Tarefa 05 - Leonardo Ratkiewicz Soares Cardoso

a) $\begin{cases} T(n) = T(n-1) + c & c \text{ constante, } n > 1 \\ T(1) = 0 \end{cases}$

$$T(n) = \cancel{T(n-1)} + c$$
$$\cancel{T(n-1)} = \cancel{T(n-2)} + c$$
$$\cancel{T(n-2)} = \cancel{T(n-3)} + c$$
$$\vdots$$
$$\cancel{T(n-(n-2))} = \cancel{T(n-(n-1))} + c$$
$$\cancel{T(n-(n-1))} = 0$$

$$T(n) = \sum_{i=1}^{n-1} c = c \cdot (n-1-1+1) = c \cdot n - c$$

b) $\begin{cases} T(n) = T(n-1) + 2^n & n \geq 1 \\ T(0) = 1 \end{cases}$

$$T(n) = \cancel{T(n-1)} + 2^n$$
$$\cancel{T(n-1)} = \cancel{T(n-2)} + 2^{n-1}$$
$$\cancel{T(n-2)} = \cancel{T(n-3)} + 2^{n-2}$$
$$\vdots$$
$$\cancel{T(n-(n-1))} = \cancel{T(n-n)} + 2^1$$
$$\cancel{T(n-n)} = 1$$

$$T(n) = \sum_{i=0}^{n} 2^i = \frac{2^{n+1} - 1}{2-1} = \boxed{\frac{2^{n+1} - 1}{1}}$$

1) $\begin{cases} T(n) = c\,T(n-1) \\ T(0) = k \end{cases}$ c, k constantes, $n > 0$

$$T(n) = c\,T(n-1)$$
$$c\,T(n-1) = c^2\,T(n-2)$$
$$c^2\,T(n-2) = c^3\,T(n-3)$$
$$c^3\,T(n-3) = c^4\,T(n-4)$$
$$\vdots$$
$$c^{n-1}\,T(n-(n-1)) = c^n\,T(n-n)$$
$$c^n\,T(n-n) = c^n . k$$

$$\boxed{T(n) = c^n . k}$$

2) $\begin{cases} T(n) = 3T(n/2) + n & n > 1 \\ T(1) = 1 \end{cases}$

$n = 2^k \qquad k = \log_2 n$

$$T(2^k) = 3T(2^{k-1}) + 3^0 . 2^k$$
$$3T(2^{k-1}) = 3^2(2^{k-2}) + 3^1 . 2^{k-1}$$
$$3^2T(2^{k-2}) = 3^3(2^{k-3}) + 3^2 . 2^{k-2}$$
$$\vdots$$
$$3^{k-1}T(2^{k-(k-1)}) = 3^k(2^{k-k}) + 3^{k-1} . 2^1$$
$$3^k T(2^{k-k}) = 3^k . 2^0$$

$$T(2^k) = \sum_{i=0}^{k} 3^i . 2^i = \sum_{i=0}^{k} 6^i = \frac{6^{k+1} - 1}{5}$$

$$T(n) = 6^{\log n + 1} - 1 = \frac{n . 2 . 3^{\log n} . 3 - 1}{5}$$

e) $\begin{cases} T(n) = 3T(n-1) - 2T(n-2) & n>1 \\ T(0) = 0 \\ T(1) = 1 \end{cases}$

$$T(n) = 3T(n-1) - 2T(n-2)$$
$$3T(n-1) = 9T(n-2) - 3\cdot 2T(n-3)$$
$$7\cdot T(n-2) = 3\cdot 7T(n-3) - 7\cdot 2T(n-4)$$
$$15\cdot T(n-3) = 3\cdot 15T(n-4) - 15\cdot 2T(n-5)$$
$$\vdots$$

$$(2^{n-1}-1)\cdot T(n-(n-2)) = 3\cdot(2^{n-1}-1)\cdot T(n-(n-1)) - 0$$
$$(2^{n}-1)\cdot \underbrace{T(n-(n-1))}_{1} = \boxed{2^{n}-1} \rightarrow \boxed{T(n) = 2^{n}-1}$$

2 - a) $\begin{cases} T(n) = T(3n/5) + n \\ T(1) = 1 \end{cases}$

$$n = \left(\frac{5}{3}\right)^k \qquad k = \log_{5/3} n$$

b)

$$T\left(\left(\frac{5}{3}\right)^k\right) = T\left(\left(\frac{5}{3}\right)^{k-1}\right) + \left(\frac{5}{3}\right)^k$$
$$T\left((5/3)^{k-1}\right) = T\left((5/3)^{k-2}\right) + (5/3)^{k-1}$$
$$T\left((5/3)^{k-2}\right) = T\left((5/3)^{k-3}\right) + (5/3)^{k-2}$$
$$\vdots$$
$$T\left((5/3)^{k-(k-1)}\right) = T\left((5/3)^{k-k}\right) + (5/3)^1$$
$$T\left((5/3)^{k-k}\right) = 1$$

$$T\left((5/3)^k\right) = \sum_{i=1}^{k} (5/3)^i = \frac{(5/3)^{k+1} - 1}{5/3 - 1}$$
$$= \frac{5/3^{\log_{5/3} n} \cdot (5/3) - 1}{5/3 - 1}$$

c) $\boxed{T\left((5/3)^m\right) = \frac{m \cdot 5/3 - 1}{5/3 - 1}}$