

MAST90104: A First Course in Statistical Learning

Week 1 Workshop/Lab

Workshop questions

1. Show that $X^T X$ is a symmetric matrix.

2. (a) Let

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

be a nonsingular 2×2 matrix. Show by direct multiplication that

$$A^{-1} = \frac{1}{ad - bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}.$$

(b) Find the inverse of

$$\begin{bmatrix} 3 & 1 \\ 4 & -2 \end{bmatrix}.$$

3. Let

$$A = \begin{bmatrix} -6 & 3 & 0 \\ 1 & -1 & 1 \end{bmatrix}$$

and

$$B = \begin{bmatrix} 0 & 6 & -1 \\ 8 & -1 & 0 \\ 0 & 2 & 3 \end{bmatrix}$$

(a) Find the product AB

(b) Does BA exist?

(c) Can we calculate $B^T A^T$? If so, what is it?

4. Let $\mathbf{x} = \begin{bmatrix} 1 & 6 & -4 & 2 \end{bmatrix}^T$ and $\mathbf{y} = \begin{bmatrix} 2 & -1 & -1 & 0 \end{bmatrix}^T$

(a) What is the norm of \mathbf{x} ?

(b) Are \mathbf{x} and \mathbf{y} orthogonal?

5. Is

$$X = \begin{bmatrix} 1 & 1 & -1 & -1 \\ 1 & -1 & 1 & -1 \\ 1 & -1 & -1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

orthogonal? If not, what value of c makes the matrix cX orthogonal?

6. For the following matrices, find the eigenvalues and eigenvectors

(a) $\begin{bmatrix} 1 & 2 \\ 0 & -1 \end{bmatrix}$

(b) $\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$

Practical questions

The following are taken from Chapter 2 of spuRs (Introduction to Scientific Programming and Simulation Using R).

1. Give R assignment statements that set the variable z to

- (a) x^{a^b}
- (b) $(x^a)^b$
- (c) $3x^3 + 2x^2 + 6x + 1$ (try to minimise the number of operations required)
- (d) the second-to-last digit of x before the decimal point (hint: use `floor(x)` and/or `%%`)
- (e) $z + 1$

2. Give R expressions that return the following matrices and vectors

- (a) (1, 2, 3, 4, 5, 6, 7, 8, 7, 6, 5, 4, 3, 2, 1)
- (b) (1, 2, 2, 3, 3, 3, 4, 4, 4, 4, 5, 5, 5, 5, 5)
- (c) $\begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}$
- (d) $\begin{pmatrix} 0 & 2 & 3 \\ 0 & 5 & 0 \\ 7 & 0 & 0 \end{pmatrix}$

3. Use R to produce a vector containing all integers from 1 to 100 that are not divisible by 2, 3, or 7.

4. Which of the following assignments will be successful? What will the vectors \mathbf{x} , \mathbf{y} , and \mathbf{z} look like at each stage?

```
rm(list = ls())
x <- 1
x[3] <- 3
y <- c()
y[2] <- 2
y[3] <- y[1]
y[2] <- y[4]
z[1] <- 0
```

5. Build a 10×10 identity matrix. Then make all the non-zero elements 5. Do this latter step in at least two different ways.