MAST 90014 - Optimisation for Industry

Individual assignment - 2024

General Instructions

The assignment is individual and will be thorougly checked for plagiarism. You are not allowed to share any documents/files regarding this activity with your colleagues.

Your answers should be submitted as three documents:

1. In Canvas: a PDF file generated via the tex template provided and a Jupyter notebook, also following the template provided.
2. In Gradescope: The filled summary results sheet provided. Numbers should be filled rounded to the closest integer, no decimals.

Your implementation in Jupyter should use the same parameters as the model in your PDF. Constraints should be presented in the same order in both documents (follow the example in the template files provided). The code should contain commands to print the answers you used to fill the tables in the summary sheet.

You should explain any new parameters you add to the problem. This includes providing values for any big M parameter.

Assignments will be marked for correctness, clarity of explanations, organisation and proper following of these instructions.

1 PPP Printing

PPP Printing Pty Ltd has approached you, a senior mathematical optimisation specialist, to assist them in determining the best base roster for the next 6-month period. The employees of the printing company is employed under the company's Enterprise Agreement.

PPP Printing Pty Ltd operates on a two-week roster cycle, i.e. the base roster repeats every fortnight. The first day of the roster is Monday.

The daily coverage requirement is outlined below (these are minimum requirements. You can roster more employees than needed at any given time):

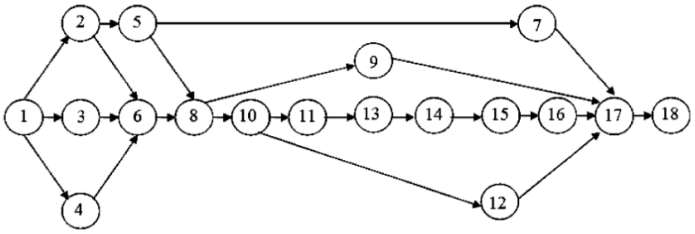
- Monday to Friday: two employees from 9am to 5pm, and two additional employees between 12pm and 3pm.
- Saturday: two employees from 9am to 4pm, and one additional employee between 11am and 2pm.
- Sunday: two employees from 9am to 2pm.

Consider the following characteristics and model as associated mixed-integer program model in order to minimise PPP's cost. For each item, unless stated otherwise, you should consider that all characteristics mentioned in previous items are still to be respected.

1. PPP has 4 employees which are paid at a base rate of $25 per hour. Employees can work a single shift a day. Shifts can have durations of 3, 4, 5, 6, 7 or 8 hours.
2. Now consider that employee 1 is a part-time students and can work a maximum of 20h per week. Employees 2 to 4 are full-time employees and, by contract, need to be assigned at least 60h per fortnight. If a employee works more than 30h per week, this is considered overtime. No employee can work more than 10h of overtime per week. Each overtime hour incurs an additional overtime penalty rate of 50% (that is, the pay is now $37.5 per hour).
3. Now consider that, by contract, full time employees must have at least one of the weekends off every fortnight. That is, they won't be available for work either on the first weekend or on the second (or on both). Also consider that an additional part-time employee is hired in this scenario.
4. By contract, no employee can work more than 3 days in a row. Implement this new requirement and check if there is a feasible solution that can be implemented with the current workforce of item 3 (that is, 2 part time and 3 full time employees). If the answer is no, would it be cheaper for PPP to hire an additional full time employee or to hire an additional part time employee? Justify your answer.

2 Assembly line

In an assembly line, the product is moved sequentially through a set of stations during its manufacturing process. At each station, some of the tasks needed for manufacturing are executed. A set of precedence constraints limit the order in which the tasks can be assigned. These precedences can be represented by a directed graph, as exemplified in the figure below.



In the figure above, there are 18 tasks to be executed and the precedence graph indicates, e.g., that task 6 can only be executed after tasks 2, 3 and 4 are completed.

In assembly lines with heterogeneous workforces, the time to execute a task depends on the worker that is assigned to its execution. For example, consider the execution times in the workers x tasks table below:

Tasks	Workers						
	h1	h2	h3	h4	h5	h6	h7
1	31	31	31	26	26	31	31
2	84	84	80	53	51	71	80
3	115	115	146	73	66	113	110
4	115	115	141	94	73	110	110
5	80	80	84	74	65	75	80
6	119	114	141	98	86	139	119
7	84	84	80	61	73	75	80
8	93	81	124	90	87	94	93
9	26	27	53	44	30	35	35
10	31	44	Inf	42	38	56	44
11	39	42	Inf	42	39	55	44
12	15	14	Inf	15	15	16	15
13	70	90	Inf	80	59	88	80
14	81	101	Inf	92	71	81	66
15	97	97	Inf	98	60	97	63
16	106	106	168	82	84	98	85
17	Inf	Inf	181	Inf	Inf	124	Inf
18	Inf	Inf	26	Inf	Inf	26	Inf

The first task can be executed in 31 units of time (say, seconds) by workers 1,2,3,6 and 7. However, workers 4 and 5 are faster for this task and can complete the job in 26s. Some workers can not execute some tasks and this is indicated by the entries 'Inf' in the table.

Each worker can only be assigned to a single station and each station receives at most a single worker. The load of a station is known as *cycle time* and corresponds to the combined execution time of all tasks assigned to that station.

1. In the problem known as type-2, the goal is to minimise the cycle time while employing all workers. That is, in the case of the example of the figures above, 7 stations will be open and a worker assigned to each one of them. Model and implement

a mixed-integer programming model to find the task and worker assignments that solve the type-2 problem.

2. In the problem known as type-1, the goal is to minimise the number of workers (equivalently, of stations) while guaranteeing that a cycle time $\overline{c}$ is respected. Model and implement a mixed-integer programming model to find the task and worker assignments that solve the type-1 problem for $\overline{c} = 300$.

3. In the provided zip file, there are a number of instances for the problem of type-2. The file has the following format: the first line contains the number of tasks in the instance, n. This is followed by n lines containing the execution times for each task. This is then followed by the precedence relations to be respected. The file ends with the line '-1 -1'.

   Run all instances in the file for the problem of type 2. Create a table in which each instance appears as a row. For each row, indicate the name of the instance (i.e., the name of the file), the number of tasks in the problem, the optimal solution value and the execution time. In case you can not solve the problem in less than 5 minutes, indicate the best incumbent solution you obtained at the end of the computational time.