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ASSIGNMENT-3

1) b) Our model is

$$y = X \begin{bmatrix} \mu \\ \tau_{RV2} \\ \tau_{RV3} \\ \eta_{B349} \\ \eta_{J052} \end{bmatrix} + \epsilon$$

Height difference $= \eta_{J052} - \eta_{B349} = 1.2944 - 3.3150$
$= -2.02$

c) For CI: $t^T = (0\ -1\ 1\ 0\ 0\ -1\ 1\ 0\ 0)$

$\beta = (\mu\ \tau_{RV2}\ \tau_{RV3}\ \eta_{B349}\ \eta_{J052}\ \xi_{RV2,B349}\ \xi_{RV3,B349}\ \xi_{RV2,J052}\ \xi_{RV3,J052})^T$

as we want

$\text{diff} = (\mu + \tau_{RV3} + \xi_{RV3,B349}) - (\mu + \tau_{RV2} + \xi_{RV2,B349})$

estimate $= t^T b = 3.47$

width $= t(1-\alpha/2) \times \sqrt{s^2\, t^T (X^TX)^{-1}\, t} = 2.03$

95% CI interval $= (1.44\ ,\ 5.5)$

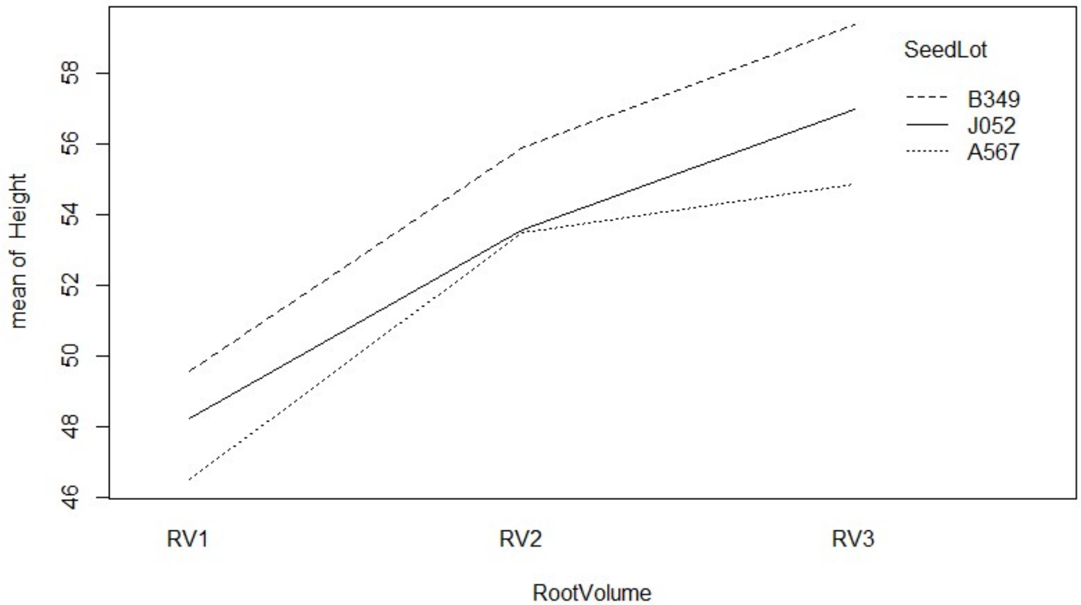
d) We test for height difference from J052 plot

$H_0: (\tau_{RV3} + \xi_{RV3,J052}) - (\tau_{RV2} + \xi_{RV2,J052}) = 0$

$C = (0\ -1\ 1\ 0\ 0\ 0\ 0\ -1\ 1)$

$dst = 0$

We get p value $= 0.0016 \Rightarrow$ We reject null hypothesis



e) The lines are not parallel ⇒ There may be interaction

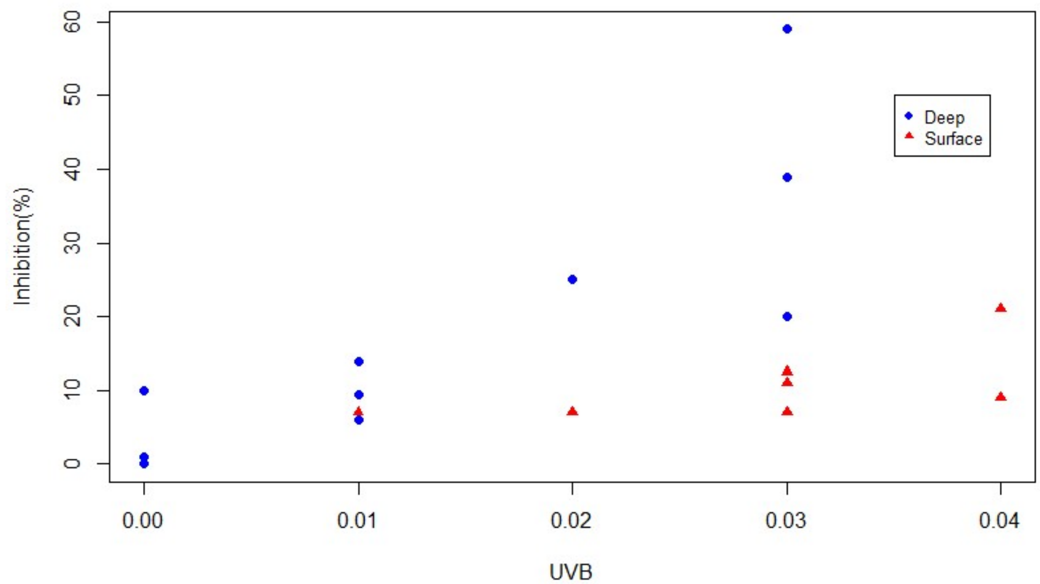
f) We test interaction model with additive model.

H_0: There is no interaction

p-value of Interaction = 0.4855

⇒ We cannot reject H_0 ⇒ There is no interaction

2a



2) b) If the effect of UVB differs, then there will be interaction between UVB & Surface factor.

We test interaction model with additive model.
H_0: There is no interaction b/w UVB & surface factor

Using ANOVA we see that $\boxed{\text{p-value} = 0.039}$ ⇒ We reject the null hypothesis ⇒ There is ~~inter~~ significant interaction.

∴ Effect of UVB does differ at surface & deep.

Other way is to make model:

$X = [\ \text{Surface} \quad \text{Deep} \quad \text{UVB} * \text{Surface} \quad \text{UVB} * \text{deep}\ ]$

with $\beta = [\ \tau_{surface} \quad \tau_{deep} \quad \cancel{\beta}\ \xi_{UVB,surface} \quad \xi_{UVB,deep}\ ]^T$

If the effect does not differ then $\xi_{UVB*Surface} = \xi_{UVB*deep}$

$C = (0, 0 \quad 1 \quad -1)$, $\delta = 0$

We test linear Hypothesis, $C\beta = \delta$

We get same p-value = 0.039

3) a) Minimize: $f(n_1, n_2, n_3, n_4, n_5, \lambda) = \sigma^2\left(\frac{4}{n_5} + \frac{1}{n_1} + \frac{1}{n_2} + \frac{1}{n_3} + \frac{1}{n_4}\right) + \lambda\left(\sum n_i - 30\right)$

$$\frac{\partial f}{\partial n_5} = \frac{-4\sigma^2}{n_5^2} + \lambda = 0$$

$$\Rightarrow n_5^2 = \frac{4\sigma^2}{\lambda}$$

for $i \neq 5$

$$\frac{\partial f}{\partial n_i} = -\frac{\sigma^2}{n_i^2} + \lambda = 0 \Rightarrow n_i^2 = \frac{\sigma^2}{\lambda}$$

$$\Rightarrow n_5^2 = 4n_i^2 \Rightarrow \boxed{n_5 = 2n_i}$$

$\sum n_i = 30$

$$\Rightarrow \frac{n_5}{2} \times 4 + n_5 = 30 \Rightarrow 3n_5 = 30 \Rightarrow \boxed{n_5 = 10}$$

$$\boxed{n_1 = 5 = n_2 = n_3 = n_4}$$

$\Rightarrow$ Placebo gets 10 units while others treatments get 5 each.

b) Output:

Treatments:	$s1 =$ 25	4	7	1	2					
	$s2 =$ 23	11	14	18	19					
	$s3 =$ 27	10	30	21	28					
	$s4 =$ 9	5	22	15	12					
Placebo,	$s5 =$ 13	17	26	8	6	20	29	3	24	16

4) a) We maximize $L = \sum_i \log\left[{}^{n_i}C_{y_i}\, p_i^{y_i} (1-p_i)^{n_i - y_i} \right]$ wrt p_i

where $n_i \rightarrow$ total units ~~for~~ for a particular dose.
$y_i \rightarrow$ tumors for a particular dose.

& $\eta_i = \alpha + \beta * dose_i + \epsilon_i$ & $p_i = \frac{1}{1+e^{-\eta_i}}$

$$L = C + \sum \left(y_i \log \frac{p_i}{1-p_i} + n_i \log(1-p_i) \right)$$

Now, $\log\left(\frac{p_i}{1-p_i}\right) = \eta_i = \alpha + \beta * dose_i$

$\Rightarrow$ We maximize, $L = \sum \left[y_i \log \eta_i + n_i \log(1+e^{\eta_i}) \right]$

After numerical optimization, we get

$$\boxed{\alpha = -3.036, \quad \beta = 0.0901}$$

$$\Rightarrow \boxed{p_i = \frac{1}{1+e^{3.036 - 0.0901 \times dose}}}$$

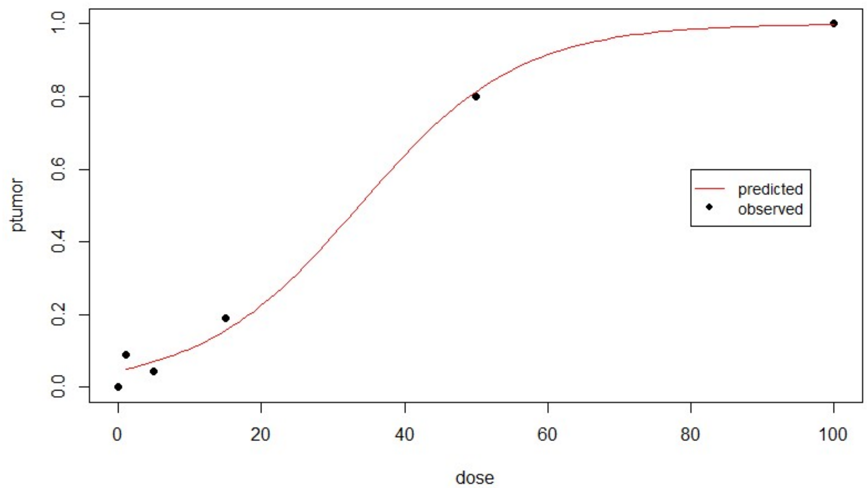
b) For CIs we need $I^{-1}(\theta)$

$I_{11} = \sum_{i=1}^{6} n_i \hat{p}_i (1-\hat{p}_i)$, $I_{21} = \sum n_i\, dose_i\, \hat{p}_i (1-\hat{p}_i) = I_{12}$

$I_{22} = \sum n_i (dose_i)^2 \hat{p}_i (1-\hat{p}_i)$

$$I(\theta)^{-1} = \begin{bmatrix} 0.232 & -0.0053 \\ -0.0053 & 0.0002 \end{bmatrix}$$

- 95% CI of $\alpha = -3.036 \pm z(0.975)\sqrt{0.232}$
  $= (-3.98, -2.09)$
- 95% CI of $\beta = 0.0901 \pm 1.96\sqrt{0.0002}$
  $= (0.0616, 0.1186)$



c) $D \approx \chi^2_{6-2} = \chi^2_4$

We get, $D = 2.897$ for our model

If dose is not significant, the null model, $p = \frac{\Sigma y_i}{\Sigma n_i}$ will be close to our model.

Null model: $\hat{p} = 0.4044$

$D_{Null} = 116.524$

$df_{Null} = 6-1 = 5$

Now, $D_{Null} - D \approx \chi^2_{5-4} = \chi^2_1$

We get p-value $= 1.58 \times 10^{-26} \Rightarrow$ Our model is significant

$\Rightarrow$ coefficient of dose is significant.

d) $b = (-3.036 \quad 0.0901)^T$

$x^* = (1 \quad 70)^T$

$$P_{70} = \frac{1}{1+e^{-x^{*T}b}} = 0.9634$$

95% CI of $\eta = x^T b \pm \text{qnorm}(0.975)\sqrt{x^T I^{-1}(\theta)\, x}$

we already calculated $I^{-1}(\theta)$

$= (1.84, 4.7)$

95% CI of $p_{70} = (\text{ilogit}(1.84), \text{ilogit}(4.7))$

$= (0.863, 0.991)$

e) $\eta = \Phi^{-1}(p)$, $p = \Phi(\eta)$

We have observed p values, use them to get η

Here, we maximize:

$$L = \sum y_i \log \frac{p_i}{1-p_i} + n_i \log(1-p_i)$$

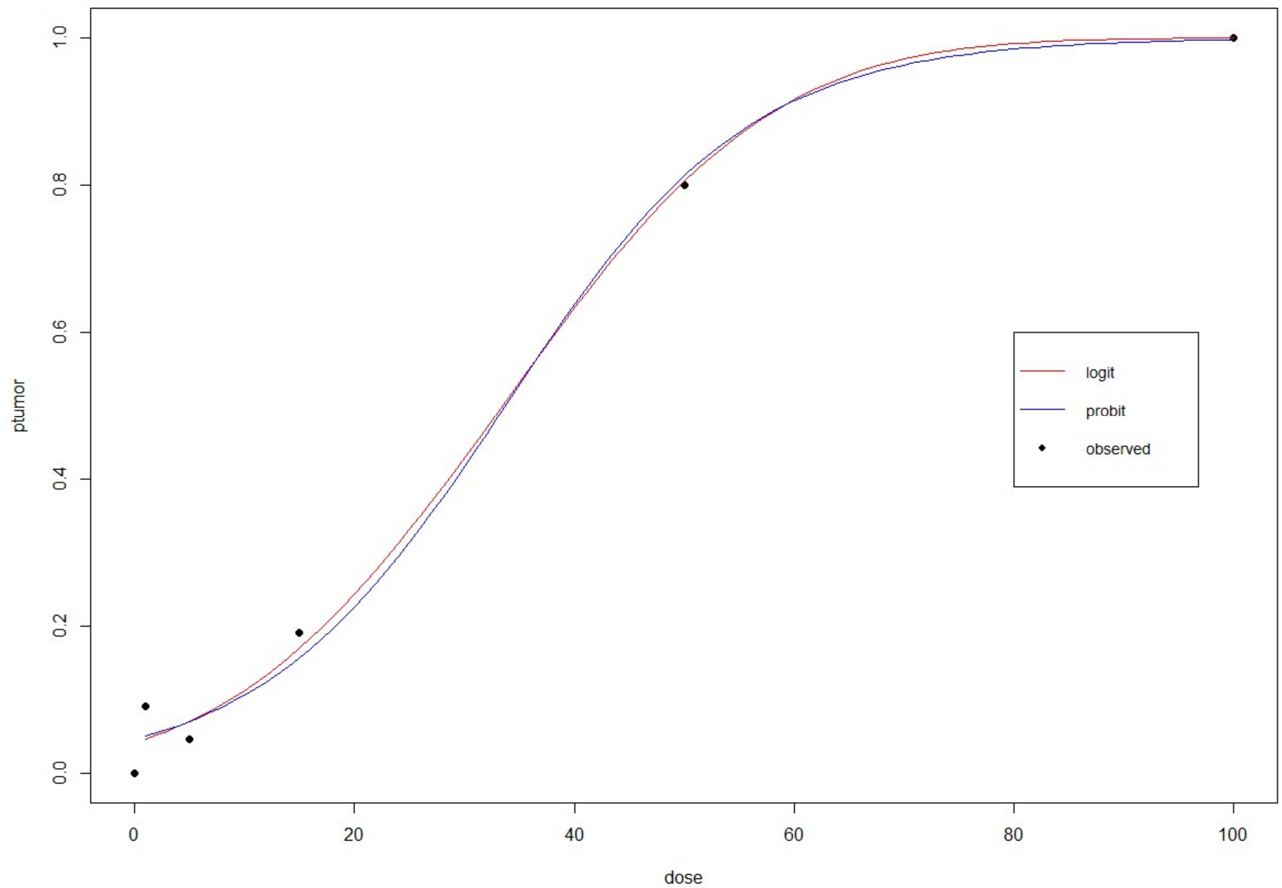
$$= \sum y_i \log \frac{\Phi(\eta_i)}{1-\Phi(\eta_i)} + n_i \log(1-\Phi(\eta_i))$$

We get, $b = (-1.736 \quad 0.519)^T$

$$\eta = -1.736 + 0.519 \times \text{dose}$$

with $p = \Phi(\eta)$

f) From plots we see that probit & logit follow very closely.



logit
probit
observed
ptumor
dose
0.0
0.2
0.4
0.6
0.8
1.0
0
20
40
60
80
100