Assignment # 1 (BS-CS: Section CY-A, B, C, D)
(CS-1005 Discrete Structures – Spring-2023)

Due Date and Time: Thursday, 23 February, 2023 (11:59 pm) Marks: 90

Instructions:

- *Late assignment will not be accepted*
- *Only the clear scan (using, e.g., CamScanner) copy of handwritten attempt will be graded, i.e., printed attempts will not be graded.*
- *Only the attempts submitted on Google classroom (till the due date & time) will be considered, i.e., the submissions that will be slided beneath instructors' office doors or submitted via email or elsewhere will not be graded.*
- *There will be no credit if the given requirements are changed.*
- *Your solution will be evaluated in comparison with the best solution.*
- *Whenever a calculation is involved, your solution should show complete steps and a final answer. There will be significant marks for the correct final answer (as far as assignments are concerned).*
- *You must write your roll number, name, and section (of your Discrete Course only) on your submitted attempt.*

Question 1: [10 Marks]

i) Construct a truth table for each formula below. Which ones are tautologies? Please justify your answer.

a) $(\sim p \vee q) \rightarrow p$

b) $(p \rightarrow q) \vee (p \rightarrow \sim q)$

c) $(p \rightarrow q) \rightarrow r$

ii) Consider a proposition P is $\underline{(A \vee B) \rightarrow C}$ and Q is $\underline{(\sim C \rightarrow \sim A) \vee (\sim C \rightarrow \sim B)}$. Which of the following best describes the relationship between P and Q? Please circle exactly one answer.

a. P and Q are equivalent.

b. P→ Q

c. Q→ P

d. All of the above

e. None of the above

Justify your answer by using a truth table and illustrate your reasoning. You can use as many columns as you need.

Question 2: [10 Marks]

i) Let A be "It is sunny", B be "it is cold", and C be "It's snowing" then translate the following into English.

a) $(B \wedge C) \rightarrow (\sim A)$

b) $B \rightarrow C \vee A$

c) $((\sim A) \wedge A) \rightarrow C$

ii) Let p: “you get 100% on the final” and q: “you will get an A”. Write all the alternate way of implication or conditional statements.

a) p implies that q.
b) If p, then q.
c) If p, q.
d) p is sufficient for q.
e) q if p.
f) q unless ~p.

Question 3: [10 Marks]

i) “The Blue team wins whenever it is raining?” Identify the hypothesis and conclusion of the statements and then rewrite its

a) Negation form
b) Inverse form
c) Converse form
d) Contrapositive form

ii) Use truth tables to verify the below logical equivalences.

a) $(p \wedge q) \leftrightarrow p \equiv p \rightarrow q$
b) $(p \wedge q) \rightarrow r \equiv p \rightarrow (\sim q \vee r)$
c) $(p \rightarrow \sim q) \wedge (p \rightarrow \sim r) \equiv \sim (p \wedge (q \vee r))$

Question 4: [10 Marks]

i) Analyze the below statement using a truth tables:

“if you get more doubles than any other player you will lose, or that if you lose you must have bought the most properties,”.

ii)Are the statements, “it will not rain or snow” and “it will not rain and it will not snow” logically equivalent? Justify your answer using a truth table.

Question 5: [10 Marks]

i) Prove the validity of the following arguments form (deduction rule).

$p \rightarrow r$
$q \rightarrow r$
r
$\therefore p \vee q$

ii) Prove the validity of the following argument form (deduction rule).

$p \rightarrow q$
$\sim p \rightarrow q$
$\therefore q$

Question 6: [10 Marks]

i) Are the statements $(p \vee q) \rightarrow r$ and $(p \rightarrow r) \vee (q \rightarrow r)$ logically equivalent? Justify your answer using truth table.

ii) Aslam was telling you what he ate yesterday afternoon. He tells you, "I had either popcorn or raisins. Also, if I had cucumber sandwiches, then I had soda. But I didn't drink soda or tea." Of course you know that Aslam is the world's worst liar, and everything he says is false. **What did Aslam eat?** Justify your answer by writing all of Aslam's statements using sentence variables (p, q, r, s, t), taking their negations, and using these to deduce what Aslam actually ate.

Question 7: [10 Marks]

Prove or disprove using Truth tables and then logical laws that:

i) $\sim(\sim r \wedge \sim q) \vee (\sim(r \wedge q) \wedge p) = \sim r \vee \sim q$

ii) $(x \wedge y \wedge \sim z) \vee (x \wedge \sim y) \vee z = x \vee y$

iii) $(p \wedge q) \leftrightarrow p \equiv p \rightarrow q$

iv) $(p \wedge q) \rightarrow r \equiv p \rightarrow (\sim q \vee r)$

v) $(p \rightarrow \sim q) \wedge (p \rightarrow \sim r) \equiv \sim(p \wedge (q \vee r))$

Question 8: [10 Marks]

Jack is murdered. You find four suspects and ask them for information.
They tell you the following things.
Tom: \Paul is the killer. Sam is the killer."
Sam: \I am not the killer. Tom is the killer."
Paul: \Tom is not the killer. Sam is the killer."
John: \I am not the killer. Tom is the killer."
You know that each of them tells you exactly one truth. There is exactly one killer. **Who is the killer?** Show your deduction carefully.

Question 9: [10 Marks]

i) Verify the following using truth table.

a) $(p \oplus q) \oplus r \equiv p \oplus (q \oplus r)$
b) $(p \oplus q) \wedge r \equiv (p \wedge r) \oplus (q \wedge r)$

ii) Test the validity of the argument using truth table:
If it snows, Paul will miss class.
It did not snow.
Therefore, Paul did not miss class.