Ch 3.1-2: (Multi)-Linear Regression

Lecture 5 - CMSE 381

Prof. Elizabeth Munch

Michigan State University
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Dept of Computational Mathematics, Science & Engineering

Fri, Sep 8, 2023

Announcements

Last time:

- Started 3.1 - Single linear regression

Announcements:

- Office Hours: Tues - Fri
- Homework #1 grades and feedback posted
- Homework #2 Due Mon, Sep 11

Covered in this lecture

- Confidence interval, hypothesis test, and p-value for coefficient estimates
- Residual standard error (RSE)
- R squared
- Setup for multiple linear regression

Section 1

Last time

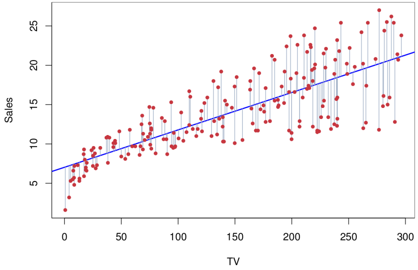
Setup

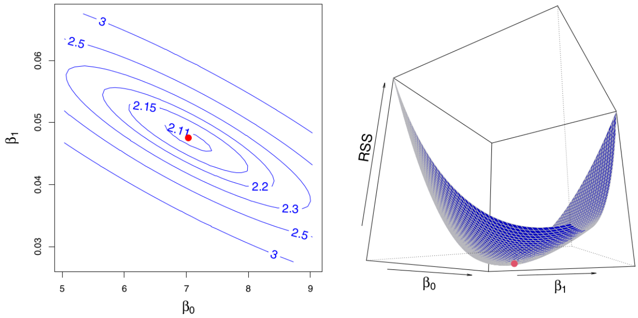
- Predict Y on a single predictor variable X

$$Y \approx \beta_0 + \beta_1 X$$

- "$\approx$" .... "is approximately modeled as"

- Given $(x_1, y_1), \cdots, (x_n, y_n)$
- Let $\hat{y}_i = \hat{\beta}_0 + \hat{\beta}_1 x_i$ be prediction for Y on ith value of X.
- $e_i = y_i - \hat{y}_i$ is the ith residual





Residual sum of squares RSS is

$$
\begin{aligned}
RSS &= e_1^2 + \cdots + e_n^2 \\
&= \sum_i (y_i - \hat{\beta}_0 - \hat{\beta}_1 x_i)^2
\end{aligned}
$$

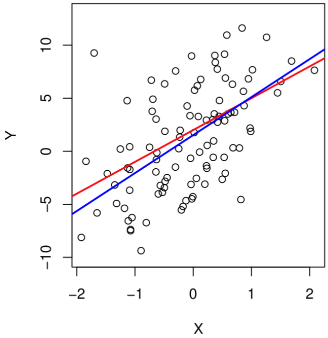
Least squares criterion

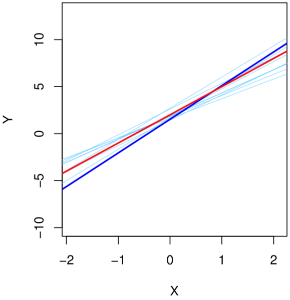
Find β_0 and β_1 that minimize the RSS.

$$
\hat{\beta}_1 = \frac{\sum_{i=1}^n (x_i - \overline{x})(y_i - \overline{y})}{\sum_{i=1}^n (x_i - \overline{x})^2}
$$

$$
\hat{\beta}_0 = \overline{y} - \hat{\beta}_1 \overline{x}
$$

Linear regression is unbiased





Section 2

New stuff on evaluating models

Variance of linear regression estimates

- Variance of linear regression estimates:

$$\mathrm{SE}(\hat{\beta}_0) = \sigma^2 \left[\frac{1}{n} + \frac{\overline{x}^2}{\sum_{i=1}^{n}(x_i - \overline{x})^2} \right]$$

$$\mathrm{SE}(\hat{\beta}_1)^2 = \frac{\sigma^2}{\sum_{i=1}^{n}(x_i - \overline{x})^2}$$

where $\sigma^2 = \mathrm{Var}(\varepsilon)$

- Residual standard error is an estimate of σ

$$RSE = \sqrt{RSS/(n-2)}$$

Confidence Interval

The 95% confidence interval for β_1 approximately takes the form

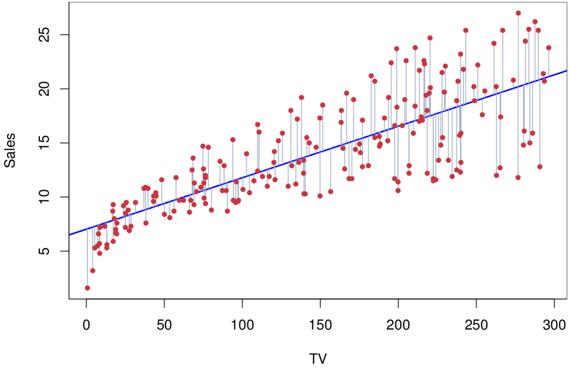
$$\hat{\beta}_1 \pm 2 \cdot \mathrm{SE}(\hat{\beta}_1)$$

Interpretation:
There is approximately a 95% chance that the interval

$$\left[\hat{\beta}_1 - 2 \cdot \mathrm{SE}(\hat{\beta}_1), \hat{\beta}_1 + 2 \cdot \mathrm{SE}(\hat{\beta}_1)\right]$$

will contain β_1 where we repeatedly approximate $\hat{\beta}_1$ using repeated samples.

CI in Advertising data



For the advertising data set, the 95% CIs are:

- β_1 :: $[0.042, 0.053]$
- β_0 :: $[6.130, 7.935]$

Hypothesis testing

H_0: There is no relationship between X and Y

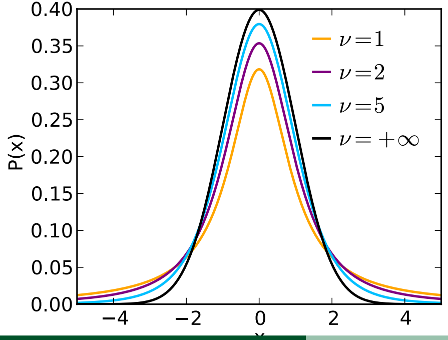
H_1: There is some relationship between X and Y

Test statistic and p-value

Test statistic:

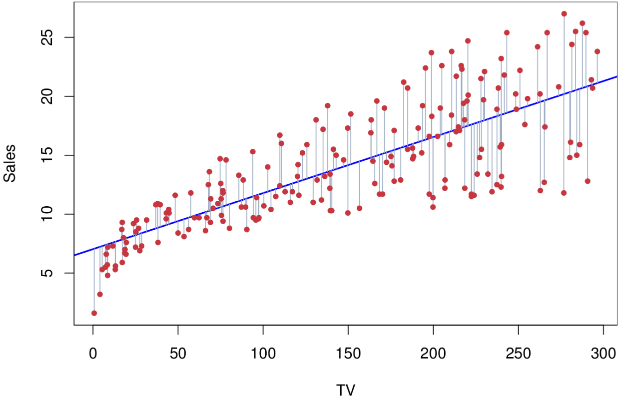
$$t = \frac{\hat{\beta}_1 - 0}{\mathrm{SE}(\hat{\beta}_1)}$$

t-distribution with $n-2$ degrees of freedom



Advertising example

	Coefficient	Std. error	t-statistic	p-value
`Intercept`	7.0325	0.4578	15.36	< 0.0001
`TV`	0.0475	0.0027	17.67	< 0.0001



Residual standard error (RSE):

$$\begin{aligned} RSE &= \sqrt{\frac{1}{n-2} RSS} \\ &= \sqrt{\frac{1}{n-2} \sum_i (y_i - \hat{y}_i)^2} \end{aligned}$$

Assessing the accuracy of the module: R^2

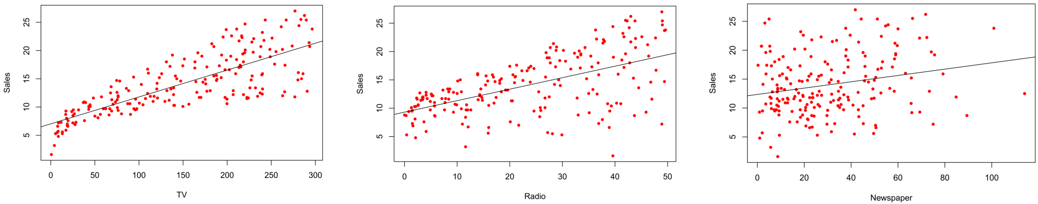
R squared:

$$R^2 = \frac{TSS - RSS}{TSS} = 1 - \frac{RSS}{TSS}$$

where total sum of squares is

$$TSS = \sum_i (y_i - \overline{y})^2$$

Advertising example



$R^2 = 0.61$ $R^2 = 0.33$ $R^2 = 0.05$

Coding group work

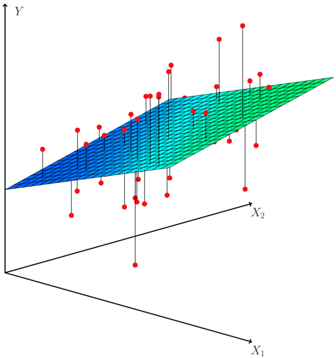
Run the section titled "Assessing Coefficient Estimate Accuracy"

Section 3

Multiple Linear Regression

Setup

$$Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \cdots \beta_p X_p + \varepsilon$$



Estimation and Prediction

Given estimates $\hat{\beta}_0, \hat{\beta}_1, \hat{\beta}_2, \cdots, \hat{\beta}_p$, prediction is

$$\hat{y} = \hat{\beta}_0 + \hat{\beta}_1 x_1 + \cdots + \hat{\beta}_p x_p$$

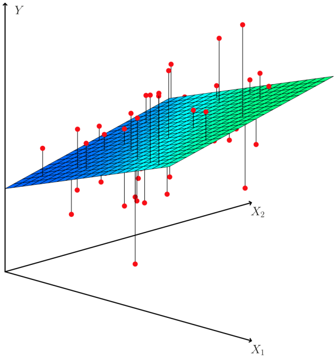
Minimize the sum of squares

$$\begin{aligned} RSS &= \sum_i (y_i - \hat{y}_i)^2 \\ &= \sum (y_i - \hat{\beta}_0 - \hat{\beta}_1 x_1 - \cdots - \hat{\beta}_p x_p) \end{aligned}$$

Coefficients are closed form but UGLY

Advertising data set example

$$\texttt{Sales} = \beta_0 + \beta_1 \cdot \texttt{TV} + \beta_2 \cdot \texttt{radio} + \beta_3 \cdot \texttt{newspaper}$$



	Coefficient
Intercept	2.939
TV	0.046
radio	0.189
newspaper	−0.001

Interpretation of coefficients

$$\texttt{Sales} = \beta_0 + \beta_1 \cdot \texttt{TV} + \beta_2 \cdot \texttt{radio} + \beta_3 \cdot \texttt{newspaper}$$

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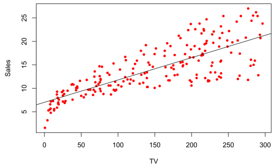
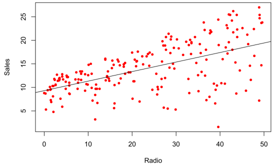
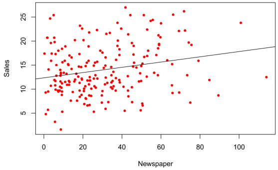
Single regression vs multi-regression



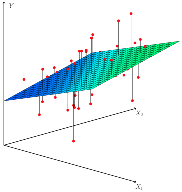
	Coefficient
Intercept	7.0325
TV	0.0475



	Coefficient
Intercept	9.312
radio	0.203



	Coefficient
Intercept	12.351
newspaper	0.055



	Coefficient
Intercept	2.939
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Correlation matrix

	TV	radio	newspaper	sales
TV	1.0000	0.0548	0.0567	0.7822
radio		1.0000	0.3541	0.5762
newspaper			1.0000	0.2283
sales				1.0000

Coding group work

Run the section titled "Multiple Linear Regression"

Next time

Lec #	Date			Reading	Homeworks	***Quizzes*** *(Note: These are not announced until after they happen)*
1	Mon	Aug 28	Intro / First day stuff / Python Review Pt 1	1		
2	Wed	Aug 30	What is statistical learning?	2.1		
3	Fri	Sep 1	Assessing Model Accuracy	2.2.1, 2.2.2	HW #1 Due	Quiz #1
	Mon	Sep 4	No class - Labor day			
4	Wed	Sep 6	Linear Regression	3.1		
5	Fri	Sep 8	More Linear Regression	3.1/3.2		
6	Mon	Sep 11	Even more linear regression	3.2.2	Hw #2 Due	
7	Wed	Sep 13	Probably more linear regression	3.3		
8	Fri	Sep 15	Linear regression coding module			
9	Mon	Sep 18	Intro to classification, Logisitic Regression	2.2.3, 4.1, 4.2, 4.3		
10	Wed	Sep 20	More logistic regression			
11	Fri	Sep 22	Multiple Logistic Regression / Multinomial Logistic Regression /Project day			
	Mon	Sep 25	***Review***			
	Wed	Sep 27	**Midterm #1**			
	Fri	Sep 29	No class - Dr Munch out of town			