Deffuzification methods

DEFUZZIFICATION

- Defuzzification is a mapping process from a space of fuzzy control actions defined over an output universe of discourse into a space of crisp (nonfuzzy) control actions.

- Defuzzification is a process of converting output fuzzy variable into a unique number .

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METHODS OF DEFUZZIFICATION

Defuzzification is the process of conversion of a fuzzy quantity into a precise quantity. Defuzzification methods include:

- **Max-membership principle,**
- **Centroid method,**
- **Weighted average method,**
- **Mean-max membership,**
- **Center of sums,**
- **Center of largest area,**
- **First of maxima, last of maxima.**

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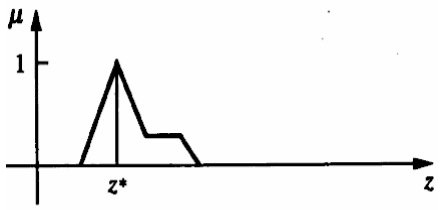
Max-membership principal, also known as height method



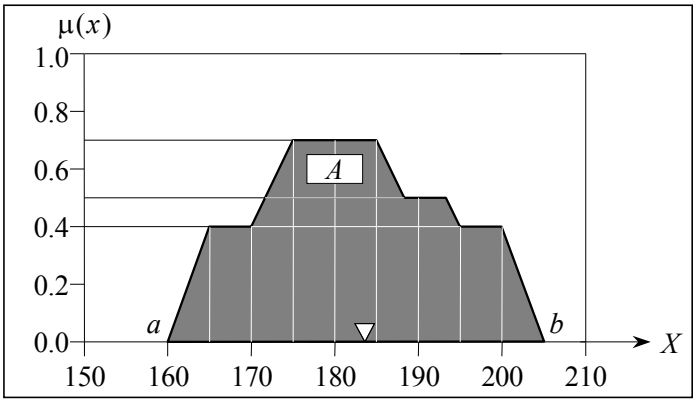
$\mu_C(z^*) \geq \mu_C(z)$ for all $z \in Z$

Centre of gravity (COG) OR Centre of Area(COA)

- There are several defuzzification methods, but probably the most popular one is the **centroid technique**. It finds the point where a vertical line would slice the aggregate set into two equal masses. Mathematically this **centre of gravity (COG)** can be expressed as:

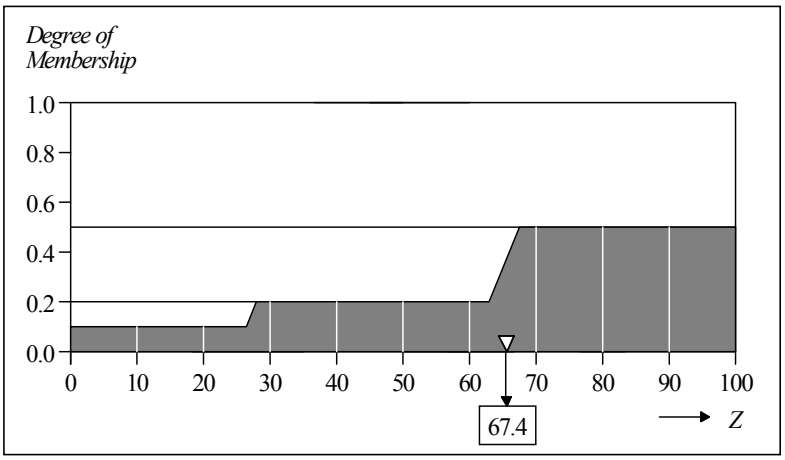
$$COG = \frac{\int_a^b \mu_A(x)\, x\, dx}{\int_a^b \mu_A(x)\, dx}$$

- Centroid defuzzification method finds a point representing the centre of gravity of the fuzzy set, *A*, on the interval, *ab*.
- A reasonable estimate can be obtained by calculating it over a sample of points.



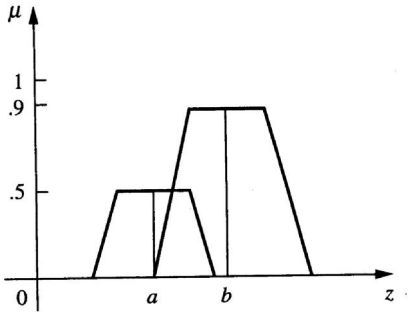
Centre of gravity (COG):

$$COG = \frac{(0+10+20)\times 0.1+(30+40+50+60)\times 0.2+(70+80+90+100)\times 0.5}{0.1+0.1+0.1+0.2+0.2+0.2+0.2+0.5+0.5+0.5+0.5} = 67.4$$



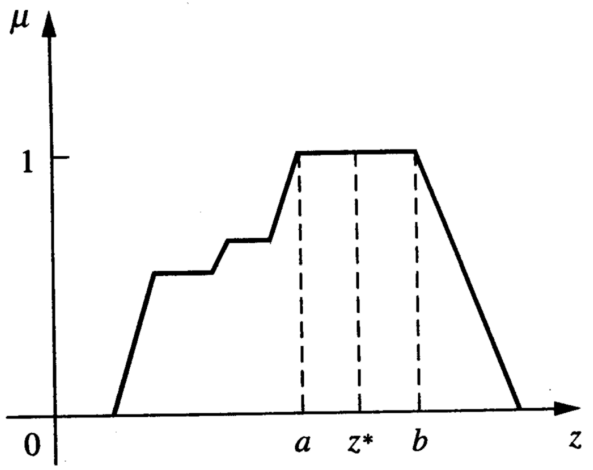
Weighted average method

- Valid for symmetrical output membership functions
- Produces results very close to the COA method
- Less computationally intensive



$$z^* = \frac{\sum \mu_C(\bar{z}) \cdot \bar{z}}{\sum \mu_C(\bar{z})} \qquad z^* = \frac{a(.5) + b(.9)}{.5 + .9}$$

Mean of Maximum(MOM)



CENTER OF SUMS

This method employs the algebraic sum of the individual fuzzy subsets instead of their unions. The calculations here are very fast but the main drawback is that the intersecting areas are added twice. The defuzzified value x^* is given by

$$x^* = \frac{\int_x x \sum_{i=1}^{n} \mu_{C_i}(x) dx}{\int_x \sum_{i=1}^{n} \mu_{C_i}(x) dx}$$

CENTER OF LARGEST AREA

This method can be adopted when the output consists of at least two fuzzy subsets which are not overlapping. The output in this case is biased towards a side of one membership function. When output fuzzy set has at least two regions then the center-of-gravity of the fuzzy subregion having the largest area is used to obtain the defuzzified value x^*. This value is given by

$$x^* = \frac{\int \mu_{C_j}(x) \cdot x \, dx}{\int \mu_{C_j}(x) \, dx}$$

where C_j is the subregion that has the largest area

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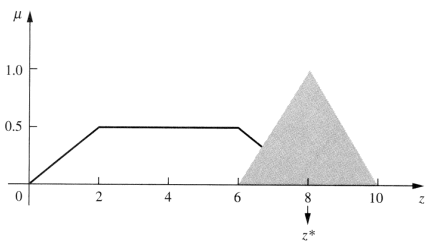
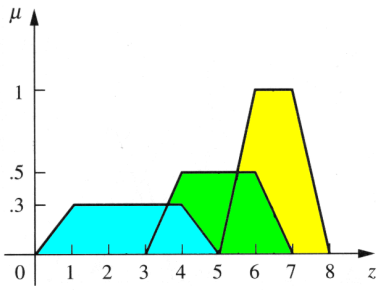
First (or last) of maxima

- Determine the smallest value of the domain with maximized membership degree

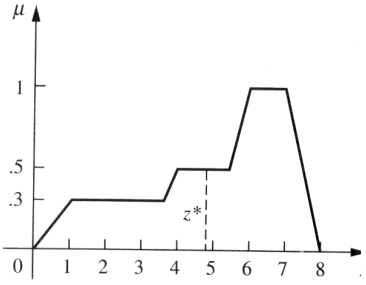


Example: Defuzzification

- Find an estimate crisp output from the following 3 membership functions



Centre of gravity (COG) OR Centre of Area(COA)



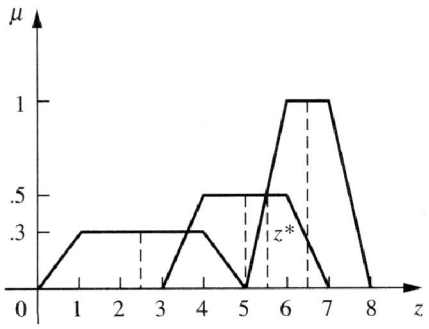
$$z^* = \frac{\int \mu_{\underline{B}}(z) \cdot z\, dz}{\int \mu_{\underline{B}}(z)\, dz} =$$

$$\left[\int_0^1 (.3z)z\, dz + \int_1^{3.6} (.3z)\, dz + \int_{3.6}^4 \left(\frac{z-3}{2}\right) z\, dz + \int_4^{5.5} (.5)z\, dz + \int_{5.5}^6 (z-5)z\, dz + \int_6^7 z\, dz + \int_7^8 (8-z)z\, dz\right]$$

$$\div \left[\int_0^1 (.3z)\, dz + \int_1^{3.6} (.3)\, dz + \int_{3.6}^4 \left(\frac{z-3}{2}\right) dz + \int_4^{5.5} (.5)\, dz + \int_{5.5}^6 (z-5)\, dz + \int_6^7 dz + \int_7^8 (8-z)\, dz\right]$$

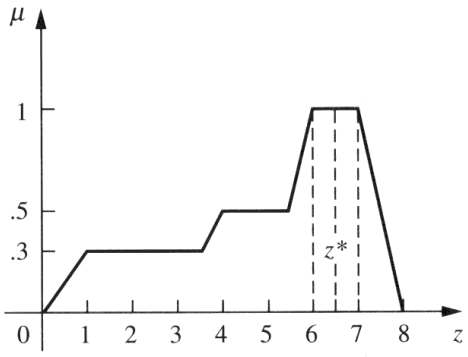
$$= 4.9 \text{ meters}$$

Weighted average method



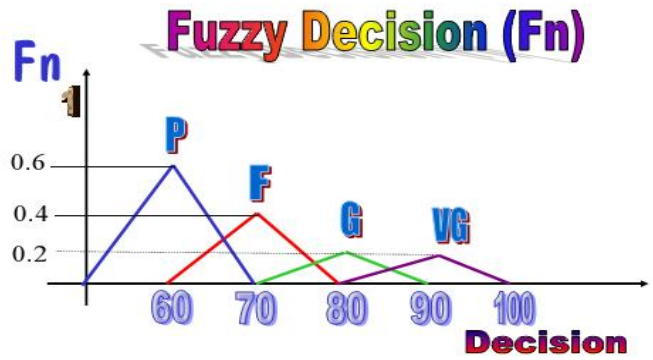
$$z^* = \frac{(.3 \times 2.5) + (.5 \times 5) + (1 \times 6.5)}{.3 + .5 + 1} = 5.41 \text{ meters}$$

Mean of Maximum(MOM)



Z* = (6+7)/2 = 6.5

FUZZY DECISION

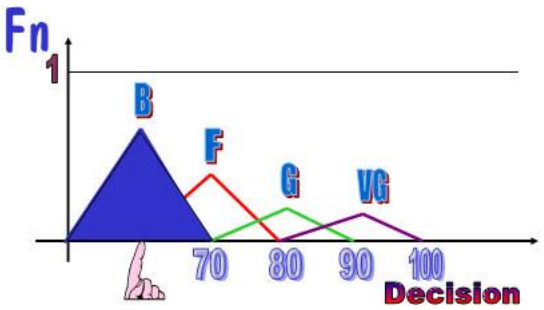


Fn = {P,F,G,VG,E}

Fn = {0.6, 0.4, 0.2, 0.2, 0}

MAX MEMBERSHIP METHOD

- Fuzzy set with the largest membership value is selected.
- Fuzzy decision: Fn = {P, F. G, VG, E}
  Fn = {0.6, 0.4, 0.2, 0.2, 0}
- Final decision (FD) = Poor Student
- If two decisions have same membership max, use the average of the two.

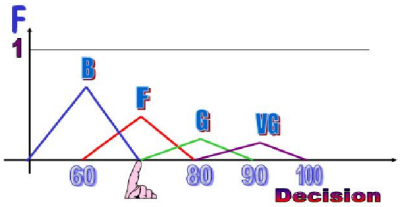


WEIGHTED AVERAGE METHOD

$$FD=\frac{\sum_i \mu_f \, fn}{\sum_i \mu_f}=\frac{\mu_E \times E+\mu_{VG} \times VG+.....}{\mu_E+\mu_{VG}+.....}$$

$$FD=\frac{0\times 100+0.2\times 90+0.2\times 80+0.4\times 70+0.6\times 60}{0.2+0.2+0.4+0.6}=70$$

Final Decision (FD) = Fair Student



METHODS OF MEMBERSHIP VALUE ASSIGNMENT

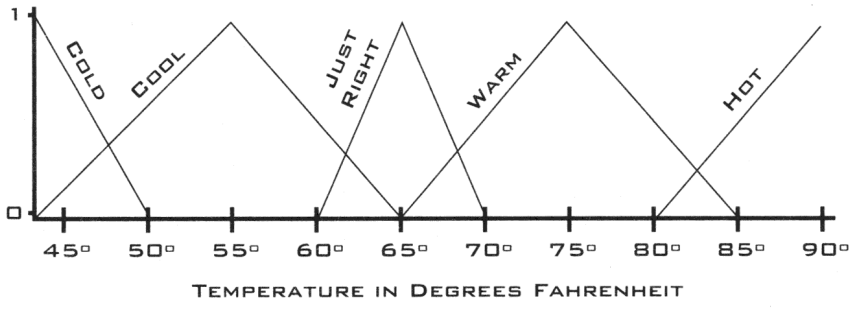
The various methods of assigning membership values are:

- Intuition,
- Inference,
- Rank ordering,
- Neural networks,
- Genetic algorithm,

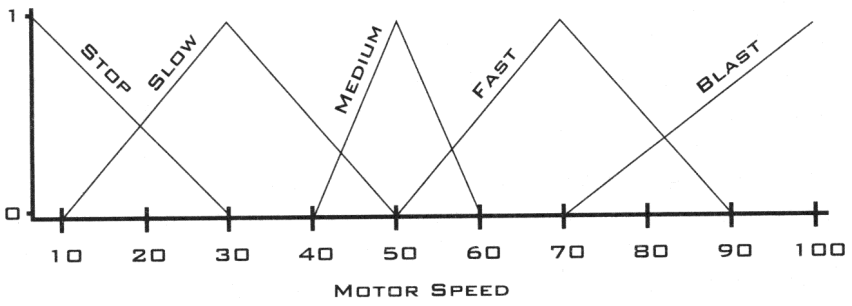
Fuzzy controller design

- Design a motor speed controller for air conditioner. Input is room temperature in the range of 40 to 90 degrees F with the desired set temp 65. Output is speed of motor normalized in 0-100 range. Design should be supported by figures wherever necessary. Clearly indicate what will be the speed when temp is just less than the set value.

Input Fuzzy sets



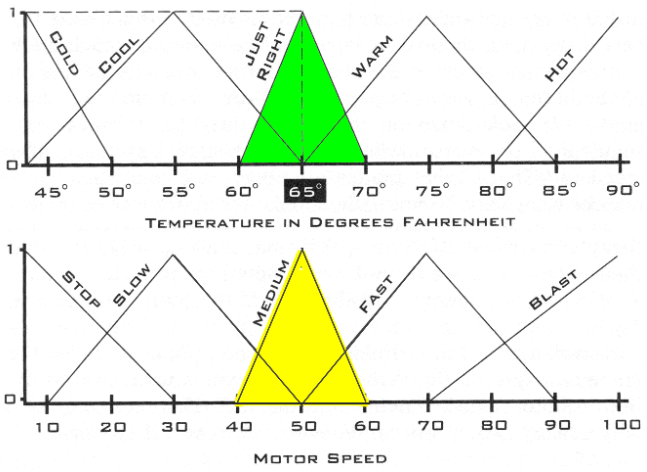
Output Fuzzy sets



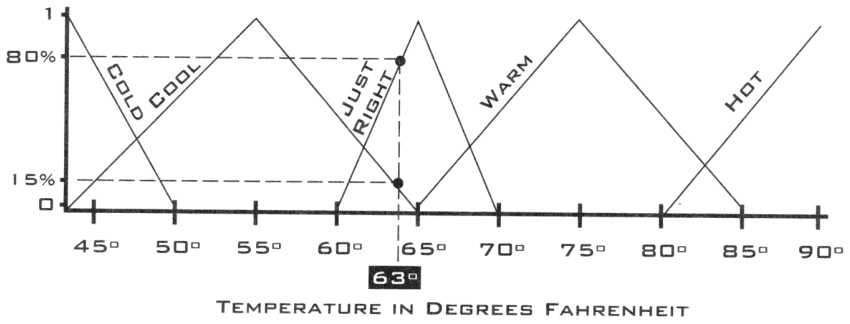
Fuzzy Rules

- If temperature is cold then motor speed is stop
- If temperature is cool then motor speed is slow
- If temperature is just right then motor speed is medium
- If temperature is warm then motor speed is fast
- If temperature is hot then motor speed is blast

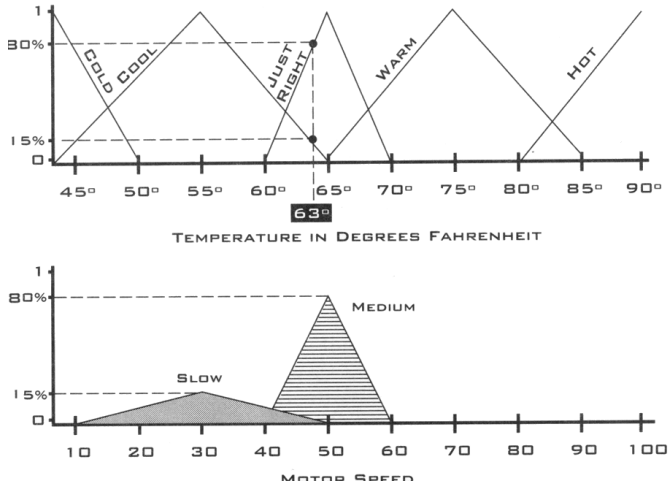
Example: temp. = 65 degree F.



Example: temp. = 63 degree F.



Fuzzy output



Fuzzy Controller for Washing Machine

- The control system senses both quality of dirt, load size and adjusts the washing cycle.Find the washing cycle time for 70% dirt and 60% load.
- Load={Small , Medium , Large}
- Dirt={Not Greasy , Med. Greasy , Greasy}
- Washing time={Very short , short , Med. , Long , Very Long}

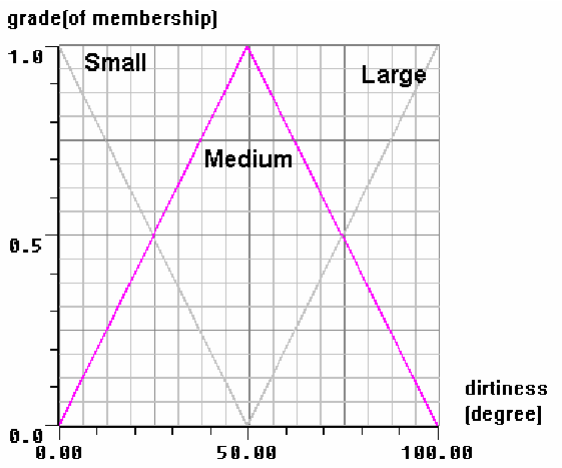
Washing Machine

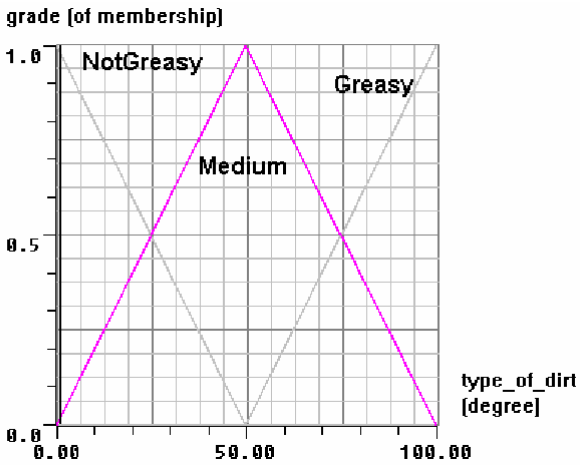
- 1. If quanity is Large and type_of_dirt is Greasy then wash_time is VeryLong;
- 2. If quanity is Medium and type_of_dirt is Greasy then wash_time is Long;
- 3. If quanity is Small and type_of_dirt is Greasy then wash_time is Long;
- 4. If quanity is Large and type_of_dirt is Medium then wash_time is Long;
- 5. If quanity is Medium and type_of_dirt is Medium then wash_time is Medium;

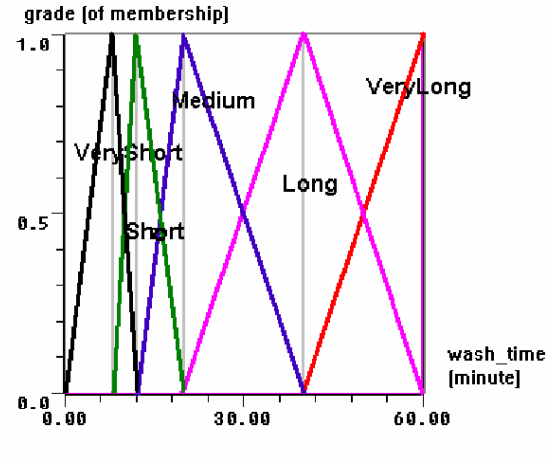
- 6. If quanity is Small and type_of_dirt is Medium then wash_time is Medium;
- 7. If quanity is Large and type_of_dirt is NotGreasy then wash_time is Medium;
- 8. If quanity is Medium and type_of_dirt is NotGreasy then wash_time is Short;
- 9. If quanity is Small and type_of_dirt is NotGreasy then wash_time is VeryShort

Rule Base

	S	M	L
N G	VS	S	M
M G	M	M	L
G	L	L	VL




grade [of membership]
1.0
0.5
0.0
NotGreasy
Greasy
Medium
type_of_dirt
[degree]
0.00
50.00
100.00


grade [of membership]
1.0
0.5
0.0
VeryShort
Short
Medium
Long
VeryLong
wash_time
[minute]
0.00
30.00
60.00

Fuzzy Controller Example

- Assume that we need to evaluate student applicants based on their GPA and GRE scores.
- For simplicity, let us have three categories for each score [High (H), Medium (M), and Low(L)]
- Let us assume that the decision should be Excellent (E), Very Good (VG), Good (G), Fair (F) or Poor (P)
- An expert will associate the decisions to the GPA and GRE score. They are then Tabulated.

Fuzzy Inference

- *Assume a student with GRE=900 and GPA=3.6*
- *A decision on the classification of the applicant is needed.*
  - *Excellent*
  - *Very good*
  - *Etc.*

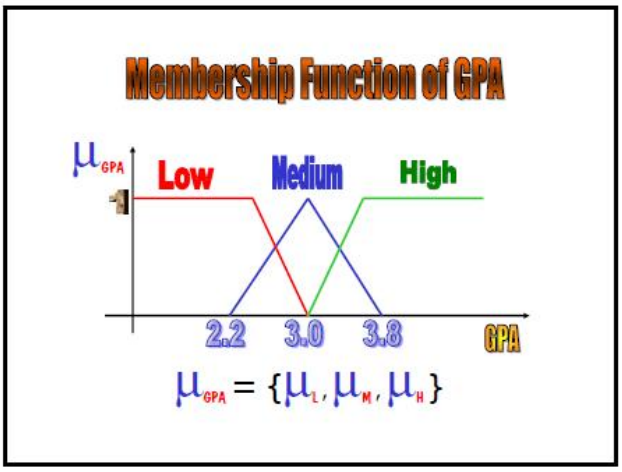
Rule Base

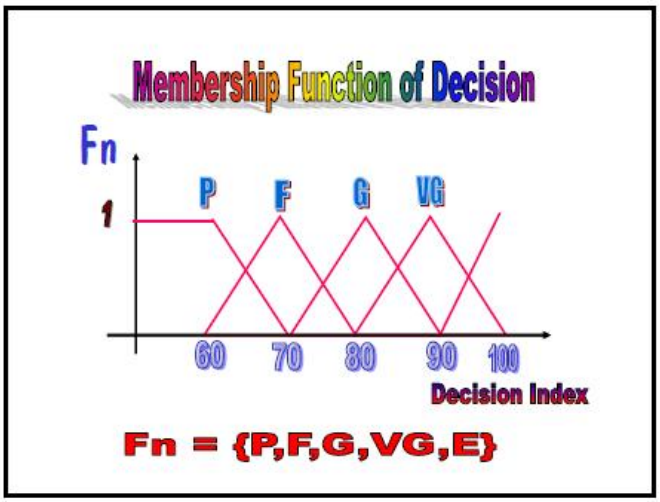
GPA \ GRE	H	M	L
H	E	VG	F
M	G	G	P
L	F	P	P

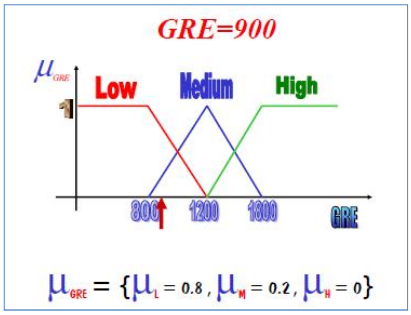
Excellent	= 95-100%
Very Good	= 90 - 94%
Good	= 80 - 89%
Fair	= 70 - 79%
Poor	= 0 - 69%

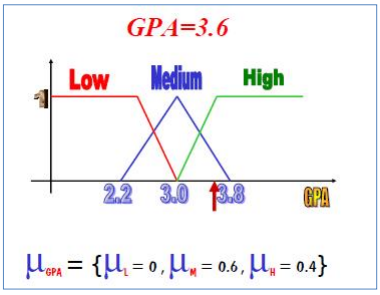
Membership Function of GRE

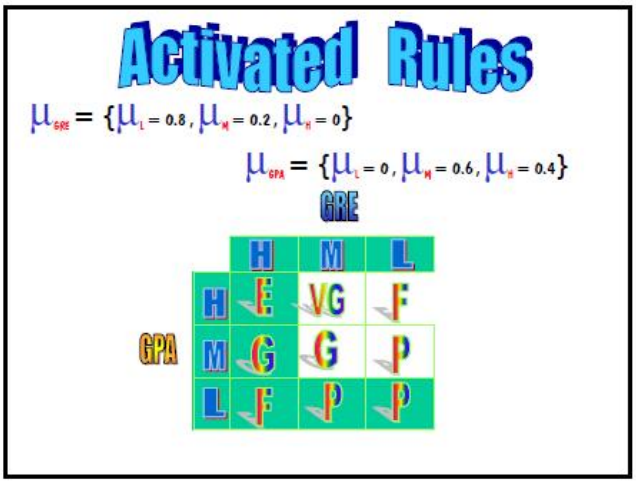
$$\mu_{GRE} = \{\mu_L, \mu_M, \mu_H\}$$


Membership Function of GPA
μ_{GPA}
Low
Medium
High
1
2.2
3.0
3.8
GPA
$\mu_{GPA} = \{\mu_L, \mu_M, \mu_H\}$

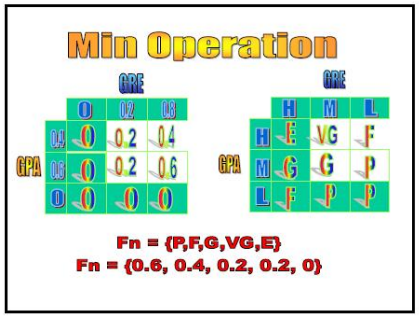

Membership Function of Decision
Fn
1
P
F
G
VG
60
70
80
90
100
Decision Index
Fn = {P,F,G,VG,E}

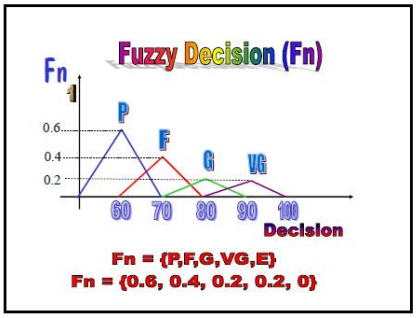

GRE=900
Low
Medium
High
800
1200
1800
GRE
$\mu_{GRE} = \{\mu_L = 0.8, \mu_M = 0.2, \mu_H = 0\}$


GPA=3.6
Low
Medium
High
2.2
3.0
3.8
GPA
$\mu_{GPA} = \{\mu_L = 0, \mu_M = 0.6, \mu_H = 0.4\}$


Activated Rules
$\mu_{GRE} = \{\mu_L = 0.8, \mu_M = 0.2, \mu_H = 0\}$
$\mu_{GPA} = \{\mu_L = 0, \mu_M = 0.6, \mu_H = 0.4\}$
GRE
H
M
L
GPA
H
E
VG
F
M
G
G
P
L
F
P
P

Fuzzy Output





- The water sprinkler system receives inputs via its sensors and determines the watering duration. The fuzzy controller uses two inputs: air temperature, and soil moisture. The system controls the watering duration by turning various valves on and off automatically.
- Temperature(cold,mild,warm,hot)
- soil moisture(Dry,moist,wet)
- watering duration(very short,short,long,verylong)

Justify the controller operation for warm temperature and dry soil moisture.