ASSIGNMENT-4

1291822

1.

a) After stepwise selection we get the following parameters.

```
[1] "(Intercept)"         "fixed.acidity"        "volatile.acidity"    "citric.acid"         "residual.sugar"
[6] "chlorides"           "free.sulfur.dioxide" "density"             "pH"                  "sulphates"
```

b) Ordinal model gives few different parameters:

```
fixed.acidity
volatile.acidity
residual.sugar
chlorides
free.sulfur.dioxide
density
pH
sulphates
alcohol
```

The ordinal model has “alcohol” as a parameter while the multinomial model does not.

The ordinal model does not have “citric.acid” as a parameter while multinomial model has.

Multinomial model has Residual Deviance: 5394.309, while ordinal model has Residual Deviance: 5533.829.

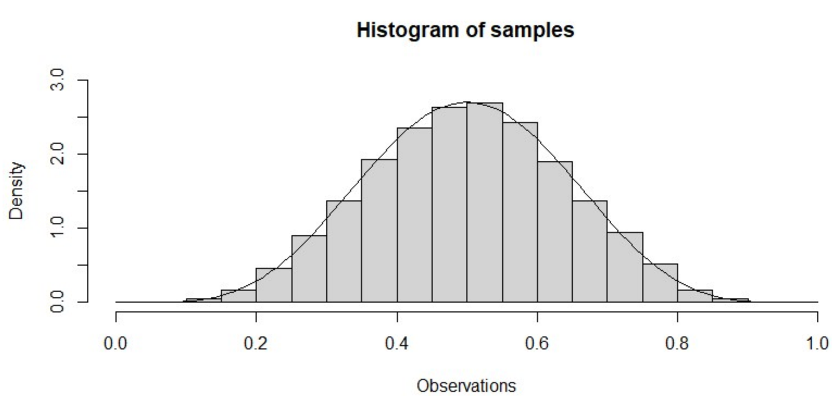
c)For multinomial model, P(bad wine) = 0.438.

For ordinal model, P(bad wine) = 0.446.

d) Odds ratio = exp[-567.89-3*0.28022-otherterms]/ exp[-567.89-5*0.28022-otherterms] =1.7514.

2.

b) The theoretical values fit the produced sample nicely.



3.

a)

3) a) $f(D|\beta_0,\beta_1,\sigma^2) = \prod_i \left(\frac{1}{\sqrt{2\pi\sigma^2}} \exp\left[-\frac{(y_i-\beta_0-\beta_1 x_i)^2}{2\sigma^2}\right]\right)$

$$f(\beta_0|D,\beta_1,\sigma^2) \propto f(D|\beta_0,\beta_1,\sigma^2) \times f(\beta_0)$$

$$\propto \prod_i \left\{ \exp\left[-\frac{(y_i-\beta_0-\beta_1 x_i)^2}{2\sigma^2}\right] \right\} \times \exp\left(\frac{-\beta_0^2}{2\sigma_\beta^2}\right)$$

$$= \exp\left[-\left(\frac{\beta_0^2}{2\sigma_\beta^2} + \frac{\Sigma(\beta_0+\beta_1 x_i - y_i)^2}{2\sigma^2}\right)\right]$$

$$= \exp\left[-\left(\frac{\beta_0^2}{2\sigma_\beta^2} + \frac{\Sigma(\beta_0^2+k_i^2-2\beta_0 k_i)}{2\sigma^2}\right)\right]; \boxed{k_i = y_i - \beta_1 x_i}$$

$$\propto \exp\left[-\left(\frac{\beta_0^2\left(\frac{\sigma^2}{\sigma_\beta^2}+n\right) - 2\beta_0 \Sigma k_i}{2\sigma^2}\right)\right]$$

$$\propto \exp\left[-\frac{n'}{2\sigma^2}\left(\beta_0^2 - \frac{2\beta_0 \Sigma k_i}{n'} + \left(\frac{\Sigma k_i}{n'}\right)^2\right)\right]; \boxed{n' = \frac{\sigma^2}{\sigma_\beta^2} + n}$$

$$= \exp\left[-\frac{\left(\beta_0 - \frac{\Sigma k_i}{n'}\right)^2}{2\sigma^2/n'}\right]$$

$$\Rightarrow \boxed{f(\beta_0|D,\beta_1,\sigma^2) \approx N\left(\frac{\Sigma k_i}{n'}, \frac{\sigma^2}{n'}\right)}$$

where $k_i = y_i - \beta_1 x_i$ & $n' = \sigma^2/\sigma_\beta^2 + n$

$\|^{ly}$ for β_1

$$f(\beta_1 \mid \beta_0, \sigma^2, D) \propto \prod_i \left\{ \exp\left[-\frac{(y_i - \beta_0 - \beta_1 x_i)^2}{2\sigma^2} \right] \right\} \exp\left(-\frac{\beta_1^2}{\sigma_\beta^2} \right)$$

$$\cancel{= \prod_i \left\{ \exp\left[-(\beta_1 - g_i) \right. \right.}$$

$$= \prod_i \left\{ \exp\left[-\frac{(x_i\beta_1 - g_i)^2}{2\sigma^2} \right] \right\} \exp\left(-\frac{\beta_1^2}{\sigma_\beta^2} \right); \; g_i = y_i - \beta_0$$

$$\propto \exp\left[-\left(\frac{\beta_1^2 \left(\Sigma x_i^2 + \frac{\sigma^2}{\sigma_\beta^2} \right) - 2\beta_1 \Sigma x_i g_i}{2\sigma^2} \right) \right]$$

$$\propto \exp\left[\frac{-\alpha}{2\sigma^2} \left(\beta_1 - \frac{\Sigma x_i g_i}{\alpha} \right)^2 \right]; \; \alpha = \Sigma x_i^2 + \frac{\sigma^2}{\sigma_\beta^2}$$

$$\Rightarrow \boxed{f(\beta_1 \mid \beta_0, \sigma^2, D) \approx N\left(\frac{\Sigma x_i g_i}{\alpha}, \frac{\sigma^2}{\alpha} \right)}$$

where, $g_i = y_i - \beta_0$ & $\alpha = \Sigma x_i^2 + \frac{\sigma^2}{\sigma_\beta^2}$

$$f(\sigma^2 \mid D, \beta_0, \beta_1) \propto (\sigma^2)^{-a-1-n/2} \exp\left[\frac{-b}{\sigma^2} - \frac{\Sigma (y_i - \beta_0 - \beta_1 x_i)^2}{2\sigma^2} \right]$$

$$= (\sigma^2)^{-a-1-n/2} \exp\left[-\frac{\left(b + \frac{\Sigma (y_i - \beta_0 - \beta_1 x_i)^2}{2} \right)}{\sigma^2} \right]$$

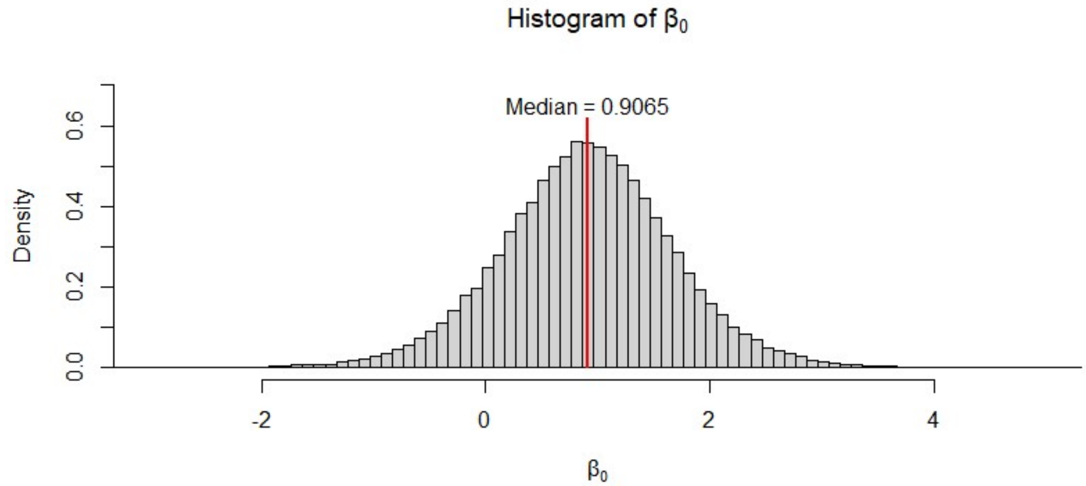
$$\boxed{= IG\left(a + \frac{n}{2}, \; b + \frac{\Sigma (y_i - \beta_0 - \beta_1 x_i)^2}{2} \right)}$$

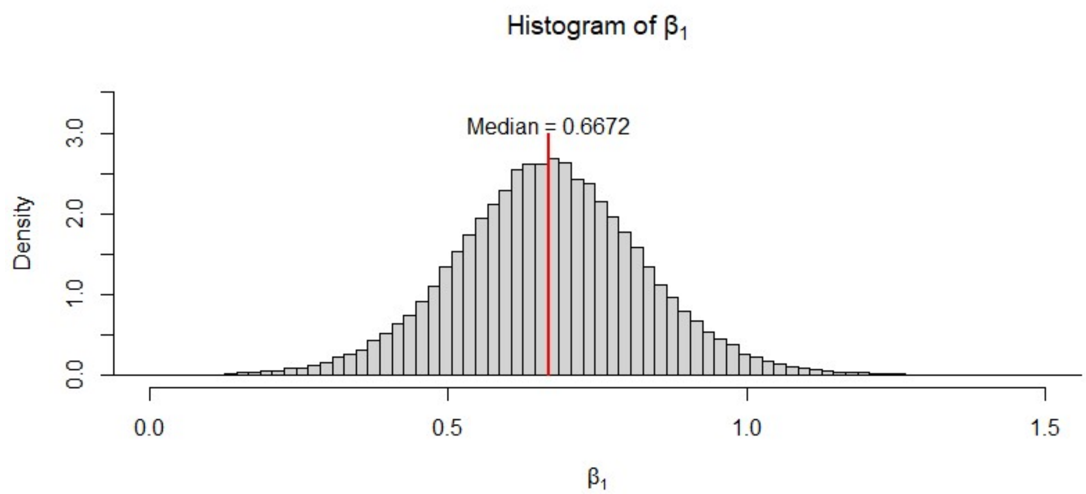
b) Posterior median:

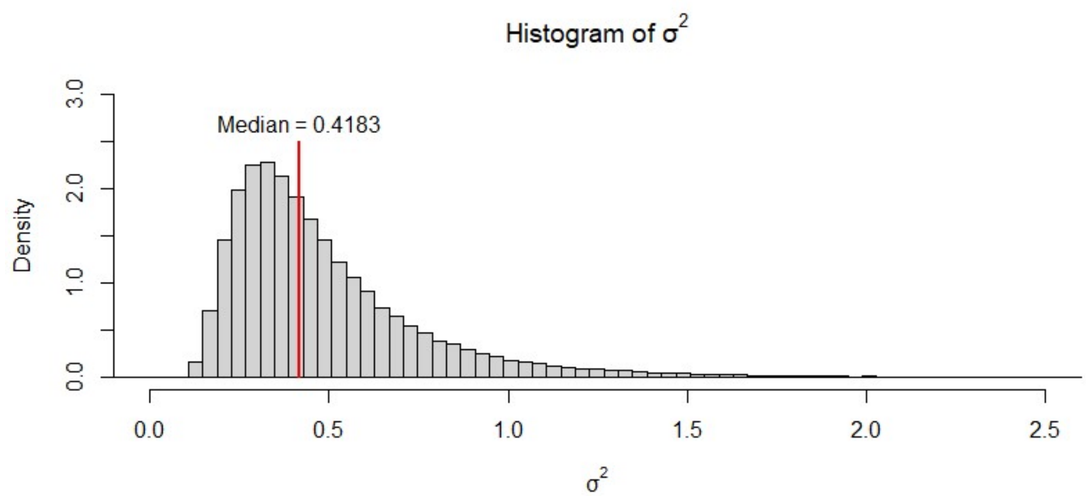
β_0=0.9065

β_1=0.6672

σ^2=0.4183


Histogram of β0
Median = 0.9065
Density
0.0
0.2
0.4
0.6
-2
0
2
4
β0


Histogram of β1
Median = 0.6672
Density
0.0
1.0
2.0
3.0
0.0
0.5
1.0
1.5
β1


Histogram of σ2
Median = 0.4183
Density
0.0
1.0
2.0
3.0
0.0
0.5
1.0
1.5
2.0
2.5
σ2