ASSIGNMENT 1

KUNAL PATEL

1291822

1.

1) a) True

b) False

eg: Matrix, $A = \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}$ is nonsingular as

$|A| = -2$, but is also not orthogonal as

$A^T A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix} = \begin{bmatrix} 5 & 11 \\ 11 & 25 \end{bmatrix} \neq I$.

c) False.

eg: Matrix, $A = \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}$ is of full rank as it is non singular but it not orthogonal as shown above.

2.

2) Let A be a ~~matrix~~ symmetric matrix, then there exists an orthogonal matrix Q, i.e., $A = Q^T \Lambda Q$

where $\lambda = \begin{bmatrix} \lambda_1 & & \\ & \ddots & \\ & & \lambda_n \end{bmatrix}$ where λ_i are eigenvalues of A.

Now, $y^T A y = (y^T Q^T) \Lambda (Q y)$

$= (Qy)^T \Lambda (Qy) = x^T \Lambda x = \sum_{i=1}^{n} \lambda_i x_i^2$

where $Qy = x$.

$\Sigma \lambda_i x_i^2 > 0$ is true for all x iff all $\lambda_i > 0$.

Since Q is orthogonal $\forall x \ \exists y : y = Q^T x$.

3.

3) a) Ay is also a MVN vector.

$$Z = Ay \sim MVN(A\mu, AVA^T)$$

New mean, $A\mu = \begin{bmatrix} 1 & -4 & 3 \\ -4 & 3 & 6 \\ 3 & 6 & 2 \end{bmatrix} \begin{bmatrix} 3 \\ 1 \\ -2 \end{bmatrix} = \begin{bmatrix} -7 \\ -21 \\ 11 \end{bmatrix}$

New variance, $V' = AVA^T = \begin{bmatrix} 39 & 40 & -11 \\ 40 & 222 & 75 \\ -11 & 75 & 175 \end{bmatrix}$

b) $E[y^T Ay] = tr(AV) + \mu^T A\mu$

$$= tr\left(\begin{bmatrix} -1 & -4 & 8 \\ -9 & 8 & 27 \\ 15 & 17 & 14 \end{bmatrix}\right) - 64$$

$$= -43$$

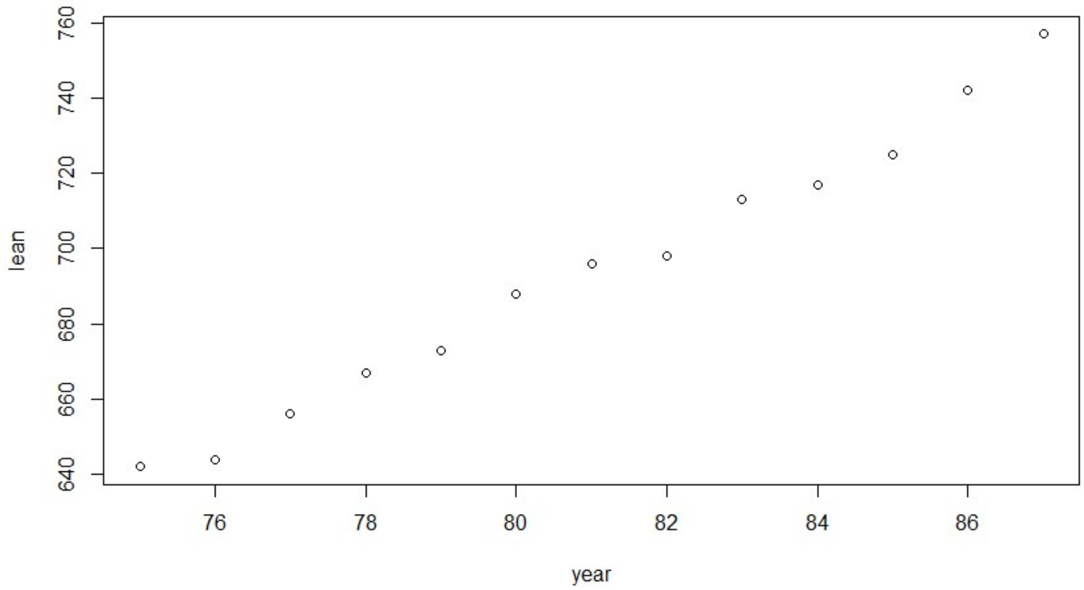
c) If $y \sim MVN(\mu, V)$ then $y^T Ay$ is noncentral χ^2 iff AV is idempotent & rank k.

Here $AV = \begin{bmatrix} -1 & -4 & 8 \\ -9 & 8 & 27 \\ 15 & 17 & 14 \end{bmatrix}$

$|AV| = -3961 \neq$ AV is not idempotent $\therefore$

$\therefore$ $y^T Ay$ does not follow non-central χ^2 distribution

4.a) Yes, the linear model is appropriate.



4) b) $\underset{13\times1}{y} = [642\ 644\ 656\ 667\ 673\ 688\ 696\ 698\ 713\ 717\ 725\ 742\ 757]^T$

$$\underset{13\times2}{X} = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 75 & 76 & 77 & 78 & 79 & 80 & 81 & 82 & 83 & 84 & 85 & 86 & 87 \end{bmatrix}^T$$

$\underset{2\times1}{\beta} = (\beta_0\ \beta_1)^T$

$\underset{13\times1}{\varepsilon} = [\varepsilon_1\ \varepsilon_2\ \varepsilon_2\ \ldots\ldots\ \varepsilon_{13}]^T$

c) Here all values of x are different, $\therefore$ it is a full rank model.

d) For the least squares solution

$$\beta = (X^TX)^{-1}X^Ty$$

Solving in R we get, $\beta = (-61.121\ \ 9.319)^T$

$\Rightarrow$ lean $= -61.121 + 9.319 \times$ year

Comparison of fitted model against true values, we see linear model fits very good.