Tarefa 04 - Leonardo Rothier Soares Cardoso

1 - a) $\sum_{i=1}^{n} i = \frac{n.(n+1)}{2}$

b) $\sum_{i=1}^{n} a^i = \frac{a^{n+1}-1}{a-1}$

c) $\sum_{i=1}^{n} i a^i = \boxed{\frac{a^{n+1}(an-n-1)+a}{(a-1)^2}}$

$\sum_{i=1}^{n} a^i = \frac{a^{n+1}-1}{a-1}$ ← faltando um (-1) pois a $i=1$ não $i=0$

$\sum_{i=1}^{n} \frac{d}{da}(a^i).a = \frac{d}{da}\left(\frac{a^{n+1}-1}{a-1}\right).a$

$\sum_{i=1}^{n} i a^i = \frac{\frac{d}{da}(a^{n+1}-1).(a-1) - (a^{n+1}-1).\frac{d}{da}(a-}{(a-1)^2}$

$= \left(\frac{(n+1).a^n.(a-1)+(-a^{n+1}+1).(1)}{(a-1)^2}\right).a$

$= \frac{(na^n + a^n).(a-1) - a^{n+1} + 1}{(a-1)^2}$

$= \frac{na^{n+1} - na^n + \cancel{a^{n+1}} - a^n - \cancel{a^{n+1}} + 1}{(a-1)^2}$

$= a.\frac{(na^{n+1} - na^n - a^n + 1)}{(a-1)^2}$

$\frac{a^{n+1}(an-n-1)+a}{(a-1)^2}$

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d) $\sum_{i=1}^{n} \frac{1}{i} = 1 + \frac{1}{2} + \frac{1}{3} + \dots + \frac{1}{n}$

$H_n \to$ Harmonic number

Exceto pelo caso de $n = 1$, o número harmônico nunca é inteiro

por definição, o número harmônico satisfaz a relação de recorrência:

$$H_{n+1} = H_n + \frac{1}{n+1}$$

Os números harmônicos satisfazem a identidade:

$$\boxed{\sum_{i=1}^{n} \frac{1}{i} = (n+1) \cdot H_n - n}$$

e) $\sum_{i=1}^{n} i 2^{-i} = \sum_{i=1}^{n} i \cdot \left(\frac{1}{2}\right)^{i}$

$$\sum_{i=1}^{n} i a^{i} = \frac{a^{n+1}(an - n - 1) + a}{(a-1)^2}$$

$a = 1/2$

$$\boxed{\sum_{i=}^{n} i\left(\frac{1}{2}\right)^{i} = \frac{\left(\frac{1}{2}\right)^{n+1} \cdot \left(\left(\frac{1}{2}\right) \cdot n - n - 1\right) + \frac{1}{2}}{\left(-\frac{1}{2}\right)^2}}$$

b) $\sum_{i=0}^{n} \frac{1}{7^i} = 1 + \frac{1}{7} + \frac{1}{49} + \dots + \left(\frac{1}{7}\right)^n$

$$\sum_{i=0}^{n} \left(\frac{1}{7}\right)^i$$

$$\sum_{i=0}^{n} a^i = \frac{a^{n+1} - 1}{a - 1}, \quad a = \frac{1}{7}$$

$$\boxed{\sum_{i=0}^{n} \left(\frac{1}{7}\right)^i = \frac{\left(\frac{1}{7}\right)^{n+1} - 1}{\frac{1}{7} - 1}}$$