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THE BEATLES

Tarefa 1

1- Considerando que a operação relevante é o número de vezes que a operação soma é executada, apresente a função de complexidade de tempo para:

a) $\sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{k=1}^{n} 3 = \sum_{i=1}^{n}\sum_{j=1}^{n} 3.(n-1+1) = \sum_{i=1}^{n}\sum_{j=1}^{n} 3n$

$T(n) = \sum_{i=1}^{n} 3n.(n-1+1) = \sum_{i=1}^{n} 3n^2 = 3n^2.(n-1+1)$
$= \boxed{3n^3}$

b) $T(n) = \sum_{i=1}^{n}\sum_{j=1}^{i}\sum_{k=1}^{j} 3 = \sum_{i=1}^{n}\sum_{j=1}^{i} 3.(j-1+1) = \sum_{i=1}^{n}\sum_{j=1}^{i} 3j$

$= \sum_{i=1}^{n} 3.\left(\frac{i.(i+1)}{2}\right) = \sum_{i=1}^{n} \frac{3i^2+3i}{2}$

$= \frac{3}{2}.\sum_{i=1}^{n} i^2 + i = \frac{3}{2}.\left(\sum_{i=1}^{n} i^2 + \sum_{i=1}^{n} i\right)$

$= \frac{3}{2}.\left(\frac{n(n+1)(2n+1)}{6} + \frac{n.(n+1)}{2}\right)$

$= \frac{n(n+1)(2n+1)}{4} + \frac{3n(n+1)}{4}$

$= \frac{n(n+1)(2n+1+3)}{4}$

$= \frac{n(n+1)(2n+4)}{4}$

c) $T(n) = \sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{k=i}^{n} 3 = \sum_{i=1}^{n}\sum_{j=1}^{n} 3\cdot(n-i+1) = \sum_{i=1}^{n}\sum_{j=1}^{n} 3n-3i+3$

$$= \sum_{i=1}^{n}\left(\sum_{j=1}^{n} 3n - \sum_{j=1}^{n} 3i + \sum_{j=1}^{n} 3\right)$$

$$= \sum_{i=1}^{n}\left(3n\cdot(n-1+1) - 3i\cdot(n-1+1) + 3\cdot(n-1+1)\right)$$

$$= 3\sum_{i=1}^{n} n^2 - in + n = 3n\sum_{i=1}^{n} n - i + 1$$

$$= 3n\cdot\left(\sum_{i=1}^{n} n - \sum_{i=1}^{n} i + \sum_{i=1}^{n} 1\right)$$

$$= 3n\cdot\left(n\cdot(n-1+1) - \frac{n\cdot(n+1)}{2} + 1\cdot(n-1+1)\right)$$

$$= 3n\cdot\left(n^2 - \frac{n^2 - n}{2} + n\right)$$

$$= 3n\cdot\left(\frac{2n^2 - n^2 - n + 2n}{2}\right)$$

$$T(n) = 3n\cdot\left(\frac{n^2 + n}{2}\right)$$

d) $T(n) = \sum_{i=1}^{n} \sum_{j=i}^{n} \sum_{k=i}^{n} 3 = \sum_{i=1}^{n} \sum_{j=i}^{n} 3.(n-i+1) = \sum_{i=1}^{n} \sum_{j=i}^{n} 3n-3i+3$

$= \sum_{i=1}^{n} \left(\sum_{j=i}^{n} 3n - \sum_{j=i}^{n} 3i + \sum_{j=i}^{n} 3 \right)$

$= \sum_{i=1}^{n} \left(3n.(n-i+1) - 3i.(n-i+1) + 3.(n-i+1) \right)$

$= \sum_{i=1}^{n} 3n^2 - 3ni + 3n - 3in + 3i^2 - 3i + 3n - 3i + 3$

$= 3 \sum_{i=1}^{n} n^2 - 2ni + 2n + i^2 - 2i + 1$

$= 3.\left(\sum_{i=1}^{n} n^2 - 2n \sum_{i=1}^{n} i + \sum_{i=1}^{n} 2n + \sum_{i=1}^{n} i^2 - 2\sum_{i=1}^{n} i + \sum_{i=1}^{n} 1 \right)$

$= 3.\left(n^3 - \cancel{2} \frac{n^2(n+1)}{\cancel{2}} + 2n^2 + \frac{n(n+1)(2n+1)}{6} - \frac{\cancel{2}n.(n+1)}{\cancel{2}} + n \right.$

$= 3.\left(\cancel{n^3} - \cancel{n^3} - n^2 + 2n^2 + \frac{n(n+1)(2n+1)}{6} \cancel{- n^2 - n} \right.$

$= 3.\left(\frac{n(n+1)(2n+1)}{6} \right)$

$= \frac{n(n+1)(2n+1)}{2} = \frac{(n^2+n)(2n+1)}{2}$

$= \frac{2n^3 + n^2 + 2n^2 + n}{2} = \boxed{\frac{\cancel{2}n^3}{\cancel{2}} + \frac{3n^2}{2} + \frac{n}{2}}$

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2) $T(n) = \sum_{i=1}^{n}\sum_{j=i}^{n}\sum_{k=i}^{j} 3 = \sum_{i=1}^{n}\sum_{j=1}^{n} 3.(j-i+1)$

$= \sum_{i=1}^{n} \left(3\sum_{j=1}^{n} j - \sum_{j=1}^{n} 3i + \sum_{j=1}^{n} 1\right)$

$= \sum_{i=1}^{n} \left(\frac{3n.(n+1)}{2} - 3i.(n-1+1) + 1.(n-1+1)\right)$

$= \sum_{i=1}^{n} \left(\frac{3n^2+3n-6in+2n}{2}\right)$

$= \frac{1}{2}\sum_{i=1}^{n} 3n^2+5n-6in$

$= \frac{n}{2}\sum_{i=1}^{n} 3n+5-6i$

$= \frac{n}{2}.\left(\sum_{i=1}^{n} 3n + \sum_{i=1}^{n} 5 - 6\sum_{i=1}^{n} i\right)$

$= \frac{n}{2}.\left(3n.(n) + 5n - \frac{6n(n+1)}{2}\right)$

$= \frac{n}{2}.\left(\frac{\cancel{6n^2} + 10n - \cancel{6n^2} - 6n}{2}\right)$

$= \frac{n}{2}.\left(\frac{4n}{2}\right) = \frac{4n^2}{4} = \boxed{n^2}$