2.4: Exact Equations.

Exact Equation

A differential expression $M(x,y)\,dx + N(x,y)\,dy$ is an exact differential, in a region R of xy-plane if it corresponds to the differential of some function $f(x,y)$ defined in R. A first order differential equation of the form

$$M(x,y)\,dx + N(x,y)\,dy = 0$$

is said to be exact equation. if the expression on the left hand side is an exact differential.

Example

$$x^2y^3\,dx + x^3y^2\,dy = 0$$

we can write the left hand side of equation as

$$d\left(\tfrac{1}{3}x^3y^3\right) = \tfrac{1}{3}(3x^2y^3)\,dx + \tfrac{1}{3}(3x^3y^2)\,dy$$

$$= x^2y^3\,dx + x^3y^2\,dy.$$

So left hand side of equation is an exact differential and Equation is an exact equation

Identifying $M(x,y)$ & $N(x,y)$

$\Rightarrow$ $M(x,y) = x^2y^3$ $\qquad$ $N(x,y) = x^3y^2$.

$\dfrac{\partial M}{\partial y} = 3x^2y^2$ $\qquad$ $\dfrac{\partial N}{\partial x} = 3x^2y^2$.

So we can conclude that

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$$

Criterion for an Exact differential

Let $M(x,y)$ and $N(x,y)$ be continuous and have continuous first partial derivatives in a rectangular region R defined by $a<x<b$, $c<y<d$, then a neccessary and sufficient condition that $M(x,y)\,dx + N(x,y)\,dy$ be an exact differential is

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}. \qquad (A)$$

Method of solution

Given the differential Equation

$$M(x,y)\,dy + N(x,y)\,dx = 0.$$

Check when the equality given in Eq (A) holds or not. If it holds then there exist a function f, for which

$$\frac{\partial f}{\partial x} = M(x,y)$$

Integrating w.r.t x

$$f(x,y) = \int M(x,y)\,dx + g(y) \qquad (1)$$

where $g(y)$ is constant of integration

~~Integrating (1) w.~~

Differentiating w.r.t y

$$\Rightarrow \frac{\partial f}{\partial y} = \frac{\partial}{\partial y}\int M(x,y)\,dx + g'(y) \qquad (2)$$

Also, we have

$$\frac{\partial f}{\partial y} = N(x,y) \qquad (3)$$

Equating Eq (2) & (3)

$$\Rightarrow \frac{\partial}{\partial y}\int M(x,y)\,dx + g'(y) = N(x,y)$$

$$g'(y) = N(x,y) + \frac{\partial}{\partial y}\int M(x,y)\,dx. \qquad (4)$$

Integrating (4) w.r.t y, we'll get $g(y)$ which can be substituted in (1) to get the solution $f(x,y)$.

Example Solve $2xy\,dx + (x^2-1)\,dy = 0$

Solution

$M(x,y) = 2xy \qquad N(x,y) = x^2-1$

Checking the equality $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$

$\frac{\partial M}{\partial y} = 2x \qquad \frac{\partial N}{\partial x} = 2x$

$\Rightarrow \quad \frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$

So given Eq. is an exact Equation. So we have.

$\frac{\partial f}{\partial x} = M(x,y) \quad (A) \qquad \frac{\partial f}{\partial y} = N(x,y) \quad (b)$

$\frac{\partial f}{\partial x} = 2xy \quad (1) \qquad \frac{\partial f}{\partial y} = x^2-1 \quad (2)$

Integrating (1) w.r.t x

$\Rightarrow \quad f(x,y) = \cancel{2}y\frac{x^2}{\cancel{2}} + g(y) \qquad (3)$

Diff. w.r.t y

$\Rightarrow \quad \frac{\partial f}{\partial y} = x^2 + g'(y)$

using Eq. (2)

$\Rightarrow \quad \cancel{x^2} - 1 = \cancel{x^2} + g'(y)$

$\Rightarrow \quad g'(y) = -1$

Integrating w.r.t y

$\Rightarrow \quad g(y) = -y + c$

using $g(y)$ in (3)

$\Rightarrow \quad f(x,y) = yx^2 - y + c$

So $x^2y - y = c$ is the implicit solution of differential Equation.

$\Rightarrow \quad y = \dfrac{c}{x^2-1} \rightarrow$ Explicit solution.

This solution is defined on any interval not containing $x=1$ & $x=-1$.

Example

Solve $\left(e^{2y} - y\cos(xy)\right)dx + \left(2xe^{2y} - x\cos(xy) + 2y\right)dy.$

Solution

$M(x,y) = e^{2y} - y\cos(xy) \qquad N = 2xe^{2y} - x\cos(xy) + 2y.$

$\dfrac{\partial M}{\partial y} = 2e^{2y} - \cos xy + yx\sin(xy) \qquad \dfrac{\partial N}{\partial x} = 2e^{2y} - \cos(xy) + xy\sin(xy)$

$\Rightarrow \quad \dfrac{\partial M}{\partial y} = \dfrac{\partial N}{\partial x}$

So given Eq. is an Exact Equation

Now

$\underbrace{\dfrac{\partial f}{\partial x} = e^{2y} - y\cos(xy)}_{(1)} \qquad \underbrace{\dfrac{\partial f}{\partial y} = 2xe^{2y} - x\cos(xy) + 2y.}_{(2)}$

Integrating (1) w.r.t x

$\Rightarrow \quad f(x,y) = \cancel{2}e^{2y}x - \cancel{y}\dfrac{\sin xy}{\cancel{y}} + g(y) \qquad (3)$

Now Diff w.r.t y

$\Rightarrow \quad \dfrac{\partial f}{\partial y} = 2x\dfrac{e^{2y}}{\cancel{2}} - (\cos xy)x + g'(y)$

using (2)

$\Rightarrow \quad \cancel{2xe^{2y}} - \cancel{x\cos(xy)} + 2y = \cancel{2xe^{2y}} - \cancel{x\cos(xy)} + g'(y)$

$$g'(y) = 2y$$

Integrating w.r.t y

$$\Rightarrow \quad g(y) = y^2 + c$$

So we get from (3) as

$$f(x,y) = xe^{2y} - \sin xy + y^2 + c.$$

So Family of solutions is

$$xe^{2y} - \sin xy + y^2 + c = 0.$$

Example

Solve $\dfrac{dy}{dx} = \dfrac{xy^2 - \cos x \sin x}{y(1-x^2)}$ $\quad y(0) = 2.$

Solution

Rearranging the differential Equation as

$$y(1-x^2)\,dy = (xy^2 - \cos x \sin x)\,dx$$

$$\Rightarrow \quad (\cos x \sin x - xy^2)\,dx + (y)(1-x^2)\,dy = 0$$

$M(x,y) = \cos x \sin x - xy^2 \qquad N(x,y) = y(1-x^2)$

$\dfrac{\partial M}{\partial y} = -2xy \qquad \dfrac{\partial N}{\partial x} = -2xy.$

So $\dfrac{\partial M}{\partial y} = \dfrac{\partial N}{\partial x}$

and given Equation is exact

Now

$$\frac{\partial f}{\partial x} = \cos x \sin x - xy^2 \qquad (1)$$

$$\frac{\partial f}{\partial y} = y(1-x^2) \qquad (2).$$

Integrating (2) w.r.t y

$$f(x,y) = \frac{y^2}{2}(1-x^2) + h(x) \qquad (3)$$

Diff. w.r.t x

$$\Rightarrow \quad \frac{\partial f}{\partial x} = -xy^2 + h'(x)$$

using Eq. (1)

$$\Rightarrow \quad \cos x \sin x - \cancel{xy^2} = -\cancel{xy^2} + h'(x)$$

$$\Rightarrow \quad h'(x) = \cos x \sin x.$$

Integrating w.r.t x

$$h(x) = \frac{\sin^2 x}{2} + c.$$

From (3)

$$\Rightarrow \quad f(x,y) = \frac{y^2}{2}(1-x^2) + \frac{\sin^2 x}{2} + c.$$

So Solution is

$$\frac{y^2}{2}(1-x^2) + \frac{\sin^2 x}{2} = -c.$$

$$y^2(1-x^2) + \sin^2 x = -2c = c_1.$$

using the condition that when $x=0$, $y=2$

$$\Rightarrow \quad 4(1) + \sin^2(0) = c_1$$

$$\Rightarrow \quad c_1 = 4$$

So Final solution is.

$$y^2(1-x^2) + \sin^2 x = 4.$$

Integrating factors

Some times we can covert a non-exact differential Equation into exact one by Multiplying it with an integrating factor $u(x,y)$ such that

$$u(x,y)\, M(x,y)\, dx + u(x,y)\, N(x,y)\, dy = 0 \qquad (1)$$

is an exact equation

For Eq (1) to be exact

$$\frac{\partial}{\partial y}(\mu M) = \frac{\partial}{\partial x}(\mu N)$$

$$\mu_y M + \mu M_y = \mu_x N + \mu N_x \qquad (2)$$

* Now to find μ, we will assume that μ is a function of one variable only.

Let suppose, $\mu = \mu(x)$ & $\mu_y = 0$

so (2) becomes

$$\mu M_y = \mu_x N + \mu N_x$$

$$\Rightarrow \quad \mu_x N = \mu M_y - \mu N_x$$

$$\boxed{\frac{\partial \mu}{\partial x} = \frac{\mu(M_y - N_x)}{N}}$$

$$\frac{d\mu}{\mu} = \frac{(M_y - N_x)}{N}\, dx$$

$$\ln \mu = \int \frac{(M_y - N_x)}{N}\, dx$$

$$\Rightarrow \quad \boxed{\mu = \exp\left(\int \frac{M_y - N_x}{N}\, dx\right)} \qquad \text{I. F.}$$

Now if $\mu = \mu(y) \Rightarrow \mu_x = 0$

so (2) $\Rightarrow$

$$\mu_y M = \mu(N_x - M_y)$$

$$\frac{d\mu}{dy} = \left(\frac{N_x - M_y}{M}\right)\mu$$

$$\Rightarrow \quad \frac{d\mu}{\mu} = \frac{N_x - M_y}{M}\, dy$$

Integrating

$$\Rightarrow \quad \ln \mu = \int \frac{N_x - M_y}{M}\, dy$$

$$\mu = \exp\left(\int \frac{N_x - M_y}{M}\, dy\right).$$

Summary

If we have

$$M(x,y)\,dx + N(x,y)\,dy = 0 \qquad (A)$$

* If $\frac{M_y - N_x}{N}$ is function of x alone, then I.F for (A) is

$$\mu(x) = e^{\int \frac{M_y - N_x}{N}\, dx} \qquad (B)$$

* If $\frac{N_x - M_y}{M}$ is function of y alone, then I.F. for (A) is

$$\mu(y) = e^{\int \frac{N_x - M_y}{M}\, dy} \qquad (C)$$

Example

$$xy\, dx + (2x^2 + 3y^2 - 20)\, dy = 0 \qquad (1)$$

Checking whether DE is exact or not

$M(x,y) = xy \qquad N(x,y) = 2x^2 + 3y^2 - 20$

$M_y = x \qquad N_x = 4x$

So $\quad M_y \neq N_x$

Now Checking

$$\frac{M_y - N_x}{N} = \frac{x - 4x}{2x^2 + 3y^2 - 20} = \frac{-3x}{2x^2 + 3y^2 - 20}$$

which is function of both x & y.

Now Checking

$$\frac{N_x - M_y}{M} = \frac{4x - x}{xy} = \frac{3x}{xy} = \frac{3}{y}$$

which is function of y only. So I.F. is

$$\mu(y) = e^{\int \frac{3}{y} dy} = e^{3 \ln y} = e^{\ln y^3} = y^3$$

xing y^3 with (1)

$\Rightarrow \quad xy^4 dx + y^3(2x^2+3y^2-20)dy = 0$

Now to Check, whether it is exact or Not.

$M(x,y) = xy^4$ $\qquad\qquad$ $N(x,y) = y^3(2x^2+3y^2-20)$

$\dfrac{\partial M}{\partial y} = 4xy^3$ $\qquad\qquad$ $\dfrac{\partial N}{\partial x} = 4xy^3$

Exercise 2.4

Determine whether given Eq. is Exact. If it is exact, Solve it.

6) $\left(2y - \frac{1}{x} + \cos 3x\right)\frac{dy}{dx} + \frac{y}{x^2} - 4x^3 + 3y\sin 3x = 0$

$$\left(2y - \frac{1}{x} + \cos 3x\right)dy + \left(\frac{y}{x^2} - 4x^3 + 3y\sin 3x\right)dx = 0$$

$M(x,y) = 2y - \frac{1}{x} + \cos 3x$ $\quad N(x,y) = \frac{y}{x^2} - 4x^3 + 3y\sin 3x$

$\frac{\partial M}{\partial y} = 2$ $\quad \frac{\partial N}{\partial x} = \frac{-2y}{x^3} - 12x^2 + 9y\cos 3x$

$$\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}$$

Thus the equation is not exact.

10 $\quad (x^3+y^3)\,dx + 3xy^2\,dy = 0$

$M(x,y) = x^3 + y^3$ $\quad N(x,y) = 3xy^2$

$\frac{\partial M}{\partial y} = 3y^2$ $\quad \frac{\partial N}{\partial x} = 3y^2$

As $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$

So given Eq. is exact. Therefore a function $f(x)$ exists such that

$$\frac{\partial f}{\partial x} = M(x,y) = x^3 + y^3 \qquad (1)$$

$$\frac{\partial f}{\partial y} = N(x,y) = 3xy^2 \qquad (2)$$

Integrating (2) w.r.t y

$$\Rightarrow f(x,y) = xy^3 + g(x) \qquad (3)$$

Diff. w.r.t x

$$\frac{\partial f}{\partial x} = y^3 + g'(x)$$

using (1)

$$x^3+\cancel{y^3} = \cancel{y^3}+g'(x)$$

$$g'(x) = x^3$$

Integrating w.r.t x

$$\Rightarrow \quad g(x) = \frac{x^4}{4}+c$$

So from (3)

$$f(x,y) = xy^3+\frac{x^4}{4}+c$$

17 $\left(\tan x - \sin x \sin y\right)dx + \cos x \cos y \, dy$

$M(x,y) = \tan x - \sin x \sin y \qquad N(x,y) = \cos x \cos y$

$\dfrac{\partial M}{\partial y} = -\sin x \cos y \qquad \dfrac{\partial N}{\partial x} = -\sin x \cos y$

So there exist a function f such that

$$\frac{\partial f}{\partial x} = \tan x - \sin x \sin y = M(x,y) \qquad (1)$$

$$\frac{\partial f}{\partial y} = \cos x \cos y = N(x,y) \qquad (2)$$

Integrating w.r.t y

$$f(x,y) = \cos x \sin y + g(x). \qquad (3)$$

Diff. w.r.t x

$$\Rightarrow \quad \frac{\partial f}{\partial x} = -\sin x \sin y + g'(x)$$

using (1)

$$\Rightarrow \quad \tan x - \sin x\cancel{\sin y} = -\sin x\cancel{\sin y}+g'(x)$$

$$g'(x) = \tan x.$$

Integrating w.r.t x

$$g(x) = -\ln|\cos x|+c$$

From Eq. 3

$$f(x,y) = \cos x \sin y - \ln|\cos x| + c$$

Solve the given IVP.

24 $\left(\frac{3y^2 - t^2}{y^5}\right)\frac{dy}{dt} + \frac{t}{2y^4} = 0$

$$\left(\frac{3y^2}{y^5} - \frac{t^2}{y^5}\right)dy + \frac{t}{2y^4}dt = 0$$

$$(3y^{-3} - t^2y^{-5})\,dy + \frac{t}{2}y^{-4}dt = 0$$

$M(t,y) = 3y^{-3} - t^2y^{-5}$ $\qquad$ $N(t,y) = \frac{t}{2}y^{-4}$

$\frac{\partial M}{\partial t} = -2ty^{-5}$ $\qquad$ $\frac{\partial N}{\partial y} = \frac{-4ty^{-5}}{2} = \frac{-2t}{y^5}$

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial y}$$

Now

$\frac{\partial f}{\partial y} = 3y^{-3} - t^2y^{-5}$ (1) $\qquad$ $\frac{\partial f}{\partial t} = \frac{t}{2}y^{-4}$ (2)

Integrating (1) w.r.t y

$$\Rightarrow f(t,y) = -3\frac{y^{-2}}{2} - t^2\frac{y^{-4}}{-4} + g(t)$$

$$= \frac{-3y^{-2}}{2} + \frac{t^2y^{-4}}{4} + g(t) \qquad (3)$$

Diff w.r.t t

$$\Rightarrow \frac{\partial f}{\partial t} = \frac{t}{2}y^{-4} + g(t)$$

From (2)

$$\frac{t}{2}\cancel{y^{-4}} = \frac{t}{2}\cancel{y^{-4}} + g'(t)$$

$$g'(t) = 0$$

Integrating

$$\Rightarrow \; g(t) = c.$$

So (3) $\Rightarrow$

$$f(t,y) = \frac{t^2}{4y^4} - \frac{3}{2y^2} + c.$$

So Implicit Solution is

$$\frac{t^2}{4y^4} - \frac{3}{2y^2} = c.$$

using initial condition. $y(1) = 1$

$$\Rightarrow \quad \frac{1}{4} - \frac{3}{2} = c \qquad \Rightarrow \quad c = -5/4.$$

$$\Rightarrow \quad \frac{t^2}{4y^4} - \frac{3}{2y^2} = -5/4.$$

Find the value of K so that given Differential Eq is exact.

28 $(6xy^3 + \cos y)\,dx + (2Kx^2y^2 - x\sin y)\,dy = 0.$

$M(x,y) = 6xy^3 + \cos y \qquad N(x,y) = 2Kx^2y^2 - x\sin y.$

$\frac{\partial M}{\partial y} = 18xy^2 - \sin y \qquad \frac{\partial N}{\partial x} = 4Kxy^2 - \sin y$

For Equation to be exact

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$$

$$\Rightarrow \quad 18xy^2 - \cancel{\sin y} = 4Kxy^2 - \cancel{\sin y}$$

$$4K\cancel{xy^2} = 18\cancel{xy^2}$$

$$K = 18/4 = 9/2.$$

30 verify that Given DE is not Exact: Multiply the DE by the indicated I.F $u(x,y)$ and verify that new Eq. is exact.

$$(x^2+2xy-y^2)\,dx + (y^2+2xy-x^2)\,dy = 0$$

$$u(x,y) = (x+y)^{-2}$$

Solution

$M(x,y) = x^2+2xy-y^2$ $\qquad$ $N(x,y) = y^2+2xy-x^2$

$\frac{\partial M}{\partial y} = 2x-2y$ $\qquad$ $\frac{\partial N}{\partial x} = 2y-2x$

$$\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}$$

So given Eq. is not exact.

Xing with given IF. i-e $u(x,y) = (x+y)^{-2}$.

$$\Rightarrow (x+y)^{-2}(x^2+2xy-y^2)\,dx + (x+y)^{-2}(y^2+2xy-x^2)\,dy = 0$$

$$(x+y)^{-2}(x^2+y^2+2xy-y^2-y^2)\,dx + (x+y)^{-2}(x^2+y^2+2xy-x^2-x^2)\,dy = 0$$

$$(x+y)^{-2}((x+y)^2-2y^2)\,dx + (x+y)^{-2}((x+y)^2-2x^2)\,dy = 0$$

$$\left(1-\frac{2y^2}{(x+y)^2}\right)dx + \left(1-\frac{2x^2}{(x+y)^2}\right)dy = 0$$

Now

$M(x,y) = 1-\frac{2y^2}{(x+y)^2}$ $\qquad$ $N(x,y) = 1-\frac{2x^2}{(x+y)^2}$

$$\frac{\partial M}{\partial y} = -\frac{4y(x+y)^2-2y^2\cdot 2(x+y)}{(x+y)^2} = -\frac{4x^2y+4xy^2}{(x+y)^2}$$

$$\frac{\partial N}{\partial x} = -\frac{4x(x+y)^2 - 2x^2 \cdot 2(x+y)}{(x+y)^4}$$

$$= -\frac{4x^2y + 4xy^2}{(x+y)^4}$$

so

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$$

∴ Eq. is exact

$$\frac{\partial f}{\partial x} = 1 - \frac{2y^2}{(x+y)^2} \quad (1) \qquad \frac{\partial f}{\partial y} = 1 - \frac{2x^2}{(x+y)^2} \quad (2)$$

Integrating w.r.t x

$$\Rightarrow \quad f(x,y) = x + \frac{2y^2}{(x+y)} + g(y)$$

$$\frac{\partial f}{\partial y} = 0 + \frac{(x+y)(4y) - 2y^2}{(x+y)^2} + g'(y)$$

$$= \frac{4xy + 4y^2 - 2y^2}{(x+y)^2} + g'(y)$$

from (2)

$$\Rightarrow \quad g'(y) + \frac{4xy + 2y^2}{(x+y)^2} = \frac{(x+y)^2 - 2x^2}{(x+y)^2}$$

$$g'(y) = \frac{x^2 + y^2 + 2xy - 2x^2 - 4xy - 2y^2}{(x+y)^2}$$

$$= \frac{-x^2 - y^2 - 2xy}{(x+y)^2} = \frac{-\cancel{(x+y)^2}}{\cancel{(x+y)^2}}$$

$$g'(y) = -1$$

$$g(y) = -y + c.$$

$$\Rightarrow \quad f(x,y) = x - y + \frac{2y^2}{x+y} + c.$$

34 $\cos x\, dx + \left(1 + \frac{2}{y}\right) \sin x\, dy = 0$

$M(x,y) = \cos x$ $\qquad N(x,y) = (1 + 2/y) \sin x.$

$\frac{\partial M}{\partial y} = 0$ $\qquad \frac{\partial N}{\partial x} = (1 + 2/y) \cos x.$

Computing $\frac{M_y - N_x}{N} = \frac{0 - \cancel{(1 + 2/y)} \cos x}{\cancel{(1 + 2/y)} \sin x} = -\cot x.$

which is function of x only.

$$e^{\int \frac{M_y - N_x}{N} dx} = e^{\int -\cot x\, dx}$$

$$= e^{-\ln|\sin x|} = e^{\ln(\sin x)^{-1}}$$

$$= \frac{1}{\sin x} = \csc x.$$

xing the I.F. with given Eq

$$\csc x \cos x\, dx + (1 + 2/y) \sin x \times \csc x\, dy = 0$$

$$\cot x\, dx + (1 + 2/y) dy = 0$$

Now

$M(x,y) = \cot x$ $\qquad N(x,y) = 1 + 2/y.$

$\frac{\partial M}{\partial y} = 0$ $\qquad \frac{\partial N}{\partial x} = 0$

So Eq. is exact Now.

$\Rightarrow \quad \frac{\partial f}{\partial x} = \cot x \qquad (1)$

$\frac{\partial f}{\partial y} = 1 + 2/y \qquad (2).$

Integrating (2)

$\Rightarrow \quad (x,y) = y + 2\ln y + g(x) \qquad (3)$

Diff w.r.t x

$\Rightarrow \quad \frac{\partial}{\partial x} = g'(x)$

From (1)

$g'(x) = \cot x$

$\Rightarrow \quad g(x) = \ln|\sin x| + c$

From (3)

$$f(x,y) = y + 2\ln y + \ln|\sin x| + c.$$

39 Show that a one parameter family of solutions of Eq.

$(4xy + 3x^2)\,dx + (2y + 2x^2)\,dy = 0$

is $\quad x^3 + 2x^2y + y^2 = c.$

soln we have

$$x^3 + 2x^2y + y^2 = c$$

Diff. w.r.t x

$\Rightarrow \quad 3x^2 + 2\left(x^2\frac{dy}{dx} + 4xy\right) + 2y\frac{dy}{dx} = 0$

$\frac{dy}{dx}(2x^2 + 2y) = -(3x^2 + 4xy)$

$\Rightarrow \quad (3x^2 + 4xy)\,dx + (2x^2 + 2y)\,dy = 0$

So $\quad x^3 + 2x^2y + y^2 = c$ is the solution of given DE.

(b) Show that the initial conditions $y(0) = -2$ & $y(1) = 1$ determine the same implicit solution

$$x^3 + 2x^2y + y^2 = c.$$

using $y(0) = -2$

$\Rightarrow \quad 0+0+4=c \Rightarrow \quad c=4.$

using $y(1)=1$

$\Rightarrow \quad 1+2+1=c \Rightarrow \quad c=4$

As we get same constant by Both ICs so we get same implicit solution.