Practicals Manual - Physics

for High Schools and Universities

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Contents

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*Practical pulled directly from past WASSCE Exams.

Click "Peace Corps Liberia" in the header of any practical to return to this table of contents. Click "Period" in the header of any practical to return to period's mini table of contents. Note that these clickable features do not work in all document viewing applications.

The author of this document made considerable efforts to minimize errors, including those of grammar, syntax, concept, algebra, analysis and solution. However caution and healthy skepticism should be exercised when referring to its content. When technology permits, please send all questions and corrections as an email to clog@ucdavis.edu or contact +231 077 851 2311.

How to Use this Document

- This document provides instruction for a set of practicals meant to be carried out at the High School and University levels in Liberia.
  - The only difference between the grade levels is the post-lab questions included at the end of each session.

- Many of the practicals included in this document are direct adaptations of past WASSCE (alternative) practical exam questions.
  - Those that have been pulled from past exams include an asterisk* in their titles.
  - Those that have not been taken from past exams are structured similarly to those that have.

- The structure of each practical is
  1. **Introduction**
     - Review of relevant concepts and formulas
  2. **Apparatus and Materials**
     - A list of all materials, tools, instruments and supplies required for the complete practical
  3. **Setup**
     - Instructions on setting up the experiment before the actual lab session
  4. **Warm Up Questions**
     - A set of questions designed to engage student's critical interaction with the relevant concepts
  5. **Procedure and Calculations**
     - Instructions on the steps to be carried out by the students during the lab session.
     - Many *Procedure and Calculation* sections include a sample table of the data to be collected.
     - These tables are formatted with various background colors as show in the table below.
       * **dark gray** for all columns and rows to be filled in before carrying out the lab
       * **white (no color)** for all columns and rows to be filled in with data collected in lab
       * **light gray** for all columns and rows to filled in with values calculated from recorded data

pre-filled header					
pre-filled values	recorded data	calculated values	calculated values	calculated values	calculated values

  6. **Data Plotting and Slope/Intercept** Determination
     - Instructions on how students should plot and process their data
     - It is the teacher's choice on whether this is done in lab or as homework
  7. **Exam Prompt** (Only for WAASCE-based Practicals)
     - Copy of prompt taken from past WAASCE exam as source for practical
  8. **Solutions to Exam Prompt** (Only for WAASCE-based Practicals)
     - Solutions to questions posed in exam prompt
  9. **Post-Lab Questions - High School**
     - Reflection questions to be answered as homework at the High School Level
  10. **Post-Lab Questions - Post-Lab Questions - University Level 1**
      - Reflection questions to be answered as homework, added only at the introductory University level or as extra credit at the High School level
  11. **Post-Lab Questions - Post-Lab Questions - University Level 2**
      - Reflection questions to be answered as homework, added only at the advanced University level or as extra credit at the High School and introductory University level

Period 1 Introduction to Physics and Properties of Matter

Period Contents

10.P1.L1 Quantities, Units and Prefixes

10.P1.L1.1 Introduction

- This may be the student's first time in a science laboratory.
- Before any practicals can occur, students must have a basic understanding of physical quantities.
  - This includes a quantity's symbol, unit and typical measuring device.
- This lab covers no actual laboratory procedure.
  - However, it is necessary before any actual labs begin.
  - Treat this material either in lecture or as an introductory lab session.

10.P1.L1.2 Warm Up Questions

1. Name some examples of physical quantities.
   - Mass, length, area, volume, time, temperature, etc
2. Why do we use units to measure physical quantities?
   - Units allow for a common method of recording, sharing and analyzing physical quantities.
3. Name some examples of units we use everyday.
   - kilometer, mile, pound, kilogram, centimeter, degrees Celsius, etc.

10.P1.L1.3 Base Quantities

1. Begin with an empty table with 6 columns and 8 rows as shown in Table 1.
2. Add the headers to each column of the first row as shown in dark gray.
3. As shown in light gray, work with the students to fill in each cell of the first column **(Base Quantity)**.
4. Work with the students to fill out columns 2 through 6 of the first row **(distance)**.
5. Repeat for the remaining six quantities **(mass, time, etc.)**.

Base Quantity	Quantity Symbol	Dimension Symbol	SI Unit	SI Unit Symbol	Measuring Devices
distance	d	L	meter	m	tape rule, ruler, *vernier caliper
mass	m	M	kilogram	kg	three beam balance
time	t	T	second	s	stop watch, clock
electric current	I	I	ampere	A	ammeter
absolute temperature	T	Θ	kelvin	K	thermometer
amount of substance	n	N	mole	mol	-
luminous intensity	I_v	J	candela	cd	*photometer

Table 1: Base Quantities

* Device used in Liberian industry, but familiarity not required for lab.

Notes to Teacher

- Don't worry if some of the measuring devices are unfamiliar to you or the students.
  - Each lab has instructions on how to use each tool when it's introduced.
- Students may confuse the symbol used for a quantity, its unit and its dimension (e.g. m, M and kg for mass).
  - Take time to distinguish the purpose of each of these symbols.

10.P1.L1.4 Derived/Secondary Quantities

1. Draw another empty table with 5 columns and 10 rows as shown in table 2.
2. Fill out this table in same process as Table 1.

Base Quantity	Quantity Symbol	Dimension Symbol	SI Unit	SI Unit Symbol	Measuring Devices
area	A	L^2	square meter	m^2	*planimeter, *theodolite
volume	V	L^3	cubic meter	m^3	graduated cylinder
density	ρ	$L^{-3}M$	kilogram per cubic meter	$\frac{kg}{m^3}$	*hydrometer
frequency	f	T^{-1}	hertz	hz	*tachometer, *hertz meter
velocity	v or $\vec{v}$	LT^{-1}	meter per second	$\frac{m}{s}$	*speedometer, *anemometer
acceleration	a or $\vec{a}$	LT^{-2}	meter per second squared	$\frac{m}{s^2}$	*accelerometer
force	F	LMT^{-2}	newton	N	spring scale, digital scale
mechanical energy	E	L^2MT^{-2}	joule	J	-
power	P	L^2MT^{-3}	watt	W	*electric wattmeter
pressure	P	$L^{-1}MT^{-2}$	pascal	Pa	barometer, manometer

Table 2: Derived Quantities

* Device used in Liberian industry, but familiarity not required for lab.

Notes to Teacher

- The devices shown with an asterisk (*) are likely never to be encountered in the school's lab.
  - However, its worth discussing them briefly to better understand their use in science and industry.
  - You may either list the devices with an asterisk (*) as shown, or simply leave space empty.
- You may ignore some of these derived quantities if you feel they may be overwhelming for the students.
  - However, be sure to introduce these quantities before any relevant labs.
  - For example, the students must be aware of the quantities of
    - velocity, acceleration and force before Newton's laws;
    - pressure before gas laws;
    - angles before vectors, etc.
- You may also ignore the symbols indicated in vector notation $(\vec{v}$ and $\vec{a})$ if they are unfamiliar to the students.

10.P1.L1.5 Unit Prefixes

1. Draw third empty table with 7 columns and 13 rows as shown in table 3.
   - Be sure to show some parts of the header over two columns, as shown with
     - "Multiplication Factor" over columns 3 and 4 and
     - "Common Example" over columns 5, 6 and 7.
   - Explain that prefixes can be added to SI base and derived units to make them larger or smaller.
   - A helpful trick is "**G**randma **M**ay **k**illed **h**er **d**ear **b**oyfriend **d**eci, **c**ouldn't **m**aster μicroscopic **n**ames"
     - Use trick to fill out the first column of the table, as shown.
   - Fill out this table in same process as Table 1.

		Multiplication Factor		**Common Example**		
Prefix	**Symbol**	**Sci. notation**	**Decimal**	**Quantity**	**Abbreviation**	**Base Equivalent**
giga-	G	10^9	1000000000	1 gigabyte	1 *GB*	1000000000 bytes
mega-	M	10^6	1000000	1 megabyte	1 *MB*	1000000 bytes
kilo-	k	10^3	1000	1 kilometer	1 *km*	1000 meters
heca-	h	10^2	100	-	-	-
deca-	d	10^1	10	-	-	-
(base)	-	10^0	1	1 meter	1 *m*	1 meter
deci-	d	10^{-1}	0.1	-	-	-
centi-	c	10^{-2}	0.01	1 centimeter	1 *cm*	0.01 meters
milli-	m	10^{-3}	0.001	1 millimeter	1 *mm*	0.001 *m*
micro-	μ	10^{-6}	0.000001	1 micrometer	1 μm	0.000001 *m*
nano-	n	10^{-9}	0.000000001	1 manometer	1 *nm*	0.000000001 *m*

Table 3: Unit Prefixes

10.P1.L1.6 Post-Lab Questions - High School

1. What's the difference between the units of a physical quantity and the algebraic symbol we use to represent the associated property?
   - We use the unit as a "scalable" amount of a given property while we use an algebraic symbol to serve as a substitute for the entire "scaled" quantity, whether it be known or unknown.
2. Are there some letters that are used for both units and their algebraic symbols?
   - Yes - **m** for both meters and mass, **s** for both seconds and displacement (sometimes), **A** for both amperes and Amplitude, etc.
3. Why do we sometimes use Greek letters for quantities and property symbols?
   - There are far more physical quantities than letters in the English (Latin) alphabet.

10.P1.L1.7 Post-Lab Questions - University Level 1

4. What's the difference between the precision and accuracy of an instrument?
   - The instrument's accuracy is the difference between the value it measures of a given property and that property's actual, true value. The instrument's precision is the difference between two or more of its measurements. Less abstractly, the instrument's precision is the minimal amount of "randomness" inherent to its measuring capacity.

10.P1.L1.8 Post-Lab Questions - University Level 2

5. The dimension of every derived physical quantity can be expressed as $L^a M^b T^c I^d \Theta^e N^f J^g$.
   i) Create a table of 10 columns 18 rows.
   ii) Populate the second columns by copying the name of each of the quantities in Table 1 and Table 2.
   iii) Populate the first column by naming the group “Base” and “Derived”, accordingly.
   iv) Populate the columns three through nine with the corresponding values of **a**, **b**, . . . , **g** for each quantity.
      • Note that these values may range from -3 to 3.
   v) Populate the tenth column with the given expression substituted with the values of a, b, . . . , g.
   vi) Briefly discuss a few general patterns of the values of a, b, . . . , g observed in the table.

	Quantity	**a**	**c**	**c**	**d**	**e**	**f**	**g**	$\mathbf{L^a M^b T^c I^d \Theta^e N^f J^g}$
Base	distance	1	0	0	0	0	0	0	$L^1 M^0 T^0 I^0 \Theta^0 N^0 J^0$
	mass	0	1	0	0	0	0	0	$L^0 M^1 T^0 I^0 \Theta^0 N^0 J^0$
	time	0	0	1	0	0	0	0	$L^0 M^0 T^1 I^0 \Theta^0 N^0 J^0$
	electric current	0	0	0	1	0	0	0	$L^0 M^0 T^0 I^1 \Theta^0 N^0 J^0$
	absolute temperature	0	0	0	0	1	0	0	$L^0 M^0 T^0 I^0 \Theta^1 N^0 J^0$
	amount of substance	0	0	0	0	0	1	0	$L^0 M^0 T^0 I^0 \Theta^0 N^1 J^0$
	luminous intensity	0	0	0	0	0	0	1	$L^0 M^0 T^0 I^0 \Theta^0 N^0 J^1$
Derived	area	2	0	0	0	0	0	0	$L^2 M^0 T^0 I^0 \Theta^0 N^0 J^0$
	volume	3	0	0	0	0	0	0	$L^3 M^0 T^0 I^0 \Theta^0 N^0 J^0$
	density	-3	1	0	0	0	0	0	$L^{-3} M^1 T^0 I^0 \Theta^0 N^0 J^0$
	frequency	0	0	-1	0	0	0	0	$L^0 M^0 T^{-1} I^0 \Theta^0 N^0 J^0$
	velocity	1	0	-1	0	0	0	0	$L^1 M^0 T^{-1} I^0 \Theta^0 N^0 J^0$
	acceleration	1	0	-2	0	0	0	0	$L^1 M^0 T^{-2} I^0 \Theta^0 N^0 J^0$
	force	1	1	-2	0	0	0	0	$L^1 M^1 T^{-2} I^0 \Theta^0 N^0 J^0$
	mechanical energy	2	1	-2	0	0	0	0	$L^2 M^1 T^{-2} I^0 \Theta^0 N^0 J^0$
	power	2	1	-3	0	0	0	0	$L^2 M^1 T^{-3} I^0 \Theta^0 N^0 J^0$
	pressure	-1	1	-2	0	0	0	0	$L^{-1} M^1 T^{-2} I^0 \Theta^0 N^0 J^0$

Table 4

General patterns:
- For all base all base quantities,
  - there is only one of a, b, . . . , g which has a non-zero value and;
  - this value is always one.
- For all derived quantities,
  - there are either combined non-zero values of a, b, . . . , g or;
  - there is only one of a, b, . . . , g which has a non-zero but,
    * this value is not equal to one.

Notes to Teacher

- Consider drawing the table for the students, complete with the header and first two columns.
- Also consider providing a work-through of one base quantity and one derived quantity to help the students understand the assignment.

10.P1.L2 Measuring Distance

10.P1.L2.1 Introduction

- This lab should help students learn to measures distances between 1 *mm* and 5 *m*.
- This lab should be carried out across three "stations".

10.P1.L2.2 Apparatus and Materials for Each Station

- Station A requires
  - 1 ruler with both inches and centimeters
  - 6 nails of the lengths 1 $^1/_8$ *in*, 1 $^1/_2$ *in*, 2 *in*, 2 $^1/_2$ *in*, 3 *in* and 4 *in*
  - plaster tape
  - 1 marker
- Station B requires
  - 1 ruler, at least 15 *cm* long
  - 1 deck of playing cards **OR** blank paper sheets, enough to form a stack of at least 1*cm*
- Station C requires
  - 1 tape rule, at least 3 *m* long

10.P1.L2.3 Setup

- Station A requires the set of nails to be labeled.
- Use the plaster tape and maker to apply a label "A", "B", "C", "D", "E" and "F" to each of the six nails as shown.

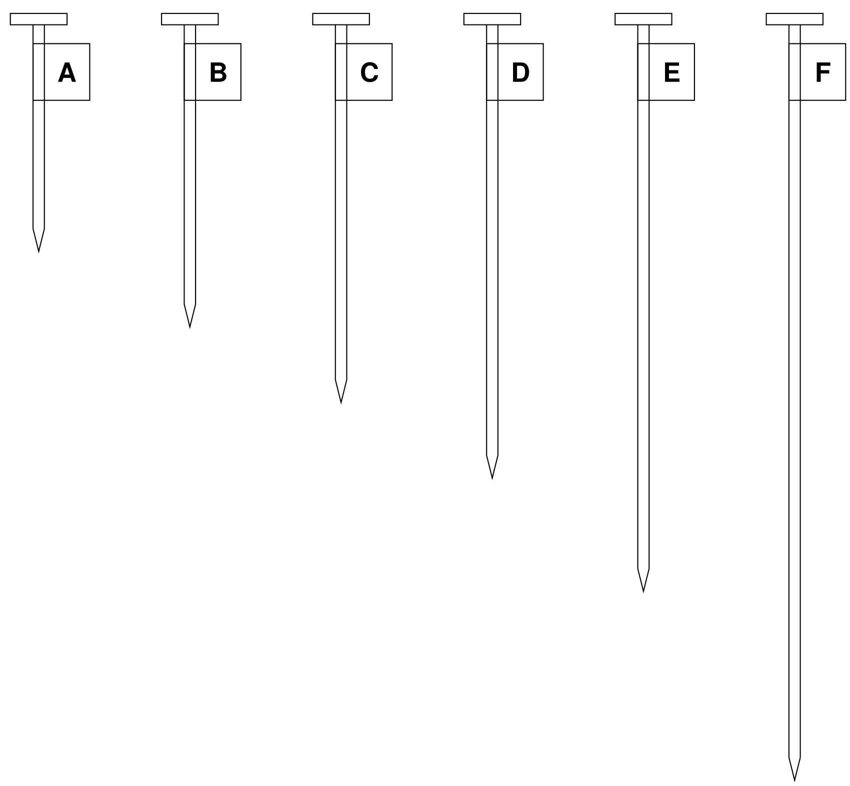


Figure 1

10.P1.L2.4 Warm Up Questions

1. What do we call the base-ten measurement system that uses centimeters, meters, etc?
   - The Metric System
2. Rulers usually have which two units?
   - Centimeters an inches
3. How many centimeters per inch?
   - 2.54 *cm* per inch
4. Which tool can we use to measure a distance larger than the length of a ruler?
   - Tape Rule
5. Express $^0/_{16}$, $^1/_{16}$, $^2/_{16}$, $^3/_{16}$, $^4/_{16}$, $^5/_{16}$, $^6/_{16}$, $^7/_{16}$, $^8/_{16}$, $^9/_{16}$, $^{10}/_{16}$, $^{11}/_{16}$, $^{12}/_{16}$, $^{13}/_{16}$, $^{14}/_{16}$, $^{15}/_{16}$, $^{16}/_{16}$, $^{17}/_{16}$ in the most simplified fractional form.
   - 0, $^1/_{16}$, $^1/_8$, $^3/_{16}$, $^1/_4$, $^5/_{16}$, $^3/_8$, $^7/_{16}$, $^1/_2$, $^9/_{16}$, $^5/_8$, $^{11}/_{16}$, $^3/_4$, $^{13}/_{16}$, $^7/_8$, $^{15}/_{16}$, 1, 1 $^1/_{16}$

10.P1.L2.5 Procedure for Station A - Measuring Nail Lengths

- Students should collect data similar to Table 1 using the steps below.

Nail	L_{cm}	$L_{cm} \times \left(\frac{1 \text{ in}}{2.54 \text{ cm}}\right)$	L_{in}	$L_{in} \times \left(\frac{2.54 \text{ cm}}{1 \text{ in}}\right)$
A	2.5	0.984	1	2.540
B	3.8	1.496	$1\frac{1}{2}$	3.810
C	5.1	2.008	2	5.080
D	6.4	2.520	$2\frac{1}{2}$	6.350
E	7.6	2.992	3	7.620
F	10.2	4.016	4	10.160

Table 1

A) Create an empty table of 5 columns and 7 rows.

B) **Row 1, Header**: Fill in the header information as shown.

C) **Column 1, Nail**: Fill in the list of nails to be measured.

D) **Column 2, L_{cm}**: Measure the length of each nail in centimeters.

E) **Column 3, $L_{cm} \times \left(\frac{1 \text{ in}}{2.54 \text{ cm}}\right)$**: Convert these measurements into inches.

F) **Column 4, L_{in}** - Measure the length of each nail again in inches.

G) **Column 5, $L_{in} \times \left(\frac{2.54 \text{ cm}}{1 \text{ in}}\right)$** - Convert these measurements into centimeters.

- Students should compare their values in column 2 with those in column 5.
- Likewise, they should compare their values in column 3 with those in column 4.
- Students should also discuss why these values are close, but don't match exactly.

- Take **caution** to measure the nail lengths properly.
  - Place the head of the nail at the "zero" mark.
  - Read the nail's length from the sharp tip.
  - Note that the centimeter side of the ruler is marked in "tenths".
    - Each small line between the "centimeter" readings is an additional 0.1 *cm*.
  - Note that the inch side of the ruler is usually marked in "sixteenths".
    - Each small line between the "inch" readings is an additional $^1/_{16}$ *in*.

Measuring in centimeters

Measuring in inches

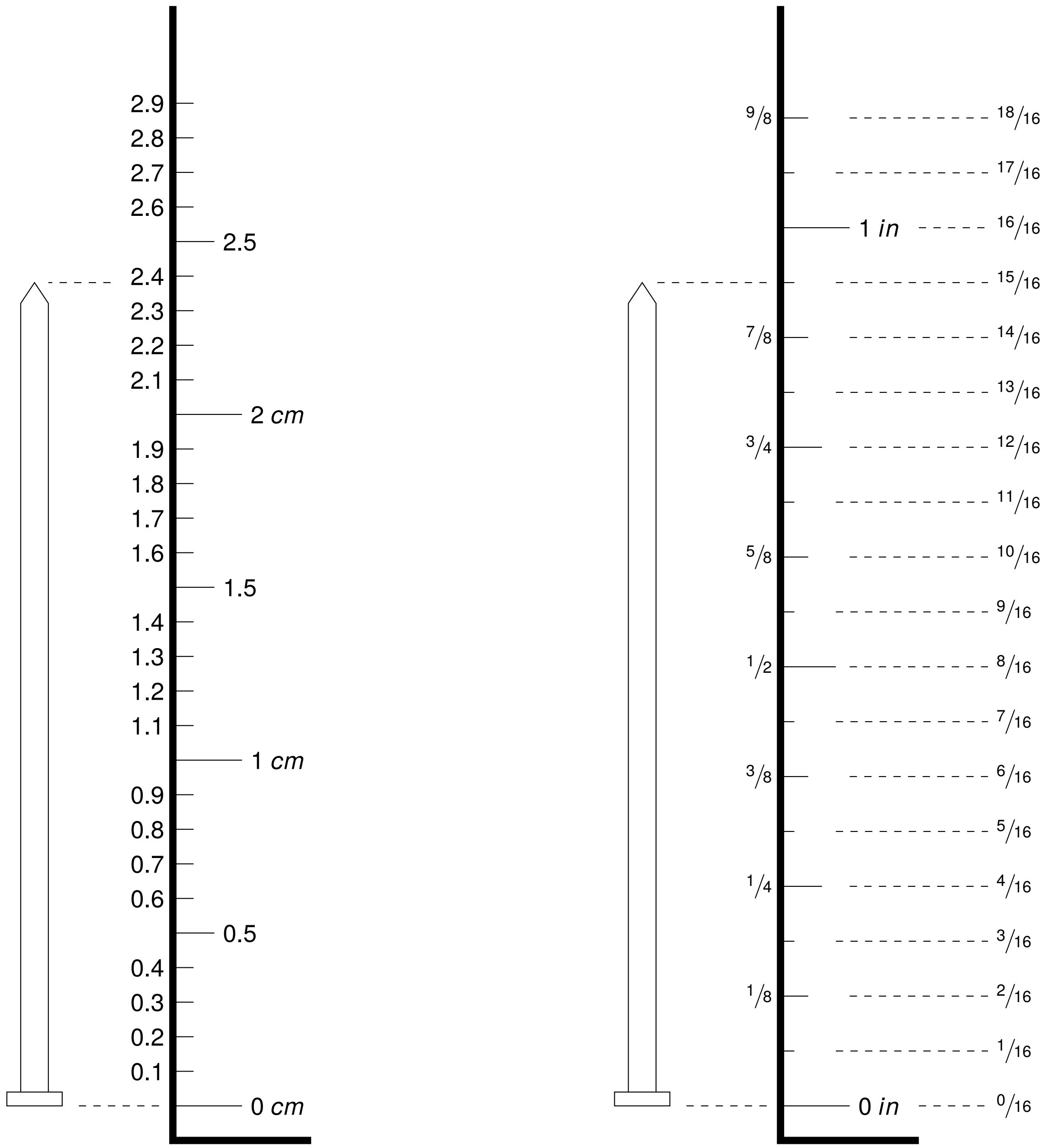


Figure 2

10.P1.L2.6 Procedure for Station B - Measuring the Thickness of a Playing Card

- Students should collect data similar to Table 2 using the steps below.

t_{total} (cm)	N_{cards}	$t_{per\ card}$ (cm)	$t_{per\ card}$ (μm)
1.5	48	0.0313	313
1.2	38	0.0316	316
1.0	32	0.0313	313
0.8	26	0.0308	308
0.6	19	0.0316	316

Table 2

A) Start with a complete deck of 52 cards

B) Create an empty table of 4 columns and 6 rows.

C) **Row 1, Header**: Fill in the header information as shown.

D) **Column 1, t_{total} (cm)**: Fill in the predetermined deck thicknesses to be measured.

E) **Column 2, N_{cards}**:
   - Remove cards one-by-one until the deck thickness t_{total} is equal to 1.5 *cm*.
   - Count the number of cards in this deck and record this value of N_{cards} for t_{total} = 1.5 *cm*.

F) **Column 3, $t_{per\ card}$ (cm)**: Calculate the thickness of a single card for each row using

$$t_{per\ card} = \frac{t_{total}}{N_{cards}} \qquad \text{(Equation 1)}$$

G) **Column 4, $t_{per\ card}$ (μm)**: Convert this measurement into micrometers using

$$t_{per\ card,\ \mu m} = t_{per\ card,\ cm}\left(\frac{10^4\ \mu m}{1\ cm}\right) \qquad \text{(Equation 2)}$$

H) Repeat steps E) through G) for t_{total} = 1.2 *cm*, 1.0 *cm*, 0.8 *cm* and 0.6 *cm*.

- Students should discuss any differences between the calculate card thicknesses.
- Students should also discuss the convenience of using the μm unit for the thickness of a single card.

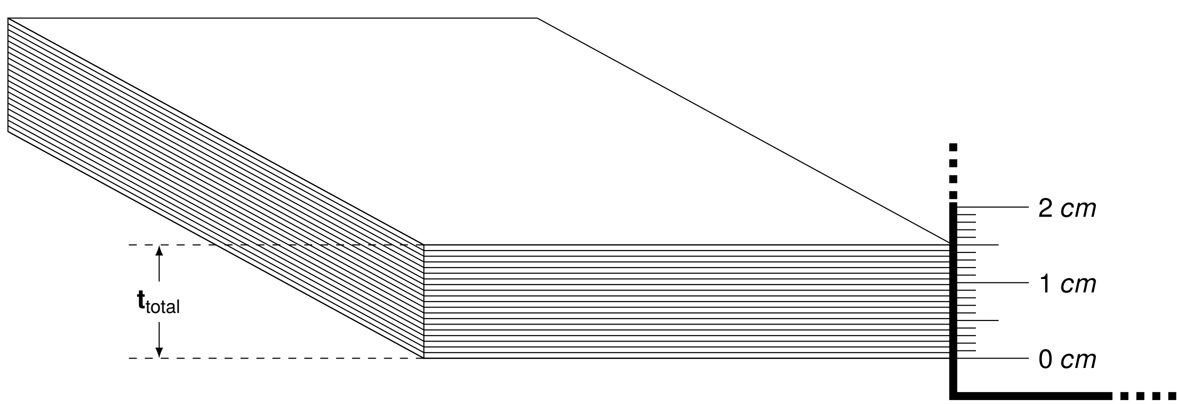


Figure 3

10.P1.L2.7 Procedure for Station C - Measuring Larger Distances

- Students should collect data similar to Table 3 using the steps below.

Distance Description	**d (cm)**	**d (m)**	**d (in)**	**d (ft)**	**d (m), converted**
Door Width	74.5	0.745	$29\frac{5}{16}$	2.443	0.74454
Door Height	249.5	2.495	$98\frac{1}{4}$	8.188	2.49556
Table Length	431.3	4.313	$169\frac{13}{16}$	14.151	4.31320
Table Width	143.8	1.438	$56\frac{5}{8}$	4.719	1.43828
Chair Height	77.5	0.775	$30\frac{1}{2}$	2.542	0.77470

Table 3

A) Create an empty table of 5 columns and 6 rows.

B) **Row 1, Header**: Fill in the header information as shown.

C) **Column 1, Distance Description**: Fill in five object lengths to measure in the room.
   - Any length less than 5 *m* will work.
   - This could be the width of the door, table height, etc.
   - The lengths and measurements shown in Table 3 are just a few examples.

D) **Column 2, d (cm)**: Measure each of these distances in centimeters.

E) **Column 3, d (m)**: Convert each distance to meters using

$$d_m = d_{cm}\left(\frac{1\ m}{100\ cm}\right) \qquad \text{(Equation 3)}$$

F) **Column 4, d (in)**: Measure each of these distances again in inches.

G) **Column 5, d (ft)**: Convert each distance to feet using

$$d_{ft} = d_{in}\left(\frac{1\ ft}{12\ in}\right) \qquad \text{(Equation 4)}$$

H) **Column 6, d (m), converted**: Convert each distance back to meters using

$$d_{m,\ converted} = d_{in}\left(\frac{2.54\ cm}{1\ in}\right)\left(\frac{1\ m}{100\ cm}\right) \qquad \text{(Equation 5)}$$

- Students should compare their values in column 3 with those in column 6.
  - They should also discuss why these values don't match exactly.

- Students should discuss how to match their values from column 5 with the "foot" markings on the tape rule.

10.P1.L2.8 Post-Lab Questions - High School

1. Which distance is easier to use, inches or centimeters?
   - Most would argue centimeters.
   - They're easier to measure and convert.
2. How many centimeters are in 3.5 meters?

$$\text{given quantity in given unit: } d = 3.5\ m$$

$$\text{applying conversion factor: } d = (3.5\ m)\left(\frac{100\ cm}{1\ m}\right)$$

$$\text{solving: } d = 350\ cm$$

3. How many centimeters are in a mile (use 1 mile = 5280 feet).

$$\text{given quantity in given unit: } d = 1\ mile$$

$$\text{applying conversion factors: } d = (1\ mile)\left(\frac{5280\ ft}{1\ mile}\right)\left(\frac{12\ in}{1\ ft}\right)\left(\frac{2.54\ cm}{1\ in}\right)$$

$$\text{solving: } d = 160,934.4\ cm$$

10.P1.L2.9 Post-Lab Questions - University Level 1

4. If a single playing card has a thickness of 310 μm, about how many playing cards are in a deck with a total thickness of 1 $^3/_8$ *in*?

$$\text{given equation for card thickness: } t_{per\ card} = \frac{t_{total}}{N_{cards}}$$

$$\text{isolating card quantity: } N_{cards} = \frac{t_{total}}{t_{per\ card}}$$

$$\text{subsituting known values: } N_{cards} = \frac{1\ ^3/_8\ in}{310\ \frac{\mu m}{card}}$$

$$\text{converting fraction to decial: } N_{cards} = \frac{1.375\ in}{310\ \frac{\mu m}{card}}$$

$$\text{applying conversion factors: } N_{cards} = \frac{(1.375\ in)\left(\frac{2.54\ cm}{1\ in}\right)}{\left(310\ \frac{\mu m}{card}\right)\left(\frac{1\ cm}{10^4\ \mu m}\right)}$$

$$\text{solving: } N_{cards} = 112.661\ cards$$

$$\text{rounding: } N_{cards} = 112\ or\ 113\ cards$$

10.P1.L2.10 Post-Lab Questions - University Level 2

5. The distance between the minor lines placed in the region between inch-marks on a ruler can be described by

$$\Delta d = \frac{1\ in}{2^n} \tag{Equation 6}$$

Where

- **Δd** is the distance between two of the smallest lines, in inches;
- **n** is the "halving number" between each inch-mark.

For example, some values of **n** and their corresponding precision are

- **n** = 0, precision = 1 inch (only inch marks are shown)
- **n** = 1, precision = $^1/_2$ inch
- **n** = 2, precision = $^1/_4$ inch
- **n** = 3, precision = $^1/_8$ inch
- **n** = 4, precision = $^1/_{16}$ inch (most common on academic rulers)

Using Equation Equation 6, determine which halving number achieves a precision smaller than one μm.

$$\text{given equation for precision: } \Delta d = \frac{1\ in}{2^n}$$

$$\text{isolating exponential term: } 2^n = \frac{1\ in}{\Delta d}$$

$$\text{isolating halving number: } n = \log_2\left(\frac{1\ in}{\Delta d}\right)$$

$$\text{substituting known values: } n = \log_2\left(\frac{1\ in}{1\ \mu m}\right)$$

$$\text{applying conversion factors } n = \log_2\left(\frac{1\ in\left(\frac{2.54\ cm}{1\ in}\right)}{1\ \mu m\left(\frac{1\ cm}{10^4\ \mu m}\right)}\right)$$

$$\text{solving: } n = 14.6325$$

$$\text{rounding: } n = 15$$

10.P1.L3 Measuring Mass

10.P1.L3.1 Introduction

- Ideally, this lab includes three methods of measuring mass, using a
  1. weight balance (or single beam balance)
  2. triple beam balance
  3. digital scale
- However, this lab can still be carried out if only one or two of the apparatus above are available.

10.P1.L3.2 Apparatus and Materials for Each Station

- There should be as many stations as there are separate apparatus available. E.g.
  - One station if the lab only has a single beam balance (See section A.2 for construction);
  - Two stations if the lab only has a single beam balance and a triple beam balance;
  - Three stations if the lab has a single beam balance, a triple beam balance and a digital scale.
- Each Station requires
  - either 1 deck of playing cards **OR** a pack of at least 20 nails, all the same length and size
- Station A requires
  - 1 single beam balance (See section A.2 for construction)
  - 1 syringe, 10 mL
  - 1 container of about $^1/_2$ L of water
  - 1 cloth for drying any spilled water
- Station B requires
  - 1 triple beam balance
- Station C requires
  - 1 digital scale

10.P1.L3.3 Warm Up Questions

1. What is the SI unit of mass?
   - The kilogram
2. What is the abbreviation of the SI unit of mass?
   - *kg*
3. If we're measuring the mass of a single nail or playing card, which unit of mass should we use?
   - gram - A kilogram is too large.
4. Is it okay to measure the mass of an object, even if the value is recorded in a unit that isn't base SI?
   - Yes, we just have to make sure we convert it before the value in any other equations.
5. What is the mass of a single milliliter of water?
   - The mass is 1 g.

10.P1.L3.4 Procedure for Station A - Single Beam Balance

- Students should collect data similar to Table 1 using the steps below.

N_{cards}	$N_{full\ syringes}$	V_f (mL)	V_T (mL)	m_T (g)	$m_{per\ card}$ (g)
10	x	3.5	16.5	16.5	1.650
20	xxx	7.0	33.0	33.0	1.650
30	xxxx	0.5	49.5	49.5	1.650
40	xxxxxx	4.0	66.0	66.0	1.650
50	xxxxxxxx	7.0	83.0	83.0	1.660

Table 1

A) Create an empty table of 7 columns and 6 rows.

B) **Row 1, Header**: Fill in the header information as shown.

C) **Column 1, $\mathbf{N_{cards}}$**: Fill in the values for the card quantities as shown.

D) **Column 2, $\mathbf{N_{full\ syinges}}$** and **Column 3, $\mathbf{V_f}$ (mL)**:
  i) Place the first quantity of cards (10) in the right container.
    – This should cause the beam to tilt down on the right.
  ii) Draw water into the syringe until it contains the full 10 *mL*.
  iii) Fill the left container slowly until the beam becomes horizontal again.
    – Don't worry if the beam tips toward the left after balancing.
    – Just be sure to stop filling when this happens.
  iv) Record the final value on the syringe as $\mathbf{V_f}$ in the 3^{rd} column when the beam has been balanced.
  v) If the beam doesn't balance after the full 10 *mL* in the syringe is released, make a mark in the 2^{nd} column to keep track, and carry on with another 10 *mL*.
  vi) After recording this volume, continue to the next card quantity (20, 30, etc).
    – Place 10 more cards in the right container.
    – Release the remaining water in the syringe into the left container.
    – Record this additional quantity of "full syringes" in the 2^{nd} column.
    – Add more water until the beam balances again.
    – Record this new value of $\mathbf{V_f}$ in the 3^{rd} column.
  vii) Repeat steps v) through vi) for all remaining quantities.

E) **Column 4,** 10 **mL** − $\mathbf{V_f}$ (**mL**): Calculate the actual volume of water placed in the left container (separate from the full syringe volumes).

F) **Column 5, $\mathbf{V_T}$ (mL)** - Calculate the total volume in the left container using

$$V_T = (10\ mL)\left(N_{full\ syringes} + 1\right) - V_f \qquad \text{(Equation 1)}$$

G) **Column 6, $\mathbf{m_T}$** (*g*) - Calculate the total mass in the left container using

$$m_T = (V_T)\left(\frac{1\ g}{1\ mL}\right) \qquad \text{(Equation 2)}$$

H) **Column 7, $\mathbf{m_{per\ card}}$** (*g*) - Calculate the mass of a single card using

$$m_{per\ card} = \frac{m_T}{N_{cards}} \qquad \text{(Equation 3)}$$

- Students should discuss the differences in calculated values of $\mathbf{m_{per\ card}}$ in the last column.

10.P1.L3.5 Procedure for Station B - Triple Beam Balance

- Students should collect data similar to Table 2 using the steps below.

N_{cards}	$m_{top\ beam}$	$m_{middle\ beam}$	$m_{bottom\ beam}$	$\sum m_{beams}$	$m_{per\ card}$ (g)
10	10	0	6.6	16.6	1.660
20	30	0	3.1	33.1	1.655
30	40	0	9.7	49.7	1.657
40	60	0	6.2	66.2	1.655
50	80	0	2.8	82.8	1.656

Table 2

A) Create an empty table of 6 columns and 6 rows.

B) **Row 1, Header**: Fill in the header information as shown.

C) **Column 1, N_{cards}**: Fill in the values for the card quantities as shown.

D) As shown in Figure 1, be sure all sliders are set to their zero positions.

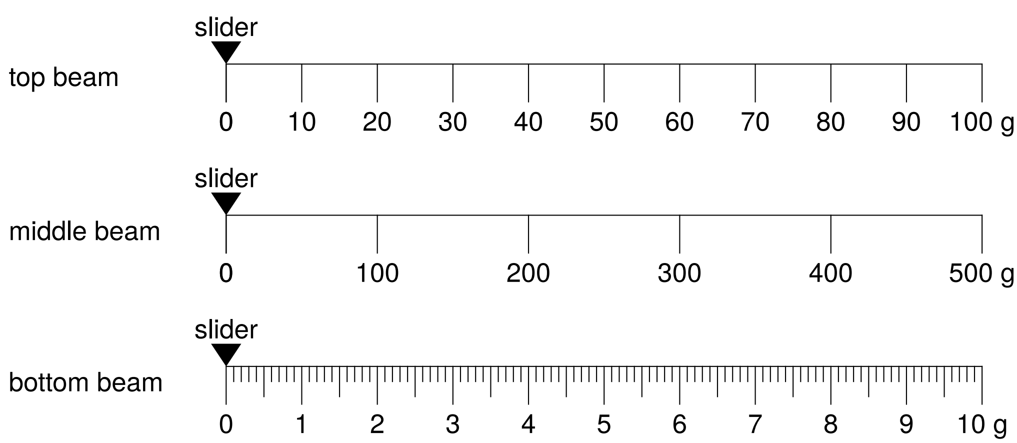


Figure 1

E) Confirm that the balance reads zero.
   - This can be observed by the arrow at the end of the beam pointing closely to the zero mark.

F) Place 10 cards on the weighing pan.
   - Observe that the arrow at the end of the beam points above zero in this unbalanced state.
   - Shift the slider on the middle beam to its closest notch to the right.
   - Observe if this causes the arrow at the end of the beam to point below zero.
   - If it **does**, this means the 100 *g* reading is *larger* than the mass being measured.
     - Shift the middle slider back to its zero mark.
   - Shift the slider on the top beam to its closest notch to the right.
   - Observe if this causes the arrow at the end of the beam to point below zero.
   - If it **does**, this means the 10 *g* reading is greater than the mass being measured.
     - The top slider should be shifted back to its zero mark.
   - If it does **not**, the 10 *g* reading indicated by the slider is *less* than the mass being measured.
     - Shift the bottom slider to the left until the arrow at the end of the beam points to zero.

G) **Columns 2, 3, and 4, $\mathbf{m_{top\ beam}}$, $\mathbf{m_{middle\ beam}}$, $\mathbf{m_{bottom\ beam}}$**: When the arrow at the end of the beam points to zero, record the slider readings for all three beams.

H) **Column 5, $\sum \mathbf{m_{beams}}$**: Sum the values across all three beams using

$$\sum m_{beams} = m_{top\ beam} + m_{middle\ beam} + m_{bottom\ beam} \qquad \text{(Equation 4)}$$

I) **Column 6, $\mathbf{m_{per\ card}}$**: Calculate the mass of a single card using

$$m_{per\ card} = \frac{\sum m_{beams}}{N_{cards}} \qquad \text{(Equation 5)}$$

J) Repeat steps F) through I) for all remaining card quantities.

- Students should
  - discuss the differences in calculated values of $\mathbf{m_{per\ card}}$ in the last column;
  - compare the values of $\mathbf{m_T}$ from Station A with $\sum \mathbf{m_{beams}}$ from Station B;
  - compare the values of $\mathbf{m_{per\ card}}$ between Stations A and B.

10.P1.L3.6 Procedure for Station C - Digital Scale

- Students should collect data similar to Table 3 using the steps below.

$\mathbf{N_{cards}}$	$\mathbf{m_T}$ (g)	$\mathbf{m_{per\ card}}$ (g)	$\mathbf{m_{per\ card}}$ (mg)	$\mathbf{m_{per\ card}}$ (kg)
10	16.67	1.667	1,667	0.001667
20	33.14	1.657	1,657	0.001657
30	49.68	1.656	1,656	0.001656
40	66.24	1.656	1,656	0.001656
50	82.79	1.656	1,656	0.001656

Table 3

A) Create an empty table of 6 columns and 6 rows.

B) **Row 1, Header**: Fill in the header information as shown.

C) **Column 1, $\mathbf{N_{cards}}$**: Fill in the values for the card quantities as shown.

D) **Column 2, $\mathbf{m_T}$ (g)**: Record the reading from the digital scale for each quantity of cards.

E) **Column 3, $\mathbf{m_{per\ card}}$ (g)**: Calculate the mass of a single card using Equation 3.

F) **Column 4, $\mathbf{m_{per\ card}}$ (mg)**: Convert this single card mass to milligrams using

$$m_{per\ card,\ mg} = \left(m_{per\ card,\ g}\right)\left(\frac{1000\ mg}{1\ g}\right) \qquad \text{(Equation 6)}$$

G) **Column 5, $\mathbf{m_{per\ card}}$ (kg)**: Convert this single card mass to kilograms using

$$m_{per\ card,\ kg} = \left(m_{per\ card,\ g}\right)\left(\frac{1\ kg}{1000\ g}\right) \qquad \text{(Equation 7)}$$

H) Repeat steps D) through G) for all remaining card quantities.

- Students should
  - compare the values of $m_{per\ card}$ between all stations.
  - discuss the reduced time and improved precision achieved with the digital scale.

10.P1.L3.7 Post-Lab Questions - High School

1. How many milligrams are in 0.056 *kg*?

$$\text{given quantity in given unit: } m = 0.056\ kg$$

$$\text{applying conversion factors: } m = (0.056\ kg)\left(\frac{1000\ g}{1\ kg}\right)\left(\frac{1000\ mg}{1\ g}\right)$$

$$\text{solving: } m = 56,000\ mg$$

2. How many pounds are in 97, 652 *mg*? (use 1 kilogram = 2.20462 (mass) pounds).

$$\text{given quantity in given unit: } m = 97,652\ mg$$

$$\text{applying conversion factors: } m = (97,652\ mg)\left(\frac{1\ g}{1000\ mg}\right)\left(\frac{1\ kg}{1000\ g}\right)\left(\frac{2.20462\ lbm}{1\ kg}\right)$$

$$\text{solving: } m = 0.215286\ lbm$$

3. Does a scale measure the mass of an object directly?
   - No, a scale measures the weight of an object.
   - Weight is the force created by the effect of gravity on its mass.
   - We assume a constant proportion between its mass and its weight

10.P1.L3.8 Post-Lab Questions - University Level 1

4. If a single playing card has a mass of 1, 670 *mg*, about how many playing cards are in a deck with a total mass of 21.7 *g*?

$$\text{given equation for card mass: } m_{per\ card} = \frac{m_t}{N_{cards}}$$

$$\text{isolating card quantity: } N_{cards} = \frac{m_t}{m_{per\ card}}$$

$$\text{subsituting known values: } N_{cards} = \frac{21.7\ g}{1,670\ mg}$$

$$\text{applying conversion factor: } N_{cards} = \frac{21.7\ g}{(1,670\ mg)\left(\frac{1\ g}{1000\ mg}\right)}$$

$$\text{solving: } N_{cards} = 12.994\ cards$$

$$\text{rounding: } N_{cards} = 13\ cards$$

10.P1.L3.9 Post-Lab Questions - University Level 2

5. Of all the ordinary mass in the observable universe, it is estimated that about 24% is composed of helium. If there exists 3.6×10^{46} *Gg* of helium in the observable universe,

a) calculate the mass of the observable universe in kilograms;

$$\text{considering given proportionality: } 24\% = \frac{m_{He}}{m_T}$$

$$\text{isolating total mass: } m_T = \frac{m_{He}}{24\%}$$

$$\text{converting percent into decimal: } m_T = \frac{m_{He}}{0.24}$$

$$\text{substituting known values: } m_T = \frac{3.6 \times 10^{46}\ Gg}{0.24}$$

$$\text{simplifying: } m_T = 1.5 \times 10^{47}\ Gg$$

$$\text{applying conversion factors: } m_T = \left(1.5 \times 10^{47}\ Gg\right)\left(\frac{10^9\ g}{1\ Gg}\right)\left(\frac{1\ kg}{10^3\ g}\right)$$

$$\text{simplifying: } m_T = 1.5 \times 10^{53}\ kg$$

b) calculate the mass of hydrogen in the observable universe in kilograms, given that there is about 3.08333 times as much hydrogen as there is helium;

$$\text{considering given proportionality: } m_H = 3.08333\,(m_{He})$$

$$\text{substituting known values: } m_H = 3.08333\left(3.6 \times 10^{46}\ Gg\right)$$

$$\text{simplifying: } m_H = 1.11 \times 10^{47}\ Gg$$

$$\text{applying conversion factors: } m_H = \left(1.11 \times 10^{47}\ Gg\right)\left(\frac{10^9\ g}{1\ Gg}\right)\left(\frac{1\ kg}{10^3\ g}\right)$$

$$\text{simplifying: } m_H = 1.11 \times 10^{53}\ kg$$

c) determine the percentage of mass in the observable universe that is neither helium nor hydrogen;

$$\text{summing all contributing mass percentages: } 100\% = \%_H + \%_{He} + \%_{other}$$

$$\text{isolating non-Hydrogen, non-Helium percentage: } \%_{other} = 100\% - (\%_H + \%_{He})$$

$$\text{substituting known values: } \%_{other} = 100\% - (3.08333\,(24\%) + 24\%)$$

$$\text{solving: } \%_{other} = 2\%$$

d) calculate the mass of non-hydrogen, non-helium matter in the observable universe in kilograms.

$$\text{expressing proportion: } m_{other} = (\%_{other})\,(m_T)$$

$$\text{subsituting known values: } m_{other} = (2\%)\left(1.5 \times 10^{53}\ kg\right)$$

$$\text{solving: } m_{other} = 3 \times 10^{51}\ kg$$

Period 2 Velocity and Acceleration

Period Contents

10.P2.L1 Measuring Time

10.P2.L1.1 Introduction

- This lab should help students learn to measure periods of time between 0.62 *s* and 5 *s*.
- A phone can provide all the time measurement tools necessary for this lab.

10.P2.L1.2 Apparatus and Materials for Each Station

- This lab should be carried out across either two or three "stations".
  - Use two stations if only a phone is available, but with no data.
  - Use three stations if a phone is available that also has access to the internet.

- Each Station requires
  - a role of plaster tape **OR** any solid object of about 150 *g*
  - a retort stand and burette clamp **OR** a hammer and a nail **OR** anything from which to hang a string/spring

- Station A requires
  - a spring (See section A.1 for construction)
  - a stopwatch **OR** a phone with a stopwatch feature

- Station B requires
  - strong, thin string, about 150 *cm*
  - a stopwatch **OR** a phone with a stopwatch feature

- Station C requires
  - strong, thin string, about 150 *cm*
  - a phone with access to the internet

10.P2.L1.3 Setup

- For station A,
  - hang the spring from a support (burette clamp, nail in a table, etc);
  - suspend the plaster tape (or any object) from the spring's bottom hook.
- For stations B and C,
  - tie two loops into both ends of the string;
  - use one loop to tie around the plaster tape (or any object);
  - hang the string and tape/object from a support using the other loop.

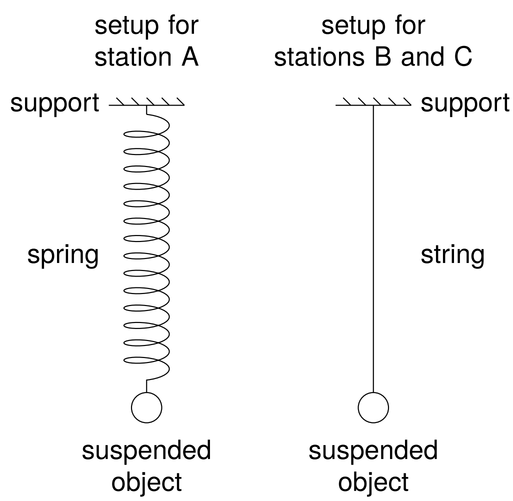


Figure 1

10.P2.L1.4 Warm Up Questions

1. What is the SI unit of time?
   - The second
2. What is the abbreviation of the SI unit of time?
   - *s*
3. Give an example of an event that happens once a day.
   - Sunrise
4. What is the amount of time, in hours between each occurrence of this event?
   - 24
5. How many times would this event occur in a week?
   - 7

10.P2.L1.5 Procedure for Station A - Average Period of an Oscillating Spring

- Students should collect data similar to Table 1 using the steps below.

$\mathbf{N_{cycles}}$	$\mathbf{T_{total}}$ (**s**)	$\mathbf{T_{per\ cycle}}$ (**s**)
10	8.1	0.812
20	16.2	0.810
30	24.3	0.811
40	32.4	0.810
50	40.4	0.808

Table 1

A) Create an empty table of 3 columns and 6 rows.

B) **Row 1, Header**: Fill in the header information as shown.

C) **Column 1, Nail**: Fill in the list of cycles to be counted, as shown.

D) Pull the suspended object downward about 5 *cm* and release it.
   – This should start a repeating, bouncing motion in the spring.

E) After a few cycles, start the stopwatch just as the object is at its lowest position.

F) Count the number of cycles as each time the object approaches this lowest position and returns.
   – Be sure to count the first cycle as complete only has bounced up after the initial release and returned back down to the lowest position.

G) Stop the stopwatch after the 10^{th} cycle.

H) **Column 2, $\mathbf{T_{total}}$** (**s**): Record the stopwatch reading in seconds.

I) **Column 3, $\mathbf{T_{per\ cycle}}$** (**s**): Calculte the period of a single cycle using

$$T_{per\ cycle} = \frac{T_{total}}{N_{cycles}} \qquad \text{(Equation 1)}$$

J) Repeat steps D) through I) for all remaining cycle quantities.

- Students should discuss the differences in calculated values of $\mathbf{T_{per\ cycle}}$ in the last column.

10.P2.L1.6 Procedure for Station B - Average Period of a Pendulum

- Students should collect data similar to Table 2 using the steps below.

N_{cycles}	T_{total} (s)	$T_{per\ cycle}$ (s)
10	26.5	2.651
20	53.1	2.653
30	79.4	2.646
40	106.1	2.652
50	132.5	2.649

Table 2

A) Create an empty table of 3 columns and 6 rows.

B) **Row 1, Header**: Fill in the header information as shown.

C) **Column 1, Nail**: Fill in the list of cycles to be counted, as shown.

D) Pull the suspended object leftward about 1.5 *m* and release it.
  – This should start a repeating, swinging motion.

E) After a few cycles, start the stopwatch when the object is at its left-most point.

F) Count the number of cycles as each time the object approaches this left-most point and returns.
  – Be sure to count the first cycle as complete when the object has swung to the right after the first release, and returned back to the left-most position.

G) Stop the stopwatch after the 10^{th} cycle.

H) **Column 2, T_{total}** (s): Record the stopwatch reading in seconds.

I) **Column 3, $T_{per\ cycle}$** (s): Calculte the period of a single cycle using Equation 1.

J) Repeat steps D) through I) for all remaining cycle quantities.

- Students should discuss the differences in calculated values of $T_{per\ cycle}$ in the last column.

10.P2.L1.7 Procedure for Station C - Direct Measurement of the Period of a Pendulum

- Use a phone with data to connect to access the link https://bulklc.github.io/periodic_sound_generator/
  – Note that this page can be loaded when there is data and used later when there isn't.

A) Be sure that the phone's sound is on.

B) Pull the suspended object leftward about 1.5 *m* and release it to start a swinging motion.

C) Adjust the period value until the pendulum is at the same location every time the sound occurs.
  – Don't worry if this location isn't the left-most , right-most or center positions.
  – The location just has to be the same for each cycle.

D) Record the pendulum's period as shown on the phone.

- Students should compare this with the values of $T_{per\ cycle}$ in the last column of Table 2 from Station B.

10.P2.L1.8 Post-Lab Questions - High School

1. How many seconds are in 37 minutes?

$$\text{given quantity in given unit: } t = 37\ min$$

$$\text{applying conversion factor: } t = (37\ min)\left(\frac{60\ s}{1\ min}\right)$$

$$\text{solving: } t = 2{,}220\ s$$

2. How many minutes are in two days?

$$\text{given quantity in given unit: } t = 2\ day$$

$$\text{applying conversion factors: } t = (2\ day)\left(\frac{24\ hr}{1\ day}\right)\left(\frac{60\ min}{1\ hr}\right)$$

$$\text{solving: } t = 2{,}880\ min$$

3. If a weather station records 5 separate storms with a total precipitation (rain) duration of 45 minutes, what is the average duration (in minutes) of a single storm event?

$$\text{given equation for period of a cycle: } T_{per\ cycle} = \frac{T_{total}}{N_{cycles}}$$

$$\text{substituting known values: } T_{per\ cycle} = \frac{45\ min}{5}$$

$$\text{solving: } T_{per\ cycle} = 9\ min$$

10.P2.L1.9 Post-Lab Questions - University Level 1

4. Consider an analog (circle) clock with three hands - the hour hand, minute hand and second hand. Create a table showing the period of each hand. Include a row for each of the three units of time. Set the first row as the header.

Hand	**T** (**s**)	**T** (**min**)	**T** (**hr**)
Hour	3,600	60.000	1.000000
Minute	60	1.000	0.016666
Second	1	0.017	0.000278

10.P2.L1.10 Post-Lab Questions - University Level 2

5. The period of a computer's processing cycle is the amount of time required to carry out one line of instruction. Consider a computer that, when left to run continuously, could complete 9.228×10^{14} lines of instruction within a single day.

   Calculate this computer's

   a) processing period in the base SI unit of time;

   $$\text{given equation for period of a cycle: } T_{per\ cycle} = \frac{T_{total}}{N_{cycles}}$$

   $$\text{substituting known values: } T_{per\ cycle} = \frac{24\ hr}{9.228 \times 10^{14}\ cycles}$$

   $$\text{applying conversion factors: } T_{per\ cycle} = \left(\frac{24\ hr}{9.228 \times 10^{14}\ cycles}\right)\left(\frac{60\ min}{1\ hr}\right)\left(\frac{60\ s}{1\ min}\right)$$

   $$\text{solving: } T_{per\ cycle} = 9.363 \times 10^{-11}\ s$$

   b) processing period in picoseconds (use 1 s = 10^{12} ps);

   $$\text{given period in base SI unit: } T_{per\ cycle} = 9.363 \times 10^{-11}\ s$$

   $$\text{applying conversion factor: } T_{per\ cycle} = \left(9.363 \times 10^{-11}\ s\right)\left(\frac{10^{12}\ ns}{1\ s}\right)$$

   $$\text{solving: } T_{per\ cycle} = 93.632\ ps$$

   c) processing speed (cycles per second) in the base unit of frequency;

   $$\text{considering frequency of cycle: } f = \frac{1}{T_{per\ cycle}}$$

   $$\text{substituting known values: } f = \frac{1}{9.363 \times 10^{-11}\ s}$$

   $$\text{solving: } f = 1.068 \times 10^{10}\ Hz$$

   d) processing speed in gigahertz.

   $$\text{given quantity in given unit: } f = 1.068 \times 10^{10}\ Hz$$

   $$\text{applying conversion factor: } f = \left(1.068 \times 10^{10}\ Hz\right)\left(\frac{1\ GHz}{10^{9}\ Hz}\right)$$

   $$\text{solving: } f = 10.68\ GHz$$

10.P2.L2 Measuring Velocity

10.P2.L2.1 Introduction

1. An object moving in a straight line has a one-dimensional path.

2. Its average velocity can be determined by considering two points, **A** and **B** along this path.

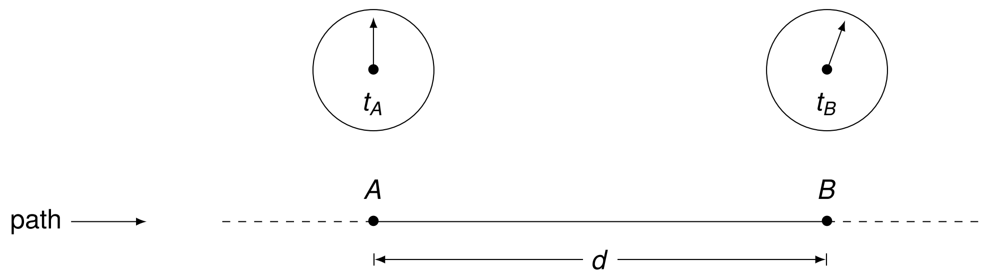


Figure 1

3. Its average velocity is the rate of the distance between the two points per the time taken to travel between them.

$$V_{AB} = \frac{d}{t_B - t_A} \qquad \text{(Equation 1)}$$

4. If the time t_A is set to be zero, this equation can be simplified as

$$V = \frac{d}{t} \qquad \text{(Equation 2)}$$

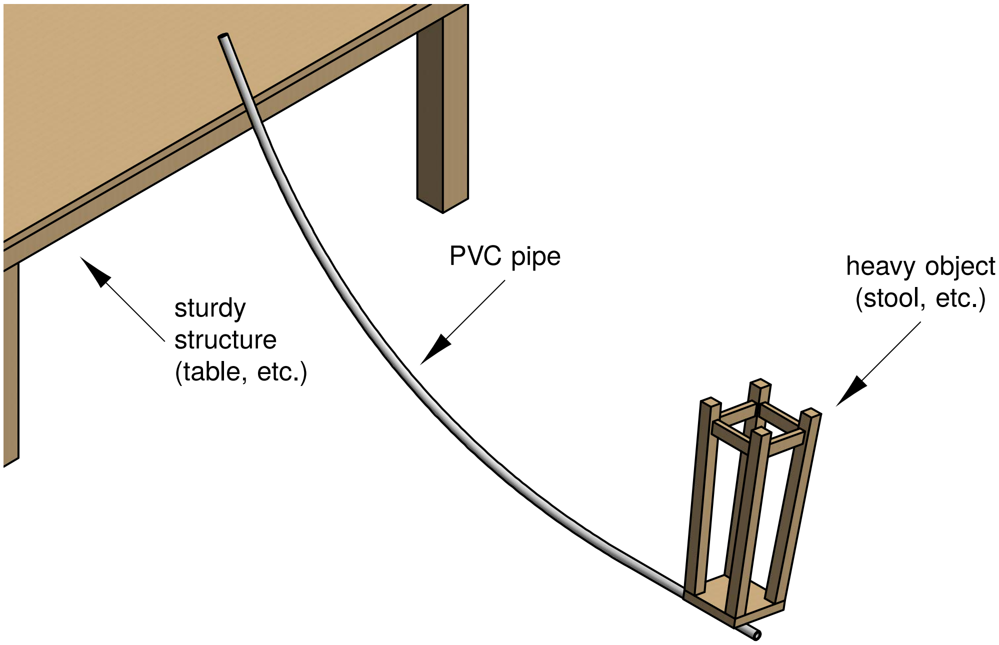
where
- **d** is the distance between points **A** and **B**;
- **t** is the time taken for the object to travel between points **A** and **B**.

10.P2.L2.2 Apparatus and Materials

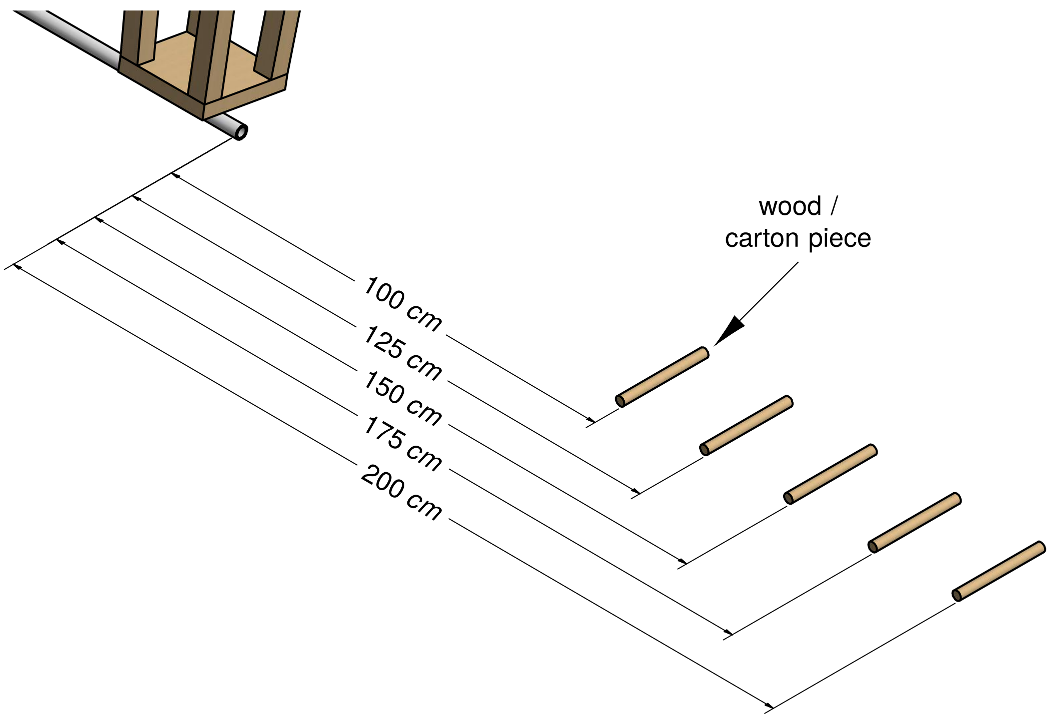
- 1 PVC pipe
  - The length should be at least 3 *m*.
  - Interior diameter should be at least 20 *mm*.
- 5 small balls
  - Each should be small enough to roll freely inside the PVC pipe.
  - These balls can be toy marbles, motorcycle engine bearings, etc.
- 1 piece of wood or carton, at least 20 *cm* long in any direction
- tape rule, at least 2 *m* long
- 1 piece of chalk
- 1 heavy object to hold the pipe in place
  - This can be a book, brick, lab stool, etc
- 1 stopwatch **OR** a phone with a stopwatch feature

10.P2.L2.3 Setup

1. Lean the PVC pipe on a steady surface such as a counter or table.
2. Allow one end of the pipe to rest on the ground.
3. Place a heavy object on the part of the pipe near the ground to hold it in place.



4. Be sure the floor area beyond the pipe's bottom end is clean and free of obstructions.
5. Use chalk to mark 5 separate distances from the bottom end of the pipe as shown below.



If students are directed to complete the University-level post-lab questions, have them also measure and record the vertical distance (height) between the inlet at the pipe's top end and the floor.

10.P2.L2.4 Warm Up Questions

1. What is the SI unit for distance?
   - meter - *m*

2. What is the SI unit for time?
   - second - *s*

3. Which physical property does a car's speedometer show?
   - speed or velocity

4. What are the units of a car's speedometer?
   - either kilometers per hour (kph) or miles per hour (mph)

5. Why don't speedometers show their readings in SI units (or in feet per second)?
   - Cars usually travel distances much greater than a few meters or feet.
     - Therefore, the distance-portion of the speed reading is usually in a larger unit like kilometers or miles.
   - Cars usually travel for durations of time much greater than a few seconds.
     - Therefore, the time-portion of the speed reading is usually in a much larger unit like hours.

10.P2.L2.5 Procedure and Calculations

- Students should collect data similar to Table 1 using the steps below.

d (**cm**)	**d** (**m**)	$\mathbf{T}_{\text{total}}$ (**s**)	$\mathbf{T}_{\text{per cycle}}$ (**s**)
100.0	1.00	3.429	0.686
125.0	1.25	4.024	0.805
150.0	1.50	4.619	0.924
175.0	1.75	5.214	1.043
200.0	2.00	5.810	1.162

Table 1

A) Create an empty table of 4 columns and 6 rows.

B) **Row 1, Header**: Fill in the header information as shown.

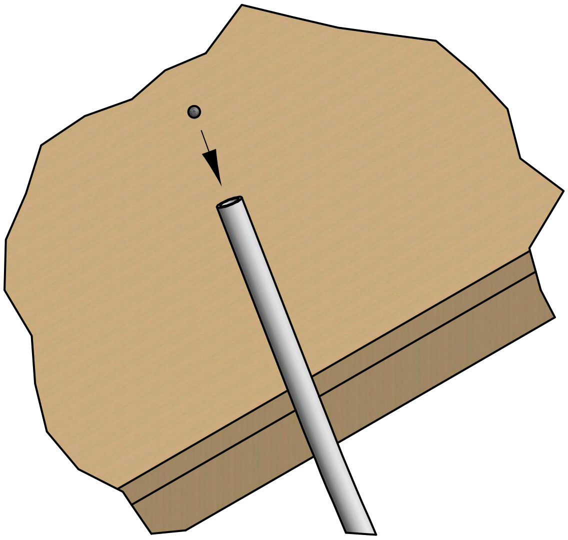
C) **Column 1, d** (**cm**): Fill in the distances from the pipe end as shown.

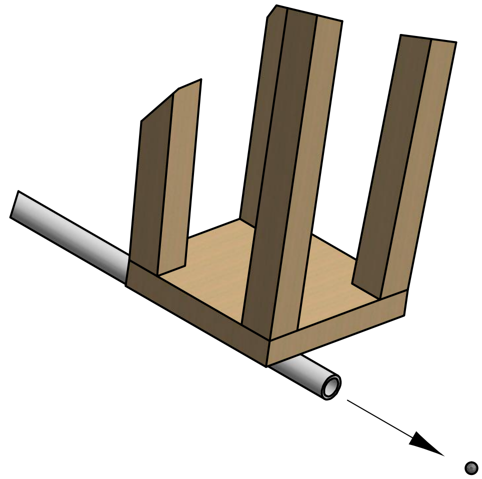
D) **Column 2, d** (**m**): Convert each of these distances to meters using

$$d_m = \left(\frac{1\ m}{100\ cm}\right) d_{cm} \qquad \text{(Equation 3)}$$

E) Drop one of the balls through the top end of the pipe.

F) Confirm that the ball exits freely from the bottom end.





G) Also confirm that the ball passes through each of the 5 markings on the floor.

H) Collect the dropped ball.

I) Place the wood / carton piece at the first marking (100 *cm*).

J) Carefully coordinate the drop of another ball with the starting of a stopwatch.

K) Drop this ball again and listen for its impact with the wood / carton piece below.

L) Drop a 2nd ball once the impact of the 1st is heard.

M) Repeat step L) for the 3rd, 4th and 5th balls.

N) Stop the stopwatch when the impact of the 5th ball is heard.

O) **Column 3, $\mathbf{T_{total}}$ (s)**: Record the total time shown on the stopwatch.

P) **Column 4, $\mathbf{T_{per\ cycle}}$ (s)**: Calculate the duration of each drop-impact cycle using

$$T_{\text{per cycle}} = \frac{T_{\text{total}}}{5} \qquad \text{(Equation 4)}$$

Q) Repeat steps I) through P) for the 4 remaining markings.

10.P2.L2.6 Data Plotting and Slope/Intercept Determination

1. Students should plot the 2^{nd} column of Table 1 against its last, similar to Figure 2 below.

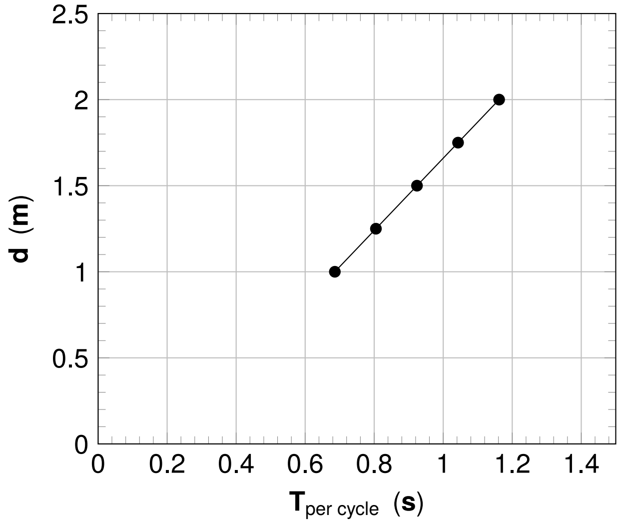


Figure 2

2. Students should then determine the graph's **slope** using

$$\text{general equation for the slope of a straight line: } s = \frac{y_f - y_i}{x_f - x_i}$$

$$\text{specifying for this practical: } s = \frac{(d)_f - (d)_i}{\left(T_{\text{per cycle}}\right)_f - \left(T_{\text{per cycle}}\right)_i}$$

$$\text{substituting known values: } s = \frac{(2\ m) - (1\ m)}{(1.162\ s) - (0.686\ s)}$$

$$\text{solving: } s = 2.1008\ \frac{m}{s}$$

3. Finally, students should then determine the graph's **horizontal intercept** using

$$\text{general equation for the horizontal intercept of a straight line: } x_o = x_i - \frac{y_i}{s}$$

$$\text{specifying for this practical: } \left(T_{\text{per cycle}}\right)_o = \left(T_{\text{per cycle}}\right)_i - \frac{(d)_i}{s}$$

$$\text{substituting known values: } \left(T_{\text{per cycle}}\right)_o = (0.686\ s) - \frac{1\ m}{2.1008\ \frac{1}{s}}$$

$$\text{solving: } \left(T_{\text{per cycle}}\right)_o = 0.210\ s$$

4. Finally, students should discuss the meaning of this horizontal intercept, including the following concepts.

   - The horizontal intercept, $\left(T_{\text{per cycle}}\right)_o$, is the time taken for the ball to travel from the top inlet to the bottom outlet of the pipe.
   - This time is essentially the same for each trial, regardless of the distance of the wood / carton piece from the pipe's bottom outlet.
   - Note this corresponds with a distance of $d = 0\ m$ from the pipe's bottom outlet.

10.P2.L2.7 Post-Lab Questions - High School

1. A car travels 91.125 kilometers in 1.35 hours. Calculate its average velocity in

   a) kilometers per hour

   $$\text{using given equation for velocity: } v = \frac{d}{t}$$

   $$\text{substituting known values: } v = \frac{91.125\ km}{1.35\ hr}$$

   $$\text{solving: } v = 67.5\ {}^{km}/_{hr}$$

   b) meters per second

   $$\text{considering velocity calculated previously: } v = 67.5\ \frac{km}{hr}$$

   $$\text{applying conversion factosr: } v = (67.5\ {}^{km}/_{hr})\left(\frac{1\ hr}{60\ min}\right)\left(\frac{1\ min}{60\ s}\right)\left(\frac{1000\ m}{1\ km}\right)$$

   $$\text{solving: } v = 18.75\ {}^{m}/_{s}$$

2. If a truck takes 4 hours to travel 360 kilometers, how many hours does it take to travel 450 kilometers?

   $$\text{using given equation for veloicty: } v = \frac{d}{t}$$

   $$\text{isolating time: } t = \frac{d}{v}$$

   $$\text{substituting known values: } t = \frac{450\ km}{\left(\frac{360\ km}{4\ hr}\right)}$$

   $$\text{solving: } t = 5\ hr$$

3. Consider two motorcycles - moto **A** and moto **B**. If moto **A** travels 50 km in 45 minutes and moto **B** travels 100 km in 90 minutes, how do their velocities compare?
   - Their velocities are the same.

   $$\text{using given equation for veloicty: } v = \frac{d}{t}$$

   $$\text{considering velocity of moto A: } v_A = \frac{d_A}{t_A}$$

   $$\text{substituting known values: } v_A = \frac{50\ km}{45\ min}$$

   $$\text{solving: } v_A = 1.11\ {}^{km}/_{min}$$

   $$\text{considering velocity of moto B: } v_B = \frac{d_B}{t_B}$$

   $$\text{substituting known values: } v_B = \frac{100\ km}{90\ min}$$

   $$\text{solving: } v_B = 1.11\ {}^{km}/_{min}$$

10.P2.L2.8 Post-Lab Questions - University Level 1

4. Consider a rolling ball that rolls down a slope from point **A** to point **B** as shown in the figure below.

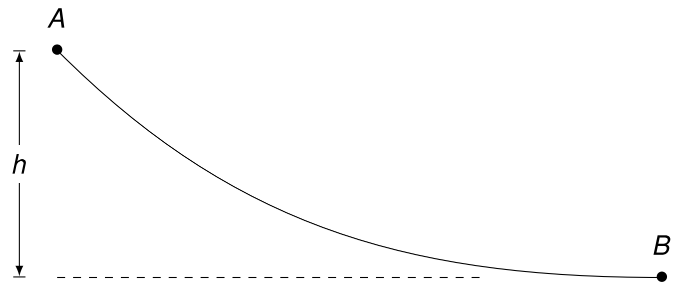


Figure 3

Derive an expression for the ball's velocity just as it passes point **B** in terms of only

- **h**, the height of point **A** above **B**;
- *g*, the acceleration of gravity.

Assume the ball is released from rest at **A** and that all energy is either potential or translational (linear) kinetic.

considering potential and kinetic energy at both points: $PE_A + KE_A = PE_B + KE_B$

$$\text{substituting equations for energy: } mg\,(h_A) + \frac{1}{2}m\,(v_A)^2 = mg\,(h_B) + \frac{1}{2}m\,(v_B)^2$$

$$\text{simplifying: } g\,(h_A) + \frac{1}{2}\,(v_A)^2 = g\,(h_B) + \frac{1}{2}\,(v_B)^2$$

$$\text{considering no initial velocity: } g\,(h_A) = g\,(h_B) + \frac{1}{2}\,(v_B)^2$$

$$\text{setting final height as zero: } g\,(h) = \frac{1}{2}\,(v_B)^2$$

$$\text{isolating velocity at B: } v_B = \sqrt{2gh}$$

5. Use the measured value of **h** from this practical to calculate the theoretical velocity of the ball just as it passes point **B**. Use $g = 9.81\ {}^{m}/{}_{s^2}$.

The solution below is for a pipe-end height of 95 *cm above the ground.*

$$\text{using equation derived previosly: } v_B = \sqrt{2gh}$$

$$\text{substituting known values: } v_B = \sqrt{2\left(9.81\frac{m}{s^2}\right)(0.95\ m)}$$

$$\text{solving: } v_B = 4.31\ {}^{m}/{}_{s}$$

6. Discuss the difference between the theoretical velocity calculated in Question 5 and the slope of the graph created in this practical.
   - The initial potential energy is converted into other forms of energy besides the translational kinetic energy.
   - These other forms include
     - the ball's rotational kinetic energy (rolling);
     - the thermal energy due to the friction between the ball and the pipe's interior surface;
     - the thermal energy due to the friction between the ball and the ground;
     - the kinetic energy transferred into the pipe itself as the ball occasionally impacts its interior surface.

10.P2.L2.9 Post-Lab Questions - University Level 2

7. Derive an expression for the ball's velocity just as it passes point **B** in the same terms and under the same assumptions as those given in Question 4. Consider also the rotational kinetic energy (KE_R) of the ball in addition to the translational kinetic energy (KE_T).

   Use

   $$KE_R = \frac{1}{2}I\omega^2 \qquad \text{(Equation 5)}$$

   where
   - $\mathbf{KE_R}$ is the rotational kinetic energy of the ball;
   - **I** is the ball's moment of intertia about the ball's axis of rotation;
   - ω is the angular velocity of the ball's rotation.

   Also use

   $$I = \frac{2}{5}mr^2 \qquad \text{(Equation 6)}$$

   where
   - **m** is the mass of the ball;
   - **r** is the radius of the ball.

   Solution Step 1 - Derive expression for angular velocity in terms of linear velocity:

   considering equation for tangential velocity of a rotating object: $v = \omega r$

   isolating angular velocity: $\omega = \dfrac{v}{r}$

   Solution Step 2 - Derive expression for linear velocity:

   considering all initial and final energies: $PE = KE_T + KE_R$

   substituting equations for energy: $mgh = \dfrac{1}{2}m\,(v_B)^2 + \dfrac{1}{2}I\omega^2$

   substituting equation for moment of inertia: $mgh = \dfrac{1}{2}m\,(v_B)^2 + \dfrac{1}{2}\left(\dfrac{2}{5}mr^2\right)\omega^2$

   simplifying: $gh = \dfrac{1}{2}\,(v_B)^2 + \dfrac{1}{2}\left(\dfrac{2}{5}r^2\right)\omega^2$

   substituting previously-derived equation for angular velocity: $gh = \dfrac{1}{2}\,(v_B)^2 + \dfrac{1}{2}\left(\dfrac{2}{5}r^2\right)\left(\dfrac{v_B}{r}\right)^2$

   simplifying: $gh = \dfrac{1}{2}\,(v_B)^2 + \dfrac{1}{5}\,(v_B)^2$

   simplifying: $gh = \dfrac{7}{10}\,(v_B)^2$

   isolating velocity at B: $v_B = \sqrt{\dfrac{10}{7}gh}$

8. Use the measured value of **h** from this practical to calculate the theoretical velocity of the ball just as it passes point **B** considering both the translational and rotational kinetic energies. Use $g = 9.81\ {}^{m}/_{s^2}$.

   The solution below is for a pipe-end height of 95 *cm above the ground.*

   using equation derived previosly: $v_B = \sqrt{\frac{10}{7} gh}$

   substituting known values: $v_B = \sqrt{\frac{10}{7}\left(9.81\frac{m}{s^2}\right)(0.95\ m)}$

   solving: $v_B = 3.64\ {}^{m}/_{s}$

9. Discuss which value is closer to the slope of the graph generated in this practical - the velocity value calculated in Question 8 or from Question 5.

   - The value for $\mathbf{v_B}$ calculated in Question 8 was more accurate.
   - However, it was still quite higher.
   - This means that a significant amount of energy is lost due
     - friction between the ball and pipe's interior;
     - impacts between the ball and the pipe's interior;
     - friction between the ball and the ground.

Period 3 Thermal Physics and Fluid Properties

Period Contents

10.P3.L1 Boyle's Law*

10.P3.L1.1 Introduction

1. An ideal gas can be modeled as air contained within a syringe.
   - When the syringe is compressed, the volume inside decreases.
   - If the syringe is plugged at the end, no air can escape.
   - If no air can escape, the mass of the air contained within stays the same.
   - As a constant mass of air decreases in volume, its pressure increases.

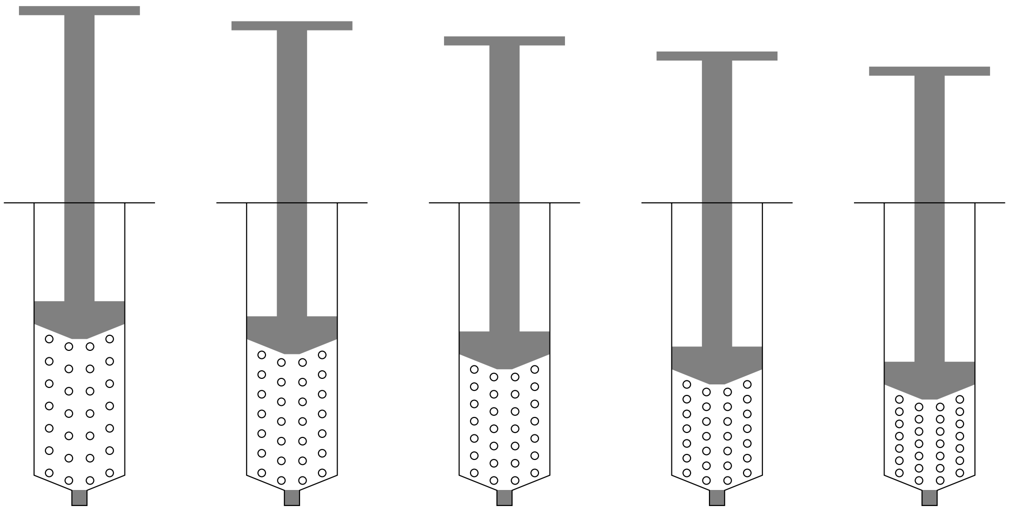


Figure 1

   - The syringe's interior pressure and volume before and after compression can be related as

$$P_1 V_1 = P_2 V_2 \qquad \text{(Equation 1)}$$

   Where
     - $\mathbf{P}_1$ is the interior pressure before compression;
     - $\mathbf{V}_1$ is the interior volume before compression;
     - $\mathbf{P}_2$ is the interior pressure after compression;
     - $\mathbf{V}_2$ is the interior volume after compression;
     - the temperature of the gas inside remains constant.

2. Equation 9 also explains that the product of contained gas' volume and pressure is constant
   - That is, assuming constant temperature,

$$P_1 V_1 = P_2 V_2 = k \qquad \text{(Equation 2)}$$

   Where
     - **k** is a constant calculated from various properties of the enclosed gas.

3. Equation 9 also presents the inverse proportionality between a contained gas' pressure and volume.
   - That is, assuming constant temperature,

$$V = \frac{k}{P} \qquad \text{(Equation 3)}$$

10.P3.L1.2 Apparatus and Materials

- 1 syringe (including needle, either 10 *mL* or 20 *mL*)
- 1 plastic (block) eraser
- 1 knife **OR** a pair of scissors
- 1 hammer
- 2 nails **OR** two chairs
- 100 *cm* of strong, thin string
- 1 bucket of at least 5 *L*
- 2 bottles of water (500 *mL*, full)
- 2 bottles of water (1.5 *L*, full)

10.P3.L1.3 Setup

1. Use a knife or pair of scissors to cut four lines / notches into the syringe finger-grip as shown in Figure 2.
2. Use a hammer and nail to puncture two holes in the finger grip.

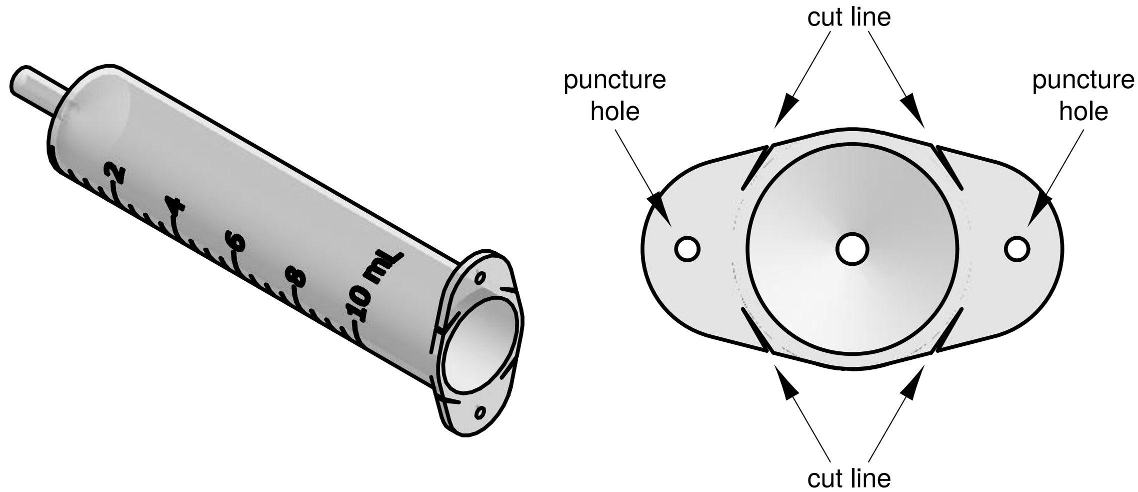


Figure 2

3. Cut two pieces of the thin, strong string, each about 30 *cm* long.
4. Make a loop out of each piece.
5. Guide one of these loops around the syringe body, through the lines cut previously, as shown in Figure 3.
6. Repeat for the other loop.
7. Use these loops to suspend the syringe from two strong supports (nails in side of a table, two chair backs, etc).

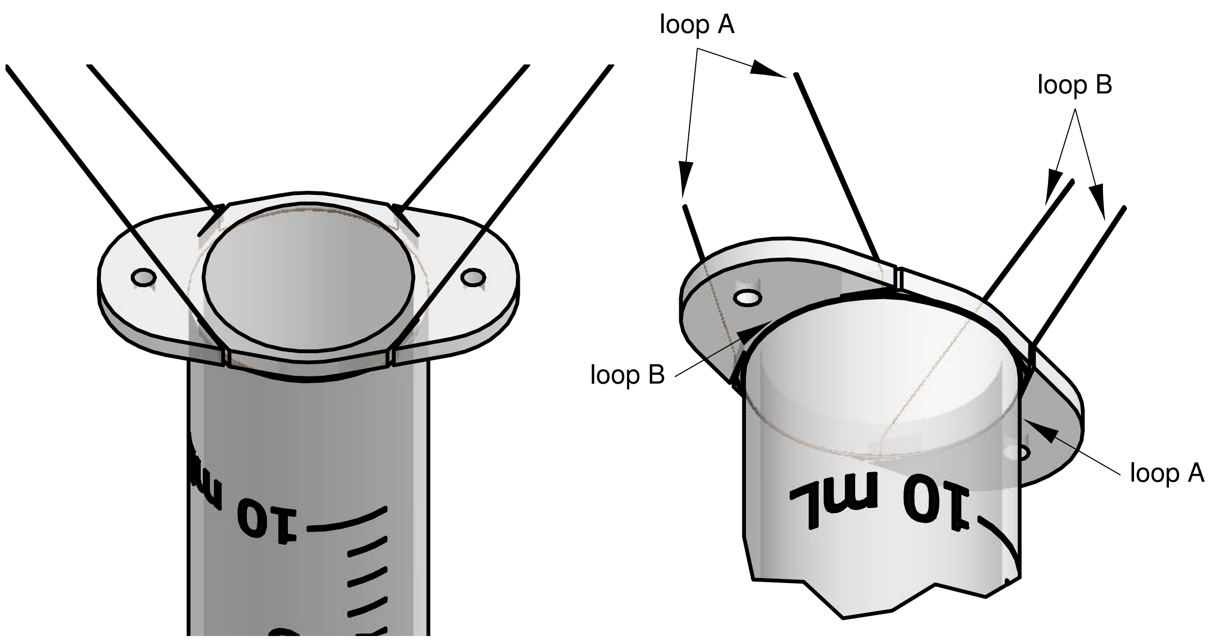


Figure 3

8. Use a knife or pair of scissors to cut two lines/notches into the syringe plunger, as shown in Figure 4.

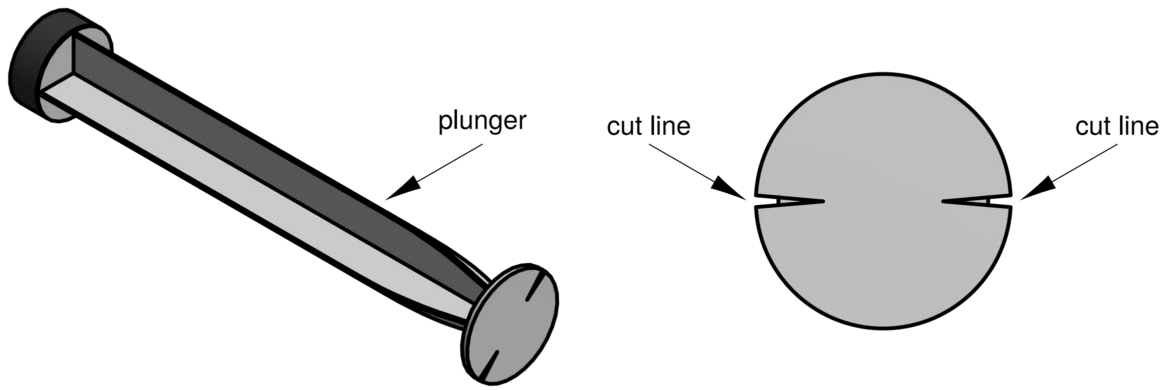


Figure 4

9. Place the plunger into the suspended syringe.
10. Cut another piece of 100 *cm* of the thin, strong string.
11. Guide the string through each of the punctured holes.
12. Guide the string into the lines cut into the plunger head.
13. Tie the ends of the string to the handle of a bucket or container.
    - Note that the container needs to be **strong** but **does not need to be water tight**.

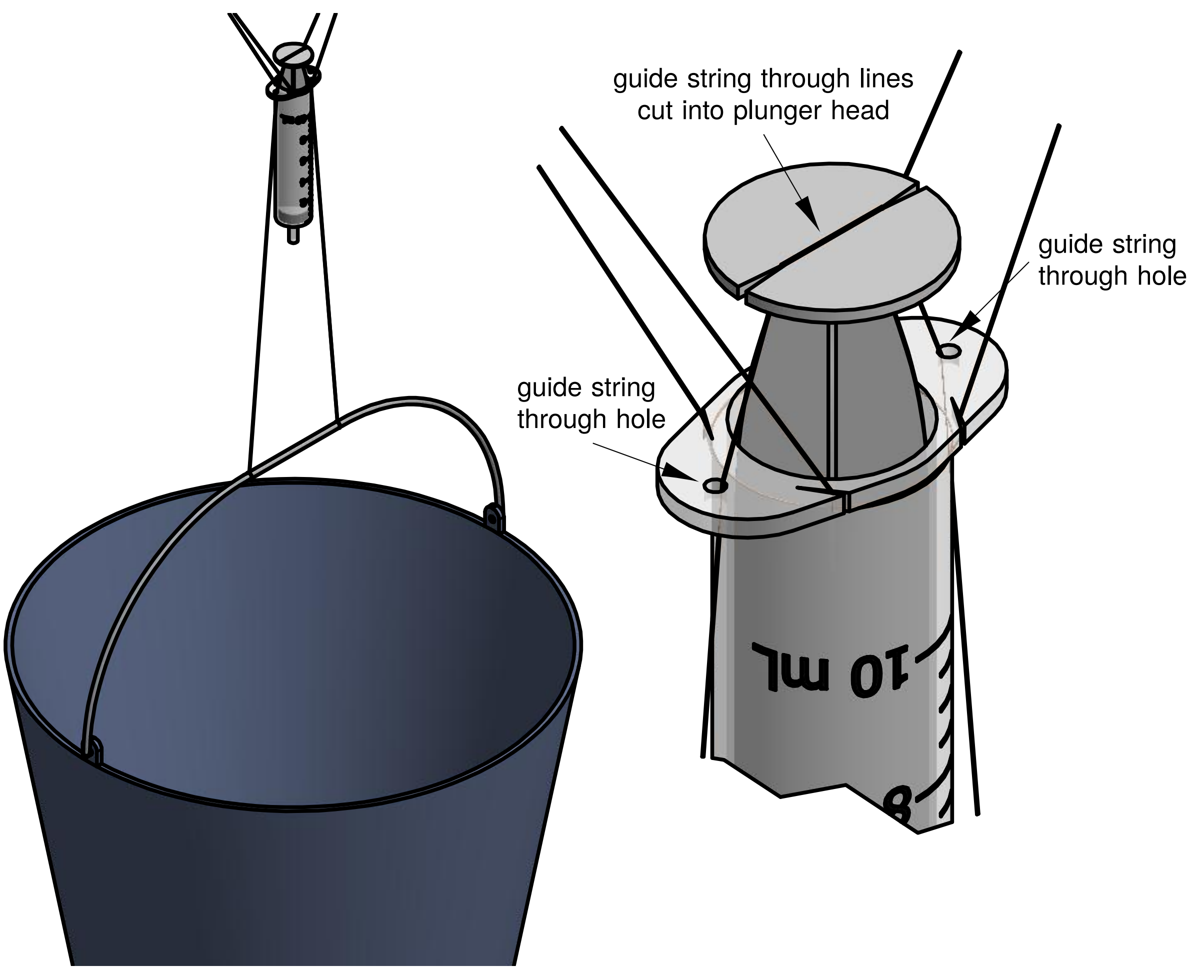


Figure 5

14. Place the syringe needle carefully into a plastic eraser.
    - Take **caution** that the needle does not push back out through the edges of the eraser.

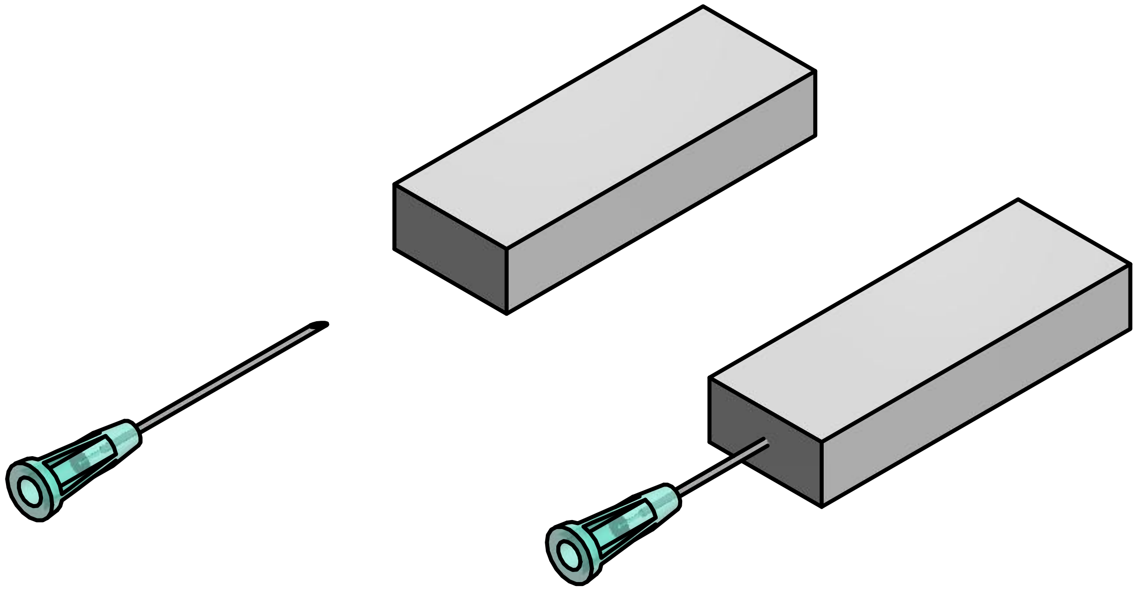


Figure 6

15. Pull the plunger up until a volume of 10 *mL* (or whichever max volume) is contained.
16. Firmly place the needle/eraser assembly onto the nozzle.
17. Release the plunger.

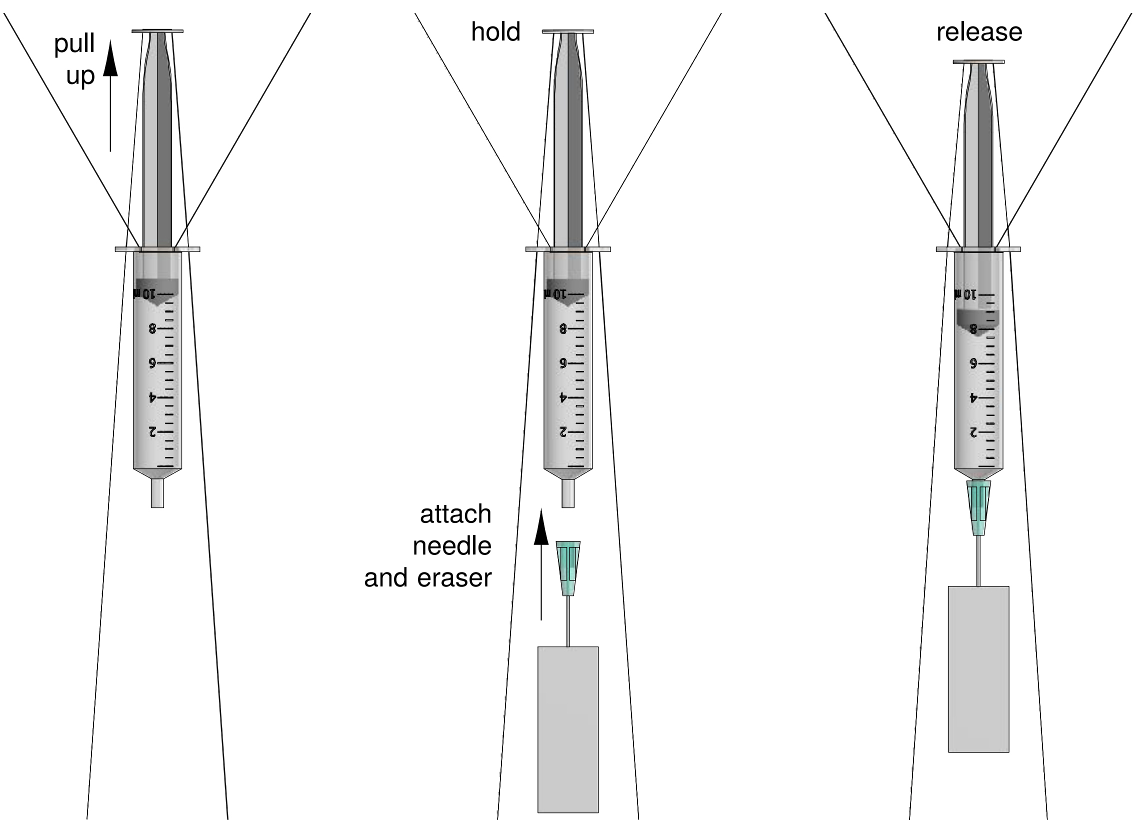


Figure 7

10.P3.L1.4 Warm Up Questions

Consider a group of 12 students, each with their own football. Each student throws their ball at the same wall at a rate of one throw per second. The area of the wall is 9 m^2.

1. If the wall area were reduced, but the student quantity and throw rate remained the same, would the impact rate increase or decrease?
   - It would increase, as the area is in the denominator.
2. If the wall area were increased, but the student quantity and throw rate remained the same, would the impact rate increase or decrease?
   - It would decrease, as the throws would be distributed over a larger area.
3. Explain how this relates to Boyle's law.
   - The quantity of students, each with their own ball, is like the quantity of molecules in an ideal gas.
   - The throw rate is like the temperature, or kinetic activity of the air molecules.
   - The size of the wall is like the volume of the object containing the ideal gas.

10.P3.L1.5 Procedure and Calculations

- Students should collect data similar to Table 1 using the steps below.

$\mathbf{N}_{500\ \mathbf{mL}}$	$\mathbf{N}_{1.5\ \mathbf{L}}$	$\mathbf{V}_{\mathbf{water}}$ (**L**)	**m** (**kg**)	**V** (**mL**)	$\mathbf{V}^{-1}$ $(\mathbf{mL}^{-1})$
1	0	0.5	0.5	6.5	0.154
2	0	1.0	1.0	5.0	0.200
0	1	1.5	1.5	4.5	0.222
0	2	3.0	3.0	3.0	0.333
2	2	4.0	4.0	2.5	0.400

Table 1

A) Create an empty table of 6 columns and 6 rows.

B) **Row 1, Header**: Fill in the header information as shown.

C) **Column 1, $\mathbf{N}_{500\ \mathbf{mL}}$**: Fill in the list of quantities of 500 mL water bottles.

D) **Column 2, $\mathbf{N}_{1.5\ \mathbf{L}}$**: Fill in the list of quantities of 1.5 L water bottles.

E) For each of the 5 combinations of water bottles,

   – **Column 3, $\mathbf{V}_{\mathbf{water}}$ (L)**: Calculate the total volume of water from the bottle combination using

$$V_{water} = (0.5\ L)\,(N_{500\ mL}) + (1.5\ L)\,(N_{1.5\ L}) \qquad \text{(Equation 4)}$$

   – **Column 4, m (kg)**: Calculate the mass of the water from the bottle combination using

$$m = (V_{water})\left(\frac{1\ kg}{1\ L}\right) \qquad \text{(Equation 5)}$$

F) Load the first bottle combination in the bucket.

G) **Column 5, V (mL)**: Record the new volume indicated on the syringe.

H) **Column 6, $\mathbf{V}^{-1}$ $(\mathbf{mL}^{-1})$**: Take the inverse of this volume using

$$V^{-1} = \frac{1}{V} \qquad \text{(Equation 6)}$$

I) Repeat steps F) through H) for all remaining bottle combinations.

10.P3.L1.6 Data Plotting and Slope/Intercept Determination

1. Students should plot the last column of Table 1 against its 4^{th}, similar to Figure 8 below.

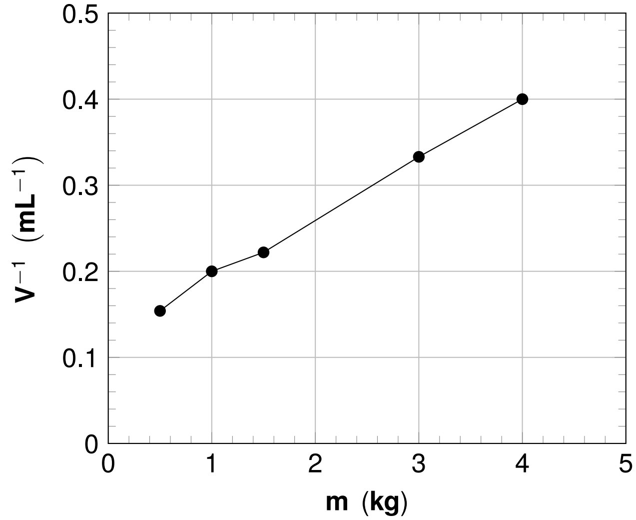


Figure 8

2. Students should then determine the graph's **slope** using

$$\text{general equation for the slope of a straight line: } s = \frac{y_f - y_i}{x_f - x_i}$$

$$\text{specifying for this practical: } s = \frac{\left(V^{-1}\right)_f - \left(V^{-1}\right)_i}{(m)_f - (m)_i}$$

$$\text{substituting known values: } s = \frac{\left(0.400\ mL^{-1}\right) - \left(0.154\ mL^{-1}\right)}{(4.0\ kg) - (0.5\ kg)}$$

$$\text{solving: } s = 0.0703\ \frac{mL^{-1}}{kg}$$

3. Students should then determine the graph's **vertical intercept** using

   general equation for the vertical intercept of a straight line: $y_o = y_i - s(x_i)$

$$\text{specifying for this practical: } \left(V^{-1}\right)_o = \left(V^{-1}\right)_i - s\,(m)_i$$

$$\text{substituting known values: } \left(V^{-1}\right)_o = \left(0.154\ mL^{-1}\right) - \left(0.0703\ \frac{mL^{-1}}{kg}\right)(0.5\ kg)$$

$$\text{solving: } \left(V^{-1}\right)_o = 0.119\ mL^{-1}$$

4. Students should then calculate **k** where $\mathbf{k} = \left[\left(\mathbf{V}^{-1}\right)_{\mathbf{o}}\right]^{-1}$.

$$\text{given equation: } k = \frac{1}{\left(V^{-1}\right)_o}$$

$$\text{substituting known values: } k = \frac{1}{0.119\ mL^{-1}}$$

$$\text{solving: } k = 8.403\ mL$$

10.P3.L1.7 Exam Prompt

Figure 9 illustrates a syringe with a petri dish fixed to the top of its plunger. A known mass, **M**, is placed on the petri dish. The piston of the syinge moves downard and the volume, **V**, of air beneath the piston is read and recorded.

The procedure is repeated for **four** other values of **m**.

Figure 10 represents the masses, $\mathbf{m_i}$, placed on the petri dish.

Figure 11 represesetents the corresponding volumes, $\mathbf{V_i}$, where $\mathbf{i} = 1,\ 2,\ 3,\ 4$ and 5.

(i) Read and record the values of $\mathbf{m_i}$.
(ii) Read and record the corresponding values of $\mathbf{V_i}$.
(iii) Evaluate $\mathbf{V}^{-1}$ for each $\mathbf{i} = 1,\ 2,\ 3,\ 4$ and 5.
(iv) Tabulate your readings.
(v) Plot a graph with **m** on the vertical axis and $\mathbf{V}^{-1}$ on the horizontal axis.
(vi) Calculate **s**, the graph's slope.
(vii) Which physical law does this experiment verify?
(viii) State **two** precautions that are necessary to ensure accurate results when performing this experiment.

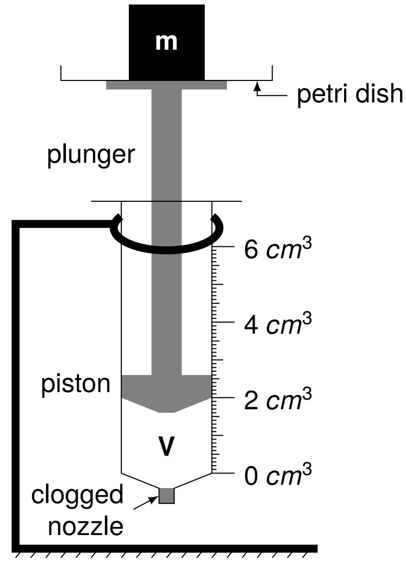


Figure 9

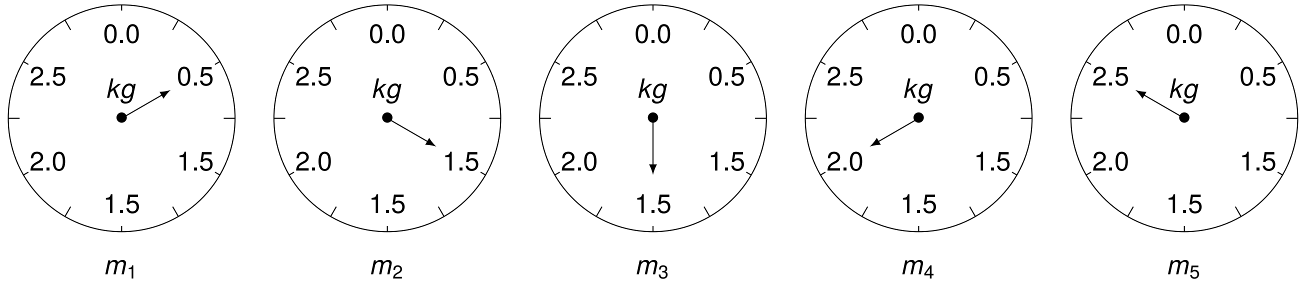


Figure 10

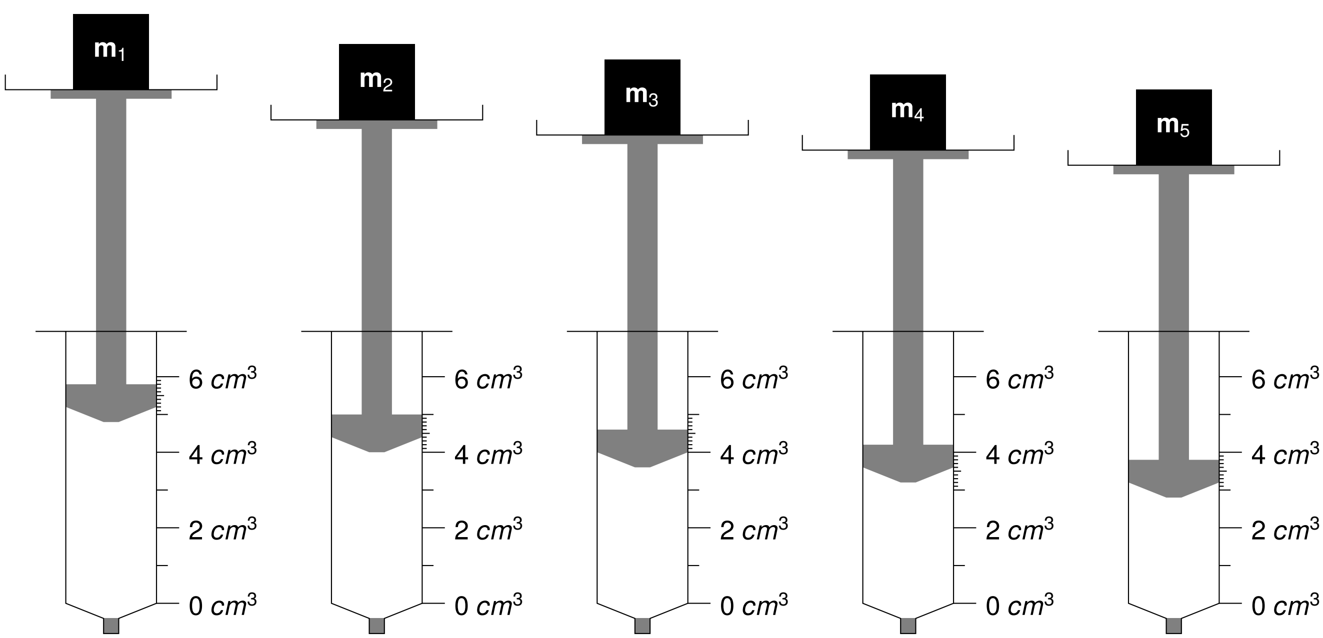


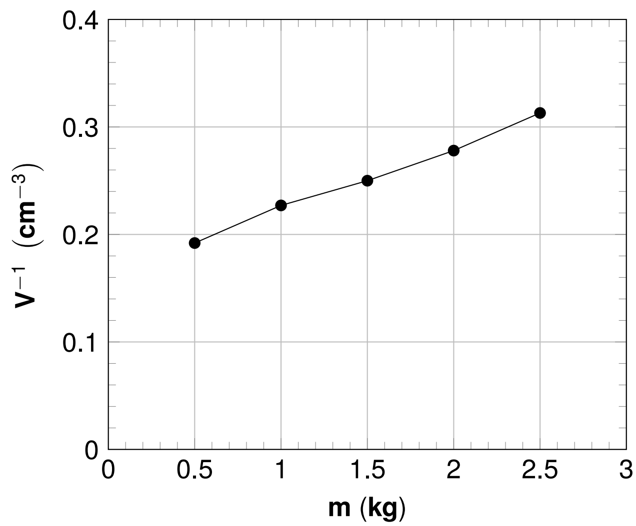
Figure 11

10.P3.L1.8 Solutions to Exam Prompt

(iv)

i	m (kg)	V (cm^3)	V^{-1} (cm^{-3})
1	0.5	5.2	0.192
2	1.0	4.4	0.227
3	1.5	4.0	0.250
4	2.0	3.6	0.278
5	2.5	3.2	0.313

(v)



(vi) *Calculating slope*

general equation for the slope of a straight line: $s = \dfrac{y_f - y_i}{x_f - x_i}$

specifying for this practical: $s = \dfrac{\left(V^{-1}\right)_f - \left(V^{-1}\right)_i}{(m)_f - (m)_i}$

substituting known values: $s = \dfrac{\left(0.313\ cm^{-3}\right) - \left(0.192\ cm^{-3}\right)}{(2.5\ kg) - (0.5\ kg)}$

solving: $s = 0.0605\ \dfrac{cm^{-3}}{kg}$

(vii) This experiment demonstrates Boyle's law, as the mass of air within the syringe experienced a pressure change that was inversely proportional to its volume change at a constant (room) temperature.

(viii) Precautions include

- ensuring the syringe nozzle is completely to allow for the assumption of constant mass;
- reading the syring volume at a 90° angle to avoid parallax error;
- ensuring minimal friction around the piston, so as to not underestimate the decrease in volume.

10.P3.L1.9 Post-Lab Questions - High School

1. Explain the effect of decreasing the volume of a fixed mass of gas at constant temperature using the kinetic molecular theory of gas.
   - A given volume of gas consists of very small particles known as molecules.
   - These particles are in constant (kinetic) motion.
   - Collisions occur frequently within this mass, either between molecules or against the container walls.
   - As the volume of the container decreases, so does the area of its walls facing the gas inside.
   - Thus, the same quantity of wall collisions is distributed along a smaller area.
   - This in turn increases the pressure exerted by the gas on its container walls.
2. Discuss the contents of the syringe in this practical. Is there anything inside?
   - There is a mass of air inside the syringe.
   - Thought it cannot be seen, there are many molecules of gas within.
   - This includes oxygen and nitrogen molecules as well as water vapor and other matter.
3. Would this practical be valid if the class warmed up between the first and last data recordings?
   - No. Boyle's law only applies to processes occurring under constant temperature.
4. If the volume of a water bottle at standard pressure and temperature is 1.5 L, calculate its interior pressure, in units of atmospheres, if it is compressed to a volume of 0.2 L.

$$\text{given equation: } P_1 V_1 = P_2 V_2$$

$$\text{isolating final pressure: } P_2 = \frac{P_1 V_1}{V_2}$$

$$\text{substituting known values: } P_2 = \frac{(1\ atm)(1.5\ L)}{0.2\ L}$$

$$\text{solving: } P_2 = 7.5\ atm$$

5. A syringe initially has a mass of $\mathbf{m_a}$ = 0.75 kg placed on its plunger. When an additional $\mathbf{m_b}$ = 1.5 kg of mass is added, what what is the ratio of its new volume to its initial?

$$\text{given equation: } P_1 V_1 = P_2 V_2$$

$$\text{isolating volume ratio: } \frac{V_2}{V_1} = \frac{P_1}{P_2}$$

$$\text{substituting equation for pressure: } \frac{V_2}{V_1} = \frac{F_1/A}{F_2/A}$$

$$\text{considering weight as only relevant force: } \frac{V_2}{V_1} = \frac{W_1/A}{W_2/A}$$

$$\text{considering mass of each object: } \frac{V_2}{V_1} = \frac{\frac{(m_a)\,g}{A}}{\frac{(m_a + m_b)\,g}{A}}$$

$$\text{simplifying: } \frac{V_2}{V_1} = \frac{m_a}{m_a + m_b}$$

$$\text{substituting known values: } \frac{V_2}{V_1} = \frac{0.75\ kg}{0.75\ kg + 1.5\ kg}$$

$$\text{solving: } \frac{V_2}{V_1} = \frac{1}{3}$$

10.P3.L1.10 Post-Lab Questions - University Level 1

6. A syringe is attached to a gauge which presents absolute pressure readings. When object **Q** is loaded on top of its plunger, the syringe contains a certain volume of air, and the gauge's reading is 1750. With the **addition** of a second object **R**, its volume is compressed to one third its initial size. Calculate the mass (in *kg*) of the second object **R** if the gauge shows readings in *mmHg*, and if the diameter of the syringe's piston is 15 *mm*. (Use 1 *mmHg* = 133.322 *Pa* and *g* = 9.81 $^{m}/_{s^{-2}}$).

given equation: $P_1V_1 = P_2V_2$

substituting equation for pressure: $\left(\frac{F_1}{A}\right)V_1 = \left(\frac{F_2}{A}\right)V_2$

considering weight as only relevant force: $\left(\frac{W_1}{A}\right)V_1 = \left(\frac{W_2}{A}\right)V_2$

considering the weight of each object: $\left(\frac{W_Q}{A}\right)V_1 = \left(\frac{W_Q + W_R}{A}\right)V_2$

expanding: $\left(\frac{W_Q}{A}\right)V_1 = \left(\frac{W_Q}{A}\right)V_2 + \left(\frac{W_R}{A}\right)V_2$

subtracting term from both sides: $\left(\frac{W_Q}{A}\right)V_1 - \left(\frac{W_Q}{A}\right)V_2 = \left(\frac{W_R}{A}\right)V_2$

factoring: $\left(\frac{W_Q}{A}\right)(V_1 - V_2) = \left(\frac{W_R}{A}\right)V_2$

re-simplifying: $P_1\,(V_1 - V_2) = \left(\frac{W_R}{A}\right)V_2$

considering mass of second object: $P_1\,(V_1 - V_2) = \left(\frac{(m_R)\,g}{A}\right)V_2$

isolating mass of second object: $m_R = (V_1 - V_2)\left(\frac{P_1A}{gV_2}\right)$

distributing: $m_R = \left(\frac{V_1}{V_2} - 1\right)\left(\frac{P_1A}{g}\right)$

considering piston area: $m_R = \left(\frac{V_1}{V_2} - 1\right)\left(\frac{P_1\left(\pi\left(\frac{D}{2}\right)^2\right)}{g}\right)$

substituting known values: $m_R = (3 - 1)\left(\frac{(1750\ mmHg)\left(\pi\left(\frac{15\ mm}{2}\right)^2\right)}{9.81\ ^{m}/_{s^2}}\right)$

applying conversion factors: $m_R = 2\left(\frac{(1750\ mmHg)\left(\pi\left(\frac{15\ mm}{2}\right)^2\right)}{9.81\ ^{m}/_{s^2}}\right)\left(\frac{133.322\ Pa}{1\ mmHg}\right)\left(\frac{1\ m}{1000\ mm}\right)^2$

solving: $m_R = 8.40568\ kg$

10.P3.L1.11 Post-Lab Questions - University Level 2

7. Using the ideal gas law ($PV = nRT$), derive a linear equation for this practical's graph as

$$V^{-1} = s\,(m_x)$$

where **s** is the slope and $\mathbf{m_x}$ is the mass of the load on top of the plunger. Use only the terms
   - **V** - the volume contained;
   - **g** - the acceleration of gravity;
   - **D** - the diameter of the piston head's cross sectional area;
   - $\mathbf{m_a}$ - the mass of air contained in the syringe;
   - $\mathbf{M_a}$ - the molecular mass of the air contained in the syringe $\left(\text{note } n = \frac{m}{M}\right)$;
   - **R** - the ideal gas constant;
   - **T** - the absolute temperature of the air contained in the syringe.

Solution

$$\text{given equation: } PV = nRT$$

$$\text{substituting equation for molecular mass: } PV = \left(\frac{m_a}{M_a}\right) RT$$

$$\text{isolating volume: } V = \frac{\left(\frac{m_a}{M_a}\right) RT}{P}$$

$$\text{inverting: } V^{-1} = \frac{P}{\left(\frac{m_a}{M_a}\right) RT}$$

$$\text{substituting equation for pressure: } V^{-1} = \frac{\frac{F}{A}}{\left(\frac{m_a}{M_a}\right) RT}$$

$$\text{simplifying: } V^{-1} = \frac{FM_a}{Am_aRT}$$

$$\text{considering weight of load on plunger: } V^{-1} = \frac{WM_a}{Am_aRT}$$

$$\text{considering mass of load: } V^{-1} = \frac{m_xgM_a}{Am_aRT}$$

$$\text{considering piston area: } V^{-1} = \frac{m_xgM_a}{(\pi r^2)\, m_aRT}$$

$$\text{considering piston diameter: } V^{-1} = \frac{m_xgM_a}{\left(\pi\left(\frac{D}{2}\right)^2\right) m_aRT}$$

$$\text{rearranging: } V^{-1} = \left(\frac{gM_a}{\left(\pi\left(\frac{D}{2}\right)^2\right) m_aRT}\right)(m_x)$$

$$\text{adopting the given form: } s = \frac{gM_a}{\left(\pi\left(\frac{D}{2}\right)^2\right) m_aRT}$$

8. Considering the slope calculated in this practical, use the expression derived in question 7 to evaluate $\mathbf{m_a}$, the mass (in grams) of air contained within the syringe. Use

   - $\mathbf{g} = 9.81\ \frac{m}{s^2}$
   - $\mathbf{M_a} = 28.97\ \frac{g}{mol}$
   - $\mathbf{D} = 1.5\ cm$
   - $\mathbf{R} = 8.314\ \frac{J}{(K)(mol)}$
   - $\mathbf{T} = 24.85°C$, (note $T_K = T_{\circ C} + 273.15$)

   Solution

$$\text{equation derived previously: } s = \frac{gM_a}{\left(\pi\left(\frac{D}{2}\right)^2\right) m_a RT}$$

$$\text{isolating mass of air: } m_a = \frac{gM_a}{\left(\pi\left(\frac{D}{2}\right)^2\right) sRT}$$

$$\text{substituting known values: } m_a = \frac{\left(9.81\ \frac{m}{s^2}\right)\left(28.97\ \frac{g}{mol}\right)}{\left(\pi\left(\frac{1.5\ cm}{2}\right)^2\right)\left(0.0703\ \frac{mL^{-1}}{kg}\right)\left(8.314\ \frac{J}{(K)(mol)}\right)(24.85 + 273.15)\ K}$$

$$\text{solving, partially: } m_a = \frac{284.196\ \frac{(g)(m)}{(s^2)\,(mol)}}{\left(1.76715\ cm^2\right)\left(0.584474\ \frac{\left(cm^{-3}\right)(J)}{(kg)\,(K)\,(mol)}\right)(298\ K)}$$

$$\text{solving, partially: } m_a = 0.923342\ \frac{(g)(m)(kg)(K)(mol)}{\left(s^2\right)(mol)\left(cm^2\right)\left(cm^{-3}\right)(J)(K)}$$

$$\text{simplifying units: } m_a = 0.923342\ \frac{(g)(m)(kg)(cm)}{\left(s^2\right)(J)}$$

$$\text{expressing Joules in base units: } m_a = 0.923342\ \frac{(g)(m)(kg)(cm)}{\left(s^2\right)\left[(m)(kg)\frac{m}{s^2}\right]}$$

$$\text{simplifying units: } m_a = 0.923342\ \frac{(g)(cm)}{m}$$

$$\text{applying conversion factor: } m_a = 0.923342\ \frac{(g)(cm)}{m}\left(\frac{1\ m}{100\ cm}\right)$$

$$\text{solving, completely: } m_a = 0.00923342\ g$$

Period 1 Motion in Two Dimensions

Period Contents

11.P1.L1 Pendulum Oscillations*

11.P1.L1.1 Introduction

1. The period of a simple pendulum can be calculated from its length using,

$$T = 2\pi\sqrt{\frac{L}{g}} \qquad \text{(Equation 1)}$$

   Where
   - **T** is the period of the pendulum's oscillation;
   - **L** is the distance between the pendulum's anchor point and its center of mass;
   - **g** is the acceleration of gravity.
2. As shown in Figure 1, an object is suspended from a support with a string.
   - This forms a simple pendulum when set into motion.
   - While the length of the pendulum itself is not measured, two relevant distances are:
     H - The vertical distance between the anchor point and some datum below (likely the floor);
     h - The vertical distance between the center of the Bob and this same datum.

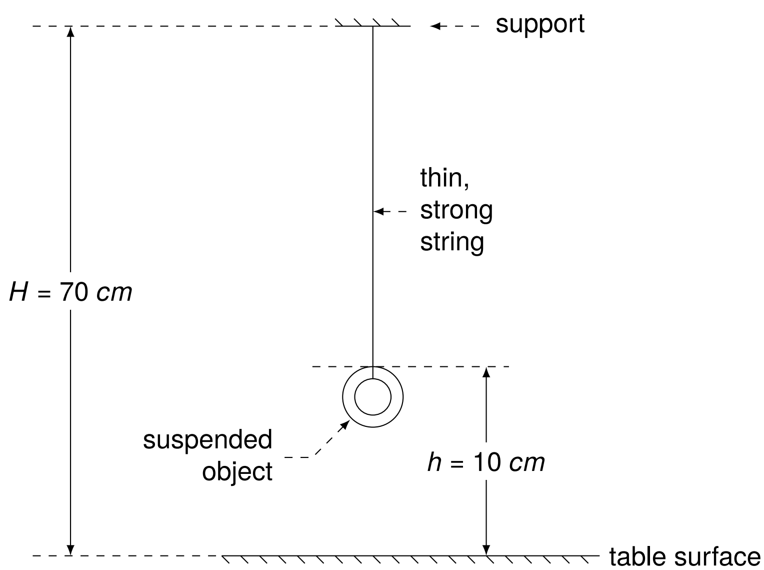


Figure 1

11.P1.L1.2 Apparatus and Materials

- 1 role of plaster tape **OR** any solid object of about 150 *g*
- 100 *cm* of strong, thin string
- 1 stopwatch **OR** 1 phone with a stopwatch feature
- 1 meter rule **OR** tape rule **OR** anything capable of measuring distances up to 100 *cm*
- 1 retort stand and 1 burette clamp **OR** 1 hammer and 1 nail **OR** anything from which to hang a string/spring

11.P1.L1.3 Setup

1. Tie two loops into both ends of the string.
2. Use one loop to tie around the plaster tape / object.
3. Hang the string and tape/object from a support (burette clamp, nail in a table, etc) using the other loop.
4. Be sure the vertical distance between the support and the flat surface below is 70 *cm*.
5. Adjust the string such that the vertical distance between its bottom loop and the flat surface is 10 *cm*.

11.P1.L1.4 Warm Up Questions

1. What is the difference between *oscillatory motion* and *rotational motion*?
   - Oscillatory motion occurs in a cyclic manner between two extremes.
   - Motion is considered "rotational" or "circular" when its trajectory can be considered as an arc of a circle with a set radius and center point.
   - While some objects can exhibit both types of motion at once others may not. Consider the following:

Description of Motion	Oscillatory	Rotational/Circular
The bob of a vertical pendulum	yes	yes
A mass suspended vertically on a spring	yes	no
A car driving on a curved road	no	yes

2. Does the period of a pendulum increase or decrease with its length? Is this relationship linear?
   - The period increases with increasing length, but not linearly.
3. Considering the relationship $\mathbf{T^p} \propto \left(\sqrt{\mathbf{L}}\right)^{\mathbf{p}}$, which value of **p** causes **L** to be raised to the first power?
   - **p** = 2

11.P1.L1.5 Procedure and Calculations

- Students should collect data similar to Table 1 using the steps below.

h (cm)	h (m)	t_a (s)	t_b (s)	t_c (s)	t_{avg} (s)	T (s)	T^2 (s^2)
10	0.10	32.29	32.47	32.61	32.46	1.62	2.633
20	0.20	29.72	29.89	30.02	29.88	1.49	2.232
30	0.30	26.91	27.06	27.18	27.05	1.35	1.830
40	0.40	23.77	23.90	24.01	23.90	1.19	1.428
50	0.50	20.15	20.26	20.35	20.25	1.01	1.025

Table 1

A) Create an empty table of 8 columns and 6 rows.
B) **Row 1, Header**: Fill in the header information as shown.
C) **Column 1, h** (**cm**): Fill in the list of heights as shown.

D) **Column 2, h (m)**: Convert each of these heights to meters using

$$h_m = h_{cm}\left(\frac{1\ m}{100\ cm}\right) \qquad \text{(Equation 2)}$$

E) Adjust the string until its point of connection with the object is **h** = 10 **cm** above the flat surface below.

F) Pull the suspended object leftward about 30 *cm* and release it.
   – This should start a repeating, swinging motion.

G) After a few cycles, start the stopwatch when the object is at its left-most point.

H) Count the number of cycles as each time the object approaches this left-most point and returns.
   – Be sure to count the first cycle as complete when the object has swung to the right after the first release, and returned back to the left-most position.

I) Stop the stopwatch after the 20^{th} cycle.

J) **Column 3, $\mathbf{t_a}$ (s)**: Record the stopwatch reading in seconds.

K) **Columns 4 and 5, $\mathbf{t_b}$ (s) and $\mathbf{t_c}$ (s)**: Repeat steps E) through J) for next two columns.

L) **Column 6, $\mathbf{t_{avg}}$ (s)**: Calculate the average duration of 20 cycles using

$$t_{avg} = \frac{t_a + t_b + t_c}{3} \qquad \text{(Equation 3)}$$

M) **Column 7, T (s)**: Calculate the average period of a single cycle using

$$T = \frac{t_{avg}}{N_{cycles}} = \frac{t_{avg}}{20} \qquad \text{(Equation 4)}$$

N) **Column 8, $\mathbf{T^2}$ $(\mathbf{s^2})$**: Calculate the square of the cycle period.

O) Repeat steps E) through N) for all remaining heights.

11.P1.L1.6 Data Plotting and Slope/Intercept Determination

1. Students should plot the last column of Table 1 against its 2^{nd}, similar to Figure 2 below.

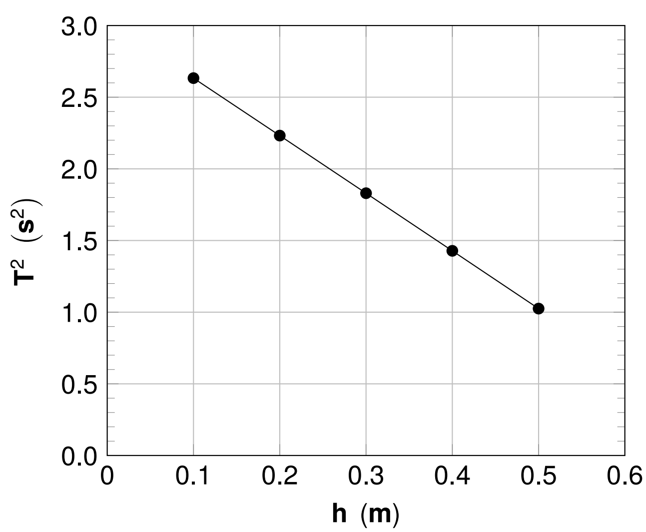


Figure 2

2. Students should then determine the graph's **slope** using

$$\text{general equation for the slope of a straight line: } s = \frac{y_f - y_i}{x_f - x_i}$$

$$\text{specifying for this practical: } s = \frac{\left(T^2\right)_f - \left(T^2\right)_i}{(h)_f - (h)_i}$$

$$\text{substituting known values: } s = \frac{\left(1.025\ s^2\right) - \left(2.633\ s^2\right)}{(0.5\ m) - (0.1\ m)}$$

$$\text{solving: } s = -4.0200\ \frac{s^2}{m}$$

3. Students should then determine the graph's **vertical intercept** using

$$\text{general equation for the vertical intercept of a straight line: } y_o = y_i - s(x_i)$$

$$\text{specifying for this practical: } \left(T^2\right)_o = \left(T^2\right)_i - s\,(h)_i$$

$$\text{substituting known values: } \left(T^2\right)_o = \left(2.633\ s^2\right) - \left(-4.0200\ \frac{s^2}{m}\right)(0.1\ m)$$

$$\text{solving: } \left(T^2\right)_o = 3.035\ s^2$$

4. Students should also calculate $\mathbf{k}_1$ and $\mathbf{k}_2$ where $\mathbf{k}_1 = \frac{4\pi^2}{\mathbf{s}}$ and $\mathbf{k}_2 = \frac{(\mathbf{T_o})^2}{\mathbf{s}}$

Calculating $\mathbf{k}_1$

$$\text{given equation: } k_1 = \frac{4\pi^2}{s}$$

$$\text{subsituting known values: } k_1 = \frac{4\pi^2}{-4.0200\ {}^{s^2}\!/_{m}}$$

$$\text{solving: } k_1 = -\ 9.811\ {}^{m}\!/_{s^2}$$

Calculating $\mathbf{k}_2$

$$\text{given equation: } k_2 = \frac{(T_o)^2}{s}$$

$$\text{subsituting known values: } k_2 = \frac{3.035\ s^2}{-4.0200\ \frac{s^2}{m}}$$

$$\text{solving: } k_2 = -\ 0.755\ m$$

11.P1.L1.7 Exam Prompt

Figure 3 illustrates a simple pendulum suspended from a distance **H**, from the floor. The center of the bob is at a distance **h**, from the floor. The bob is set into oscillation and the time, **t**, for 20 complete oscillations is taken and recorded.The procedure is repeated **four** more times for varying **h**, and the corresponding values of **t**.

Figure 4 and Figure 5 show the values of $\mathbf{h_i}$ and $\mathbf{t_i}$ respectively, where **i** = 1, 2, 3, 4 and 5.

(i) Measure and record the raw values of $\mathbf{h_{raw}}$ of $\mathbf{h_i}$.
(ii) Convert $\mathbf{h_{raw}}$ in (i) above to actual values $\mathbf{h_{real}}$ using the given scale.
(iii) Read and record the corresponding values of **t**.
(iv) In each case, evaluate the period **T**.
(v) Also evaluate $\mathbf{T}^2$ in **each** case.
(vi) Tabulate your readings.
(vii) Plot a graph with $\mathbf{T}^2$ on the vertical axis and $\mathbf{h_{real}}$ on the horizontal axis, starting both axes at $(0, 0)$.
(viii) Determine the slope, **s**, of the graph.
(ix) Determine the value of $\mathbf{T_o}^2$ when $h = 0$.
(x) Evaluate α) $\mathbf{k}_1 = \dfrac{4\pi^2}{\mathbf{s}}$ take $\pi^2 = 10$; β) $\mathbf{k}_2 = \dfrac{\mathbf{T_o}^2}{\mathbf{s}}$
(xi) State **two** precautions that are necessary to ensure accurate results when performing this experiment.

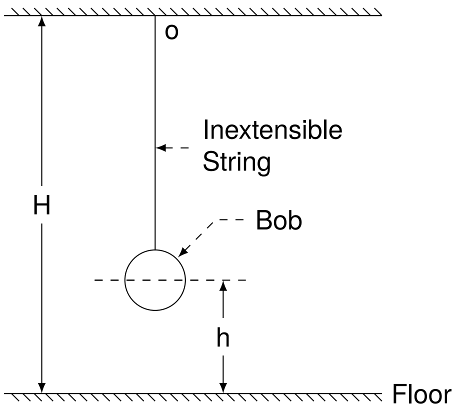


Figure 3

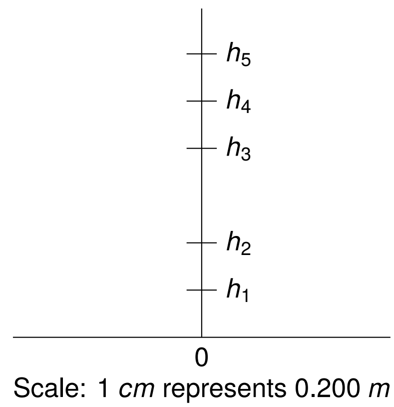


Figure 4

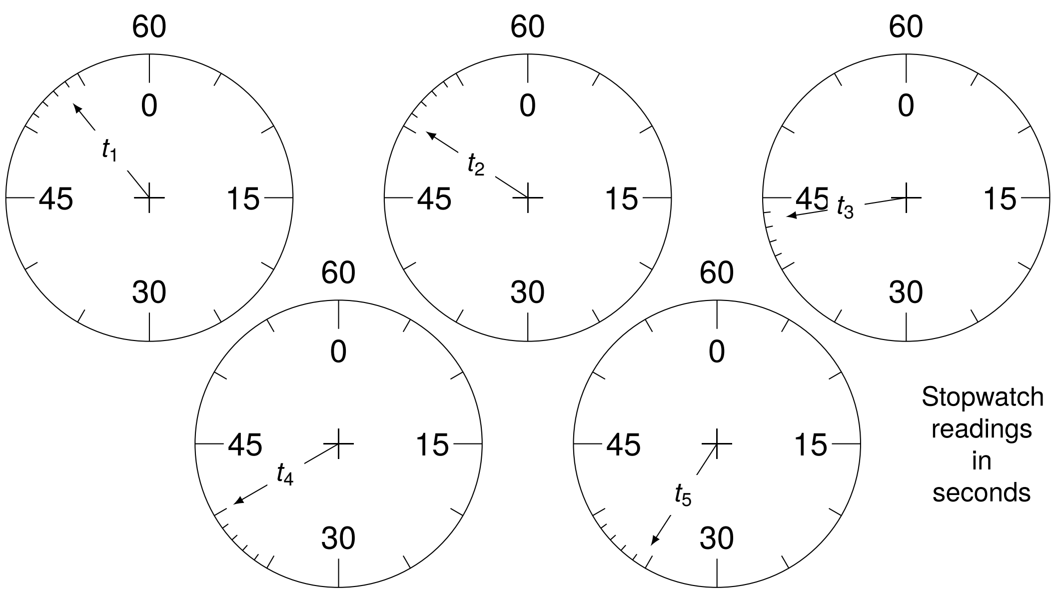


Figure 5

11.P1.L1.8 Solutions to Exam Prompt

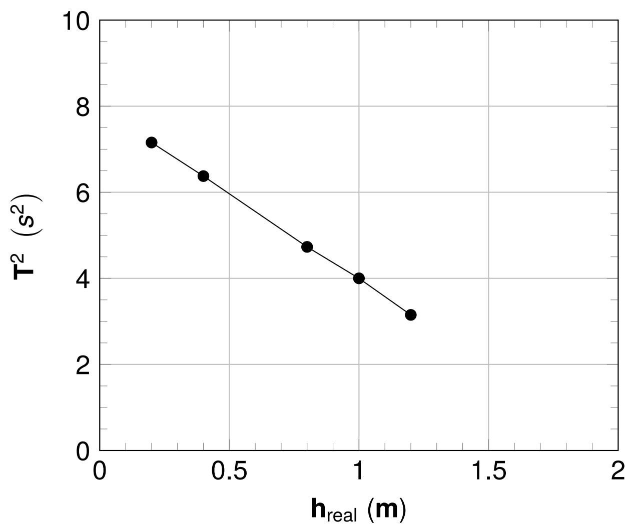
(ii) Use $h_{real} = \left(\frac{0.2\ m}{1\ cm}\right) h_{raw}$

(iv) Use $T = \frac{t}{N_{cycles}} = \frac{t}{20}$.

(vi)

i	$\mathbf{h_{raw}}$ (cm)	$\mathbf{h_{real}}$ (m)	t (s)	T (s)	$\mathbf{T^2}$ $(\mathbf{s^2})$
1	1.0	0.20	53.5	2.68	7.16
2	2.0	0.40	50.5	2.53	6.38
3	4.0	0.80	43.5	2.18	4.73
4	5.0	1.00	40.0	2.00	4.00
5	6.0	1.20	35.5	1.78	3.15

(vii)



(viii)

general equation for the slope of a straight line: $s = \frac{y_f - y_i}{x_f - x_i}$

specifying for this practical: $s = \frac{(T^2)_f - (T^2)_i}{(h_{real})_f - (h_{real})_i}$

substituting known values: $s = \frac{(3.15\ s^2) - (7.16\ s^2)}{(1.2\ m) - (0.2\ m)}$

solving: $s = -4.0100\ \frac{s^2}{m}$

(ix)

general equation for the vertical intercept of a straight line: $y_o = y_i - s(x_i)$

specifying for this practical: $\left(T^2\right)_o = \left(T^2\right)_i - s\,(h_{real})_i$

substituting known values: $\left(T^2\right)_o = \left(7.16\ s^2\right) - \left(-4.0100\ \frac{s^2}{m}\right)(0.2\ m)$

solving: $\left(T^2\right)_o = 7.962\ s^2$

(x)

α)

given equation: $k_1 = \dfrac{4\pi^2}{s}$

subsituting known values: $k_1 = \dfrac{4\pi^2}{-4.0100\ {}^{s^2}/{}_{m}}$

solving: $k_1 = -\ 9.835\ {}^{m}/{}_{s^2}$

β)

given equation: $k_2 = \dfrac{(T_o)^2}{s}$

subsituting known values: $k_2 = \dfrac{7.962\ s^2}{-4.0100\ \frac{s^2}{m}}$

solving: $k_2 = -\ 1.986\ m$

(xi) Precautions include

- measuring the period over multiple oscillations to reduce random error;
- performing the experiment in a room with no wind to reduce external forces on the bob;
- measuring *h* to the center of the bob's mass to consider the pendulum's total effective length.

11.P1.L1.9 Post-Lab Questions - High School

1. What would be the effect on the pendulum's period if the distance **H** shown in Figure 1 were increased, but the length of the string remained the same?
   - Given $\mathbf{L} = \mathbf{H} - \mathbf{h}$, any change in the value of **H** would be offset by an equal change in each of the values of **h** measured.
   - That is, the pendulum's length would remain unchanged, and so would its period.
2. If this practical were carried out on the surface of the moon, would the periods be longer or shorter? Explain.
   - Given $\mathbf{T} \propto \sqrt{\mathbf{L}}$ and $\mathbf{T} \propto \sqrt{\frac{1}{\mathbf{g}}}$, a decrease in the acceleration of gravity would increase the pendulum's period.
   - Given $\mathbf{g_{moon}} < \mathbf{g_{earth}}$ the pendulum's periods on the moon would be longer.
3. Why are we able to consider the length of the pendulum to be the distance from the support to the suspended object? That is, why was the mass of the string not considered?
   - The mass of the suspended object is much greater than that of the string and so the string's mass is assumed negligible. Therefore, the center of mass of the entire system is assumed to be coincident with that of the suspended object itself.
4. If a pendulum were to oscillate within a field of acceleration equal to that of earth's gravity, what would its length be (in meters) if its period were an entire day? Assume exactly 24 hours per day.

$$\text{using equation for the period of a pendulum: } T = 2\pi\sqrt{\frac{L}{g}}$$

$$\text{isolating pendulum length: } L = g\left(\frac{T}{2\pi}\right)^2$$

$$\text{substituting known values: } L = \left(9.81\ \frac{m}{s^2}\right)\left(\frac{24\ hr}{2\pi}\right)^2$$

$$\text{converting units: } L = \left(9.81\ \frac{m}{s^2}\right)\left(\frac{(24\ hr)\left(\frac{60\ min}{1\ hr}\right)\left(\frac{60\ s}{1\ min}\right)}{2\pi}\right)^2$$

$$\text{solving: } L = 1{,}855{,}669{,}494\ m = 1.86 \cdot 10^9\ m$$

11.P1.L1.10 Post-Lab Questions - University Level 1

5. The bob of a simple pendulum oscillates with an amplitude of 2×10^{-2} m and a period of 0.5 s. Calculate the speed of the bob as it passes through the equilibrium position. Take π = 3.14.
   - The following solution uses subscript “ex” to refer to the bob at either of its extreme positions, left or right, and “eq” to refer to the equilibrium position, in the middle.
   - Note that the value given for the pendulum’s period is irrelevant.

$$\text{considering energy balance of the bob: } PE_{ex} + KE_{ex} = PE_{eq} + KE_{eq}$$

$$\text{substituting formulae: } mgh_{exe} + \frac{1}{2}m\left(v_{exe}\right)^2 = mgh_{eq} + \frac{1}{2}m\left(v_{eq}\right)^2$$

$$\text{simplifying: } gh_{exe} + \frac{1}{2}\left(v_{exe}\right)^2 = gh_{eq} + {}^1/_2\left(v_{eq}\right)^2$$

$$\text{]at its extrema, the speed of the bob is zero: } gh_{exe} = gh_{eq} + \frac{1}{2}\left(v_{eq}\right)^2$$

$$\text{axes established such that height is zero at equilibrium position: } gh_{exe} = \frac{1}{2}\left(v_{eq}\right)^2$$

$$\text{isolating speed: } v_{eq} = \sqrt{2gh_{ex}}$$

$$\text{substituting known values: } v_{eq} = \sqrt{2\left(9.81\ \frac{m}{s^2}\right)\left(2 \times 10^{-2}\ m\right)}$$

$$\text{solving: } v_{eq} = 0.63\ \frac{m}{s}$$

11.P1.L1.11 Post-Lab Questions - University Level 2

6. Using the given equation for the period of a pendulum, derive an equation for this practical’s graph as

$$T^2 = sh + (T_o)^2$$

where **s** is the slope and $(\mathbf{T_o})^2$ is the vertical intercept. Use only the terms **T**, **H**, **h**, **g** and π.

$$\text{using equation for the period of a pendulum: } T = 2\pi\sqrt{\frac{L}{g}}$$

$$\text{substituting distance } h:\ T = 2\pi\sqrt{\frac{H-h}{g}}$$

$$\text{squaring both sides: } T^2 = \pm 4\pi^2\left(\frac{H-h}{g}\right)$$

$$\text{given } g < 0,\ \text{considering negative case: } T^2 = 4\pi^2\left(\frac{h-H}{g}\right)$$

$$\text{expanding: } T^2 = h\left(\frac{4\pi^2}{g}\right) - H\left(\frac{4\pi^2}{g}\right)$$

$$\text{adopting the given form, } s = \frac{4\pi^2}{g} \text{ and } (T_o)^2 = -H\left(\frac{4\pi^2}{g}\right) \text{ when } g < 0$$

7. Which physical property does $\mathbf{k}_1$ from this practical represent? Prove this algebraically and discuss whichever sign ($\pm$) its value takes.

   - $\mathbf{k}_1$ represents the acceleration of gravity:

$$\text{using representation of slope derived previously: } s = \frac{4\pi^2}{g}$$

$$\text{isolating acceleration of gravity: } g = \frac{4\pi^2}{s} = k_1, \text{ where } s < 0$$

   - Given an up-positive/down-negative convention, this property's negative value indicates that the acceleration of gravity is downward, away from the pendulum's anchor point, towards the center of the earth.

8. Which physical property does $\mathbf{k}_2$ from this practical represent? Prove this algebraically and discuss the sign ($\pm$) its value takes.

   - $\mathbf{k}_2$ represents the total vertical distance **H** from the pendulum's anchor point to the bottom datum from which the measurements are taken.

$$\text{using equivalence derived previously: } T^2 = h\left(\frac{4\pi^2}{g}\right) - H\left(\frac{4\pi^2}{g}\right)$$

$$\text{substituting slope derived previously: } h: \; T^2 = h\,(s) - H\,(s)$$

$$\text{considering bob located at datum, where } h = 0: \; {T_o}^2 = -H(s)$$

$$\text{isolating total height: } -H = \frac{{T_o}^2}{s}, \text{ where } s < 0$$

   - Given an up-positive/down-negative convention, this property takes on negative values as it measures distance downward, from the pendulum's anchor point.

11.P1.L2 Combining Parallel Components of Multiple Vectors

11.P1.L2.1 Introduction

1. As shown in Figure 1, a string can be used to connect two springs if they're supported at points **A** and **B**.
   - If no load hangs at the center **C**, the spring/string combination on either side has an initial length of $\mathbf{L_o}$.

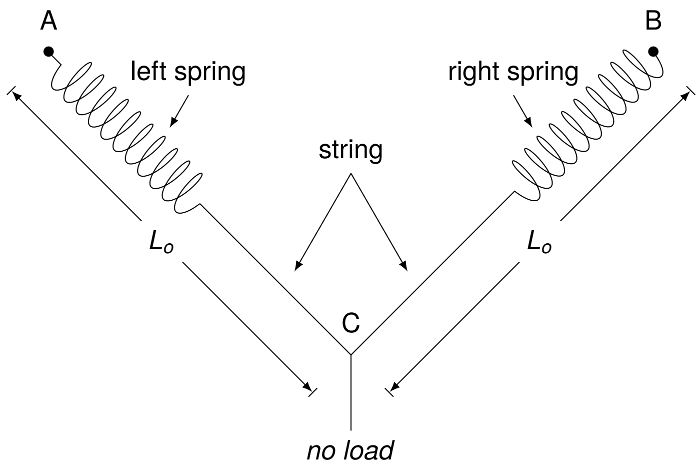


Figure 1

2. As a load of mass **m** is applied, the springs extend.
   - This causes θ, the vertical-string angle on either side, to decrease.

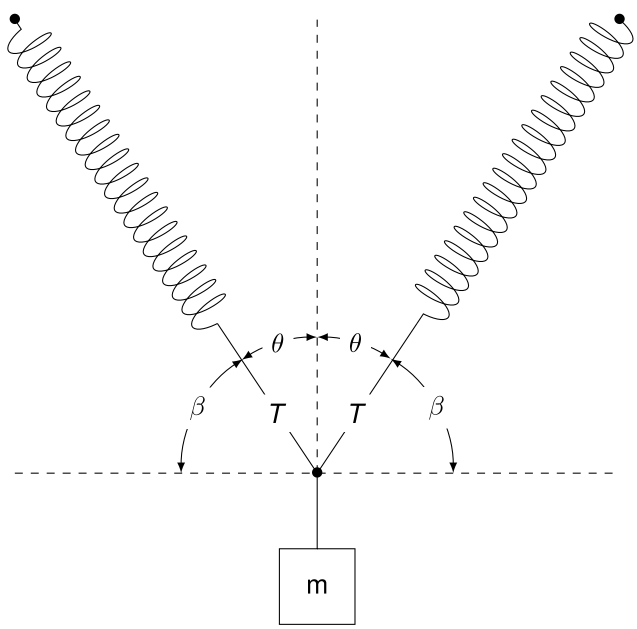


Figure 2

3. The vertical component of the tension **T** in either spring is then

$$T_y = T\cos(\theta) \qquad \text{(Equation 1)}$$

4. As a vector's angle is often measured from the horizontal, Equation 1 can be modified as

$$T_y = T\cos(90^\circ - \beta) = T\sin(\beta) \qquad \text{(Equation 2)}$$

5. The tension in the spring can be expressed as a function of the mass applied and the angle θ as

$$\text{summing all vertical vectors: } F_{\text{weight}} = 2T_y$$

$$\text{substituting equation for weight: } mg = 2T_y$$

$$\text{substituting equation for vertical component: } mg = 2\,(T\cos(\theta))$$

$$\text{isolating tension: } T = \frac{mg}{2\cos(\theta)} \qquad \text{(Equation 3)}$$

6. As shown in Figure 3, one can consider **d**, the horizontal distance between the supports.
   - It is assumed that the load mass **m** is hung at the setup's center, at $^{d}/_{2}$ from either support.

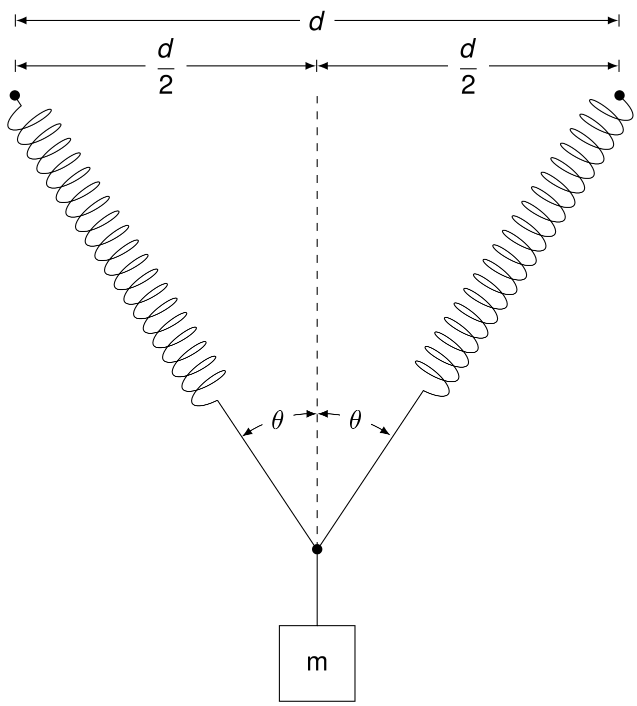


Figure 3

   - The angle θ can be related to the load's mass **m** using **d** and **k**, the Hooke's constant of either spring, and **g**, the acceleration of gravity as

$$\frac{1}{\sin(\theta)} = \left(\frac{m}{\cos(\theta)}\right)\left(\frac{g}{dk}\right) + \frac{2L_o}{d} \qquad \text{(Equation 4)}$$

   - This practical is based on Equation 4, which is derived in question 11 of the post lab material.

11.P1.L2.2 Apparatus and Materials

- 1 knife
- 1 protractor
- 1 set of incremental masses (totaling at least 400 *g*) **OR**
  - 1 syringe **AND** about 500 *mL* of water **AND** a container (with handle) for the water
- 1 hammer and 3 nails **OR**
  - any set of 3 sturdy supports (3 burette clamps and 3 retort stands, etc)
- 2 springs, each with about 25 coils (See section A.1 for construction)
- 200 *cm* of strong, thin string

11.P1.L2.3 Setup

1. Use a knife to cut two holes in a protractor as shown in Figure 4.
   - Cut one hole at the center.
   - Cut another hole near the left-most 180° mark.

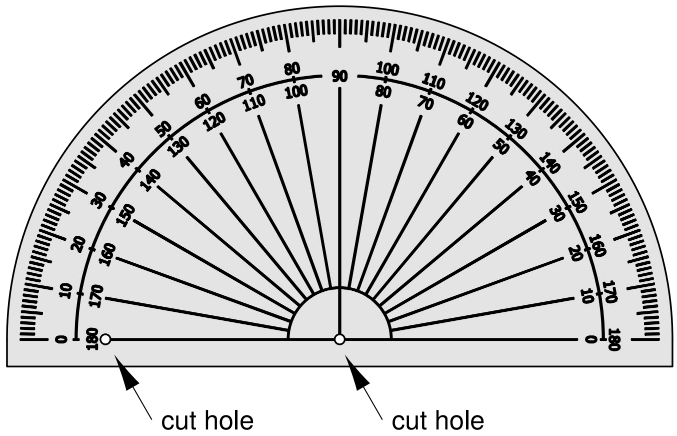


Figure 4

2. Create three support points, each spaced 30 *cm* apart.
   - These can be nails in the side of a table, burette clamps, etc.

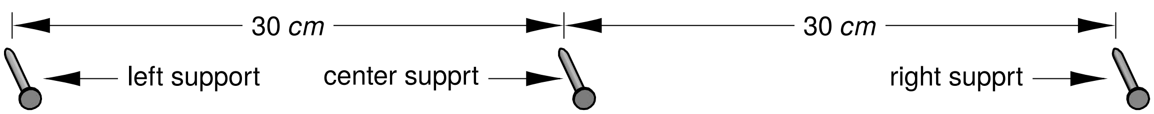


Figure 5

3. Cut a piece of string about 60 *cm* long.
4. Tie a loop into each end the string.
5. Attach one end of the loop to a spring.
6. Hang this spring from the support on the left.

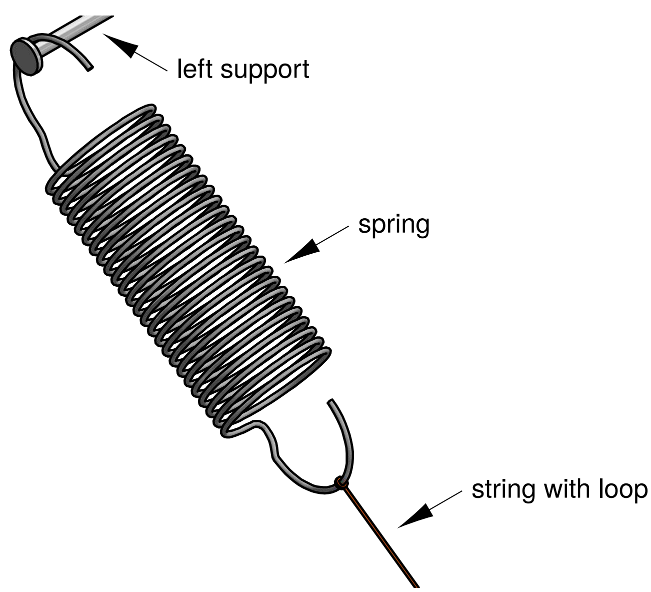


Figure 6

7. Pull this string through the protractor's 180° hole.
    - Be sure to draw the string through from the back of the protractor, as shown in Figure 7.

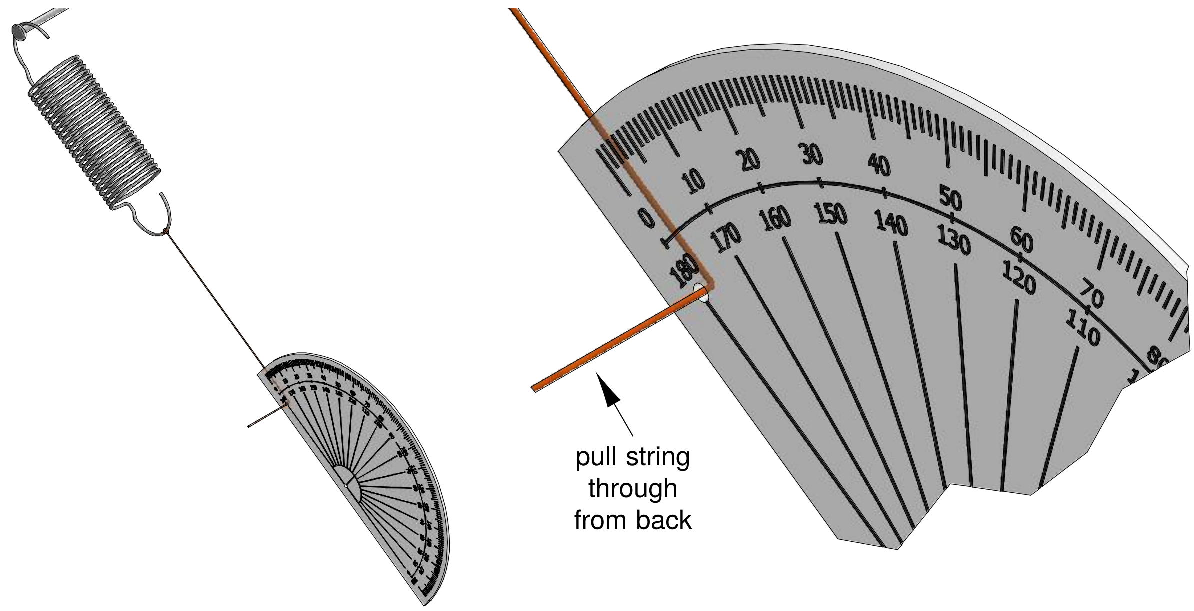


Figure 7

8. Pull the string back through the protractor's center hole.
    - Be sure to pull the string through from the front towards the back of the protractor, as shown in Figure 8.

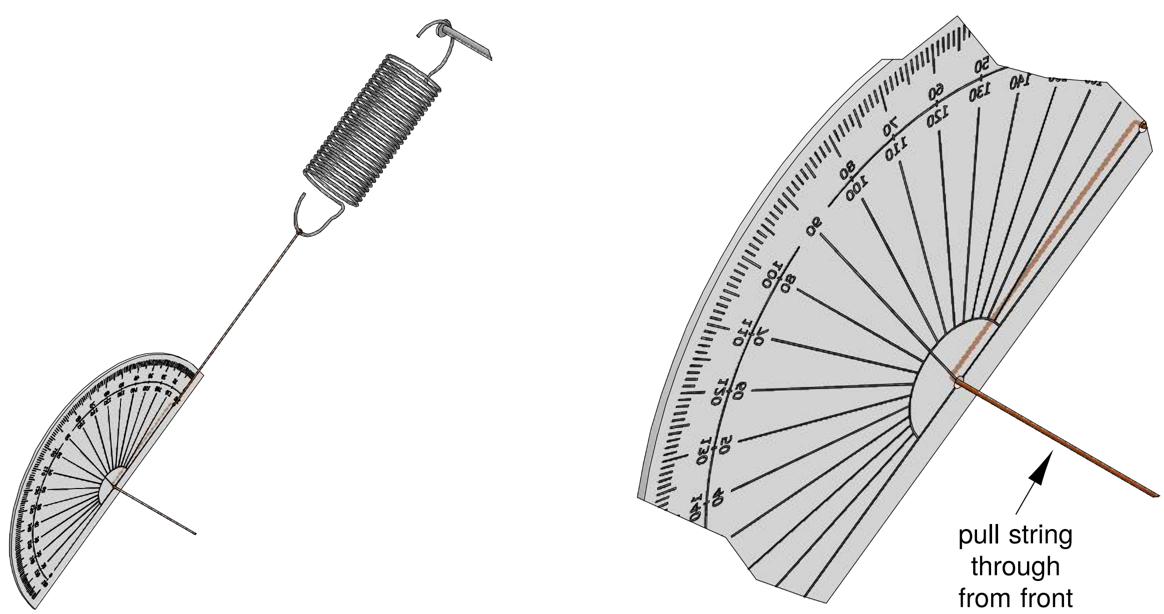


Figure 8

9. Cut a second piece of string about 30 *cm* long.
10. Tie a loop into each and of the string.

11. Pull the first string through a loop of the second, as shown in Figure 9.

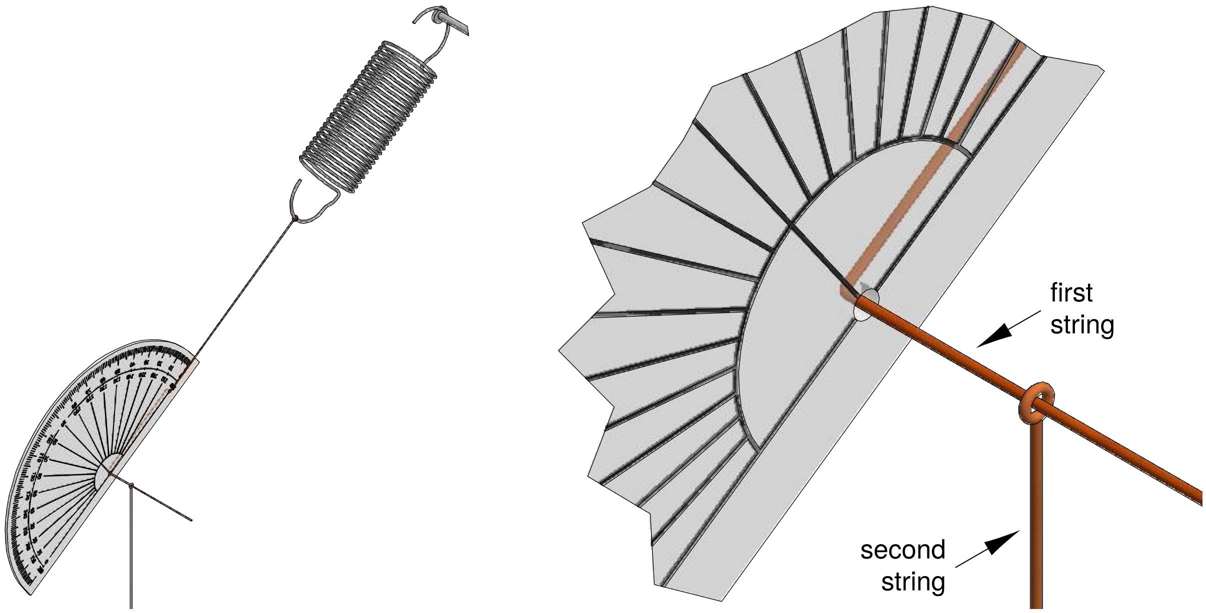


Figure 9

12. Attach the second loop of the first string to another spring.
13. Hang this spring from the right support.
    - Be sure this string passes behind the protractor towards the right support.
14. Suspend a small container from the remaining loop of the second string.
    - This container can be an empty water bottle, small bucket, etc.

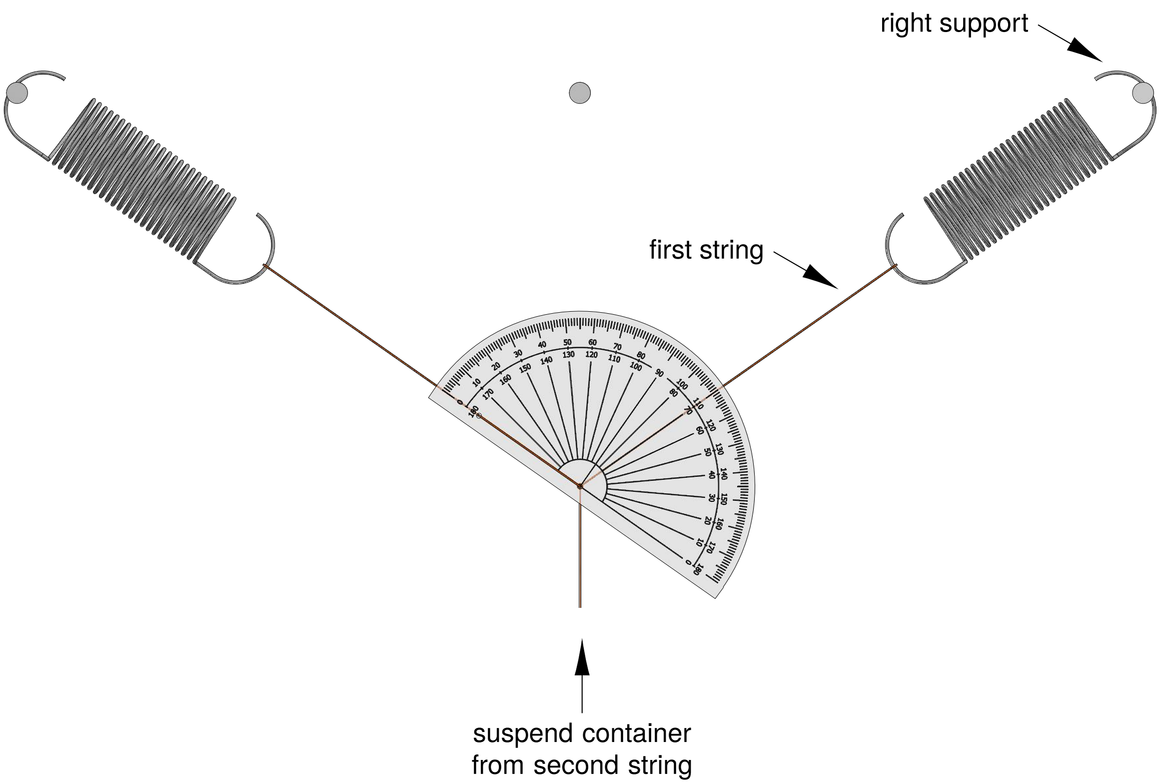


Figure 10

15. Cut a third piece of string about 100 *cm* long.
16. Tie a loop into each end of the string.
17. Tie one loop of the third string to a heavy object such as a bottle of water.
18. Use the third string's remaining loop to hang the heavy object from the center support, as shown in Figure 11.

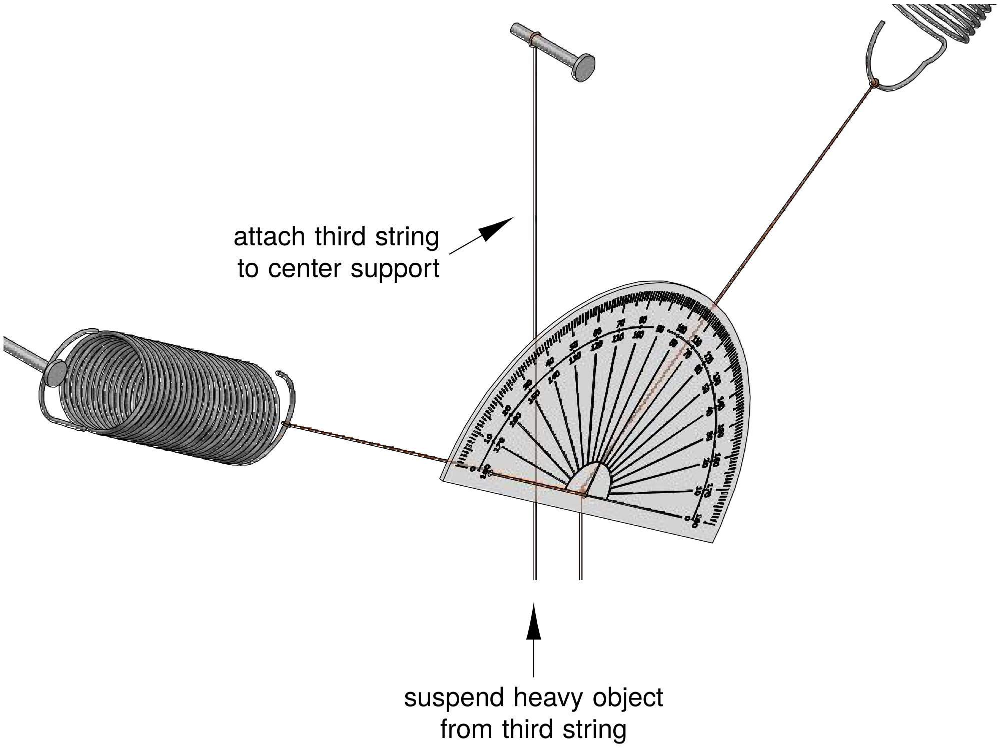


Figure 11

19. Allow the heavy object to hang behind the initial assembly.
20. Observe the protractor's center at a 90° angle, as shown in Figure 12.
21. Shift the protractor along the first string until the strings overlap from this view.

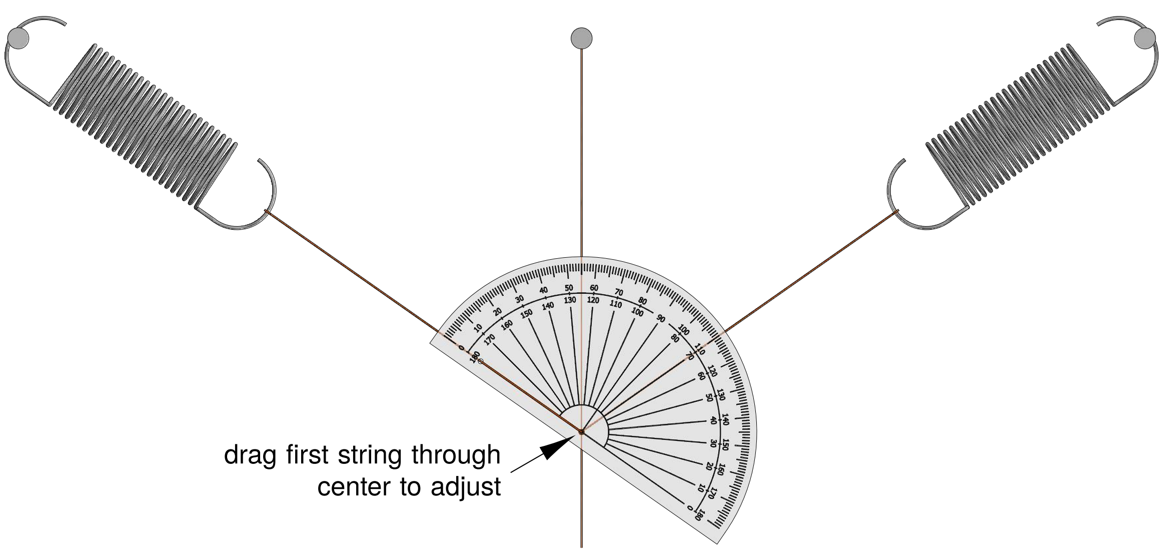


Figure 12

22. Remove the heavy object and third string after centering the protractor.

11.P1.L2.4 Warm Up Questions

1. Which unit is frequently used to measure angles?
   - The degree (°). *Radians is also a correct, but not relevant in this practical.*
2. If θ is an angle, do the values of $\sin(\theta)$ or $\cos(\theta)$ have any units?
   - No, the sine and cosine of any angle are unitless / dimensionless. *(They're scalars.)*
3. Consider a string attached to two horizontal points, **A** and **B**. An object is hanging from the middle of the string, at a horizontal distance of $^{AB}/_2$ from either point. How is the object's load distributed to each point?
   - The load is distributed evenly to each point.

11.P1.L2.5 Procedure and Calculations

- Students should collect data similar to Table 1 using the steps below.

m (**g**)	**m** (**kg**)	α (°)	θ (°)	$\cos(\theta)$	$\frac{\mathbf{m}}{\cos(\theta)}$ (**kg**)	$\sin(\theta)$	$\frac{1}{\sin(\theta)}$
100	0.10	101	50.5	0.636	0.157	0.772	1.296
150	0.15	97	48.5	0.663	0.226	0.749	1.335
200	0.20	95	47.5	0.676	0.296	0.737	1.356
250	0.25	92	46.0	0.695	0.360	0.719	1.390
400	0.40	86	43.0	0.731	0.547	0.682	1.466

Table 1

A) Create an empty table of 8 columns and 6 rows.

B) **Row 1, Header**: Fill in the header information as shown.

C) **Column 1, m** (**g**): Fill in the list of different masses to be loaded.

D) **Column 2, m** (**kg**): Convert each of these masses to kilograms using

$$m_{kg} = m_g \left(\frac{1\ kg}{1000\ g} \right) \qquad \text{(Equation 5)}$$

E) Load the first mass.

F) **Column 3,** α (°): Record the angle indicated by the string's alignment behind the protractor.
   - For example, the angle indicated in Figure 13 is α = 97°

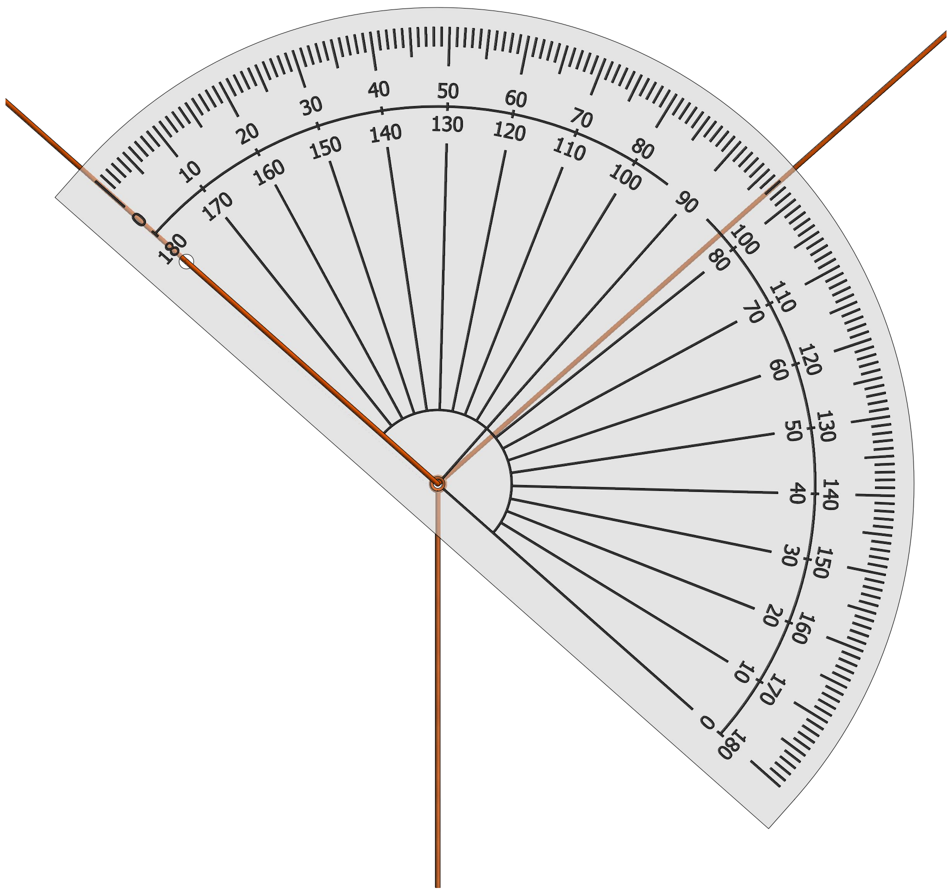


Figure 13

G) **Column 4,** θ (°): Divide this angle in half using

$$\theta = \frac{\alpha}{2} \qquad \text{(Equation 6)}$$

H) **Column 5,** $\cos(\theta)$: Calculate the cosine of this half-angle.

I) **Column 6,** $\frac{\mathbf{m}}{\cos(\theta)}$ (**kg**): Calculate the quotient of the mass (in kilograms) and the cosine of this half angle.

J) **Column 7,** $\sin(\theta)$: Calculate the sine of this half-angle.

K) **Column 8,** $\frac{1}{\sin(\theta)}$: Calculate the inverse of the sine of this half-angle.

L) Repeat steps E) through K) for all remaining masses.

11.P1.L2.6 Data Plotting and Slope/Intercept Determination

1. Students should plot the last column of Table 1 against its 6^{th}, similar to Figure 14 below.

   *Note to Teacher: Consider **not** starting the vertical axis at zero.*
   - *Rather, set the min and max values only to whichever increments of 0.1 allow all data to be plotted.*

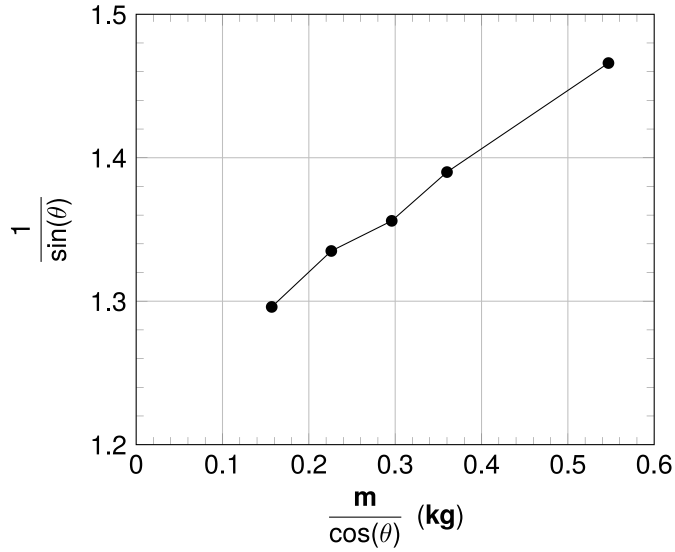


Figure 14

2. Students should then determine the graph's **slope** using

$$\text{general equation for the slope of a straight line: } s = \frac{y_f - y_i}{x_f - x_i}$$

$$\text{specifying for this practical: } s = \frac{\left(\frac{1}{\sin(\theta)}\right)_f - \left(\frac{1}{\sin(\theta)}\right)_i}{\left(\frac{m}{\cos(\theta)}\right)_f - \left(\frac{m}{\cos(\theta)}\right)_i}$$

$$\text{substituting known values: } s = \frac{(1.466) - (1.296)}{(0.547\ kg) - (0.157\ kg)}$$

$$\text{solving: } s = 0.4359\ \frac{1}{kg}$$

3. Students should then determine the graph's **vertical intercept** using

$$\text{general equation for the vertical intercept of a straight line: } y_o = y_i - s(x_i)$$

$$\text{specifying for this practical: } \left(\frac{1}{\sin(\theta)}\right)_o = \left(\frac{1}{\sin(\theta)}\right)_i - s\left(\frac{m}{\cos(\theta)}\right)_i$$

$$\text{substituting known values: } \left(\frac{1}{\sin(\theta)}\right)_o = (1.296) - \left(0.4359\ \frac{1}{kg}\right)(0.157\ kg)$$

$$\text{solving: } \left(\frac{1}{\sin(\theta)}\right)_o = 1.228$$

4. Students should then calculate γ where

$$\gamma = \frac{g}{sd} \quad \text{(Equation 7)}$$

and
   - **s** is the graph's slope;
   - **g** is the acceleration of gravity (assume $g = 9.81\ ^{m}/_{s^2}$);
   - **d** is the distance between the left and right support.

as follows

$$\text{given equation: } \gamma = \frac{g}{sd}$$

$$\text{subtituting known values: } \gamma = \frac{\left(9.81\ \frac{m}{s^2}\right)}{\left(0.4359\ \frac{1}{kg}\right)(0.6\ m)}$$

$$\text{solving: } \gamma = 37.509\ \frac{kg}{s^2}$$

5. Finally, students should derive the most simplified expression for the units of **k**, a Hookean material's spring constant, entirely in base units.

$$\text{considering units of a spring constant: } [k] = \frac{N}{m}$$

$$\text{substituting base units: } [k] = \frac{kg\frac{m}{s^2}}{m}$$

$$\text{simplifying: } [k] = \frac{kg}{s^2}$$

11.P1.L2.7 Post-Lab Questions - High School

Figure 15 shows a string under tension **T**, with an angle θ against the vertical.

1. Find the ratio of $\frac{T_y}{T}$ when $\theta = 30°$.

$$\text{applying cosine: } \frac{T_y}{T} = \cos(\theta)$$

$$\text{substituting known value: } \frac{T_y}{T} = \cos(30°)$$

$$\text{solving: } \frac{T_y}{T} = 0.866$$

2. For which value of θ is the vertical component $\mathbf{T_y}$ equal to half of **T**? (assume $0° < \theta < 90°$)

$$\text{applying cosine to angle: } \cos(\theta) = \frac{T_y}{T}$$

$$\text{isolating angle: } \theta = \cos^{-1}\left(\frac{T_y}{T}\right)$$

$$\text{substituting known value: } \theta = \cos^{-1}\left(\frac{1}{2}\right)$$

$$\text{solving: } \theta = 60°$$

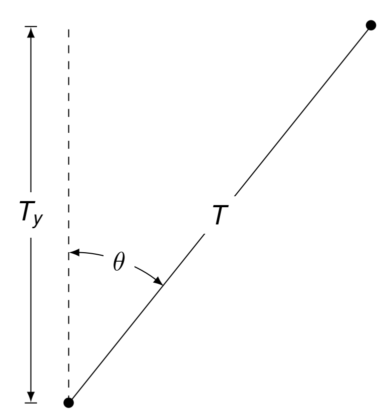


Figure 15

11.P1.L2.8 Post-Lab Questions - University Level 1

Figure 16 shows two springs attached to two strings. Both spring/string combos are symmetrically angled by θ against the vertical. The angle between each is α. They both support the load $\mathbf{F_W}$ equally with their respective tensions $\mathbf{T_L}$ and $\mathbf{T_R}$.

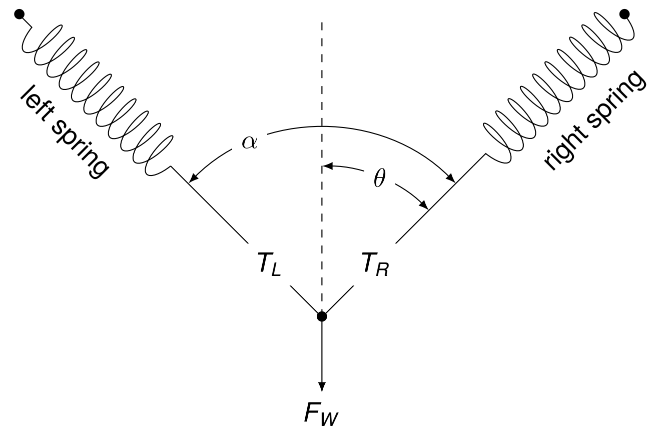


Figure 16

3. What is the relationship between θ, the angle of either string against the vertical and α, the inter-string angle?

$$\alpha = \frac{\theta}{2}$$

4. Consider $\mathbf{T_{L}}_y$ and $\mathbf{T_{R}}_y$, the vertical components of the tensions within the left and right spring/string combos, respectively. What is the relationship between these components and $\mathbf{F_W}$?

$$T_{Ly} = T_{Ry} = \frac{F_W}{2}$$

5. Using the relationship developed in questions 3 and 4, derive an expression for $\mathbf{T_L}$ in terms of θ and $\mathbf{F_W}$

$$\text{applying cosine: } \cos(\theta) = \frac{T_{Ly}}{T_L}$$

$$\text{substituting equivalence derived previously: } \cos(\theta) = \frac{\left(\frac{F_w}{2}\right)}{T_L}$$

$$\text{isolating left tension: } T_L = \frac{\left(\frac{F_W}{2}\right)}{\cos(\theta)}$$

$$\text{simplifying: } T_L = \frac{F_W}{2\cos(\theta)}$$

6. Using the expression developed in question 5, what is the relationship between $\mathbf{T_L}$, $\mathbf{T_R}$ and $\mathbf{T}$, where $\mathbf{T}$ is the general term for the tension of either spring.

$$T_L = T_R = T = \frac{F_W}{2\cos(\theta)}$$

7. Modify the expression developed in question 6, express $\mathbf{T}$ in terms of θ as well as $\mathbf{m}$, the mass of the weight load and $\mathbf{g}$, the acceleration of gravity.

$$T = \frac{mg}{2\cos(\theta)}$$

11.P1.L2.9 Post-Lab Questions - University Level 2

Figure 17 shows the same setup as Figure 16 with the following additions and modifications.

- **d** - the horizontal distance between the supports;
- **L** - the length of either spring/string combination;
- **T** - the tension in either spring/string combination.

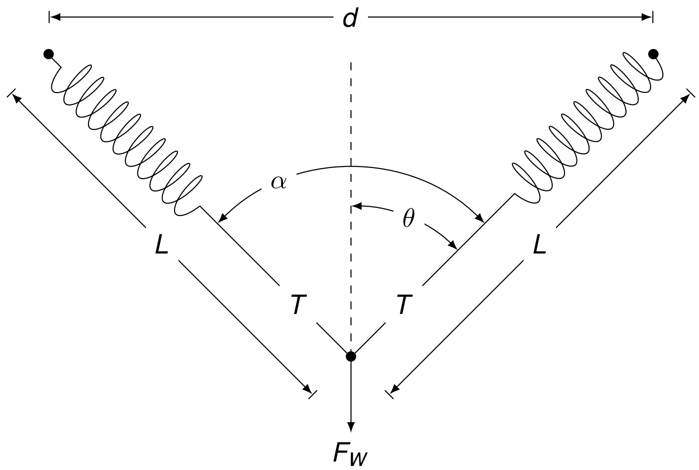


Figure 17

8. Express the tension **T** of either side in terms of **L** as well as **k**, the spring's constant and $\mathbf{L_o}$, the initial length of either spring/string under no added load.

$$\text{considering Hooke's law: } F = k\Delta x$$

$$\text{specifying for this practical: } T = k\Delta L$$

$$\text{considering initial and variable lengths: } T = k\,(L - L_o)$$

9. Use the expressions developed in questions 7 and 8 to express the extension $\mathbf{L} - \mathbf{L_o}$ in terms of **m**, **g**, **k** and θ.

$$\text{setting both previously derived equivalences equal: } T = \frac{mg}{2\cos(\theta)} = k\,(L - L_o)$$

$$\text{isolating extension: } L - L_o = \frac{mg}{2k\cos(\theta)}$$

10. Apply sine about angle θ to derive an expression for **L** in terms of **d** and θ.

$$\text{applying sine: } \sin(\theta) = \frac{\left(\frac{d}{2}\right)}{L}$$

$$\text{isolating length: } L = \frac{\left(\frac{d}{2}\right)}{\sin(\theta)}$$

$$\text{simplifying: } L = \frac{d}{2\sin(\theta)}$$

11. Use the expressions developed in questions 9 and 10 to derive a linear equation for this practical's graph as

$$\frac{1}{\sin(\theta)} = s\left(\frac{m}{\cos(\theta)}\right) + c$$

where **s** is the slope and **c** is the vertical intercept. Use only the terms

- θ - the angle of either spring/string combination against the vertical;
- **m** - the mass hung from the setup;
- **g** - the acceleration of gravity;
- **d** - the horizontal distance between the left and right supports;
- **k** - the spring constant of either spring;
- $\mathbf{L_o}$, the initial length of either spring/string combination.

Solution

$$\text{using equivalence derived previously: } L - L_o = \frac{mg}{2k\cos(\theta)}$$

$$\text{substituting previously derived equation for length: } \frac{d}{2\sin(\theta)} - L_o = \frac{mg}{2k\cos(\theta)}$$

$$\text{isolating sine term: } \frac{d}{2\sin(\theta)} = \frac{mg}{2k\cos(\theta)} + L_o$$

$$\text{isolating inverse of sine: } \frac{1}{\sin(\theta)} = \left(\frac{2}{d}\right)\left(\frac{mg}{2k\cos(\theta)} + L_o\right)$$

$$\text{distributing: } \frac{1}{\sin(\theta)} = \frac{mg}{dk\cos(\theta)} + \frac{2L_o}{d}$$

$$\text{rearranging: } \frac{1}{\sin(\theta)} = \left(\frac{m}{\cos(\theta)}\right)\left(\frac{g}{dk}\right) + \frac{2L_o}{d}$$

$$\text{adopting the given form: } s = \frac{g}{dk} \text{ and } c = \frac{2L_o}{d}$$

12. Considering the expression derived in question 11, which quotient can be calculated from the slope term **s** assuming known values of **g** and **d**? Prove this algebraically.

- **s** can be used to calculate **k**, the spring constant of either side.

$$\text{using equivalence derived previously: } s = \frac{g}{dk}$$

$$\text{isolating spring constant: } k = \frac{g}{sd}$$

13. Considering the expression derived in question 11, which quotient can be calculated from the intercept term **c** assuming a known value of **d**? Prove this algebraically.

- **c** can be used to calculate $\mathbf{L_o}$, the initial length of either spring/string combination.

$$\text{using equivalence derived previously: } c = \frac{2L_o}{d}$$

$$\text{isolating initial length: } L_o = \frac{cd}{2}$$

Period 1 Optics: Refraction and Dispersion

Period Contents

12.P1.L1 Refraction in Rectangular Glass Prisms*

12.P1.L1.1 Introduction

1. The angles of incidence and refraction within a rectangular prism are related to the interior and exterior indices of refraction as

$$(n_{exterior}) \sin(i) = (n_{interior}) \sin(r) \qquad \text{(Equation 1)}$$

2. Assuming the outside material is air and the inside material is glass, this equation can be modified as

$$(n_{air}) \sin(i) = \left(n_{glass}\right) \sin(r) \qquad \text{(Equation 2)}$$

3. Assuming the refractive index of air can be approximated as that of a vacuum, where $n_{vacuum} = 1$, this equation can be simplified as

$$\frac{\sin(i)}{\sin(r)} = n \qquad \text{(Equation 3)}$$

Where

- **i** is the angle of incidence;
- **r** is the angle of refraction;
- **n** is the refractive index of the material, in this case glass, and where $\mathbf{n_{air}}$ is assumed to be 1.

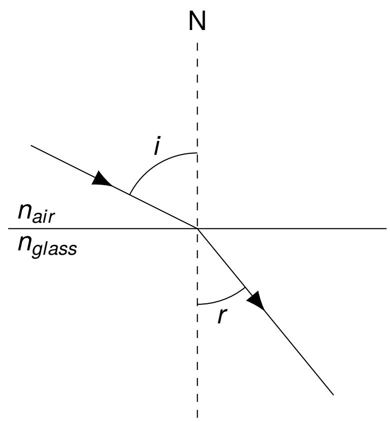


Figure 1

12.P1.L1.2 Apparatus and Materials

- 1 blank sheet of paper
- 2 pieces of carton, each big enough for the paper sheet to be attached
- 1 protractor
- 1 ruler (at least 15 *cm*)
- 1 rectangular glass prism (block)
- 4 pins (either textile pins, syringe needles, etc)
- 1 role of tape (either clear/white or plaster)

12.P1.L1.3 Setup

1. Use tape to attach a sheet of blank paper to two equally-sized pieces of carton.
2. Place the glass rectangular prism large-face-down around the center of the paper.
   - Be sure that the prism is placed such that its longest edge is horizontal.
3. Trace the outline of the prism on the paper and remove the block.
4. Create a small dot along the top edge of the rectangle outline, 1 *cm* from the left edge.
5. Place the label **"O"** just below and left of this dot.

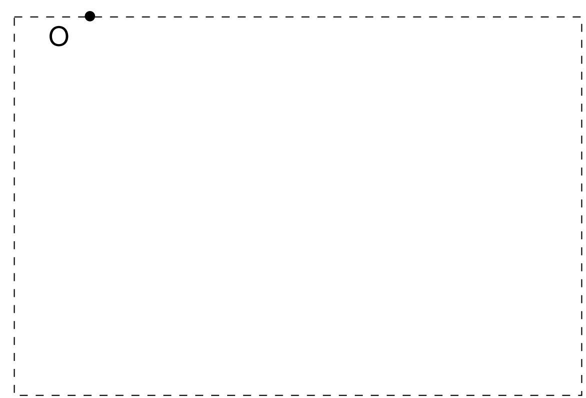


Figure 2

6. Use a protractor centered at **O** to draw six lines angled 15°, 30°, 45°, 60°, 75° and 90° from the long edge.
   - Make the line last line at 90° dashed, and place an **N** above it.

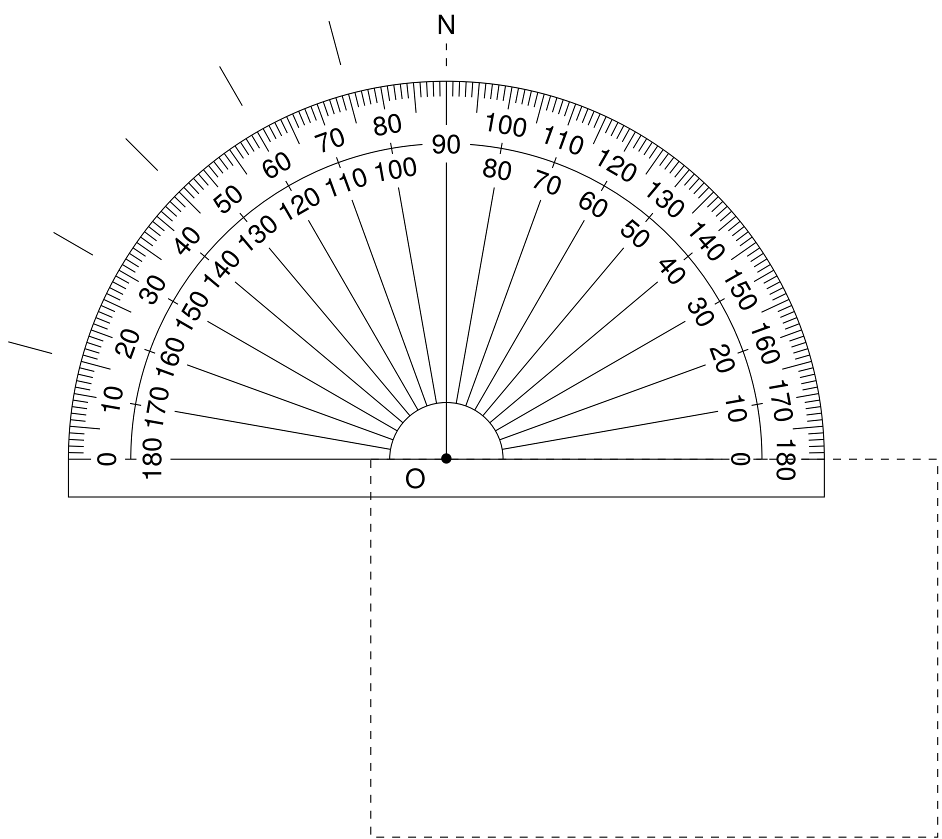


Figure 3

7. Remove the protractor.
8. Use a ruler to trace the lines inward so that they all intersect at **O**.
9. Place two dots on each of these lines and label them leftward form **N** as $\mathbf{P}_1$, $\mathbf{Q}_1$ through $\mathbf{P}_5$, $\mathbf{Q}_5$.

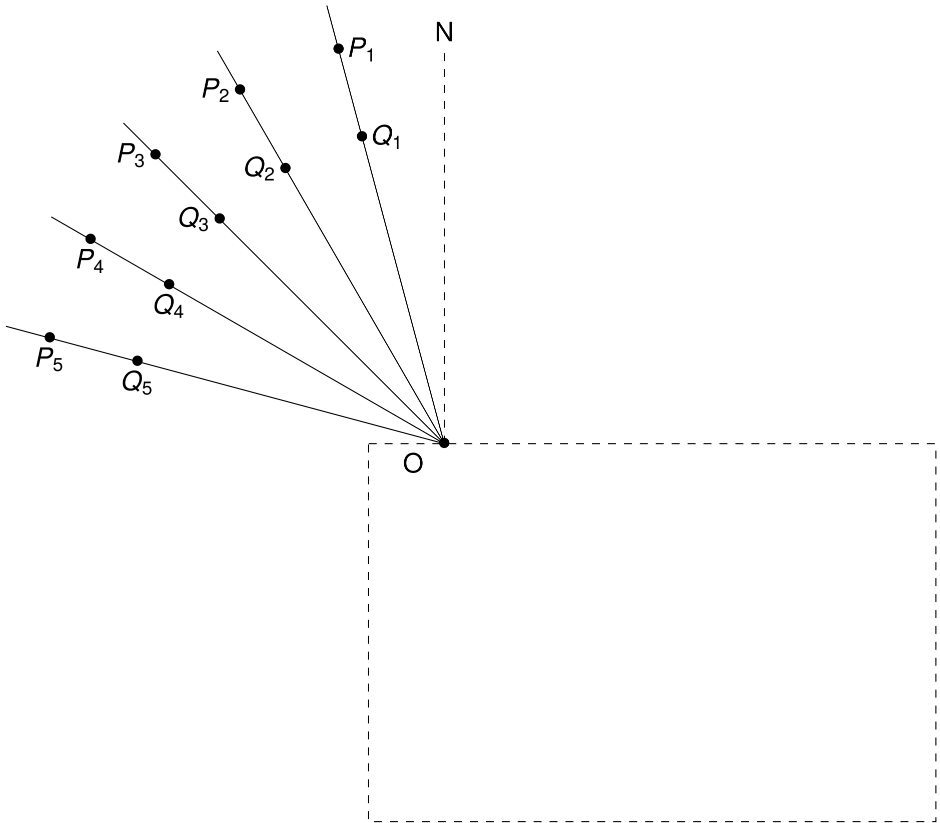


Figure 4

12.P1.L1.4 Warm Up Questions

1. When entering a material of increased refractive index, does light move faster or slower?
   - It moves slower.
2. If an object's refractive index concerns the speed of light, why can't we simply measure a photon's speed with a meter rule and timing device?
   - The speed of light is far too high to be measured with such devices.
3. Are reflection and refraction synonyms? If not, what is the difference between these two phenomena?
   - They are not. Reflection occurs when light returns from a surface at the same angle to the normal at which it approached. Refraction occurs when light enters a new object at a different angle to the normal at which it approached.
4. As light enters an object of higher refractive index, does its bend towards or away from the normal line?
   - It bends towards the normal line.
5. If we wish to draw a straight line, what is the minimum number of points needed to do so?
   - Two.

12.P1.L1.5 Procedure and Calculations

- Students should collect data similar to Table 1 using the steps below.

Case	**i** (°)	$\sin^2$ (**i**)	**L** (**cm**)	$\mathbf{L}^{-2}$ $(\mathbf{cm}^{-2})$
1	15	0.067	5.2	0.037
2	30	0.250	5.4	0.034
3	45	0.500	5.8	0.030
4	60	0.750	6.2	0.026
5	75	0.933	6.7	0.022

Table 1

A) Create an empty table of 5 columns and 6 rows.
B) **Row 1, Header**: Fill in the header information as shown.
C) **Column 1, Case**: Fill in the case numbers 1 through 5 as shown.
D) **Column 2, i** (°): Measure the angle **i** between line **ON** and each of the lines $\mathbf{P}_1\mathbf{Q}_1$ through $\mathbf{P}_2\mathbf{Q}_2$.
E) **Column 3,** $\sin^2$(**i**): Calculate the square of the sine of each angle of incidence **i**.
F) Place one pin at point $\mathbf{P}_1$ and another at point $\mathbf{Q}_1$.
G) Place the glass block within the outline on the paper.
H) Look into the bottom face of the prism.

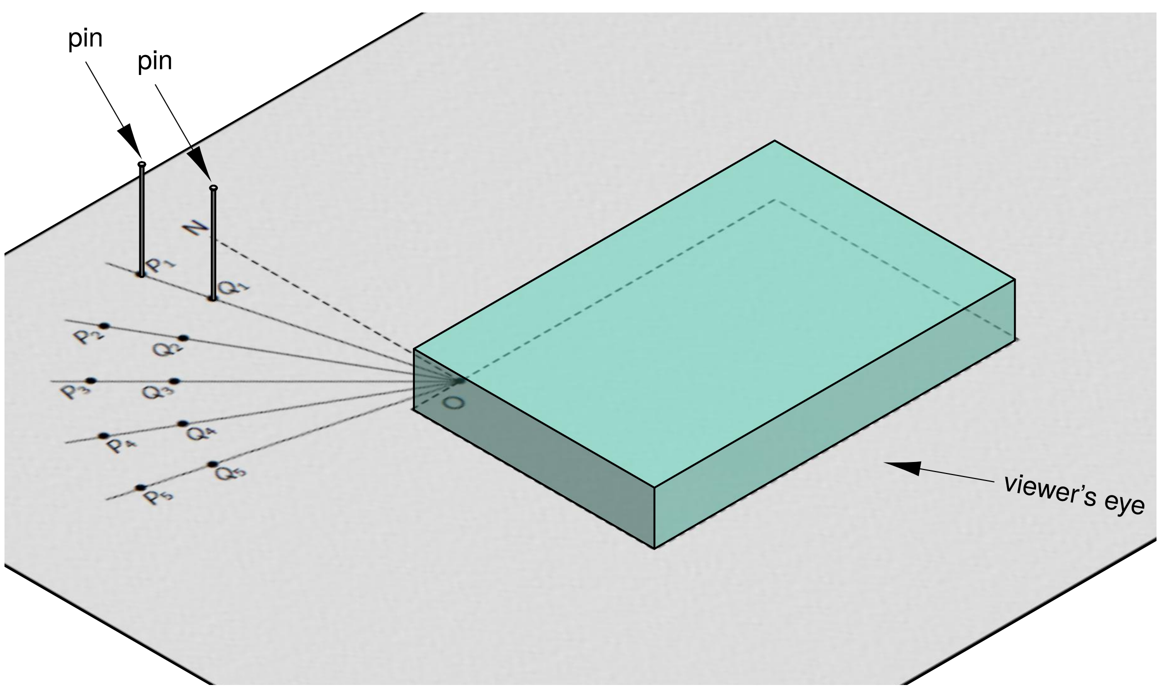


Figure 5

I) Shift your head left or right until the pins at P_1 and Q_1 overlap when viewed through the prism.
   - Figure 6 shows an incorrect viewing position because the pins are not overlapping.

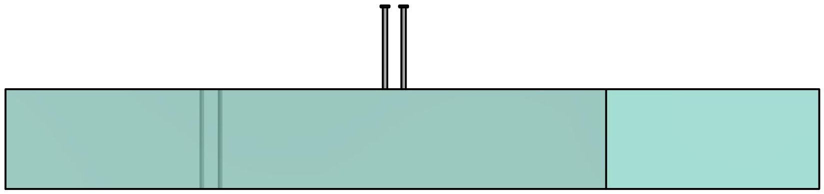


Figure 6

   - Figure 7 shows a correct viewing position because the pins overlap.

Figure 7

J) Once a correct viewing position is found, place a pin in front of the bottom face to block the appearance of the pins in the prism.

Figure 8

K) Keep this angle of vision, and place another pin along the bottom face.

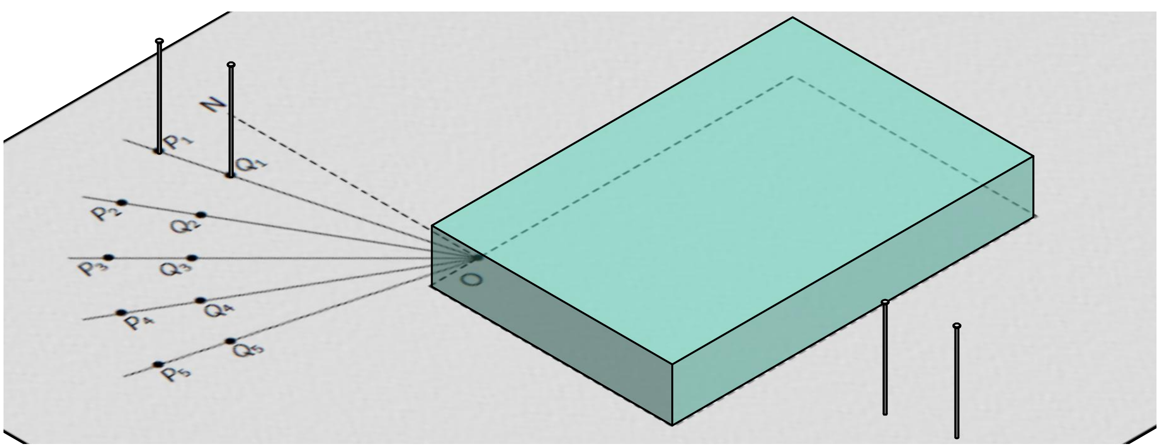


Figure 9

L) Remove all four pins, as well as the prism.

M) Place dots over the holes left by each of the two bottom pins, and label each $\mathbf{R}_1$ and $\mathbf{S}_1$.

N) Draw a line connecting points $\mathbf{R}_1$ $\mathbf{R}_2$ with the bottom edge of the outline.

O) Draw a line connecting **O** to the point where line $\mathbf{S}_1\mathbf{R}_1$ touches the bottom edge.

P) Label this line as $\mathbf{L}_1$.

Q) **Column 4, L (cm)**: Use the rule to measure the length of line L_1 in cm.

R) **Column 5, $\mathbf{L}^{-2}$ $(\mathbf{cm}^{-2})$**: Calculate the inverse of the square of the length of this line using

$$L^{-2} = \left(\frac{1}{L}\right)^2 \qquad \text{(Equation 4)}$$

S) Repeat steps F) through R) for points $\mathbf{P}_2$, $\mathbf{Q}_2$ through $\mathbf{P}_5$, $\mathbf{Q}_5$.

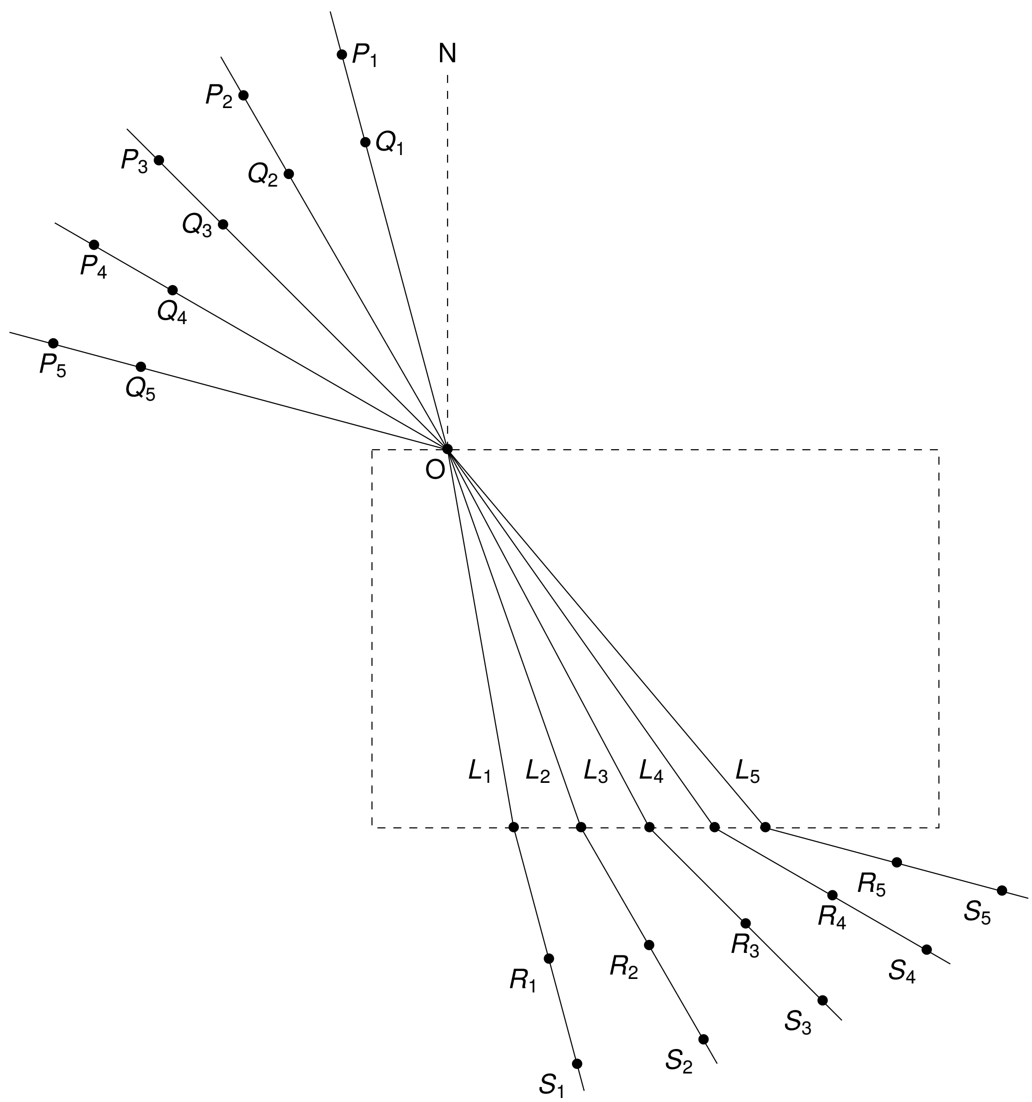


Figure 10

12.P1.L1.6 Data Plotting and Slope/Intercept Determination

1. Students should plot the last column of Table 1 against its 3^{rd}, similar to Figure 11 below.

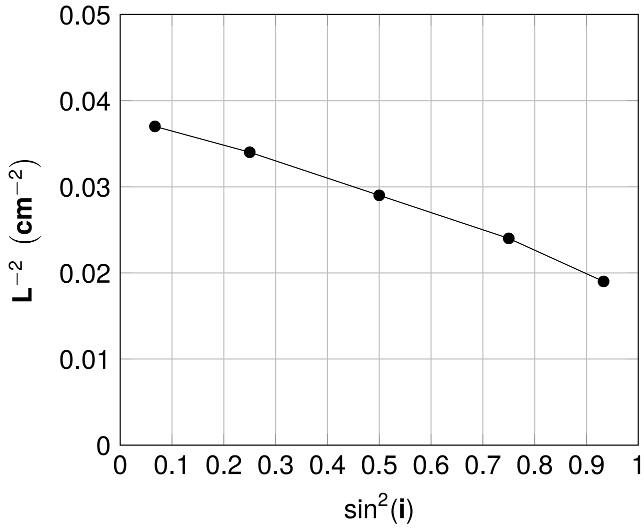


Figure 11

2. Students should then determine the graph's **slope** using

$$\text{general equation for the slope of a straight line: } s = \frac{y_f - y_i}{x_f - x_i}$$

$$\text{specifying for this practical: } s = \frac{\left(L^{-2}\right)_f - \left(L^{-2}\right)_i}{\left(\sin^2(i)\right)_f - \left(\sin^2(i)\right)_i}$$

$$\text{substituting known values: } s = \frac{\left(0.022\ cm^{-2}\right) - \left(0.037\ cm^{-2}\right)}{(0.933) - (0.067)}$$

$$\text{solving: } s = -0.0173\ cm^{-2}$$

3. Students should then determine the graph's **vertical intercept** using

   general equation for the vertical intercept of a straight line: $y_o = y_i - s(x_i)$

$$\text{specifying for this practical: } \left(L^{-2}\right)_o = \left(L^{-2}\right)_i - s\left(\sin^2(i)\right)_i$$

$$\text{substituting known values: } \left(L^{-2}\right)_o = \left(0.037\ cm^{-2}\right) - \left(-0.0173\ cm^{-2}\right)(0.067)$$

$$\text{solving: } \left(L^{-2}\right)_o = 0.038\ cm^{-2}$$

4. Finally, students should determine **k**, the quotient between the graph's **vertical intercept** and its **slope** using

$$\text{given quotient: } k = \frac{\left(L^{-2}\right)_o}{s}$$

$$\text{substituting known values: } k = \frac{0.038\ cm^{-2}}{-0.0173\ cm^{-2}}$$

$$\text{solving: } -2.197$$

12.P1.L1.7 Exam Prompt

Figure 12 illustrates the outline **ABCD** of a rectangular glass prism traced on a drawing paper. **P** and **Q** are points on a ray of light incident at point **O** on face **AB** of the prism and forming an angle **i** with the normal at **O**. The emergent ray is traced through face **CD** of the prism and located by points **R** and **S**. Points **R** and **S** are joined and produced to meet **CD** at **X**. The path of the ray in the prism is indicated by the line **OX=L**, creating an angle of refraction **r** with the normal at **O**.

The procedure is repeated for various angles of incidence **i**. **Each** time, the path **OX=L** of the ray is measured.

Figure 13 shows the positions of points **P, Q, R** and **S** for **five** different angles of incidence of the ray of light.

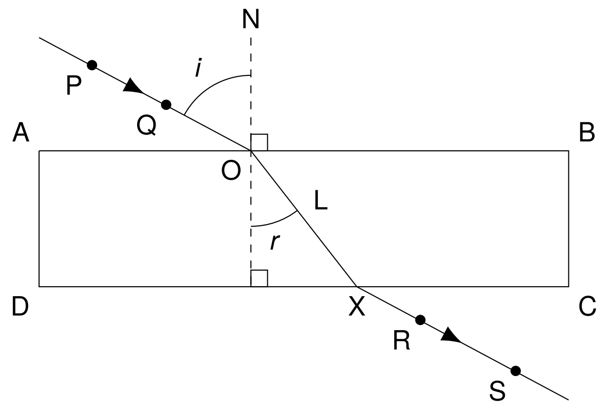


Figure 12

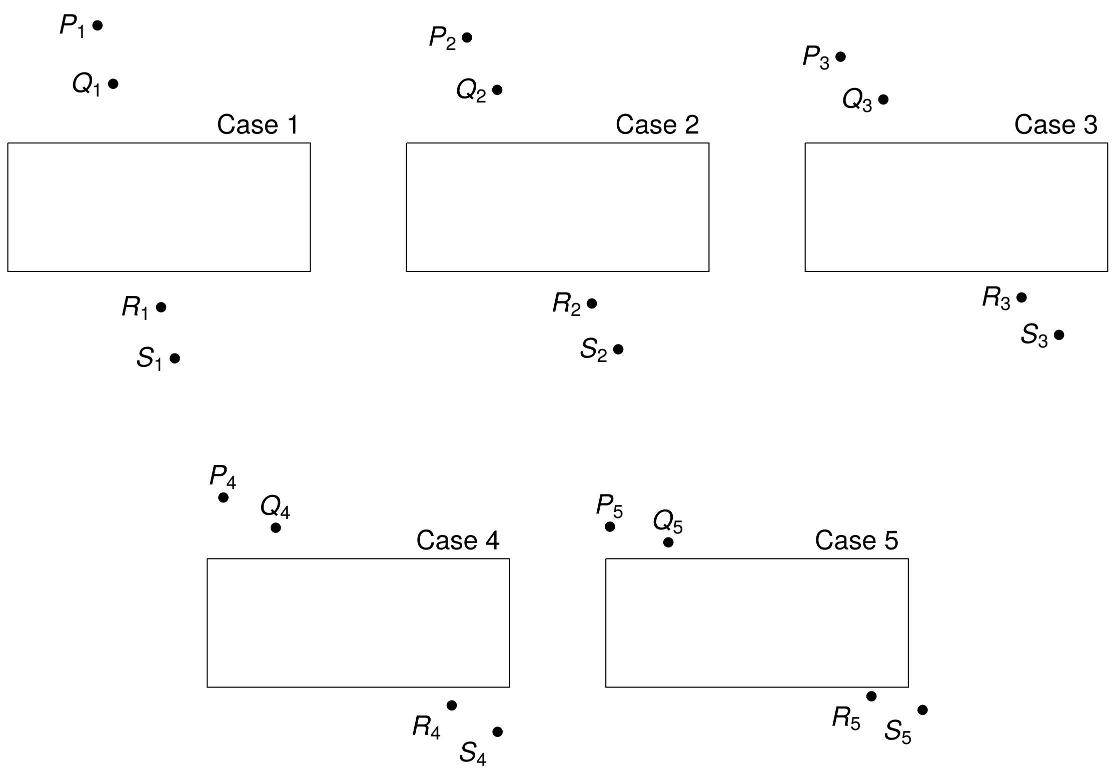


Figure 13

(i) In **each** case, draw lines **PQ** and **RS**. Produce **PQ** to meet face **AB** of the prism at **O**. Also, produce **RS** to meet face **CD** of the prism at **X**.

(ii) Draw a normal to **AB** at **O**. Measure and record the angle of incidence **i**. Evaluate $\sin^2(\mathbf{i})$ in **each** case.

(iii) Join **OX**. Measure and record **L** = **OX** and evaluate $\mathbf{L}^{-2}$ in **each** case.

(iv) Tabulate your readings.

(v) Plot a graph of $\mathbf{L}^{-2}$ on the vertical axis and $\sin^2(\mathbf{i})$ on the horizontal axis.

(vi) Determine the slope, **s**, of the graph and the intercept, **c**, on the vertical axis.

(vii) Evaluate $\mathbf{k} = \frac{\mathbf{c}}{\mathbf{s}}$.

(viii) State **two** precautions that are necessary to ensure accurate results when performing this experiment.

12.P1.L1.8 Solutions to Exam Prompt

See Figure 14 for steps (i) through (iii).

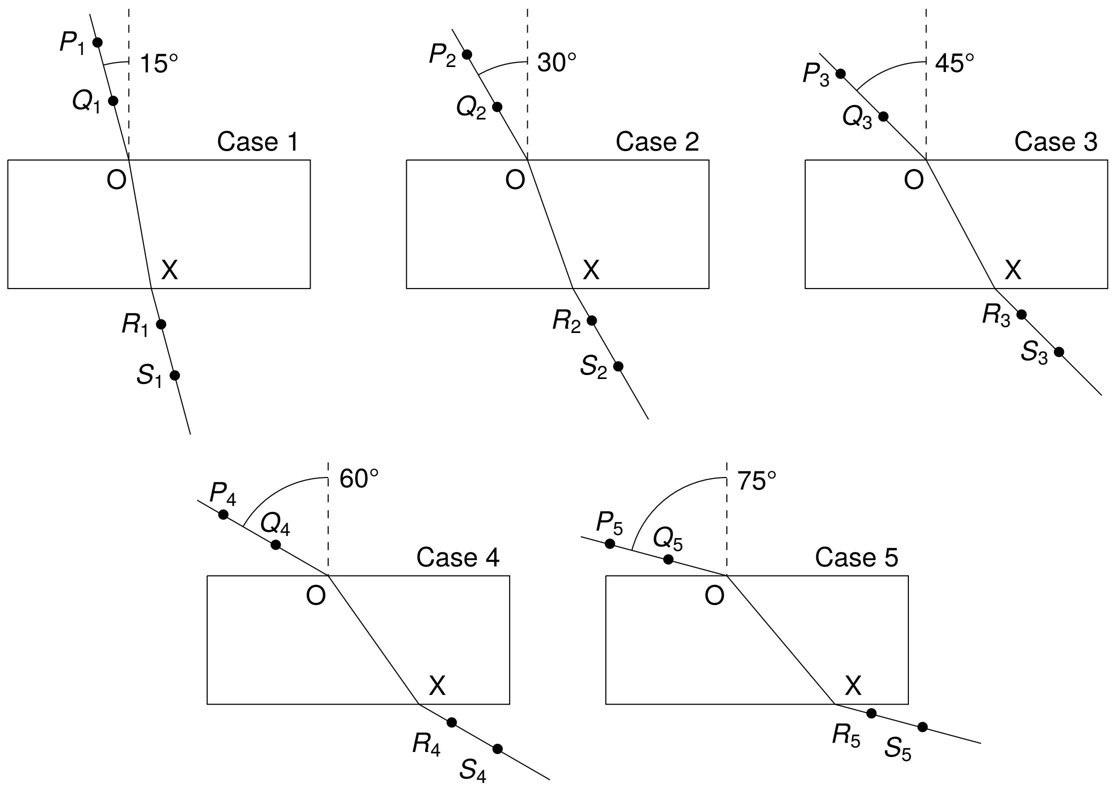
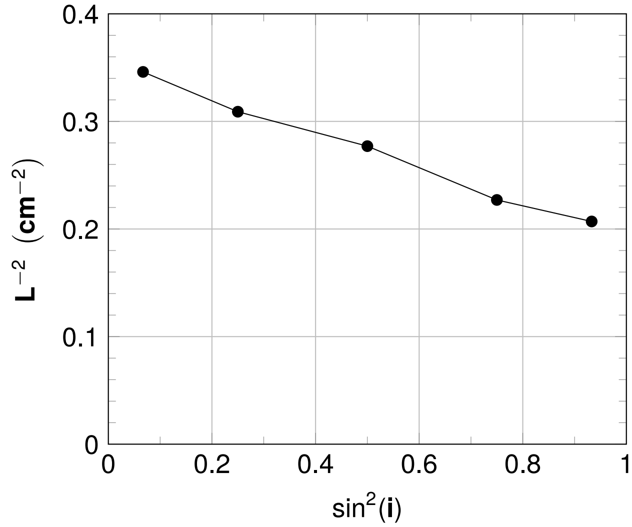


Figure 14

(iv)

Case	**i** (°)	$\sin^2(\mathbf{i})$	**L cm**	$\mathbf{L}^{-2}$ ($\mathbf{cm}^{-2}$)
1	15	0.067	1.7	0.346
2	30	0.250	1.8	0.309
3	45	0.500	1.9	0.277
4	60	0.750	2.1	0.227
5	75	0.933	2.2	0.207

(v)



(vi) *Slope*

general equation for the slope of a straight line: $s = \dfrac{y_f - y_i}{x_f - x_i}$

specifying for this practical: $s = \dfrac{\left(L^{-2}\right)_f - \left(L^{-2}\right)_i}{\left(\sin^2(i)\right)_f - \left(\sin^2(i)\right)_i}$

substituting known values: $s = \dfrac{\left(0.207\ cm^{-2}\right) - \left(0.346\ cm^{-2}\right)}{(0.933) - (0.067)}$

solving: $s = -0.161\ cm^{-2}$

Vertical Intercept

general equation for the vertical intercept of a straight line: $y_o = y_i - s(x_i)$

specifying for this practical: $\left(L^{-2}\right)_o = \left(L^{-2}\right)_i - s\left(\sin^2(i)\right)_i$

substituting known values: $\left(L^{-2}\right)_o = \left(0.346\ cm^{-2}\right) - \left(-0.161\ cm^{-2}\right)(0.067)$

solving: $\left(L^{-2}\right)_o = 0.357\ cm^{-2}$

(vii)

$$k = \frac{c}{s} = \frac{0.357\ cm^{-2}}{-0.161\ cm^{-2}} = -2.217$$

(viii) Precautions include

- angle readings taken perpendicular to the prism's thickness to minimize parallax error;
- using thin markers / pins to approximate the path of straight ray of light.

12.P1.L1.9 Post-Lab Questions - High School

1. If the prism were the same shape, but instead made of diamond (n = 2.417), would we expect the values of $\mathbf{L}_1 \ldots \mathbf{L}_5$ to increase or decrease?
   - They would increase. As the refractive index $n = {}^{n_{interior}}/_{n_{vacuum}}$ increases, so does the angle of refraction **r**. As **r** increases, so does the length of the path through the prism **L**.
2. Why didn't we place the prism with the broad face oriented vertically to make the pins easier to see?
   - This would have reduced the variance across $\mathbf{L}_1 \ldots \mathbf{L}_5$ as refraction would have occurred only along the thickness of the glass block. This would have concentrated the random error in our measurements.
3. When we view objects through glass windows, why don't they usually appear offset from their actual locations?
   - In such cases, the thickness of the window is often too small to cause any noticeable shift in any object's apparent location.

12.P1.L1.10 Post-Lab Questions - University Level 1

4. State *Snell's law* of refraction in words.
   - The ratio of the sines of the angles of incidence and refraction is equivalent to the reciprocal of the ratio of the indices of refraction.
5. State *Snell's law* algebraically.

$$\frac{\sin(\theta_1)}{\sin(\theta_2)} = \frac{n_2}{n_1} \quad \text{or} \quad \frac{\sin(i)}{\sin(r)} = \frac{n_{interior}}{n_{exterior}} \quad \text{or} \quad \frac{\sin(i)}{\sin(r)} = n \text{ where } n_{exterior} \approx 1$$

6. Differentiate between *reflection*, *refraction*, *diffraction* and *dispersion*.
   - **Reflection** is the return of a wave from an inter-medium interface at the same angle against the normal from which it is incident.
   - **Refraction** is the change in a wave's direction and speed as it enters one medium from another.
   - **Diffraction** is the bending of waves around obstacles and openings, within the same medium.
   - **Dispersion** is the separation of a beam of light into its constituent components as enters through one face of a different medium and exits through another, non-parallel face back into the original medium.

	Reflection	**Refraction**	**Diffraction**	**Dispersion**
Return from Media Interface	X			
Change in θ against N		X	X	X
Change in Medium		X		*not ultimately*
Beam Separation				X

12.P1.L1.11 Post-Lab Questions - University Level 2

7. Using the given equation for refraction in a rectangular prism, derive a linear equation for this practical's graph as

$$L^{-2} = s\left(\sin^2(i)\right) + c$$

where **s** is the slope and **c** is the vertical intercept. Use only the terms **L**, **n**, **i** and **h**, the height of the prism's broad face as shown in Figure 15.

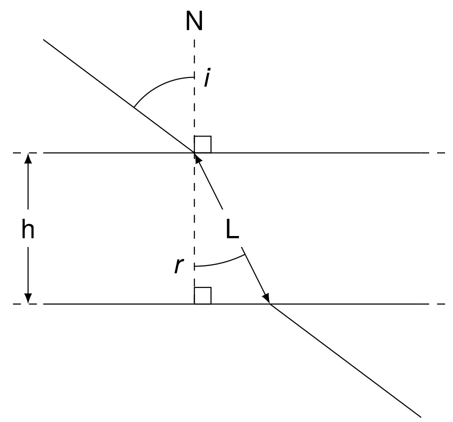


Figure 15

using equation for the index of refraction: $\dfrac{\sin(i)}{\sin(r)} = n$

isolating term involving angle of refraction: $\sin(r) = \dfrac{\sin(i)}{n}$

squaring both sides: $\sin^2(r) = \dfrac{\sin^2(i)}{n^2}$

substituting Pythagorean identity: $1 - \cos^2(r) = \dfrac{\sin^2(i)}{n^2}$

substituting trigonometric identity: $1 - \left(\dfrac{h}{L}\right)^2 = \dfrac{\sin^2(i)}{n^2}$

isolating dependent variable: $L^{-2} = \dfrac{1}{h^2}\left(1 - \dfrac{\sin^2(i)}{n^2}\right)$

expanding: $L^{-2} = -\left(\dfrac{1}{nh}\right)^2 \sin^2(i) + \dfrac{1}{h^2}$

adopting the given form, $s = -\dfrac{1}{(nh)^2}$ and $c = \dfrac{1}{h^2}$

8. Considering the expression derived in Problem 7 above, which dimension of the prism can be calculated from the intercept term **c**? Prove this algebraically.
   - **c** can be used to calculate **h**, the height of the prism's broad face.

considering equivalence derived previously: $c = \dfrac{1}{h^2}$

isolating prism face height: $h = \dfrac{1}{\sqrt{c}}$

9. Which physical property does **k** from this practical represent? Prove this algebraically.
   - **k** can be used to calculate the prism's index of refraction.

$$\text{using equation presented in problem: } k = \frac{c}{s}$$

$$\text{substituting equivalences derived previously: } k = \frac{\frac{1}{h^2}}{-\frac{1}{(nh)2}}$$

$$\text{simplifying: } k = -n^2$$

$$\text{isolating index of refraction: } n = \sqrt{-k}$$

10. If this same glass prism were embedded in a diamond medium ($n = 2.417$), would the effect of refraction be observable for all angles of incidence ($0° < i < 90°$)? If no, what would the maximum angle of incidence be for refraction to still occur within the glass?
    - Given $n_{glass} < n_{diamond}$, refraction would only occur in the glass for all $i < i_{critical}$. For all $i > i_{critical}$, the light would be reflected back out into the diamond.

$$\text{using equation for Snell's Law: } n_1 \sin(i) = n_2 \sin(r)$$

$$i_{crit} \text{ occurs when } r \text{ is perpendicular to Normal: } n_1 \sin(i_{crit}) = n_2 \sin(90°)$$

$$\text{simplifying: } n_1 \sin(i_{crit}) = n_2$$

$$\text{isolating critical angle of incidence: } i_{crit} = \sin^{-1}\left(\frac{n_1}{n_2}\right)$$

$$\text{subtituting known values: } i_{crit} = \sin^{-1}\left(\frac{1.5}{2.417}\right)$$

$$\text{solving: } i_{crit} = 38.36°$$

12.P1.L2 Converging Lenses*

12.P1.L2.1 Introduction

1. When light travels from an object through a biconvex lens, an image is formed.

2. This image is real when the object is placed beyond (before) the lens' focal point.

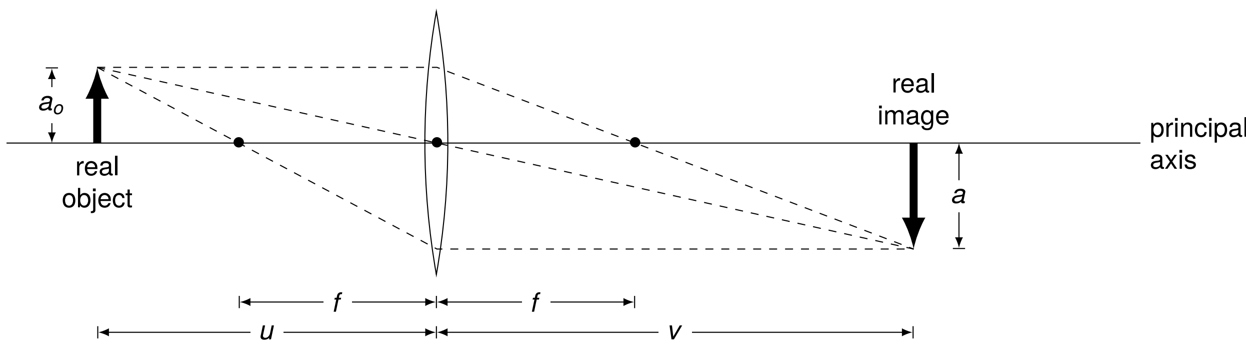


Figure 1

3. The size of the real image is related to the size of the object as

$$M = \frac{a}{a_o} = \frac{v}{u} \qquad \text{(Equation 1)}$$

where
- $\mathbf{a_o}$ is the size of the object;
- **a** is the size of the image;
- **u** is the distance from the object to the lens;
- **v** is the distance from the object to the image;
- **M** is the magnification of the object.

4. The object-lens distance is related to the image-lens distance as

$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v} \qquad \text{(Equation 2)}$$

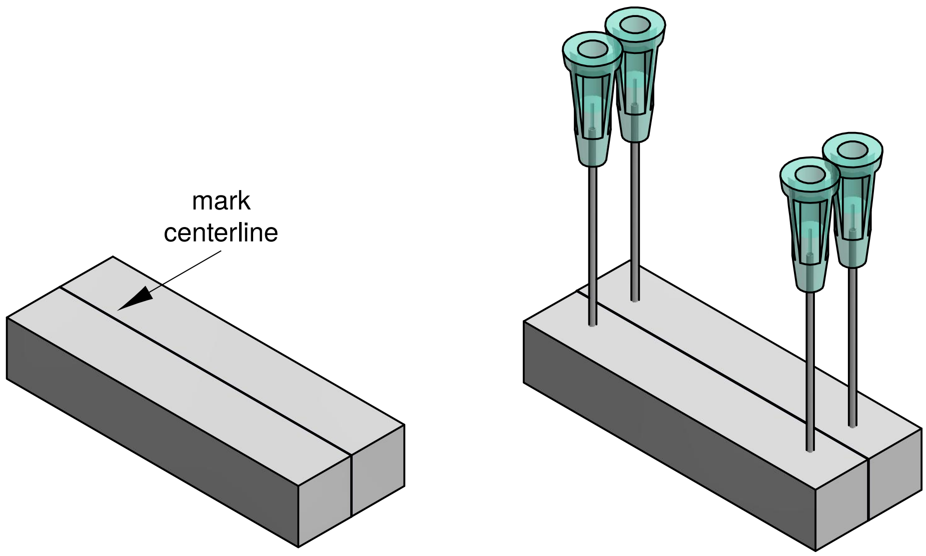
where
- **f** is the focal length of the lens.

12.P1.L2.2 Apparatus and Materials

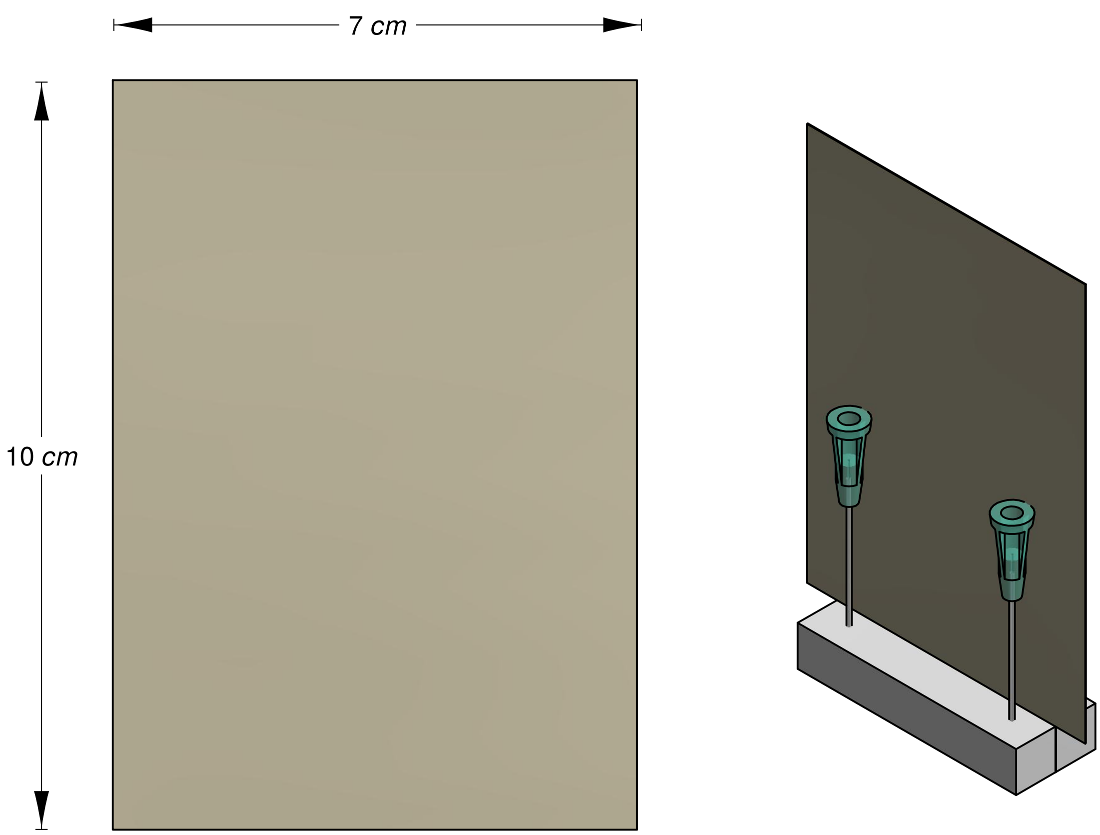
- 1 syringe (10 *mL*) with needle
- 3 additional syringe needles
- 1 additional plastic (block eraser)
- carton piece - at least 10 *cm* by 7 *cm*
- 1 rubber band
- 1 thin, biconvex lens, (See section A.3 for construction)
- tape rule, at least 1 *m* long
- 1 ruler, at least 15 *cm* long
- light source (phone, torch, etc.)

12.P1.L2.3 Setup

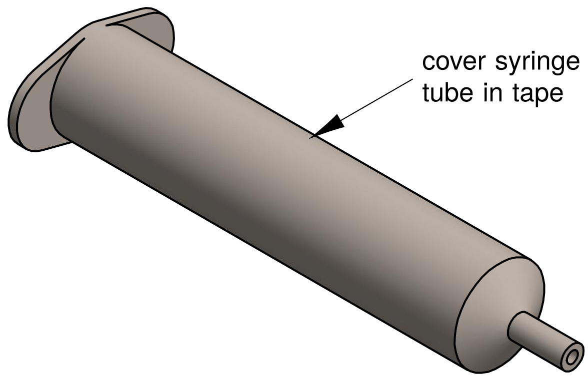
1. Use a pen to mark the center-line of a block eraser.
2. Place four syringe needles on either side of this center-line.



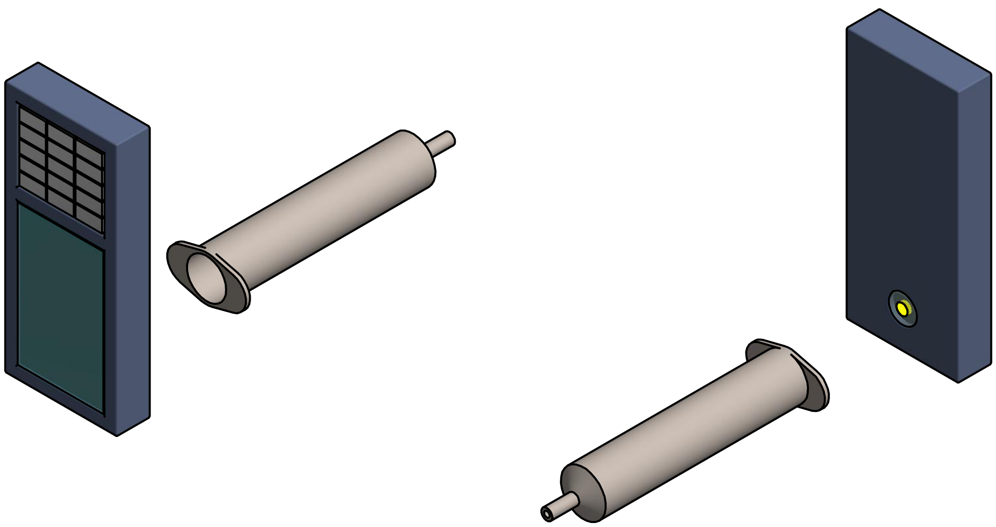
3. Cut a piece of carton to the dimensions shown below.
4. Place this carton piece into the eraser/needle stand.



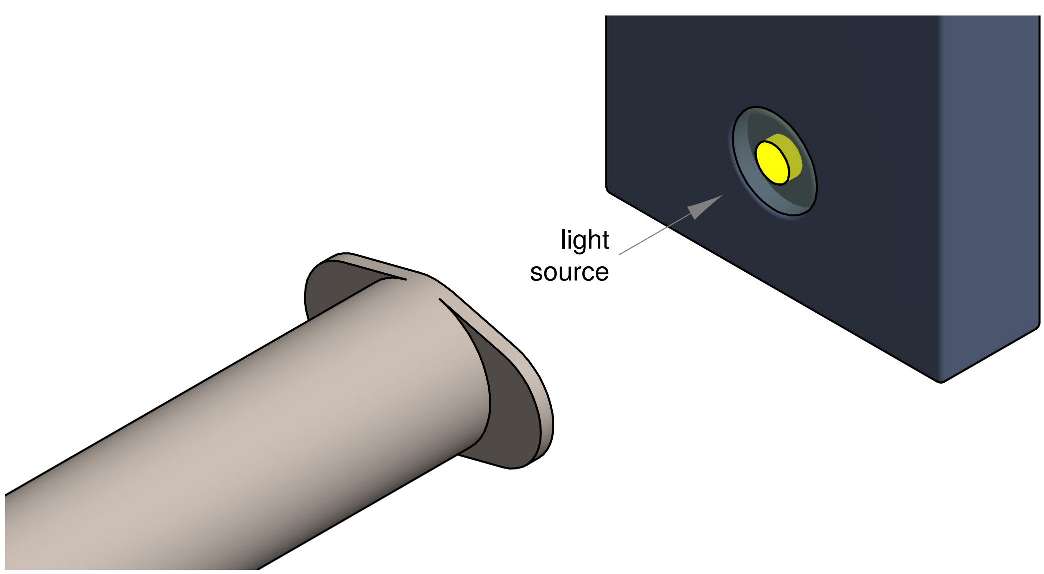
5. Wrap a syringe tube with plaster tape.



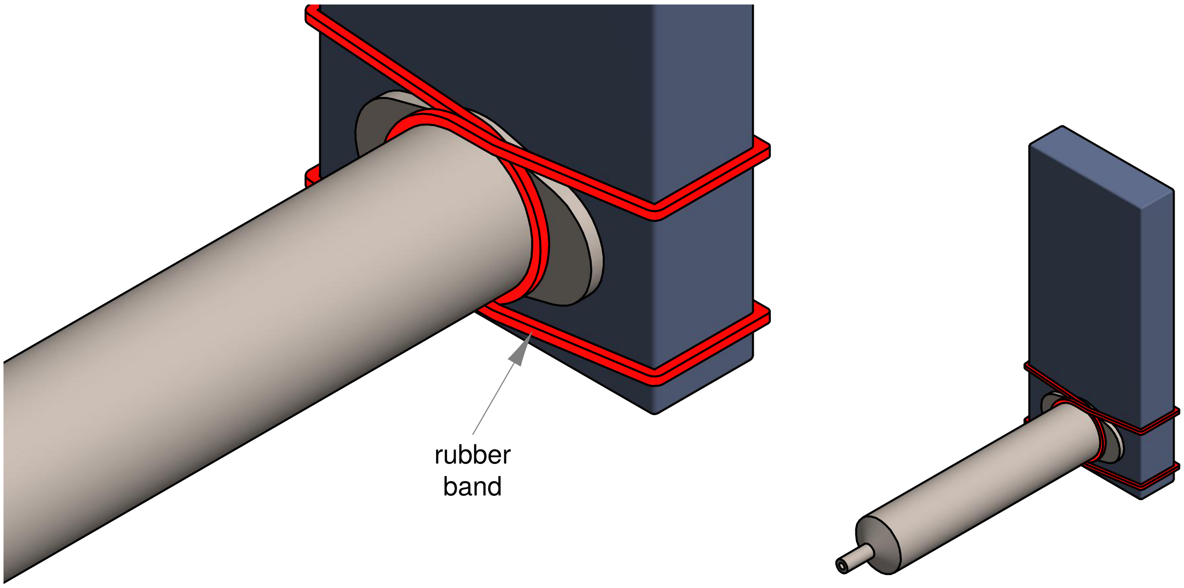
6. Line the syringe tube up with the light source.
   - Note that while the image below shows a phone, any light source will do.



7. Position the wide opening of the syringe tube over the light source.

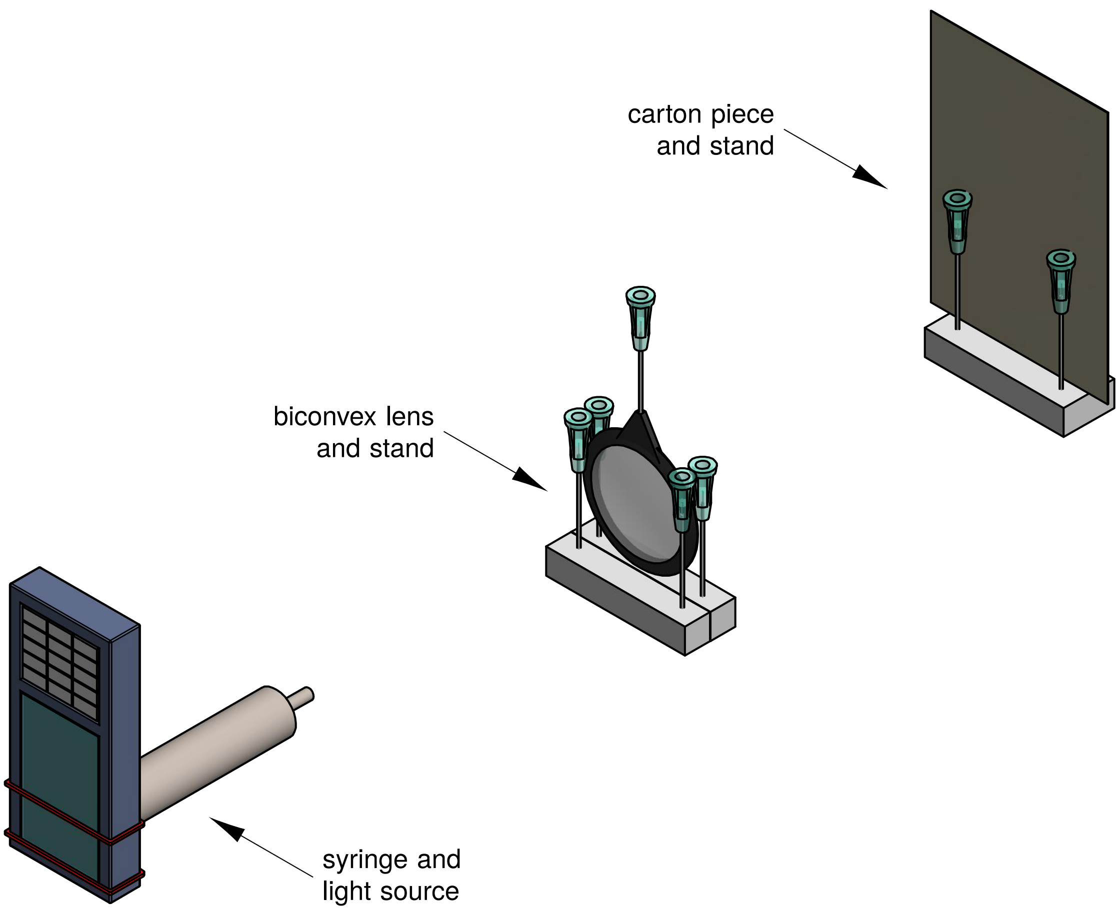


8. Use a rubber band to secure the syringe tube in place as shown.



9. Place the light source on one side of the biconvex lens.

10. Place the carton piece and its stand on the other side.



12.P1.L2.4 Warm Up Questions

1. Describe the path of a single ray of light within a constant medium.
   - A single ray of light travels in a straight line within a single medium.

2. Does the magnification of a lens have any units?
   - No, **M** is one distance divided by another.
   - Therefore, it is a dimensionless ratio.

3. Which physical phenomenon causes a light ray's path to change direction?
   - Refraction

4. What are three examples of a transparent medium whose index of refraction is different than air?
   - *Possibilities include:*
   - Water
   - Glass
   - Oil

5. Which type of lens does a single drop of water resemble more, biconvex or biconcave?
   - Biconvex

12.P1.L2.5 Procedure and Calculations

- Students should collect data similar to Table 1 using the steps below.

u (**cm**)	**v** (**cm**)	**a** (**cm**)	**f** (**cm**)	**M**	$\mathbf{M}^{-1}$
12.0	63.8	1.1	10.100	5.5	0.182
13.0	45.3	0.7	10.101	3.5	0.286
14.0	36.3	0.5	10.104	2.5	0.400
15.0	30.9	0.4	10.098	2.0	0.500
20.0	20.4	0.2	10.099	1.0	1.000

Table 1

A) Create an empty table of 6 columns and 6 rows.

B) **Row 1, Header**: Fill in the header information as shown.

C) **Column 1, u** (**cm**): Fill in the distance between the syringe nozzle and lens as shown.

   – If the lens' focal length is known, set these values of **u** at 1.2, 1.3, 1.4, 1.5 and 2 times the focal length.

   – Note the focal length of the lens constructed in Section A.3 is about 10 *cm*.

     * This is the focal length used to determine the values of **u** in Table 1.

D) Turn on the light source.

E) Place the lens at the first position of **u**, shown in column 1.

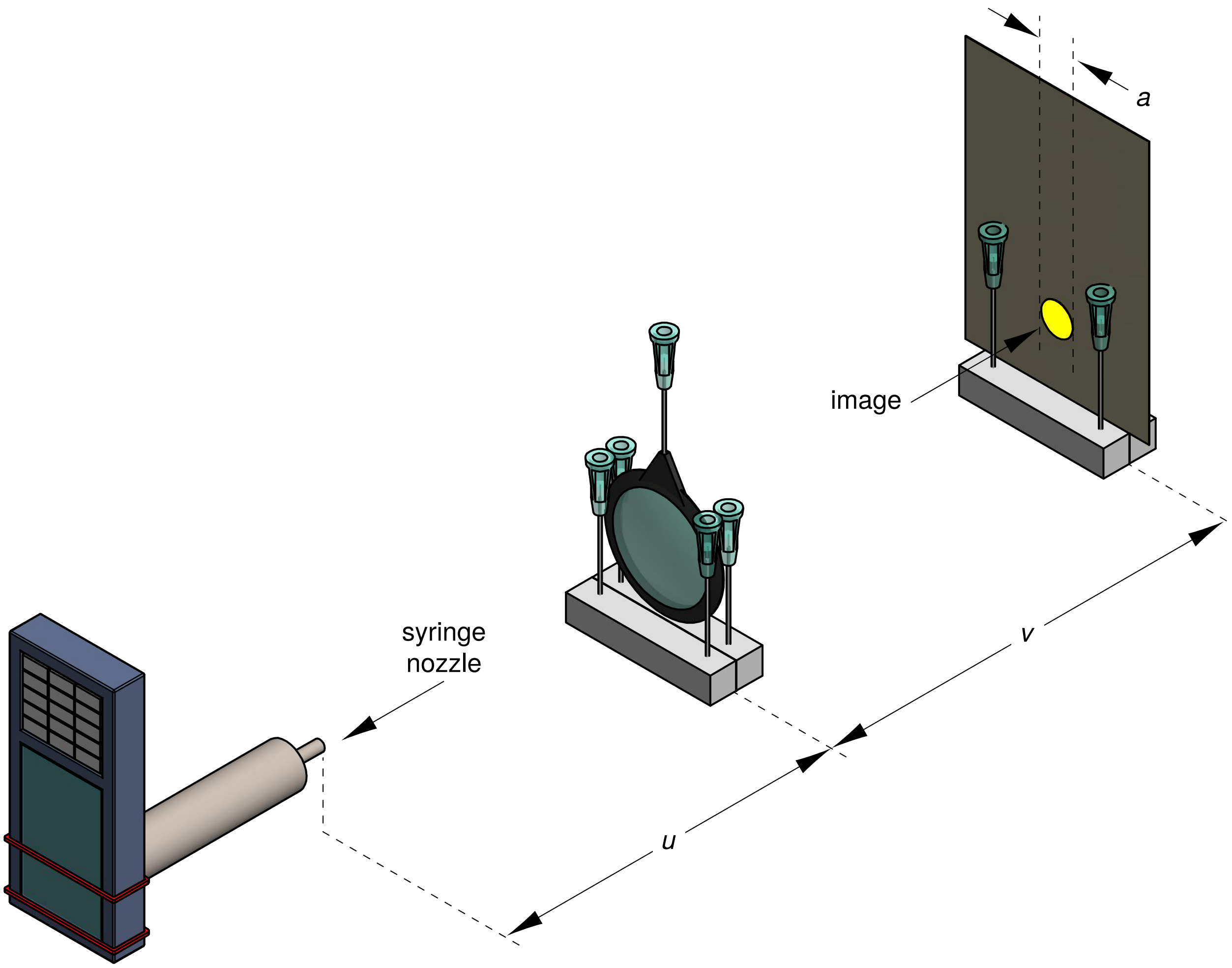
F) Adjust the angle of the light source until its light forms an image on the carton, through the lens.

G) Adjust the carton piece's location by shifting the block eraser back and fourth.

H) Continue this adjustment until the circular image on the carton piece is as small and sharp as possible.

I) Once this maximum image sharpness and minimum size is achieved,
   - **Column 2, v** (**cm**): use a tape rule to measure and record the distance from the lens to the carton.
   - **Column 3, a** (**cm**): use a ruler to measure the horizontal width of the image formed.

J) Repeat steps E) through I) for all remaining lens positions.



K) Turn off the light source.

L) **Column 4, f** (**cm**): Calculate the focal length according to each lens position using

$$f = \frac{uv}{u + v} \qquad \text{(Equation 3)}$$

M) **Column 5, M**: Calculate the image's magnification for each lens position using

$$M = \frac{a}{a_o} \qquad \text{(Equation 4)}$$

where a_o is the diameter of the nozzle outlet, which is about 2 *mm*, or 0.2 *cm* for a 10 *mL* syringe.

N) **Column 6, M^{-1}**: Calculate the inverse of the image's magnification for each lens position using

$$M^{-1} = \frac{1}{M} \qquad \text{(Equation 5)}$$

12.P1.L2.6 Data Plotting and Slope/Intercept Determination

1. Students should plot the last column of Table 1 against its 1^{st}, similar to Figure 2 below.

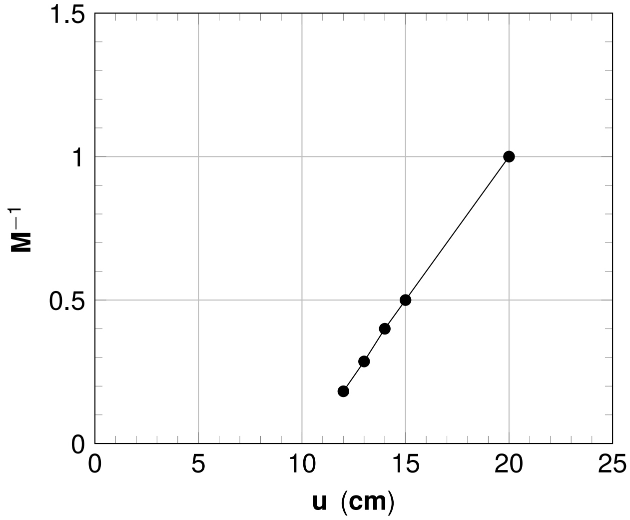


Figure 2

2. Students should then determine the graph's **slope** using

$$\text{general equation for the slope of a straight line: } s = \frac{y_f - y_i}{x_f - x_i}$$

$$\text{specifying for this practical: } s = \frac{\left(M^{-1}\right)_f - \left(M^{-1}\right)_i}{(u)_f - (u)_i}$$

$$\text{substituting known values: } s = \frac{(1.0) - (0.182)}{(20.0\ cm) - (12.0\ cm)}$$

$$\text{solving: } s = 0.1023\ \frac{1}{cm}$$

3. Students should then determine the graph's **vertical intercept** using

$$\text{general equation for the vertical intercept of a straight line: } y_o = y_i - s(x_i)$$

$$\text{specifying for this practical: } \left(M^{-1}\right)_o = \left(M^{-1}\right)_i - s\,(u)_i$$

$$\text{substituting known values: } \left(M^{-1}\right)_o = (0.182) - \left(0.1023\ \frac{1}{cm}\right)(12.0\ cm)$$

$$\text{solving: } \left(M^{-1}\right)_o = -1.045$$

4. Finally, students should then determine the graph's **horizontal intercept** using

$$\text{general equation for the horizontal intercept of a straight line: } x_o = x_i - \frac{y_i}{s}$$

$$\text{specifying for this practical: } (u)_o = (u)_i - \frac{\left(M^{-1}\right)_i}{s}$$

$$\text{substituting known values: } (u)_o = (12.0\ cm) - \frac{0.182}{0.1023\ \frac{1}{cm}}$$

$$\text{solving: } (u)_o = 10.221\ cm$$

12.P1.L2.7 Exam Prompt

Figure 3 illustrates an illuminated object of size $\mathbf{a_o}$ as well as a mounted converting lens and a screen. After placing the object at a distance of **u** before the lens, the screen is adjusted until a sharp image of the object appears on the screen. The size of the image, **a**, and the corresponding lens-image distance, **v**, are measured and recorded. The procedure is repeated for **four** other values of **u**.

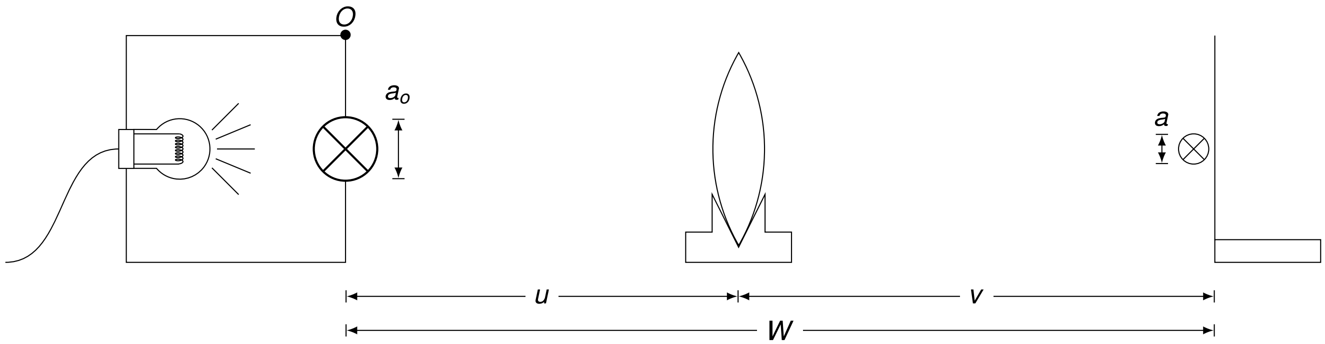


Figure 3

Figure 4 shows the object size $\mathbf{a_o}$ as well as the corresponding images sizes $\mathbf{a_i}$ where **i** = 1, 2, 3, 4 and 5.

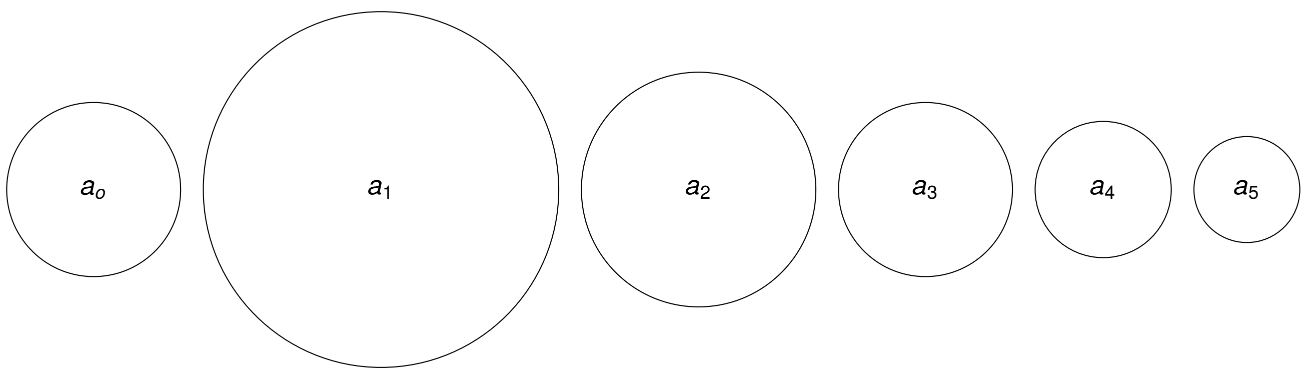
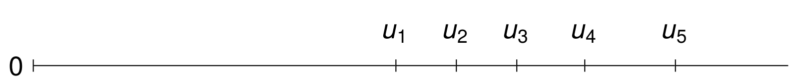


Figure 4

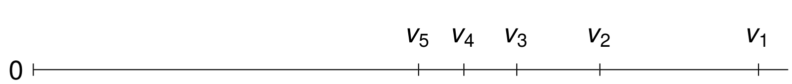
Figure 5 represents the distances **u** of the object before the lens.



Scale: 1 *cm* represents 5 *cm*

Figure 5

Figure 6 represents the corresponding distances **v** of the image beyond the lens.



Scale: 1 *cm* represents 5 *cm*

Figure 6

(i) Read and record the object size $\mathbf{a_o}$.
(ii) Read and record the image size **a**.
(iii) For each case, evaluate **M** where

$$M = \frac{a}{a_o} \qquad \text{(Equation 6)}$$

(iv) Evaluate $\mathbf{M}^{-1}$ for each case.
(v) Measure and record the distances $\mathbf{u}_{\text{raw}}$ from Figure 5.
(vi) For each case, evaluate the real value **u** from each measured value of $\mathbf{u}_{\text{raw}}$.
(vii) Measure and record the corresponding values of $\mathbf{v}_{\text{raw}}$ from Figure 6.
(viii) For each case, evaluate the real value **v** from each measured value of $\mathbf{v}_{\text{raw}}$
(ix) For each case, evaluate **k** where

$$k = \frac{uv}{u + v} \qquad \text{(Equation 7)}$$

(x) Tabulate your readings.
(xi) Plot a graph with $\mathbf{M}^{-1}$ on the vertical axis and **u** on the horizontal axis.
(xii) Determine the slope, **s**, of the graph and the intercept, **c**, on the vertical axis.
(xiii) Determine the value of **u** for which $\mathbf{M}^{-1} = 0$.
(xiv) State **two** precautions that are necessary to ensure accurate results when performing this experiment.

12.P1.L2.8 Solutions to Exam Prompt

(i) $\mathbf{a_o} = 2.3\ cm$
(vi) Use $\mathbf{u} = (\mathbf{u_{raw}}) \left(\frac{5\ cm}{1\ cm}\right)$ (viii) Use $\mathbf{v} = (\mathbf{v_{raw}}) \left(\frac{5\ cm}{1\ cm}\right)$

(x)

Case	a (cm)	M	$\mathbf{M}^{-1}$	$\mathbf{u_{raw}}$ (cm)	u (cm)	$\mathbf{v_{raw}}$ (cm)	v (cm)	k (cm)
1	4.7	2.043	0.489	4.8	24.0	9.6	48.0	15.999
2	3.1	1.348	0.742	5.6	28.0	7.5	37.5	16.031
3	2.3	1.000	1.000	6.4	32.0	6.4	32.0	15.999
4	1.8	0.783	1.278	7.3	36.5	5.7	28.5	16.003
5	1.4	0.609	1.643	8.5	42.5	5.1	25.5	15.936

(xi)

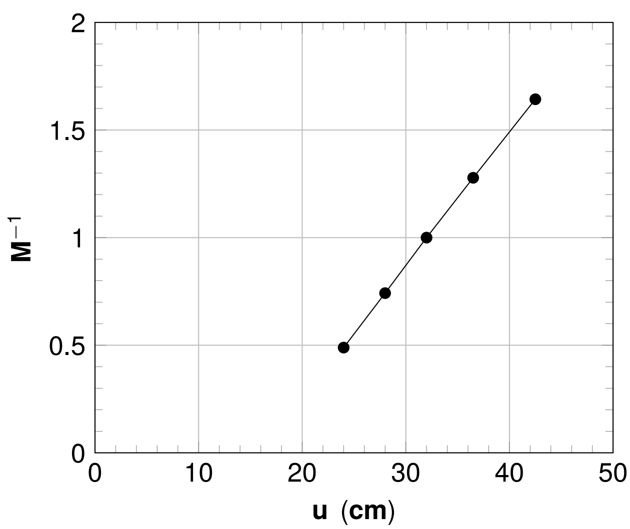


Figure 7

(xii)

Slope

general equation for the slope of a straight line: $s = \dfrac{y_f - y_i}{x_f - x_i}$

specifying for this practical: $s = \dfrac{\left(M^{-1}\right)_f - \left(M^{-1}\right)_i}{(u)_f - (u)_i}$

substituting known values: $s = \dfrac{(1.643) - (0.489)}{(42.5\ cm) - (24.0\ cm)}$

solving: $s = 0.0624\ \dfrac{1}{cm}$

Vertical Intercept

general equation for the vertical intercept of a straight line: $y_o = y_i - s(x_i)$

specifying for this practical: $\left(M^{-1}\right)_o = \left(M^{-1}\right)_i - s\,(u)_i$

substituting known values: $\left(M^{-1}\right)_o = (0.489) - \left(0.0624\ \dfrac{1}{cm}\right)(24.0\ cm)$

solving: $\left(M^{-1}\right)_o = -1.008$

using given variable for intercept: $c = -1.008$

(xiii) *Horizontal Intercept*

general equation for the horizontal intercept of a straight line: $x_o = x_i - \dfrac{y_i}{s}$

specifying for this practical: $(u)_o = (u)_i - \dfrac{\left(M^{-1}\right)_i}{s}$

substituting known values: $(u)_o = (24.0\ cm) - \dfrac{0.489}{0.0624\ \dfrac{1}{cm}}$

solving: $(u)_o = 16.163\ cm$

(xiv) Precautions include

- reading the distances from the meter at a 90° angle to minimize parallax error;
- ensuring the lens is upright, at a 90° angle to the distances being measure;
- ensuring the images formed on the screen are sharp and clear;
- cleaning the lens itself to make eliminate any inherent blurriness.

12.P1.L2.9 Post-Lab Questions - High School

1. When the object is placed beyond (before) the lens' focal length,
   a) does an increase in **u** cause an increase or decrease in **v**?
      - It causes a decrease in **v**.

   b) does an increase in **u** cause an increase of decrease in the image's size?
      - It causes a decrease in the image's size.

2. An object 2.3 *mm* in size is placed 17 *cm* in front of a biconvex lens whose focal length is 12 *cm*.
   a) Calculate the lens-image distance of the image formed on the other side from the lens.

$$\text{using given equation for object/lens/image distances: } \frac{1}{f} = \frac{1}{u} + \frac{1}{v}$$

$$\text{isolating lens-image distance term: } \frac{1}{v} = \frac{1}{f} - \frac{1}{u}$$

$$\text{taking inverse: } v = \left(\frac{1}{f} - \frac{1}{u}\right)^{-1}$$

$$\text{substituting known values: } v = \left(\frac{1}{12\ cm} - \frac{1}{17\ cm}\right)^{-1}$$

$$\text{solving: } v = 40.8\ cm$$

   b) Calculate the magnification of the image formed on the other side of the lens.

$$\text{using given equation magnification: } M = \frac{v}{u}$$

$$\text{substituting known values: } M = \frac{40.8\ cm}{17\ cm}$$

$$\text{solving: } M = 2.4$$

   c) Calculate the size of the image formed.

$$\text{using given equation magnification: } M = \frac{a}{a_o}$$

$$\text{isolating image size: } a = M\,(a_o)$$

$$\text{substituting known values: } a = 2.4(2.3\ mm)$$

$$\text{solving: } a = 5.52\ cm$$

12.P1.L2.10 Post-Lab Questions - University Level 1

3. Considering the same setup in Question 2, calculate the object's distance from the lens if it forms a sharp image 25 *cm* beyond the lens.

$$\text{using given equation for object/lens/image distances: } \frac{1}{f} = \frac{1}{u} + \frac{1}{v}$$

$$\text{isolating object-lens distance term: } \frac{1}{u} = \frac{1}{f} - \frac{1}{v}$$

$$\text{taking inverse: } u = \left(\frac{1}{f} - \frac{1}{v}\right)^{-1}$$

$$\text{substituting known values: } u = \left(\frac{1}{12\ cm} - \frac{1}{25\ cm}\right)^{-1}$$

$$\text{solving: } u = 23.077\ cm$$

12.P1.L2.11 Post-Lab Questions - University Level 2

4. Using Equation 1 and Equation 2, derive an expression for **M**, the image magnification in terms of only **f**, the lens' focal length and **u**, the distance from the object to the lens.

$$\text{starting with second equation: } \frac{1}{f} = \frac{1}{u} + \frac{1}{v}$$

$$\text{isolating lens-image distance term: } \frac{1}{v} = \frac{1}{f} - \frac{1}{u}$$

$$\text{creating common denominator: } \frac{1}{v} = \frac{u}{uf} - \frac{f}{fu}$$

$$\text{combining like terms: } \frac{1}{v} = \frac{u-f}{uf}$$

$$\text{taking inverse: } v = \frac{uf}{u-f}$$

$$\text{substituting into first equation: } M = \frac{\frac{uf}{u-f}}{u}$$

$$\text{simplifying: } M = \frac{f}{u-f}$$

5. Using the equation derived in Question 4, derive a linear equation for this practical's graph as

$$M^{-1} = s\,(u) + c$$

where **s** is the slope and **c** is the vertical intercept. Use only the terms **u** and **f**.

$$\text{using equation derived previously: } M = \frac{f}{u-f}$$

$$\text{taking inverse: } M^{-1} = \frac{u-f}{f}$$

$$\text{expanding quotient: } M^{-1} = \frac{u}{f} - \frac{f}{f}$$

$$\text{simplifying: } M^{-1} = \left(\frac{1}{f}\right) u - 1$$

$$\text{adopting the given form, } s = \frac{1}{f} \text{ and } c = -1$$

6. Considering the expression derived in question 5 above, which lens property can be calculated from **s**, the slope? Prove this algebraically.
   - **s** can be used to calculate **f**, the lens' focal length.

$$\text{considering equivalence derived previously: } s = \frac{1}{f}$$

$$\text{isolating focal length: } f = \frac{1}{s}$$

Period 2 Direct Current Electricity

Period Contents

12.P2.L1 Ohm's Law*

12.P2.L1.1 Introduction

1. An applied electric potential (voltage) "pushes" electric current through a resistive element.
   - The intensity of this current is proportional to the voltage applied.

$$V \propto I \qquad \text{(Equation 1)}$$

   - This constant of proportionality is referred to as the element's "resistance".

$$V = IR \qquad \text{(Equation 2)}$$

   Where
   - **V** is the voltage drop across the element;
   - **I** is the intensity of the electric current flowing through the element
   - **R** is the resistance of the element.

   - This equation also presents the inverse proportionality between the current through an element and its resistance under a given electric potential / voltage.

$$I = \frac{V}{R} \qquad \text{(Equation 3)}$$

2. Resistivity is a material property, much like density or elasticity.
   - For a cylindrical (tube) body of a given resistivity, its resistance is proportional to its conductive length **L** and inversely proportional to its cross-sectional area $\mathbf{A_{cs}}$, as detailed in Figure 1.

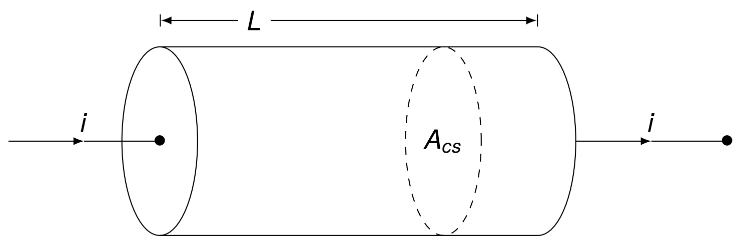


Figure 1

   - This constant of proportionality is referred to as the element's resistivity, or ρ.

$$R = \rho \frac{L}{A_{cs}} \qquad \text{(Equation 4)}$$

   Where
   - **R** is the total resistance along the conductive path;
   - ρ is the element's resistivity;
   - **L** is the length of the conductive path;
   - $\mathbf{A_{cs}}$ is the cross-sectional area normal to the conductive path.

   - Assuming the cross sectional area is constant along the conductive path, the resistance of an element can be considered as a function of its length as

$$R = \left(\frac{\rho}{A_{cs}}\right) L \qquad \text{(Equation 5)}$$

   Where the quotient $\frac{\rho}{\mathbf{A_{cs}}}$ is the resistance per unit length of the element.

3. An "ideal" voltage source provides an electromotive force (**emf**) with zero resistance across its connections.
   - A realistic voltage source has some internal resistance.
   - This can be modeled as an internal ideal source in series with a resistive element, as shown in Figure 2.

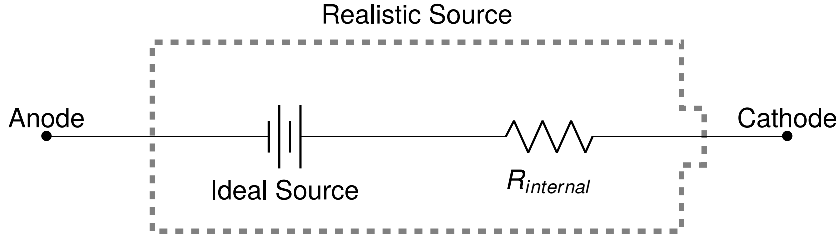


Figure 2

12.P2.L1.2 Apparatus and Materials

- 1 voltage source of about 3 *V* (See section A.4)
- 300 *cm* of Nichrome wire
- 1 digital electric multi-meter **OR**
  - 1 analog ammeter
  - (optional) 1 analog voltmeter and/or ohmeter
- 1 resistor between 30 Ω and 100 Ω
- 30 *cm* of loose wire
- 1 ruler (at least 15 *cm*)
- 1 role of plaster Tape
- 1 dark marker
- 1 pencil and eraser
- 1 piece of carton, about 30 *cm* by 21 *cm*

12.P2.L1.3 Setup

1. Use the pencil to draw a grid on the piece of carton as shown in Figure 3.
   - Space the lines 2 *cm* apart.
   - Draw the grid to be 20 *cm* wide and 20 *cm* high.
   - This should result in 11 horizontal lines and 11 vertical lines.

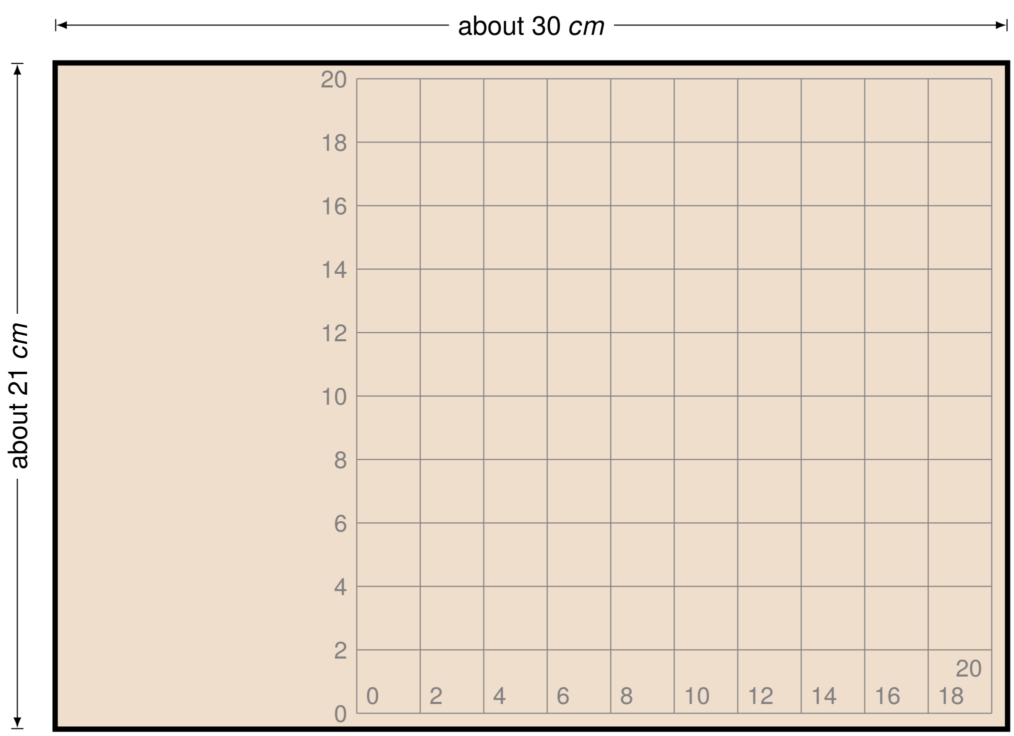


Figure 3

2. Use the marker to make dots at the line intersections shown in Figure 4.

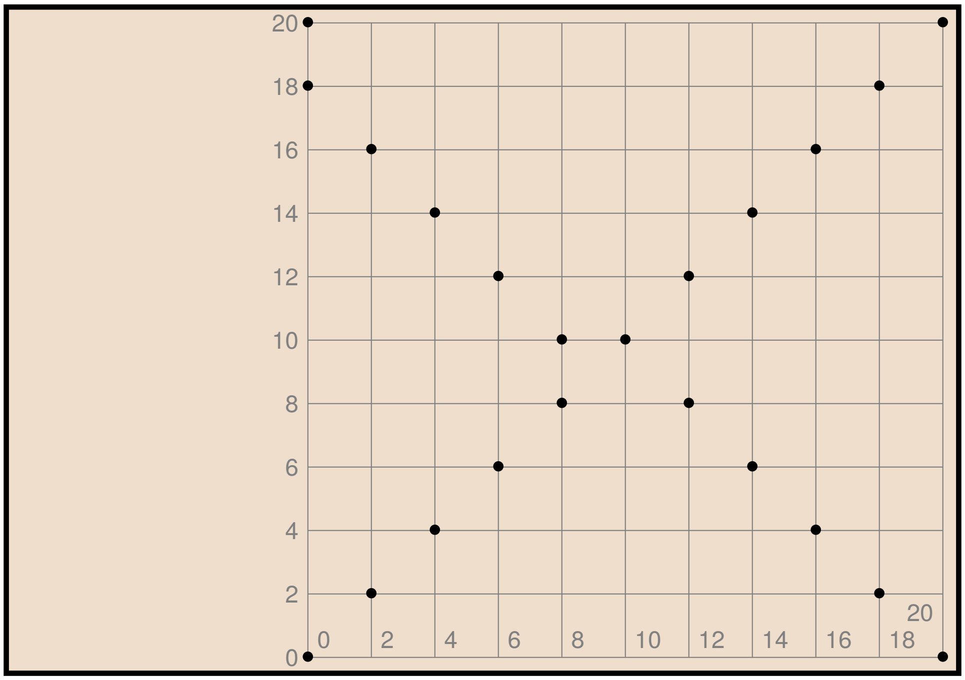


Figure 4

3. Use the eraser to remove the horizontal and vertical lines as well as any number labels.
4. Use the Nichrome wire to connect each of the dots in a spiral fashion.
    - Start in the upper-left corner.
    - Work clockwise, inwards.
    - At each dot, use tape to fix the wire in place, but leave the 90° bend exposed.
5. At each dot/bend, label the distance along the wire from the spiral's starting point **P**.

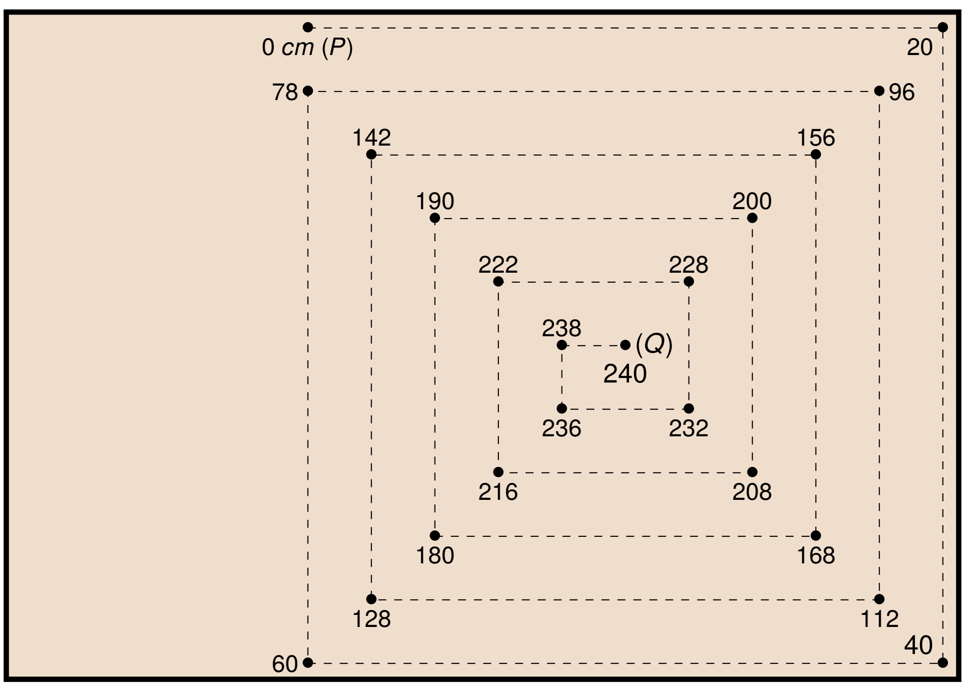


Figure 5

6. Attach the resistor to the Nichrome wire before node **P**.
7. Attach the positive lead/wire of the ammeter to the other end of the resistor.
8. Attach the negative lead/wire of the ammeter to the positive lead of the voltage source.
9. Let the wire **X** attached to the voltage source's negative lead rest away from the carton.
   - Take **caution** that this negative lead doesn't touch any part of the circuit on the carton.

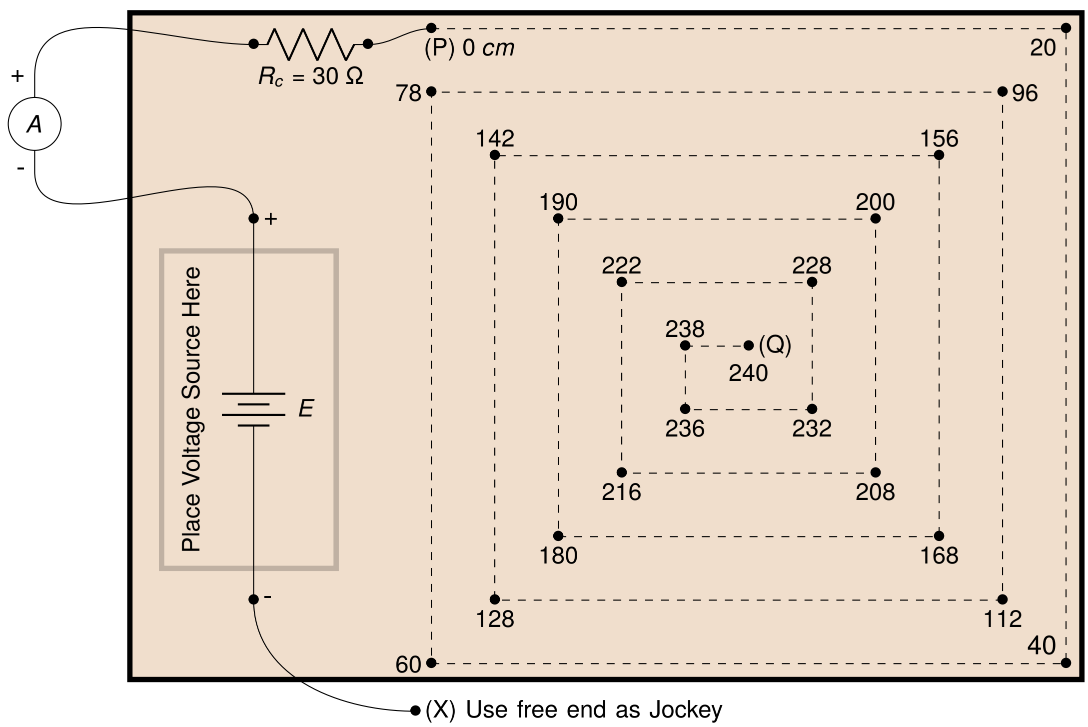


Figure 6

12.P2.L1.4 Warm Up Questions

1. Consider a taxi driver who's willing to transport a single passenger 15 *km* along a dry, dirt road for 150 *LD*.
   a) What rate of money per distance per passenger is the driver charging?

$$\frac{(150\ LD)}{(15\ km)(1\ \text{passenger})} = \frac{10\ LD}{(km)(\text{passenger})}$$

   b) If the driver were willing to transport the same single passenger 20 *km* along the same road for 200 *LD*, would the money-per-distance-per passenger rate change?

$$\text{No, the rate would be the same: } \frac{(200\ LD)}{(20\ km)(1\ \text{passenger})} = \frac{10\ LD}{(km)(\text{passenger})}$$

   c) If the driver will willing to transport two passengers along the same road for 200 *LD*, but for a distance of only 10 *km*, would the money-per-distance-per passenger rate change?

$$\text{No, the rate would be the same: } \frac{(200\ LD)}{(10\ km)(2\ \text{passengers})} = \frac{10\ LD}{(km)(\text{passenger})}$$

   d) After a serious rainstorm, the road has become very muddy, and the driver is only willing to transport a single passenger 15 *km* for 300 *LD*. Would the money-per-distance-per passenger rate change?

$$\text{Yes, it would have increased: } \frac{(300\ LD)}{(15\ km)(1\ \text{passenger})} = \frac{20\ LD}{(km)(\text{passenger})}$$

e) A few months later, the road now has cold-tar, and the driver is willing to transport a single passenger 15 *km* for only 75 *LD*. Would the money-per-distance-per passenger rate change?

$$\text{Yes, would have decreased: } \frac{(75\ LD)}{(15\ km)(1\text{ passenger})} = \frac{5\ LD}{(km)(\text{passenger})}$$

f) Explain how this relates to Ohm's law.
- If the passengers are like current and the money is like voltage, the money-per-distance-per-passenger rate is analogous to resistance.

  trip cost = (passenger quantity)(money-per-distance-per-passenger rate)

  similarly, $V = IR$
- Also, considering just the resistance in Ohm's law,
  - the trip cost (voltage) increases with the trip length (length of conductive element);
  - the trip cost increases with road difficulty (resistivity of conductive element).

12.P2.L1.5 Procedure and Calculations

- Students should collect data similar to Table 1 using the steps below.

L (**cm**)	**L** (**m**)	**i** (**mA**)	**i** (**A**)	$\mathbf{i}^{-1}$ $(\mathbf{A}^{-1})$
20	0.20	64.6	0.0646	15.480
112	1.12	24.9	0.0249	40.161
156	1.56	19.2	0.0192	52.083
208	2.08	15.1	0.0151	66.225
240	2.40	13.4	0.0134	74.626

Table 1

A) Create an empty table of 5 columns and 6 rows.

B) **Row 1, Header**: Fill in the header information as shown.

C) **Column 1, L** (**cm**): Fill in the list of wire lengths as shown.

D) **Column 2, L** (**m**): Convert each of these lengths to meters using

$$L_m = L_{cm}\left(\frac{1\ m}{100\ cm}\right) \quad \text{(Equation 6)}$$

E) Touch the Jockey (**X**) to the Nichrome wire corner at the 20 **cm** node.

F) **Column 3, i** (**mA**): Record the ammeter's current reading.
- Remove the Jockey (**X**) from the Nichrome wire as soon as the recording is taken.

G) **Columns 4, i** (**A**): Convert this current reading to amperes using

$$i_A = i_{mA}\left(\frac{1\ A}{1000\ mA}\right) \quad \text{(Equation 7)}$$

H) **Columns 5,** $\mathbf{i}^{-1}$ $(\mathbf{A}^{-1})$: Take the inverse of this current reading using

$$i^{-1} = \frac{1}{i} \quad \text{(Equation 8)}$$

I) Repeat steps E) through H) for all remaining wire positions.

12.P2.L1.6 Data Plotting and Slope/Intercept Determination

1. Students should plot the last column of Table 1 against its 2^{nd}, similar to Figure 7 below.

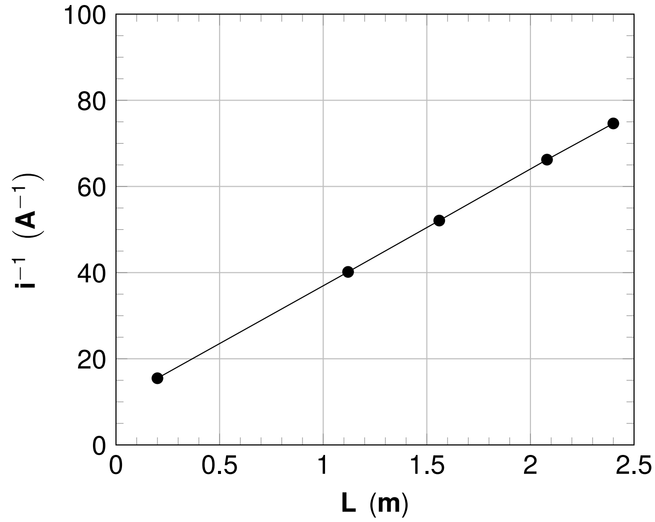


Figure 7

2. Students should then determine the graph's **slope** using

$$\text{general equation for the slope of a straight line: } s = \frac{y_f - y_i}{x_f - x_i}$$

$$\text{specifying for this practical: } s = \frac{\left(i^{-1}\right)_f - \left(i^{-1}\right)_i}{(L)_f - (L)_i}$$

$$\text{substituting known values: } s = \frac{\left(74.63\ A^{-1}\right) - \left(15.48\ A^{-1}\right)}{(2.4\ m) - (0.2\ m)}$$

$$\text{solving: } s = 26.8864\ \frac{A^{-1}}{m}$$

3. Students should then determine the graph's **vertical intercept** using

general equation for the vertical intercept of a straight line: $y_o = y_i - s(x_i)$

$$\text{specifying for this practical: } \left(i^{-1}\right)_o = \left(i^{-1}\right)_i - s\,(L)_i$$

$$\text{substituting known values: } \left(i^{-1}\right)_o = \left(15.48\ A^{-1}\right) - \left(26.8864\ \frac{A^{-1}}{m}\right)(0.2\ m)$$

$$\text{solving: } \left(i^{-1}\right)_o = 10.103\ A^{-1}$$

4. Students should then use the voltmeter to measure the voltage of the source $\mathbf{V_s}$.
    - If no voltmeter is available, the source voltage can be approximated from the number of batteries using

$$V_{s,\ approx} = (1.65\ V)\,(N_{batteries}) \qquad \text{(Equation 9)}$$

Where
- $\mathbf{V_{s,\ approx}}$ is the approximate voltage of the source;
- $\mathbf{N_{batteries}}$ is the quantity of batteries (Size AA, AAA, C or D).

5. Students should then calculate $\mathbf{k}_1$ and $\mathbf{k}_2$ where $\mathbf{k}_1 = \mathbf{V_s}(\mathbf{s})$ and $\mathbf{k}_2 = \mathbf{V_s}\left(\mathbf{i}^{-1}\right)_{\mathbf{o}}$.

   (*Note the following solution assumes a measured* $\mathbf{V_s} = 3.3\ V$)

   Calculating $\mathbf{k}_1$

$$\text{given equation: } k_1 = V_s(s)$$

$$\text{substituting known values: } k_1 = (3.3\ V)\left(26.8864\ \frac{A^{-1}}{m}\right)$$

$$\text{solving: } k_1 = 88.725\ V\left(\frac{A^{-1}}{m}\right)$$

$$\text{simplifying units: } k_1 = 88.725\ \frac{\Omega}{m}$$

   Calculating $\mathbf{k}_2$

$$\text{given equation: } k_2 = V_s\left(i^{-1}\right)_o$$

$$\text{substituting known values: } k_2 = (3.3\ V)\left(10.103\ A^{-1}\right)$$

$$\text{solving: } k_2 = 33.34\ V\left(A^{-1}\right)$$

$$\text{simplifying units: } k_2 = 33.34\ \Omega$$

6. Students should then determine the resistance $\mathbf{R_c}$ of the constant resistor connected to the Nichrome wire.

   - If a digital multimeter is available, use its ohmeter feature.
   - If not, students can determine the resistance from the bands on the resistor.
   - As a last resort, the students can be told the resistor's resistance directly.

7. Students should then calculate $\mathbf{R_{int}} = \mathbf{k}_2 - \mathbf{R_c}$.

   (*Note the following solution assumes a calculated* $\mathbf{k}_2 = 33.34\ \Omega$ *and* $\mathbf{R_c} = 30\ \Omega$)

$$\text{given equation: } R_{int} = k_2 - R_c$$

$$\text{substituting known values: } R_{int} = 33.34\ \Omega - 30\ \Omega$$

$$\text{solving: } R_{int} = 3.34\ \Omega$$

12.P2.L1.7 Exam Prompt

Figure 8 represents an electric circuit set up to determine the internal resistnace, **r**, of a dry cell of **e.m.f.** = 1.5 V.

With the circuit closed and the circuit element **Z** making contact with wire **PQ** at **X** such that **PX** = **d**, the current **I** is measured and recorded.

The procedure is repeated for **five** other positions of **X**.

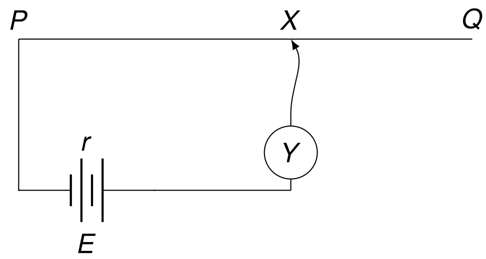
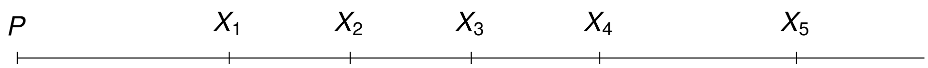


Figure 8

Figure 9 shows the various lengths $\mathbf{d_i}$ and Figure 10 shows the corresponding current readings $\mathbf{I_i}$ where **i** = 1, 2, 3, 4, 5 and 6.



Scale: 1 *cm* represents 2 *cm*

Figure 9

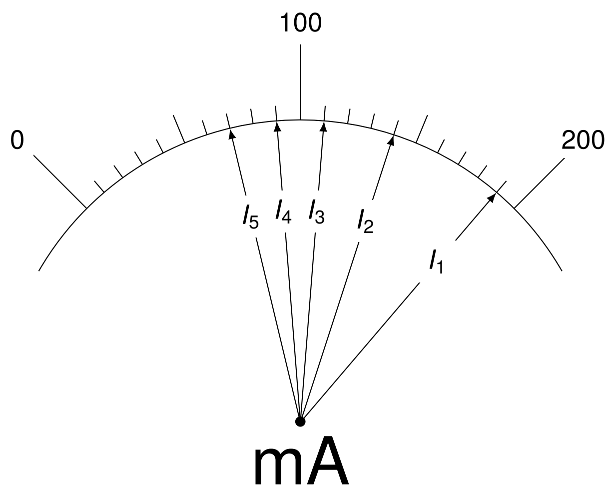


Figure 10

(i) Identify the circuit elements.

(ii) For **each** case,

a) record the raw values of the distance $\mathbf{d_{i,\ raw}}$ of position $\mathbf{X_i}$ from **P** along the wire **XP**;

b) use $\mathbf{d_{i,\ raw}}$ to determine $\mathbf{d_{i,\ real}}$;

c) record the value of $\mathbf{I_i}$ in the units shown;

d) convert the value of $\mathbf{I_i}$ to SI units;

e) evaluate $\mathbf{I}^{-1}$ using the value of $\mathbf{I_i}$ in SI units.

(iii) Tabulate your readings.

(iv) Plot a graph of $\mathbf{I}^{-1}$ on the vertical axis against $\mathbf{d_{real}}$, starting both axes from the origin (0, 0).

(v) Calculate **s**, the slope of the graph.

(vi) Calculate **c**, the vertical intercept of the graph.

(vii) Calculate $\mathbf{k}_1$ = (1.5 **V**)(**s**) and $\mathbf{k}_2$ = (1.5 **V**)(**c**) .

(viii) State **two** precautions that are necessary to ensure accurate results when performing this experiment.

12.P2.L1.8 Solutions to Exam Prompt

(i)

- **E** is the source of voltage, or dry cell battery in this case;
- **Y** is an ammeter, connected in series with the battery;
- **PQ** is bare resistance wire (likely Nichrome);
- **X** is the jockey, which is made to touch the bare resistance wire at different points between **P** and **Q**.

(ii)

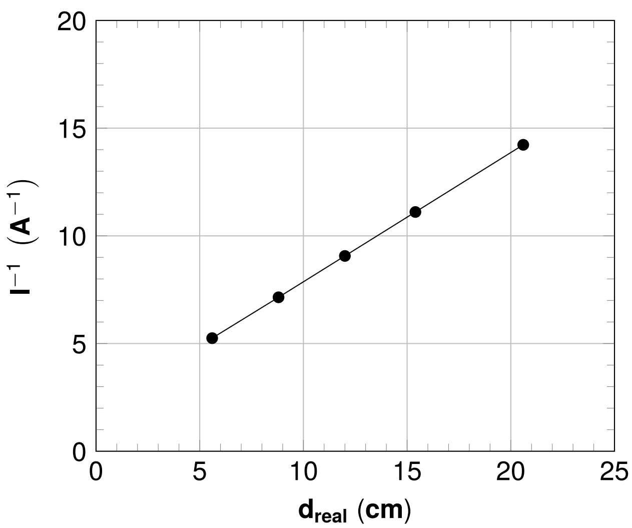
b) Use $\mathbf{d}_{\mathbf{i},\ \mathbf{real}} = \left(\dfrac{2\ cm}{1\ cm}\right)\mathbf{d}_{\mathbf{i},\ \mathbf{raw}}$

d) Use $\mathbf{I}_{\mathbf{i},\ \mathbf{A}} = \left(\dfrac{1\ A}{1000\ mA}\right)\mathbf{I}_{\mathbf{i},\ \mathbf{mA}}$

(iii)

i	$\mathbf{d}_{\mathbf{raw}}$ (cm)	$\mathbf{d}_{\mathbf{real}}$ (cm)	I (mA)	I (A)	$\mathbf{I}^{-1}$ ($\mathbf{A}^{-1}$)
1	2.8	5.6	190	0.190	5.251
2	4.4	8.8	140	0.140	7.147
3	6.0	12.0	110	0.110	9.067
4	7.7	15.4	90	0.090	11.107
5	10.3	20.6	70	0.070	14.227

(iv)



(v)

general equation for the slope of a straight line: $s = \dfrac{y_f - y_i}{x_f - x_i}$

specifying for this practical: $s = \dfrac{\left(i^{-1}\right)_f - \left(i^{-1}\right)_i}{(d_{real})_f - (d_{real})_i}$

substituting known values: $s = \dfrac{\left(14.227\ A^{-1}\right) - \left(5.251\ A^{-1}\right)}{(20.6\ cm) - (5.6\ cm)}$

solving: $s = 0.5984\ \dfrac{A^{-1}}{cm}$

(vi)

general equation for the vertical intercept of a straight line: $y_o = y_i - s(x_i)$

$$\text{specifying for this practical: } \left(i^{-1}\right)_o = \left(i^{-1}\right)_i - s\,(d_{real})_i$$

$$\text{substituting known values: } \left(i^{-1}\right)_o = \left(5.251\ A^{-1}\right) - \left(0.5984\ \frac{A^{-1}}{cm}\right)(5.6\ cm)$$

$$\text{solving: } \left(i^{-1}\right)_o = 1.900\ A^{-1}$$

(vii) *Calculating* $\mathbf{k}_1$

$$\text{given equation: } k_1 = (1.5\ V)(s)$$

$$\text{substituting known values: } k_1 = (1.5\ V)\left(0.5984\ \frac{A^{-1}}{cm}\right)$$

$$\text{solving: } k_1 = 0.898\ V\left(\frac{A^{-1}}{cm}\right)$$

$$\text{simplifying units: } k_1 = 0.898\ \frac{\Omega}{cm}$$

Calculating $\mathbf{k}_2$

$$\text{given equation: } k_2 = (1.5\ V)\left(i^{-1}\right)_o$$

$$\text{substituting known values: } k_2 = (1.5\ V)\left(1.900\ A^{-1}\right)$$

$$\text{solving: } k_2 = 2.85\ V\left(A^{-1}\right)$$

$$\text{simplifying units: } k_2 = 2.85\ \Omega$$

(viii) Precautions include
- being sure the ammeter presents a zero reading before connection to minimize non-zero error;
- (if analog) reading the ammeter at a 90° angle to minimize parallax error;
- confirming all circuit connections are secure to minimize contact resistance in circuit components.

12.P2.L1.9 Post-Lab Questions - High School

1. A wire can be thought of as a long, thin prism of which shape cross section - square, triangle, circle, or other?
   - It can be thought of as a circular-based prism, also know as a cylinder. *See Figure 1.*
2. Does resistance along the bare resistance wire increase or decrease with length between contact points?
   - Resistance increases with length between contact points, as evidenced by a decrease in current.
3. Given a constant value of **R**, is the relationship between **V** and **I** proportional?
   - Yes, $V \propto I$
4. Given a constant value of **I**, is the relationship between **V** and **R** proportional?
   - Yes, $V \propto R$
5. Given a constant value of **V**, is the relationship between **R** and **I** proportional?
   - No, $R \propto I^{-1}$ (or $I^{-1} \propto R$).
6. Which of the reflection questions 3 through 5 is relevant to the data collected in this practical?
   - Question 5 - The voltage of the source **V** was kept constant, while the resistance **R** was varied. The value of $\mathbf{I}^{-1}$ was in turn proportional to these variations in **R**.

12.P2.L1.10 Post-Lab Questions - University Level 1

7. Create a table comparing a) negative, linear proportionality in to b) inverse proportionality.

	Negative, Linear Proportionality	**Inverse Proportionality**
General equation	$y = sx + c$ where $s < 0$	$y = {}^{k}/_{x}$ where $k > 0$ and $x > 0$
Trends downwards	X	X
Graph is a straight line	X	
Graph is curved		X
Passes through origin (0, 0)	*only when* $c = 0$	X
Example using Ohm's law	$V = R\,(I_o - I_x)$ where **R** is the slope	$I = {}^{V}/_{R}$ where **V** is constant

12.P2.L1.11 Post-Lab Questions - University Level 2

8. Using the given equation for Ohm's, derive a linear equation for this practical's graph as

$$I^{-1} = s\,(d) + c$$

where **s** is the slope and **c** is the vertical intercept. Use only the terms

- **E** - the source voltage;
- **I** - the current through the ammeter (and circuit at-large);
- $\mathbf{R_{int}}$ - the internal resistance of the source;
- $\mathbf{R_c}$ - the resistance of the constant resistor placed before the bare resistance wire;
- **d** - the length of bare resistance wire contributing to its variable effective resistance;
- $\dfrac{\rho}{\mathbf{A_{cs}}}$ - the resistance per unit length of the bare resistance wire.

Solution

adapting equation for Ohm's law to this practical: $E = IR$

considering all relevant resistances: $E = I\,(R_c + R_{int} + R_{br\ wire})$

considering resistivity of bare resistance wire: $E = I\left(R_c + R_{int} + d\left(\dfrac{\rho}{A_{cs}}\right)\right)$

dividing both sides by current: $\dfrac{E}{I} = R_c + R_{int} + d\left(\dfrac{\rho}{A_{cs}}\right)$

isolating inverse current: $I^{-1} = \dfrac{R_c + R_{int} + d\,({}^{\rho}/_{A_{cs}})}{E}$

expanding: $I^{-1} = d\left(\dfrac{{}^{\rho}/_{A_{cs}}}{E}\right) + \dfrac{R_c + R_{int}}{E}$

adopting the given form: $s = \dfrac{{}^{\rho}/_{A_{cs}}}{E}$ and $c = \dfrac{R_c + R_{int}}{E}$

9. Considering the expression derived in Problem 8 above, which circuit component property can be calculated from the intercept term **c** assuming known values of **E** and $\mathbf{R_c}$?
   a) Prove this algebraically.
      - **c** can be used to calculate $\mathbf{R_{int}}$, the internal resistance of the source.

$$\text{using equivalence derived previously: } c = \frac{R_c + R_{int}}{E}$$

$$\text{isolating internal resistance of source: } R_{int} = cE - R_c$$

   b) Evaluate this property using the actual values determined in this practical.

$$\text{using equivalence derived previously: } R_{int} = cE - R_f$$

$$\text{substituting known values: } R_{int} = \left(10.103\ A^{-1}\right)(3.3\ V) - (30\ \Omega)$$

$$\text{solving: } R_{int} = 3.34\ \Omega$$

10. Considering the expression derived in Problem 8 above, which quotient can be calculated from the slope term **s** assuming known values of **E**?
   a) Prove this algebraically.
      - **s** can be used to calculate ρ/A_{cs}, the resistance per unit length of the bare resistance wire.

$$\text{using equivalence derived previously: } s = \frac{\rho/A_{cs}}{E}$$

$$\text{isolating resistance per unit length: } \frac{\rho}{A_{cs}} = sE$$

   b) Evaluate this property using the actual values determined in the lab.

$$\text{using equivalence derived previously: } \frac{\rho}{A_{cs}} = sE$$

$$\text{substituting known values: } \frac{\rho}{A_{cs}} = \left(26.8864\ \frac{A^{-1}}{m}\right)(3.3\ V)$$

$$\text{solving: } \frac{\rho}{A_{cs}} = 88.7251\ \frac{\Omega}{m}$$

12.P2.L2 Voltage Dividers*

12.P2.L2.1 Introduction

1. When two circuit components $\mathbf{R}_1$ and $\mathbf{R}_2$ are arranged in series and connected to an electromotive force (source of voltage) as shown in Figure 1, the voltage drop across either competent is

$$V_{R1} = V_{AB} = V_{emf}\left(\frac{R_1}{R_1 + R_2}\right) \quad \text{(Equation 1)}$$

$$V_{R2} = V_{BC} = V_{emf}\left(\frac{R_2}{R_1 + R_2}\right) \quad \text{(Equation 2)}$$

Where
- $\mathbf{V}_1$ and $\mathbf{V_{AB}}$ are the resistance across the first resistive component;
- $\mathbf{V}_2$ and $\mathbf{V_{BC}}$ are the resistance across the second resistive component;
- $\mathbf{R}_1$ is the resistance of the first resistive component;
- $\mathbf{R}_2$ is the resistance of the second resistive component;
- $\mathbf{V_{emf}}$ is the voltage of the electromotive source.

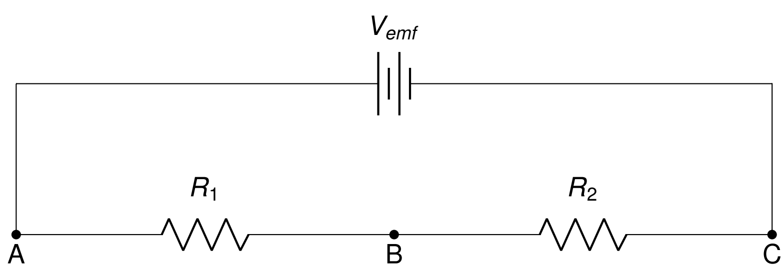


Figure 1

2. Resistivity is a material property, much like density or elasticity.
   - Assuming constant cross sectional and material make-up, the resistance of a bare resistance wire can be considered as a function of its length as

$$R = \left(\frac{\rho}{A_{cs}}\right) L \quad \text{(Equation 3)}$$

   Where
     - $\mathbf{R}$ is the total resistance along the conductive path;
     - $\frac{\rho}{\mathbf{A_{cs}}}$ is the conductive path's resistance per unit length;
     - $\mathbf{L}$ is the length of the conductive path.
   - See introduction to Lab 12.P2.L1 on Ohm's law for further detail on resistivity.

12.P2.L2.2 Apparatus and Materials

- 1 voltage source of about 3 *V* (See section A.4)
- 300 *cm* of Nichrome wire
- 1 digital electric multi-meter **OR**
  - 1 analog voltmeter
  - (optional) 1 analog ohmeter
- 1 resistor between 300 Ω and 1000 Ω
- 30 *cm* of loose wire
- 1 ruler (at least 15 *cm*)
- 1 role of plaster Tape
- 1 dark marker
- 1 pencil and eraser
- 1 piece of carton, about 30 *cm* by 21 *cm*

12.P2.L2.3 Setup

Note: The same wire spiral from Lab 12.P2.L1 may be reused for this practical.

- If this spiral is reused, it needs to be modified to match the setup shown in figure 5.
- This can be accomplished with the following changes:

  A) Exchange the constant resistor $\mathbf{R_c}$ for one having a resitance of 500 Ω
  - If 500 Ω is not available, anything between 300 Ω and 1000 Ω works.

  B) Connect the constant resistor $\mathbf{R_c}$ directly to the battery's positive terminal.

  C) Connect the voltmeter in parallel to the constant resistor $\mathbf{R_c}$.
  - This is done by touching each of the voltmeter's probes to each of the resistor's lead wires.

If the sprial from Lab 12.P2.L1 is not available, continue to Step 1 below.

1. Use the pencil to draw a grid on the piece of carton as shown in Figure 2.
   - Space the lines 2 *cm* apart.
   - Draw the grid to be 20 *cm* wide and 20 *cm* high.
   - This should result in 11 horizontal lines and 11 vertical lines.

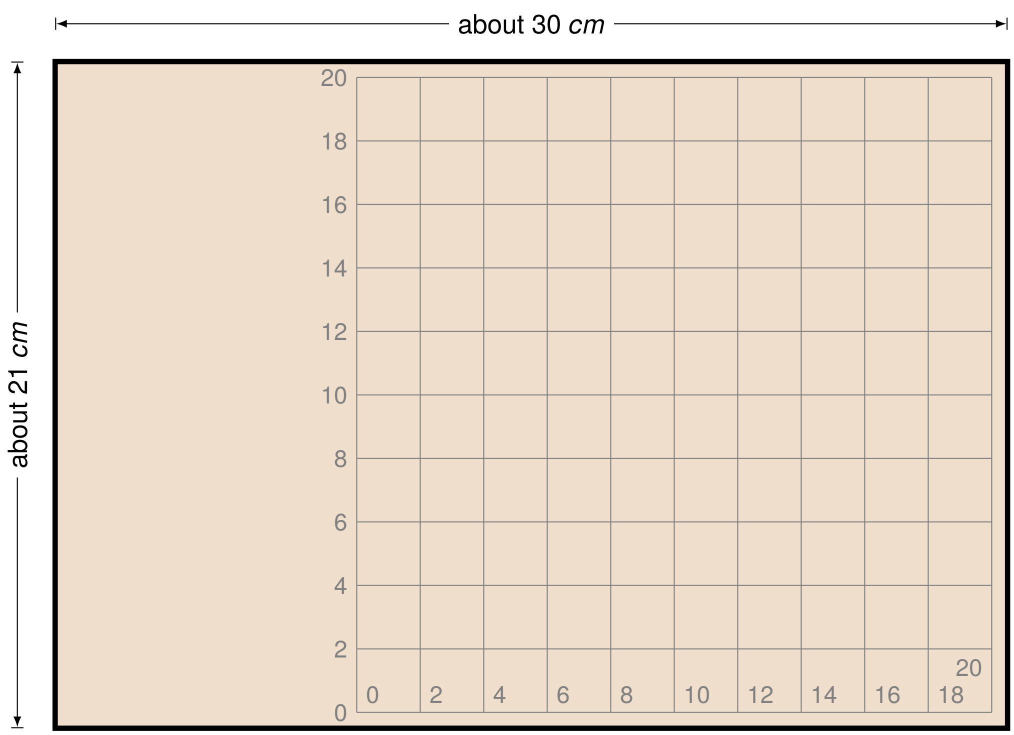


Figure 2

2. Use the marker to make dots at the line intersections shown in Figure 3.

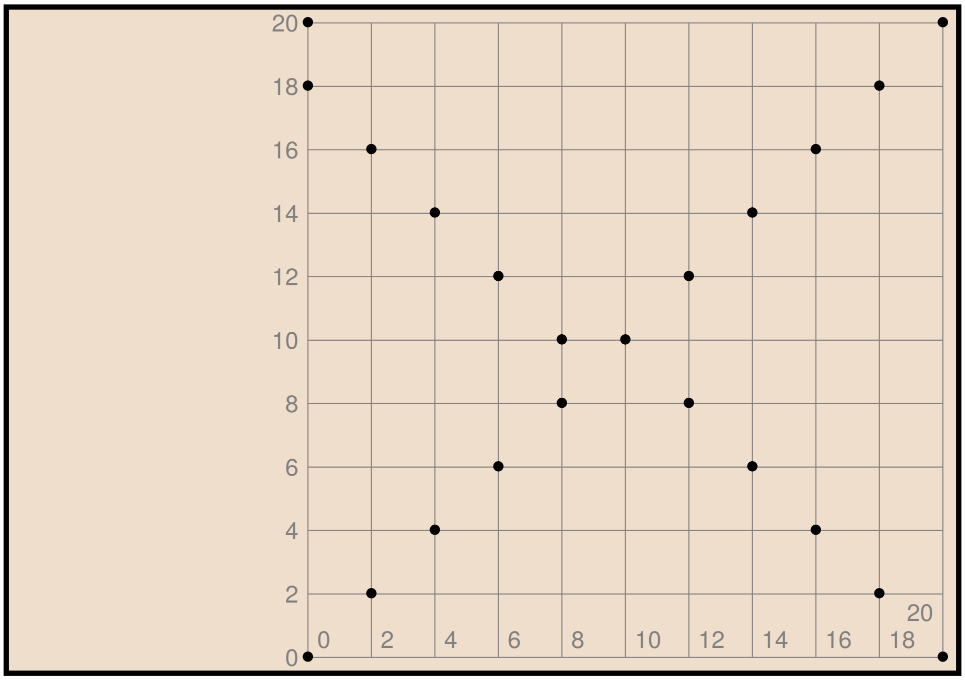


Figure 3

3. Use the eraser to remove the horizontal and vertical lines as well as any number labels.
4. Use the Nichrome wire to connect each of the dots in a spiral fashion.
   - Start in the upper-left corner.
   - Work clockwise, inwards.
   - At each dot, use tape to fix the wire in place, but leave the 90° bend exposed.
5. At each dot/bend, label the distance along the wire from the spiral's starting point **P**.

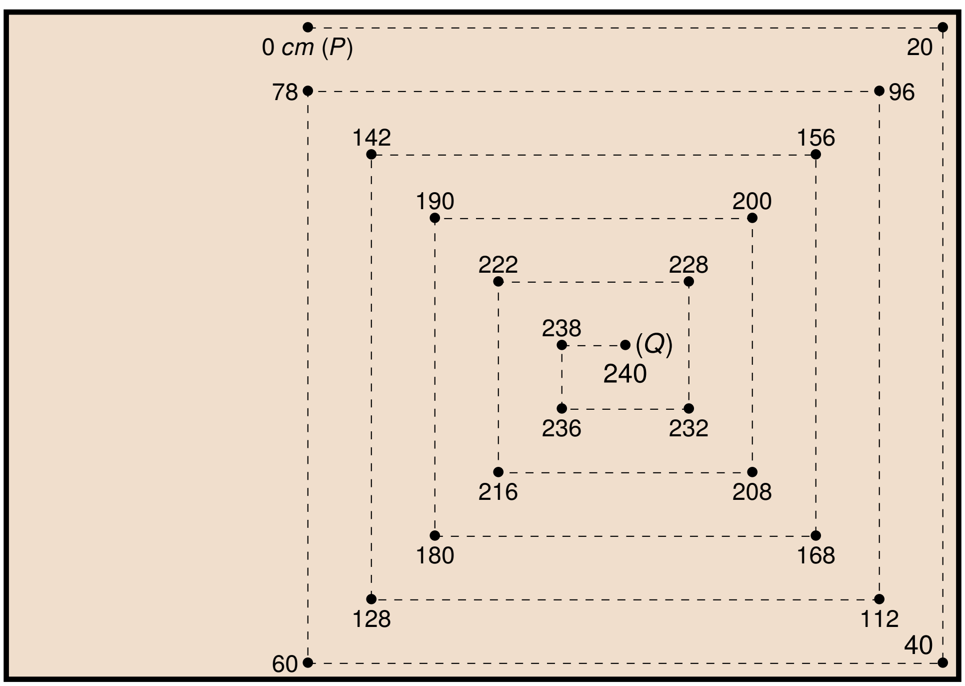


Figure 4

6. Attach the resistor to the Nichrome wire before node **P**.
7. Attach the other end of the resistor to the positive terminal of the voltage source.
8. Attach the positive lead/wire of the voltmeter to the end of the resistor closer to **P**.
9. Attach the negative lead/wire of the voltmeter to the other end of the resistor.
10. Let the wire **X** attached to the voltage source's negative lead rest away from the carton.
    - Take **caution** that this negative lead doesn't touch any part of the circuit on the carton.

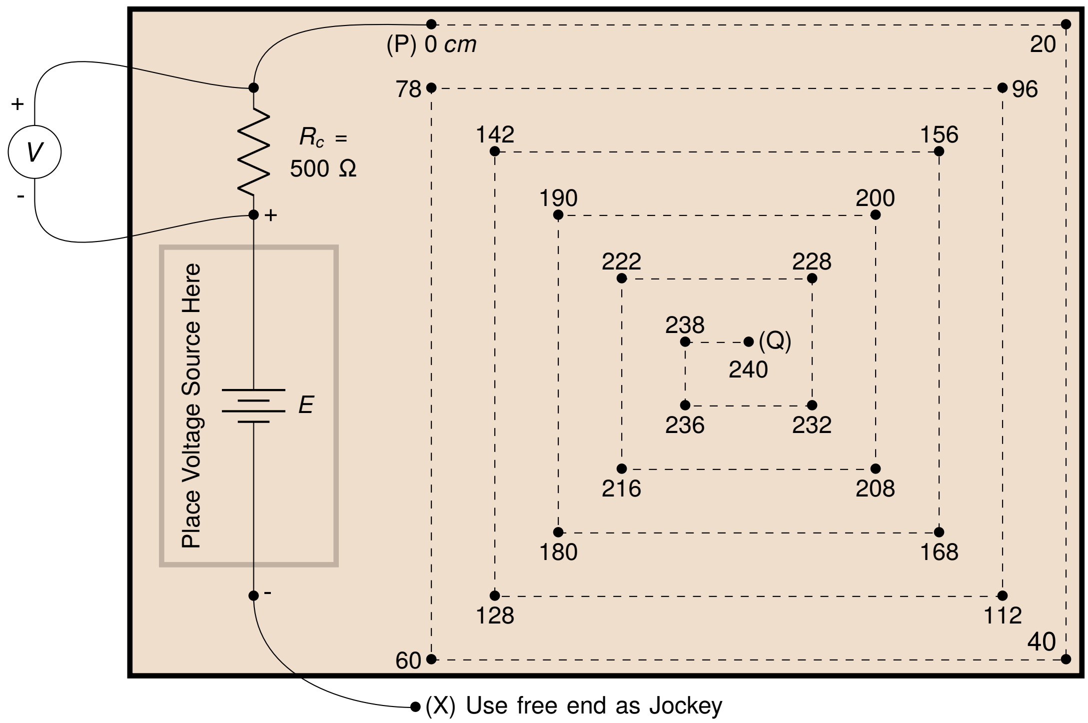


Figure 5

12.P2.L2.4 Warm Up Questions

1. Why isn't the resistance of the wires also considered in this practical's circuit?
    - The resistance of all wires should be far less than that of the resistor and bare resistance wire. Therefore, the share of the source voltage which is dissipated across the wires should be negligible.
2. If the resistance $\mathbf{R}_1$ in Figure 1 were significantly *less* than that of $\mathbf{R}_2$, would *most* of the voltage from $\mathbf{V}_{\mathbf{emf}}$ be dissipated across $\mathbf{R}_1$ or $\mathbf{R}_2$?
    - Most would be dissipated across $\mathbf{R}_2$, as the coefficient $\frac{\mathbf{R}_2}{\mathbf{R}_1 + \mathbf{R}_2}$ would be greater than half.
3. If the resistance $\mathbf{R}_1$ in Figure 1 were significantly *less* than that of $\mathbf{R}_2$, but they were arranged in parallel, as opposed to in series, would *most* of the voltage from $\mathbf{V}_{\mathbf{emf}}$ be dissipated across $\mathbf{R}_1$, $\mathbf{R}_2$, or other?
    - The voltage drop across each would be the same, as is the case for all resistors in parallel.
4. Consider the following terms present in Equation 1:
    - $\mathbf{V}_{\mathbf{AB}}$
    - $\mathbf{V}_{\mathbf{emf}}$
    - $\mathbf{R}_1$
    - $\mathbf{R}_2$
    - $\frac{\mathbf{R}_1}{\mathbf{R}_1+\mathbf{R}_2}$

    If, in this practical, $\mathbf{R}_2$ is considered as the variable resistance along a bare resistance (Nichrome) wire,

    a) Which of these terms are assumed constant?
        - $\mathbf{V}_{\mathbf{emf}}$ and $\mathbf{R}_1$

    b) Which of these terms are assumed variable?
        - $\mathbf{V}_{\mathbf{AB}}$, $\mathbf{R}_2$ and $\frac{\mathbf{R}_1}{\mathbf{R}_1+\mathbf{R}_2}$

12.P2.L2.5 Procedure and Calculations

- Students should collect data similar to Table 1 using the steps below.

L (**cm**)	**L** (**m**)	**V** (**V**)	**V** $\left(\mathbf{V}^{-1}\right)$
20	0.20	3.10	0.323
112	1.12	2.69	0.372
156	1.56	2.53	0.395
208	2.08	2.37	0.422
240	2.40	2.28	0.439

Table 1

A) Create an empty table of 4 columns and 6 rows.

B) **Row 1, Header**: Fill in the header information as shown.

C) **Column 1, L** (**cm**): Fill in the list of wire lengths as shown.

D) **Column 2, L** (**m**): Convert each of these lengths to meters using

$$L_m = L_{cm}\left(\frac{1\ m}{100\ cm}\right) \qquad \text{(Equation 4)}$$

E) Touch the Jockey (**X**) to the Nichrome wire corner at the 20 **cm** node.

F) **Column 3, V** (**V**): Record the voltmeter's voltage reading.
   - Remove the Jockey (**X**) from the Nichrome wire as soon as the recording is taken.

G) **Columns 4, V**$^{-1}$ $\left(\mathbf{V}^{-1}\right)$: Take the inverse of this voltage reading using

$$V^{-1} = \frac{1}{V} \qquad \text{(Equation 5)}$$

H) Repeat steps E) through G) for all remaining wire positions.

12.P2.L2.6 Data Plotting and Slope/Intercept Determination

1. Students should plot the last column of Table 1 against its 2nd, similar to Figure 6 below.

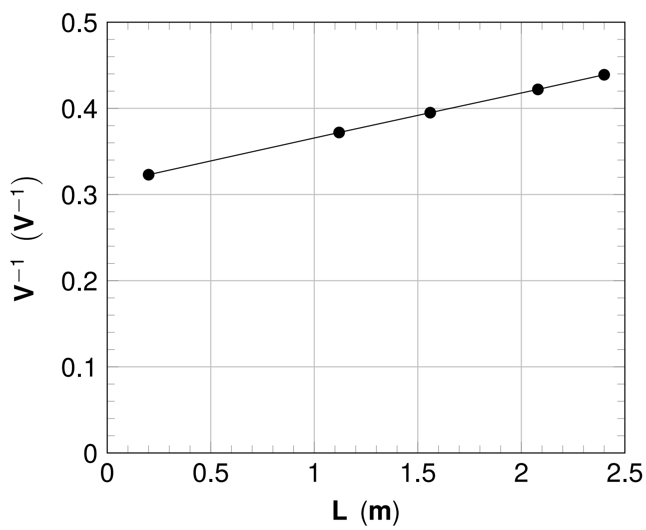


Figure 6

2. Students should then determine the graph's **slope** using

$$\text{general equation for the slope of a straight line: } s = \frac{y_f - y_i}{x_f - x_i}$$

$$\text{specifying for this practical: } s = \frac{\left(V^{-1}\right)_f - \left(V^{-1}\right)_i}{(L)_f - (L)_i}$$

$$\text{substituting known values: } s = \frac{\left(0.439\ V^{-1}\right) - \left(0.323\ V^{-1}\right)}{(2.4\ m) - (0.2\ m)}$$

$$\text{solving: } s = 0.0527\ \frac{V^{-1}}{m}$$

3. Students should then determine the graph's **vertical intercept** using

   general equation for the vertical intercept of a straight line: $y_o = y_i - s(x_i)$

$$\text{specifying for this practical: } \left(V^{-1}\right)_o = \left(V^{-1}\right)_i - s\,(L)_i$$

$$\text{substituting known values: } \left(V^{-1}\right)_o = \left(0.323\ V^{-1}\right) - \left(0.0527\ \frac{V^{-1}}{m}\right)(0.2\ m)$$

$$\text{solving: } \left(V^{-1}\right)_o = 0.312\ V^{-1}$$

4. Students should then calculate $\mathbf{k}_1$ where $\mathbf{k}_1 = \left[\left(\mathbf{V}^{-1}\right)_{\mathbf{o}}\right]^{-1}$.

$$\text{given equation: } k_1 = \frac{1}{\left(V^{-1}\right)_o}$$

$$\text{substituting known values: } k_1 = \frac{1}{0.312\ V^{-1}}$$

$$\text{solving: } k_1 = 3.205\ V$$

5. Students should then determine the resistance $\mathbf{R_c}$ of the constant resistor connected to the Nichrome wire.
   - If a digital multimeter is available, use its ohmeter feature.
   - If not, students can determine the resistance from the bands on the resistor.
   - As a last resort, the students can be told the resistor's resistance directly.
6. Students should then calculate $\mathbf{k}_2$ where $\mathbf{k}_2 = (\mathbf{s})\,(\mathbf{k}_1)\,(\mathbf{R_c})$.

   (*Note the following solution assumes a calculated* $\mathbf{k}_1 = 3.205\ V$ *and* $\mathbf{R_c} = 500\ \Omega$)

$$\text{given equation: } k_2 = (s)\,(k_1)\,(R_c)$$

$$\text{substituting known values: } k_2 = \left(0.0527\frac{V^{-1}}{m}\right)(3.205\ V)\,(500\ \Omega)$$

$$\text{solving: } k_2 = 84.452\ \frac{\Omega}{m}$$

12.P2.L2.7 Exam Prompt

A battery **E**, a standard resistor **R**, a bare resistance wire **XY** and a jockey are connected as shown in Figure 7.

A voltmeter is used to measure the potential difference, **V** across the standard resistor for various lengths **L**, of the wire.

The jockey is made to touch the wire at **P** and the length **L** = **XP** is measured and recorded. The procedure is **repeated** for **five** other values of **L** to obtain their corresponding values of **V**.

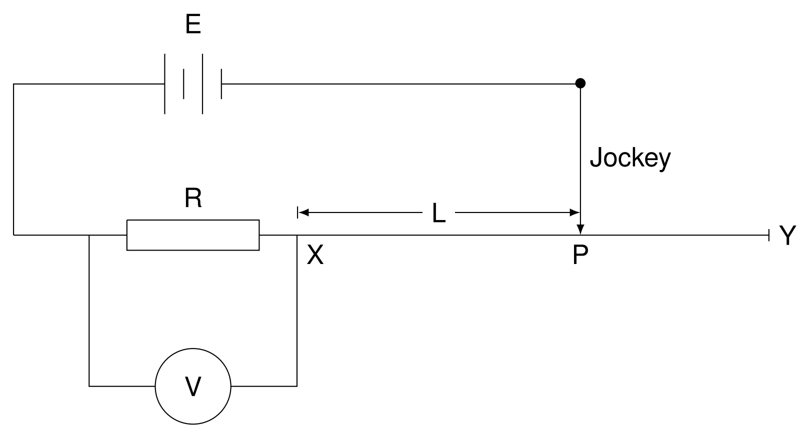
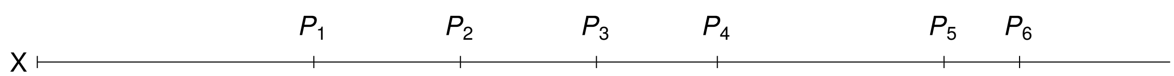


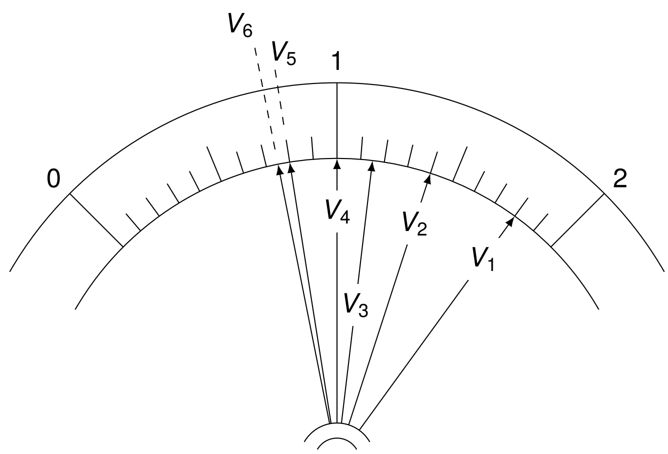
Figure 7

Figure 8 shows the lengths $\mathbf{L_i}$ = $\mathbf{XP_i}$ of the wire while Figure 9 shows the corresponding potential differences $\mathbf{V_i}$ where **i** = 1, 2, 3, 4, 5 and 6, respectively.



Scale: 1 *cm* represents 5 *cm*

Figure 8



Voltmeter (V)

Figure 9

(i) Measure and record the raw values of the length $\mathbf{L_{raw}}$ of wire.
(ii) Convert the raw values $\mathbf{L_{raw}}$ in (i) above to actual values of $\mathbf{L_{real}}$ using the given scale.
(iii) Read and record the corresponding values of the potential differences $\mathbf{V_i}$.
(iv) Evaluate $\mathbf{V}^{-1}$.
(v) Tabulate your readings.
(vi) Plot a graph with $\mathbf{V}^{-1}$ on the vertical axis and $\mathbf{L_{real}}$ on the horizontal axis, starting both axes from $(0, 0)$.
(vii) Determine the slope, **s**, of the graph.
(viii) Determine the value of **V** when **L** = 0 *cm*.
(ix) State **two** precautions that are necessary to ensure accurate results when performing this experiment.

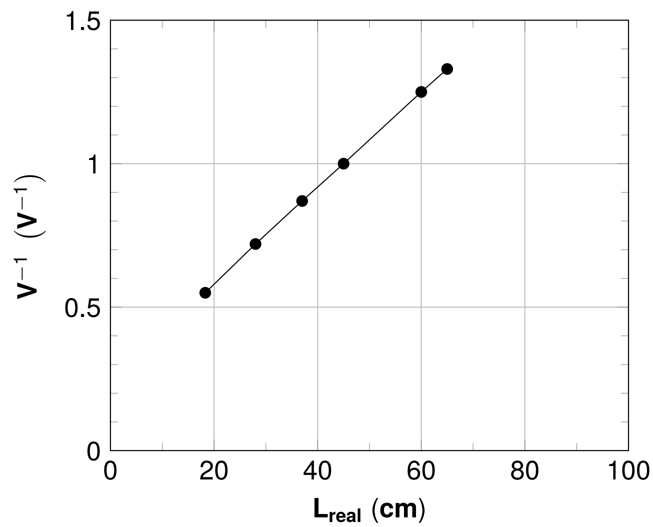
12.P2.L2.8 Solutions to Exam Prompt

(ii) Use $\mathbf{L_{real}} = \left(\frac{5\ cm}{1\ cm}\right) h_{raw}$

(v)

i	$\mathbf{L_{raw}}$ (cm)	$\mathbf{L_{real}}$ (cm)	V (V)	$\mathbf{V^{-1}}$ ($\mathbf{V^{-1}}$)
1	3.7	18.3	1.80	0.55
2	5.6	28.0	1.40	0.72
3	7.4	37.0	1.15	0.87
4	9.0	45.0	1.00	1.00
5	12.0	60.0	0.80	1.25
6	13.0	65.0	0.75	1.33

(vi)



(vii)

general equation for the slope of a straight line: $s = \frac{y_f - y_i}{x_f - x_i}$

specifying for this practical: $s = \frac{\left(V^{-1}\right)_f - \left(V^{-1}\right)_i}{(L)_f - (L)_i}$

substituting known values: $s = \frac{\left(1.33\ V^{-1}\right) - \left(0.55\ V^{-1}\right)}{(65.0\ cm) - (18.3\ cm)}$

solving: $s = 0.0167\ \frac{V^{-1}}{cm}$

(viii)

problem setup

$$\text{considering known inverse: } V = \frac{1}{V^{-1}}$$

$$\text{considering inverse's value at given location: } V = \frac{1}{\left(V^{-1}\right)_o} \text{ when } L = 0\ cm$$

solving for vertical intercept

$$\text{general equation for vertical intercept of straight line: } y_o = y_i - s(x_i)$$

$$\text{specifying for this practical: } \left(V^{-1}\right)_o = \left(V^{-1}\right)_i - s\,(L)_i$$

$$\text{substituting known values: } \left(V^{-1}\right)_o = \left(0.55\ V^{-1}\right) - \left(0.0167\ \frac{V^{-1}}{cm}\right)(18.3\ cm)$$

$$\text{solving: } \left(V^{-1}\right)_o = 0.244\ V^{-1}$$

using vertical intercept value to answer original question

$$\text{substituting known values: } V = \frac{1}{0.244\ V^{-1}} \text{ when } L = 0\ cm$$

$$\text{solving: } V = 4.098\ V \text{ when } L = 0\ cm$$

(ix) Precautions include
- being sure the voltmeter presents a zero reading before connection to minimize non-zero error;
- (if analog) reading the voltmeter at a 90° angle to minimize parallax error;
- confirming all circuit connections are secure to minimize contact resistance in circuit components.

12.P2.L2.9 Post-Lab Questions - High School

1. Why do we refer to these circuits as “voltage dividers”?
   - The total voltage/emf of the source is being “divided” across each resistor.
2. Considering this practical’s graph, what value of **p** renders the expression $\mathbf{V^p} \propto \mathbf{L}$ valid?
   - **p** = -1, which is why we took the inverse of **V** to obtain a linear relationship with **L**.
3. Consider two resistor’s arranged in series $\mathbf{R_a}$ = 3 Ω and $\mathbf{R_b}$ = 4 Ω.
   a) Calculate α, the ratio of $\mathbf{R_a}$ over the sum $\mathbf{R_a}$+$\mathbf{R_b}$.

$$\text{given equation: } \alpha = \frac{R_a}{R_a + R_b}$$

$$\text{subtituting known values: } \alpha = \frac{3\ \Omega}{3\ \Omega + 4\ \Omega}$$

$$\text{solving: } \alpha = 0.429$$

   b) Calculate the voltage drop across $\mathbf{R_a}$ if a voltage of $\mathbf{V_T}$ = 14 V is applied across the resistor pair.

$$\text{voltage drop across first resistor is ratio of total voltage: } V_a = \alpha V_T$$

$$\text{subtituting known values: } V_a = 0.429(14\ V)$$

$$\text{solving: } V_a = 6\ V$$

12.P2.L2.10 Post-Lab Questions - University Level 1

4. Calculate the current in the circuit when **XP** = 60 *cm* if the *emf* of the battery in this practical is 5 *V*, the voltage drop across **R** is 3 *V*, and the resistance per unit length of the bare resistance wire is 2 Ωm^{-1}.

$$\text{considering voltage drop across both resistive elements: } E = V_{resistor} + V_{br\ wire}$$

$$\text{substituting Ohm's law for wire: } E = V_{resistor} + (I_{br\ wire})(R_{br\ wire})$$

$$\text{assuming current is constant throughout circuit: } E = V_{resistor} + I\,(R_{br\ wire})$$

$$\text{isolating current: } I = \frac{E - V_{resistor}}{R_{br\ wire}}$$

$$\text{considering resistivity of wire: } I = \frac{E - V_{resistor}}{(L_{br\ wire})\left(\frac{\rho}{A_{cs}}\right)}$$

$$\text{substituting known values: } I = \frac{(5\ V) - (3\ V)}{(60\ cm)\left(\frac{1\ m}{100\ cm}\right)\left(2\ \frac{\Omega}{m}\right)}$$

$$\text{solving: } I = 1.667\ A$$

12.P2.L2.11 Post-Lab Questions - University Level 2

5. Using the given equation for voltage division, derive a linear equation for this practical's graph as

$$V^{-1} = s\,(L) + c$$

where **s** is the slope and **c** is the vertical intercept. Use only the terms

- **E** - the source voltage;
- **V** - the voltage drop across the resistor;
- $\mathbf{R_c}$ - the resistance of the fixed resistor;
- **L** - the length of bare resistance wire contributing to its variable effective resistance;
- $\frac{\rho}{\mathbf{A_{cs}}}$ - the resistance per unit length of the bare resistance wire.

Solution

$$\text{adapting equation for voltage division to this practical: } V = E\left(\frac{R_c}{R_c + R_{br\ wire}}\right)$$

$$\text{considering resistivity of bare resistance wire: } V = E\left(\frac{R_c}{R_c + \left(\frac{\rho}{A_{cs}}\right)L}\right)$$

$$\text{inverting both sides: } V^{-1} = \frac{R_c + \left(\frac{\rho}{A_{cs}}\right)L}{ER_c}$$

$$\text{expanding: } V^{-1} = \frac{L}{ER_c}\left(\frac{\rho}{A_{cs}}\right) + \frac{R_c}{ER_c}$$

$$\text{simplifying and rearranging: } V^{-1} = \left[\left(\frac{1}{ER_c}\right)\left(\frac{\rho}{A_{cs}}\right)\right](L) + \frac{1}{E}$$

$$\text{adopting the given form: } s = \left(\frac{1}{ER_C}\right)\left(\frac{\rho}{A_{cs}}\right) \text{ and } c = \frac{1}{E}$$

6. Considering the expression derived in Problem 5 above, which circuit component property can be calculated from the intercept term **c**?

   a) Prove this algebraically.

      - **c** can be used to calculate **E**, the voltage of the source.

   using equivalence derived previously: $c = \frac{1}{E}$

   isolating source voltage: $E = \frac{1}{c}$

   b) Evaluate this property using values determined during this practical.

   using equivalence derived previously: $E = \frac{1}{c}$

   substituting known values: $E = \frac{1}{0.312\ V^{-1}}$

   solving: $E = 3.205\ V$

7. Considering the expression derived in Problem 5 above, which quotient can be calculated from the slope term **s** assuming known values of **E** and $\mathbf{R_c}$?

   a) Prove this algebraically.

      - **s** can be used to calculate $\frac{\rho}{\mathbf{A_{cs}}}$, the resistance per unit length of the bare resistance wire.

   using equivalence derived previously: $s = \left(\frac{1}{ER_c}\right)\left(\frac{\rho}{A_{cs}}\right)$

   isolating resistance per unit length: $\frac{\rho}{A_{cs}} = sER_c$

   b) Evaluate this property using values determined during this practical.

   using equivalence derived previously: $\frac{\rho}{A_{cs}} = sER$

   substituting known values: $\frac{\rho}{A_{cs}} = \left(0.0527\ \frac{V^{-1}}{m}\right)(3.205\ V)(500\ \Omega)$

   solving: $\frac{\rho}{A_{cs}} = 84.452\ \frac{\Omega}{m}$

8. Evaluate the resistivity of the bare resistance wire using values determined during this practical as well as the following information:

   - Bare resistance wire is Nichrome, 36 gauge
   - The diameter of a wire can be determined from its American Wire Gauge (AWG) number using

   $$D = e^{2.1104-0.11594(n)}$$

   Where
   - **D** is the wire diameter, in *mm*;
   - **n** is the gauge number (dimensionless).

   Solution

   using equivalence derived previously: $\frac{\rho}{A_{cs}} = 84.452\ \frac{\Omega}{m}$

   isolating resistivity: $\rho = \left(84.452\ \frac{\Omega}{m}\right) A_{cs}$

   susbtituting formula for circle area: $\rho = \left(84.452\ \frac{\Omega}{m}\right) \left(\frac{\pi}{4} D^2\right)$

   susbtituting AWG diameter formula: $\rho = \left(84.452\ \frac{\Omega}{m}\right) \left(\frac{\pi}{4} \left[\left(e^{2.1104-0.11594(n)}\right)(mm)\right]^2\right)$

   applying conversion factor: $\rho = \left(84.452\ \frac{\Omega}{m}\right) \left(\frac{\pi}{4} \left[\left(e^{2.1104-0.11594(n)}\right)(mm)\left(\frac{1\ m}{1000\ mm}\right)\right]^2\right)$

   susbtituting known values: $\rho = \left(84.452\ \frac{\Omega}{m}\right) \left(\frac{\pi}{4} \left[\left(e^{2.1104-0.11594(36)}\right)(mm)\left(\frac{1\ m}{1000\ mm}\right)\right]^2\right)$

   solving: $\rho = 0.0000010699\ \Omega m = 1.07 \cdot 10^{-6}\ \Omega m$

App. A Assembly of Commonly-Used Devices

A.1 Spring

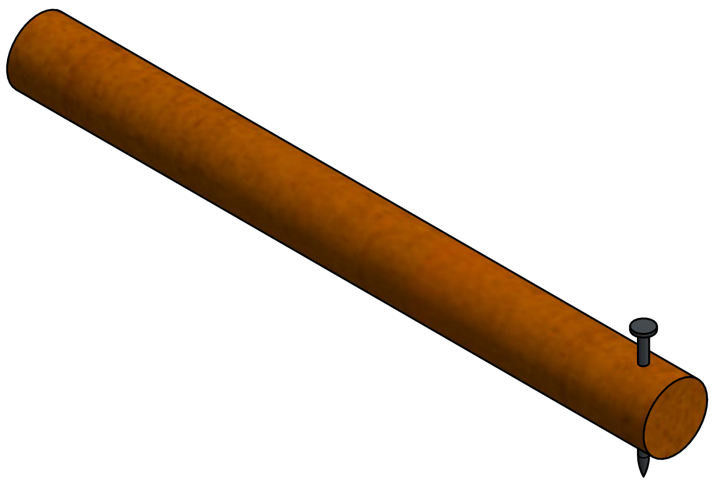
A.1.1 Introduction

- A spring can be used for demonstrations of Hooke's Law as well as oscillation demonstrations.
- Assembly requires

  a) a portion of a broom handle, about 20 *cm* long
  b) tire wire, at least 3 *m* long
  c) a pair of pliers or cutters to cut the tire wire
  d) 1 nail, any size
  e) a hammer

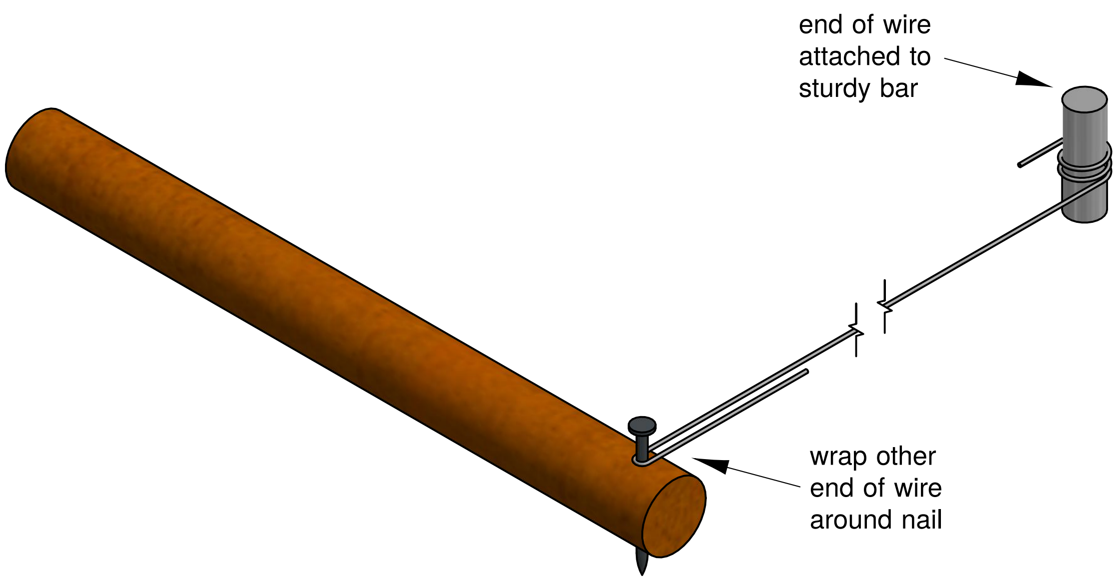
A.1.2 Assembly

A) Hammer a nail into one end of the wooden rod.

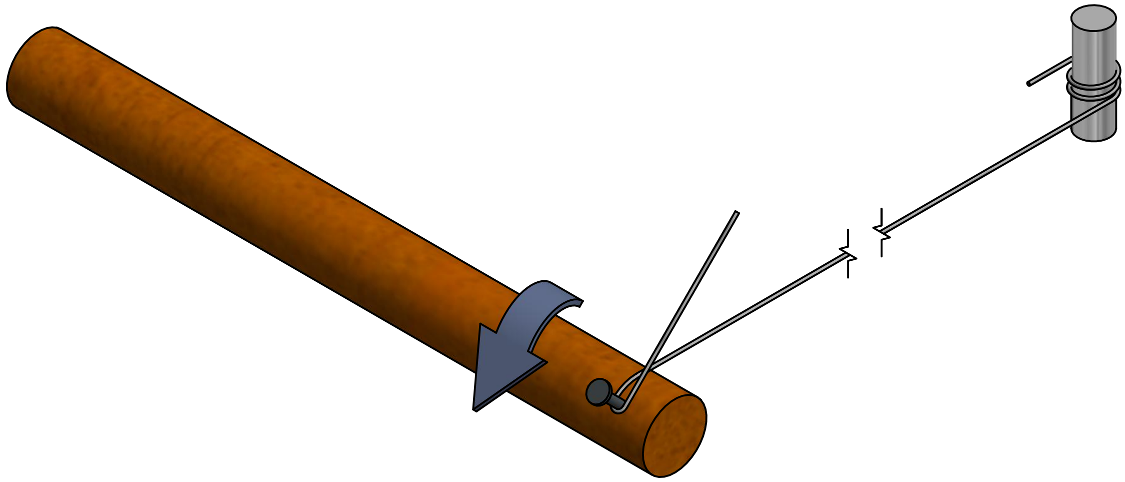


B) Wrap one end of the wire around a sturdy bar.
  - This could be a window's security bars, a plumbing pipe, etc.

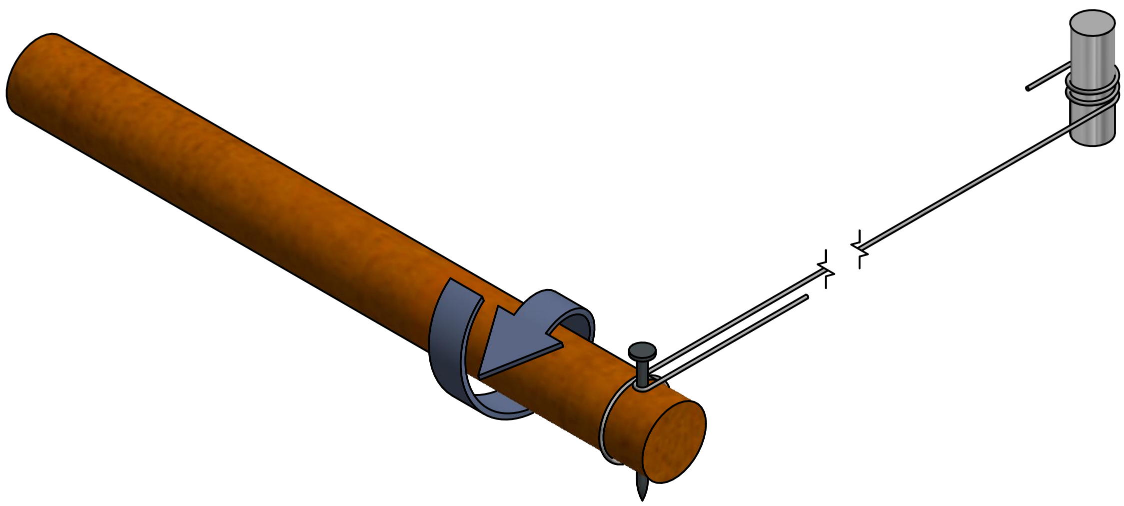
C) Wrap the other end around the nail on the rod.



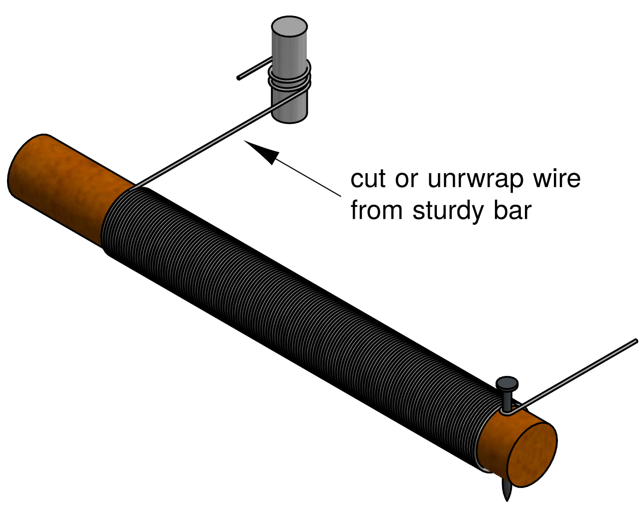
D) Use the rod to pull tight on the wire.
E) Roll the rod away from the sturdy connection.



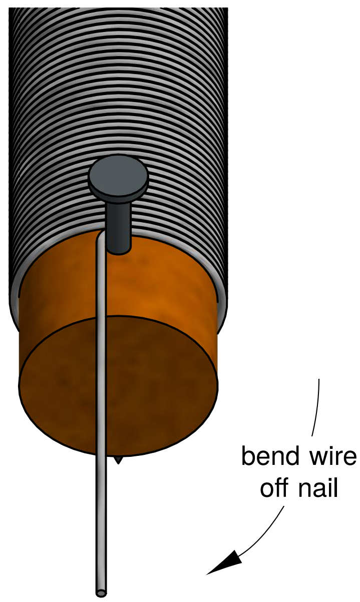
F) Continue rolling the rod to form loops along its length.

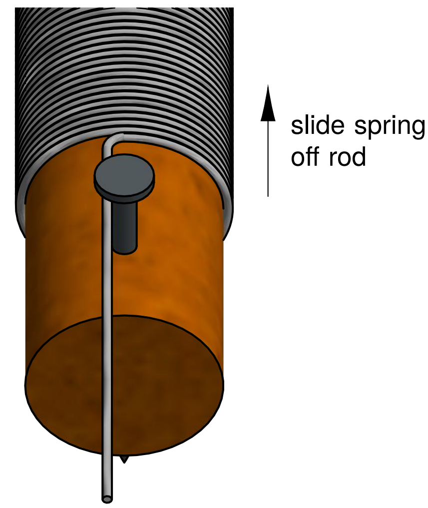


G) Continue until only about 5 *cm* of wire remains
H) Cut or unwrap the wire from the sturdy bar.

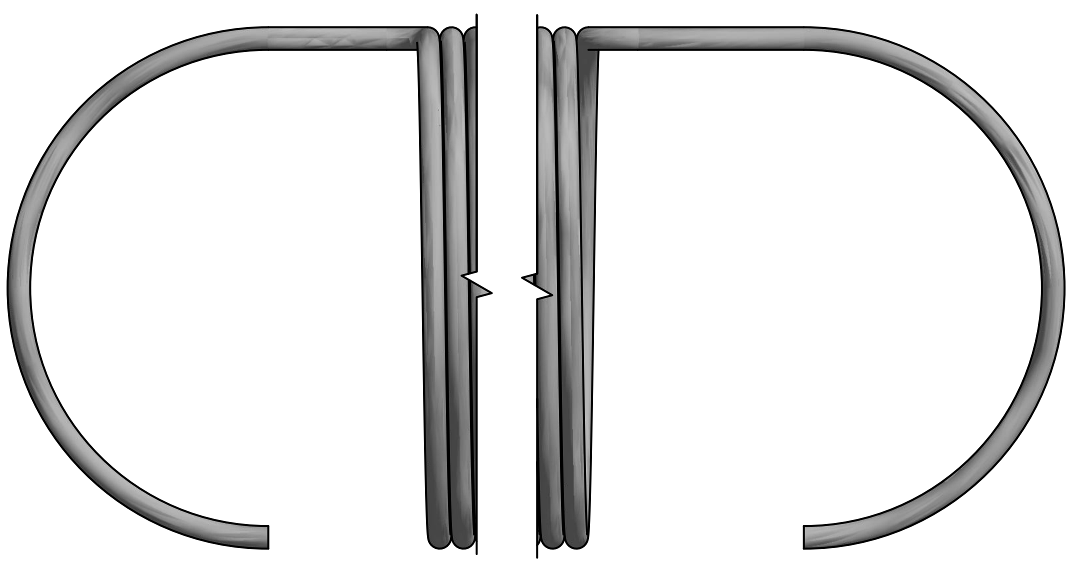


I) Bend the wire away from the nail.
J) Slide the spring off the rod.





K) Bend the remaining wire into a hook on both ends of the spring.



A.2 Beam Balance

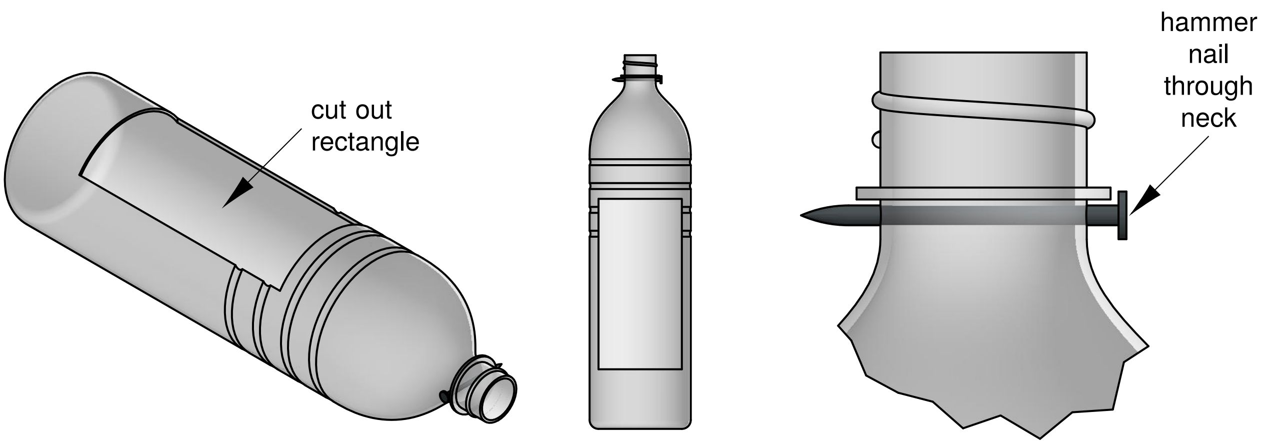
A.2.1 Introduction

- A single beam balance (or just beam balance) is a simple tool that measures the weight or mass of an object.
- Assembly requires

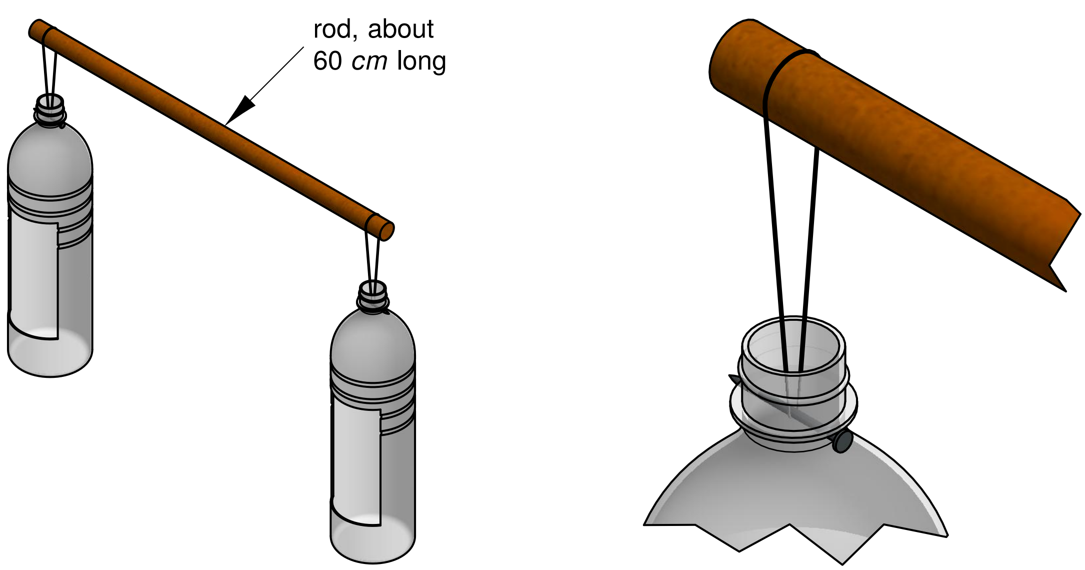
  a) 1 broom handle, about 120 *cm* long
  b) a wood saw (or anything to cut the broom handle)
  c) strong, thin string, about 100 *cm*
  d) a pair of scissors or knife to cut the string
  e) 2 large water bottles, each 1.5 *L*
  f) 2 nails, each 5 *cm* long (or longer than 3 *cm*)
  g) a hammer
  h) tape (plaster or electric)
  i) a ruler (at least 30 *cm* long)

A.2.2 Assembly

A) Use the scissors to cut a large rectangular window into each bottle.
B) Use the hammer to puncture a nail through the neck of each bottle.



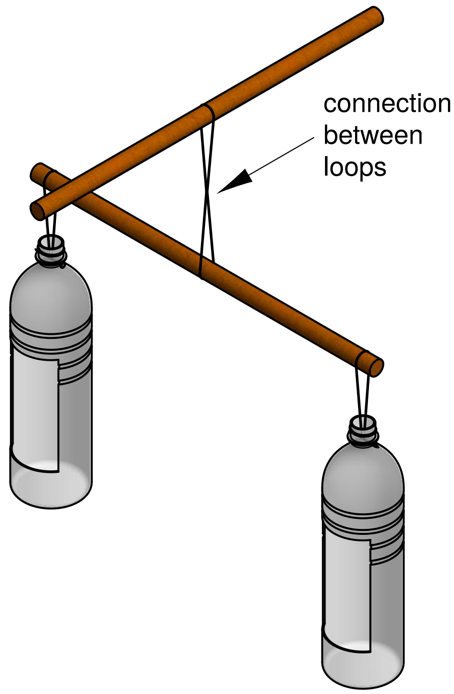
C) Cut the broom handle in half to create two rods, each about 60 *cm* long.
   - Take the rod with the threaded end (where the brush attaches) and set it aside.
D) Cut two pieces of string, each about 20 *cm* long.
E) Use the string pieces to tie the bottles on each end of the non-threaded rod.

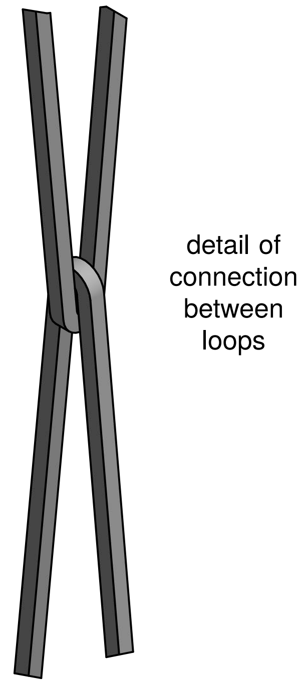


F) Cut two more pieces of string, each about 20 *cm* long.

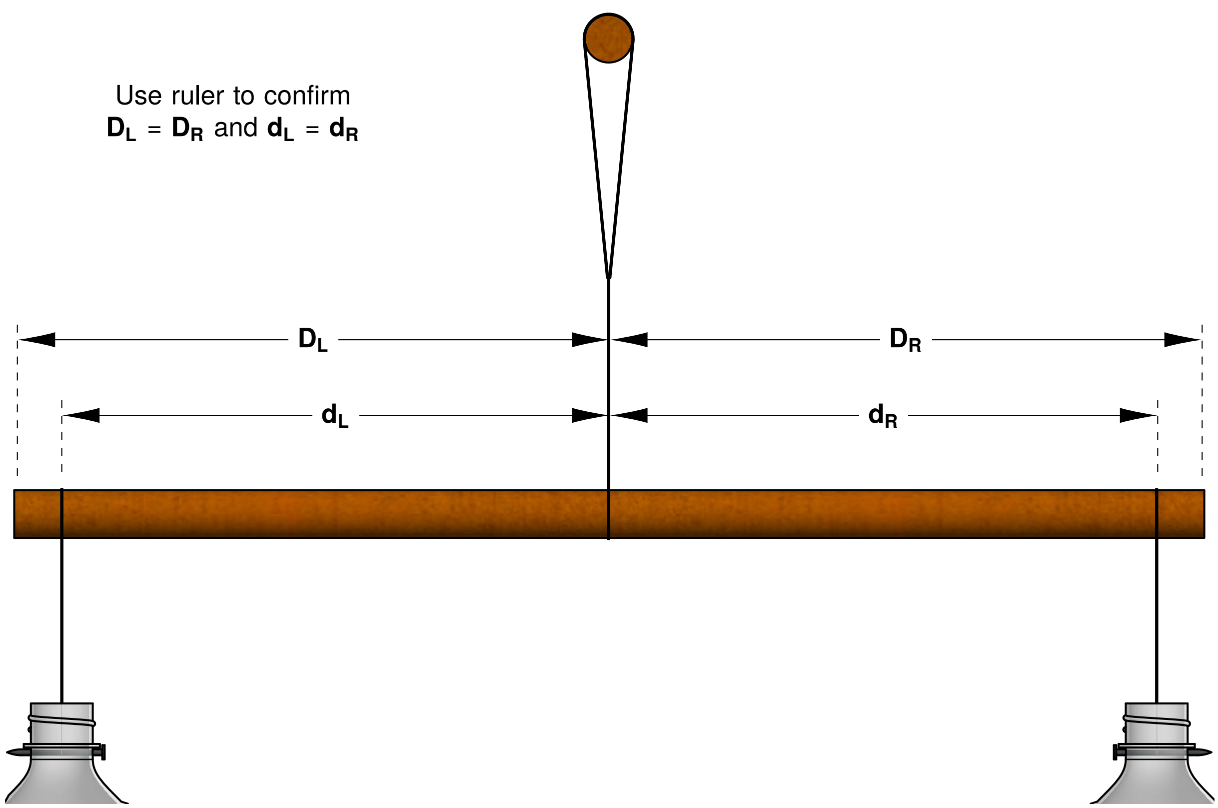
G) Form two connecting loops with each piece.
  - Make sure the loops are connected as shown in the detail below.

H) Use these interconnected loops to attach the two rods.
  - The loop connection should cause the bottom rod to hang at a right angle to the top rod.



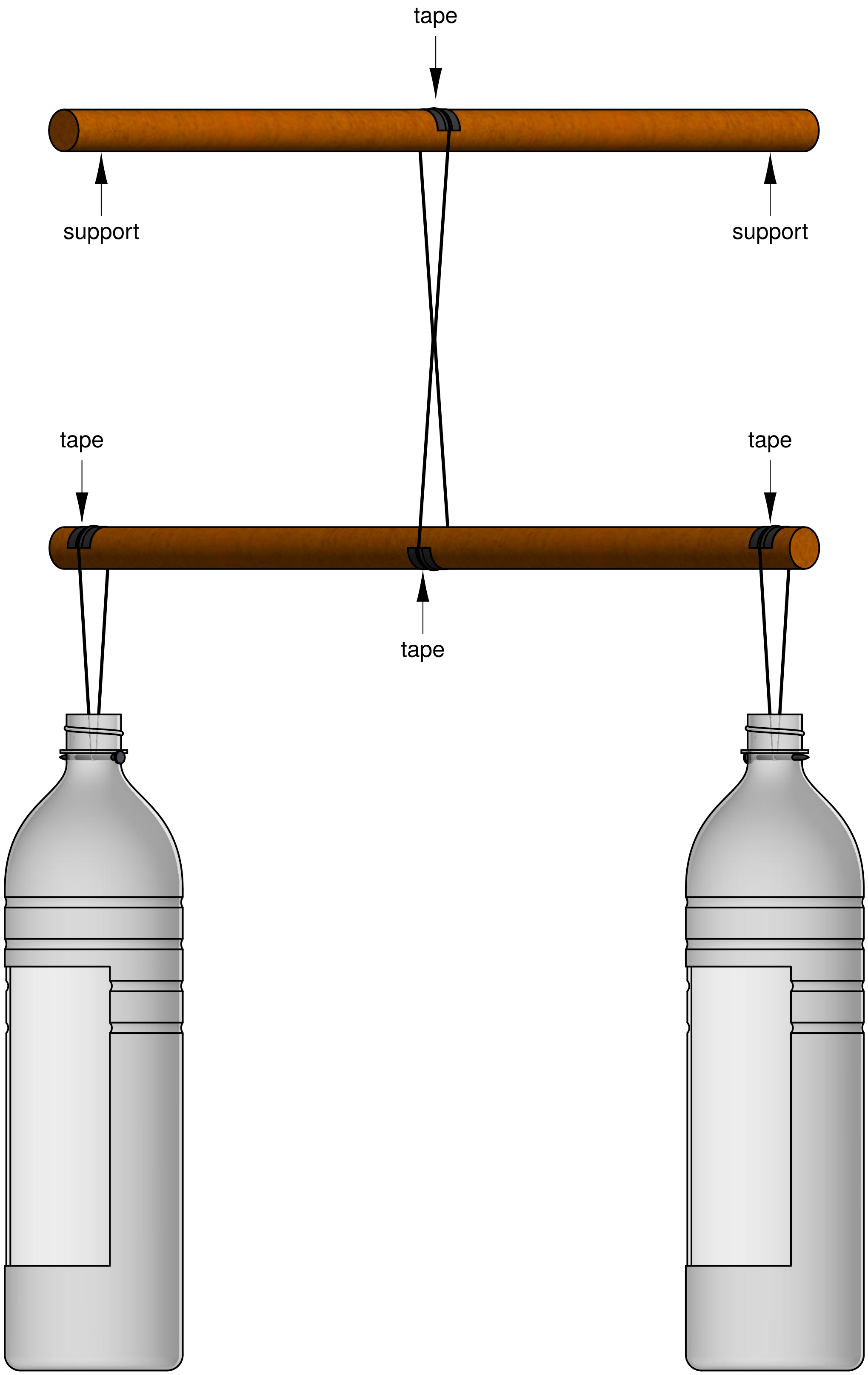


I) Be sure the bottom rod is placed such that all weight is balanced about its center.



J) Once the system is balanced, place tape at all points of connection between string and rod.

K) Use the setup by holding the top rod by hand or resting it across two supports (buckets, chairs, etc).



A.3 Biconvex Lens

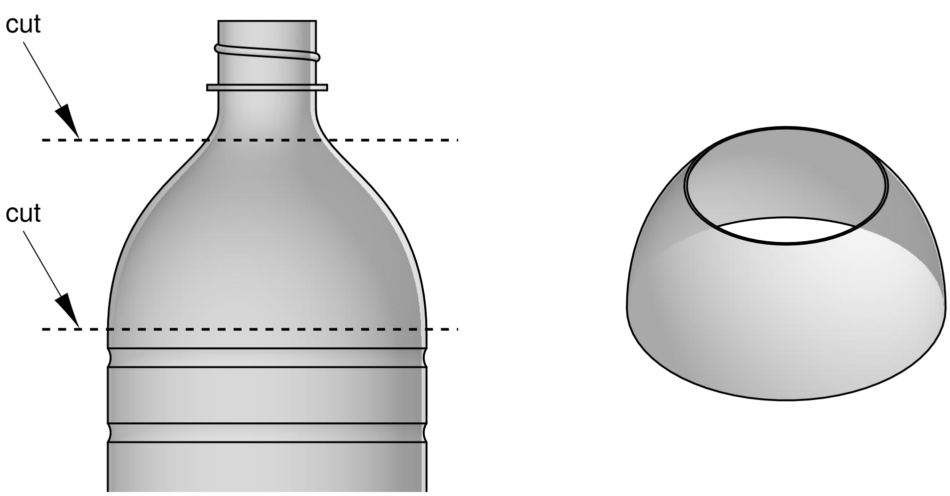
A.3.1 Introduction

- A biconvex lens is a converging lens that can be used to magnify images.
- Assembly requires

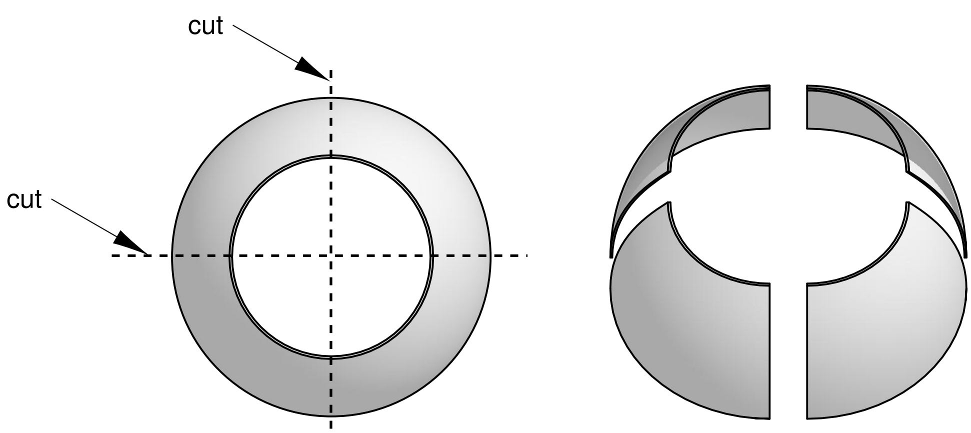
  a) a 1.5 *L* bottle of water
  b) 1 syringe with needle
  c) 4 additional syringe needles
  d) 1 plastic (block) eraser
  e) electric tape
  f) 1 pair of scissors **OR** a knife

A.3.2 Assembly

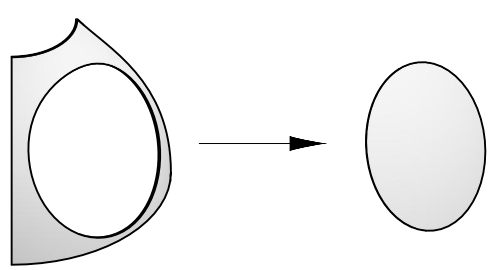
A) Use a pair of scissors or a knife to cut the 1.5 *L* bottle as shown.



B) Cut the bottle piece into four equal pieces.

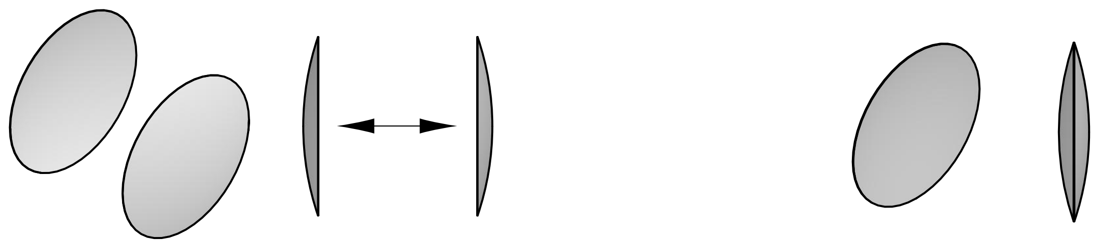


C) Cut two 4.5 *cm* diameter circles from two of the four pieces.

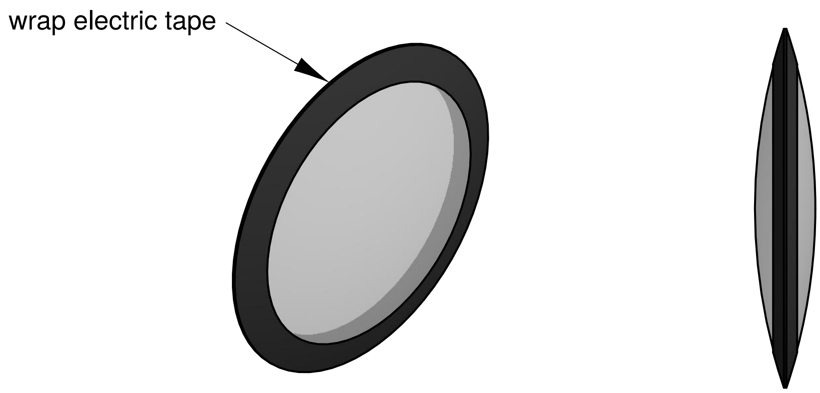


(cut two circles from two pieces)

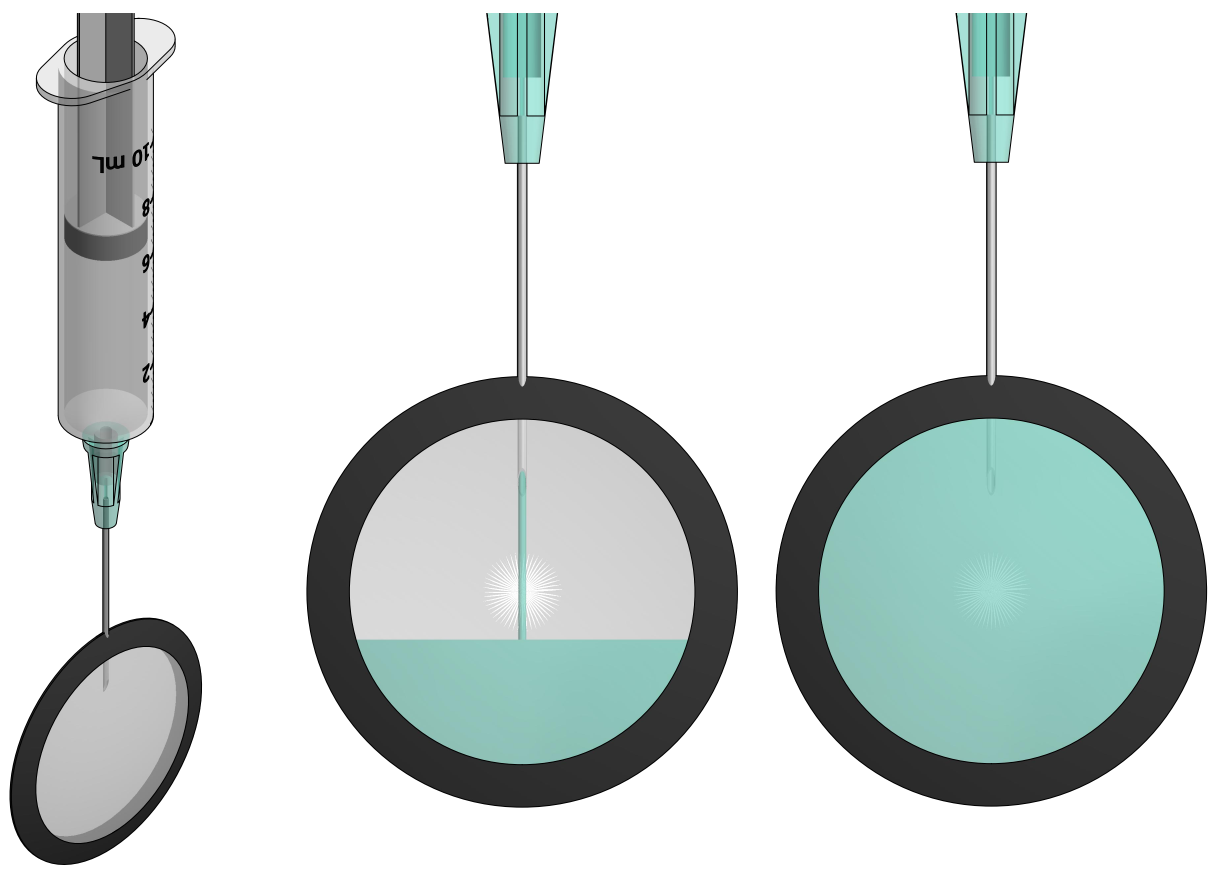
D) Place the two circles next to each other, in opposite directions.



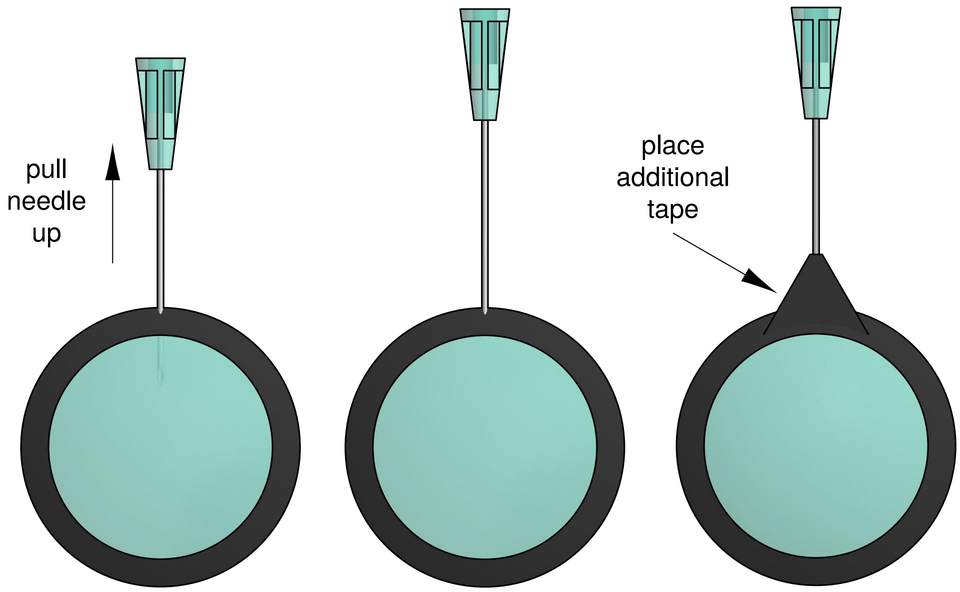
E) Tightly wrap electric tape around the circumference of the assembled circles.



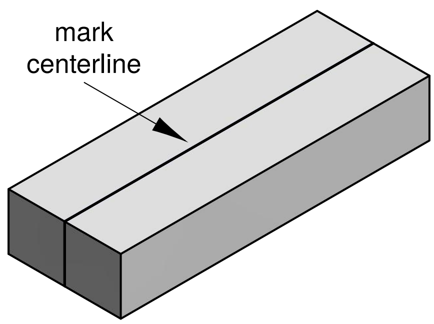
F) Use a syringe to inject water into the assembly.



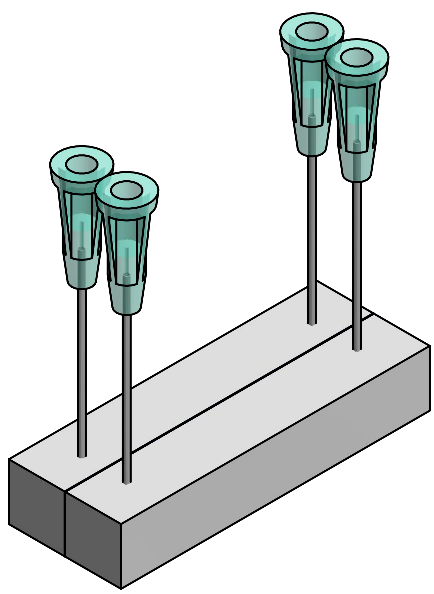
G) Pull the needle up until its tip is hidden by the tape.
H) Place additional electric tape around the needle's insertion point.



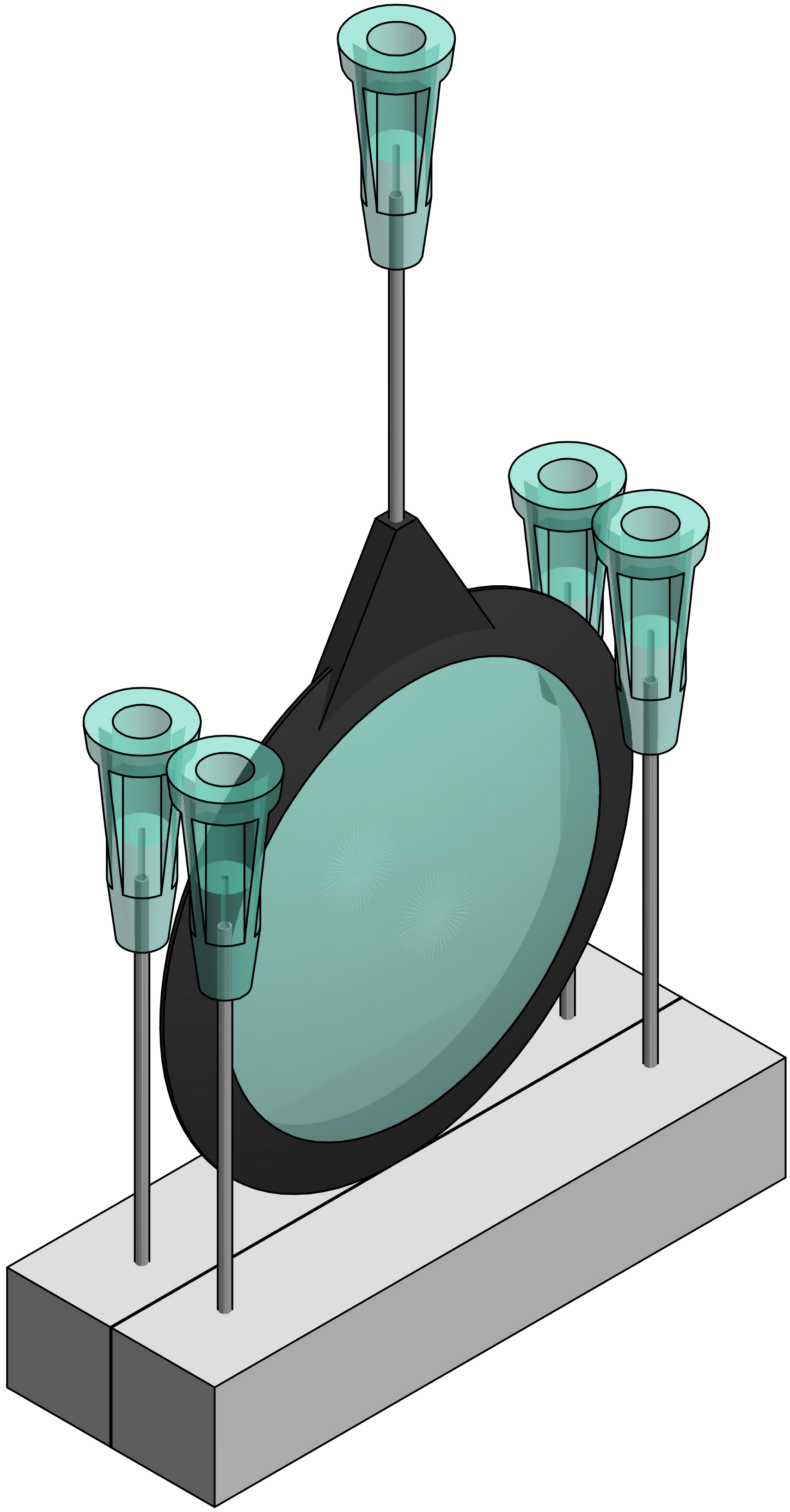
I) Use a pen to mark the center-line of a block eraser.



J) Place four needles on either side of the center line, near the eraser's ends.



K) Guide the lens assembly in between the needles.



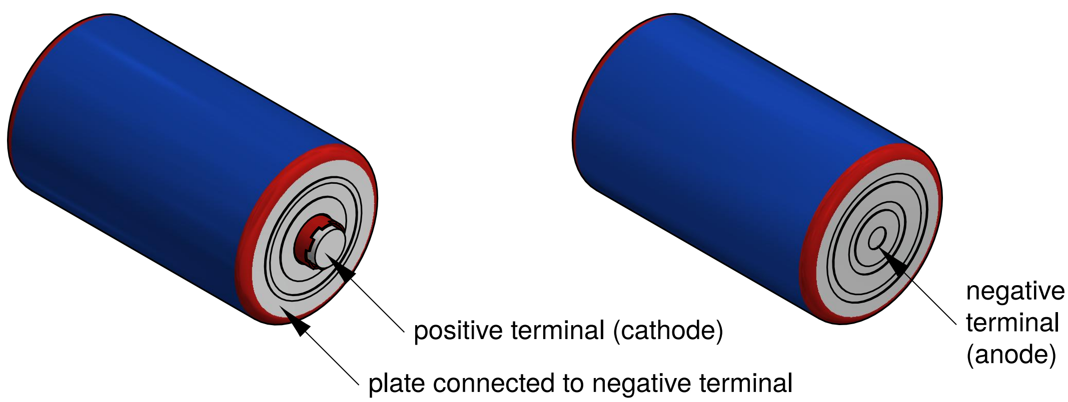
A.4 DC Voltage Source

A.4.1 Introduction

- Many of the practicals involving circuits require some source of Direct Current (DC) Voltage. This can be achieved with simple batteries.
  - Battery sizes “AA”, “AAA”, “C” and “D” all have the same voltage of approximately 1.5 V.
  - The larger size provide their voltage for a longer amount of time.
  - Therefore, it is recommended to use “D” size batteries to save on time and construction materials.
- Assembly requires
  - a) two or more batteries
  - b) plaster tape
  - c) a beer bottle cap
  - d) rubber band
  - e) two pieces of wire, each about 10 cm long

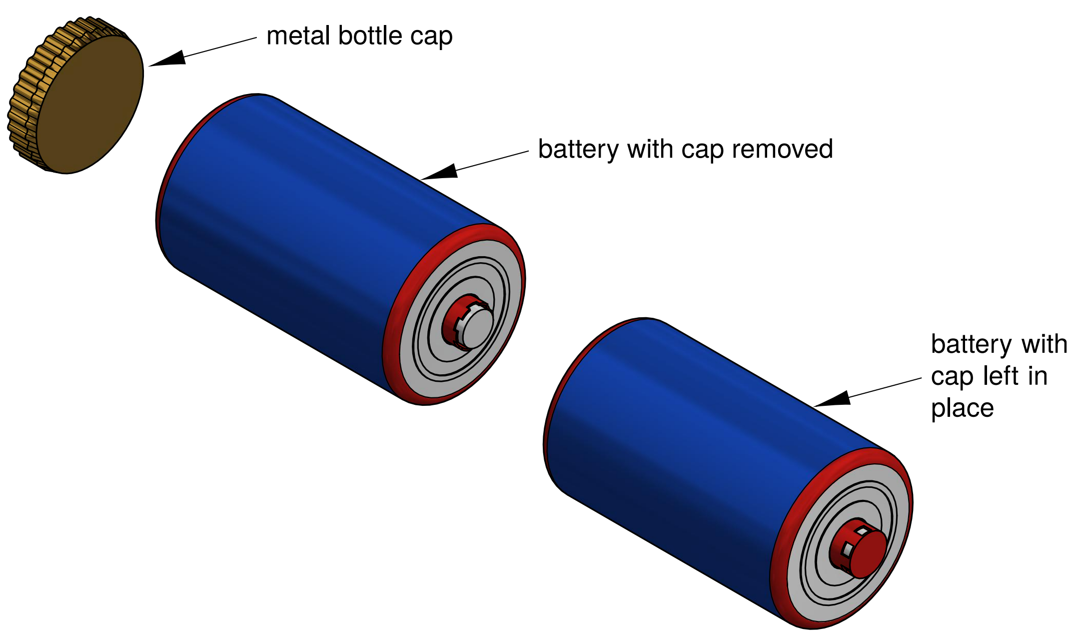
A.4.2 Battery Anatomy

- Each battery has a positive and negative terminal.
- Take **caution** that no contact is made between a battery’s top plate and its positive lead.

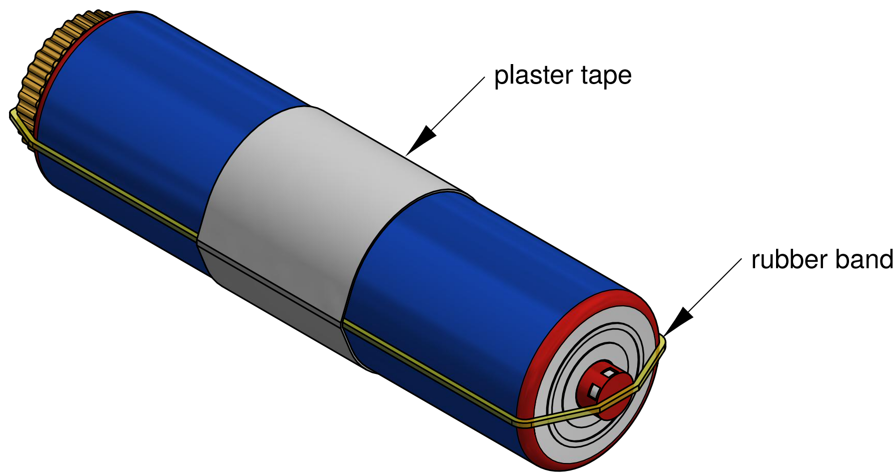


A.4.3 Assembly

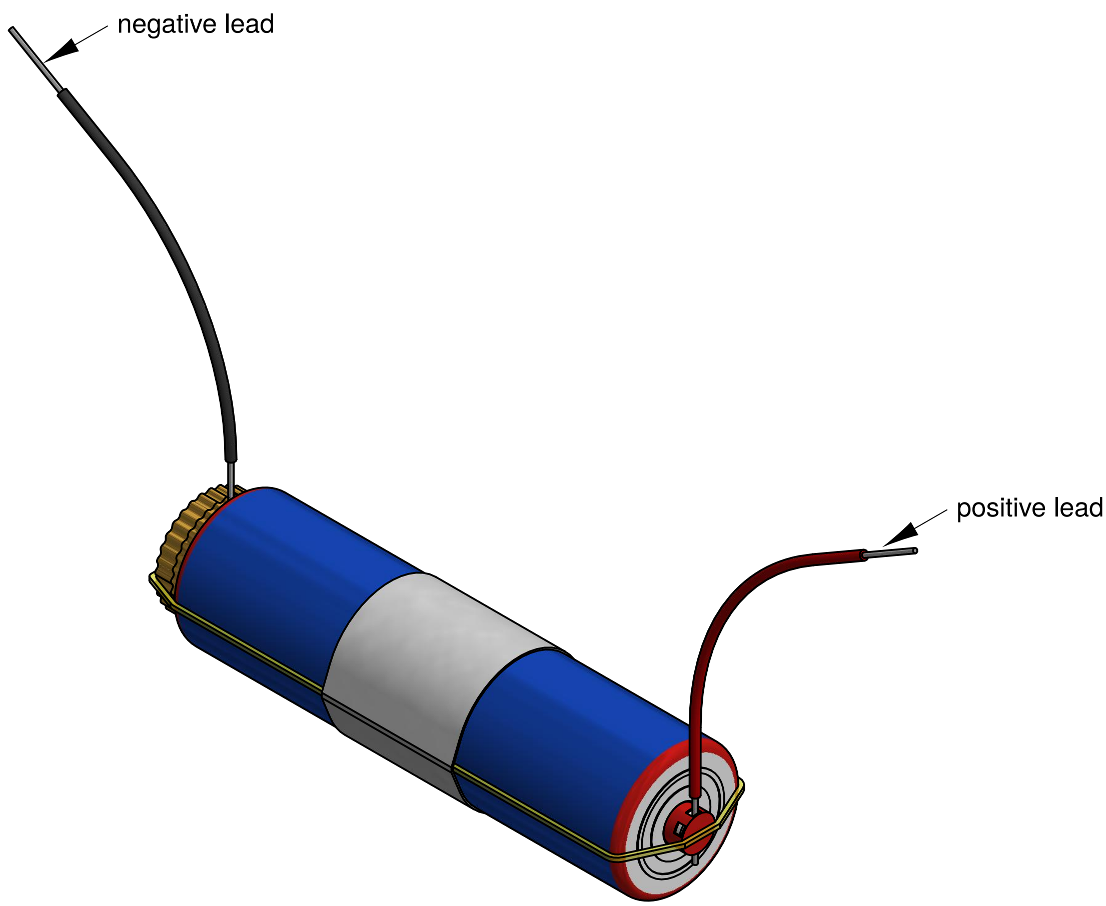
A) Remove the plastic cap from one battery and arrange it with the cap and other battery as shown.



B) Place the rubber band around the three components, and tape it in place as shown.



C) Jam two wires into both exposed terminals of the assembly.



- **Caution** must be taken to make sure the exposed ends of these two wires do not touch.
- Similar assemblies can be made with 3 or more batteries.
- Each additional battery adds approximately 1.5 V to the total effective source.
- The assembly shown above should have a voltage just above 3 V.