4.4 Undetermined Coefficient

Superposition Approach

one of the method used to calculate the particular solution is method of undetermined coefficient

* The idea is to guess the form of particular solution depending on the type of input function $g(x)$.

* This method is limited to Linear DE with

1. The coefficient of DE a_i $i = 0, 1, \ldots, n$ are constant

2. $g(x)$ is a constant, exponential function $e^{\alpha x}$, a sine or cosine function $\sin \beta x$, or $\cos \beta x$ or finite sums and product of these functions.

Example General solution using undetermined coefficient

Solve $y'' + 4y' - 2y = 2x^2 - 3x + 6$.

Solution Solving the associated homogeneous Eq.

$$y'' + 4y' - 2y = 0$$

Auxiliary Eq.

$$\Rightarrow m^2 + 4m - 2 = 0$$

$$\Rightarrow \quad m_1 = -2 - \sqrt{6} \qquad m_2 = -2 + \sqrt{6}$$

So complementary solution is

$$y_c = c_1 e^{-(2+\sqrt{6})x} + c_2 e^{(-2+\sqrt{6})x}$$

* For particular solution,

As $g(x)$ is a quadratic polynomial, so assume that particular solution is of the form

$$y_p = Ax^2 + Bx + c$$

$$y_p' = 2Ax + B$$

$$y_p'' = 2A$$

Given DE is

$$y_p'' + 4y_p' - 2y_p = 2x^2 - 3x + 6$$

using the calculated values

$$\Rightarrow \quad 2A + 4(2Ax+B) - 2(Ax^2+Bx+C) = 2x^2-3x+6$$

$$\Rightarrow \quad 2A + 8Ax + 4B - 2Ax^2 - 2Bx - 2C = 2x^2 - 3x + 6$$

comparing the coefficients of like powers.

x^2: $\quad -2A = 2 \quad \Rightarrow \quad A = -1.$

x: $\quad 8A - 2B = -3$

$\Rightarrow \quad 8\times -1 - 2B = -3 \quad \Rightarrow \quad -2B = 5 \quad \Rightarrow B = -5/2.$

x^0:

$$2A + 4B - 2C = 6.$$

$$2\times(-1) + 4(-5/2) - 2C = 6$$

$$-2 - 10 - 2C = 6 \quad \Rightarrow \quad -2C = 18 \Rightarrow \quad C = -9.$$

So

$$y_p = -x^2 - 5/2\, x - 9.$$

& General solution is

$$y = y_c + y_p$$

$$= c_1 e^{-(2+\sqrt{6})x} + c_2 e^{(-2+\sqrt{6})x} - x^2 - \frac{5}{2}x - 9$$

Example. Find a particular solution of

$$y'' - y' + y = 2\sin 3x.$$

Solution. Guess for particular solution is

$y_p = A\cos 3x + B\sin 3x.$

$y_p' = -3A\sin 3x + 3B\cos 3x.$

$y_p'' = -9A\cos 3x - 9B\sin 3x.$

using in given DE

$\Rightarrow \quad -9A\cos 3x - 9B\sin 3x + 3A\sin 3x - 3B\cos 3x$

$+ A\cos 3x + B\sin 3x = 2\sin 3x.$

comparing coefficients of $\cos 3x$ & $\sin 3x$

$\underline{\cos 3x}$

$-9A - 3B + A = 0$

$\Rightarrow \quad -8A - 3B = 0 \Rightarrow \quad A = \frac{-3}{8}B \qquad (1)$

$\underline{\sin 3x}$

$-9B + 3A + B = 2$

$\Rightarrow \quad -8B + 3A = 2$

using (1)

$-8B + 3 \times \frac{-3}{8}B = 2$

$-8B - \frac{9}{8}B = 2 \Rightarrow \quad (-64 - 9)B = 16$

$B = \frac{-16}{73}.$

From (1)

$A = \frac{-3}{8} \times \frac{-16}{73} = \frac{6}{73}$

So particular Solution is

$y_p = \frac{6}{73}\cos 3x - \frac{16}{17}\sin 3x.$

Example Forming y_p by superposition

Solve $y'' - 2y' - 3y = 4x - 5 + 6xe^{2x}.$

Solution of homogeneous DE is

$$y'' - 2y' - 3y = 0$$

$$\Rightarrow \quad m^2 - 2m - 3 = 0$$

$$\Rightarrow \quad m = -1 \qquad m = 3$$

So $\quad y_c = c_1 e^{-x} + c_2 e^{3x}$

Particular solution

Here we have

$$g = g_1(x) + g_2(x) = (4x-5) + 6xe^{2x}$$

$$= \text{Polynomial} + \text{Exponential}$$

Let $\quad y_p = Ax + B + Cxe^{2x} + De^{2x}$

$$y_p' = A + C(e^{2x} + 2xe^{2x}) + 2De^{2x}$$

$$y_p'' = 2Ce^{2x} + 2C(e^{2x} + 2xe^{2x}) + 4De^{2x}$$

using in given DE

$$\Rightarrow 4Ce^{2x} + 4Cxe^{2x} + 4De^{2x} - 2A - 2Ce^{2x} - 4xCe^{2x}$$

$$- 4De^{2x} - 3Ax - 3B - 3Cxe^{2x} - 3De^{2x}$$

$$= 4x - 5 + 6xe^{2x}$$

Coefficient of x

$$-3A = 4 \Rightarrow \quad A = -4/3$$

Coefficient of x^0

$$-3B - 2A = -5$$

$$\Rightarrow \quad -3B + 8/3 = -5 \quad \Rightarrow \quad B = 23/9$$

Coefficient of e^{2x}

$$4C + \cancel{4D} - 2C - \cancel{4D} - 3D = 0$$

$$\Rightarrow \quad 2C - 3D = 0$$

Coefficient of xe^{2x}

$$= -4C + 4C - 3C = 6 \Rightarrow C = -2.$$

& $D = -4/3$

So
$$y_p = -\frac{4}{3}x + \frac{23}{9} - 2xe^{2x} - \frac{4}{3}e^{2x}.$$

General solution

$$y = C_1 e^{-x} + C_2 e^{3x} - \frac{4}{3}x + \frac{23}{9} - 2xe^{2x} - \frac{4}{3}e^{2x}.$$

A Glitch in the method.

Case 1

No function assumed in the particular solution is a solution of homogeneous Eq.

Example Determine the form of particular solution of

$$y'' - 8y' + 25y = 5x^3 e^{-x} - 7e^{-x}.$$

Solution

$$g(x) = (5x^3 - 7)e^{-x}.$$

So
$$y_p = (Ax^3 + Bx^2 + Cx + D)e^{-x}.$$

b) $y'' + 4y = x\cos x.$

$$y_p = (Ax + B)\cos x + (Cx + E)\sin x.$$

Form rule of case 1

The form of y_p is a linear combination of all linearly independent function that are generated by repeated differentiations of $g(x)$.

Example

$$y'' - 9y' + 14y = 3x^2 - 5\sin 2x + 7xe^{6x}.$$

* Corresponding to $3x^2$

$$y_{p_1} = Ax^2 + Bx + C$$

* corresponding to $-5\sin 2x$

$$y_{p_2} = E\cos 2x + F\sin 2x.$$

* corresponding to $7xe^{6x}$

$$y_{p_3} = (Gx+H)e^{6x}$$

So particular solution is of the form

$$y = y_{p_1} + y_{p_2} + y_{p_3}$$

$$= Ax^2 + Bx + C + E\cos 2x + F\sin 2x + (Gx+H)e^{6x}$$

Case II

A function in the assumed particular solution is also a solution of the associated homogeneous DE.

Example Particular Solution case II.

Find a particular solution of $y'' - 2y' + y = e^x$.

Solution. Associated homogeneous DE

$$y'' - 2y' + y = 0$$

$$m^2 - 2m + 1 = 0 \Rightarrow (m-1)^2 = 0$$

$$m_1 = 1, \quad m_2 = 1$$

$$\Rightarrow \quad y(x) = c_1 e^x + c_2 x e^x$$

Here $y_p = Ae^x$ cannot be chosen because e^x is already a solution part of complementary solution. Also xe^x cannot be chosen so

$$y_p = Ax^2 e^x.$$

$$\Rightarrow \quad y_p' = A(x^2 e^x + 2xe^x)$$

$$y_p'' = A(x^2 e^x + 2xe^x + 2xe^x + 2e^x)$$

$$y_p'' = A(x^2e^x + 4xe^x + 2e^x)$$

using in given DE

$$\Rightarrow \cancel{Ax^2e^x} + \cancel{4Axe^x} + 2Ae^x - \cancel{2Ax^2e^x} - \cancel{4Axe^x}$$

$$+ \cancel{Ax^2e^x} = e^x.$$

Comparing coefficients of e^x

$$\Rightarrow A = 1/2.$$

So $y_p = \frac{1}{2}x^2e^x$

Multiplication rule for case II

If any y_p contains term that duplicate terms in y_c, then y_p must be multiplied with x^n, when n is the smallest integer that eliminates duplication.

Example An initial value problem.

Solve $y'' + y = 4x + 10\sin x \qquad y(\pi) = 0, \; y'(\pi) = 2$

Solution Solution of associated homogeneous equation is

$$y'' + y = 0$$

$$\Rightarrow m^2 + 1 = 0 \qquad m = \pm i$$

$$y_c = c_1\cos x + c_2\sin x.$$

Now $g(x) = 4x + 10\sin x.$

which is sum of linear polynomial & sine function so

$$y_p = Ax + B + C\cos x + E\sin x.$$

$$y_p' = A - C\sin x + E\cos x.$$

$$y_p'' = -C\cos x - E\sin x.$$

$y_p'' + y_p = 4x + 10\sin x$

$-C\cancel{\cos x} - \cancel{E}\sin x + Ax + B + \cancel{C\cos x} + \cancel{E}\sin x = 4x + 10\sin x$

It will not give correct answer as there is duplication of terms $\sin x$ & $\cos x$ .. So we can assume y_p as

$$y_p = Ax + B + Cx\cos x + Ex\sin x$$

$$y_p' = A + C(-x\sin x + \cos x) + E(x\cos x + \sin x)$$

$$y_p'' = C(-x\cos x - \sin x - \sin x) + E(-x\sin x + \cos x + \cos x)$$

Now

$$y_p'' + y_p = 4x + 10\sin x$$

$$-\cancel{Cx\cos x} - 2C\sin x - \cancel{Ex\sin x} + 2E\cos x + Ax + B + \cancel{Cx\cos x} + \cancel{Ex\sin x} = 4x + 10\sin x$$

$$2E\cos x - 2C\sin x + Ax + B = 4x + 10\sin x$$

Comparing coefficients of x $\Rightarrow A = 4$. Coefficient of x^0 $\Rightarrow B = 0$

$\underline{\sin x}$: $\Rightarrow -2C = 10 \Rightarrow C = -5$

$\underline{\cos x}$: $\Rightarrow 2E = 0 \Rightarrow E = 0$

$$y_p = 4x - 5x\cos x$$

$$y = y_c + y_p$$

$$= c_1\cos x + c_2\sin x + 4x - 5x\cos x$$

Applying condition

$y(\pi) = 0$

$\Rightarrow \quad c_1 \cos\pi + c_2 \sin\pi + 4\pi - 5\pi\cos\pi = 0$

$\Rightarrow \quad c_1 = 9\pi$

$y'(x) = -c_1 \sin x + c_2 \cos x + 4 + 5x\sin x - 5\cos x.$

$y'(\pi) = 2$

$\Rightarrow \quad y'(\pi) = -c_1 \overset{0}{\cancel{\sin\pi}} + c_2\cos\pi + 4 + \cancel{5\pi}\overset{0}{\sin\pi} - 5\cos\pi = 2$

$-c_2 \cancel{\cos\pi} = 2 - 9$

$\Rightarrow \quad c_2 = 7.$

So solution of IVP

$y = 9\pi\cos x + 7\sin x + 4x - 5x\cos x.$

Solve $\quad y'' - 6y' + 9y = 6x^2 + 2 - 12e^{3x}$

Here $\quad y_c = c_1 e^{3x} + c_2 x e^{3x}$

&

$y_p = Ax^2 + Bx + C + Ee^{3x}$

To avoid repeatation of e^{3x} & xe^{3x}, y_p takes the form.

$y_p = Ax^2 + Bx + C + Ex^2e^{3x}.$

$y_p' = 2Ax + B + E(3x^2e^{3x} + 2xe^{3x})$

$y_p'' = 2A + E(9x^2e^{3x} + 6xe^{3x} + 6xe^{3x} + 2e^{3x})$

$= 2A + E(9x^2e^{3x} + 12xe^{3x} + 2e^{3x})$

$y_p'' - 6y_p' + 9y_p = 6x^2 + 2 - 12e^{3x}$

$2A + 9Ex^2e^{3x} + 12Exe^{3x} + 2Ee^{3x} - 12Ax - 6B - 6E(3x^2e^{3x} + 2xe^{3x})$

$+ 9Ax^2 + 9Bx + 9C + 9Ee^{3x} = 6x^2 + 2 - 12e^{3x}$

$9Ax^2 + (-12A + 9B)x + 2A - 6B + 9C + 2e^{3x}E = 6x^2 + 2 - 12e^{3x}$

$\underline{e^{3x}}$ $\quad 2E = -12 \Rightarrow \quad E = -6.$

$\underline{x^2}$ $\quad 9A = 6 \Rightarrow \quad A = 2/3$

$\underline{x}$ $\quad -12A + 9B = 0$

$\Rightarrow \quad B = 8/9.$

x^0

$2A - 6B + 9C = 2$

$C = 2/3.$

so

$$y_p = \frac{2}{3}x^2 + \frac{8}{9}x + \frac{2}{3} - 6x^2 e^{3x}.$$

Trial particular solution

$g(x)$	Form of y_p
1) 1	A
2) $5x+7$	$Ax+B$
3) $3x^2-2$	Ax^2+Bx+C
4) x^3-x+1	Ax^3+Bx^2+Cx+E
5) $\sin 4x$	$A\cos 4x + B\sin 4x.$
6) $\cos 4x$	$A\cos 4x + B\sin 4x.$
7) e^{5x}	Ae^{5x}
8) $(9x-2)e^{5x}$	$(Ax+B)e^{5x}$
9) x^2e^{5x}	$(Ax^2+Bx+C)e^{5x}$
10) $e^{3x}\sin 4x.$	$(A\sin 4x + B\cos 4x)e^{3x}$
11) $5x^2\sin 4x$	$(Ax^2+Bx+C)\cos 4x + (Ex^2+Fx+G)\sin 4x$
12) $xe^{3x}\cos 4x.$	$(Ax+B)e^{3x}\cos 4x + (Cx+E)e^{3x}\sin 4x.$

Exercise 4.4.

Solve the given DE by undetermined coefficient.

6) $y'' - 8y' + 20y = 100x^2 - 26xe^x$

Associated homogeneous Equation is.

$y'' - 8y' + 20y = 0$

Auxiliary Eq. is

$m^2 - 8m + 20 = 0$

$m = \dfrac{8 \pm \sqrt{64 - 80}}{2} = 4 \pm 2i$

$y(x) = e^{4x}(c_1 \cos 2x + c_2 \sin 2x)$.

Let particular solution has the form

$y_p = Ax^2 + Bx + C + Dxe^x + Ee^x$

$y_p' = 2Ax + B + D(xe^x + e^x) + Ee^x$.

$y_p'' = 2A + D(xe^x + e^x + e^x) + Ee^x$.

$= 2A + xe^x D + 2e^x D + Ee^x$

using in

$y'' - 8y' + 20y = 100x^2 - 26xe^x$

$2A + xe^x D + 2e^x D + Ee^x - 16Ax - 8B - 8xe^x D - 8e^x$

$- 8Ee^x + 20Ax^2 + 20Bx + 20C + 20xe^x D$

$+ 20Ee^x = 100x^2 - 26xe^x$.

comparing coefficients.

x^2: $20A = 100 \Rightarrow A = 5$

x: $20B - 16A = 0 \Rightarrow B = 4$.

x^0: $2A - 8B + 20C = 0 \Rightarrow 20C = -2 \times 5 + 8 \times 4 = -10 + 32$

$= 22$.

$C = \dfrac{22}{20} = \dfrac{11}{10}$

xe^x $\quad 13D = -26 \Rightarrow \quad D = -2$

e^x $\quad 13E - 6D = 0 \Rightarrow \quad E = \frac{-12}{13}$

So

$$y_p = 5x^2 + 4x + \frac{11}{10} - 2xe^x - \frac{12}{13}e^x$$

$$y = y_c + y_p$$

19 $\quad y'' + 2y' + y = \sin x + 3\cos 2x. \qquad (1)$

Associated homogeneous Eq. is

$$y'' + 2y' + y = 0$$

$$m^2 + 2m + 1 = 0 \Rightarrow (m+1)^2 = 0$$

$$m_1 = -1 \qquad m_2 = -1.$$

$$y(x) = c_1 e^{-x} + c_2 x e^{-x}.$$

$$y_p = A\sin x + B\cos x + C\cos 2x + D\sin 2x$$

$$y_p' = A\cos x - B\sin x - 2C\sin 2x + 2D\cos 2x$$

$$y_p'' = -A\sin x - B\cos x - 4C\cos 2x - 4D\sin 2x$$

using in (1)

$\Rightarrow -A\sin x - B\cos x - 4C\cos 2x - 4D\sin 2x + 2A\cos x - 2B\sin x$
$\quad - 4C\sin 2x + 4D\cos 2x + A\sin x + B\cos x + C\cos 2x + D\sin 2x$
$\quad = \sin x + 3\cos 2x.$

comparing coefficients

$\sin x$ $\quad \cancel{-A} - 2B \cancel{+A} = 1 \Rightarrow B = -1/2$

$\cos x$ $\quad \cancel{-B} + 2A \cancel{+B} = 0 \Rightarrow A = 0.$

$\sin 2x$ $\quad -4D - 4C + D = 0 \Rightarrow -3D - 4C = 0 \Rightarrow 3D + 4C = 0.$

$\cos 2x$ $\quad -4C + 4D + C = 3 \Rightarrow 4D - 3C = 3$

Solving $D = -\frac{4}{3}C$.

$\Rightarrow$ $4D - 3C = 3$

$\Rightarrow$ $4 \times -\frac{4}{3}C - 3C = 3$ $\Rightarrow$ $\frac{-16}{3}C - 3C = 3$.

$-25C = 9 \Rightarrow$ $C = -9/25$

AnD

$$D = \frac{12}{25}$$

$$y_p = -\frac{1}{2}\cos x - \frac{9}{25}\cos 2x + \frac{12}{25}$$

25 $y^{(4)} + 2y'' + y = (x-1)^2$

$$y^{(4)} + 2y'' + y = x^2 - 2x + 1$$

Associated homogeneous Eq.

$$y^{(4)} + 2y'' + y = 0$$

Auxiliary Eq. is

$$m^4 + 2m^2 + 1 = 0$$

$$(m^2+1)^2 = 0$$

$m = \pm i$ $\qquad$ $m = \pm i$

$$y(x) = C_1 (\cos x) + C_2 \sin x + C_3 x\cos x + C_4 x \sin x.$$

For particular solution, let

$$y_p = Ax^2 + Bx + C$$

$y_p' = 2Ax + B$ $\qquad$ $y_p'' = 2A$

$y_p''' = 0$ $\qquad$ $y_p^{(4)} = 0$.

using in given DE

$\Rightarrow$ $0 + 4A + Ax^2 + Bx + C = x^2 - 2x + 1$

comparing coefficients.

x^2: $A = 1$.

x: $B = -2$.

x^0: $4A + C = 1$

$\Rightarrow C = -3$

So $y_p = x^2 - 2x - 3$.

& general soln is

$$y = y_c + y_p$$

$$= c_1 \cos x + c_2 \sin x + c_3 x \cos x + c_4 x \sin x + x^2 - 2x - 3.$$

Solve the given IVP.

30 $y'' + 4y' + 4 = (3+x)e^{-2x}$ $y(0) = 2$, $y'(0) = 5$

Associated homogeneous Eq. is

$$y'' + 4y' + 4 = 0$$

Auxiliary Eq. is

$$m^2 + 4m + 4 = 0$$

$$(m+2)^2 = 0$$

$m_1 = -2$ $m_2 = -2$

$$y(x) = c_1 e^{-2x} + c_2 x e^{-2x}.$$

For particular solution

$$y_p = (Ax + B) e^{-2x}.$$

$y_p' =$ As xe^{-2x} & e^{-2x} are already in complementary solution so

$$y_p = Ax^3 e^{-2x} + Bx^2 e^{-2x}.$$

$$y_p' = (3Ax^2 + 2Bx) e^{-2x} + (-2Ax^3 - 2Bx^2) e^{-2x}$$

$$y_p'' = (6Ax + 2B) e^{-2x} + 2(-6Ax^2 - 4Bx) e^{-2x} + (4Ax^3 + 4Bx^2) e^{-2x}$$

$$y_p'' = 4Ax^3 + (-12A + 4B)x^2 + (6A - 8B)x + 2B$$

Inserting in given DE

$$\left(4Ax^3 + (-12A+4B)x^2 + (6A-8B)x + 2B\right)e^{-2x}$$
$$+4\left((3Ax^2+2Bx)e^{-2x} + (-2Ax^3 - 2Bx^2)e^{-2x}\right)$$
$$+4\left(Ax^3+Bx^2\right)e^{-2x} = (3+x)e^{-2x}$$

coefficient of xe^{-2x}

$$6A - 8B + 8B = 1$$
$$\Rightarrow \quad A = 1/6.$$

coefficient of e^{-2x}.

$$2B = 3 \quad \Rightarrow \quad B = 2/3.$$

$$y_p = \frac{1}{6}x^3e^{-2x} + \frac{3}{2}x^2e^{-2x}.$$

General solution is

$$y = y_c + y_p$$
$$y = c_1e^{-2x} + c_2xe^{-2x} + \frac{1}{6}x^3e^{-2x} + \frac{3}{2}x^2e^{-2x}$$

using $y(0) = 2$

$$\Rightarrow \quad c_1 = 2.$$

$$y' = -2c_1e^{-2x} - 2c_2xe^{-2x} + c_2e^{-2x} + \frac{1}{2}x^2e^{-2x}$$
$$-\frac{1}{3}x^3e^{-2x} + 3xe^{-2x} - 3x^2e^{-2x}$$

$$y'(0) = 5$$
$$\Rightarrow \quad 2c_1 + c_2 = 5 \quad \Rightarrow \quad c_2 = 9.$$

so

$$y = 2e^{-2x} + 9xe^{-2x} + \frac{1}{6}x^3e^{-2x} + \frac{3}{2}x^2e^{-2x}$$

34 $\frac{d^2x}{dt^2} + \omega^2 x = F_0 \cos \gamma t$ $\quad x(0)=0, \; x'(0)=0$

Solving associated homogeneous Eq.

$x(t) = c_1 \cos\omega t + c_2 \sin\omega t$

Let particular solution is of the form

$x_p = A \cos \gamma t + B \sin \gamma t$

$x_p' = -A\gamma \sin\gamma t + B\gamma \cos\gamma t$

$x_p'' = -A\gamma^2 \cos\gamma t - B\gamma^2 \sin\gamma t$

using in given DE

$\Rightarrow \; -A\gamma^2 \cos\gamma t - B\gamma^2 \sin\gamma t + \omega^2 A \cos\gamma t + \omega^2 B \sin\gamma t$

$= F_0 \cos\gamma t$

$A(\omega^2-\gamma^2)\cos\gamma t + B(\omega^2-\gamma^2)\sin\gamma t = F_0 \cos\gamma t$.

comparing coefficients of $\cos\gamma t$

$\Rightarrow \; A(\omega^2-\gamma^2) = F_0$

$A = \frac{F_0}{\omega^2-\gamma^2}$

comparing coefficient of $\sin\gamma t$

$\Rightarrow \; B = 0$

So $\; x_p = \frac{F_0}{\omega^2-\gamma^2} \cos\gamma t$

General solution is

$x(t) = c_1 \cos\omega t + c_2 \sin\omega t + \frac{F_0}{\omega^2-\gamma^2} \cos\gamma t$

$x(0) = 0$

$\Rightarrow \; 0 = c_1 + \frac{F_0}{\omega^2-\gamma^2} \cos 0$

$$c_1 = \frac{-F_0}{\omega^2-\gamma^2}$$

$$x'(t) = c_2\omega\cos\omega t \pm c_1\omega\sin\omega t + \frac{F_0}{\omega^2-\gamma^2}(-\gamma\sin\gamma t)$$

$x'(0) = 0$

$\Rightarrow \quad c_2 = 0$

So

$$x(t) = -\frac{F_0}{\omega^2-\gamma^2}\cos\omega t + \frac{F_0}{\omega^2-\gamma^2}\cos\gamma t$$

40 $\quad y'' + 3y = 6x \qquad y(0)+y'(0)=0, \; y(1)=0.$

Associated homogeneous Eq. D

$y''+3y=0$

$m^2+3=0 \qquad m = \pm\sqrt{3}\,i$

$y(x) = c_1\cos\sqrt{3}x + c_2\sin\sqrt{3}x$

Particular solution is

$y_p = Ax+B$

$y_p' = A, \qquad y_p'' = 0$

$\Rightarrow \quad 3Ax+3B = 6x$

$\Rightarrow \quad A=2, \quad B=0$

$y_p = 2x.$

$\Rightarrow \quad y = c_1\cos\sqrt{3}x + c_2\sin\sqrt{3}x + 2x$

$y'(x) = -\sqrt{3}c_1\sin\sqrt{3}x + \sqrt{3}c_2\cos\sqrt{3}x + 2.$

Now $y(0)+y'(0)=0$

$\Rightarrow \quad c_1\cos 0 + c_2\sin 0 + 0 - \sqrt{3}c_1\sin 0 + \sqrt{3}c_2\cos 0 + 2 = 0$

$c_1 + \sqrt{3}c_2 + 2 = 0$

$y(1)=0 \Rightarrow$

$c_1 \cos\sqrt{3} + c_2 \sin\sqrt{3} = -2.$

Solving these two Eqs for c_1 & c_2 as

$$c_1 = \frac{2\cos\sqrt{3} - 2}{\sin\sqrt{3} - \sqrt{3}\cos\sqrt{3}}$$

$$c_2 = \frac{2\sqrt{3} - 2\sin\sqrt{3}}{\sin\sqrt{3} - \sqrt{3}\cos\sqrt{3}}$$

So

$$y(x) = \frac{(2\sqrt{3} - 2\sin\sqrt{3})}{\sin\sqrt{3} - \sqrt{3}\cos\sqrt{3}} \cos\sqrt{3}\,x + \frac{(2\cos\sqrt{3} - 2)\sin\sqrt{3}\,x}{\sin\sqrt{3} - \sqrt{3}\cos\sqrt{3}}$$

$$+ 2x.$$