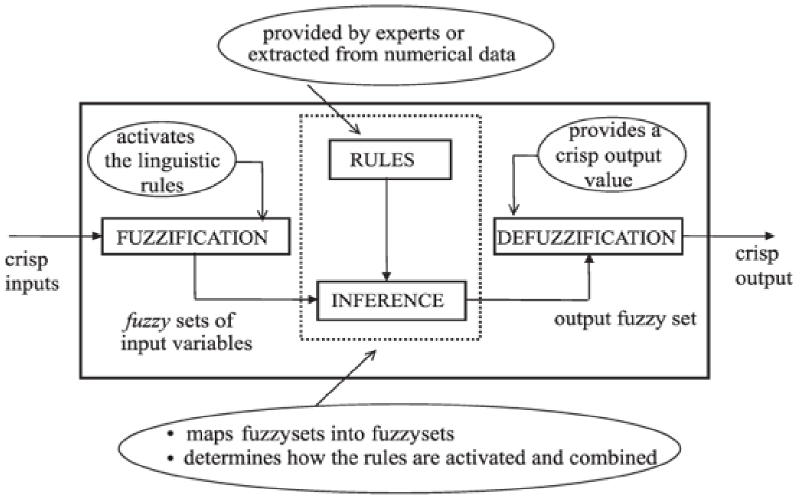
Sugeno fuzzy inference



http://www.scielo.br/scielo.php?script=sci_arttext&pid=S0104-40362007000300009

FIS

1. Fuzzy Inference Systems
   a) Ebrahim Mamdani (University of London)
   b) Sugeno, or TSK (Takagi/Sugeno/Kang)

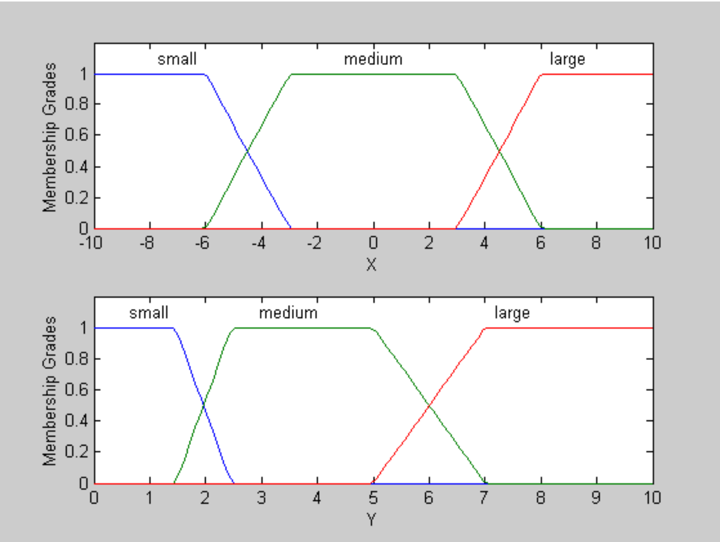
Mamdani Fuzzy models) (cont.)

– Example #1

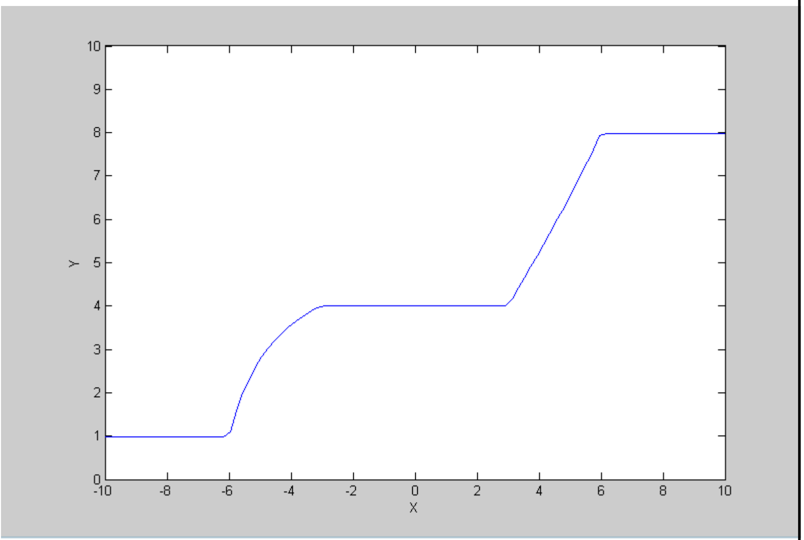
Single input single output Mamdani fuzzy model with 3 rules:

If X is small then Y is small $\rightarrow R_1$
If X is medium then Y is medium $\rightarrow R_2$
Is X is large then Y is large $\rightarrow R_3$

X = input $\in [-10, 10]$
Y = output $\in [0, 10]$
Using max-min composition (R_1 o R_2 o R_3) and centroid defuzzification, we obtain the following overall input-output curve



Single input single output antecedent & consequent MFs



Overall input-output curve

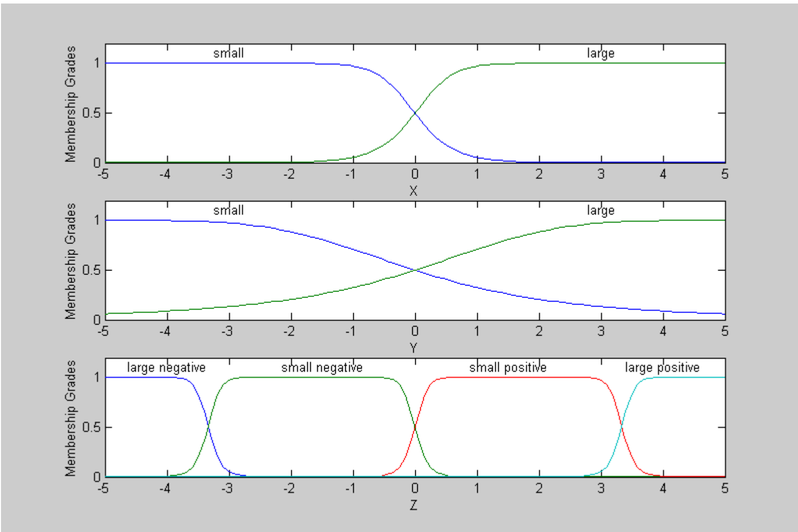
Mamdani Fuzzy models (cont.)

– Example #2

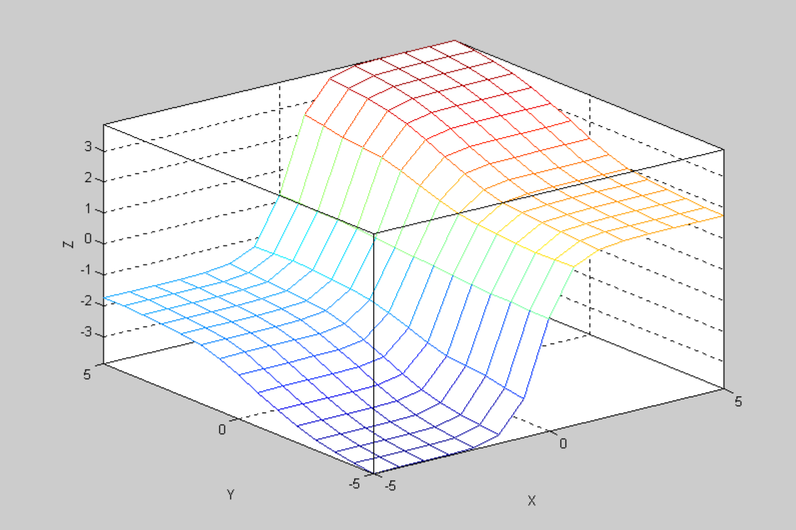
Two input single-output Mamdani fuzzy model with 4 rules:

If X is small & Y is small then Z is negative large
If X is small & Y is large then Z is negative small
If X is large & Y is small then Z is positive small
If X is large & Y is large then Z is positive large

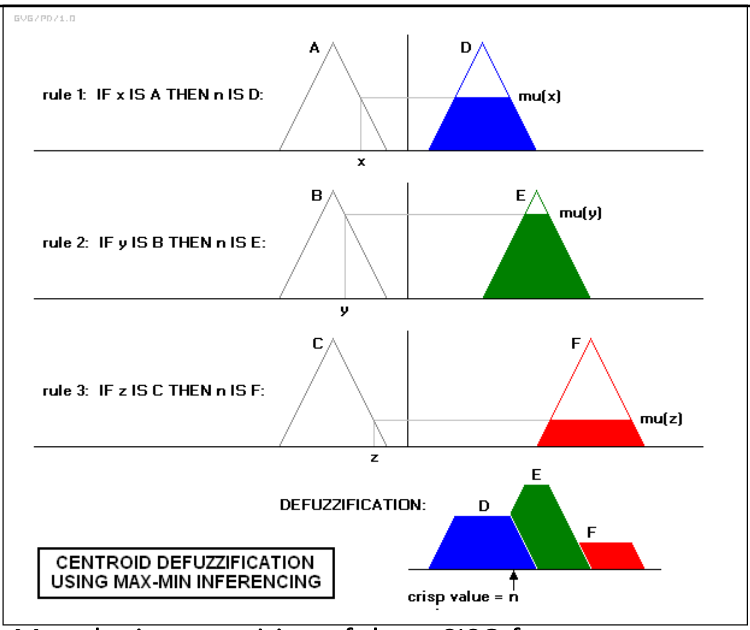
X = [-5, 5]; Y = [-5, 5]; Z = [-5, 5] with max-min composition & centroid defuzzification, we can determine the overall input output surface



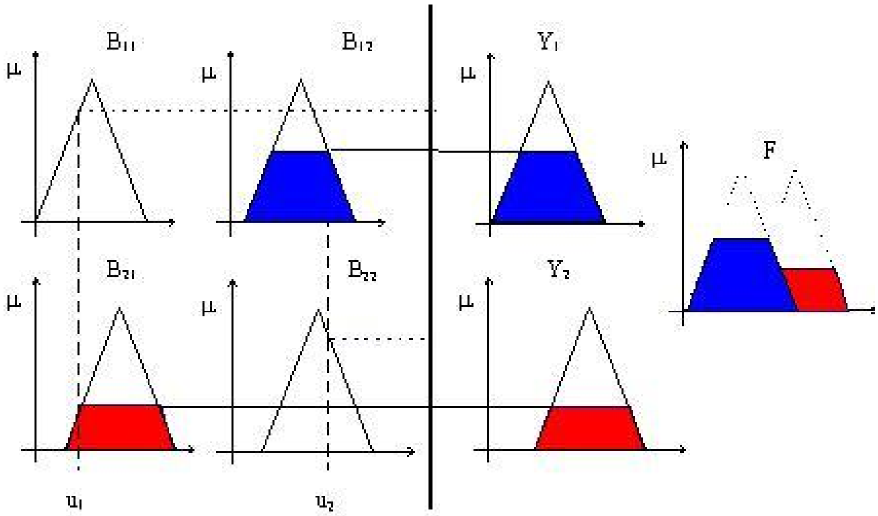
Two-input single output antecedent & consequent MFs



Overall input-output surface

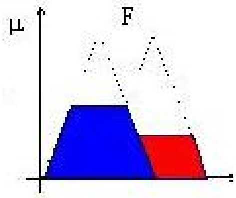


Mamdani composition of three SISO fuzzy outputs
http://en.wikipedia.org/wiki/Fuzzy_control_system

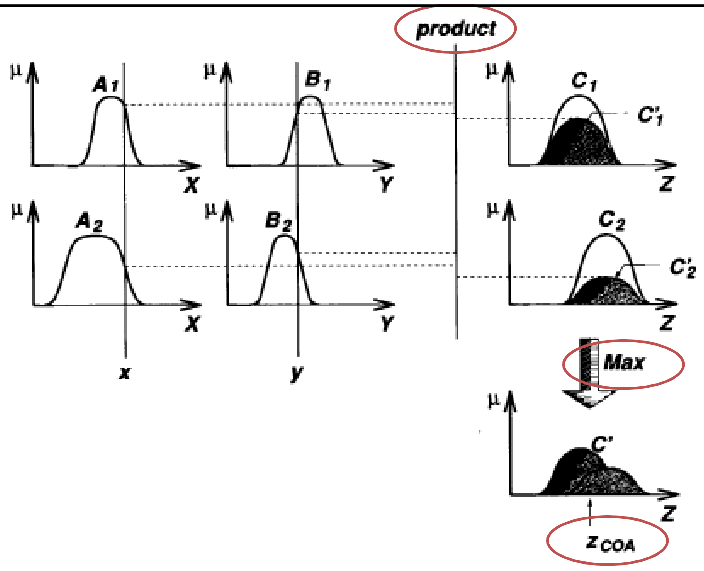


Mamdani composition of two-rule fuzzy system max-min inferencing

http://aragorn.pb.bialystok.pl/~radev/logic/logrozm.htm



Defuzzification can be computationally expensive. What is the center of area of this fuzzy set?



What would point-wise summation look like?

Sugeno fuzzy inference

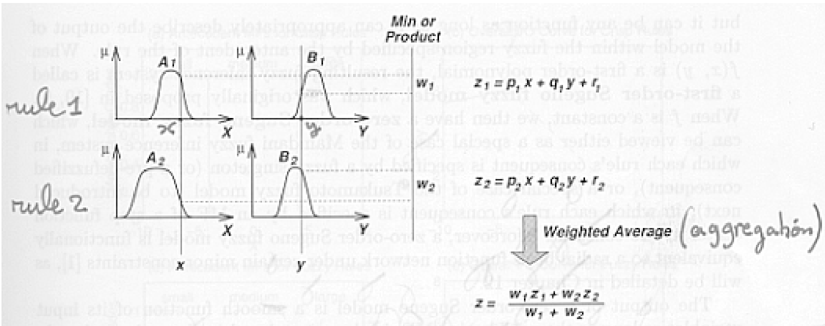
- Mamdani-style inference, requires us to find the centroid of a two-dimensional shape by integrating across a continuously varying function. In general, this process is not computationally efficient.
- **Michio Sugeno** suggested to use a single spike, a *singleton*, as the membership function of the rule consequent. A **singleton**, or more precisely a **fuzzy singleton**, is a fuzzy set with a membership function that is unity at a single particular point on the universe of discourse and zero everywhere else.

Sugeno-style fuzzy inference is very similar to the Mamdani method. Sugeno changed only a rule consequent. Instead of a fuzzy set, he used a mathematical function of the input variable. The format of the **Sugeno-style fuzzy rule** is

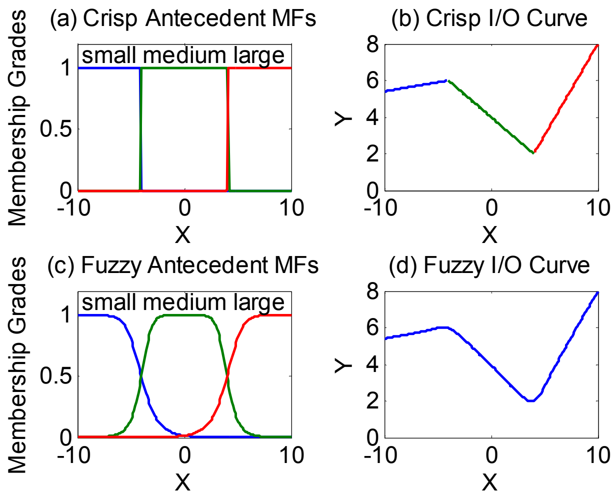
IF *x* is *A*
AND *y* is *B*
THEN *z* is *f* (*x, y*)

where *x*, *y* and *z* are linguistic variables; *A* and *B* are fuzzy sets on universe of discourses *X* and *Y*, respectively; and *f* (*x, y*) is a mathematical function.

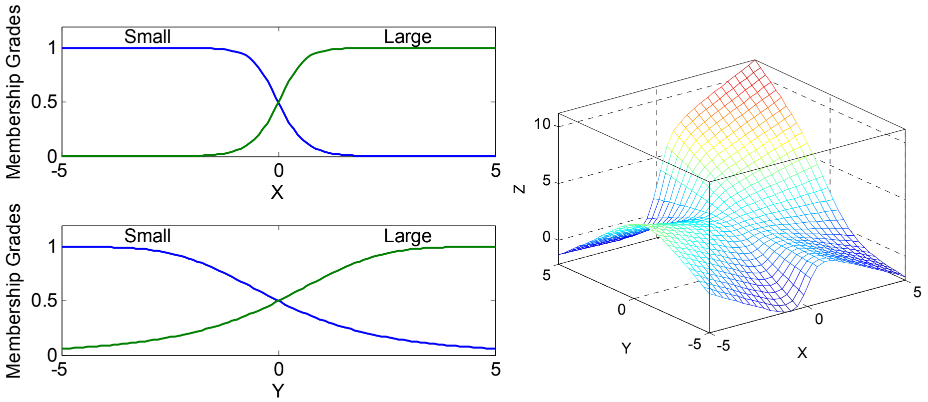
- If f(.,.) is a first order polynomial, then the resulting fuzzy inference is called a first order Sugeno fuzzy model
- If f(.,.) is a constant then it is a zero-order Sugeno fuzzy model (special case of Mamdani model)
- Case of two rules with a first-order Sugeno fuzzy model
  - Each rule has a crisp output
  - Overall output is obtained via weighted average
  - No defuzzyfication required



The Sugeno fuzzy model



- If x is small then y = 0.1x + 6.4
- If x is medium then y = –0.5x + 5
- If x is large then y = x – 2



- If x is small and y is small then $z = -x + y + 1$
- If x is small and y is large then $z = -y + 5$
- If x is large and y is small then $z = -x + 3$
- If x is large and y is large then $z = x + y + 2$

$R_1 \rightarrow (x \wedge s)$ & $(y \wedge s) \rightarrow w_1$

$R_2 \rightarrow (x \wedge s)$ & $(y \wedge l) \rightarrow w_2$

$R_3 \rightarrow (x \wedge l)$ & $(y \wedge s) \rightarrow w_3$

$R_4 \rightarrow (x \wedge l)$ & $(y \wedge l) \rightarrow w_4$

Aggregated consequent $\rightarrow F[(w_1, z_1); (w_2, z_2); (w_3, z_3); (w_4, z_4)]$

= weighted average

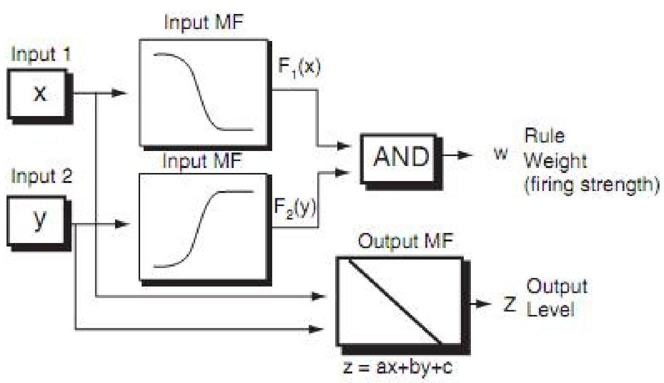
The most commonly used **zero-order Sugeno fuzzy model** applies fuzzy rules in the following form:

IF ***x* is *A***
AND ***y* is *B***
THEN ***z* is *k***

where *k* is a constant.

In this case, the output of each fuzzy rule is constant. All consequent membership functions are represented by singleton spikes.

Sugeno Rule



Mamdani Fuzzy Inference

Project Funding: Adequate , Marginal , Inadequate

Project Staffing : Small , Large

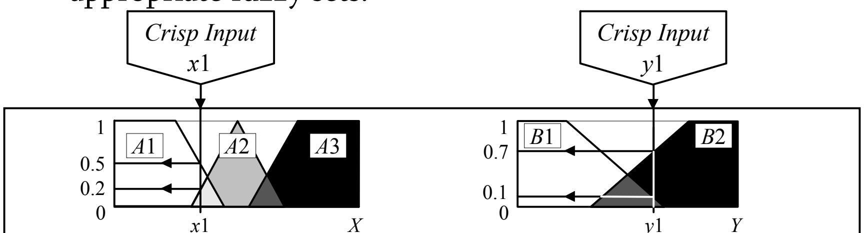
Risk: Low , Normal ,high

We examine a simple two-input one-output problem that includes three rules:

<u>Rule: 1</u>		<u>Rule: 1</u>			
IF	x is A3	IF	project_funding		is adequate
OR	y is B1	OR	project_staffing		is small
THEN	z is C1	THEN	risk		is low
<u>Rule: 2</u>		<u>Rule: 2</u>			
IF	x is A2	IF	project_funding		is marginal
AND	y is B2	AND	project_staffing		is large
THEN	z is C2	THEN	risk		is normal
<u>Rule: 3</u>		<u>Rule: 3</u>			
IF	x is A1	IF	project_funding		is inadequate
THEN	z is C3	THEN	risk		is high

Step 1: Fuzzification

- The first step is to take the crisp inputs, x1 and y1 (*project funding* and *project staffing*), and determine the degree to which these inputs belong to each of the appropriate fuzzy sets.



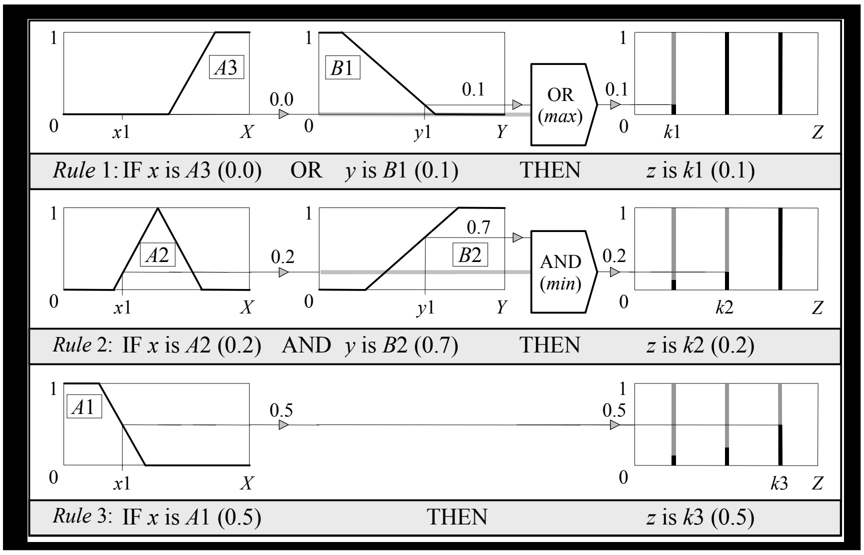
$\mu_{(x = A1)} = 0.5$

$\mu_{(x = A2)} = 0.2$

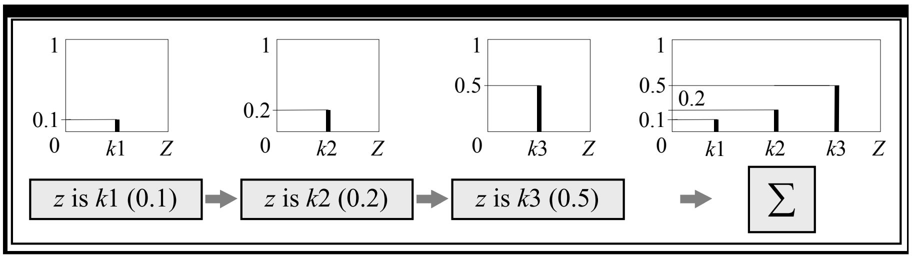
$\mu_{(y = B1)} = 0.1$

$\mu_{(y = B2)} = 0.7$

Sugeno-style rule evaluation



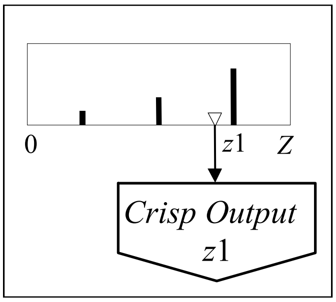
Sugeno-style aggregation of the rule outputs



Weighted average (WA):

$$WA = \frac{\mu(k1)\times k1 + \mu(k2)\times k2 + \mu(k3)\times k3}{\mu(k1)+\mu(k2)+\mu(k3)} = \frac{0.1\times 20 + 0.2\times 50 + 0.5\times 80}{0.1+0.2+0.5} = 65$$

Sugeno-style defuzzification



How to make a decision on which method to apply – Mamdani or Sugeno?

- Mamdani method is widely accepted for capturing expert knowledge. It allows us to describe the expertise in more intuitive, more human-like manner. However, Mamdani-type fuzzy inference entails a substantial computational burden.
- On the other hand, Sugeno method is computationally effective and works well with optimisation and adaptive techniques, which makes it very attractive in control problems, particularly for dynamic nonlinear systems.

Advantages of the Sugeno Method

- It is computationally efficient.
- It works well with linear techniques (e.g., PID control).
- It works well with optimization and adaptive techniques.
- It has guaranteed continuity of the output surface.
- It is well suited to mathematical analysis.

Advantages of the Mamdani Method

- It is intuitive.
- It has widespread acceptance.
- It is well suited to human input.