UNIVERSITY OF MINNESOTA: TWIN CITIES

CE 4511 HYDRAULIC STRUCTURES

Final

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1) Scour Question

As discussed during a lecture on scours as part of the May 2nd lecture of CEGE 4511: *Hydraulic Structures*, the scour at bridge piers are most commonly attributed to both

- Horse shoe vorticies
- Wake vorticies

2) Vortex Drop Shafts

While vortex drop shafts include many drawback such as

- likeliness to entrain air;
- likeliness to cause cavitation;
- likeliness loose discharge capacity,

they are still produced as features in many hydraulic structures due to their:

- practical installment as a secondary discharge structures in many systems;
- tight control on contribution to total system discharge as a function of inlet height;
- relatively low need to develop local conditions during construction (economical).

3) Cavitation Check with Tainter Gate

3.0) Given

The tainter gate system shown in figure 1 is given. As shown, the properties are as follows:

$$
\begin{aligned}
\text{system width}: &\quad b = 30ft \\
\text{gate opening}: &\quad G_0 = 2ft \\
\text{upstream pool surface elevation}: &\quad H_w = 1032 \\
\text{sill elevation}: &\quad z_{\text{sill}} = 1014ft \\
\text{shaft/axis elevation}: &\quad z_{\text{shaft}} = 1028ft \\
\text{gate radius}: &\quad r = 25ft \\
\text{water temperature}: &\quad T = 65^\circ F \\
\text{acceleration of gravity}: &\quad g = 32.2\frac{ft}{s^2} \\
\text{weight - density of water at } T = 65^\circ F: &\quad \rho = 62.336588\frac{lbs}{ft^3} \\
\text{atmospheric pessure}: &\quad P_a = 14.5psi
\end{aligned}
$$

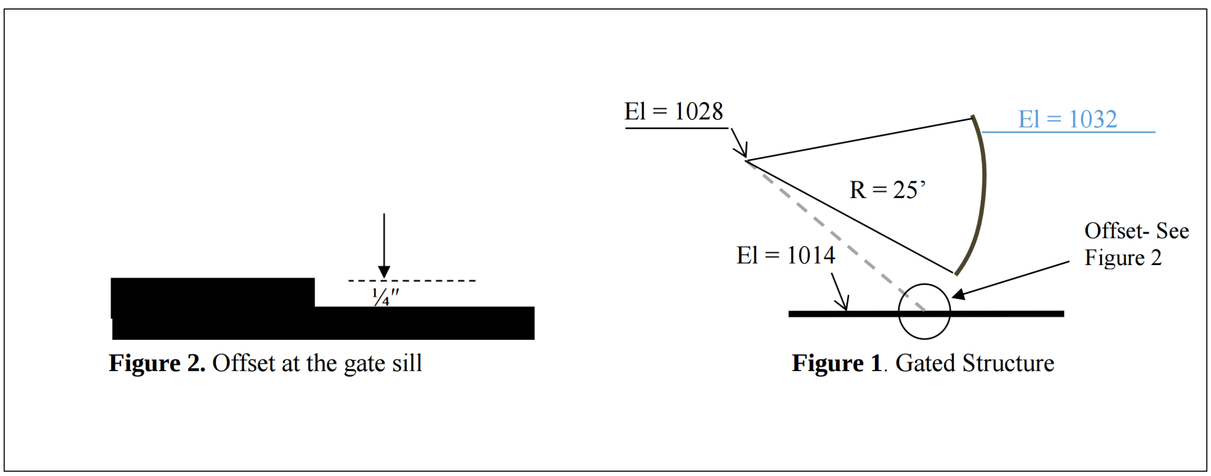

Figure 1: Given gate system

3.a) Discharge

Figure 2 presents the information required to compute discharge through the system, assuming flow is controlled by the gate lip.

- **Assumption :** Constraining flow regime is that of controlled.
- **Assumption :** The $\frac{X}{H_d} = 0$ curve will be read, given the flat sill.
- **Assumption :** There are no tailwatering/submergence effects of concern.

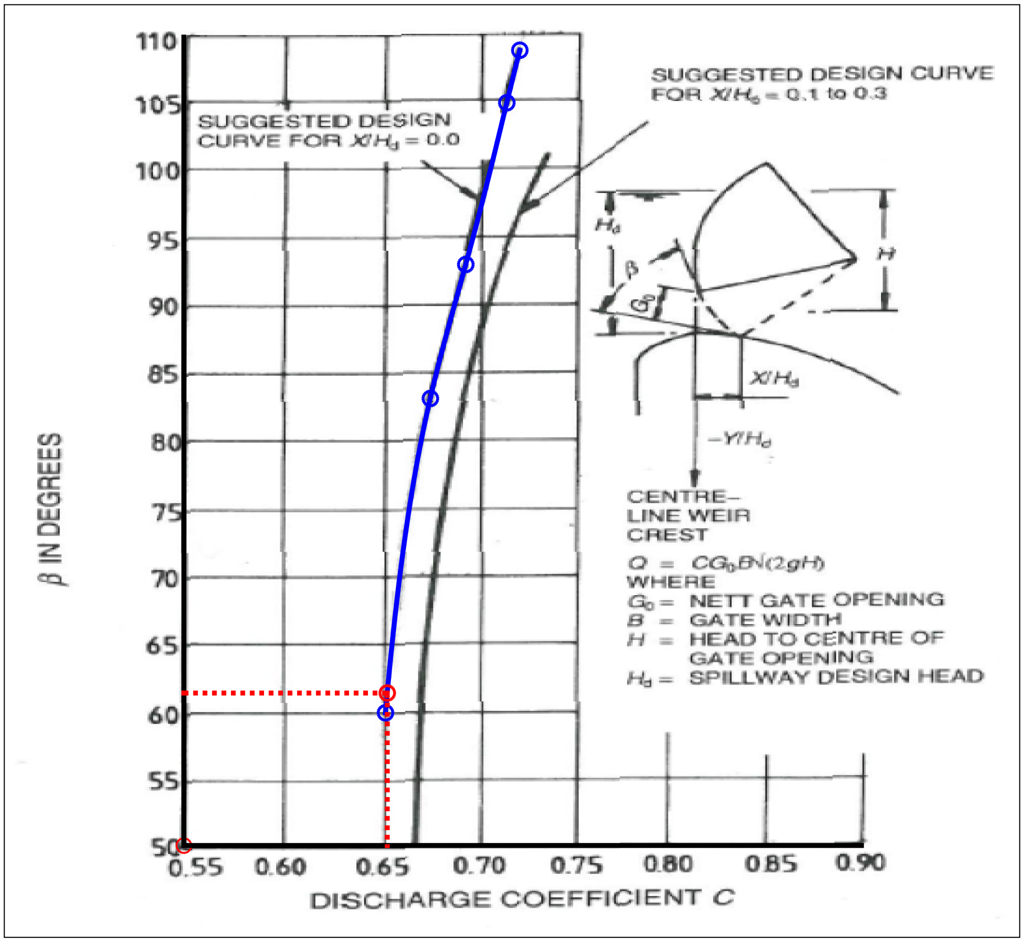

Figure 2: Controlled system discharge *USACE*

The discharge coefficient is read from this figure as follows:

- As shown in figure 3, the shaft elevation (z_{shaft}), gate opening (G_0), gate radius (r) and sill elevation (z_2) are all used to develop a function of β in terms of G_0.

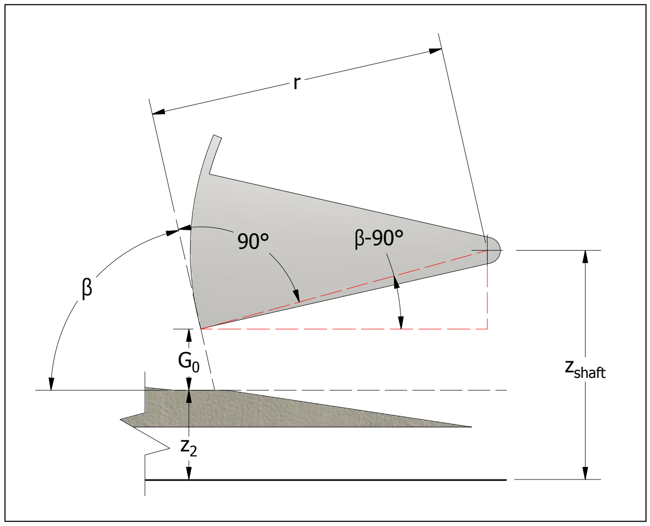

Figure 3: β calculation detail

- A right triangle is drawn, as shown in red, for which the hypotenuse is r and both the acute angle and shorter leg length can be equated:

$$\sin(\beta - 90^\circ) = \frac{z_{\text{shaft}} - G_0 - z_2}{r} \tag{1}$$

- This can be simplified, knowing $\sin(\theta - 90^\circ) = \cos(\theta)$

$$\cos(\beta) = \frac{z_{\text{shaft}} - G_0 - z_2}{r} \tag{2}$$

- β can then be isolated:

$$\beta = \cos^{-1}\left(\frac{z_{\text{shaft}} - G_0 - z_2}{r}\right) \tag{3}$$

- Using the equation, β is calculated:

$$\beta = \cos^{-1}\left(\frac{1028ft - 2ft - 1014ft}{25ft}\right) = 61.3147^\circ$$

- As shown in figure 2, this known ordinate of β is used to read the discharge coefficient as the abscissa:

$$C = 0.6551$$

With this, the lip-controlled discharge is computed with the given equation:

$$Q = (C)(G_0)\,b\sqrt{2g\left(H_w - z_{\text{sill}} - \frac{G_0}{2}\right)} \rightarrow \boxed{Q = 1300.5cfs} \tag{4}$$

3.b) Downstream Hydraulic Depth

The depth of flow just downstream of the gate lip is calculated as follows:

$$\text{coefficient of contraction calculated as :} \quad C_c = 1 - 0.75\left(\frac{\beta}{90^\circ}\right) + 0.36\left(\frac{\beta}{90^\circ}\right)^2 = 0.6561$$

$$\text{depth downstream of lip is calculatedA :} \quad y_2 = C_c G_0 = (0.6561)(2ft) \rightarrow \boxed{y_2 = 1.3123ft}$$

3.c) Cavitation Index

Given the $\frac{1}{4}in$ offset shown in figure 1, the cavitation index σ is calculated as follows:

$$\text{atmospheric pressure converted to local units :} \quad P_a = 14.5\frac{lbs}{in^2}\frac{144in^2}{1ft^2} = 2088\frac{lbs}{ft^2}$$

$$\text{ambient temperature converted to celcius :} \quad T_c = \frac{5}{9}\left(T_f - 32^\circ\right) = 18.3333^\circ C$$

$$\text{Antoinne's eq for vap. press., converting to local units:} \quad P_v = \left(10^{8.07131-\frac{1730.63}{233.426+T_c}}\right)2.784501 = 43.8443\frac{lbs}{ft^2}$$

$$\text{velocity is calculated from depth downstream of lip :} \quad V = \frac{Q}{(b)y_2} = 33.0336fps$$

$$\text{water weight density is converted to mass density :} \quad \rho = \frac{62.336588\frac{lbs}{ft^3}}{32.2\frac{ft}{s^2}} = 1.9359\frac{slugs}{ft^3}$$

$$\text{cavitation index is calculated :} \quad \sigma = \frac{P_a - P_v}{\frac{1}{2}\rho V^2} \rightarrow \boxed{\sigma = 1.9353}$$

3.d) Checking for Cavitation

Figure 4 presents $H - V$ curves for incipient cavitation. When regarding the curve provided for the given offset of $\frac{1}{4}in$, as highlighted in blue, the following comparison is made:

$$\text{considering the total pressure head : } \quad H = H_w - z_{\text{sill}} = 1032ft - 1014ft = 18ft$$
$$\text{and the previously calculated velocity at the sill : } \quad V = 33.0336fps\text{both shown in red}$$

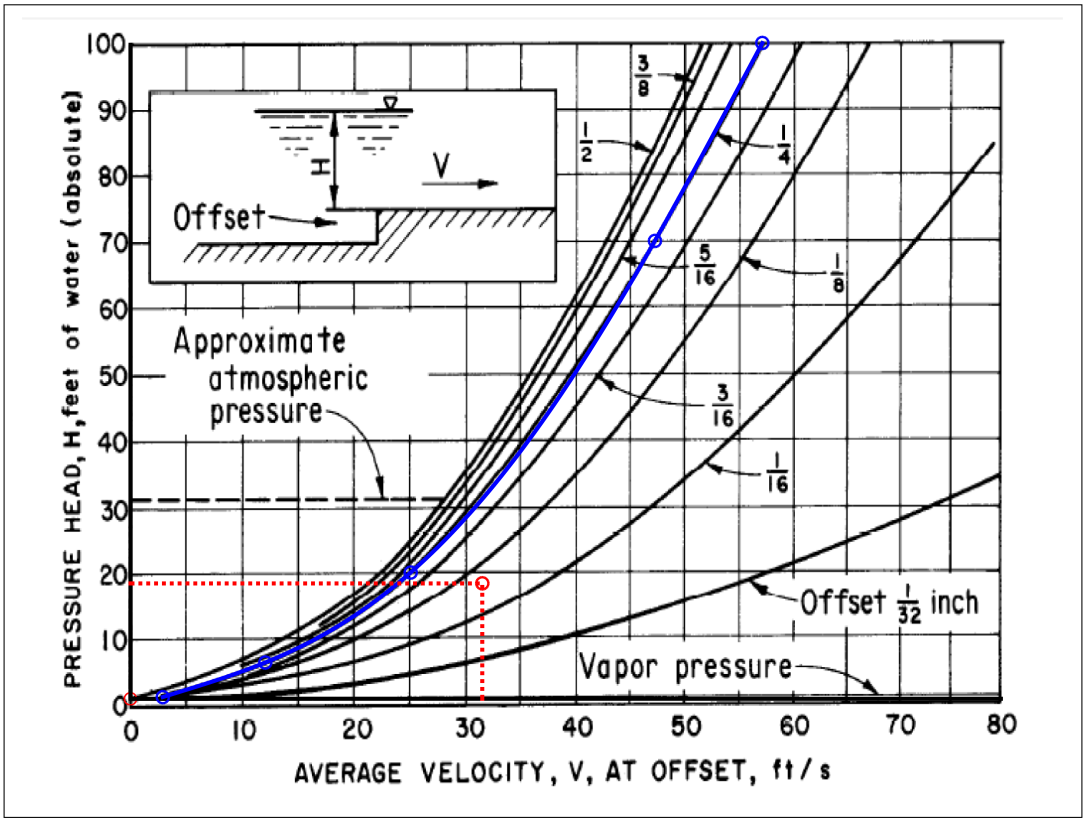

Figure 4: *H* - *V* curves for given offset *HDC*

It can be seen that the pressure head at which incipient, or /emphjust beginning to form cavitation for this offset is above the pressure head of this scenario. That is:

$$H_{\text{actual}} < H_{\text{incipient cavitation}}$$

Therefore, it is inferred that there is no cavitation under this condition.

4) Principal Spillway

4.) Given

The spillway system shown in figure 5 is given. It consists of a square, drop vortex shaft and a a culvert running under an embankment. The discharge is to be determined through the system for each of the given upstream water surface elevations of $WSE_a = 797.5 ft$ and $HW_b = 799.0 ft$. The following is also given/inferred/assumed:

$$
\begin{aligned}
\text{pipe is circular, with diameter}: &\quad D = 3 ft \\
\text{side length of interior shaft square}: &\quad d_i = 8 ft \\
\text{opening width}: &\quad w_0 = 6 ft \\
\text{opening height}: &\quad H_0 = 1 ft \\
\text{opening quantity}: &\quad N = 4 \\
\text{wall thickness}: &\quad t_w = 1.5 \\
\text{opening invert elevation}: &\quad z_0 = 797 ft \\
\text{shaft bed elevation}: &\quad z_1 = 790 ft \\
\text{bed pipe length}: &\quad L = 200 ft \\
\text{bed slope}: &\quad s_0 = 2.5\% \\
\text{acceleration of gravity}: &\quad g = 32.2 \frac{ft}{s^2} \\
\text{mannings n for concrete:} &\quad n = 0.012 \\
\text{inlet condition, square edge with headwall}: &\quad k_e = 0.4
\end{aligned}
$$

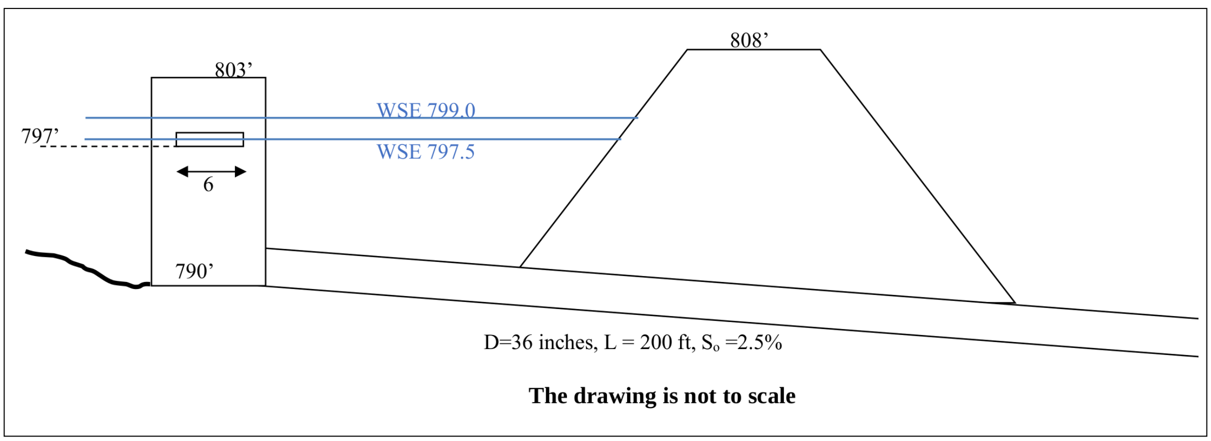

Figure 5: Given discharge system

4.a) Lower WSE

4.a.1) Discharge Through Openings

Given that the first water surface elevation is $Hw_a = 797.5ft$, the height of the energy line above the bottom edge of the shaft openings is calculated:

$$H_e = Hw_a - z_0 = 797.5ft - 797.0ft = 0.5ft$$

With this, the discharge through each of the 4 openings can be calculated by treating each opening sill as a **broad crested weir**:

$$Q = N\left[\frac{2}{3}\sqrt{\frac{2}{3}g}\left(C_d L H^{\frac{3}{2}}\right)\right] \tag{5}$$

Where:

$$\begin{aligned} \text{the length of each weir is sills' width :} \quad & L = w_0 \\ C_d = 0.85 : \quad & \text{if } \frac{H}{H+P} < 0.35 \text{ and } \frac{H}{t_w} \leq 0.35 \\ \text{where dam height is represented as window height above head :} \quad & P = z_0 - z_1 = 7ft \end{aligned}$$

This equation is based on a few assumptions:

- **Assumption**: Velocity head in the inlet pool is zero: $He = H$.
- **Assumption**: The dam depth of the opening on the side of the shaft with the tube exiting is unaffected by the tube's presence: $P_1 = P_2 = P_3 = P_4$.
- **Assumption**: Flow enters identically for all four openings.

In order to validate the choice of this equation, as well as its discharge coefficient, the given criteria for use are tested:

$$\begin{aligned} \frac{H}{H+P} = \frac{0.5ft}{0.5ft + 7ft} &= 0.667 < 0.35 \quad \rightarrow \text{verified} \\ \frac{H}{t_w} = \frac{0.5ft}{1.5ft} &= 0.333 < 0.35 \quad \rightarrow \text{verified} \end{aligned}$$

Thus, the discharge through the windows at the lower headwater elevation (WSE_a) are calculated as:

$$Q_{0a} = 4\left[\frac{2}{3}\sqrt{\frac{2}{3}32.2\frac{ft}{s^2}}\left(0.856ft0.5^{\frac{3}{2}}\right)\right] \rightarrow \boxed{Q_{0a} = 22.2780\ cfs}$$

4.a.2) Discharge Through Pipe

With the inlet discharge through the openings known, the critical and normal depths in the pipe can be solved as follows:

critical depth

$$\begin{aligned} \text{critical depth is guessed} \quad & y_c = \text{some value} \\ \text{critical angle is calculated :} \quad & \theta_c = 2\cos^{-1}\left[1 - 2\left(\frac{y_c}{D}\right)\right] \\ \text{critical area is calculated :} \quad & A_c = \left(\frac{D^2}{8}\right)\left[\theta_c - \sin\left(\theta_c\right)\right] \\ \text{critical top width is calculated :} \quad & T_c = D\sin\left(\frac{\theta}{2}\right) \\ \text{critical depth is solved for :} \quad & \frac{{Q_{0a}}^2 T_c}{g{A_c}^3} = 1 \end{aligned}$$

This process yields:

$\mathbf{y_c}$ (ft)	θ_c (radians)	$\mathbf{A_c}$ (ft^2)	$\mathbf{T_c}$ (ft)	$\mathbf{Fr_c}$ -
1.518	3.166	3.589	3.000	1.000

Table 1: Critical depth results for lower WSE

normal depth

$$\begin{aligned} \text{normal depth is guessed} \quad & y_n = \text{some value} \\ \text{normal angle is calculated :} \quad & \theta_n = 2\cos^{-1}\left[1 - 2\left(\frac{y_n}{D}\right)\right] \\ \text{normal area is calculated :} \quad & A_n = \left(\frac{D^2}{8}\right)\left[\theta_n - \sin\left(\theta_n\right)\right] \\ \text{normal wetted perimeter is caclulated :} \quad & P_n = \left(\frac{D}{2}\right)\theta_n \\ \text{normal hydraulic radius is calculated :} \quad & R_n = \frac{A_n}{P_n} \\ \text{normal depth is solved for :} \quad & Q_{0a} = \frac{1.486}{n} A_n {R_n}^{\frac{2}{3}} \sqrt{s_0} \end{aligned}$$

Which yields:

$\mathbf{y_n}$ (ft)	θ_n (radians)	$\mathbf{A_n}$ (ft^2)	$\mathbf{P_n}$ (ft)	$\mathbf{R_n}$ (ft)
0.898	2.3156	1.778	3.473	0.511

Table 2: Normal depth results for lower WSE

- Therefore, given that for WSE_a, $y_c > y_n$ for this lower WSE, the culvert has a steep slope for this lower WSE.
- In turn, this steepness indicates that the tunnel's flow is inlet controlled for this lower WSE.
- **Assumption**: Given that the system is inlet controlled, the flow enters the culvert at approximately the critical depth (y_c) and travels along an s_2 profile.
- **Assumption**: Tailwatering/submergence effects are negligible.
- With that determination, the following inlet control equations, for weir and orifice, respectively, apply:

$$H_{w_{i.c.}} = \begin{cases} y_c + (1+k_e)\frac{Q^2}{2gA_c{}^2} & \text{for } \frac{H_w}{D} < 1.4 \\ \frac{\left(\frac{Q}{C_d A_0}\right)^2}{2g} & \text{for } \frac{H_w}{D} > 1.4 \end{cases} \tag{6}$$

Where:

$$\begin{aligned} A_0 = & \quad \frac{\pi D^2}{4} \\ C_d = & \quad 0.6 \text{ for rough approximations} \end{aligned}$$

- **Assumption**: The first of these inlet controlled equations assumes that the entrance from the square drop shaft to the circular tunnel has square edges with a headwall.
- Thus, in order to check which equation applies, the first is calculated with known discharge:

$$H_{w,ic,weir} = y_c + (1+k_e)\frac{Q^2}{2gA_c{}^2} = 2.3558 ft$$

- and checked according to given criteria:

$$\frac{H_{w,ic,weir,a}}{D} = \frac{2.3558 ft}{3 ft} = 0.7853 \rightarrow \text{weir verified for this flow}$$

- Next, the flow through the pipe is calculated by treating the shaft openings as inconsequential, and thereby applying the headwater elevation at the openings to the culvert:

$$\frac{H_{w,a}}{D} = \frac{7.5 ft}{3.0 ft} = 2.5 \rightarrow \text{orifice flow}$$

$$\begin{aligned} \text{using known head}: \quad & H_{w,ic,ori,a} = \frac{\left(\frac{Q}{C_d A_0}\right)^2}{2g} = WSE_a - z_0 = 7.5 ft \\ \text{solving for discharge}: \quad & \boxed{Q_{ori,a} = 93.2089\ cfs} \end{aligned}$$

- Thus, the discharge for this lower water surface elevation is determined to be the lesser of discharges only considering the culvert and that only considering the shaft openings.

$$\begin{aligned} Q_a = \min\left[Q_{0a} \quad Q_{ori,a}\right] = \min[22.2780 cfs \quad 93.2089\ cfs] \\ \rightarrow \boxed{Q_a = Q_{WSE=797.5ft} = 22.2780\ cfs} \end{aligned}$$

4.b) Higher WSE

4.b.1) Discharge Through Openings

- For the second, higher water surface elevation, the headwater submerges the openings, and they therefore act as vertical gates.
- Therefore, the following formula applies:

$$\begin{aligned}
\text{stage-discharge equation for high elevation opening :} \quad & Q = C_d A \sqrt{2gH} \\
\text{where discharge coefficient is calculated :} \quad & C_d = \frac{C_c}{\left(1 + C_c \frac{a}{H}\right)^{\frac{1}{2}}} \\
\text{the head is referenced against the opening center :} \quad & H = WSE_2 - z_0 + \frac{1}{2}H_0 = 799.0 - 797.0 - \frac{1}{2}(1ft) = 1.5ft \\
\text{the gate opening is expressed :} \quad & a = H_0 = 1ft \\
\text{the area is calcualted :} \quad & A = ba = (w_0)(H_0) = (6ft)(1ft) = 6ft^2 \\
\text{and contraction coefficicient can be } \textbf{assumed} \text{ :} \quad & C_c = 0.6
\end{aligned}$$

- Thus, the discharge coefficient is calculated:

$$C_d = \frac{C_c}{\left(1 + C_c \frac{a}{H}\right)^{\frac{1}{2}}} = \frac{0.6}{\left(1 + 0.6 \frac{1ft}{1.5ft}\right)^{\frac{1}{2}}} = 0.5071ft$$

- And the discharge is calculated:

$$Q_{0b} = N\left[C_d A \sqrt{2gH}\right] = 4\left[(0.5071ft)\left(6ft^2\right)\sqrt{2\left(32.2\frac{f}{s^2}\right)(1.5ft)}\right]$$

$$\rightarrow \quad \boxed{Q_{0b} = 119.6154\ cfs}$$

4.b.2) Discharge Through Pipe

Using the same solving process as in section 4.a.2, the following values are determined for the critical and normal depths in the pipe:

$\mathbf{y_c}$ (ft)	θ_c (radians)	$\mathbf{A_c}$ (ft^2)	$\mathbf{T_c}$ (ft)	$\mathbf{Fr_c}$ -
2.947	5.7526	7.041	0.785	1

Table 3: Critical depth results for upper WSE

$\mathbf{y_n}$ (ft)	θ_n (radians)	$\mathbf{A_n}$ (ft^2)	$\mathbf{P_n}$ (ft)	$\mathbf{R_n}$ (ft)
2.976	5.9249	7.060	8.901	0.793

Table 4: Normal depth results for upper WSE

- In this case, given that $y_n > y_c$, the system is outlet controlled for the discharge resulting from the openings under the higher WSE
- The outlet controlled discharge is then calculated as follows:

$$\begin{aligned}
\text{stage-discharge equation}: \quad & H_w = h_0 - s_0 L + \left(1 + ke + \frac{2gn^2 L}{k_n{}^2 R_n{}^{\frac{4}{3}}}\right) \frac{Q^2}{2gA_0{}^2} \\
\text{where } \textbf{assumption}: \quad & h_0 = \frac{y_c + D}{2} \\
\text{and}: \quad & k_n = 1.49 \\
\text{headwater is set to higher WSE} \quad & H_w = 9\ ft \\
\text{and discharge is solved with known } H_w: \quad & H_w = H_w
\end{aligned}$$

This process yields:

$$\boxed{Q_{oc,b} = 44.4700\ cfs}$$

- Thus, the discharge for this higher WSE is the lesser between outlet controlled pipe discharge, and the discharge resulting from treating the openings as submerged, vertical gates.

$$\boxed{Q_{WSE=799ft} = 44.4700\ cfs}$$

4.c) Flaws in Design

There are indeed flaws in this design, including:

- Intake does not including trash racks at the openings, which could lead to serious damage or loss of discharge capacity;
- Structure does not have vortex-inhibiting features, thereby incurring the risk of loss of discharge capacity if swirling occurs in the event of asymmetrical inlet flow.

5) WES Spillway

5.) Given

The standard WES spillway in figure 6 is given. As shown, the properties are as follows:

$$
\begin{aligned}
\text{design head}: &\quad H_d = 15ft \\
\text{upstream bed elevation}: &\quad z_{us} = 925ft \\
\text{crest elevation}: &\quad z_{cr} = 950ft \\
\text{upstream pool surface elevation}: &\quad H_w = 952ft \\
\text{straight slope run per rise of 1}: &\quad \alpha = 0.9 \\
\text{downstream bed elevation}: &\quad z_{ds} = 915ft \\
\text{tailwater elevation}: &\quad T_w = \text{basin length}: \qquad L_b = 35ft \\
\text{system width}: &\quad b = 100ft \\
\text{concrete roughness}: &\quad k_s = 0.002ft \\
\text{acceleration of gravity}: &\quad 32.2\frac{ft}{s^2}
\end{aligned}
$$

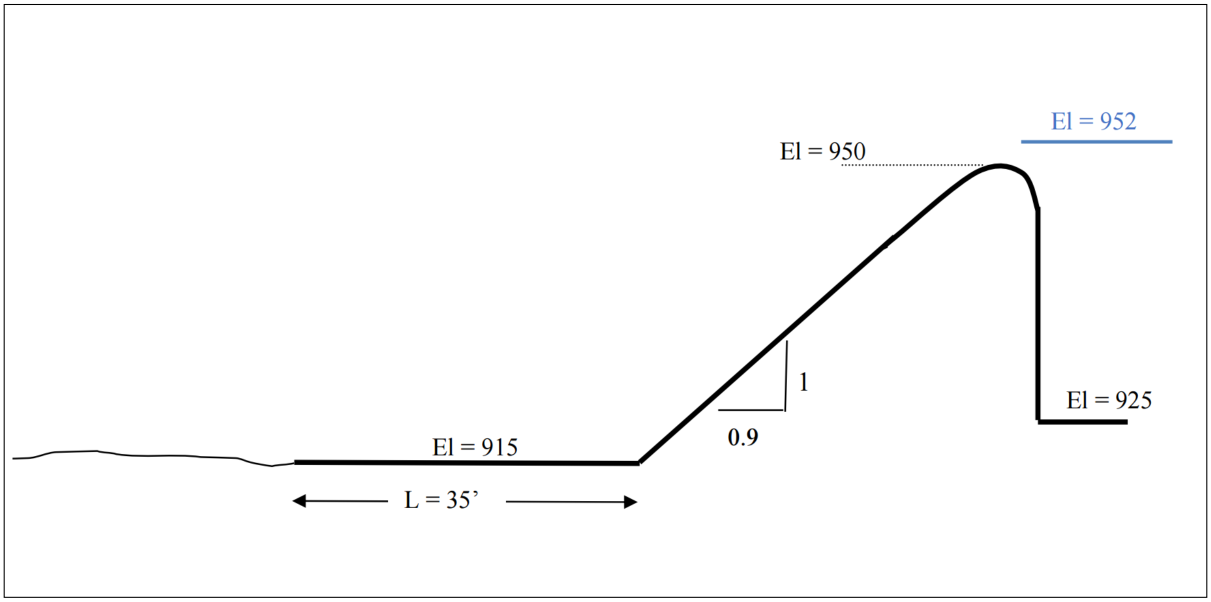

Figure 6: Given spillway

With these known values, the following preliminary calculations are carried out:

$$
\begin{aligned}
\text{dam height}: &\quad P = z_{cr} - z_{us} = 950ft - 925ft = 25ft \\
\text{crest energy head}: &\quad H_e = H_w - z_{cr} = 952ft - 950ft = 2ft
\end{aligned}
$$

- **Assumption**: This calculation of H_e assumes no upstream pool velocity head.

5.a) Discharge

The report *HDC-111-3* provides the information for discharge computation over the given spillway shown in figure 7. Given the previously state assumption of no velocity head, h_0 is set to zero, and $H_e = 2ft$. Therefore, in order to determine the discharge coefficient C from this chart, the following ordinate calculation is carried out:

$$\frac{He}{H_d} = \frac{2ft}{15ft} = 0.1333ft$$

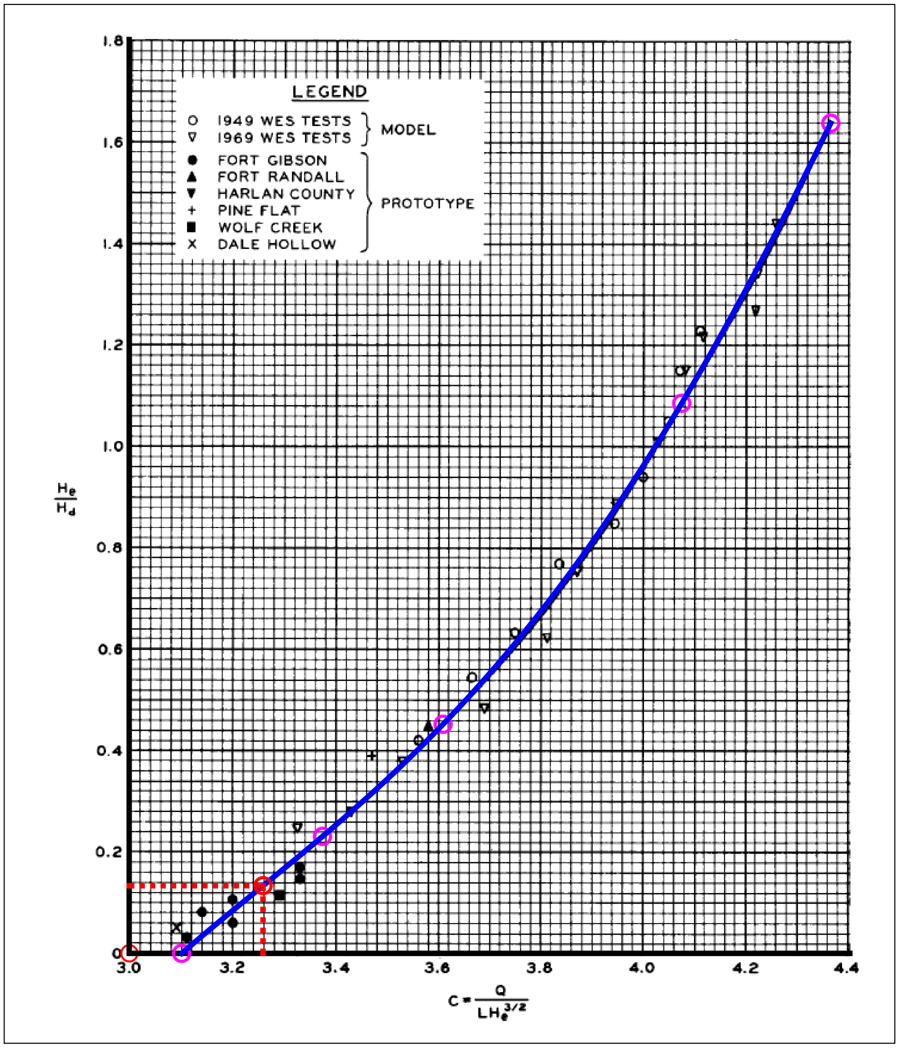

Figure 7: C chart reading *HDC 111-3*

And the abscissa is read as:

$$C = 3.26$$

And the discharge is a calculated as follows:

$$Q = (C)b\,(H_e)^{\frac{3}{2}} = 3.26(100ft)(2ft)^{\frac{3}{2}} = 922.0641cfs \rightarrow \boxed{Q = 922.0641\ \frac{ft^3}{s}}$$

5.b) Bulking Detection

With the design head known, the tangency point is determined as follows:

$$\text{using eq. 1 from HDC 111-1 :} \quad X_T = (H_d)\,1.096\left(\frac{1}{\alpha}\right)^{1.176} = (15ft)\,1.096\left(\frac{1}{0.9}\right)^{1.176} \rightarrow \boxed{X_T = 18.6086ft}$$

$$\text{using eq. 2 from HDC 111-1 :} \quad Y_T = (H_d)\,0.592\left(\frac{1}{\alpha}\right)^{2.176} = (15ft)\,0.592\left(\frac{1}{0.9}\right)^{2.176} \rightarrow \boxed{Y_T = 11.1682ft}$$

Then, the length of the curved portion of the crest (L_c) is read from figure 8. This is first done by calculating the abscissa:

$$\frac{X_T}{H_d} = \frac{18.6086ft}{15ft} = 1.2406$$

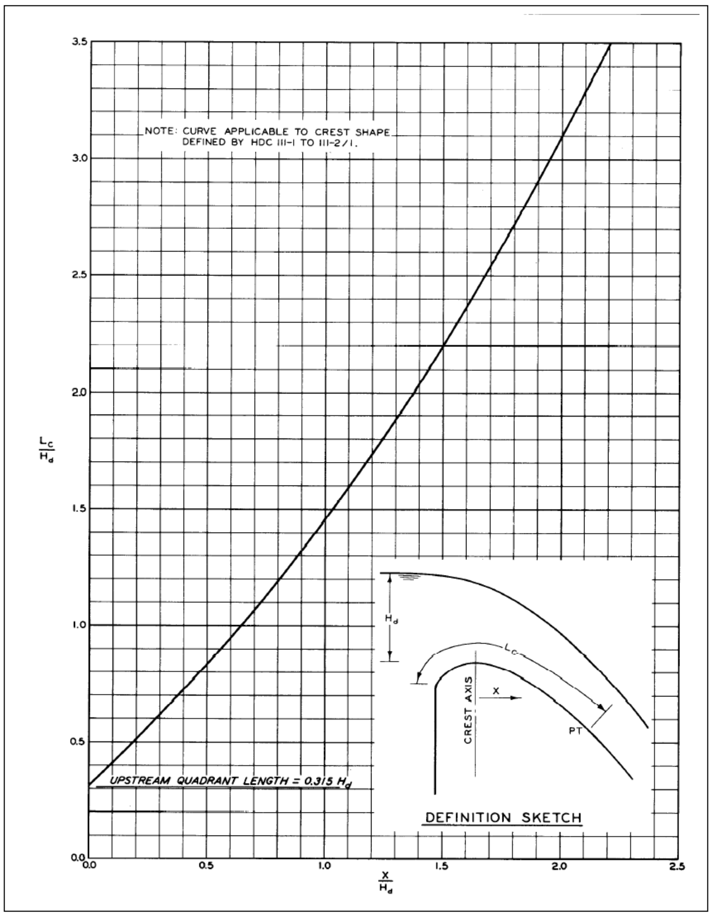

Figure 8: $\frac{L_c}{H_d}$ chart *HDC=111-18/1*

Then, this is used to read:

$$\frac{L_c}{H_d} = 1.7854 \rightarrow \boxed{L_c = 26.7803ft}$$

The length of the tangent, straight portion of the spillway (L_t) is then calculated:

$$L_t = (z_{cr} - z_{ds} - Y_T)\sqrt{0.9^2 + 1^2} \rightarrow \boxed{L_t = 32.0625 ft}$$

Thus, the full length along the spillway to the toe (L) is calculated:

$$L = L_c + L_t \rightarrow \boxed{L = 58.8428 ft}$$

The boundary layer thicknesses are then calculated as follows:

$$\text{using eq. 7 from HDC 111-18 :} \quad \delta = L(0.08)\left(\frac{L}{k_s}\right)^{-0.233} \rightarrow \boxed{\delta = 0.4281 ft}$$

$$\text{using eq. 8 from HDC 111-18 :} \quad \delta_1 = 0.18\delta = 0.0771 ft$$

$$\text{using eq. 9 from HDC 111-18 :} \quad \delta_3 = 0.22\delta = 0.0942 ft$$

Then, the potential flow depth (d_p) is calculated as follows:

$$\begin{aligned}
\text{the total head at the toe is calculated :} &\quad H_w - z_{ds} \\
\text{the toe velocity is guessed :} &\quad U = \text{value} \\
\text{potential flow depth is then equated :} &\quad d_p = \frac{H_T - \frac{U^2}{2g}}{\cos\left[\tan^{-1}\left(\frac{1}{\alpha}\right)\right]} \\
\text{discharge per linear width is equated :} &\quad q = U d_p \\
\\
\text{this is compared to calculated discharge per linear width :} &\quad q = \frac{Q}{b} \\
\text{toe velocity is solved satisfying equal } q \text{ :} &\quad \text{solve } U \text{ by setting } q = q \\
\\
\text{this yeilds a toe velocity :} &\quad U = 48.7304 fps \\
\text{which in turn yields a potential flow depth at toe :} &\quad d_p = \frac{H_T - \frac{U^2}{2g}}{\cos\left[\tan^{-1}\left(\frac{1}{\alpha}\right)\right]} = 0.1892 ft \\
\text{actual depth at toe is then solved :} &\quad d = d_p + \delta_1 = 0.1892 ft + 0.0771 ft \rightarrow \boxed{d = 0.2663 ft}
\end{aligned}$$

Thus, it is determined:

$$\boxed{\text{bulking of flow does occur over spillway :} \quad \delta > d}$$

Using the previous steps, various trial values of L are used to determine which length along the spillway yields a $\delta = d$. Using solver, this approach yields:

$$\boxed{L_{\text{inception}} = 26.1615 ft}$$

Thus, given that $L_{\text{inception}} < L_c$, it is determined that the inception point occurs along the curved portion of the spillway.

The point of inception is relativley close to the tangent point. Therefore, in order to determine the inception point, in the following approximation is made:

- The slope of the spillway between inception point and tangent point is roughly the same as that of the straight, tangential portion.

In order to determine the inception point, a line is traced back from the tangent point, as shown in figure 9.

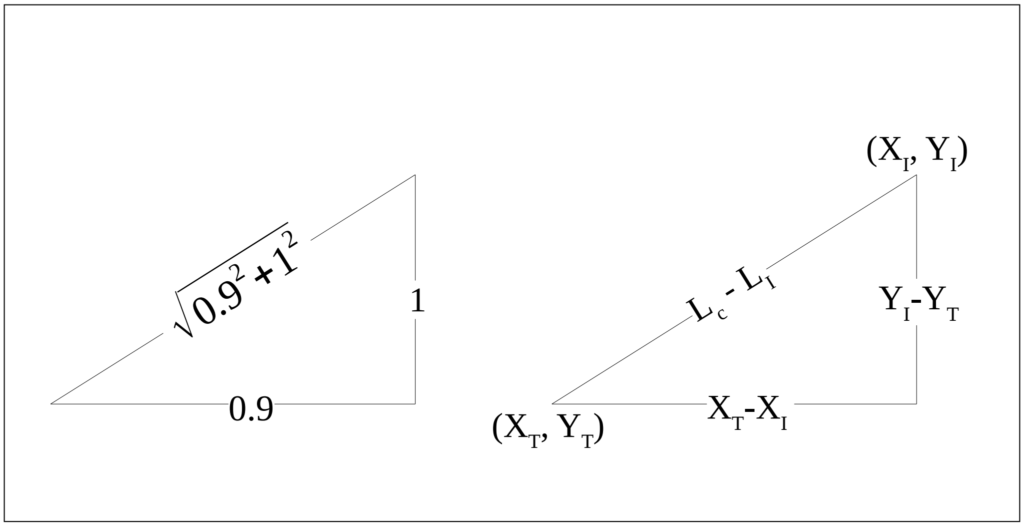

Figure 9: Triangular approximation for inception point

Therefore, in the inception point is solved as follows:

$$\frac{L_c - L_I}{\sqrt{0.9^2+1^2}} = \frac{X_T - X_I}{0.9} \quad \rightarrow X_I = X_T - 0.9\frac{L_c - L_I}{\sqrt{0.9^2+1^2}} \rightarrow \boxed{X_I = 18.1946\ ft}$$

$$\frac{L_c - L_I}{\sqrt{0.9^2+1^2}} = \frac{Y_I - Y_T}{1} \quad \rightarrow Y_I = \frac{L_c - L_I}{\sqrt{0.9^2+1^2}} + Y_T \rightarrow \boxed{Y_T = 11.6281\ ft}$$

5.c) Water Depth at Toe

Knowing the toe thickness of flow is equal to the entrained δ, the toe depth is solved using a triangular approximation, as shown in figure 10.

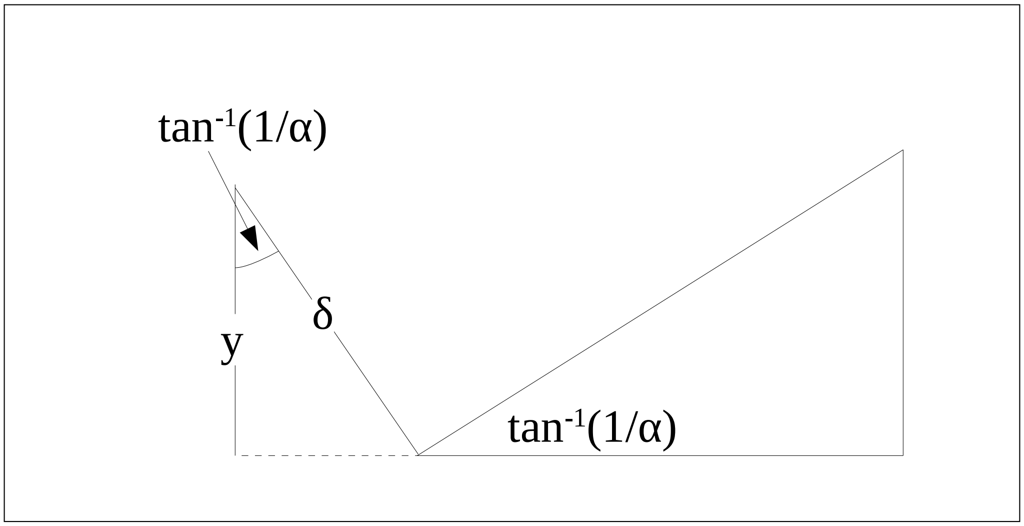

Figure 10: Triangular approximation of toe depth

Therefore, the flow depth is solved as follows:

$$y = \delta \cos\left[\tan^{-1}\left(\frac{1}{\alpha}\right)\right] \Longrightarrow \boxed{y_{toe} = 0.2864 ft}$$

5.d) Location of Jump Start

With the given tailwater elevation of $918.1ft$, the tailwater depth (y_3) is calculated, assuming no change in apron and downstream bed elevation, as follows:

- **Assumption**: natural, downstream bed elevation is the same as apron bed elevation: $\left(z_{ds,apron} = z_{ds,natural}\right)$

$$\text{hydraulic depth is calculated :} \quad y_3 = T_w - z_{ds} = 918.1ft - 915ft = 3.1ft$$

A hydraulic jump between the toe and tailwater is checked as follows:

$$\text{the toe Froude number is checked :} \quad Fr_{\text{toe}} = \frac{\frac{q}{y_{\text{toe}}}}{\sqrt{gy_{\text{toe}}}} = 10.6011$$

$$\text{the tailwater Froude number is checked :} \quad Fr_3 = \frac{\frac{q}{y_3}}{\sqrt{gy_3}} = 0.2977$$

- **Assumption**: Since the Froude numbers between the toe and the tailwater indicate a change in flow criticality from super to sub, there is a hydraulic jump somewhere between the two locations.

The properties of the hydraulic jump are calculated as follows:

$$\text{the equation for conjugate depth is considered :} \quad y_3 = \frac{y_2}{2}\left[\sqrt{1+8{Fr_2}^2} - 1\right]$$

$$\text{the equation for froude number before the jump is considered :} \quad Fr_2 = \frac{\frac{q}{y_2}}{\sqrt{gy_2}}$$

$$\text{solver is used to determine which } y_2 \text{ sets } y_3 \text{ equal to known value :} \quad y_3 = y_3$$

$$\text{this yields :} \quad y_2 = 0.4398$$

$$Fr_2 = 5.5712$$

- It should be noted at this point that this Fr_2 is higher than that recommended for the type I basin given (recommended $Fr_{\text{type I}} < 2.5$).

With this known y_2, the direct step method is used to determine the length between the toe and the jump start using the following equations:

$$\text{depth}: \quad y \tag{7}$$

$$\text{area}: \quad A = yb \tag{8}$$

$$\text{wetted perimeter}: \quad P = 2y + b \tag{9}$$

$$\text{hyraulic radius}: \quad R = \frac{A}{P} \tag{10}$$

$$\text{mean velocity}: \quad V = \frac{q}{y} \tag{11}$$

$$\text{total energy head}: \quad E = y + \frac{V^2}{2g} \tag{12}$$

$$\text{friction slope}: \quad s_f = \left(\frac{nQ}{1.49AR^{\frac{2}{3}}}\right)^2 \tag{13}$$

	y (ft)	**A** (ft^2)	**P** (ft)	**R** (ft)	**V** $(\frac{ft}{s})$	**E** (ft)	$\mathbf{s_f}$ -
toe (1)	0.2864	28.641	100.5728	0.2848	32.1939	16.3803	0.4211
pre-jump (2)	0.4398	43.9800	100.8796	0.4360	20.9655	7.2652	0.1012

A

Table 5: Direct Step Table

This process involves the following assumptions:

- **Assumption**: Only one iteration of the direct step method is needed given the relatively short anticipated inter-depth distance.
- **Assumption**: The stilling basin is perfectly horizontal ($s_0 = 0$).
- **Assumption**: Manning's n for concrete is set to $n = 0.013$.

With this, the distance between the toe and pre-jump location is solved as follows:

$$\text{average friction slope calculated}: \quad \overline{s_f} = \frac{s_{f_1} + s_{f_2}}{2} = \frac{0.4211 + 0.1012}{2} = -0.2612$$

$$\text{step distance is calculated}: \quad dl = \frac{E_2 - E_1}{-\overline{s_f}} = \frac{6.2952ft - 16.3803}{0.2494} = 34.9026ft$$

Thus, it is determined that $\boxed{\text{the jump starts 0.0974 ft downstream of the toe}}$.

5.e) Length of Rollers

The following formulas are given for the roller length of a hydraulic jump (L_r):

$$\frac{L_r}{y_2} = -12 + 160 \tanh\left(\frac{Fr_2}{20}\right) \quad \text{for } \frac{y_2}{b} < 0.1 \tag{14}$$

$$\frac{L_r}{y_2} = -12 + 100 \tanh\left(\frac{Fr_2}{12.5}\right) \quad \text{for } 0.1 < \frac{y_2}{b} < 0.7 \tag{15}$$

These equations are given for the following assumptions:

- **Assumption**: The system cross section is perfectly rectangular.
- **Assumption**: The apron bed is perfectly flat, established before $s_0 = 0$.

To determine which formula to use, the ratio $\frac{y_2}{b}$ is calculated:

$$\frac{y_2}{b} = 0.0044 \rightarrow \text{eq. 14 is used}$$

Thus, roller length is calculated:

$$L_r = 0.4398\,ft \left[-12 + 160 \tanh\left(\frac{5.5712}{20}\right)\right] \rightarrow \boxed{L_r = 13.8324\,ft}$$