

Using stub impedance — 2 mts view (TT)
20 mts final



NAME: _____ STD.: _____ SEC.: _____ ROLL NO.: _____ SUB.: _____

S. No.	Date	Title	Page No.	Teacher's Sign / Remarks
		Show for RW dominant mode of propag" is TE ₁₀ and write down values of cutoff freq & cut-off wavelength at dominant mode		
Q.)		Design single parallel, stub with $Z_L = 35 - j47.5 \Omega$ $Z_0 = 50\Omega$	OPEN	
Q.)		Design single series open stub with $Z_L = 90 + j70 \Omega$, $Z_0 = 50\Omega$		
Q.)		Design short parallel stub $Z_L = 200 + j300 \Omega$ $Z_0 = 300\Omega$		
Q.)		parallel open $Z_L = 150 + j200 \Omega$, $Z_0 = 75\Omega$		
Q.)		short series stub, $Z_L = 35 \pm j50 \Omega$, $Z_0 = 50\Omega$		
		Single stub - 5 mts		
		2 mts		
		- why waveguide does not support TEM - Waveguide excitation - Draw TE, TM patterns - Deriv" of TE, TM eq"s or numerically - Stub comparison (single, double, triple) - Compare rect. & circular waveguides - App's of microwave components - E-plane, H-plane, magic tee - deriv & expln		

Adv of Microwave:

- Very high BW as freq ↑ so guard bands possible
- with inc. in freq. directivity ↑ so security ↑
- size of components ↓ as freq ↑

Limitations:

- Sophisticated equipment is required
- High cost
- Atmospheric losses are high

App's:

- mobile phones
- GPS systems

$$\nabla \cdot \vec{E} = \frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z}$$

$$\nabla \times \vec{E} = \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ E_x & E_y & E_z \end{vmatrix}$$

$$\vec{P} = \vec{E} \times \vec{H}$$

$$\vec{E} = E_x \hat{a}_x + E_y \hat{a}_y + E_z \hat{a}_z$$

$$\nabla^2 \vec{E} = \gamma^2 \vec{E} \quad \text{— Hertzian Eq'}$$

$$\gamma = \alpha + j\beta$$

Attenuation constant $\frac{2\pi}{\beta}$

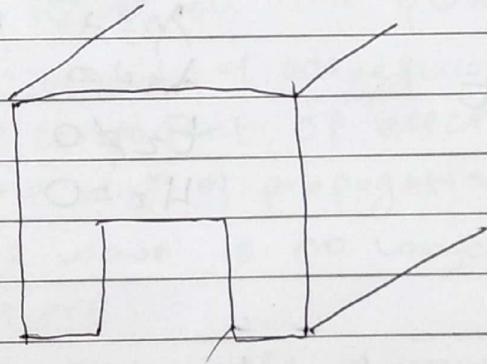
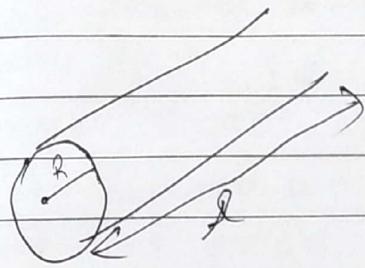
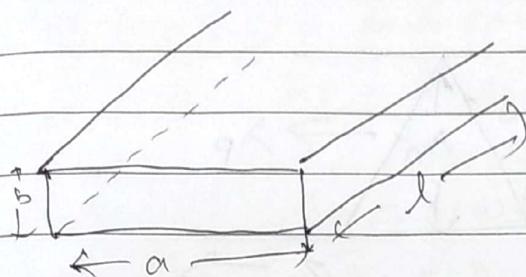
In TL ϕ_{vs} is the thin wave range the two suffer from cross talk
so we use waveguide

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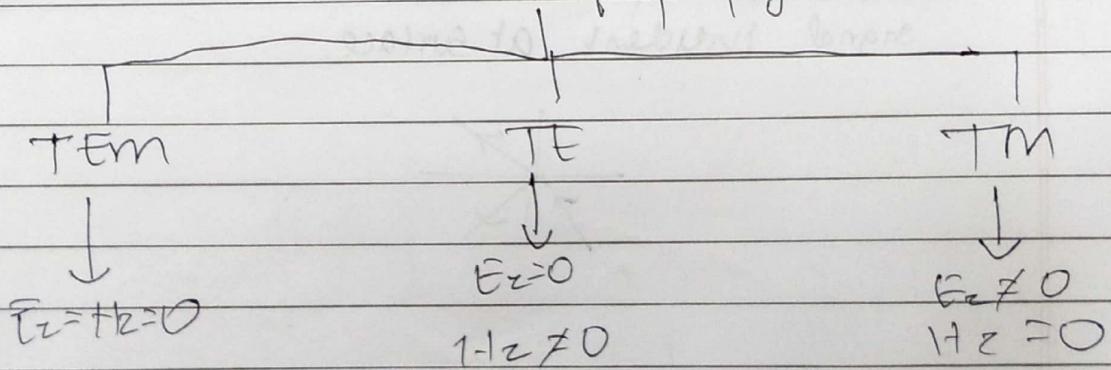
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* Waveguide:



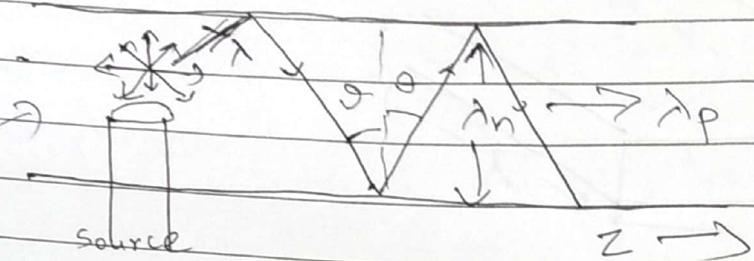
mode of propagation



(Assume 'z' as dirⁿ of propagⁿ)

Wave propagation through waveguide.

Cross section of waveguide



Signal enters waveguide to waveguide will get reflected by inner wall of waveguide another signal will come out

$$\lambda_p = \frac{\lambda}{\sin \theta}$$

$$\lambda_n = \frac{\lambda}{\cos \theta}$$

$$\theta = 0^\circ$$

$$\lambda_p = \infty$$

$$\lambda_n = \lambda$$

$$E_z = 0$$

$$H_z \neq 0$$

TE

$$\theta = 90^\circ$$

$$\lambda_p = \lambda$$

$$\lambda_n = \infty$$

$$E_z \neq 0$$

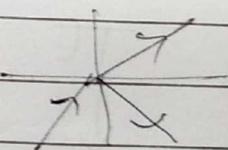
$$H_z = 0$$

TM

losses due to:

→ the conductive surfaces that absorb parallel and \perp components.

→ due to impedance mismatch not all the signal incident at surface.



- When signal connected to coaxial probe it will radiate in all 360° . Let's resolve the incident signal λ in two components : i) λ_p and parallel to walls of waveguide
$$\lambda_p = \lambda / \sin \theta$$
 where θ is angle of incidence
ii) λ_n perpendicular to dirⁿ of propogⁿ
$$\lambda_n = \lambda / \cos \theta$$
- When $\theta = 0^\circ$, $\Rightarrow \lambda_n = \lambda$, $\Rightarrow \lambda_p = \infty$, this indicates that normal component exist and parallel component doesn't exist i.e. $E_z = 0, H_z \neq 0$ i.e. T.E. mode
- When $\theta = 90^\circ$, $\Rightarrow \lambda_n = \infty$, $\Rightarrow \lambda_p = \lambda$, this indicates that normal component does not exist and parallel component exist i.e. $E_z \neq 0, H_z = 0$ i.e. TM mode
- Thus, when wave travel longitudinally down the waveguide the plane waves are reflected from walls of waveguide. This process result in a component of electric or magnetic field in the dirⁿ of propagation.
∴ The wave is no longer transverse electro-magnetic nature.
- Thus, any shape of waveguide supports T.M. or T.E. mode of propagation.

for TE mode eq's:

- ① start with $\nabla \times \vec{E} = -j\omega \vec{H}$
 $\nabla \times \vec{H} = j\omega \vec{E}$

resolve from determinants

- ② Boundary cond's are

- i) $E_x = 0$ at $y=0$
- ii) $E_y = 0$ at $x=a$
- iii) $E_x = 0$ at $y=b$
- iv) $E_y = 0$ at $x=0$

- ③ $H_z = (A \sin(k_n x) + B \cos(k_n x)) (C \sin(k_y y) + D \cos(k_y y)) e^{jBz}$

wing i.)

$$\frac{\partial H_z}{\partial y} = 0 \Big|_{y=0} \quad \text{for } E_x \text{ to be zero}$$

for TM mode
we have
boundary
conditions.

$$\text{so } [A(+) + B(-)] [C(-) - D(+)] = 0$$

$$\therefore C = 0$$

wing ii.)

directly we have
we have
boundary
conditions.

$$\frac{\partial H_z}{\partial x} = 0 \Big|_{x=0} \quad \text{for } E_y \text{ to be zero}$$

$$\text{so } [A(+) + B(-)] [C(+) - D(+)] = 0$$

$$\therefore A = 0$$

$$\text{so } H_z = B D \cos(k_n x) \cos(k_y y)$$

(4) using eq's from ① find E_x, E_y, H_x, H_y

⑤

wing iii.)

$$\frac{\partial H_z}{\partial y} = 0 \Big|_{y=b} \quad \text{for } E_x = 0$$

and $x=0$

$$\text{so get } B = -D \cos(k_n x) \sin(k_y b)$$

$$k_y b = n\pi$$

wing iv.)

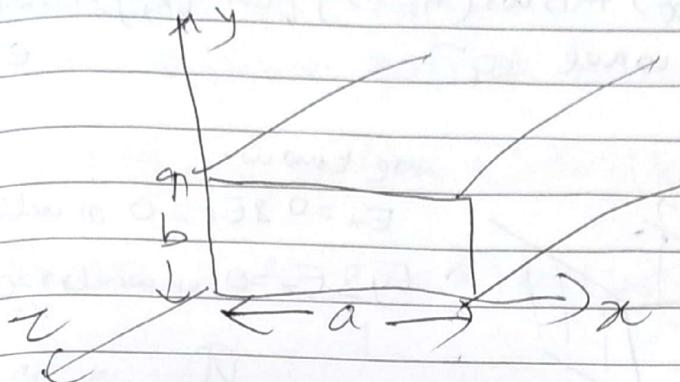
$$\frac{\partial H_z}{\partial x} = 0 \Big|_{x=a} \quad \text{for } E_y = 0$$

and $y=0$

$$\therefore k_y a = m\pi$$

TE mode:

Consider a rectangular waveguide



We know, $\nabla \times \vec{E} = -j\omega \mu \vec{H}$ & $\nabla \times \vec{H} = j\omega \epsilon \vec{E}$

In determinant form:

$$\begin{vmatrix} \text{an} & \text{ay} & \text{az} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ E_x & E_y & E_z \end{vmatrix} = -j\omega \mu \vec{H}$$

$$\begin{vmatrix} \text{an} & \text{ay} & \text{az} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ H_x & H_y & H_z \end{vmatrix} = j\omega \epsilon \vec{E}$$

We replace $\frac{\partial}{\partial z}$ with the propagation constant γ , $\gamma = \alpha + j\beta_g$ ($\alpha = 0$ for lossless waveguide)

$$\gamma = -j\beta_s$$

From determinant,

$$\frac{\partial}{\partial y} E_z + j\beta_g E_y = -j\omega \mu H_x$$

$$-\frac{\partial}{\partial x} E_z - j\beta_g E_x = -j\omega \mu H_y$$

$$\frac{\partial}{\partial x} E_y - \frac{\partial}{\partial y} E_x = -j\omega \mu H_z$$

$$\frac{\partial}{\partial y} H_z + j\beta_g E_y = j\omega \epsilon E_n$$

$$-\frac{\partial}{\partial x} H_z - j\beta_g H_x = j\omega \epsilon E_y$$

$$\frac{\partial}{\partial x} H_y - \frac{\partial}{\partial y} H_x = j\omega \epsilon E_z$$

$$H_x = -\frac{j\beta_3}{k_c^2} \frac{\partial H_z}{\partial x}$$

$$H_y = -\frac{j\beta_3}{k_c^2} \frac{\partial H_z}{\partial y}$$

$$\frac{\partial}{\partial z} = \gamma_g = \alpha_g + j\beta_g \quad \text{prop. along } z \text{ direction}$$

$$H_z = [A \sin(k_x x) + B \cos(k_x x)] [C \sin(k_y y) + D \cos(k_y y)] e^{-j\beta_3 z}$$

↓ soln of wave eq'

$$E_x = 0 \text{ at } y=0$$

$$\because E_x > 0 \text{ requires } \frac{\partial H_z}{\partial y} > 0 \text{ at } y=0$$

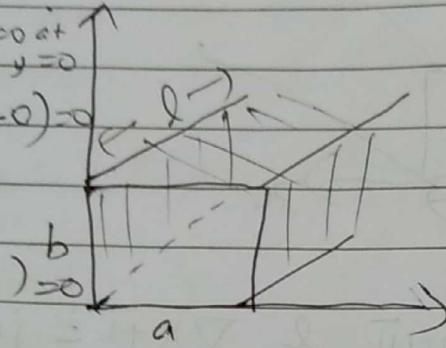
$$\frac{\partial H_z}{\partial y} \Big|_{y=0} = () (C+0) = 0$$

$$\therefore C = 0$$

$$\frac{\partial E_y}{\partial x} \geq 0 \text{ at } x=0$$

$$\frac{\partial H_z}{\partial x} \Big|_{x=0} = (A+0)(C) = 0$$

$$\therefore A = 0$$



we know

$$E_x = 0 \text{ & } E_z = 0 \text{ at wall } y=0$$

$$E_y \text{ & } E_z = 0 \text{ at walls } x=a$$

lets use 1st boundary,

$$x=y=0 \text{ (is a metallic wall)}$$

$$\therefore H_z = 0$$

$$\Rightarrow 0 = [A(0) + B] [C(0) + D] e^{-j\beta_3 z}$$

$$H_z = B \cos(k_x x) \cdot D \cos(k_y y) e^{-j\beta_3 z}$$

As B & D are constants, let represent the constant amplitude of magnetic field along z -direction.

$$H_{0z} = B \cdot D = C$$

$$H_z = H_{0z} \cos(k_x x) \cdot \cos(k_y y) e^{-j\beta_3 z}$$

$$E_x = -\frac{j\omega \mu}{k_c^2} \cdot \frac{\partial}{\partial y} H_z \quad \left[k_c = \sqrt{\omega \epsilon - \beta_3^2} \right]$$

$$E_x = \frac{j\omega \mu}{k_c^2} k_y H_{0z} \cos(k_x x) \sin(k_y y) e^{-j\beta_3 z}$$

(V) represent electric & magnetic field propagation
with rectangular waveguide

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Similarly put expression for H_z in F_y, H_x &
 H_y & solve partial differentiation

$$\rightarrow E_x = E_{ox} \cdot k_y \cos(k_x \cdot x) \sin(k_y \cdot y) e^{j\beta_y z}$$

$$\rightarrow E_y = E_{oy} \cdot k_x \sin(k_x \cdot x) \cos(k_y \cdot y) e^{-j\beta_y z}$$

$$E_z = 0$$

$$\rightarrow H_x = H_{ox} \cdot k_y \cdot \sin(k_x \cdot x) \cdot \cos(k_y \cdot y) \cdot e^{j\beta_y z}$$

$$\rightarrow H_y = H_{oy} \cdot k_y \cdot \cos(k_x \cdot x) \sin(k_y \cdot y) e^{-j\beta_y z}$$

$$\rightarrow H_z = H_{oz} \cdot \cos(k_x \cdot x) \cos(k_y \cdot y) e^{j\beta_y z}$$

To find k_x & k_y :

$$E_x = 0 \text{ at } x=0 \text{ & } y=b$$

$$0 = E_{ox} \cdot k_y \cdot 1 \cdot \sin(k_y \cdot b) e^{j\beta_y z}$$

$$k_y \sin(k_y \cdot b) = 0$$

$$k_y b \cdot \sin(k_y b) = 0$$

$$k_y b = n\pi$$

$$\boxed{k_y = \frac{n\pi}{b}}$$

$$n = 0, 1, 2, \dots$$

n represent no. of half wave variations
along dirⁿ of b .

→ Similarly if we consider horizontal wall then
 $E_y = 0$ at $x=a$ & $y=0$

$$0 = E_{oy} \cdot k_x \cdot \sin(k_x \cdot a) \cos(\omega t) e^{-j\beta_y z}$$

$$\sin(k_x \cdot a) = 0$$

$$\boxed{k_x = \frac{m\pi}{a}}$$

m represents no. of half wave
variations along dirⁿ of a .

Substitute k_x & k_y in eqⁿ (6)

(II)

JM model of propagation

$$E_x = E_{ox} \cos\left[\frac{m\pi}{a}x\right] \sin\left[\frac{n\pi}{b}y\right] e^{-j\beta_z z}$$

Similarly,

$$E_y = E_{oy} \sin\left[\frac{m\pi}{a}x\right] \cos\left[\frac{n\pi}{b}y\right] e^{-j\beta_z z}$$

$$E_z = E_{oz} \sin\left[\frac{m\pi}{a}x\right] \sin\left[\frac{n\pi}{b}y\right] e^{-j\beta_z z}$$

$$H_x = H_{ox} \sin\left[\frac{m\pi}{a}x\right] \cos\left[\frac{n\pi}{b}y\right] e^{-j\beta_z z}$$

$$H_y = H_{oy} \cos\left[\frac{m\pi}{a}x\right] \sin\left[\frac{n\pi}{b}y\right] e^{-j\beta_z z}$$

$$H_z = 0$$

→ Propagation constant (γ_g) & cut-off frequency (f_c):

it describes wave propagation through
waveguide along positive z direction.

$$\gamma_g^2 = \gamma^2 +$$

$$\gamma^2 = \gamma^2 + k_x^2 + k_y^2$$

γ propagation constant in free space

Let's replace $k_x^2 + k_y^2 = k_c^2$

$$\gamma_g^2 = \gamma^2 + k_c^2$$

We know that,

$$\gamma = \alpha + j\beta$$

FOR LOSSLESS Waveguide $\alpha = 0$

$$\gamma = j\beta$$

$$\gamma^2 = -\beta^2 = -\omega^2 \mu \epsilon$$

$$\therefore \gamma_g^2 = -\omega^2 \mu \epsilon + k_c^2$$

$$\gamma_g = \pm \sqrt{\omega^2 \mu \epsilon - k_c^2}$$

there are 3 possibilities for propg'n constant:

Possibility I: No propagation through waveguide.

$$\gamma_g = 0$$

$$0 = \omega^2 \mu \epsilon - k_c^2$$

$$\omega = k_c = \sqrt{\frac{k_x^2 + k_y^2}{\mu \epsilon}}$$

$$2\pi f = \sqrt{\frac{k_x^2 + k_y^2}{\mu \epsilon}}$$

PUTTING values of k_x & k_y

$$f_c = \frac{1}{2\pi\sqrt{\mu\epsilon}} \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2} \quad \text{--- (1)}$$

eqn (1) is cut-off freq for SW in T_{Emn} mode
which is lowest frequency at which propagation
of electric and magnetic field just starts

$$\beta^2 = k_c^2 - \omega^2$$

$$k_c = \omega \sqrt{\mu \epsilon}$$

$$k_c = \omega_c \sqrt{\mu \epsilon} = \sqrt{\left(\frac{\omega_c}{\alpha}\right)^2 + \left(\frac{\omega_c}{\beta}\right)^2}$$

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Possibility II: Wave propagates through媒質

Wave will propagate if $\gamma g > 0$

$$\omega^2 \mu \epsilon > k_c^2$$

$$\gamma g = \pm j \beta_3 = \pm j \omega \sqrt{\mu \epsilon} \sqrt{1 - \left(\frac{k_c}{f}\right)^2}$$

$$\therefore \beta g = \omega \sqrt{\mu \epsilon} \sqrt{1 - \left(\frac{k_c}{f}\right)^2}$$

Possibility III: Wave attenuation instead of propagation

If $\omega^2 \mu \epsilon < k_c^2$, γg becomes real quantity. As already we learn that real quantity of propog. constant is attenuation. This happens when $f < f_c$ when $f < f_c$

$$\gamma g = \pm \alpha g = \pm \omega \sqrt{\mu \epsilon} \sqrt{\left(\frac{k_c}{f}\right)^2 - 1}$$

* Phase velocity (V_g) for TEM₀₀ mode:

along the z dirn is through WG

$$V_g = \frac{\omega}{\beta_g} = \frac{\omega}{\omega\sqrt{\mu\epsilon}\sqrt{1-(\frac{f_c}{f})^2}}$$

Waveguide Impedance (Z_g)

$$Z_g = \frac{E_x}{H_y} = - \frac{E_y}{H_x}$$

put E_x & H_y from (I-1)

$$Z_g = \frac{\omega\mu}{\beta_g}$$

write β_g from ⑨

$$Z_g = \frac{\omega\mu}{\omega\sqrt{\mu\epsilon}\sqrt{1-(\frac{f_c}{f})^2}}$$

$$Z_g = \mu V_g$$

$$Z_g = \sqrt{\frac{\mu}{\epsilon}} \frac{1}{\sqrt{1-(\frac{f_c}{f})^2}}$$

$$Z_g = \frac{N_0}{\sqrt{1-(\frac{f_c}{f})^2}}$$

[intrinsic impedance]

$$N_0 = \sqrt{\frac{\mu}{\epsilon}}$$

* Guide wavelength (λ_g):

$$\lambda_g = \frac{2\pi}{\beta_g}$$

$$\lambda_0 = \frac{c}{f}$$

$$\lambda_g = \frac{2\pi}{\sqrt{\mu_0 \epsilon_0} \sqrt{1 - (\frac{f_c}{f})^2}}$$

frequency
of guide

Replace $\frac{2\pi}{\sqrt{\mu_0 \epsilon_0}}$ by λ

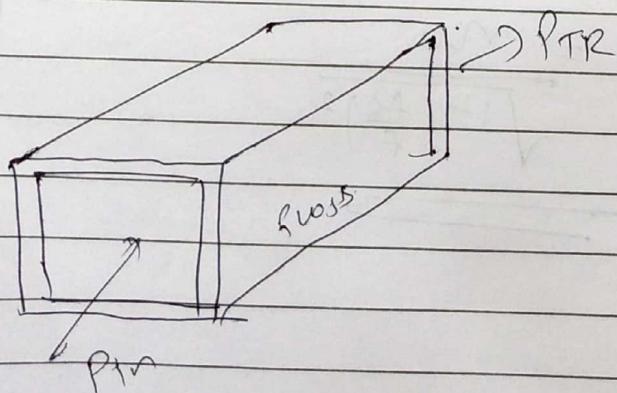
$$\lambda_g = \frac{\lambda_0}{\sqrt{1 - (\frac{f_c}{f})^2}} \quad (13)$$

expression (13) concludes λ_g is a fn
of guide dimensions a & b as f_c is
 a fn of a & b

$$\lambda_{gmn} = \frac{2ab}{\sqrt{m^2 b^2 + n^2 a^2}} = \frac{c}{f_c}$$

*more d &
a constant
a wavelet*

* Power Transmitted:



$$P_{in} = P_{TR} + P_{loss}$$

if we assume lossless WG

$$P_{in} = P_{TR}$$

Acc. to Poynting theorem

$$P_{in} = P_{TR} = \oint \frac{1}{2} (\mathbf{E} \times \mathbf{H}^*) dS$$

For lossless waveguide,

$$P_{TR} = \frac{1}{2 Z_g} \int_0^a |E|^2 da = \frac{Z_g}{2} \int_0^a |H|^2 da$$

$$Z_g = \frac{E_x}{H_y} = -\frac{E_y}{H_x}$$

$$|E|^2 = |E_x|^2 + |E_y|^2$$

$$|H|^2 = |H_x|^2 + |H_y|^2$$

$$P_{TRmn} = \frac{\sqrt{1 - (f_c/f)^2}}{2 n_0} \int_0^a \int_0^b |E|^2 dy dx$$

(14)

Various RW parameters for TM_{mn} mode of propagation:

• Propagation constant

$$\beta_g = \omega / \mu \epsilon \sqrt{1 - \left(\frac{f_c}{f}\right)^2} \quad (15)$$

• Cut-off frequency

$$f_c = \frac{1}{0.774 \sqrt{\mu \epsilon}} \sqrt{\left(\frac{m\pi}{b}\right)^2 + \left(\frac{n\pi}{a}\right)^2} \quad (16)$$

• Guide wavelength:

$$\lambda_g = \frac{\lambda_0}{\sqrt{1 - \left(\frac{f_c}{f}\right)^2}} \quad (17)$$

$$\lambda_{gmn} = \frac{2ab}{\sqrt{m^2 b^2 + n^2 a^2}}$$

• Group velocity:

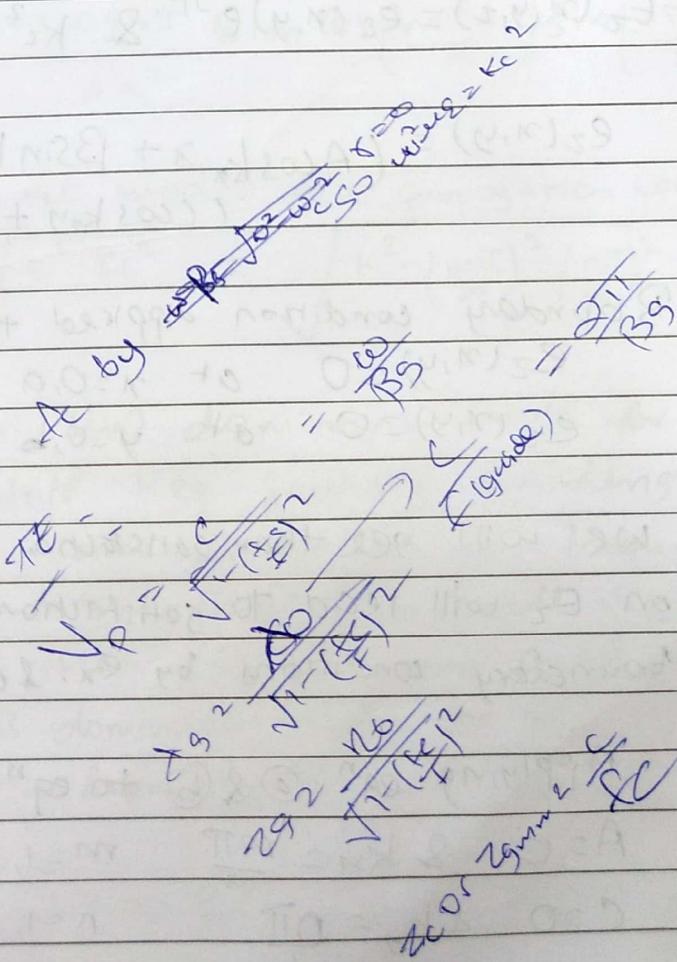
$$V_g = \frac{V_p}{\sqrt{1 - \left(\frac{f_c}{f}\right)^2}} \quad (18)$$

• Wavelength impedance:

$$Z_g = \frac{\beta_g}{\omega \mu} = \eta_0 \sqrt{1 - \left(\frac{f_c}{f}\right)^2} \quad (19)$$

- Power transmitted.

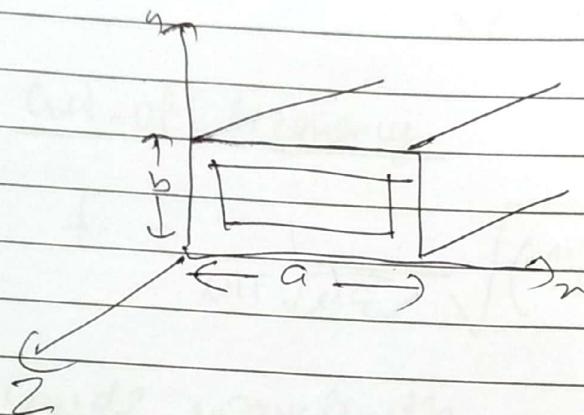
$$P_{TR} = \frac{\eta_0}{2\sqrt{1-(E_F)^2}} \int_0^a \int_0^b [E_x^2 + E_y^2] dx dy \quad (20)$$



(Q.1.) Analysis of RW in TM mode of propagation.

⇒ TM modes are characterized by fields with
 $H_z = 0, E_z \neq 0$

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + k_c^2 \right) E_z(x, y) = 0$$



$$E_z(x, y, z) = e_z(x, y) e^{j\beta z} \quad \text{and} \quad k_c^2 = k^2 - \beta^2$$

$$e_z(x, y) = (A \cos k_x x + B \sin k_x x) \cdot (C \cos k_y y + D \sin k_y y)$$

Boundary condition applied to e_z :

$$e_z(x, y) = 0 \quad \text{at} \quad x=0, a \quad \text{--- (a)}$$

$$e_z(x, y) = 0 \quad \text{at} \quad y=0, b \quad \text{--- (b)}$$

We will see that satisfaction of these conditions on E_z will lead to satisfaction of the boundary conditions by E_x & E_y .

Applying eqn (a) & (b) to eqn (2);

$$A=0 \quad \text{and} \quad k_x = \frac{m\pi}{a} \quad m=1, 2, 3, \dots$$

$$C=0 \quad \text{and} \quad k_y = \frac{n\pi}{b} \quad n=1, 2, 3, \dots$$

Sol" for E_z reduces to:

$$E_z(x, y, z) = B_{mn} \sin\left(\frac{m\pi x}{a}\right) \sin\left(\frac{n\pi y}{b}\right) e^{-j\beta z}$$

B_{mn} - arbitrary amplitude constant

The transversal field components for TM_{mn} mode can be computed as:

~~$$Ex = -j \frac{\beta m\pi}{ak_c^2} B_{mn} \cos\left(\frac{m\pi x}{a}\right) \sin\left(\frac{n\pi y}{b}\right) e^{-j\beta z}$$~~

~~$$Ey = -j \frac{\beta n\pi}{bk_c^2} B_{mn} \sin\left(\frac{m\pi x}{a}\right) \cos\left(\frac{n\pi y}{b}\right) e^{-j\beta z}$$~~

~~$$Hx = j \frac{\omega_0 \epsilon n\pi}{bk_c^2} B_{mn} \sin\left(\frac{m\pi x}{a}\right) \cos\left(\frac{n\pi y}{b}\right) e^{j\beta z}$$~~

~~$$Hy = -j \frac{\omega_0 \epsilon m\pi}{ak_c^2} B_{mn} \cos\left(\frac{m\pi x}{a}\right) \sin\left(\frac{n\pi y}{b}\right) e^{j\beta z}$$~~

As for TE modes the propagation constant is,

$$\beta = \sqrt{k^2 - k_c^2} = \sqrt{k^2 - \left(\frac{m\pi}{a}\right)^2 - \left(\frac{n\pi}{b}\right)^2}$$

Q.2) Show for RW dominant mode for propagation is TE₁₀, cutoff freq. & cut-off wavelength.

⇒ The mode with lowest cutoff frequency is dominant mode. Since TE₁₀ mode is minimum possible mode that gives non-zero field expression for RW, it is dominant mode for RW with $a > b$ & so dominant frequency is,

$$f_{c(10)} = \frac{1}{2a\sqrt{\epsilon}}$$

$$\text{Q) } f_{cmn} = \frac{k_c}{2\pi \sqrt{\epsilon_r}} = \frac{1}{2\pi \mu \epsilon_0} \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2}$$

- We observe TE₀₀ mode is min. possible mode that gives non-zero expression of E & H fields and delivers max power.

- For TE₁₀:

$$\text{cutoff wave no. } k_c = \pi/a$$

$$\text{propag'n constant } \beta = \sqrt{k^2 - (\pi/a)^2}$$

Notes:

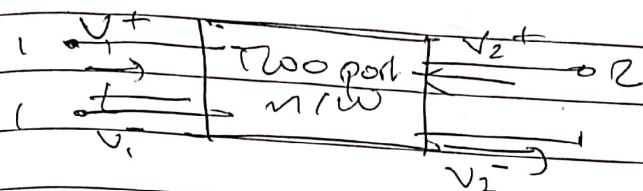
Dominant mode: The mode of RW which has lowest cutoff frequency, highest cutoff wavelength and delivers max. possible power.

For TE₀₀:



Scattering parameter:

- S parameters deal with waves that are directional in nature. They are a ratio of reflected wave at one port to incident wave provided the remaining ports are perfectly matched.



$$- |S_{11}| = \frac{V_1^-}{V_1^+} \text{ if } p_2 \text{ is perfectly matched}$$

↓ reflection coeff or p_1

$$- |S_{12}| = \frac{V_1^-}{V_2^+}$$

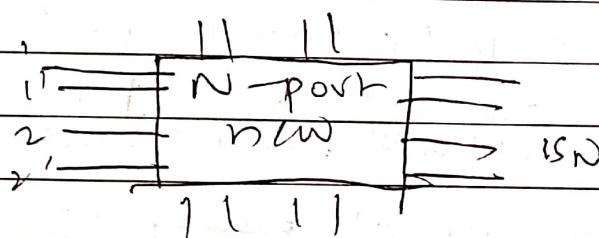
↓ reverse transmission coeff.

$$- |S_{21}| = \frac{V_2^-}{V_1^+}$$

↓ F/W transmission coeff.

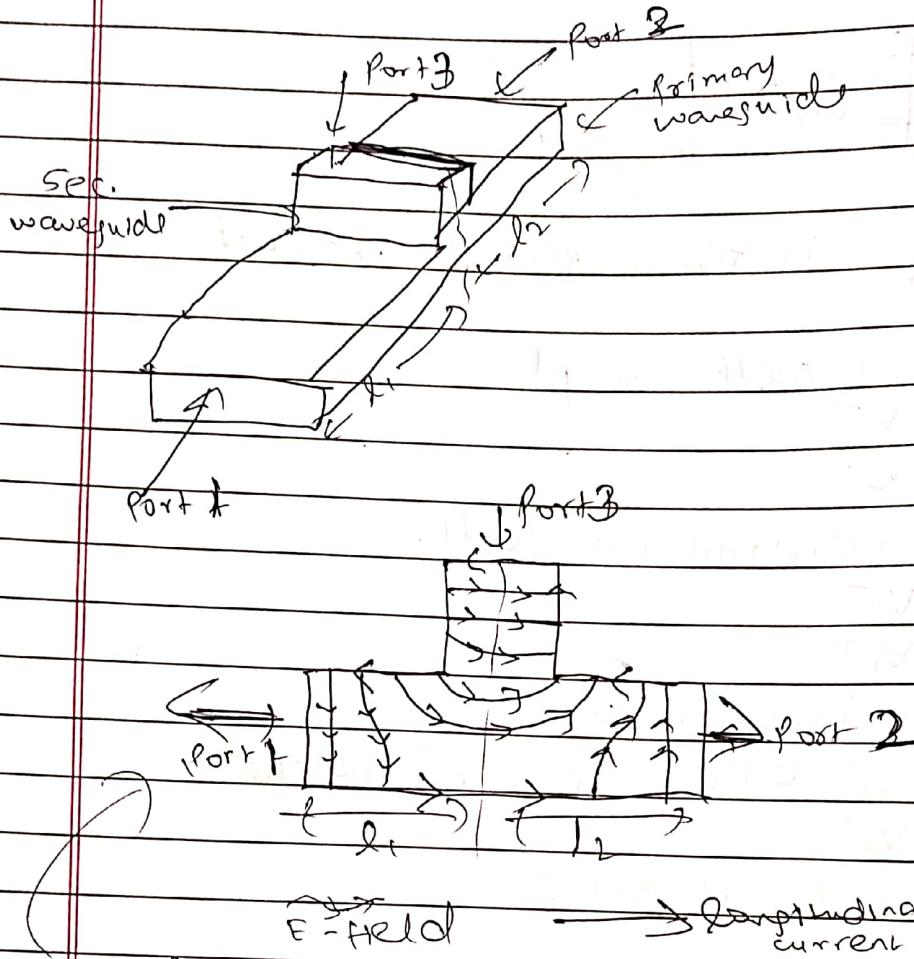
$$- |S_{22}| = \frac{V_2^-}{V_2^+} \text{ if } p_1 \text{ is perfectly matched}$$

↓ refl. coeff. at port 2



Waveguide Junction:

Eplane Tee:



- If energy is up through port 3 then half power available at port 1 & half at 2 but of opposite phase as they bend due to bending signal at p1 & p3 180° out of phase.
- If power is up at power port 2 and port 1 in phase & equal amp then zero power at port 3 as port 2 & 3 are themselves 180° out of phase.
- If power is up at port 2 & 180° out of phase & equal amp. then double power available at port 3.

Since 3 ports so scattering matrix will be 3×3

$$[S] = \begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{21} & S_{22} & S_{23} \\ S_{31} & S_{32} & S_{33} \end{bmatrix}$$

\rightarrow assume $d_1 = d_2$ & port-3 is perfectly matched i.e. $S_{33}=0$
 \rightarrow since $d_1=d_2$ & power at port 2 & 1 will be 180° out of phase w.r.t. port 3
 $S_{23} = -S_{13}$

\rightarrow From symmetry property:

$$S_{ij} = S_{ji} \text{ when } i \neq j$$

$$S_{12} = S_{21}$$

$$S_{13} = S_{31}$$

$$S_{23} = S_{32}$$

$$\therefore [S]_2 = \begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{12} & S_{22} & -S_{13} \\ S_{13} & -S_{13} & 0 \end{bmatrix}$$

\rightarrow Now, from unitary property

$$[S][S]^* = [I]$$

$$\left[\begin{array}{c} \\ \\ \end{array} \right] \left[\begin{array}{ccc} S_{11}^* & S_{12}^* & S_{13}^* \\ S_{12}^* & S_{22}^* & -S_{13}^* \\ S_{13}^* & -S_{13}^* & 0 \end{array} \right] = \left[\begin{array}{ccc} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{array} \right]$$

Let,

$$R_1 C_1 = |S_{11}|^2 + |S_{12}|^2 + |S_{13}|^2 = 1 \quad (1)$$

$$R_2 C_2 = |S_{12}|^2 + |S_{22}|^2 + |S_{13}|^2 = 1 \quad (2)$$

$$R_3 C_3 = |S_{13}|^2 + |S_{23}|^2 = 1 \quad (3)$$

$$R_3 C_1 = S_{13} S_{11}^* - S_{13} S_{12}^* = 0 \quad (4)$$

From (1) (2)

$$|S_{11}| = |S_{22}|$$

From (3)

$$\boxed{S_{13} = \frac{1}{\sqrt{2}}}$$

From (4)

$$S_{13} [S_{11}^* - S_{12}] = 0$$

$$\therefore S_{11}^* = S_{12}^*$$

$$\boxed{S_{11} = S_{12} = S_{22}}$$

From above we get,

$$|S_{11}|^2 + |S_{11}|^2 \neq \frac{1}{2} = 1$$

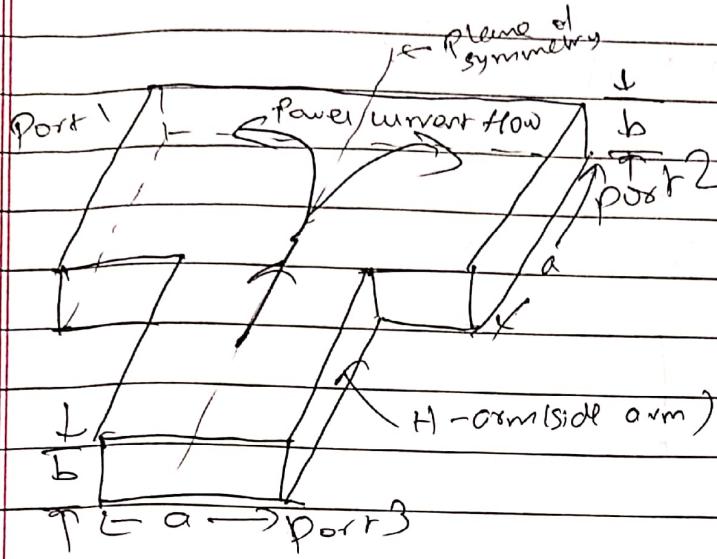
$$\underline{\underline{S_{11} = \frac{1}{2}}}$$

all $\times \frac{1}{\sqrt{2}}$
 $\times \sqrt{2}$

 $[S]$ matrix for E-plane TEE

$$[S] = \begin{bmatrix} 1/2 & 1/2 & 1/\sqrt{2} \\ 1/2 & 1/2 & 1/\sqrt{2} \\ 1/\sqrt{2} & -1/\sqrt{2} & 0 \end{bmatrix}$$

H-plane or Shunt Tee:



- If a TE mode incident at port 3, the electric field line is \perp to broad side so no bending & thus no reversal of phase occurs & thus equal & in phase signal available at p_1 & p_2
- If signals fed at p_1 & p_2 are in phase then they are added at p_3
- If signals fed at p_1 & p_2 are out of phase then they are cancelled at p_3

$$[S] = \begin{bmatrix} 1/2 & -1/2 & 1/\sqrt{2} \\ 1/2 & 1/2 & 1/\sqrt{2} \\ 1/\sqrt{2} & 1/\sqrt{2} & 0 \end{bmatrix}$$

Port 3: perfectly matched

$$S_{33} = 0$$

only S_{12}

power at port 1 & 2 will be equal & in phase

$$S_{13} = S_{23}$$

Due to symmetry,

$$S_{ij} = S_{ji} \quad i \neq j$$

$$[S][S]^* = I$$

$$\begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{12} & S_{22} & S_{23} \\ S_{13} & S_{23} & 0 \end{bmatrix} \begin{bmatrix} S_{11}^* & S_{12}^* & S_{13}^* \\ S_{12}^* & S_{22}^* & S_{23}^* \\ S_{13}^* & S_{23}^* & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$R_1 C_1 = |S_{11}|^2 + |S_{12}|^2 + |S_{13}|^2 = 1 \quad \text{--- (1)}$$

$$R_2 C_2 = |S_{12}|^2 + |S_{22}|^2 + |S_{13}|^2 = 1 \quad \text{--- (2)}$$

$$R_3 C_3 = |S_{13}|^2 + |S_{23}|^2 + 0 = 1$$

$$R_3 C_1 = S_{13} S_{11}^* + S_{13} S_{12}^* = 0$$

$$\therefore S_{11} = -S_{12} \quad \text{--- (3)}$$

$$S_{13} = \frac{1}{\sqrt{2}}$$

From eqn (1)

$$2|S_{11}|^2 + \frac{1}{2} = 1$$

$$S_{11} = \pm \frac{1}{2}$$

$$\therefore S_{12} = -S_{11} = \pm \frac{1}{2}$$

$$\text{Also, } S_{11} = S_{22}$$

$$\therefore S_{22} = 1$$

~~2~~

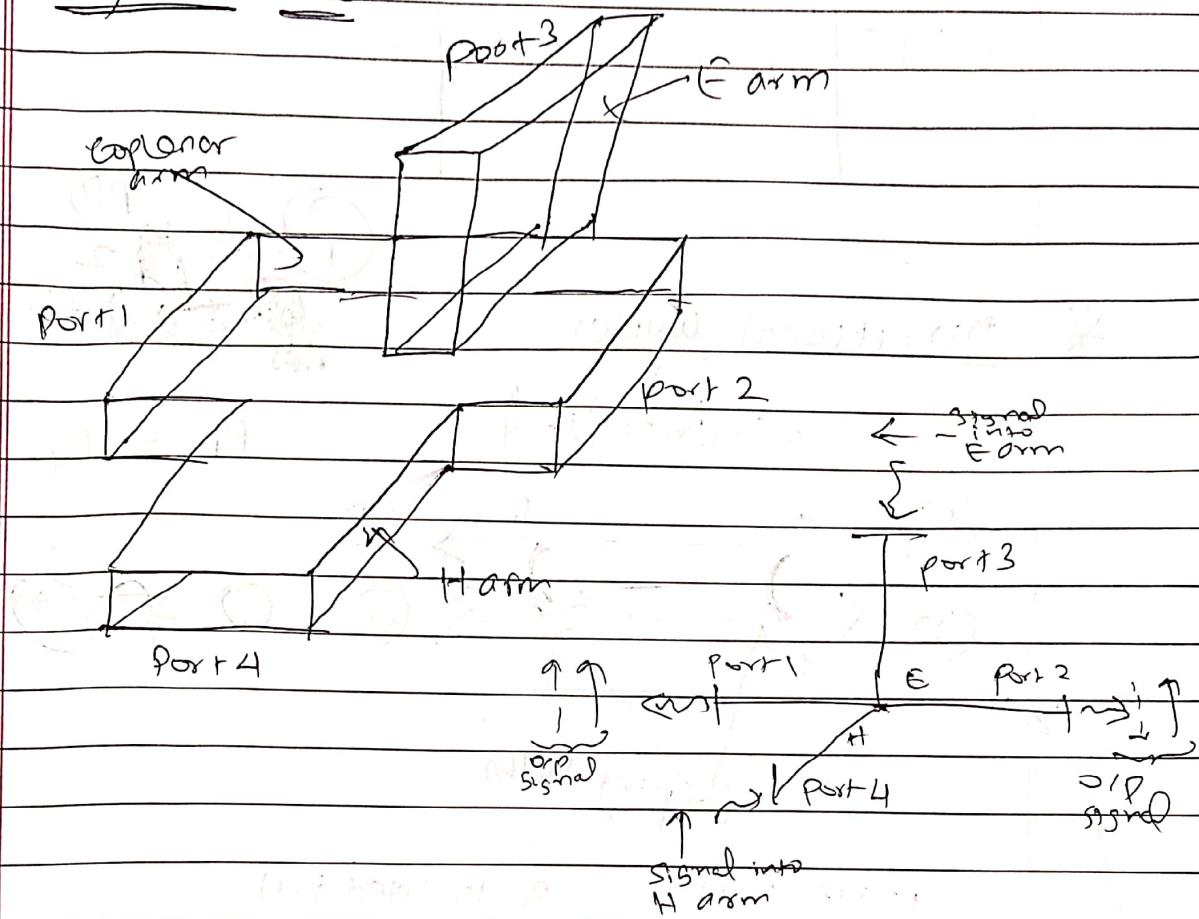
$$\therefore [S] = \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} & 1/\sqrt{2} \\ -1/\sqrt{2} & 1/\sqrt{2} & 1/\sqrt{2} \\ 1/\sqrt{2} & 1/\sqrt{2} & 0 \end{bmatrix}$$

~~[S]~~ ~~[S]~~

$$\therefore [S] = \begin{bmatrix} 1/2 & 1/2 & 1/\sqrt{2} \\ -1/2 & 1/2 & 1/\sqrt{2} \\ 1/\sqrt{2} & 1/\sqrt{2} & 0 \end{bmatrix}$$

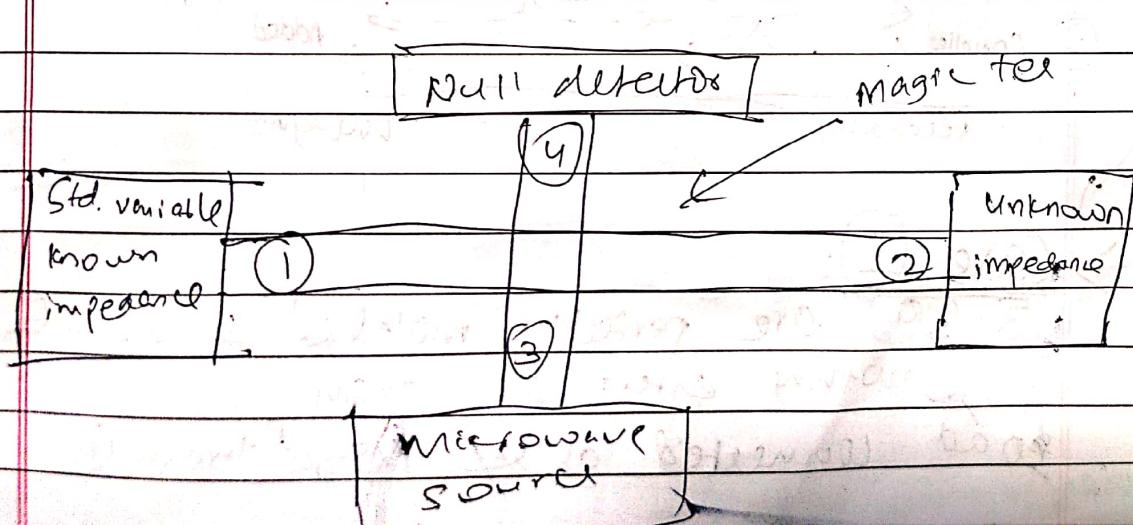
- If applied at p_3 then the signal divides & one of these goes to equal amp at p_1 & p_2 , p_4 uncoupled
- If applied at p_4 then signal at p_1 & p_2 equal & ~~CLASSMATE~~ ^{CLASSMATE} while p_3 uncoupled
- When inphase signals applied at p_1 & p_2 max signal at p_4 & min at p_3
- When out of phase signals applied at p_1 & p_2 max. signal at port 3 & min. at port 4

Hybrid Tee:

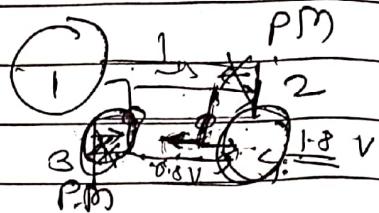


App's:

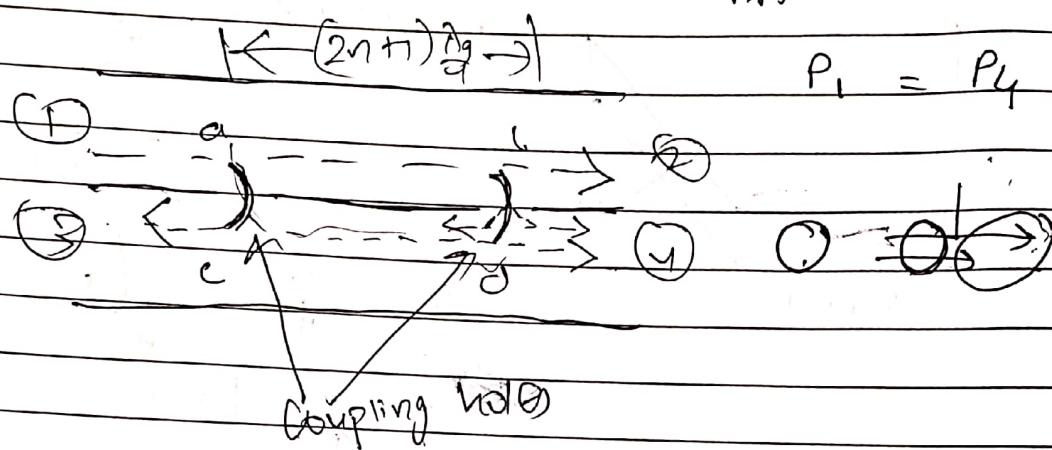
- E plane Tee
- H plane Tee
- Isolator (when port 1 & 2 matched so no refln there and if port 3 does not appear at 4 & vice versa)
- Unknown imp. measurement:



$$[S] = \begin{bmatrix} & \\ & \\ & \end{bmatrix}$$

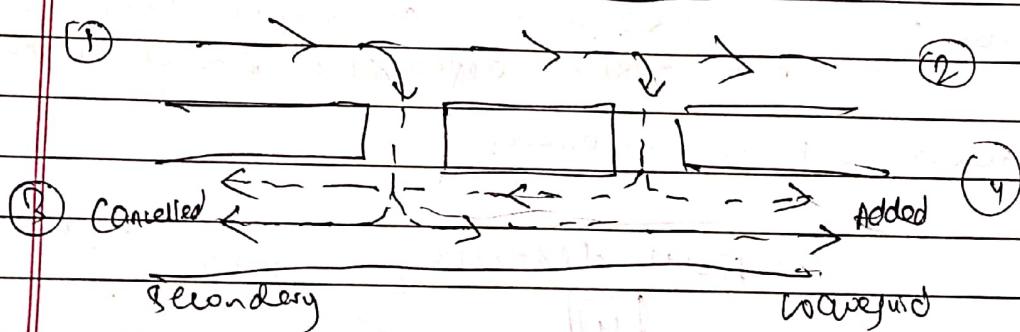


* Directional coupler:



1: input port 3: isolated port
2: direct port 4: coupled port

Primary $\xleftarrow{(2n+1)\frac{\lambda}{2}}$ waveguide



\Rightarrow Case ①:

③ & ④ are perfectly matched so no refl^n
& nothing enters from them

load connected at ② & signal through ① -

Case (1)



the path diff. b/w (1) & (2)
is $2(\frac{\lambda}{4})$ ~~cancel~~ $= 180^\circ$
out of phase
(~~opp~~ signals)
cancel each other.

Via hole signal will enter sec. waveguide & will flow to port (3) & some to port (4) & nothing comes back from them so nothing enters from holes from sec. to primary.

\therefore Some amt. of power loss occurs b/w IIP 80/p.

$$\text{Insertion loss} = 10 \log \left(\frac{P_1}{P_2} \right) \text{dB}$$

\rightarrow Case (2):



Port (2) & (3) are perfectly matched
iIP at (1), OIP taken at port (4)

both (1) & (2) travel same distance so path diff. = 0 & thus in phase & get added.

as (3) is perfectly matched so no refln can happen
nor from (2)

So energy reaching (4) will be (1) $\xrightarrow[\text{hole}]{} (4)$

+ (1) $\xrightarrow[\text{hole}]{} (4)$ & both in phase as in same dir

$$\text{Coupling coefficient} = 10 \log \left(\frac{P_1}{P_4} \right) \text{dB}$$

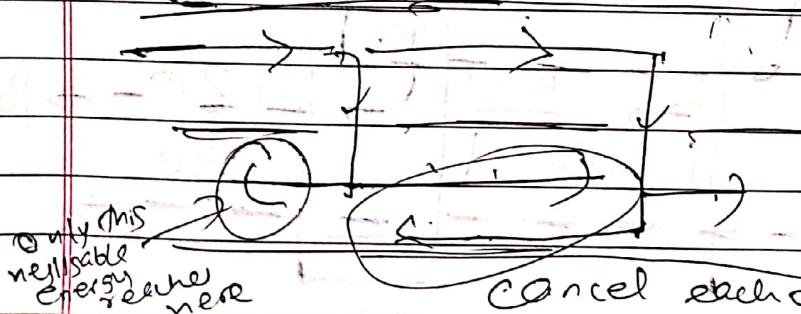
$P_1 \neq P_4$ as some energy goes to ports (2) & (3)

\rightarrow Case (3):

X Ports (2) & (1) are perfectly matched

iIP at port (1), OIP at port (3).

in theory no signal reaches port (3).



Only this
negligible
energy reaches here

through reflection
at port (3)
cancel each other as out of phase

SO appn,

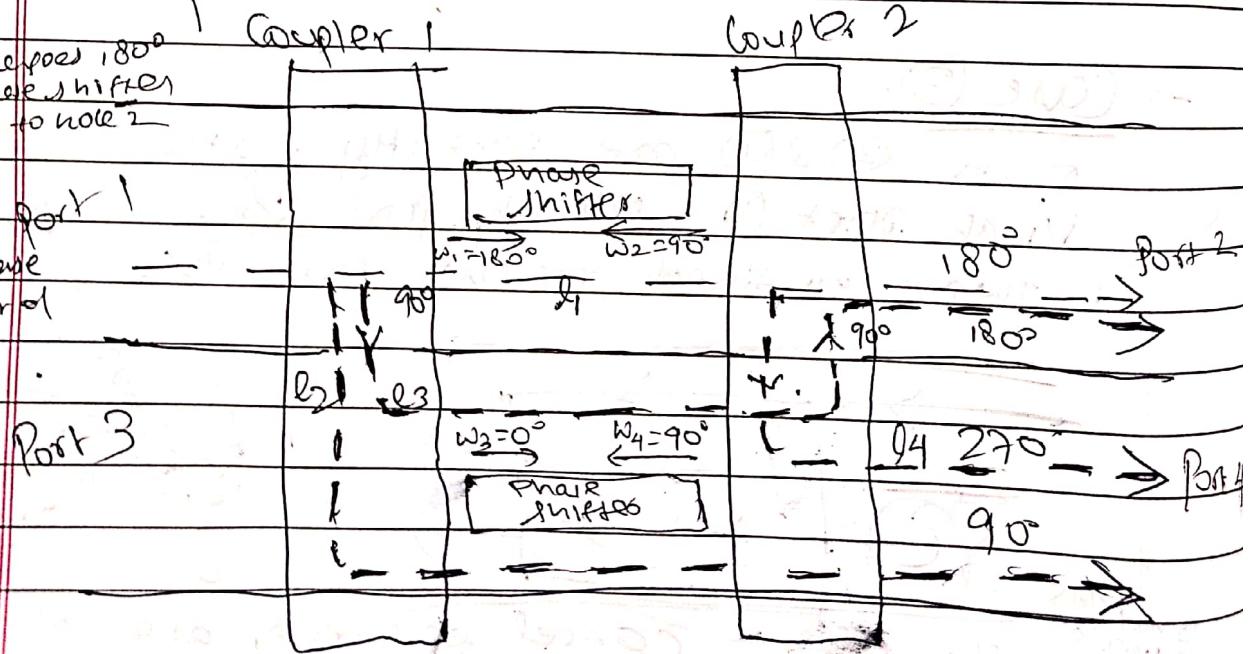
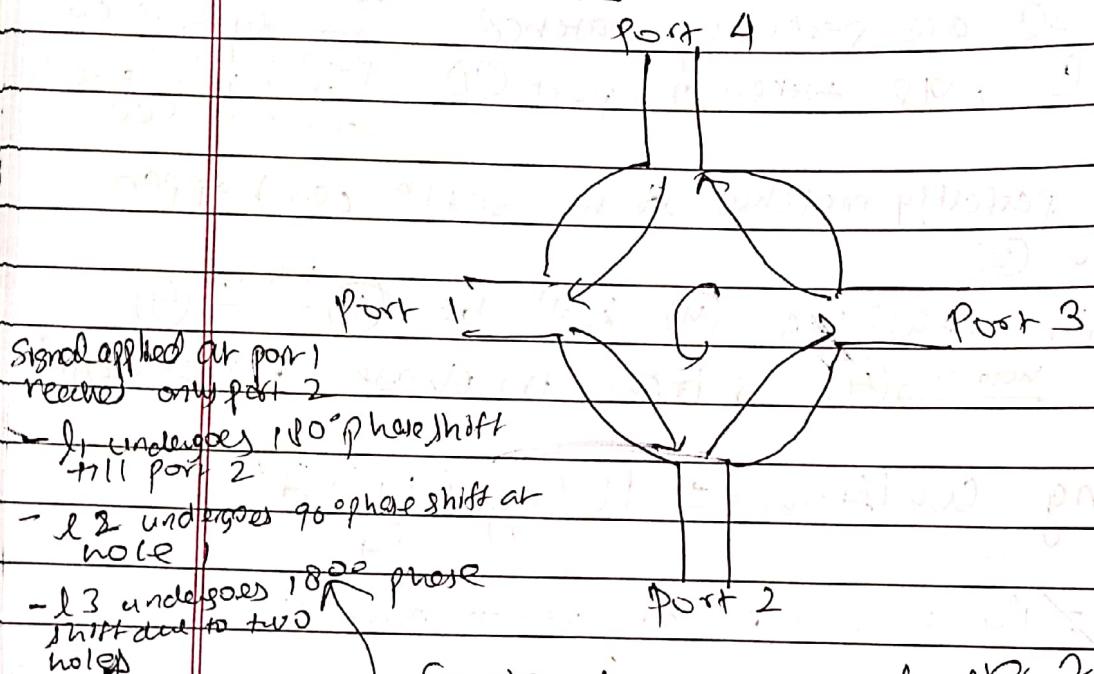
to get off as it is then case ①

- small amt. of coupled energy then case ②

• isolator used as in case ③.

∴ w.r.t. i/p port, the def'n of other ports are defined.

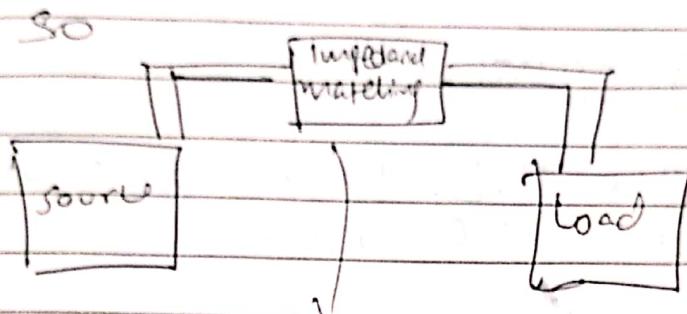
* Circulators:



*



$Z_S \neq Z_L$ so $P_L \neq P_{in}$



We use L-C Configur."

$$X_L = 2\pi f L \quad X_C = \frac{1}{2\pi f C} \quad \text{so we vary either } Z_L \text{ or } Z_C.$$

Impedance matching techniques:

- Wing Iris
- Wing stub
- Wing Quarter wave Transformer

→ Wing ~~source~~ Iris:

$$Z_g \propto a, b$$

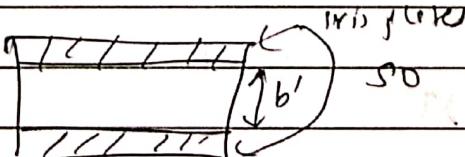
to make $Z_g = Z_L$ vary either a, b for Z_g

Iris is a metallic plate inserted along dim. a or b

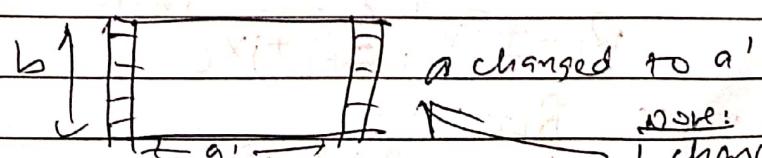
e.g.:

$$\frac{c}{\epsilon_0 A} = \frac{\sigma}{d}$$

~~charge due to plates~~
so Z_g changes
~~as ϵ & σ changes~~
~~thus Z_g is changing~~



so b changed to b'



a changed to a'

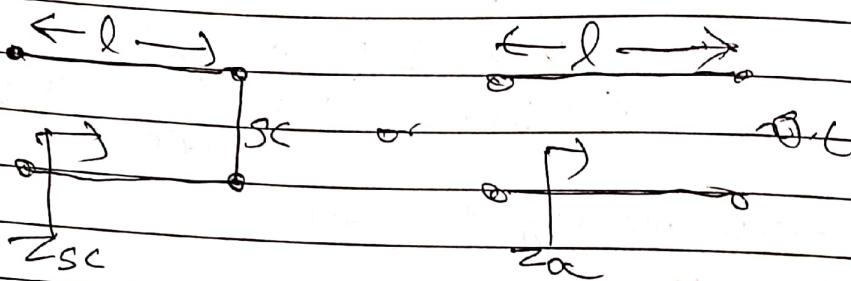
~~note: L changes due to plates
so X_L changes &
thus Z_g varied.~~

Note:

We don't we changing a or b as it changes mode of operation so we will rise along third dimension i.e. L .

Using Stub:

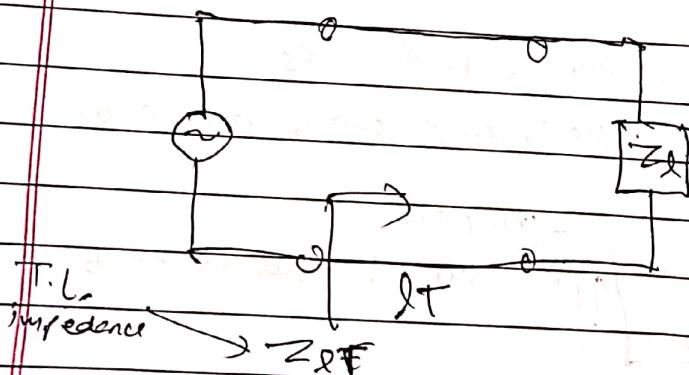
It is like T.L. but instead of load it is either O.C. or S.C.



$$Z_{sc} = jR_0 \tan(\beta l)$$

$$Z_{oc} = -jR_0 \cot(\beta l)$$

acts like +
inductor acts like
capacitor.

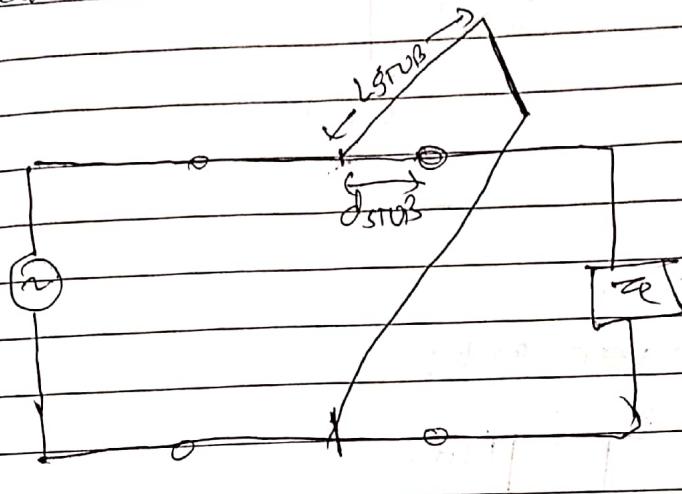


$Z_{LT} \neq Z_L$ so we stub to match them.
 $R_{LT} + jX_{LT} \neq R_L + jX_L$

1) To make $R_{LT} = R_L$

2) To make $jX_{LT} = ? jX_L \Rightarrow jX_{LT} = -jX_C$
 So that they cancel each other's effect

$\& jX_{LT} = -jX_L$
Now, to make $R_{LT} = R_L$, we try to use S.C. Stub
now question is where to connect stub & how to
connect w.r.t. load (series or parallel).

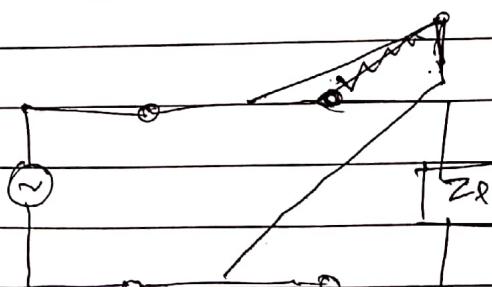


d_{STUB} = dist. of
stub from load
at which we
get matching,
by making real
part of imp. equal
& imm. part are
compensated.

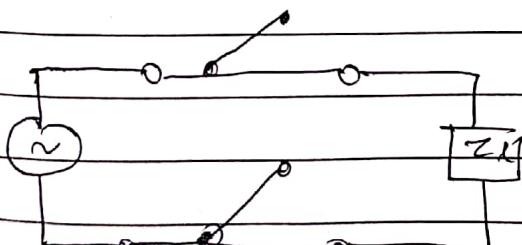
Start moving stub ends from load &
stop at dist. d_{STUB} when $Z_{LT} = Z_L$ as we
are moving Z_{LT} changes as it is dependent on length.
so at dist d_{STUB} we get imp. matching.

Four cases:

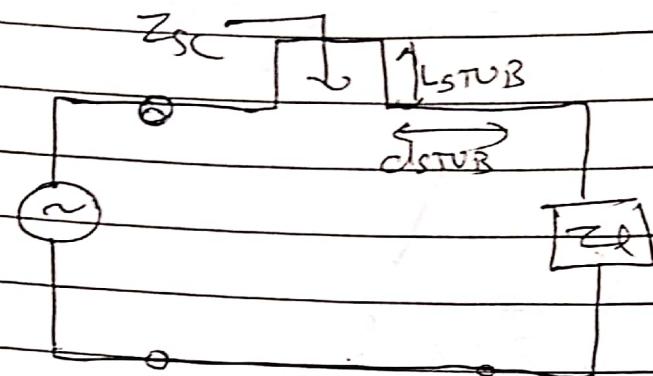
→ Shunt short circuited stub:



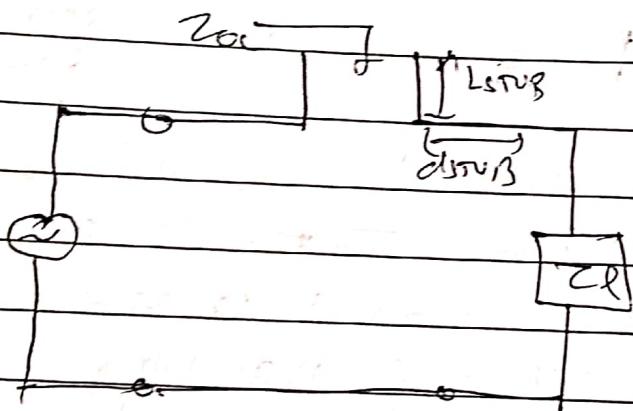
→ Shunt open stub:



→ Series Short stub:



→ series open stub:



~~If nothing given then we S.C. parallel stub.~~

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Q) Design single stub for sake of impedance matching for the load $Z_L = 15 + j10 \Omega$ whose char. imp. is 50Ω

\Rightarrow Note:

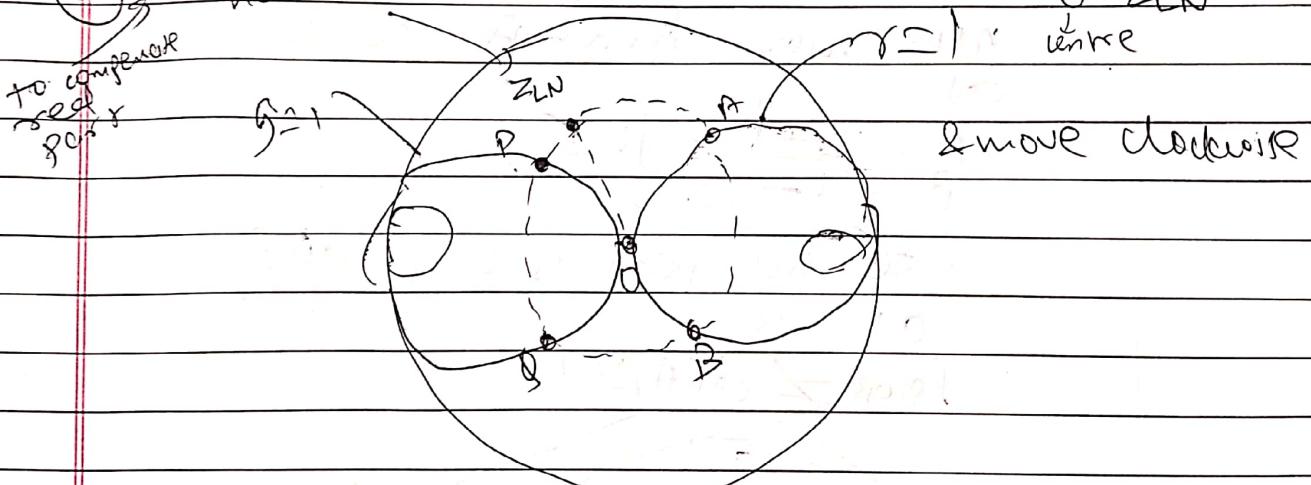
for series stub we Z chart

for parallel stub we Y chart

$$\textcircled{1} \quad Z_{LN} = \frac{15 + j10}{50} = 0.3 + j0.2$$

~~d_{stub}~~: now to make 0.3Ω to 1Ω so first draw a constant vswr circle with radius

$O - Z_{LN}$
centre



So at A, B, P, D real part becomes

so in clockwise dirⁿ $d_{stub} = d(Z_{LN}, A)$

~~or~~ $d(Z_{LN}, B)$

~~or~~ (d_{LN}, θ)

~~or~~ $d_{LN}, P)$

We take least distance b/w Z_{LN}

~~point on Z grid where $r=1$~~

so even if P was b/w Z_{LN} & A ^{so now A or B both}
we won't take $d_{stub} = d(Z_{LN}, P)$

Thus, $d_{stub} = d(Z_{LN}, A)$ but $d_{stub} \neq d(Z_{LN}, B)$
as at posⁿ B admitt $Z = 1 \oplus j0.2 \times$ ^{so have} $\frac{1}{A}$ to take

\therefore so if $z_{in} = x \pm iy$
 $d_{Stub} = d(z_{in}, \text{point})$
where point $= \frac{1}{x \pm iy}$

on z_{solid} not
 y_{solid}

(3) I_{Stub} : to compensate im. part.

now we get solⁿ point for
 I_{Stub} at A so start at
point A.

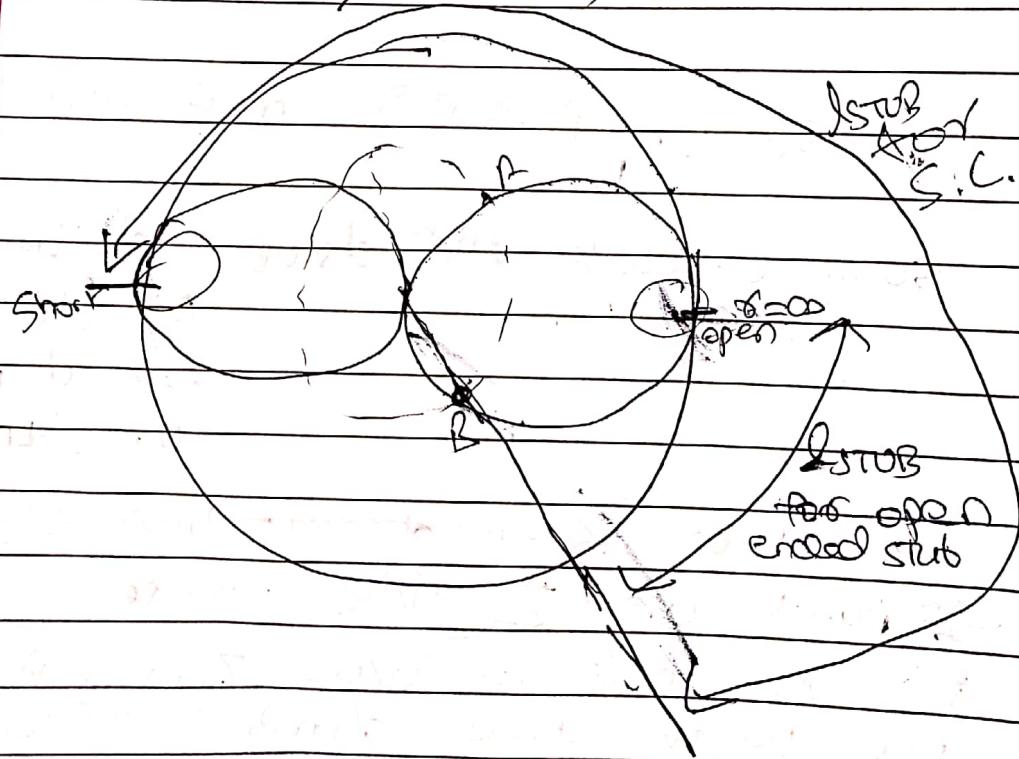
now move from A to B
(a)

$$A = \cancel{1 + j0.2}$$

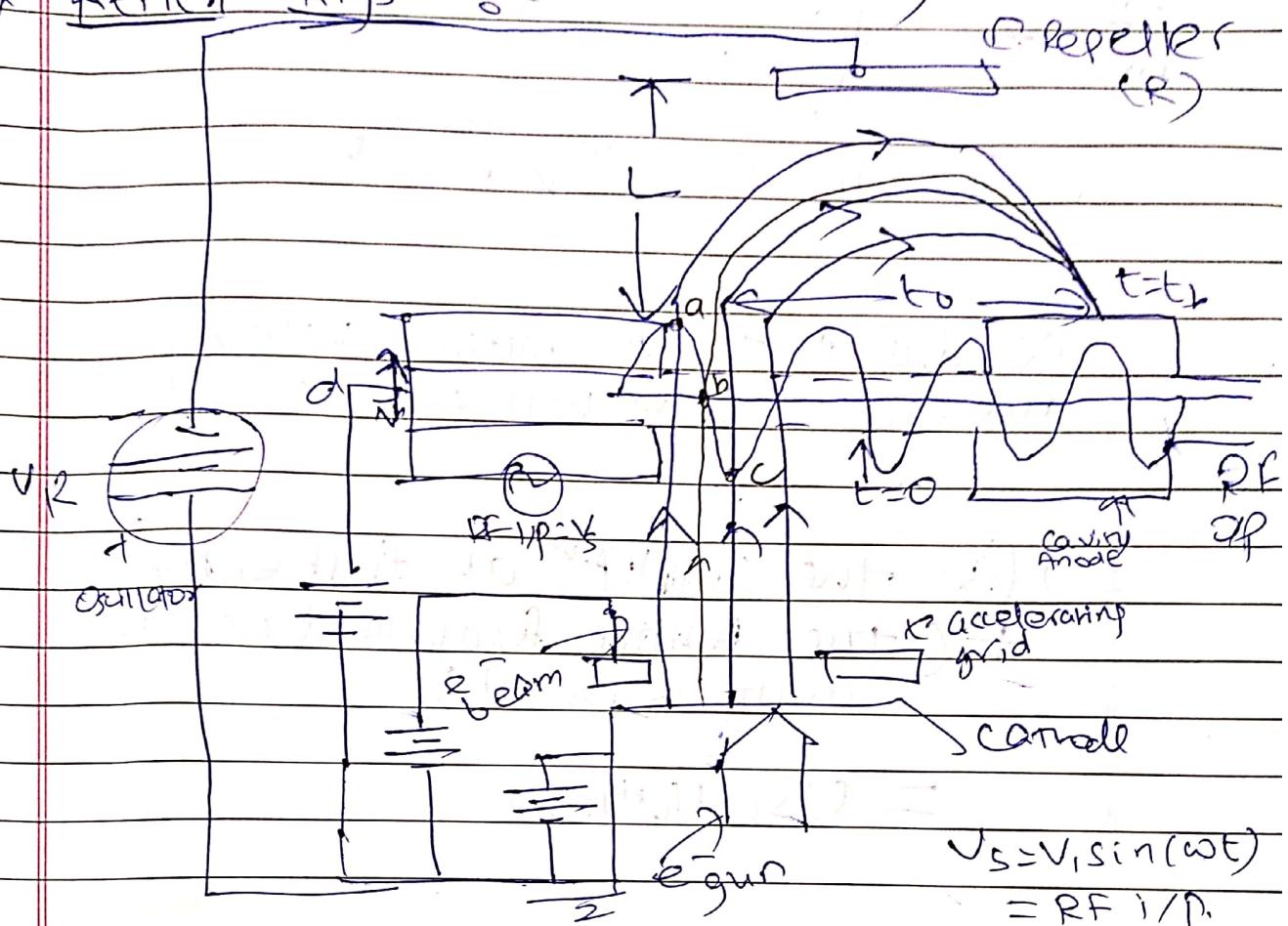
$$B = 1 - j0.2$$

so im. part is compensated
at B

$$d_{Stub} \neq d(A, B)$$



* Reflex Klystron (acts as oscillator)



- The e^- beam will be repelled by $-ve$ voltage at repeller & inside cavity will either come back in phase if add to velocity modulated signal in cavity & if out of phase then subtract with the velocity modulated signal in cavity.
- The phase of repelled signal will be decided by $-ve$ voltage V_r at repeller of length L .
- Accelerating grid to form bunch of e^- into an e^- beam
- $d \rightarrow$ dimension of reentrant cavity
- e-gun produces constant d.c. Electric field & a variable electric field inside cavity interior.

noise
or
variations
etc.
etc.

this will accelerate
the constant e-beam

no effect this will decelerate
the constant e-beam

so due to this interaction velocity
modulated beam will exit cavity.

\Rightarrow (So the amplif at first entry)
+ (the positive feedback back into
cavity (in phase)).

Oscillation

- as noise in cavity could be random so
four, four tough to determine.
- RF O/P will be in MHz
- so RF i/p will be kept in KHz
and will be kept sinusoidally periodic.

\Rightarrow PRF of Tuner

four

Tuning:

• Electronic tuning
more VR than

less VR than

mechanical tuning

change L

as $L \propto f^2$ so f
changes so four freq.
at which five f/G changes

→ Tuning:

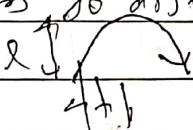
change PRF or four

• Electronic tuning:

- more VR then repulsion more & dist. travelled by beam less.



- less VR repulsion less so dist. more



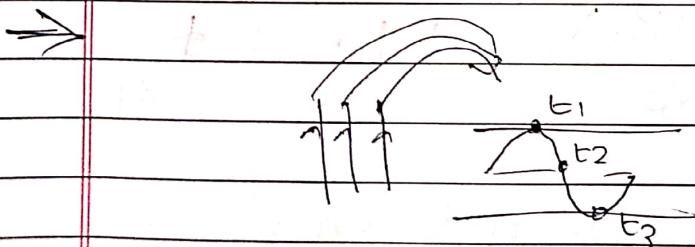
wavelled by beam
as dist. changes $\propto \frac{1}{f}$

so four changed & thus electronically tuned.

→ To change PRF: change the amp of e^- beam from e^- gun. & thus PRF will change.

• Mechanical Tuning:

changing length (L) of repeller & thus repulsion varies & thus as above four can be changed.



if e^- beam comes at t_1 then P_{max}

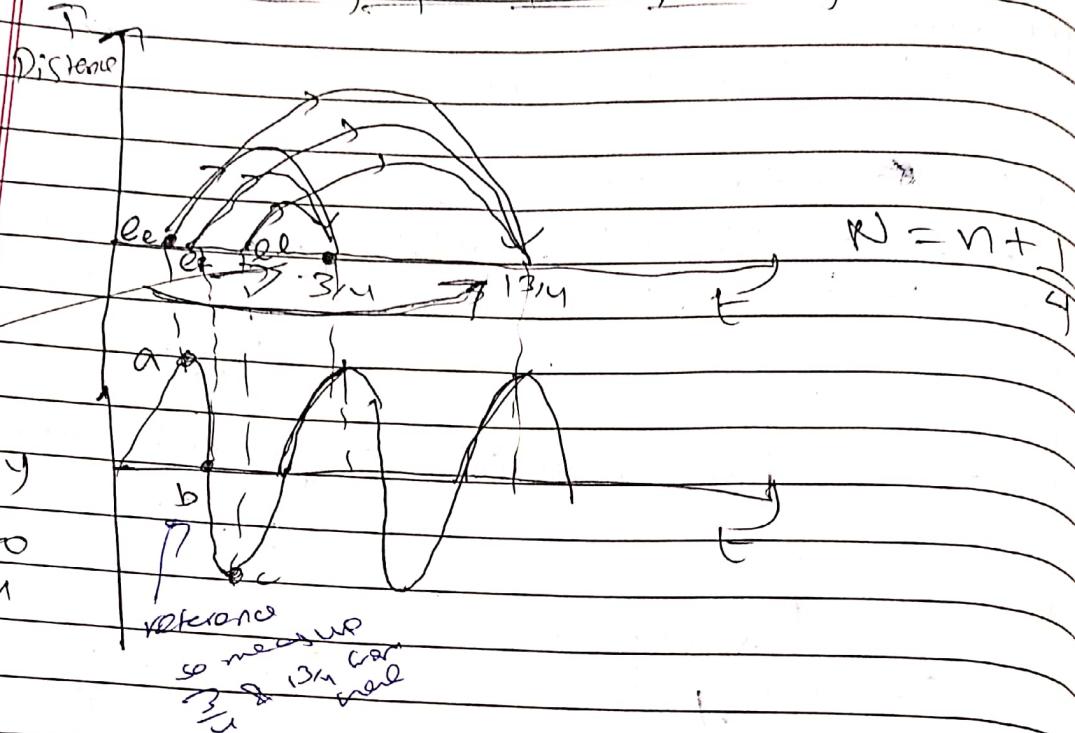
if e^- beam comes at t_3 then P_{min}

Conclusion: the e^- beam is allowed to come back inside coming at t_1 instant for max. power, this is bunching.

Theory: The bunched e⁻s can deliver more power to cavity at any instant corresponds to the peak of RF field. i.e. $t_0 = (n + \frac{3}{4})T$ where $n = 0, 1, 2, \dots$

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→ Aggregate graph for Reflex Klystron:



- So
- bunching
- done only
- at V_{RF}
- cycles to
- get more
- power

→

$$P_{OUT\ RF} = \frac{N_0 I_0 \pi^2 (X) [V_B + V_R]}{2 \pi f_{OUP} L}$$

$f_{OUP} \rightarrow$ O/P freq.

$N_0 \rightarrow$ beam (cathode voltage)

$V_R \rightarrow$ repeller voltage

$L \rightarrow$ length of drift space.

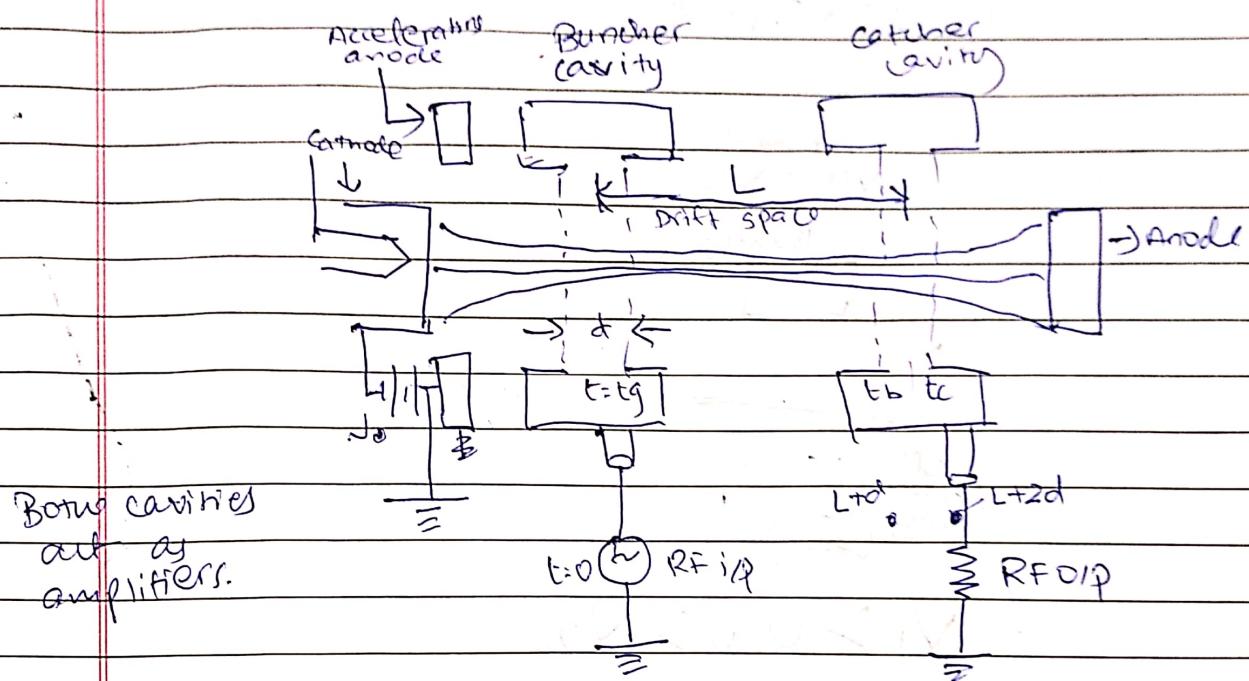
$X \rightarrow$ bunching coeff.

$$X = \pi N \beta, V \quad ; \quad N = n + 1$$

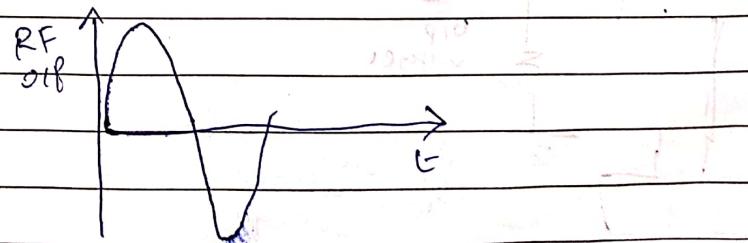
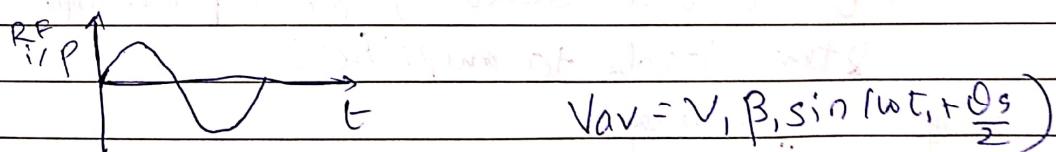
$V_1 \rightarrow$ peak amp. of RF voltage

$$\text{four} = \frac{[V_0 + V_R]N}{L \sqrt{V_0} 6.74 \times 10^{-2}} \text{ MHz}$$

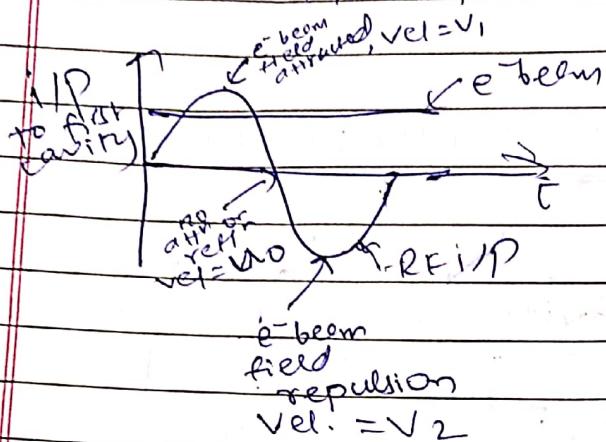
* ^{multi} Two cavity klystron (amplifier)



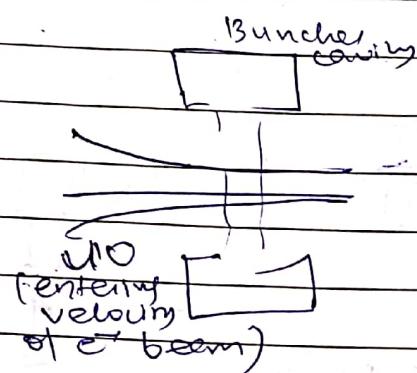
for buncher cavity, two rfs rfi p & beam o/p of this will travel distance L to catcher cavity where it gets amplified.



The e^- beam has a constant DC level V_0 .
The RF i/p is periodic



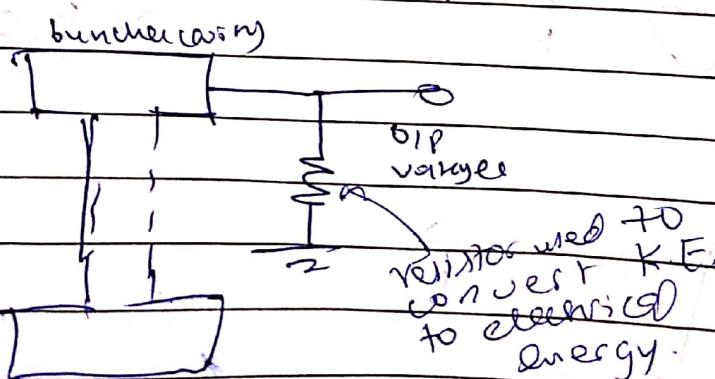
In each period the first e^- s are slowed down & the last e^- s are sped up symmetrically in bunching.



$$V_1 > U_0 > V_2$$

$$\therefore \frac{1}{2}mv_1^2 - \frac{1}{2}mu_0^2 = \frac{1}{2}mu_2^2$$

\therefore Velocity modⁿ occurs in first cavity on e^- beam caused by RF i/p. to be amplified, & thus leads to amplificⁿ.



- Role of d:

$$d = n\lambda$$

wavelength of RF i/p

$$n \leq 5$$

so as if $n > 5$ then resonant frequency very high

so e^- may dissipate and cause heating rather than amplif.

- Accelerating anode:

used to concentrate and bunch e^- s to a beam.

- Catcher cavity:

Two electric fields present, the constant DC level field due to original e^- beam &

The velocity modulated beam from first cavity.

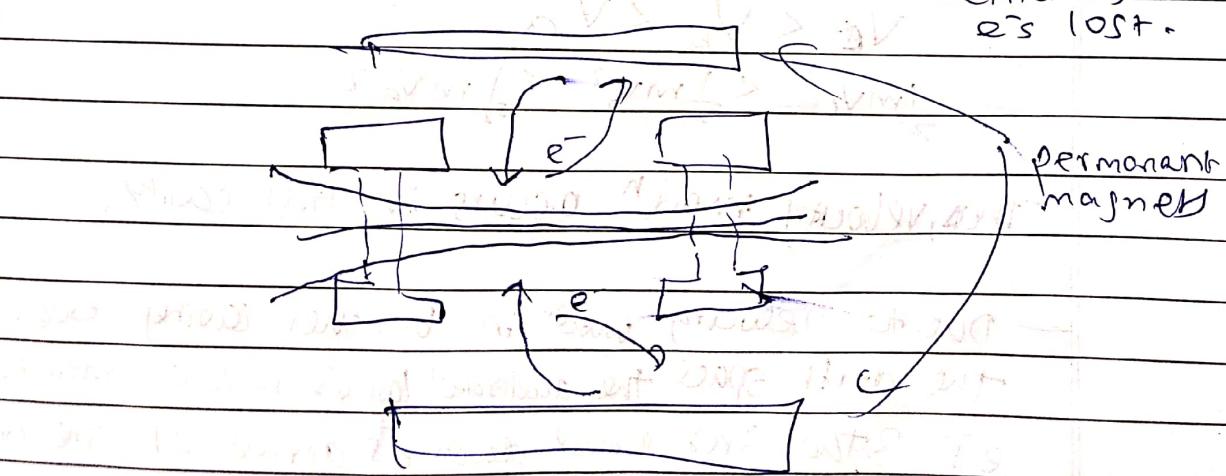
This acts as a second amplifier and as some intervals of first buncher cavity/amplifier.

- Anode: used to collect any escaping e^-

permanant magnets placed above & below cavities

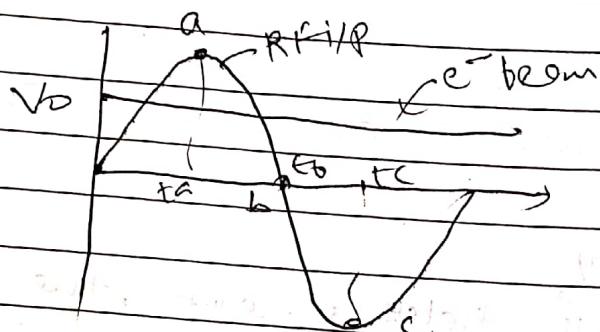
to return the escaping e^- s to beam. & helps

prevent heating / thermal runaway. & also inc. efficiency as no e^- s lost.



Opns:

- The e^- s are emitted in a continuous stream at cathode & accelerated and concentrated to a beam by accelerating anodes
- The buncher cavity has two i/p's : the RF i/p to be amplified & the e^- beam
RF i/p voltage is sinusoidally varying.



The e^- s in each period :

- the first e^- s are at $t = t_c$ are repelled by $-V_B$ RF voltage & thus slow down.
- the e^- s in middle at point $t = t_b$ are not affected.
- the e^- s entering last at $t = t_a$ are attracted by $+V_B$ RF voltage & thus accelerated & thus velocity increased.

$$V_B = V_0 \text{ (initial velocity)}$$

$$V_B < V_b < V_a$$

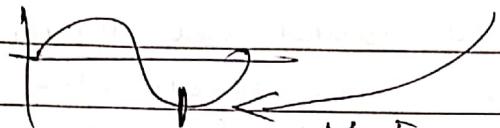
$$\frac{1}{2}mv_c^2 < \frac{1}{2}mv_b^2 < \frac{1}{2}mv_a^2$$

Thus, velocity modⁿ occurs in first cavity.

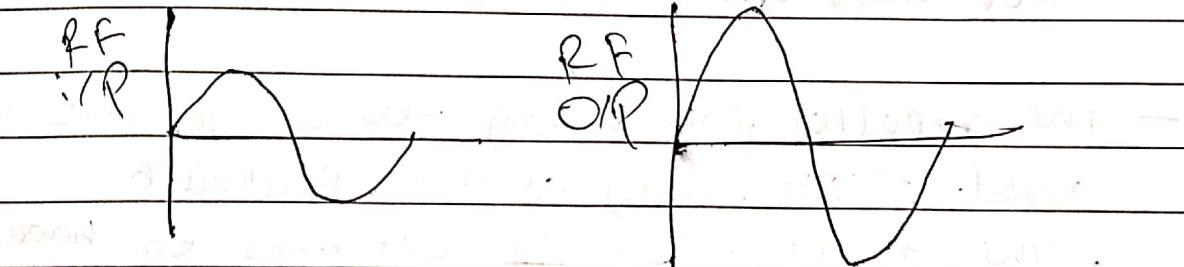
- Due to velocity modⁿ in buncher cavity over the drift space the accelerated last e^- s, middle unaffected e^- s & the first slowed down e^- s arrive at same time

at the catcher cavity in a bunch. This is called bunching effect.

- In the catcher cavity the density modulated wave is then slowed down by



thus loses its K.E. & this K.E. is used up to converted to P.E. & lead to amplification to produce amplified RF O/P



Thus the energy is conserved & the energy is increased due to bunching effect.

Thus the bunching effect produces higher energy & higher current. The bunching effect is done to get more energy & more current. The bunching effect is done to get more energy & more current.

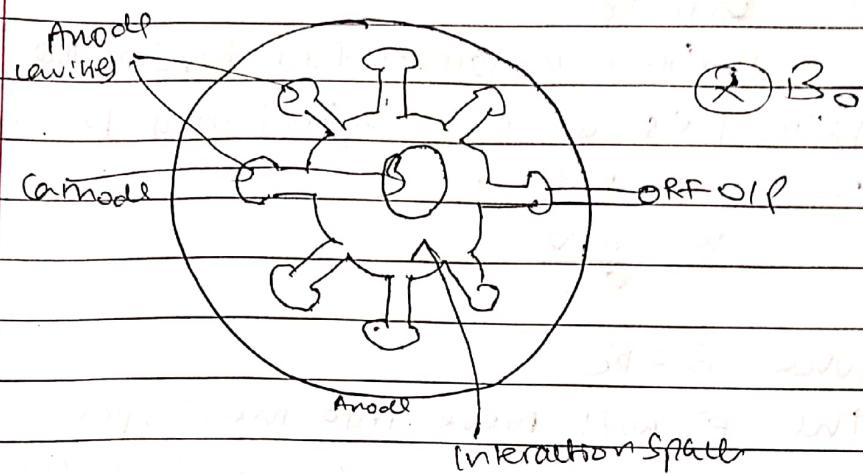
Oper' of Electron Klystron:

- The e^- beam emitted from cathode is accelerated by anode towards the cavity.
- We assume an initial ac. field in cavity due to noise, etc. and causes velocity modulation
 - The e^- s at a are accelerated due to +ve half cycle of RF field
 - The e^- s at b travel at original velocity
 - The e^- s at c will be retarded due to -ve half cycle of RF field
- The repeller plate having -ve voltage, used to repel e^- s to cavity as the feedback
 - The accelerated early electrons e_a have a long return time.
 - The slowed down late electrons e_l have shortest return time to cavity.
 - The electrons 'b' are reference e^- s e_r .

Thus, e^- s e_l catch up with e_a & e_r & thus reenter the cavity as a bunch

- The repeller distance L & voltages adjusted so that the e^- bunch are received on the +ve peak of cavity RF voltage cyl, & thus, the bunch loses its K.E.

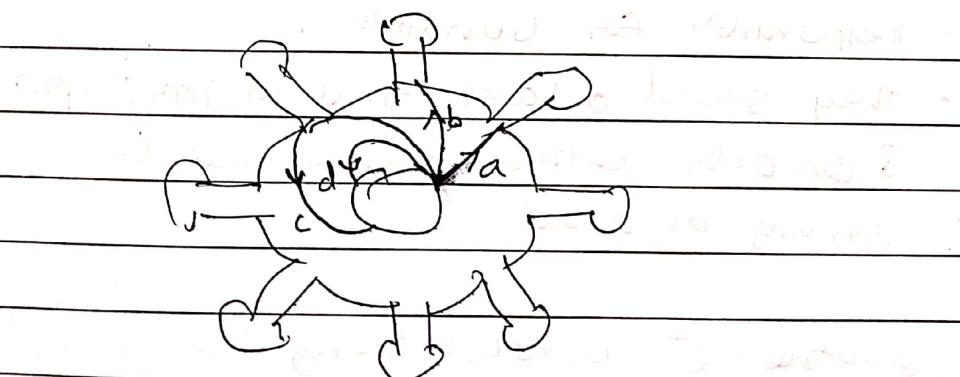
* Cylindrical Magnetron:



- It consists of a cylindrical cathode at centre & thick cylindrical block of anode. The anode block has a no. of holes & slots which act as resonant cavity.
- The space b/w anode & cathode is interⁿ space
- Electric field produced by app. dc voltage b/w anode & cathode
From anode to cathode
- The mag. field produced by permanent magnet is vertical to plane.
- The cathode emits electrons in interⁿ region b/w anode & cathode & interact with field.

Operⁿ:

- 1.) With static electric & magnetic fields.



Case 1: If mag. Field absent i.e. $B=0$ then e^- will travel straight from cathode to anode due to radial force acting on it as in trajectory (a)

CASE 1: when $B < B_c$

It will exert a lateral force bending the path of e^- 's curvilinear by trajectory b.
radius of path is;

$$R = \frac{mv}{eB}$$

CASE 2: when $B = B_c$

The e^- will move into interⁿ space & just graze around surface of cathode (c)

CASE 3: when $B > B_c$

the e^- 's experience a greater rotational force & may return back to cathode & cause heating of cathode

2.) with Active RF field:

for " " there are three kinds of e^- s emitted.

CASE 1: when oscillations are present an e^- a

is slowed down by tangential comp. of RF field

This e^- transfer their energy to RF field
and are favored e^- s.

- responsible for bunching.
- they spend a large time in interⁿ space & can orbit cathode several times before arriving at anode.

CASE 2: another e^- b, takes energy from RF field and is accelerated which cause it to bend more sharply, spend little time in interⁿ space.

- These are unfavoured e^- s. & do not help in bunching.
- cause ball heating of cathode

Note: type 'a' e⁻s spend more time in intern space than type 'b' e⁻s so more energy given to oscillations & true oscillations are sustained.

case 3). Bunching (Phase focusing)

$$\Delta t = \frac{2\pi}{\omega} = 2\pi \times 10^{-16} \text{ sec}$$

$$\Delta \theta =$$

$$\Delta \phi =$$

$$\Delta \psi =$$

$$\Delta \phi = \Delta \theta + \Delta \psi$$

Max. phase shift = $\pi/2$ (approx.)

if $\Delta \theta = \pi/2$ then $\Delta \phi = \pi/2$

$$2\pi (\sin(\pi/2)) = 1 = 36^\circ$$

$$F_B =$$

$$= \frac{eV_0}{2mL^2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} = \frac{eV_0}{8mL^2}$$

$$F_B =$$

$$= \frac{eV_0}{8mL^2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} = \frac{eV_0}{32mL^2}$$

velocity moduln:

Due to potential diff V_0 b/w anode & cathode,
e's form a current density beam with
velocity u_0 .

$$u_0 = \sqrt{2eV_0/m}$$

e = charge or e^-

m = mass of e^-

time taken by beam to cross gap
'd' is transit time & angle through
gap 'd' is

$$\begin{aligned} \text{transit time} &= t_2 - t_1 \\ &= t_g \\ &= \frac{d}{u_0} \end{aligned}$$

$$\text{transit angle} = \theta_g = \cot g$$

i/p given to buncher cavity is RF i/p.

Avg. RF i/p in gap of buncher cavity is,

$$V_{av} = \frac{1}{t_g} \int_{t_1}^{t_2} V_i \sin(\omega t) dt$$

$$= \frac{V_i}{\cot g} (\cos(\omega t_1) - \cos(\omega t_2))$$

$$= \frac{V_i}{\omega t_g} \left[2 \sin\left(\frac{\omega t_1 + \omega t_2}{2}\right) \sin\left(\frac{\omega t_1 - \omega t_2}{2}\right) \right]$$

$$= \frac{V_i}{(\omega t_g)/2} \left[\sin\left(\frac{\omega t_1 + \theta_g}{2}\right) \sin\left(\frac{\omega t_g}{2}\right) \right]$$

$$\text{Let } \beta_1 = \frac{\sin(\theta_g/2)}{\theta_g/2} = \text{buncher cavity beam coupling coefficient}$$

Note: If U_0 constant so $\frac{d}{dt} = \text{constant}$, it \Rightarrow inc. t inc. & the energy of e^- will ~~classmate~~ decrease & true total K.E. ~~reduces~~^{Date}

B_1 : $\frac{1}{2}mv_0^2$ is same for each e^- but no. of e^- in bunch of e^- decreasing with inc. in t . \Rightarrow total energy ~~reduces~~^{Date} as total no. of e^- reaching end of cavity reduces.

$$\therefore V_{av} = V_0 \beta_1 \sin(\omega t, +\theta g/2)$$

$\beta_1 \Rightarrow$ it tells w/ the number of electrons that are able to reach end of cavity having mass m and desired velocity. e^- s that dissipate their energy and cannot reach end of cavity they do not contribute to amplitude.

Note: $U_0 \propto V_0$ so amplif depends on cathode voltage. Also no. of e^- s emitted $\propto V_0$ so amplif

Limits: $V_0 \text{ min} \Rightarrow$ min. V_0 required for all e^- s to reach from cathode to anode.

$$V_{0 \text{ min}} \Rightarrow U_0 = \frac{d}{t} = d \times f_{RF \text{ input}}$$

so $V_{0 \text{ min}}$ defined by freq. of RF signal

& also $V_{0 \text{ min}}$ defined by thermal runaway as $V_0 \uparrow$ so no. of e^- also heating \uparrow thus $V_{0 \text{ min}}$ restricted.

Let $U_{av} = \text{velocity of } e^- \text{ at mid of gap}$

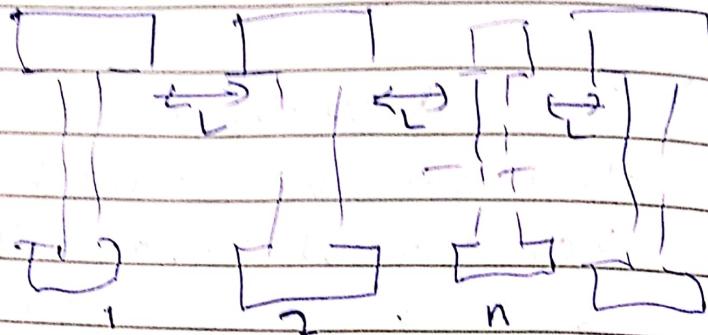
$$U_{av} = \sqrt{V_0 + V_{av}} = U_0 \sqrt{1 + \beta_1 \sin(\omega t, +\theta g/2)}$$

Let $m = \frac{V_1 \beta_1}{V_0} = \text{depth of modulation}$

$$U_{av} = \sqrt{1 + m \sin(\omega t, +\theta g/2)}$$

Now to increase gain:

array
a cascaded
amplifier



We can increase the gain by increasing no. of cavities

$$\beta = \beta_1 \beta_2 \dots \beta_n$$

- but problem is that as drift no. of cavities increase the drift space b/w the cavities cause more no. of dissipated P's to cause heating & thermal runaway

- as Gain \propto B.W. = constant

so if Gain \uparrow then B.W. \downarrow
so puts a higher limit on no. of cavities.

Thus, instead of increasing no. of cavities we can use multiple two cavity klystrons,

but limit is large no. of klystron will need large area to install