Diminished Radix Complement

Number N, in base r having n-digits

$(r-1)$'s complement of N

$= (r^n - 1) - N$

① $546700 = N$

$r = 10$

$n = 6$

$r^n = 10^6$

$$\begin{array}{r} 1000000 \\ -\quad 1 \\ \hline 999999 \\ 546700 \\ \hline 453299 \end{array}$$

② $N = 012398$

$r = 10$

$r^n = 10^6$

$$\begin{array}{r} 1000000 \\ -\quad 1 \\ \hline 999999 \\ 012398 \\ \hline 987601 \end{array}$$

s complement: $(r^n - 1) - N$

$N = 1011000$

[short cut: invert bits]

$n = 7$
$r = 2$

$(2^7 - 1) - N$

$2^7\ 2^6\ 2^5\ 2^4\ 2^3\ 2^2\ 2^1\ 2^0$

$\cancel{1}0000000$

$-\qquad\qquad 1$

$2^7 - 1 \quad 1111111$

$-\ 1011000$

0100111

2 $N = 0101101$

[short cut: invert bits]

$n = 7$
$r = 2$

$2^7\ 2^6\ 2^5\ 2^4\ 2^3\ 2^2\ 2^1\ 2^0$

10000000

$-\qquad\qquad 1$

$[2^7 - 1] \quad 1111111$

0101101

1010010

Radix Complement

r's complement is defined as

$[r^n - N]$

(r-1)'s complement is defined as

$[r^n - 1] - N$

r's complement = (r-1)'s complement + 1

$$(r^n - N) = \underbrace{r^n - 1 - N} + 1$$

N	r	(r-1)'s complement	r's complement
012398	10	987601	987602
546700	10	453299	453300
1011000	2	0100111	0101000
0101101	2	1010010	1010011
76304	8	01473	01474
~~54~~ ABCDEF	16	543210	543211

The direct method of subtraction used in elementary school uses the borrow concept

- when subtraction is implemented with digital hardware, the complements method is more efficient

The subtraction of two n-digit unsigned numbers M-N in base r can be done as follows:

$$M + (r^n - N) = M - N + r^n$$

$\boxed{\text{if } M \geq N}$

M
+ (r's complement of N)

$\underbrace{\quad}_{\text{discard any carry}}$ $\underbrace{\text{Answer}}$

$\boxed{\text{if } M \leq N}$

M
+ (r's complement of N)

(r's complement of N−M)

↓

Take R's complement

↓

$\underbrace{\quad}_{\text{no carry is produced}}$ $\underbrace{\text{Answer}}$

$r^n - (N-M)$

9's complement subtraction

$\boxed{72532 - 3250}$

72532
+ 9's complement of 3250

$$\begin{array}{r} 99999 \\ -03250 \\ \hline 96749 \end{array}$$

$$\begin{array}{r} 72532 \\ +96749 \\ \hline 169281 \\ +1 \\ \hline 69282 \end{array}$$

add the end around carry

$\boxed{03250 - 72532}$

03250
+ (9's complement of 72532)

$$\begin{array}{r} 99999 \\ -72532 \\ \hline 27467 \end{array}$$

$$\begin{array}{r} 03250 \\ +27467 \\ \hline 30717 \end{array}$$

→ 9's complement

$$\begin{array}{r} 99999 \\ -30717 \\ \hline -(69282) \end{array}$$

10's complement subtraction

$\boxed{72532 - 3250}$

$$\begin{array}{r} 99999 \\ -03250 \\ \hline 96749 \\ +1 \\ \hline 96750 \end{array}$$

72532

+ (10's complement of 03250)

$$\begin{array}{r} 72532 \\ +96750 \\ \hline 1\,69282 \end{array}$$

69282 Answer

drop the end carry.

$\boxed{3250 - 72532}$

$$\begin{array}{r} 99999 \\ -72532 \\ \hline 27467 \\ +1 \\ \hline 27468 \end{array}$$

03250

+ (10's complement of 72532)

$$\begin{array}{r} 03250 \\ +27468 \\ \hline 30718 \end{array}$$

Take 10's complement of answer.

$$\begin{array}{r} 99999 \\ 30718 \\ \hline 69281 \\ +1 \\ \hline \end{array}$$

Answer = $\boxed{-69282}$

1's complement Subtraction

X = 1010100 Y = 1000011

X − Y

1010100
\+ [1's complement of Y] → 0111100

```
 1 1 1 1
 1010100
+0111100
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(1)0010000   add end around carry → 0010000 + 1
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↳ drop the end carry

Y − X

1000011
\+ [1's complement of X] → 0101011

~~0111100~~

```
     1 1
 1000011
+0101011
--------
 1101110
```

Take 1's complement → −[0010001]

r's complement subtraction

X = 1010100 Y = 1000011

X − Y

0111101

  1010100
+ [2's complement of Y]

  1010100
  0111101
  (1)0010001

drop end carry

Y − X

  1000011
+ [2's complement of X]

  1010100
  0101011
      +1
  0101100

  1000011
+ 0101100
  1101111

Take 2's complement

−[0010001]

- advantages of 2's complement over 1's complement requires an extra addition to add end carry.
- 1's complement has 2 representations for 0.