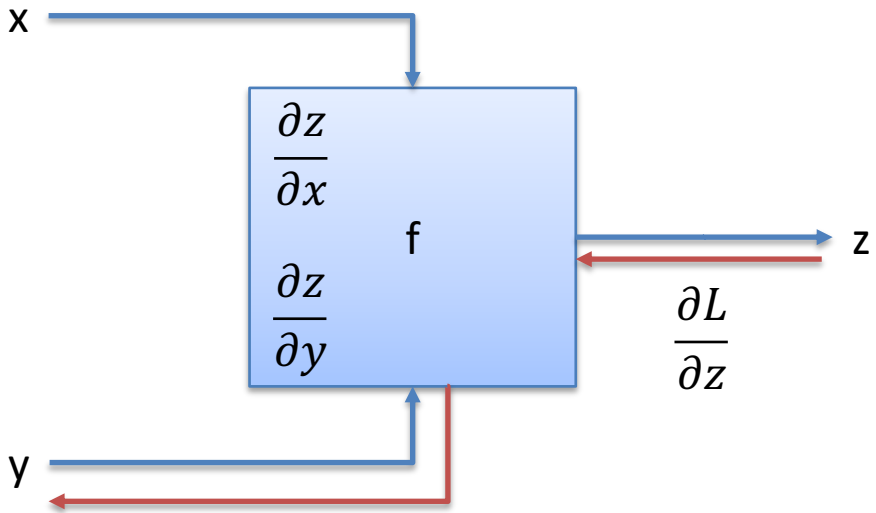
$$q = x + y$$

$$f = z\,q$$

Chain rule: $\frac{\partial f}{\partial x} = \frac{\partial f}{\partial q}\,\frac{\partial q}{\partial x} = z$



Обратное распространение ошибки

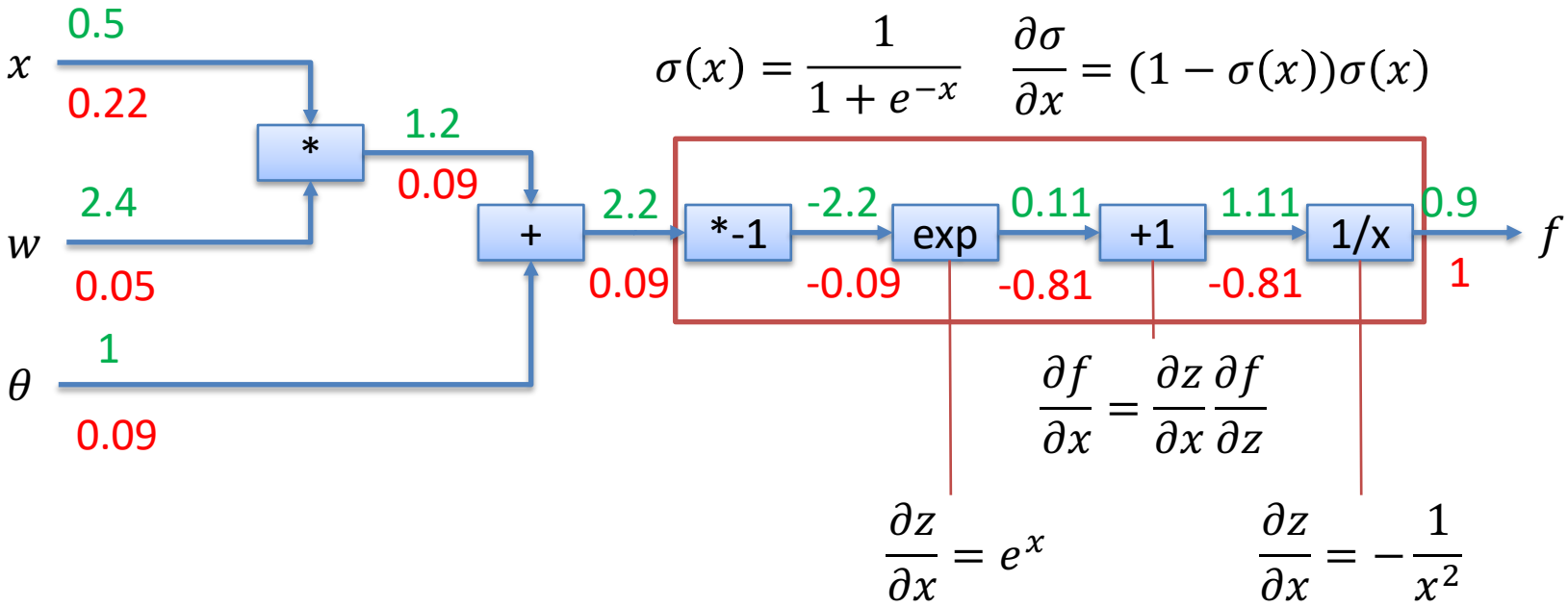


$$\frac{\partial L}{\partial y} = \frac{\partial L}{\partial z}\frac{\partial z}{\partial y}$$

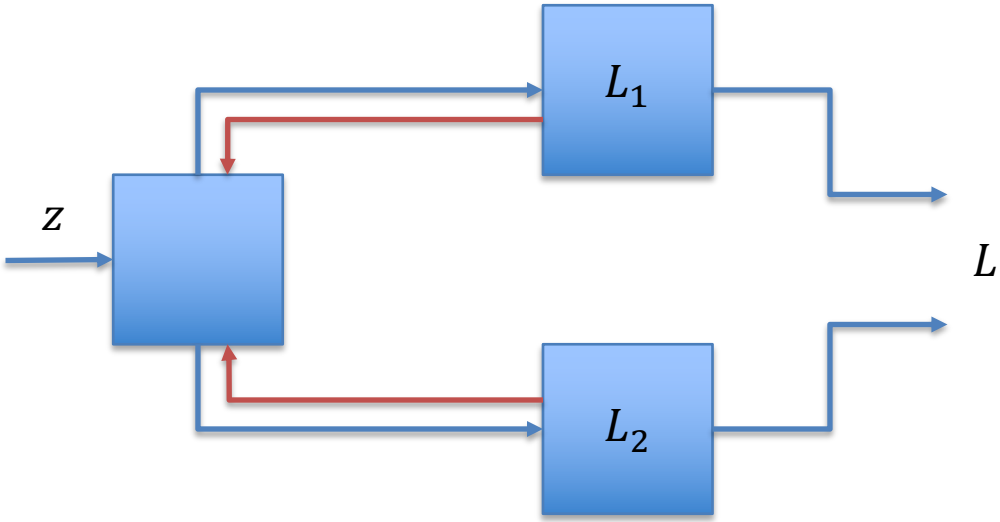
Пример посложнее

ТЕХНОАТОМ

$$f(x, w, \theta) = \frac{1}{1 + e^{-(xw+\theta)}}$$



Разветвление выхода



$$\frac{\partial L}{\partial z} = \frac{\partial L_1}{\partial z} + \frac{\partial L_2}{\partial z}$$

Матричные операции

ТЕХНОАТОМ

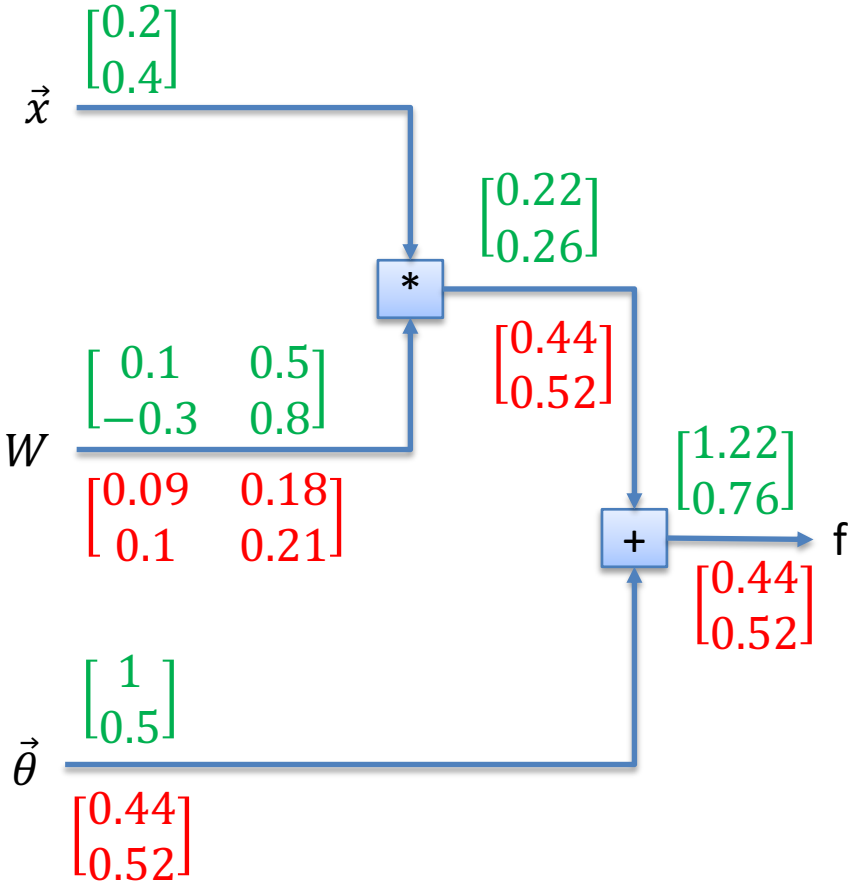
$$f(\vec{x}, W, \vec{\theta}) = W\vec{x}+\vec{\theta}$$

$$\vec{q} = W\vec{x} = \begin{pmatrix} W_{1,1}x_1 + \cdots + W_{1,n}x_n \\ \cdots \\ W_{n,1}x_1 + \cdots + W_{n,n}x_n \end{pmatrix}$$

$$\vec{f} = \vec{q} + \vec{\theta}$$

$$\frac{\partial f}{\partial w_{i,j}} = \sum_k \frac{\partial f}{\partial q_k}\frac{\partial q_k}{\partial w_{i,j}} = \frac{\partial f}{\partial q_i}x_j$$

$$\frac{\partial q_k}{\partial w_{i,j}} = 1_{k=i}x_j$$



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$$\vec{f} = \vec{q} + \vec{\theta}$$

$$\frac{\partial f}{\partial x_i} = \sum_k \frac{\partial f}{\partial q_k}\frac{\partial q_k}{\partial x_i}$$

$$\frac{\partial q_k}{\partial x_i} = W_{k,i}$$