Fuzzy rules and inference

Linguistic variables

- Temp={cool,normal,warm,hot]
- Each one is fuzzy set
- Temperature is fuzzy variable. How do we represent Linguistic variable ?
- Dynamic range and Membership function
- Other ex:age,speed,height etc.

Rule based systems
Fuzzy If-Then Rules

- A way to represent knowledge
- General format:

  If x is A then y is B

  If {premise} then {conclusion}
- Examples:
  - If pressure is high, then volume is small.
  - If the road is slippery, then driving is dangerous.
  - If a tomato is red, then it is ripe.
  - If the speed is high, then apply the brake a little.

Fuzzy inference

- This knowledge in the form of /if then rules can be used to derive inferences
- From the known rules can we define new rules?
- Rule 1:If x is A then y is B

  Everything is known
- Rule 2:If x is A’ then y is B’
- From information derived from Rule 1 is it possible to derive conclusion for Rule 2.

Fuzzy inference

- B’ can be found out using composition operation
- $B' = A' \circ R$ where R is relation matrix.

Fuzzy implication relation

- If p then q
- If <u>x is A </u>then <u>y is B</u>

        p   q

- Implication rule 1:
- If p is true then to say q is false is impossible

  ie p is true and q is false is false. From this rule

  $$\mu_R = \max[\,(1- \mu_P),\ \mu_q]$$

Fuzzy Implications

- Mamdani implication(most commonly used)
- When If-Then rules are locally true then
- P→q means p п q is true.
- So $\mu_R = \min [\mu_P , \mu_q]$
- Using various implication rules relation matrix can be calculated
- From relational matrix inference can be derived using composition rule of inference.
- Eg: max-min rule, max product rule

Example 1

- If x is A then y is B.
- A={ $\frac{0.2}{1}$, $\frac{0.5}{2}$, $\frac{0.7}{3}$ }
- B={ $\frac{0.6}{5}$, $\frac{0.8}{7}$, $\frac{0.4}{9}$ }
- Infer B' for another rule If x is A' then y is B'.
- If A' ={ $\frac{0.5}{1}$, $\frac{0.9}{2}$, $\frac{0.3}{3}$ } using Mamdani implication.

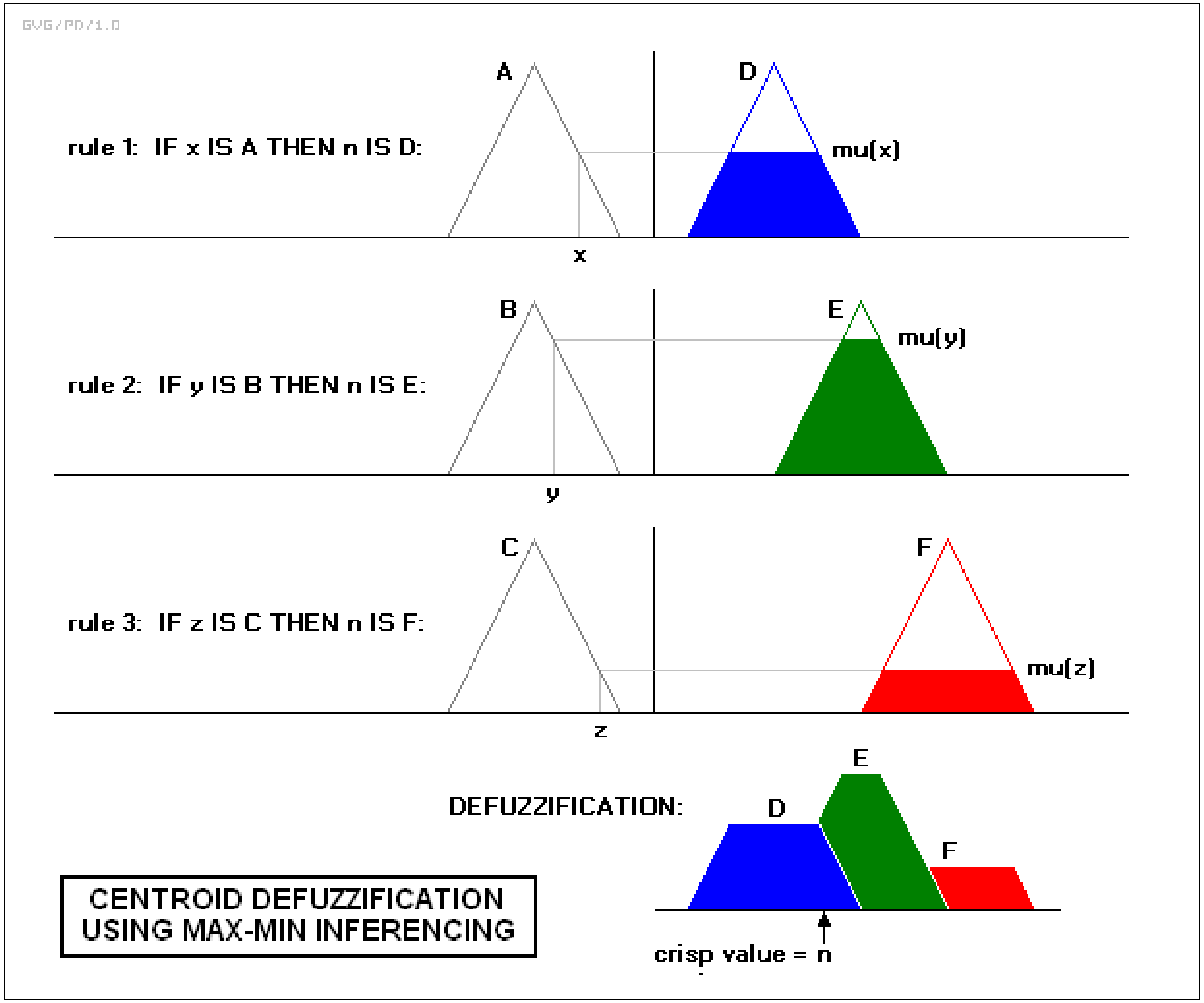
Example 2

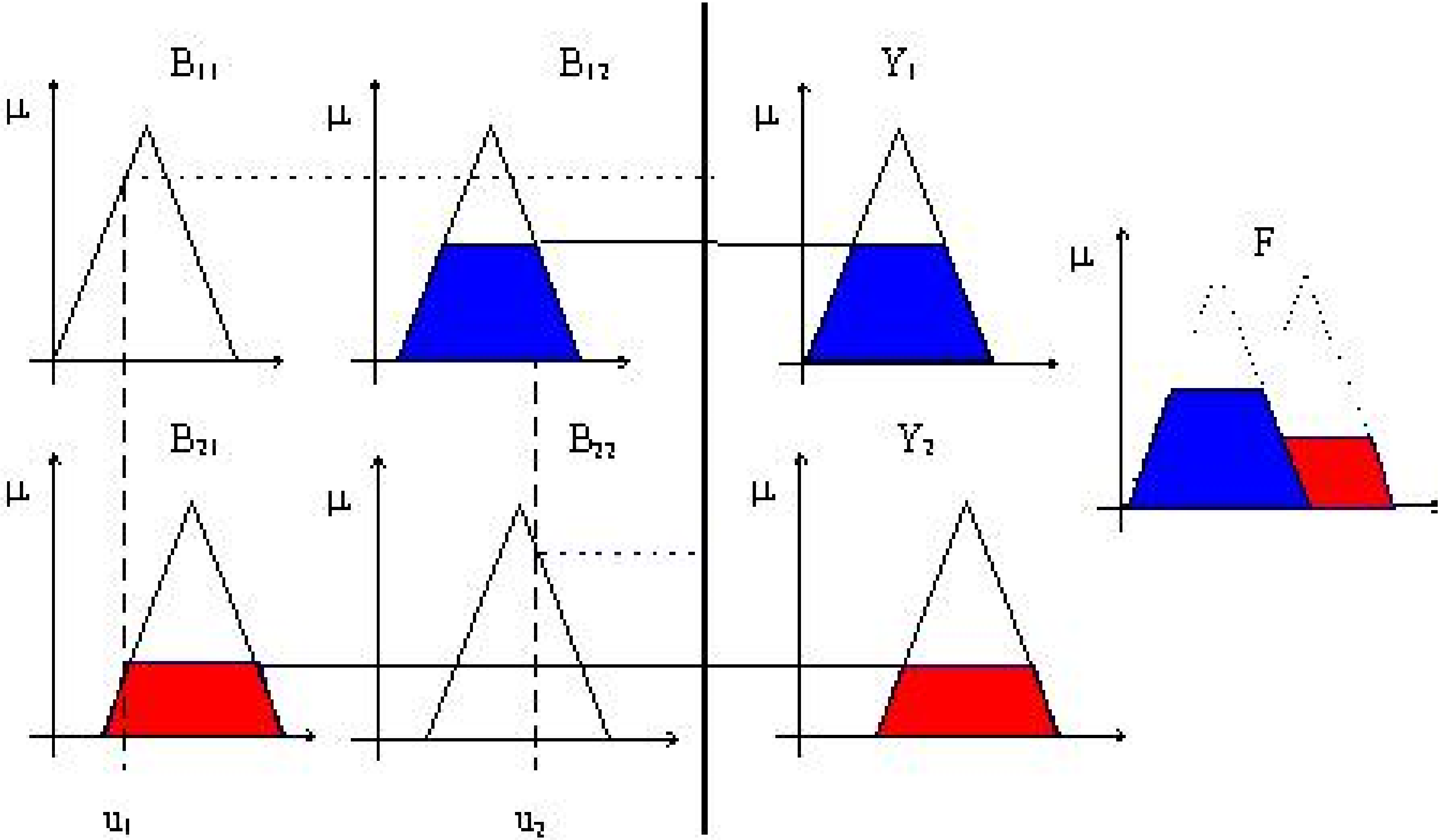
- If temperature is HOT then fan should FAST.If temperature is MODERATELY HOT then fan should run MODERATELY FAST
- H={ $\frac{0.4}{70}, \frac{0.6}{80}, \frac{0.8}{90}, \frac{0.9}{100}$ }
- F= { $\frac{0.3}{1}, \frac{0.5}{2}, \frac{0.7}{3}, \frac{0.9}{4}$ }
- H'={ $\frac{0.2}{70}, \frac{0.4}{80}, \frac{0.6}{90}, \frac{0.8}{100}$ }

Example 3

- If height is tall then speed is high
- If height is med then speed is moderate
- If height is above average what is the speed?
- H1= $\{\frac{0.5}{5}, \frac{0.8}{6}, \frac{1}{7}\}$, S1= $\{\frac{0.4}{5}, \frac{0.7}{7}, \frac{0.9}{9}\}$
- H2= $\{\frac{0.8}{5}, \frac{0.7}{6}, \frac{0.6}{7}\}$, S2= $\{\frac{0.7}{5}, \frac{0.6}{7}, \frac{0.5}{9}\}$
- H'= $\{\frac{0.5}{5}, \frac{0.9}{6}, \frac{0.8}{7}\}$, S'=???
- S'= above normal= max(H'$\circ$R1 , H'$\circ$ R2)

  = H' $\circ$ max(R1,R2)


GVG/PD/1.0
A
D
rule 1: IF x IS A THEN n IS D:
mu(x)
x
B
E
mu(y)
rule 2: IF y IS B THEN n IS E:
y
C
F
rule 3: IF z IS C THEN n IS F:
mu(z)
z
E
DEFUZZIFICATION:
D
F
CENTROID DEFUZZIFICATION
USING MAX-MIN INFERENCING
crisp value = n



Mamdani composition of two-rule fuzzy system max-min inference

Fuzzy Controller

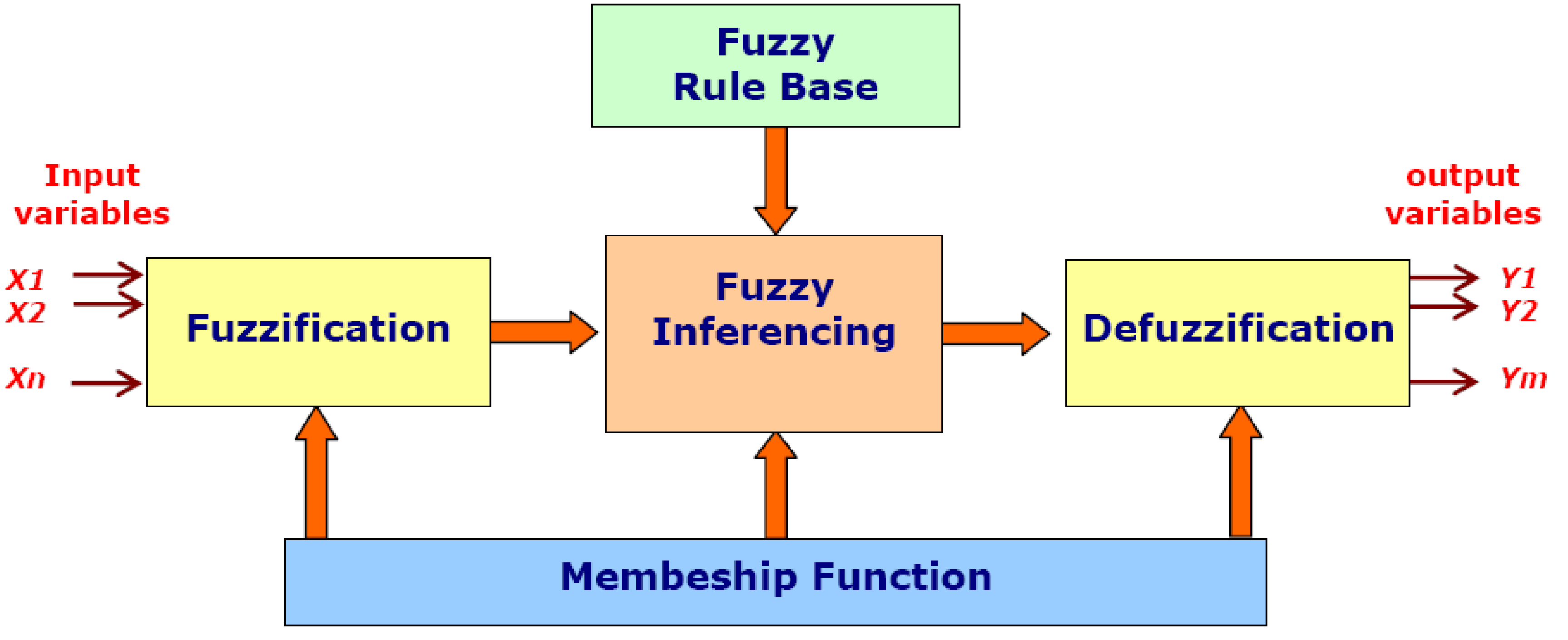


Fig. Elements of Fuzzy System

Fuzzy Controller

Fuzzy System elements

- **Input Vector** : $X = [x_1 , x_2, . . . x_n]^T$ are crisp values, which are transformed into fuzzy sets in the fuzzification block.
- **Output Vector** : $Y = [y_1 , y_2, . . . y_m]^T$ comes out from the defuzzification block, which transforms an output fuzzy set back to a crisp value.
- **Fuzzification** : a process of transforming crisp values into grades of membership for linguistic terms, "far", "near", "small" of fuzzy sets.

Fuzzy Controller

- **Fuzzy Rule base** : a collection of propositions containing linguistic variables; the rules are expressed in the form:

  If (x is A) AND (y is B) . . . . THEN (z is C)

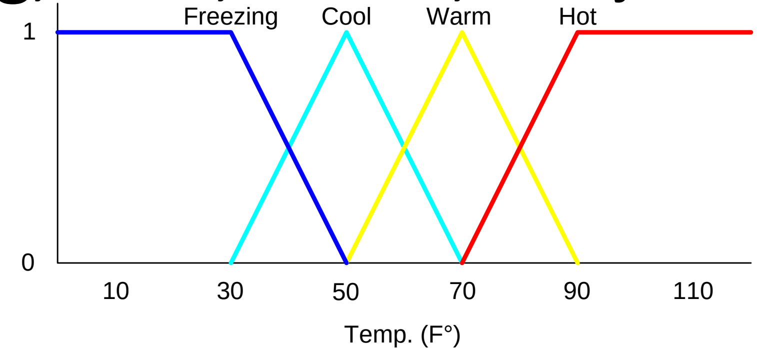
  where **x, y** and **z** represent variables (e.g. distance, size) and **A, B** and **Z** are linguistic variables (e.g. `far', `near', `small').
- **Membership function** : provides a measure of the degree of similarity of elements in the universe of discourse **U** to fuzzy set.
- **Fuzzy Inferencing** : combines the facts obtained from the Fuzzification with the rule base and conducts the Fuzzy reasoning process.
- **Defuzzyfication:** Translate results back to the real world values.

Fuzzy Control

- Fuzzy Control combines the use of fuzzy linguistic variables with fuzzy logic
- Example: Speed Control
- How fast am I going to drive today?
- It depends on the weather.

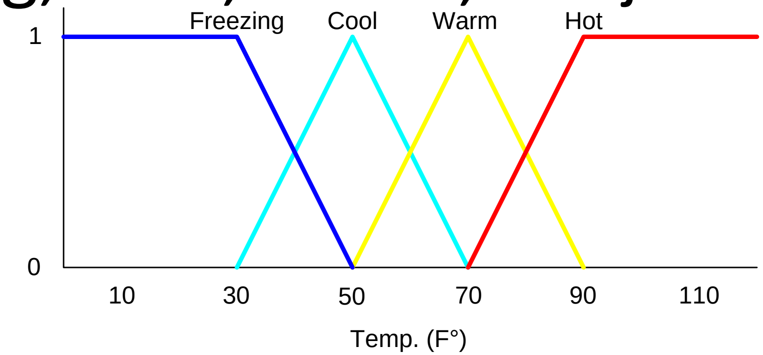
Inputs: Temperature

- Temp: {Freezing, Cool, Warm, Hot}

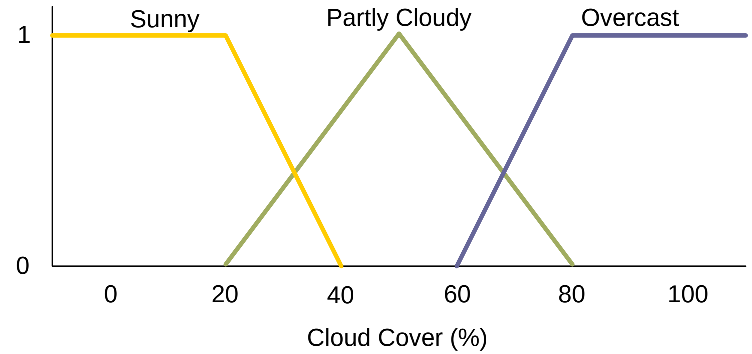


Inputs: Temperature, Cloud Cover

- Temp: {Freezing, Cool, Warm, Hot}

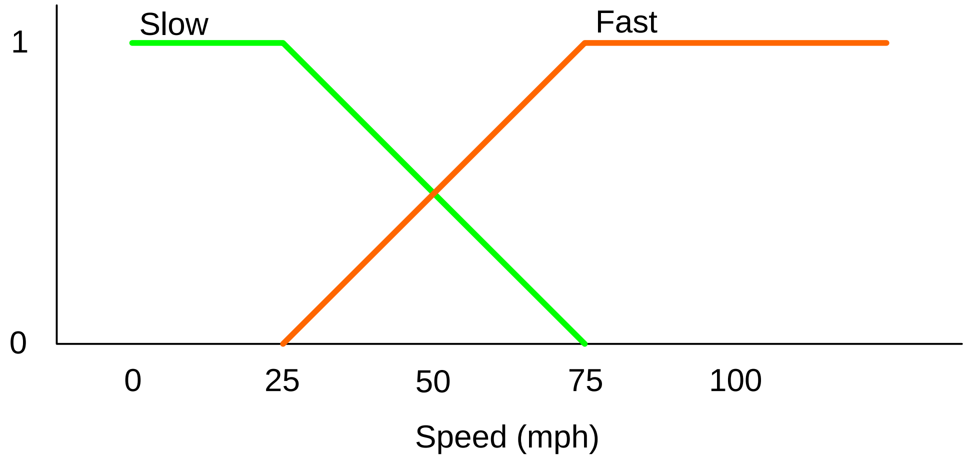


- Cover: {Sunny, Partly, Overcast}



Output: Speed

- Speed: {Slow, Fast}



Rules

- If it's Sunny and Warm, drive Fast

  $Sunny(Cover) \wedge Warm(Temp) \Rightarrow Fast(Speed)$

- If it's Cloudy and Cool, drive Slow

  $Cloudy(Cover) \wedge Cool(Temp) \Rightarrow Slow(Speed)$

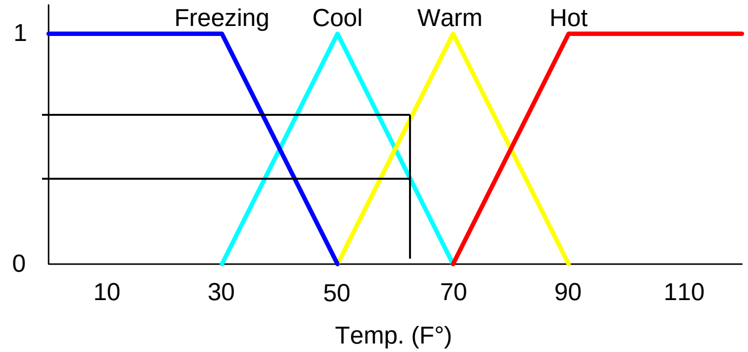
- Driving Speed is the combination of output of these rules...

Example Speed Calculation

- How fast will I go if it is
  - 65 F°
  - 25 % Cloud Cover ?

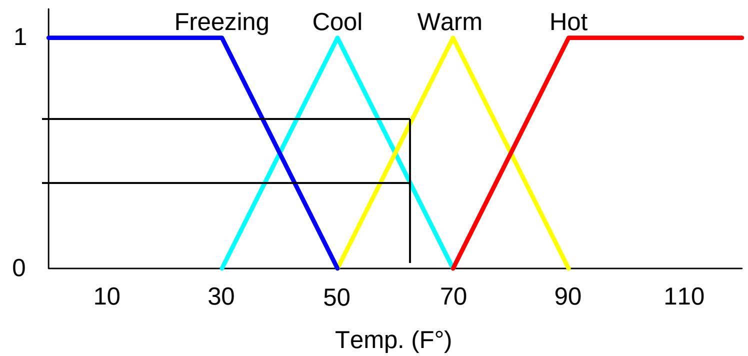
Fuzzification:
Calculate Input Membership Levels

- 65 F° $\Rightarrow$ Cool = 0.4, Warm= 0.7

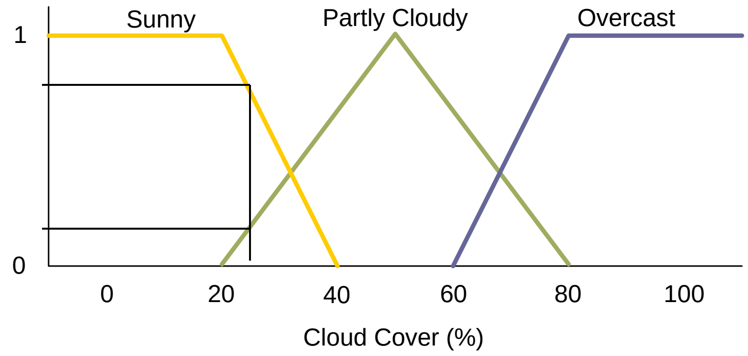


Fuzzification:
Calculate Input Membership Levels

- 65 F° ⇒ Cool = 0.4, Warm= 0.7



- 25% Cover ⇒Sunny = 0.8, Cloudy = 0.2



...Calculating...

- If it's Sunny and Warm, drive Fast

$$\text{Sunny(Cover)} \wedge \text{Warm(Temp)} \Rightarrow \text{Fast(Speed)}$$

$$0.8 \wedge 0.7 = 0.7$$

$$\Rightarrow \textbf{Fast = 0.7}$$

- If it's Cloudy and Cool, drive Slow

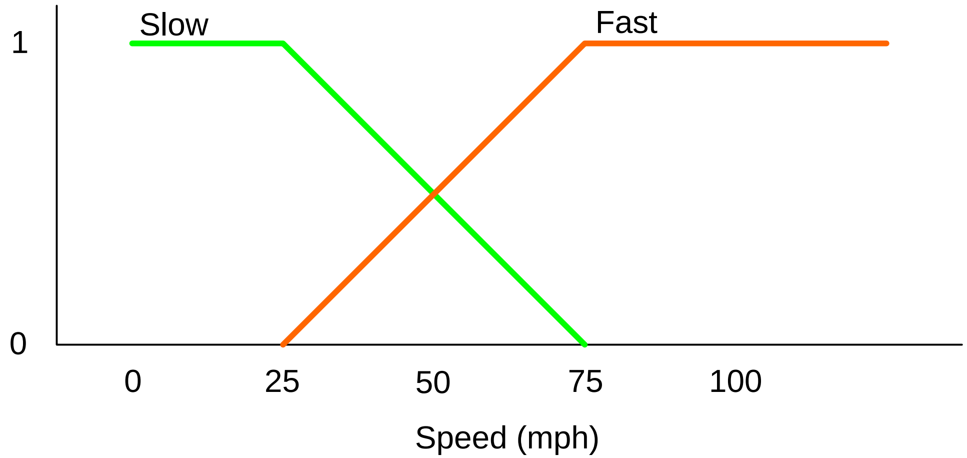
$$\text{Cloudy(Cover)} \wedge \text{Cool(Temp)} \Rightarrow \text{Slow(Speed)}$$

$$0.2 \wedge 0.4 = 0.2$$

$$\Rightarrow \textbf{Slow = 0.2}$$

Defuzzification: Constructing the Output

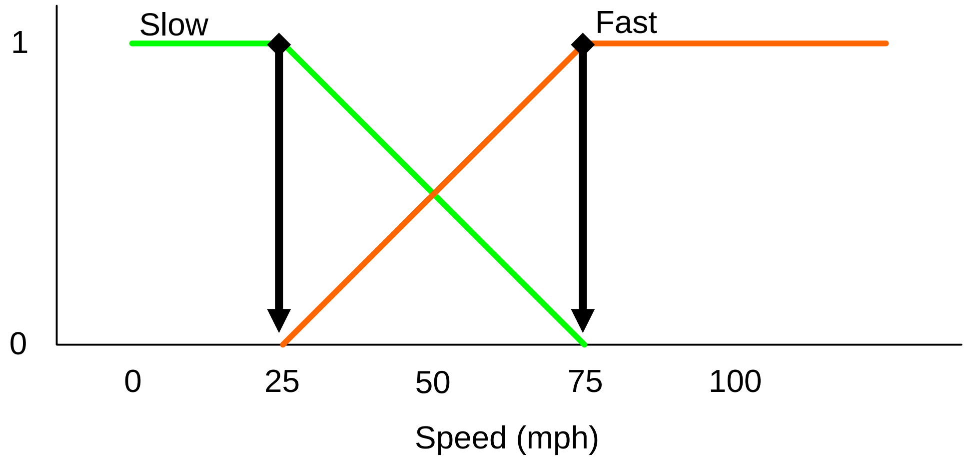
- Speed is 20% Slow and 70% Fast



- Find centroids: Location where membership is 100%

Defuzzification: Constructing the Output

- Speed is 20% Slow and 70% Fast



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Defuzzification: Constructing the Output

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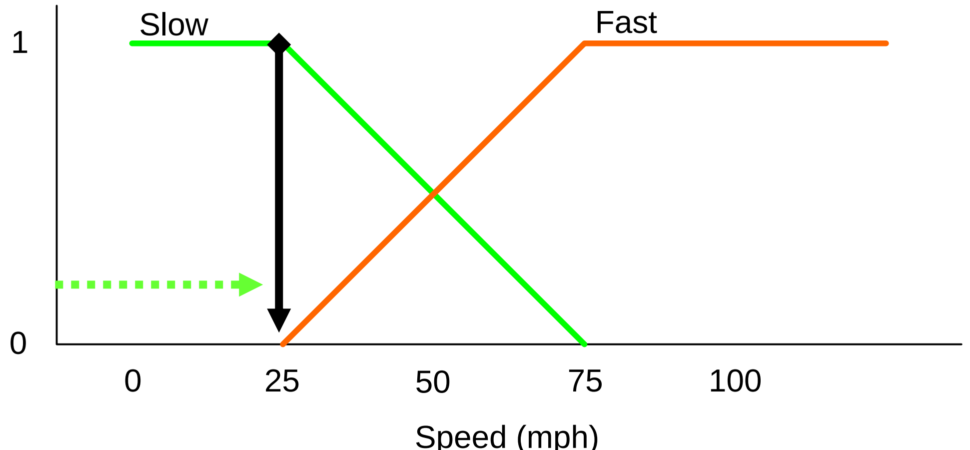
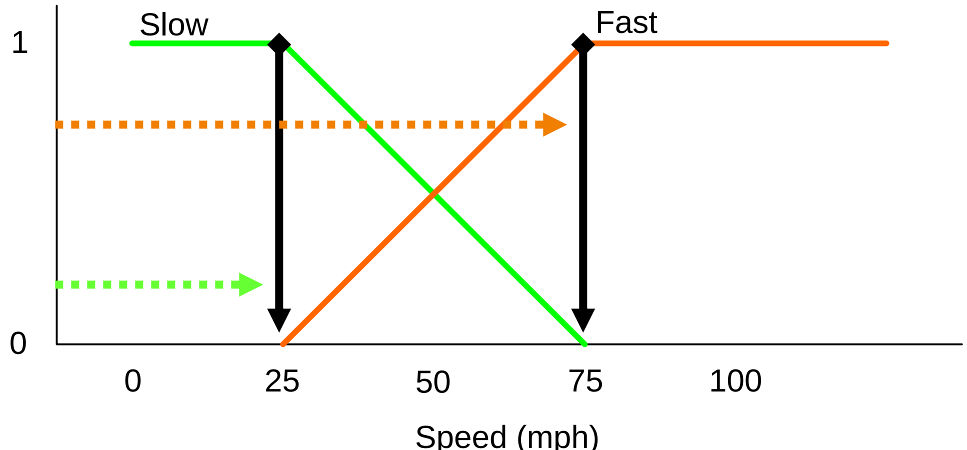


- Speed = weighted mean
  = (2*25+...

Defuzzification: Constructing the Output

- Speed is 20% Slow and 70% Fast



- Speed = weighted mean

  = (2*25+7*75)/(9)

  = 63.8 mph

Fuzzy Inference

- The most commonly used fuzzy inference technique is the so-called **Mamdani** method.

- In 1975, Professor Ebrahim Mamdani of London University built one of the first fuzzy systems to control a steam engine and boiler combination. He applied a set of fuzzy rules supplied by experienced human operators.

Mamdani Fuzzy Inference

- The Mamdani-style fuzzy inference process is performed in four steps:

1. Fuzzification of the input variables
2. Rule evaluation (inference)
3. Aggregation of the rule outputs (composition)
4. Defuzzification.

Mamdani Fuzzy Inference

Project Funding: Adequate , Marginal , Inadequate

Project Staffing : Small , Large

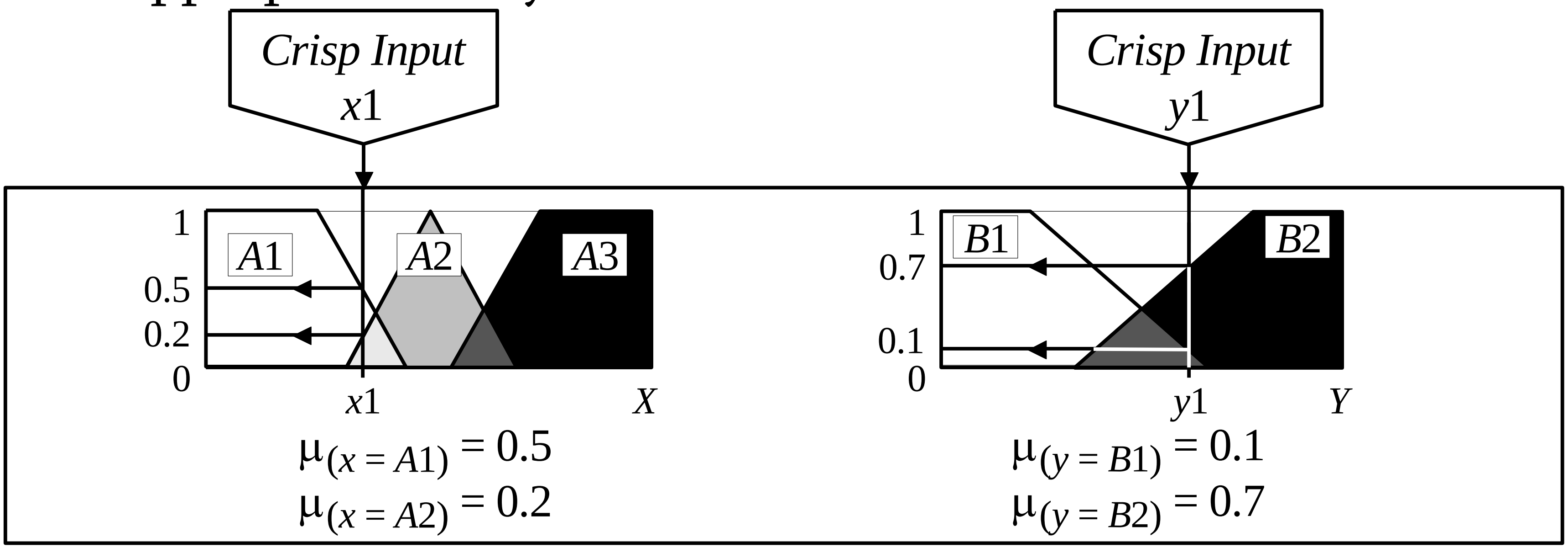
Risk: Low , Normal ,high

We examine a simple two-input one-output problem that includes three rules:

<u>Rule: 1</u>		<u>Rule: 1</u>		
IF	x is A3	IF	project_funding	is adequate
OR	y is B1	OR	project_staffing	is small
THEN	z is C1	THEN	risk	is low
<u>Rule: 2</u>		<u>Rule: 2</u>		
IF	x is A2	IF	project_funding	is marginal
AND	y is B2	AND	project_staffing	is large
THEN	z is C2	THEN	risk	is normal
<u>Rule: 3</u>		<u>Rule: 3</u>		
IF	x is A1	IF	project_funding	is inadequate
THEN	z is C3	THEN	risk	is high

Step 1: Fuzzification

- The first step is to take the crisp inputs, x1 and y1 (*project funding* and *project staffing*), and determine the degree to which these inputs belong to each of the appropriate fuzzy sets.



Step 2: Rule Evaluation

- The second step is to take the fuzzified inputs, $\mu_{(x=A1)} = 0.5$, $\mu_{(x=A2)} = 0.2$, $\mu_{(y=B1)} = 0.1$ and $\mu_{(y=B2)} = 0.7$, and apply them to the antecedents of the fuzzy rules.

- If a given fuzzy rule has multiple antecedents, the fuzzy operator (AND or OR) is used to obtain a single number that represents the result of the antecedent evaluation.

- This number (the truth value) is then applied to the consequent membership function.

Step 2: Rule Evaluation

RECALL:

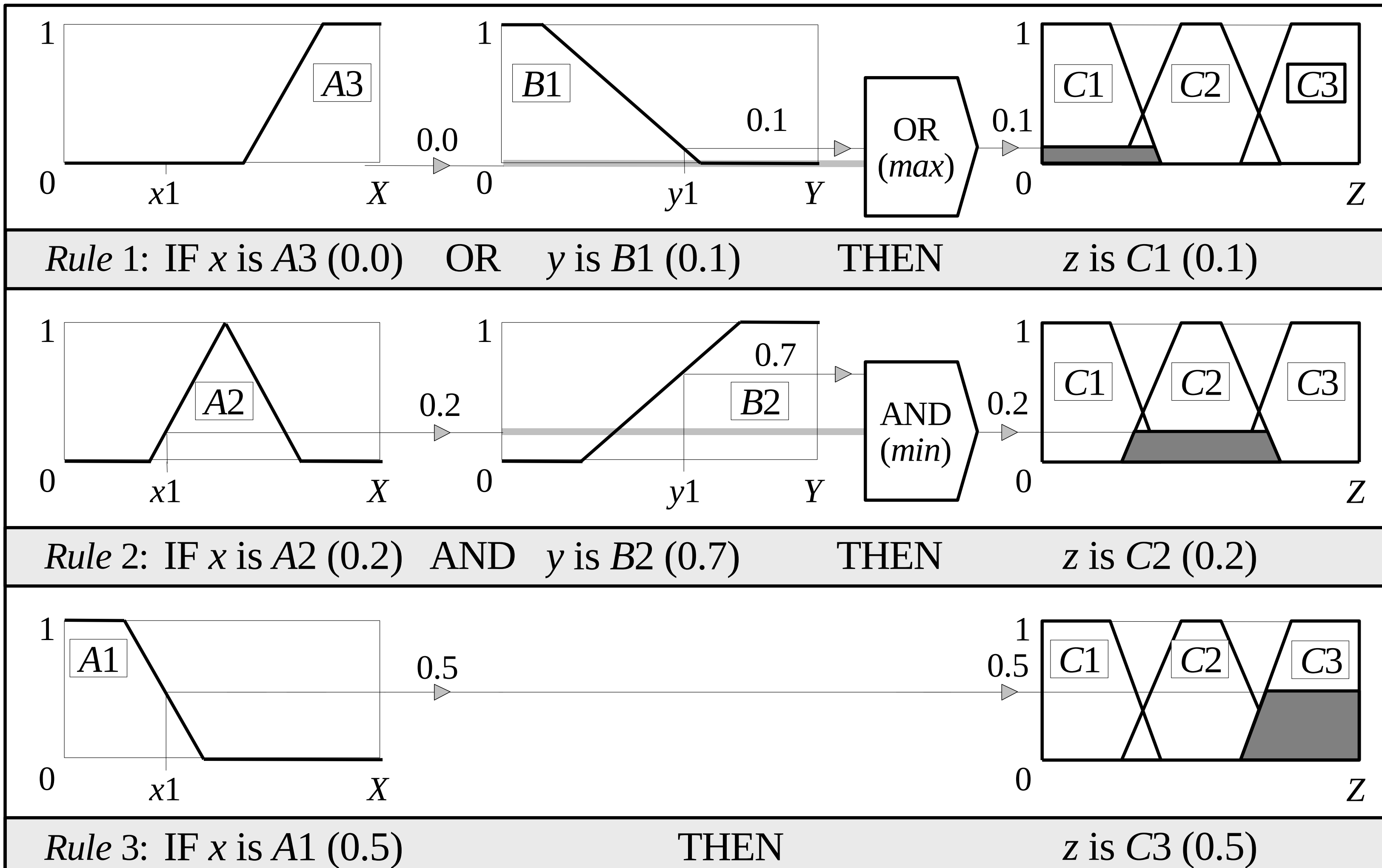
To evaluate the disjunction of the rule antecedents, we use the OR fuzzy operation. Typically, fuzzy expert systems make use of the classical fuzzy operation union:

$$\mu_{A \cup B}(x) = \max\,[\mu_A(x),\ \mu_B(x)]$$

Similarly, in order to evaluate the conjunction of the rule antecedents, we apply the AND fuzzy operation intersection:

$$\mu_{A \cap B}(x) = \min\,[\mu_A(x),\ \mu_B(x)]$$

Step 2: Rule Evaluation



Step 2: Rule Evaluation

- Now the result of the antecedent evaluation can be applied to the membership function of the consequent.

- There are two main methods for doing so:
  - Clipping
  - Scaling

Step 2: Rule Evaluation

- The most common method of correlating the rule consequent with the truth value of the rule antecedent is to cut the consequent membership function at the level of the antecedent truth. This method is called **clipping** (alpha-cut).

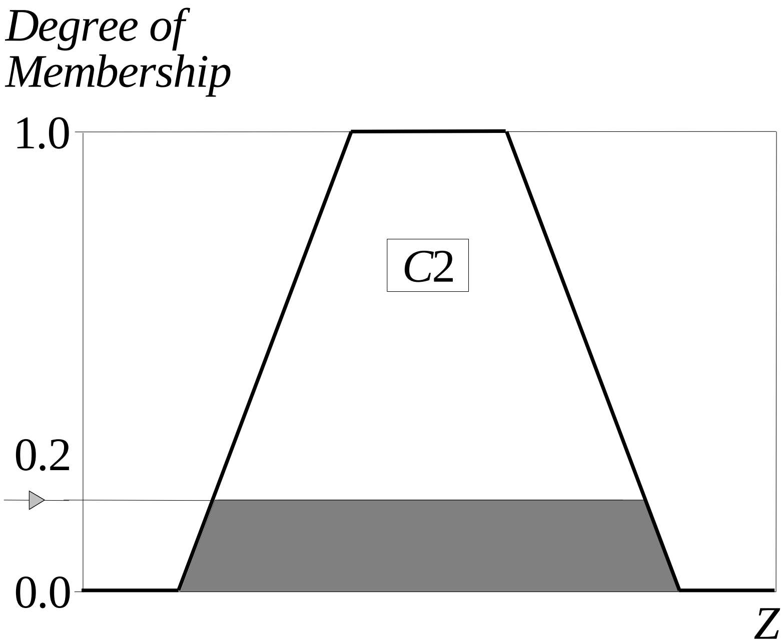
- Since the top of the membership function is sliced, the clipped fuzzy set loses some information.

- However, clipping is still often preferred because it involves less complex and faster mathematics, and generates an aggregated output surface that is easier to defuzzify.

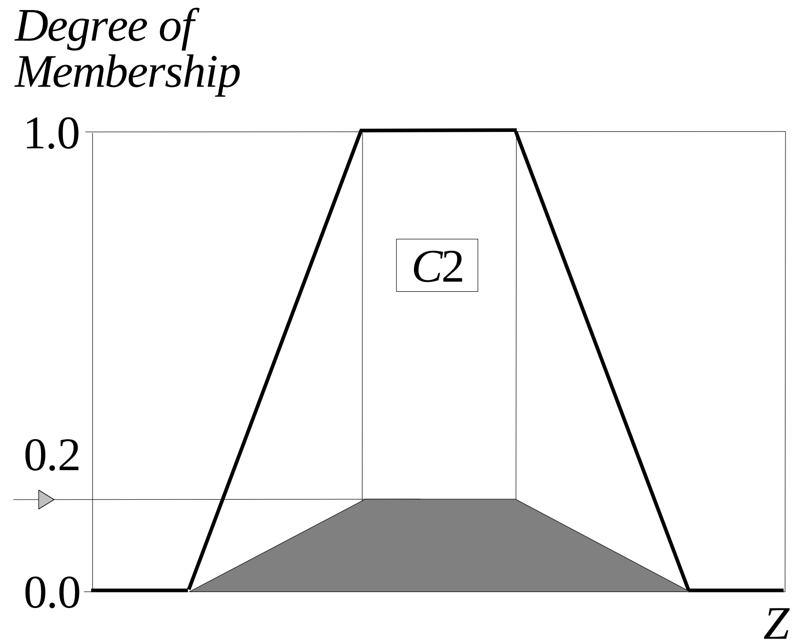
Step 2: Rule Evaluation

- While clipping is a frequently used method, **scaling** offers a better approach for preserving the original shape of the fuzzy set.
- The original membership function of the rule consequent is adjusted by multiplying all its membership degrees by the truth value of the rule antecedent.
- This method, which generally loses less information, can be very useful in fuzzy expert systems.

Step 2: Rule Evaluation



clipping



scaling

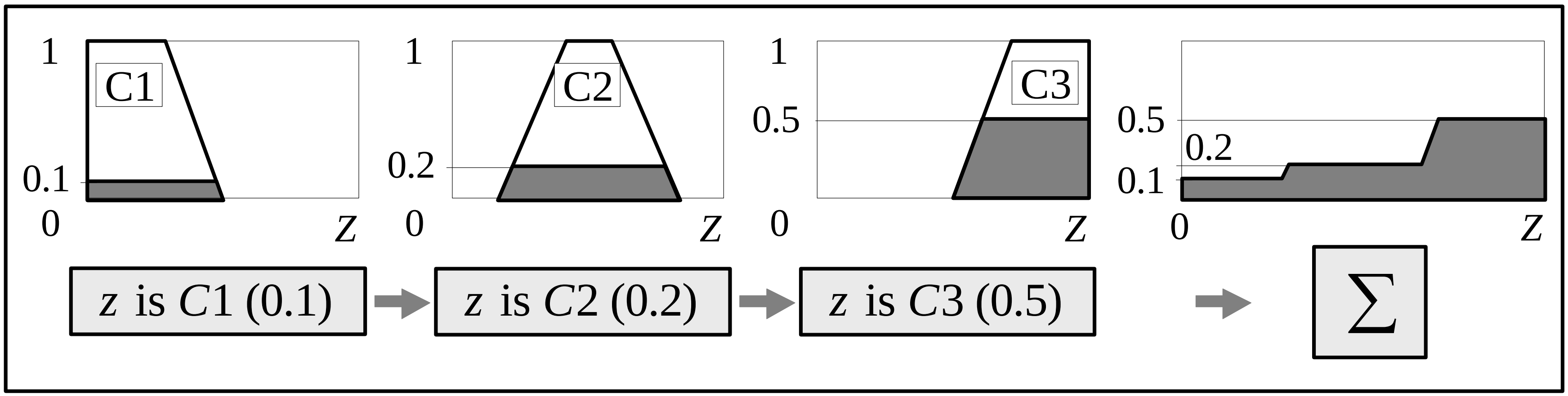
Step 3: Aggregation of the rule outputs

- Aggregation is the process of unification of the outputs of all rules.

- We take the membership functions of all rule consequents previously clipped or scaled and combine them into a single fuzzy set.

- The input of the aggregation process is the list of clipped or scaled consequent membership functions, and the output is one fuzzy set for each output variable.

Step 3: Aggregation of the rule outputs



Step 4: Defuzzification

- The last step in the fuzzy inference process is defuzzification.

- Fuzziness helps us to evaluate the rules, but the final output of a fuzzy system has to be a crisp number.

- The input for the defuzzification process is the aggregate output fuzzy set and the output is a single number.

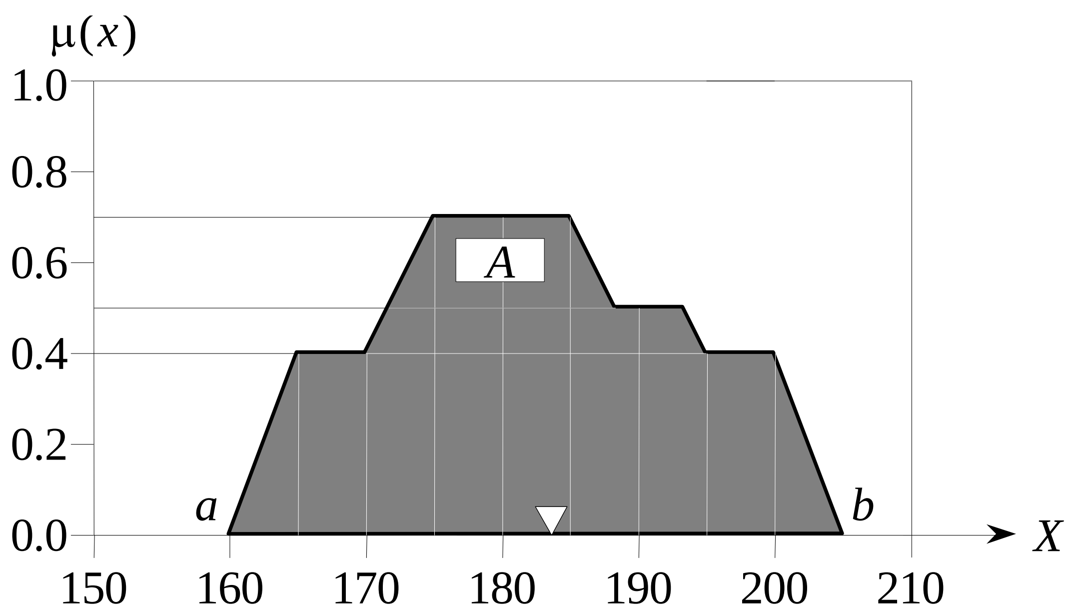
Step 4: Defuzzification

- There are several defuzzification methods, but probably the most popular one is the **centroid technique**. It finds the point where a vertical line would slice the aggregate set into two equal masses. Mathematically this **centre of gravity** (**COG**) can be expressed as:

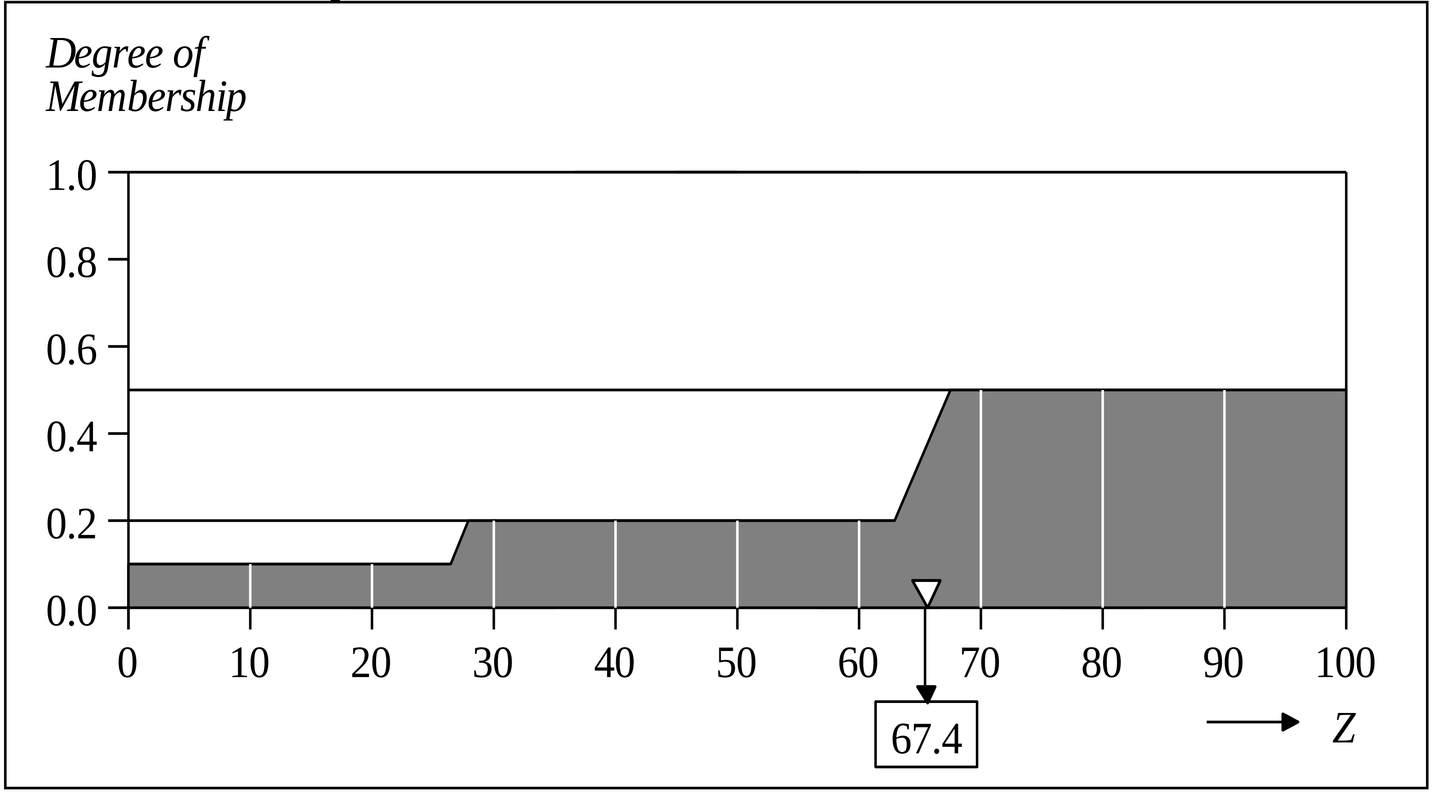
$$COG = \frac{\int_a^b \mu_A(x)\, x\, dx}{\int_a^b \mu_A(x)\, dx}$$

Step 4: Defuzzification

- Centroid defuzzification method finds a point representing the centre of gravity of the fuzzy set, *A*, on the interval, *ab*.
- A reasonable estimate can be obtained by calculating it over a sample of points.



Step 4: Defuzzification



$$COG = \frac{(0+10+20)\times 0.1 + (30+40+50+60)\times 0.2 + (70+80+90+100)\times 0.5}{0.1+0.1+0.1+0.2+0.2+0.2+0.2+0.5+0.5+0.5+0.5} = 67.4$$