Types of friezes

We could apply a finite block repeatedly to obtain an infinite frieze (See Fig.1)

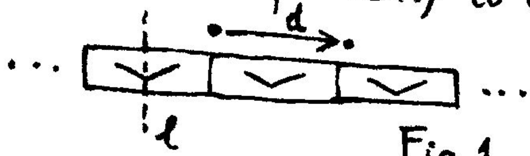
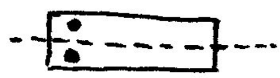


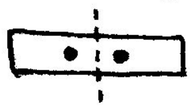
Fig.1

This frieze is symmetric w.r.t. the dashed line ℓ, or, in another view point, it is un-changed under the operation of folding this paper along the line ℓ. Such an operation (which keeps the frieze un-changed) is called directly a symmetry of the frieze. The translation d towards the right side (d in Fig.1) is a symmetry of the frieze.

The collection of all the operations which keep the frieze un-changed is called the symmetry group of the frieze. If two friezes have the same symmetry group, then they share the same pattern of symmetry; otherwise, distinct patterns of symmetry.

The possible symmetry groups of such friezes could be exhausted by asking: which operations are in its symmetry group. Besides the translations which translate the initial block (e.g. the arrow in Fig.1), all the possible symmetries of such a frieze are:

[redacted] a longitudinal folding, e.g.  ;

a transversal folding, e.g.  ;

a rotating of 180°, e.g.  ;

or a longitudinal glide, e.g. [redacted]  .

(We know that all the distance-keeping operations on the plane are foldings and their compositions, i.e. they are of the following four types: foldings, translations, rotations, or glides)

All the possible symmetry groups of friezes could be exhausted by the following table:

(Considering which types of operations are contained in the symmetry group of the frieze)

Longit. folding?	yes								no							
transver. folding?	yes				no				yes				no			
rotation 180°?	yes		no		yes		no		yes		no		yes		no	
glide ?	y	n	y	n	y	n	y	n	y	n	y	n	y	n	y	n
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
						(†)	(*)									

(*). Case 7 contains the operations presented in Fig.2, and a possible frieze of this pattern is shown in Fig.3.

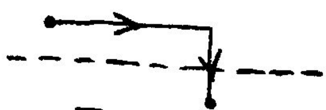


Fig.2

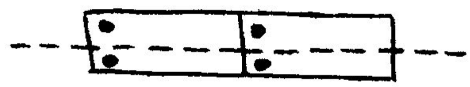


Fig.3

The symbol for this case is f--g, where:

the first 'f' indicates there is a longitudinal folding;

the '_' at the second place indicates there is no transversal folding;

'_' third there is no rotation of 180°;

the 'g' at the last place indicates there is glide.

(†). Case 6 (symboled as f-r-) is impossible:

In Fig.4 & Fig.5, each dashed line represents the operation of folding along it.

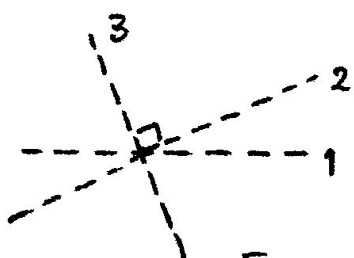


Fig.4

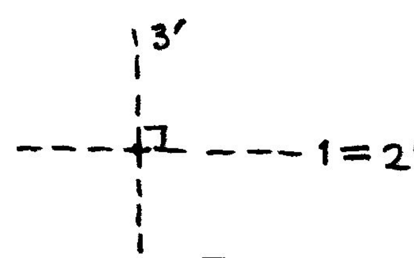


Fig.5

In Fig.4, 1 is a longitudinal folding,

composition of 2 and 3 is a rotation of 180°.

We could rotate 2 and 3 together such that 2 and 1 coincide (Fig.5)

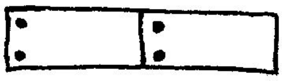
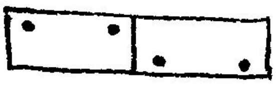
Thus, we have: 1 2 3 (successive operations composed)

= 1 2' 3'

= 3' (a transversal folding)

which contradicts the case 6 (which should have no transversal folding)

We could analize all the 16 cases in the same manner as we did for the case 7 and case 6, as follows (where the impossible cases are left empty):

Case #	symbol	possible frieze
1	ffrg	
2	ffr-	
3	ff-g	
4	ff--	
5	f-rg	
6	f-r-	
7	f--g	
8	f---	
9	-frg	
10	-fr-	
11	-f-g	
12	-f--	
13	--rg	
14	--r-	
15	---g	
16	----