Tarefa 03 - Leonardo Rotilner Soares Cardoso

1_

i) limite $k = 1$,

$$\lim_{n \to \infty} \frac{\log n}{n^k} = \frac{\infty}{\infty} \text{ - indeterminado}$$

L'Hospital: $$\lim_{n \to \infty} \frac{\frac{1}{n \cdot \ln(2)}}{k n^{k-1}} = \lim_{n \to \infty} \frac{1}{n \ln(2)} \cdot \frac{1}{k n^{k-1}}$$

$$= \lim_{n \to \infty} \frac{1}{k \ln(2)} \cdot \frac{1}{n^k} = \frac{1}{k \ln(2)} \cdot 0 = 0$$

Logo, $\log n = o(n^k)$

Se $\log n = o(n^k)$ então $\log \log n = o(n^k)$ e

$$\boxed{\log^k n = o(n^k)}$$

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ii) tendo $k = 1$

$\lim_{n \to \infty} \frac{n}{c^n}$, $\lim_{n \to \infty} (c^n) = +\infty$, $c > 1$

logo,

$\lim_{n \to \infty} \frac{n}{c^n} = \frac{\infty}{\infty}$ = indeterminado

L'H:

$\lim_{n \to \infty} \frac{1}{\ln(c) \cdot c^n} = \lim_{n \to \infty} \frac{1}{\ln(c)} \cdot \frac{1}{c^n}$

$= \frac{1}{\ln(c)} \cdot 0 = \boxed{0}$, então

$n = o(c^n)$

se $n = O(c^n)$ então

$n^2 = O(c^n)$ e $n^k = O(c^n)$

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iii) $\lim_{n\to\infty} \frac{c^n}{c^{n/2}} = \lim_{n\to\infty} c^{n-n/2} = \lim_{n\to\infty} c^{n/2} = \infty$

Logo, $\boxed{c^n = \omega(c^{n/2})}$

$\boxed{c^n = \Omega(c^{n/2})}$

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iv) $\lim_{n\to\infty} \frac{\log(n!)}{\log(n^n)} = \lim_{n\to\infty} \frac{\log\left(\frac{n}{e}\right)^n}{\log n^n}$

$= \lim_{n\to\infty} \frac{\cancel{n}\log\left(\frac{n}{e}\right)}{\cancel{n}\log n} = \lim_{n\to\infty} \frac{\log n - \log e}{\log n}$

$= \lim_{n\to\infty} 1 - \frac{\log(e)}{\log n} = 1 - 0 = \boxed{1}$

Logo, $\log(n!) = \Theta(\log(n^n))$

$\Rightarrow \log(n!) = \Theta(\log(n^n))$,

$\log(n!) = O(\log(n^n))$ e

$\log(n!) = \Omega(\log(n^n))$

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v)

tomando $k=1$,

$$\lim_{n\to\infty} \frac{\log^2 n}{\log n} = \frac{\infty}{\infty} \rightarrow \text{indeterminação}$$

$$\text{L'H: } \lim_{n\to\infty} \frac{\frac{1}{\ln(2).n.\ln(n)}}{\frac{1}{\ln(2).n}} = \lim_{n\to\infty} \frac{\ln(2).\cancel{n}}{\ln(2).\cancel{n}.\ln(n)}$$

$$= \lim_{n\to\infty} \frac{1}{\ln(n)} = \boxed{0}$$

logo temos que,

$$\log^2 n = o(\log n)$$

se $\log^2 n = o(\log^k n)$, então
$\log^{k+1} n = o(\log^k n)$

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vi)

$$\lim_{n\to\infty} \frac{c^c}{(c+1)^c} = \frac{c^c}{(c+1)^c} \Rightarrow \text{uma constante}$$

Logo temos que

$$c^c = \Theta((c+1)^c)$$

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a) B1 e B2

$$\lim_{n\to\infty} \frac{n^{\frac{1}{\log n}}}{\ln\ln n} = \lim_{n\to\infty} \frac{n^{\log_n 2}}{\ln^2 n} = \lim_{n\to\infty} \frac{2}{\ln^2 n}$$

$$= \lim_{n\to\infty} 2 \cdot \frac{1}{\ln^2 n} = 2 \cdot 0 = 0$$

Logo temos que

$$n^{\frac{1}{\log n}} = o(\ln\ln n)$$

b) B2 e B3

$$\lim_{n\to\infty} \frac{\ln\ln n}{(\ln n)^2} = \frac{\infty}{\infty} \Rightarrow \text{indeterminação}$$

$$\text{L'H: } \lim_{n\to\infty} \frac{\frac{d}{dn}(\ln\ln n)}{2 \cdot \frac{\ln(n)}{n}} = \frac{\frac{1}{\ln(n)} \cdot \frac{1}{n}}{\frac{2\ln(n)}{n}} = \frac{1}{2\ln(n) \ln \ldots}$$

$$= \lim_{n \to \infty} \frac{1}{2(\ln(n))^2} = \frac{1}{\infty} \cdot 0 = 0$$

logo temos que,

$\ln\ln(n) = o((\ln(n))^2)$

c) g_3 e g_4

$$\lim_{n \to \infty} \frac{(\ln n)^2}{n} = \frac{\infty}{\infty} \text{, indeterminação}$$

$$L'H: \lim_{n \to \infty} \frac{2 \cdot \ln(n) \cdot \frac{1}{n}}{1} = \lim_{n \to \infty} \frac{2\ln(n)}{n} = \frac{\infty}{\infty}$$

$$L'H: \lim_{n \to \infty} \frac{2 \cdot \frac{1}{n}}{1} = \lim_{n \to \infty} \frac{2}{n} = 0$$

$$\boxed{(\ln(n))^2 = o(n)}$$

d) g_4 e g_5

$$\lim_{n \to \infty} \frac{n}{2^{\log n}} = \lim_{n \to \infty} \frac{n}{n} = 1$$

$$\boxed{n = \Theta(2^{\log(n)})}$$

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e) g5 e g6

$$\lim_{n\to\infty} \frac{2^{\log(n)}}{n\log n} = \lim_{n\to\infty} \frac{\cancel{n}}{\cancel{n}\log n} = 0$$

$$\boxed{2^{\log(n)} = o(n\log(n))}$$

f) g6 e g7

$$\lim_{n\to\infty} \frac{n\log(n)}{\log(n!)} = \lim_{n\to\infty} \frac{n\log(n)}{\log\left(\frac{n}{2}\right)^n}$$

$$= \lim_{n\to\infty} \frac{\cancel{n}\log(n)}{\cancel{n}\log\left(\frac{n}{2}\right)} = \frac{\infty}{\infty} \to \text{indeterminado}$$

$$\text{L'H: } \lim_{n\to\infty} \frac{\frac{1}{\ln(2)\cdot n}}{\frac{1}{\ln(2)\cdot n}} = \lim_{n\to\infty} \frac{\ln(2)\cdot n}{\ln(2)\cdot n} = \boxed{1}$$

$$\boxed{n\log(n) = \Theta(\log(n!))}$$

g) g7 e g8

$$\lim_{n\to\infty} \frac{\log(n!)}{n^2} = \lim_{n\to\infty} \frac{\log\left(\frac{n}{e}\right)^n}{n^2} = \frac{\infty}{\infty} \text{ indet.}$$

$$\text{L'H: } \lim_{n\to\infty} \frac{\frac{1}{\ln(2)\cdot n}}{2n} = \lim_{n\to\infty} \frac{1}{\ln(2)\cdot n\cdot 2n} = \lim_{n\to\infty} \frac{1}{\ln(2)\cdot 2n^2}$$

$$= \frac{1}{\ln(2)}\cdot 0 = 0$$

$$\boxed{\log(n!) = o(n^2)}$$

h) g8 e g9

$$\lim_{n \to \infty} \frac{n^2}{4^{\log n}} = \lim_{n \to \infty} \frac{n^2}{2^{2\log n}} = \lim_{n \to \infty} \frac{n^2}{2^{\log n^2}}$$

$$\lim_{n \to \infty} \frac{n^2}{n^2} = \boxed{1}$$

$$\boxed{n^2 = \Theta(4^{\log n})}$$

) g9 e g10

$$\lim_{n \to \infty} \frac{4^{\log n}}{\left(\frac{3}{2}\right)^n} = \lim_{n \to \infty} \frac{n^2}{\frac{3^n}{2^n}} = \lim_{n \to \infty} \frac{n^2 \cdot 2^n}{3^n}$$

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j) g10 e g11

$\lim_{n\to\infty} \dfrac{\left(\frac{3}{2}\right)^n}{2^n} = \lim_{n\to\infty} \dfrac{\frac{3^n}{2^n}}{2^n} = \lim_{n\to\infty} \dfrac{3^n}{2^{2n}} = \lim_{n\to\infty} \dfrac{3^n}{4^n}$

$\lim_{n\to\infty} \left(\dfrac{3}{4}\right)^n$, $\lim_{x\to+\infty} (a^x) = 0,\ -1 < a < 1$

$\frac{3}{4}$

Logo, $\lim_{n\to\infty} \left(\dfrac{3}{4}\right)^n = \boxed{0}$

$$\boxed{\left(\dfrac{3}{2}\right)^n = o\left(2^n\right)}$$

k) g11 e g12

$\lim_{n\to\infty} \dfrac{2^n}{e^n} = \lim_{n\to\infty} \left(\dfrac{2}{e}\right)^n$, $\lim_{x\to\infty} (a^x) = 0,\ -1 < a$

$\frac{2}{e}$

Logo, $\lim_{n\to\infty} \left(\dfrac{2}{e}\right)^n = \boxed{0}$

$$\boxed{2^n = o\left(e^n\right)}$$