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Operations Research.

1. Formulate real problems.
2. Understand mathematical tools to solve
3. Use mathematical software for solving.

OR = "Optimization with Constraints"

Optimization - Selecting best solution from a set of solutions
Optimization is possible only when there are a number of solutions to a single problem.

Plants		Production time per batch made per week P1	P2	Available Time
1	Al frames & hardware	1	0	4
2	Wooden frames	0	2	12
3.	Glass	3	2	18
Profit		3000	5000	

Objective : Maximize the TOTAL Profit
Variables :
X_1 = No of batches of P1
X_2 = No of batches of P2
Objective Function :

$$Max(f) = 3000(X1) + 5000(X2)$$

Constraints : $X_1, X_2 \geq 0$

$$X_1(1) + 0(X_2) \leq 4 \Rightarrow X_1 \leq 4$$
$$0(X_1) + 2(X_2) \leq 12 \Rightarrow 2X_2 \leq 12$$
$$3(X_1) + 2(X_2) \leq 18$$

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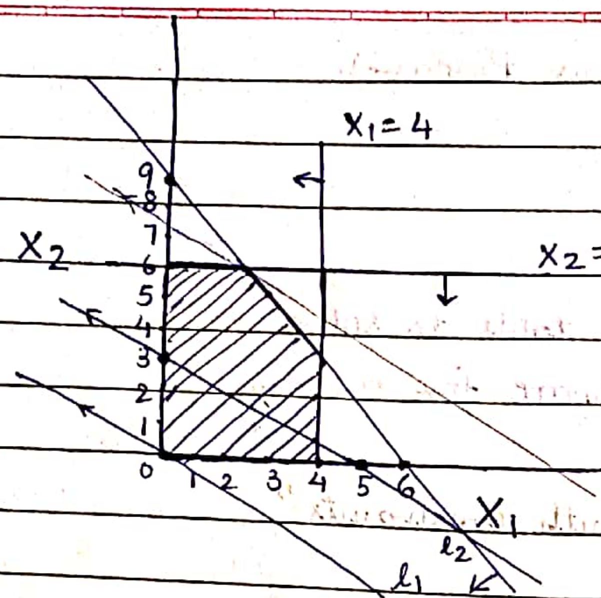


$3X_1 + 2X_2 = 18$

$X_1 = 0 \quad X_2 = 9$

$X_1 = 6 \quad X_2 = 0$

Let $3000X_1 + 5000X_2 = 15000$

$X_1 = 0 \quad X_2 = 3$

$X_1 = 5 \quad X_2 = 0$

Let $3000X_1 + 5000X_2 = 0$.

$X_1 = X_2 = 0$

Contour Line – Any point on line satisfies the equation
$\therefore$ l_1 & l_2 are the two contour lines

An inequality constraint divides the graph into two parts (halves) i.e. valid half & invalid half
The feasible region is intersection of all valid half spaces.

Solve LP by graphical means.

$$\text{Max } z = 4x_1 + 5x_2$$

s.t $-x_1 + x_2 \le 4$

$x_1 + x_2 \le 6$

$x_1, x_2 \ge 0$. $\Rightarrow$ Nonnegativity Constraints

Graphical.

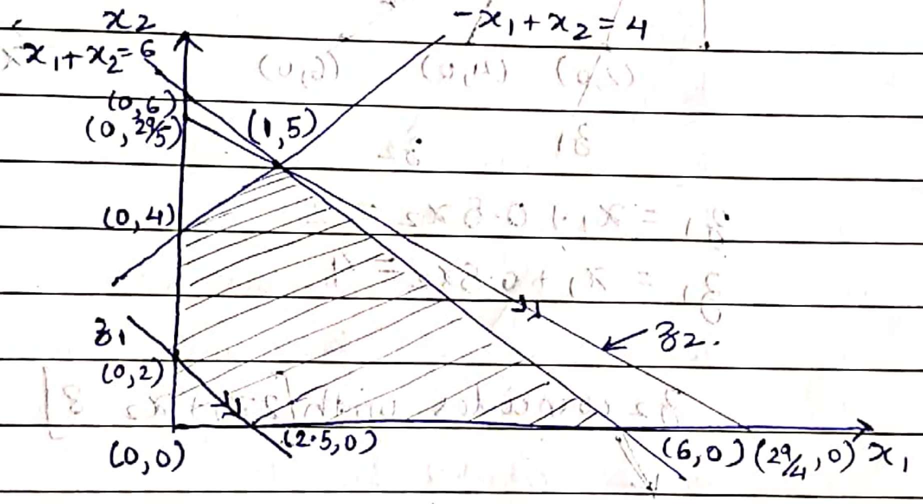
$-x_1 + x_2 = 4$

$x_1 + x_2 = 6$

Let $z_1 = 4x_1 + 5x_2 = 10$.

Let. $z_2 = 4x_1 + 5x_2 = 29$.

$\Rightarrow$ Objective functions.



$z_1 \parallel z_2$ Because x_1 & x_2 plane & cut by parallel planes z_1 & z_2

Solve

$$\text{Max } z = x_1 + 0.5x_2.$$

s.t $\quad 2x_1 + 3x_2 \le 12$

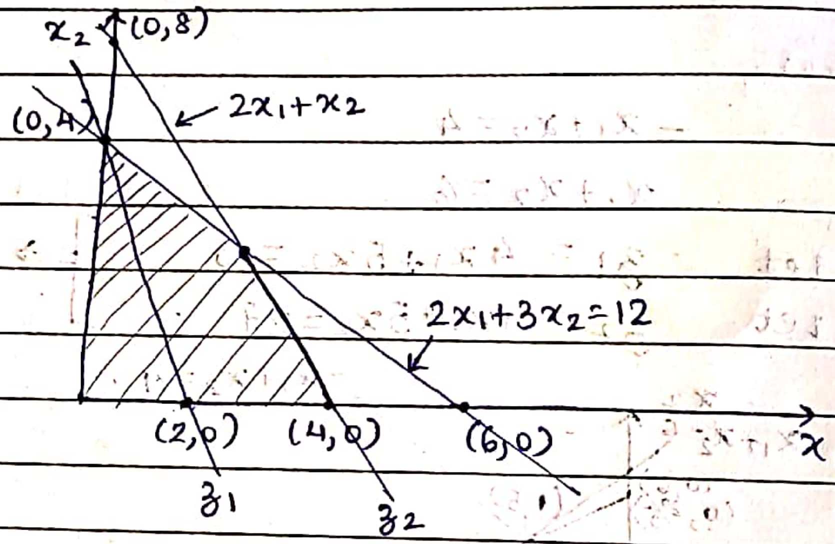
$2x_1 + x_2 \le 8.$

$x_1, x_2 \ge 0.$

$2x_1 + 3x_2 = 12$

$2x_1 + x_2 = 8$

Graph.



$$z_1 = x_1 + 0.5x_2 = 2$$
$$z_1 = x_1 + 0.5x_2 = 4$$

z_2 coincides with $[2x_1 + x_2 = 8]$

$\therefore$ Optimum doesn't occur at one corner but at two corner

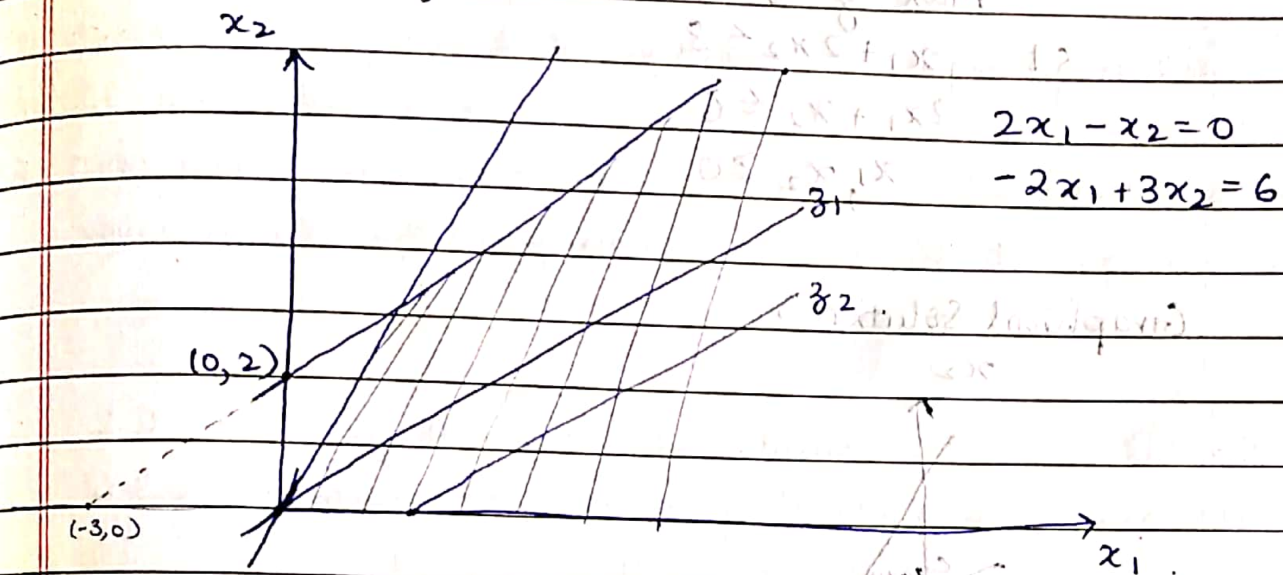
Solve.

$$\text{Max } z = x_1 - 2x_2$$

s.t. $\quad 2x_1 - x_2 \geq 0$

$\quad -2x_1 + 3x_2 \leq 6$

$\quad x_1, x_2 \geq 0$



Let $z_1 = x_1 - 2x_2 = 0$.

$z_2 = x_1 - 2x_2 = 2$.

No maximum possible.

Unbounded feasible Region

↓

doesn't mean no optimization.

Moving towards unbounded.

↓

No optimum

Moving ~~towards~~ in opposite direction

↓

Optimum possible.

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* Simplex Method:-

Consider the following Linear Program.

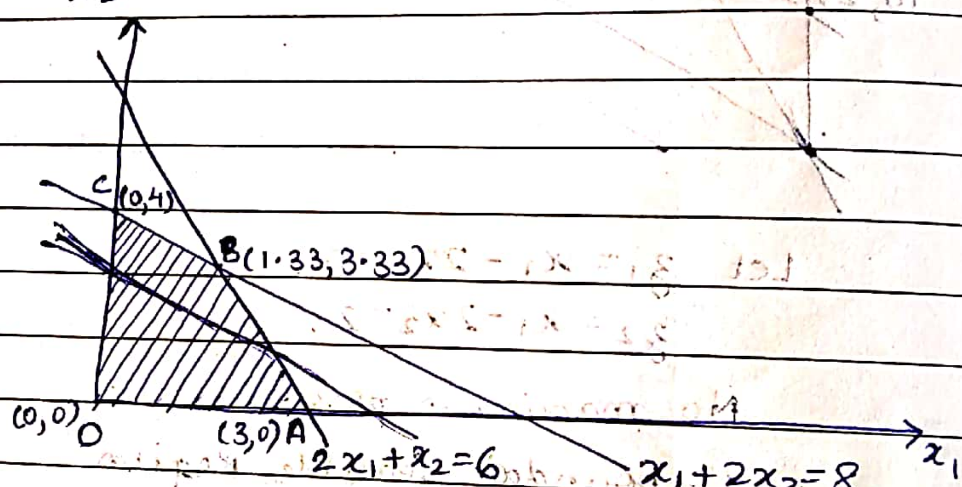
$$\text{Max } z = 3x_1 + 4x_2 .$$

$$\text{s.t} \quad x_1 + 2x_2 \le 8$$

$$2x_1 + x_2 \le 6$$

$$x_1, x_2 \ge 0$$

Graphical Solution.



$$z = 3x_1 + 4x_2 = 12$$

$$z = 3x_1 + 4x_2 = 0$$

$$x_1^* = 1.33$$

$$x_2^* = 3.33$$

$$z^* = 17.33$$

★ Optimum will always occur at corners.

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→ Simplex method is based on mathematical conclusion that for a well formulated Linear Program, the optimum will always occur at a vertex of the feasible space/region.

1. Start at a vertex of the feasible space.
2. Check if it is indeed the optimum (Optimality Test)
3. If it is, then stop!
4. Otherwise, go to another vertex which improves the value of the objective function, & check for the optimum.

2D → Easy to see feasible region.
higher dimension → Geometry doesn't help, but algebra will be useful.

∴ For analysis of better problem.
Convert inequalities to equalities:

$$x_1 + 2x_2 + S_3 = 8$$
$$2x_1 + x_2 + S_4 = 6$$

Variables 'S_3' & 'S_4' are known as slack variables. because they represent the 'SLACKNESS' between LHS & RHS.
However, 'S_3' & 'S_4' must satisfy non negativity constraint.

Restating the problem:

$$\text{Max } z = 3x_1 + 4x_2$$

S.t. $x_1 + 2x_2 + S_3 = 8$ No. of variables $= 4 = n$

$2x_1 + x_2 + S_4 = 6$ No. of equations $= 2 = m$

$x_1, x_2, S_3, S_4 \geq 0$

Infinite Solutions ∵ $n > m$ provided equations are independent & consistent.

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The equations can be represented in matrix.

$$\begin{bmatrix} 1 & 2 & 1 & 0 \\ 2 & 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ S_3 \\ S_4 \end{bmatrix} = \begin{bmatrix} 8 \\ 6 \end{bmatrix}$$

Basis factors
↳ used to represent any vector by points/group of point

* Out of those infinite solutions, we want that particular solution which maximizes the objective function, while being feasible.

1. Feasibility represents
   → Solution satisfies equations.
   → Variables are non negative

* Consider the characteristic features of the vertices of the feasible region:

Vertex	x_1	x_2	Active Constraints	S_3	S_4
O	0	0	$x_1=0$ $x_2=0$	8	6
A	3	0	$x_2=0$ $2x_1+x_2=6$	5	0
B	1.33	3.33	$x_1+2x_2=8$ $2x_1+x_2=6$	0	0
C	0	4	$x_1=0$ $x_1+2x_2=8$	0	2

We need to select $(n-m)$ out of total n variables to assign zeroes.
∴ ${}^{n}C_{n-m}$ possible combinations

The m variables, whose solution comes from the basis columns, are called basic variables; the (n−m) variables, which are assigned zeroes called the non basic variables.

For the current problem, we can write

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad \text{non-basic variables.}$$

Then solving for the basic variables, we get:

$$\begin{bmatrix} S_3 \\ S_4 \end{bmatrix} = \begin{bmatrix} 8 \\ 6 \end{bmatrix} \quad \text{basic variables.}$$

Basic	x_1	x_2	S_3	S_4	b
S_3	1	2	1	0	8
S_4	2	1	0	1	6
z	−3	−4	0	0	0

$\Rightarrow -3x_1 - 4x_2 + z = 0$

The selection of the pivot column representing the non basic variable & pivot row, for the basic variable are shown below:

Pivot Element

(One where basic vector needs to be created)

	basic	x_1	x_2	S_3	S_4	b	Ratio
Pivot Row ⇒	S_3	1	2	1	0	8	4 ⇐ b/x_2
	S_4	2	1	0	1	6	6 ⇐ b/x_2
	z	−3	−4	0	0	0	

⇑

Pivot column

Min $z = 2x_1 + 3x_2$

St. $x_1 + 2x_2 \geq 10$

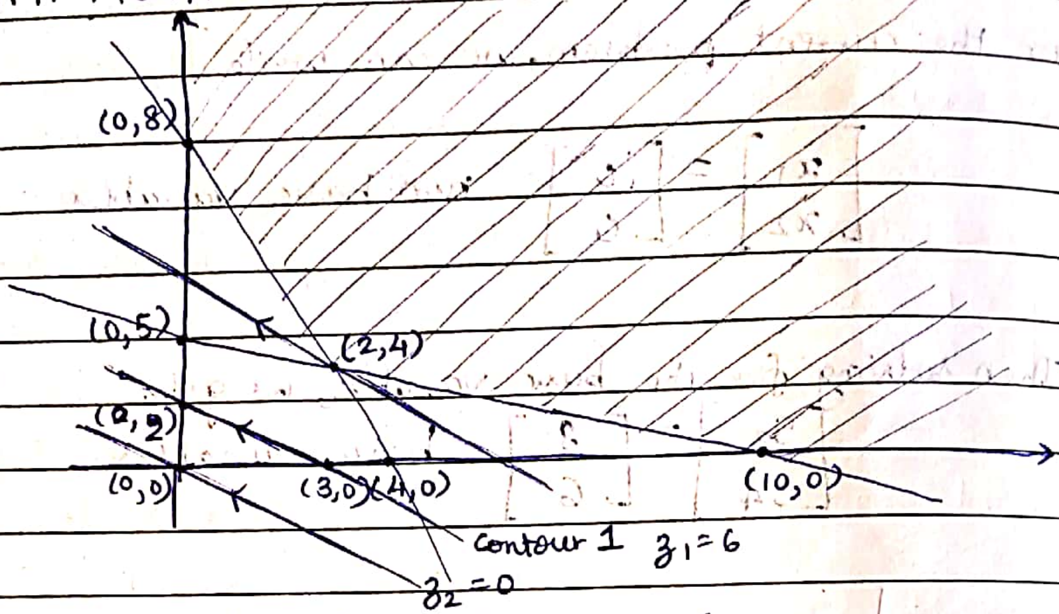
$2x_1 + x_2 \geq 8$

$x_1, x_2 \geq 0$.

$x_1 + 2x_2 = 10$

$2x_1 + x_2 = 8$

GRAPH METHOD:



Let $2x_1 + 3x_2 = 6$. $z_1 = 6$

$2x_1 + 3x_2 = 0$, $z_2 = 0$

Optimum at $(2,4)$ $\boxed{z = 2(2) + 3(4) = 16}$

SIMPLEX METHOD:

Step 1: Convert inequalities to equalities.

Since inequalities are $\geq$ we need to use "Surplus Variables"

$x_1 + 2x_2 - S_3 = 10$

$2x_1 + x_2 - S_4 = 8$

$S_3, S_4 \rightarrow$ Surplus variables,

$x_1, x_2, S_3, S_4 \geq 0$.

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Step 2: Catch a corner

Lets try setting $\left.\begin{array}{l} x_1 = 0 \\ x_2 = 0 \end{array}\right\}$ Non basic Variables.

Eqn structure.

$$\begin{array}{cccc} x_1 & x_2 & S_3 & S_4 \end{array}$$
$$\begin{bmatrix} 1 & 2 & -1 & 0 \\ 2 & 1 & 0 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ S_3 \\ S_4 \end{bmatrix} = \begin{bmatrix} 10 \\ 8 \end{bmatrix}$$

We get solution as.

$\left.\begin{array}{l} S_3 = -10 \\ S_4 = -8 \end{array}\right\}$ Basic Variables

Since this solution is not acceptable because of -ve values we need to ~~have~~ add artificial variables to equations causing problems.

$$\therefore \quad x_1 + 2x_2 - S_3 + r_5 = 10$$
$$2x_1 + x_2 - S_4 + r_6 = 8$$

6 variables 2 equations $\Rightarrow$ set 4 variables = 0.

Lets try setting

$x_1 = 0$ $\quad S_3 = 0$ $\Rightarrow$ Non basic
$x_2 = 0$ $\quad S_4 = 0$ Variables.

$\therefore$ $r_5 = 10$ $\Longrightarrow$ Basic
$r_6 = 8$ Variables.

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Phase I

Objective - To get a Basic Feasible Solution (BFS) in to Original + Surplus Variables.

∴ Create a new objective function.

Min $w = r_5 + r_6 \Rightarrow -r_5 - r_6 + w = 0$

Non basic variables

Basic	x_1	x_2	s_3	s_4	r_5	r_6	b	Ratio
r_5	1	2	-1	0	1	0	10	$10/1 = 10$
Pivot Row → r_6	2	1	0	-1	0	1	8	$8/2 = 4$
w	3	3	-1	-1	0	0	18	

Represents basic variable will become non basic

$3x_1 + 3x_2 - s_3 - s_4 + \underbrace{r_5 + r_6}_{w} = 18$

⇑
Pivot column (Represents non basic variable will become basic).

For min look for most +ve coefficient
For max look for most -ve coefficient

Select the smallest +ve ratio.

Since '2' is the pivot element.
Divide row 2 by 2

	x_1	x_2	s_3	s_4	r_5	r_6	b
r_5	1	2	-1	0	1	0	10
$x_1 \leftarrow r_6$	1	1/2	0	-1/2	0	1/2	4
w	3	3	-1	-1	0	0	18

Row 1 - Row 2 ⇒ Row 1
Row 3 - 3 × Row 2 ⇒ Row 3.

		x_1	x_2	s_3	s_4	r_5	r_6	b	Ratio
Pivot Row ⇒	r_5	0	3/2 → Pivot ele.	−1	1/2	1	−1/2	6	$6/\frac{3}{2} = 4$
x_2 ⇐	r_6	1	1/2	0	−1/2	0	1/2	4	$4/\frac{1}{2} = 8$
	w	0	3/2	−1	1/2	0	−3/2	6	

→ r_5

⇑ most +ve

Pivot column

Pivot element $= 3/2$.

∴ Multiply row 1 by $2/3$.

r_5	0	1	−2/3	+1/3	2/3	−1/3	4
r_6	1	1/2	0	−1/2	0	1/2	4
w	0	3/2	−1	1/2	0	−3/2	6.

Row 2 − 1/2 Row 1 = Row 2.

Row 3 − 3/2 Row 1 = Row 3.

	x_1	x_2	s_3	s_4	r_5	r_6	b
x_2 ⇐ r_5	0	1	−2/3	1/3	2/3	−1/3	4.
x_1 ⇐ r_6	1	0	1/3	−2/3	−1/3	2/3	2
w	0	0	0	0	−1	−1	0

$r_6 = 0$, $r_5 = 0$.

∴ $x_1 = 2$ $x_2 = 4$

① This is the optimum corner because there is no non basic variable with a +ve coefficient in the last row.

② Since $w = 0$ at the optimum, that means the original problem has BFS. It also means the artificial variables can be dropped now.

$$\left.\begin{array}{r} s_3 = 0 \\ s_4 = 0 \\ r_5 = 0 \\ r_6 = 0 \end{array}\right\} \text{Non basic.} \qquad \left.\begin{array}{l} x_1 = 2 \\ x_2 = 4 \end{array}\right\} \text{basic.}$$

Phase II :

Basic	x_1	x_2	s_3	s_4	b
x_1	0	1	$-2/3$	$1/3$	4
x_2	1	0	$1/3$	$-2/3$	2
z	0	0	$-4/3$	$-1/3$	16

$$z = 2x_1 + 3x_2.$$

convert z as a function of non basic variables s_3 & s_4.

$$x_2 - 2/3\, s_3 + 1/3\, s_4 = 4.$$

$$\boxed{x_2 = 4 + 2/3\, s_3 - 1/3\, s_4}$$

$$x_1 + 1/3\, s_3 - 2/3\, s_4 = 2$$

$$\therefore \boxed{x_1 = 2 - 1/3\, s_3 + 2/3\, s_4}$$

$$\therefore z = 2\left[2 - \frac{1}{3}s_3 + \frac{2}{3}s_4\right] + 3\left[4 + \frac{2}{3}s_3 - \frac{1}{3}s_4\right]$$

$$= 4 - \frac{2}{3}s_3 + \frac{4}{3}s_4 + 12 + 2s_3 - s_4.$$

$$\therefore \boxed{z = 16 + \frac{4}{3}s_3 + \frac{1}{3}s_4}$$

$$-\frac{4}{3}s_3 - \frac{1}{3}s_4 + z = 16$$

$\therefore$ Optimum $\Rightarrow$ $x_2^* = 4$ $\quad x_1^* = 2$ $\quad z^* = 16$

The table is the first table to be referred for Phase II
This is already optimum at

$$x_2^* = 4$$

$$\therefore x_1^* = 2$$

$$z^* = 16$$

1. If at the end of Phase I $w \neq 0$, it means the original problem has no BFS.

HiV^{A,B,C} example.

Profit = Income − Cost.

Income → Sale of Products.

x_A = kg of A produces per day.

x_B = — " — B — " —

x_C = — " — C — " —

$$\text{Income} = \frac{x_A(1.15)}{0.5} + \frac{x_B(1.25)}{0.5} + \frac{x_C(1.2)}{0.5}$$

$$\text{Income} = 2x_A(1.15) + 2x_B(1.25) + x_C(1.2)(2) \ \$/\text{day}$$

Cost

	Strawberries	Grapes	Apples
A	0.5	–	0.5
B	0.25	0.25	0.5
C	–	0.4	0.6
Availability	200 × 750	600 × 100	500 × 150 kg/day

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For strawberries:

$$0.5x_A + 0.25x_B \leq 200 \times 750$$ Cost $= \frac{200}{750}$ \$/kg of pure 'S' juice

For grapes.

$$0.25x_B + 0.4x_C \cancel{+0.5x_C} \leq 600 \times 100$$ Cost $= \frac{100}{600}$

For apples.

$$0.5x_A + 0.5x_B + 0.6x_C \leq 500 \times 150$$ Cost $= \frac{90}{500}$

Objective: Maximize the profit i.e.

Max $\cancel{Z = 2.3x_A + 2.5x_B + 2.4x_C}$

$$\text{St.} \quad 0.5x_A + 0.25x_B \leq 150000$$
$$0.25x_B + 0.4x_C \leq 60000$$
$$0.5x_A + 0.5x_B + 0.6x_C \leq 75000$$
$$x_A, x_B, x_C \geq 0$$

$$\text{Total cost} = \frac{200}{750}(0.5x_A + 0.25x_B) + \frac{1}{6}(0.25x_B + 0.4x_C) + \frac{9}{50}(0.5x_A + 0.5x_B + 0.6x_C)$$

$$P = \text{Income} - \text{cost}$$
$$= \left[2.3 - \left(\frac{1}{5} + \frac{9}{100}\right)\right]x_A + \left[2.5 - \left(\frac{1}{15} + \frac{1}{24} + \frac{9}{100}\right)\right]x_B + \left[2.4 - \left(\frac{1}{15} + \frac{27}{250}\right)\right]x_C$$

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$\therefore$ $P = 2.07\, x_A + 2.302\, x_B + 2.225\, x_c$.

Max. $P = 2.0767\, x_A + 2.3016\, x_B + 2.2253\, x_c$

non basic

Basic	x_A	x_B	x_c	S_4	S_5	S_6	b	Ratio
Pivot row $\Rightarrow$ S_4	0.5	$\boxed{0.25}$	0	1	0	0	150000	60000
S_5	0	0.25	0.4	0	1	0	60000	240000
S_6	0.5	0.5	0.6	0	0	1	75000	150000
z	-2.07	-2.30	-2.22	0	0	0	0	

$\Uparrow$ Pivot column.

$\Uparrow$ Select lowest +ve ratio.

Divide Row1 by 0.25. Row 2 = ~~0.25~~

x_A	x_B	x_c	S_4	S_5	S_6	b
2	1	0	4	0	0	600000

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THE BIG M METHOD.

Min $z = 6x_1 + 5x_2 + Mr_5 + Mr_6$

$\because$ Min we add Mr_5 & Mr_6.

But for Max we subtract Mr_5, Mr_6.

$$z = 6x_1 + 5x_2 + M(8 - 4x_1 - 2x_2 + S_3) + M(6 - 2x_1 - 3x_2 + S_4)$$

$$= 6x_1 + 5x_2 + M(14 - 6x_1 - 5x_2 + S_3 + S_4)$$

$$= (6 - 6M)x_1 + (5 - 5M)x_2 + M(S_3) + M(S_4) + 14M$$

NB

	Basic	x_1	x_2	S_3	S_4	r_5	r_6	b	Ratio
Pivot ~~row~~ $\Rightarrow$	r_5	$\boxed{4}$ Pivot element	2	−1	0	1	0	8	$8/4 = 2$
	r_6	2	3	0	−1	0	1	6	$6/2 = 3$
	z	6M−6	5M−5	−M	−M	0	0	14M	
		⇑ Pivot column							⇑ Select min

for min = Select most +ve.

Row1 = Row1/4 Row 3 = Row 3 − (6M−6) Row1

Row 2 = Row 2 − 2 Row 1

	Basic	x_1	x_2	S_3	S_4	r_5	r_6	b	Ratio
	x_1	1	$1/2$	$-1/4$	0	$1/4$	0	2	4
Pivot Row $\Rightarrow$	r_6	0	$\boxed{2}$	$1/2$	−1	$-1/2$	1	2	1
	z	0	2M−2	$M/2 - 3/2$	−M	$-\frac{6M}{4} + 6/4$	0	2M+12	
			⇑ Pivot column						

$m - 5$

$-3m + 3$

$-M + \frac{6M}{4} + \frac{6}{4}$

$\frac{2M}{4} + \frac{3}{2}$

Row 2 / 2.

Row1 − ½ Row 2 = Row 1.

Row 3 − (2M − 2) Row2 = Row 3.

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	x_1	x_2	s_3	s_4	r_5	r_6	b
x_1	1	0	$-3/8$	$1/4$	$3/8$	$-1/4$	$3/2$
x_2	0	1	$1/4$	$-1/2$	$-1/4$	$1/2$	1
z	0	0	-1	-1	$1-M$	$1-M$	14

$\frac{M}{2} - \frac{3}{2} - \frac{2M}{4\,2} + \frac{2\,1}{4\,2}$ $\frac{1}{2} - \frac{3}{2}$ $- M + M - 1$

$2M+12 - 2M-12$

$-\frac{3}{2}M + \frac{3}{2} + \frac{M}{2} - \frac{1}{2}$ $-1 - M$

Optimum $z^* = 14$.

$x_1^* = 3/2$

$x_2^* = 1$

$z = 6\left(\frac{3}{2}\right) + 5(1) = 14$.

At the optimal table, if the artificial variable remains with a positive value, it implies that the original problem doesn't have a BFS.

In programming, the value of M plays a major role in modelling & implementation.

HW

1. Create problem with open FS & check how simplex indicates the unboundedness.

2. Create a problem where a constraint gives a −ve ratio & interpret this graphically.

A city is planning to improve the tax base by demolishing a condemned area & blg a new housing complex. 1.5 million budget.

2 phases → No of houses to be demolised
→ No of -11- to be built.

① As many as 300 houses can be demolished at a cost of $2000 per unit with an area of 0.25 acre.

② Lot sizes for new 1BHK, 2, 3, 4 BHK units are 0.18, 0.28, 0.4 & 0.5 acre respectively. Streets, open spaces & utilities account for 15% of available acrage.

③ Tax levied per unit of 1, 2, 3, 4 BHK units are \$1000, \$1900, \$2700 & \$3400 respectively.

④ ~~Singles~~ In new development, triple & quadruple units account for at least 25% of the total, single units must be at least 20% & double units must be atleast 10% of the total units.

⑤ Construction cost per unit of single, double, triple & quadruple units are \$50000, \$70000, \$130,000 & \$160000 respectively. Financing from a local bank can amount to a max of \$15 million.

How many units of each type should be constructed to max tax collection.

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max.

Objective fn : $z = x_1 + 1.9x_2 + 2.7x_3 + 3.4x_4$

thousand $

x_5 = No. of houses to be demolished.

1BHK, 2, 3, 4 BHK $\Rightarrow x_1, x_2, x_3, x_4$.

Land available for new complex $= 0.85 \times 0.25\, x_5$.

Constraint on land.

$0.18x_1 + 0.28x_2 + 0.4x_3 + 0.5x_4 \le 0.2125x_5$

Constraint on money

$50x_1 + 70x_2 + 130x_3 + 160x_4 + 2x_5 \le 15000$

$x_3 + x_4 \ge 0.25(x_1 + x_2 + x_3 + x_4)$

$x_1 \ge 0.2(x_1 + x_2 + x_3 + x_4)$

$x_2 \ge 0.1(x_1 + x_2 + x_3 + x_4)$

$x_1, x_2, x_3, x_4 \ge 0$. $x_5 \le 300$.

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A company produces 2 models of electronic gadgets that use registers, capacitors & IC chips.

Resources	Unit Resource Requirement Model-1	Model-2	Max-Available
Resistors	2	3	1200
Capacitors.	2	1	1000
IC chips.	0	4	800
Unit Price ($)	3	4	

1. Determine the optimum product mix.
   [Hint: Obj - max income]
2. Determine status of each resource.
3. Determine worth of each resource.

Variables :- $x_1 \to$ No of units of Model 1
$x_2 \to$ —— " —— 2

Z = Max(Income)

$$\boxed{Z = 3x_1 + 4x_2} \quad (\$)$$

S.t
$$2x_1 + 3x_2 \le 1200 \quad \text{Resistors}$$
$$2x_1 + x_2 \le 1000 \quad \text{Capacitors}$$
$$4x_2 \le 800 \quad \text{IC Chips.}$$
$$x_1, x_2 \ge 0$$

$$2x_1 + 3x_2 + S_3 = 1200$$
$$2x_1 + x_2 + S_4 = 1000$$
$$4x_2 + S_5 = 800$$

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Basic	x_1	x_2	S_3	S_4	S_5	b	Ratio
S_3	2	3	1	0	0	1200	$\frac{1200}{3} = 400$
S_4	2	1	0	1	0	1000	$\frac{1000}{1} = 1000$
Pivot Row ⟹ S_5	0	$\boxed{4}$	0	0	1	800	$\frac{800}{4} = 200$
z	-3	-4	0	0	0	0	

⇑ Pivot column

NBV: S_3, S_4, S_5 BV: x_1, x_2

$$x_{NB} = \begin{bmatrix} S_3 \\ S_4 \\ S_5 \end{bmatrix} = \begin{bmatrix} 1200 \\ 1000 \\ 800 \end{bmatrix} \qquad x_B = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$R_3/4$. $R_1 \to R_1 - 3R_3$

$R_2 \to R_2 - R_3$

$R_4 \to R_4 + 4R_3$

swap

Basic	x_1	$\cancel{x_2}$ S_5	S_3	S_4	S_5	b	Ratio
Pivot Row ⟹ S_3	$\boxed{2}$	0	1	0	$-3/4$	600	$600/2 = 300$
S_4	2	0	0	1	$-1/4$	800	$800/2 = 400$
x_2 $\cancel{S_5}$	0	1	0	0	$1/4$	200	$200/0 = \infty$
z	-3	0	0	0	1	800	

⇑ Pivot column

NBV. BV.

$$x_{NB} = \begin{bmatrix} x_1 \\ S_5 \end{bmatrix} = \begin{bmatrix} -3 \\ 0 \end{bmatrix} \qquad x_B = \begin{bmatrix} S_3 \\ S_4 \\ x_2 \end{bmatrix} = \begin{bmatrix} 600 \\ 800 \\ 200 \end{bmatrix}$$

$z = 800$.

R1/2.

$R_2 \rightarrow R_2 - 2R1$

$R_3 \rightarrow$

Swap

S_3

Basic	~~x_1~~	x_2	S_3	S_4	S_5	b.	Ratio
x_1 ~~S_3~~	1	0	1/2	0	-3/8	300	$300/{-3/8}$ = -ve
S_5. ~~S_4~~	0	0	-1	1	$\boxed{1/2}$	200	400 ⇐
x_2	0	1	0	0	1/4	200.	800
z	0	0	3/2	0	-1/8	1700	

⇑ Pivot column.

NBV. BV

$$x_{NBV} = \begin{bmatrix} S_3 \\ \\ S_5 \end{bmatrix} = \begin{bmatrix} 3/2 \\ \\ -1/8 \end{bmatrix} \qquad x_B = \begin{bmatrix} x_1 \\ x_2 \\ S_4 \end{bmatrix} = \begin{bmatrix} 300 \\ 200 \\ 200 \end{bmatrix}$$

of c → objective function coefficient.

$z = 1700$.

$R_2 \rightarrow 2R_2$.

	x_1	x_2	S_3	S_4	S_5	b.
x_1	1	0	-1/4	3/4	0	450.
S_5	0	0	-2	2	1	400
x_2	0	1	1/2	-1/2	0	100
z	0	0	5/4	1/4.	0	1750

NBV BV

$$x_{NB} = \begin{bmatrix} S_3 \\ S_4 \end{bmatrix} = \begin{bmatrix} 5/4 \\ 1/4 \end{bmatrix} \qquad x_B = \begin{bmatrix} x_1 \\ S_5 \\ x_2 \end{bmatrix} = \begin{bmatrix} 450 \\ 400 \\ 100 \end{bmatrix}$$

$Z = 1750$.

Optimum Mix.

$x_1^* = 450$ $\quad Z^* = 1750$ \$

$x_2^* = 100$

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Ans2. $S_3 = S_4 = 0$
i.e R & C are completely used.

$S_5 = 400$

$\therefore$ IC chips remaining $= 800 - 400 = 400$.

Ans3- Worth of
$R = 5/4$ \$
$C = 1/4$ \$
$IC = 0$ \$

Worth → By increasing inventory of a particular resource by what value will 'z' increase

$$2x_1 + 3x_2 + S_3 = 1200 + \delta b_1$$
$$2x_1 + x_2 + S_4 = 1000 + \delta b_2$$
$$4x_2 + S_5 = 800 + \delta b_3$$
$$x_1, x_2 \geq 0$$

$$
\begin{array}{c|cccccc|ccc}
 & x_1 & x_2 & s_3 & s_4 & s_5 & b & \delta b_1 & \delta b_2 & \delta b_3 \\
\hline
x_1 & 2 & 3 & 1 & 0 & 0 & {}^{450}\,1200 & \cancel{1}(-\tfrac14) & \cancel{0}(\tfrac34) & \cancel{0}(0) \\
s_5 & 2 & 1 & 0 & 1 & 0 & {}^{400}\,1000 & \cancel{0}(-2) & \cancel{1}(2) & \cancel{0}(1) \\
x_2 & 0 & 4 & 0 & 0 & 1 & {}^{100}\,800 & \cancel{0}(\tfrac12) & \cancel{0}(-\tfrac12) & \cancel{1}(0) \\
\hline
 & -3 & -4 & 0 & 0 & 0 & 0 & 5/4 & 1/4 & 0
\end{array}
$$

$$
\begin{array}{ccc}
s_3 & s_4 & s_5 \\
\end{array}
\begin{bmatrix} -1/4 & 3/4 & 0 \\ -2 & 2 & 1 \\ 1/2 & -1/2 & 0 \end{bmatrix}
$$

$$x_1 = 450 - \frac{1}{4}\delta b_1 + \frac{3}{4}\delta b_2 \geq 0$$

$$s_5 = 400 - 2\delta b_1 + 2\delta b_2 - \delta b_3 \geq 0$$

$$x_2 = 100 + \frac{1}{2}\delta b_1 - \frac{1}{2}\delta b_2 \geq 0$$

For changes in the inventory of resistors alone we set $\delta b_2 = \delta b_3 = 0$ so we get,

$$450 - \frac{1}{4}\delta b_1 \geq 0 \qquad 450 \geq \frac{1}{4}\delta b_1 \qquad 8$$

$$400 - 2\delta b_1 \geq 0 \qquad 400 \geq 2\delta b_1$$

$$100 + \frac{1}{2}\delta b_1 \geq 0 \qquad +100 \not\leq \frac{\delta b_1}{2}$$

$$\boxed{-200 \leq \delta b_1 \leq 200}$$

For capacitors alone $\delta b_1 = \delta b_3 = 0$

$\overset{150}{450} + \frac{3}{4}\delta b_2 \geq 0$. $\delta b_2 \geq -600$

$400 + 2\delta b_2 \geq 0$ $\delta b_2 \geq -200$

$100 - \frac{\delta b_2}{2} \geq 0$. $\delta b_2 \leq 200$.

$$\boxed{-200 \leq \delta b_2 \leq 200}$$

For I C alone $\delta b_1 = \delta b_2 = 0$

$400 + \delta b_3 \geq 0$.

$$\boxed{\delta b_3 \geq -400}$$

$$\boxed{Z = 1750 + \frac{5}{4}\delta b_1 + \frac{1}{4}\delta b_2}$$

Solve the following LPP.

Min $Z = 5x_1 + 6x_2$.

s.t $4x_1 + 8x_2 \geq 32$

$6x_1 + 4x_2 \geq 24$.

$x_1, x_2 \geq 0$.

Use both, the two phase & the Big M methods. Can you estimate which is more efficient.

$x_1, x_2 \geq 0$.

$4x_1 + 8x_2 - S_3 = 32$

$6x_1 + 4x_2 - S_4 = 24$

BV. NBV.

$\begin{bmatrix} S_3 \\ S_4 \end{bmatrix} \begin{matrix} = -32 \\ = -24. \end{matrix}$ $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

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Phase I :

$$4x_1 + 8x_2 - S_3 + r_5 = 32.$$
$$6x_1 + 4x_2 - S_4 + r_6 = 24$$

NBV

Basic	x_1	x_2	S_3	S_4	r_5	r_6	b	Ratio
Pivot Row ⟹ r_5	4	$\boxed{8}$	-1	0	1	0	32	$32/8 = 4$
r_6	6	4	0	-1	0	1	24.	$24/4 = 6$
w	+10	+12	-1	-1	0	0	56.	

⇑ Pivot column.

BV.

$r_5 = 32$ Min. $w = r_5 + r_6$.

$r_6 = 24$.

$$r_5 = 32 - 4x_1 - 8x_2 + S_3$$
$$r_6 = 24 - 6x_1 - 4x_2 + S_4$$
$$w = 56 - 10x_1 - 12x_2 + S_3 + S_4.$$
$$10x_1 + 12x_2 - S_3 - S_4 + w = 56.$$

$R_1 \to R_1/8$. $R_2 \to R_2 - 4R_1$.

$R_3 \to R_3 - 12R_1$

Basic	x_1	x_2	S_3	S_4	r_5	r_6	b	Ratio.
x_2	$1/2$	1	$-1/8$	0	$1/8$	0	4	8
Pivot row ⟹ r_6	$\boxed{4}$	0	$1/2$	-1	$-1/2$	1	8.	2
w	4	0	$1/2$	-1	$-3/2$	0	8.	2

⇑ Pivot column.

BV.   NBV.

$x_2 = 4$ $x_1 = 4$

$r_6 = 8$ $S_3 = 1/2$

$w = 8$ $S_4 = -1$

$r_5 = -3/2$.

$\frac{24}{8}$ $-1 + \frac{12}{8}$ $-\frac{12}{8}$ $56 - \frac{48}{8}$ $\frac{4}{1/2}$ $\frac{1/2}{4}$

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$R2/4$. $R_1 \rightarrow R_1 - R2/2$ $R_3 \rightarrow R_3 - 4R2$.

Basic	x_1	x_2	s_3	s_4	r_5	r_6	b	Ratio
x_2	0	1	$-3/16$	$1/8$	$3/16$	$-1/8$	3	
x_1	1	0	$1/8$	$-1/4$	$-1/8$	$1/4$	2.	
w	0	0	0	0	-1	-1	0	

$\frac{3}{8} \cdot \frac{1}{16}$

$\frac{1}{8} + \frac{1}{16}$

$-\frac{3}{2} + \frac{1}{2}$

Phase II :

Basic	x_1	x_2	s_3	s_4	b
x_2	0	1	$-3/16$	$1/8$	3
x_1	1	0	$1/8$	$-1/4$	2
z	0	0	$-1/2$	$-1/2$	28

$$z = 5x_1 + 6x_2.$$

$$x_2 - 3/16\, s_3 + 1/8\, s_4 = 3$$
$$x_1 + 1/8\, s_3 - 1/4\, s_4 = 2$$

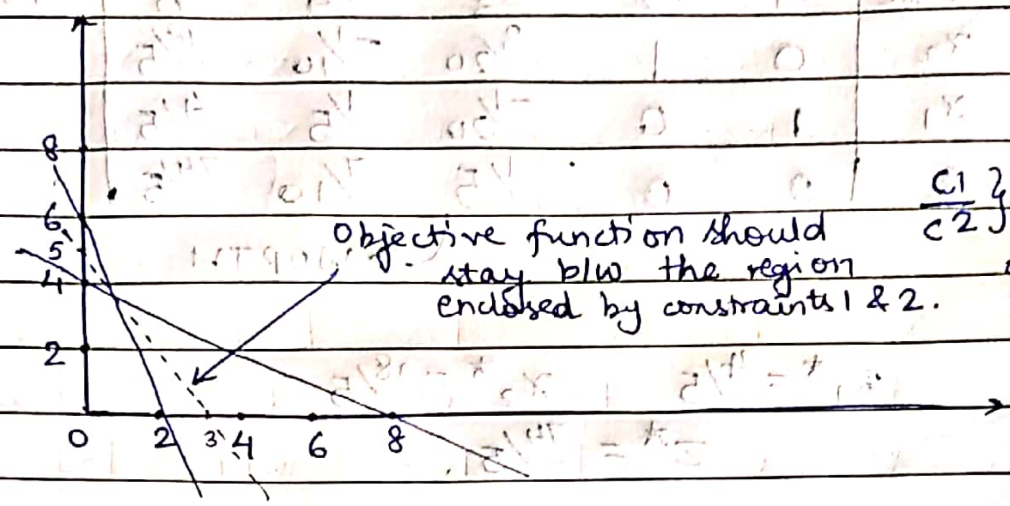
$$x_2 = 3/16\, s_3 - 1/8\, s_4 + 3$$
$$x_1 = -1/8\, s_3 + 1/4\, s_4 + 2.$$

$$z = 5\left(\frac{-1}{8} s_3 + \frac{1}{4} s_4 + 2\right) + 6\left(\frac{3}{16} s_3 - \frac{1}{8} s_4 + 3\right)$$

$$z = \frac{1}{2} s_3 + \frac{1}{2} s_4 + 28$$

Since $s_3, s_4 \rightarrow$ NBV $= -$ve & we require most +ve for min problem.
This means we have reached optimum
$z^* = 28$ $x_1^* = 2$ $x_2^* = 3$

* Sensitivity Analysis:-



Objective function should stay b/w the region enclosed by constraints 1 & 2.

$\frac{c_1}{c_2}$ } should be b/w the slopes of constraint 1 & 2.

Max $Z = 5x_1 + 3x_2$

Subject to $4x_1 + 8x_2 \leq 32$

$6x_1 + 2x_2 \leq 12$

$x_1, x_2 \geq 0$

Basic	x_1	x_2	S_3	S_4	b	Ratio
S_3	4	8	1	0	32	$32/4 = 8$
Pivot Row ⇒ S_4	$\boxed{6}$	2	0	1	12	$12/6 = 2$
z	−5	−3	0	0	0	

⇑ Pivot column

~~R~~ R2/6, R1 → R1 − 4R2, R3 → R3 + 5R1

Basic	x_1	x_2	S_3	S_4	b	Ratio
Pivot Row ⇒ S_3	0	$\boxed{20/3}$	1	$-2/3$	24	~~$\frac{24}{20/3}$~~ $72/20 = 36/10 = 18/5$
x_1	1	$1/3$	0	$1/6$	2	6
z	0	$-4/3$	0	$5/6$	10	

⇑ Pivot column

R1/$20/3$

~~Basic~~. R2 → R2 − $1/3$ R1

R3 → R3 + $4/3$ R1

$8-\frac{4}{3}$

$\frac{32}{8}$ .4

$\frac{3+5}{3}$

$-\frac{Z}{3}$ 20 10 $\frac{3}{2}$ 6 24×3 20 +0 5

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Basic	x_1	x_2	S_3	S_4	b	Ratio.
x_2	0	1	$3/20$	$-1/10$	$18/5$	
x_1	1	0	$-1/20$	$1/5$	$4/5$	
z	0	0	$1/5$	$7/10$	$74/5$	

$\underbrace{1/5 \quad 7/10}_{\text{WORTH}}$

$-\frac{1}{5}\times\frac{3}{20}$

$\frac{1}{6}+\frac{1}{30}$

$2-\frac{1}{3}\times\frac{18}{5}$

$$x_1^* = 4/5 \qquad x_2^* = 18/5$$
$$z^* = 74/5.$$

first stage:

$4x_1 + 8x_2 \leq 33 \longrightarrow$ No of capacitor
$6x_1 + 24x_2 \leq 12 \longrightarrow$ No. of transistor.

$4x_1 + 8x_2 = 33$
$2x_1 + 8x_2 = 4$

∴ On solving

$$x_1^* = \frac{3}{4} \qquad x_2^* = \frac{15}{4}$$

$$z_1^* = 15$$

$$\therefore z^* - z_1^* = 15 - \frac{74}{5} = \boxed{0.2}$$

worth of capacitor

Conclusion: The no. of capacitors can be varied from 8 to 48 with the optimum remaining the same

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Second stage:

$$6x_1 + 2x_2 \le 13$$

$$\therefore \quad 4x_1 + 8x_2 = 32$$

$$6x_1 + 2x_2 = 13$$

On solving.

$$x_1^* = 1 \qquad x_2^* = \frac{28}{8} = \frac{7}{2}$$

$$\therefore \quad Z_2^* = 15.5$$

$$\therefore \quad Z^* - Z_2^* = \boxed{0.7}$$

⇑
Worth of transistor.

$$\boxed{8 \le b_2 \le 48}$$

LOOK FOR THIS IN THE FINAL TABLEAU.