Tarefa - 09 Leonardo Rothier Soares Cardoso

a) $T(n) = 2T(n/2) + n - 1$

$a = 2$,
$b = 2$,
$f(n) = n - 1$

$\begin{cases} f(n) = n - 1 \\ n^{\log_2^2} = n^1 \end{cases}$

Caso 2

$f(n) = \Theta(n^{\log_b^a})$

$n = \Theta(n^1)$ ok

$\boxed{T(n) = \Theta(n \cdot \log n)}$

b) $T(n) = 3T(n/2) + n$

$a = 3$,
$b = 2$, $f(n) = n$

$\begin{cases} f(n) = n \\ n^{\log_2^3} = n^{1,584} \end{cases}$

Caso 1

$f(n) = O(n^{\log_2^3 - \varepsilon})$

$n = O(n^{1,584 - \varepsilon})$; $\varepsilon = 0,584$

$n = O(n^1)$ ok

$T(n) = \Theta(n^{1,584})$

c) $T(n) = 4T(n/2) + n^2$

$\begin{cases} f(n) = n^2 \\ n^{\log_2^4} = n^2 \end{cases}$

$a = 4,$
$b = 2,$
$f(n) = n^2$

Case 2

$f(n) = \Theta(n^2)$
$n^2 = \Theta(n^2)$ ok

$\boxed{T(n) = \Theta(n^2 . \log n)}$

d) $T(n) = 4T(n/2) + n^3$

$a = 4,$
$b = 2$
$f(n) = n^3$

$\begin{cases} f(n) = n^3 \\ n^{\log_2^4} = n^2 \end{cases}$

Case 3

$n^3 = \Omega(n^{2+\varepsilon})$; $\varepsilon = 1$
$n^3 = \Omega(n^3)$ ok

$a . f(n/b) \leq c\, f(n)$
$4 f(n/2) \leq c . f(n)$
$4 . \left(\frac{n^3}{8}\right) \leq c . n^3$
$\frac{\cancel{n^3}}{2} \leq c . \cancel{n^3} \rightarrow \boxed{c = 1/2}$

$\boxed{T(n) = \Theta(n^3)}$

2.

a) $\begin{cases} T(n) = 2T\left(\frac{n}{2}\right) + n\log n \\ T(1) = 0 \end{cases}$

$\begin{cases} f(n) = n\log n \\ n^{\log_2 2} = n^1 \end{cases}$

(caso 3)

Deveria funcionar para o caso 3 por $n\log n$ ser assintoticamente maior que n mas polinomialmente isso não procede.

Logo resolveremos a recorrência

$$n = 2^k, \quad k = \log_2 n$$

$$T(2^k) = 2T(2^{k-1}) + 2^k \cdot k$$
$$2T(2^{k-1}) = 2^2 T(2^{k-2}) + 2^k \cdot (k-1)$$
$$2^2 T(2^{k-2}) = 2^3 T(2^{k-3}) + 2^k \cdot (k-2)$$
$$\vdots$$
$$2^{(k-2)} T(2^{(k-(k-2))}) = 2^{k-1} T(2^{k-(k-1)}) + 2^k \cdot (2)$$
$$2^{(k-1)} T(2^1) = 2^{k-k} T(2^0) + 2^k \cdot 1$$

$$T(2^k) = \sum_{i=1}^{k} 2^k \cdot i = 2^k \cdot \frac{(k \cdot (k+1))}{2}$$

$$= n \cdot \frac{(\log n (\log n + 1))}{2}$$

$$= \boxed{\frac{n\log n \cdot (\log n + 1)}{2}}$$

$$2^k \cdot T(1) = 0 + 2^k \cdot 0$$

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b) $\begin{cases} T(n) = 4T\left(\frac{n}{2}\right) + n^2\sqrt{n} \\ T(1) = 1 \end{cases}$

$\begin{cases} f(n) = n^2\sqrt{n} \\ n^{\log_2 4} = n^2 \end{cases}$

Caso 3

$f(n) = \Omega\left(n^{\log_b a + \varepsilon}\right)$

$n^{5/2} = \Omega\left(n^{2+\varepsilon}\right)$; $\varepsilon = 0{,}1$

$n^{2{,}5} = \Omega\left(n^{2{,}1}\right)$ ok

$a \cdot f\left(\frac{n}{b}\right) \leq c\, f(n)$; $c < 1$

$4 \cdot f\left(\frac{n}{2}\right) \leq c \cdot f(n)$

$4 \cdot \frac{\cancel{n^{5/2}}}{2^{5/2}} \leq c \cdot \cancel{n^{5/2}}$

$\frac{4}{2^{5/2}} \leq c \rightarrow c \geq 0{,}71$ $\quad c = 1$ ok

$T(n) = \Theta\left(n^2\sqrt{n}\right)$

c) $\begin{cases} T(n) = 3T\left(\frac{n}{2}\right) + n \\ T(1) = 1 \end{cases}$

$\begin{cases} f(n) = n \\ n^{\log_2 3} = n^{1,584} \end{cases}$

Caso 1

$f(n) = O(n^{\log_b a - \varepsilon})$

$n = O(n^{1,584 - \varepsilon})$; $\varepsilon = 0,584$

$n = O(n^{1})$ ok

$T(n) = \Theta(n^{1,584})$