

# Superplural Logic

MoL Thesis Defence

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plural logic?

# Plural logic

FOL

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FOL +  $xx, yy, \dots$

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FOL +  $xx, yy, \dots$  +  $\preccurlyeq$

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 $\exists xx \textit{Apple}(xx)$

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FOL +  $xx, yy, \dots$  +  $\preceq$

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collective predication

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$\{\text{Russell}, \text{Whitehead}\}$  ?

# Singularisation

Problem with singular surrogates

- ▶ changing the subject
- ▶ Russelian paradox

# Ideology

‘...abandoning the fetish for the singular that pervades contemporary decadent Western ontology.’

Richard Sharvy, 1980

# Outline

What are superplurals?

Why superplurals?

Why not?

- Naturalness

- Intelligibility

Alternative accounts

Innocence



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# Superplurals

- ▶ ‘Russell and Whitehead, and Hilbert and Bernays’
- ▶ ‘the Boswell Sisters and the Mills Brothers’
- ▶ ‘the twin primes’

# Supersuperplurals

- ▶ ‘the Yankees and the Red Sox, and the Giants and the Braves’

# Literature

Hazen (1997), Linnebo (2003), Uzquiano (2004), McKay (2006), Rayo (2006), Linnebo & Nicolas (2008), Florio (2010), Oliver & Smiley (2013), Ben-Yami (2013), Rieppel (2015), Simons (1982, forthcoming)

# Notation

	plural variables	superplural variables
Simons (1982)	$h, k, l; u, v, w$	
Burgess & Rosen (1997)	$xx, yy, zz$	
Linnebo (2003)	$xx, yy, zz$	
Rayo (2006)	$xx, yy, zz$	$xxx, yyy, zzz$
McKay (2006)	$X, Y, Z$	$XX, YY, ZZ$
Yi (2006)	$xs, ys, zs$	
Nicolas (2008)	$xs, ys, zs$	$xss, yss, zss$
Oliver & Smiley (2013)	$\mathbf{x}, \mathbf{y}, \mathbf{z}$	$\mathbf{x}^2, \mathbf{y}^2, \mathbf{z}^2$

previous notations for plural and superplural terms

# Notation

	predicate	reading
Simons (1982)	$\in$	is or is among
	$\in$	is/are or is/are among
Burgess & Rosen (1997)	$==$	is or is among
Linnebo (2003)	$\prec$	is among
Burgess (2004)	$\alpha$	is or is among
Rayo (2006)	$\prec$	is or is among
	$\sim$	is/are or is/are among
McKay (2006)	K	is or is among
	A	is/are or is/are among
Yi (2006)	H	is or is among
	$\sqsubseteq$	is/are or is/are among
Nicolas (2008)	$\angle$	is/are or is/are among
Oliver & Smiley (2013)	$\preceq$	is/are or is/are among
Florio (2014)	$\prec$	is among
Simons (2016)	$\eta$	is one of

previous notations for the inclusion predicate

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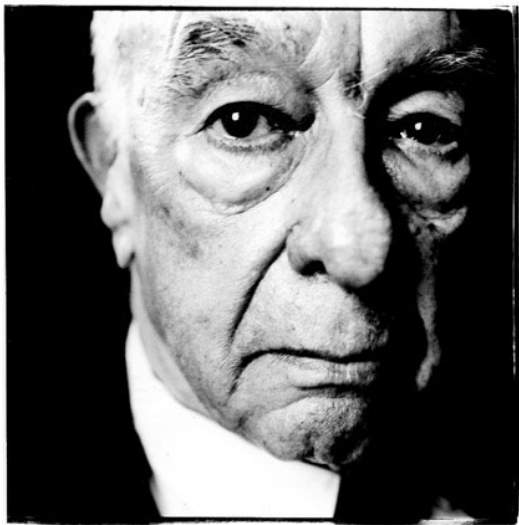
# Why superplurals?

- occurrence in natural language



# Why superplurals?

- ▶ occurrence in natural language
- ▶ ontological innocence



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# Two objections

## Intelligibility

higher-level plural quantification is unintelligible

# Two objections

## Naturalness

there are no genuine examples of superplurals in natural language

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# Examples

- ▶ ‘Russell and Whitehead, and Hilbert and Bernays wrote multivolume logic books together.’
- ▶ ‘The Boswell Sisters and the Mills Brothers gave a joint concert.’
- ▶ ‘The twin primes are infinite in number.’

# Strategies for paraphrase

- ▶ partial singularisation (groups)
- ▶ conjunctive analysis (distributive)
- ▶ ordinary plural analysis (lists)
- ▶ multigrade predicates (superplurals)



# ... and why they fail

*The boys and the girls played against each other.*

# Finnish

VALUE	SINGULAR	PLURAL
1	yksi	yhdet
2	kaksi	kahdet
3	kolme	kolmet
4	neljä	neljät
5	viisi	viidet
6	kuusi	kuudet
7	seitsemän	seitsemät
8	kahdeksan	kahdeksat
9	yhdeksään	yhdeksät
10	kymmenen	kymmenet
pair, couple	pari	parit
a few	muutama	muutamat
many	moni	monet
several	usea	useat
a few, not many	harva	harvat

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# Intelligibility

- collapse

# Intelligibility

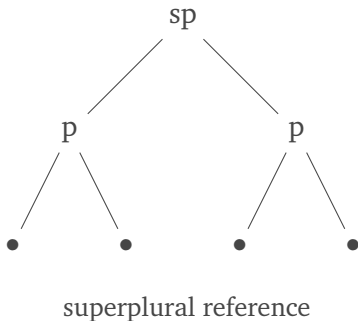
- ▶ collapse
- ▶ collapse vs. singularisation

# Iteration

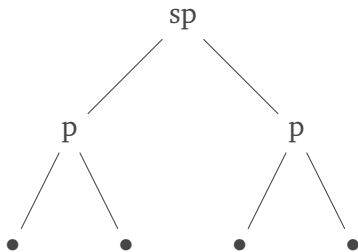
A natural question that arises is whether the step from the singular to the plural can be iterated. Are there terms that stand to ordinary plural terms the way ordinary plural terms stand to singular terms? Let's call such terms superplurals. A superplural term would thus, loosely speaking, refer to several 'pluralities' at once, much as an ordinary plural term refers to several objects at once.

Linnebo & Nicolas, 2008, p. 186

# Which picture?



# Which picture?

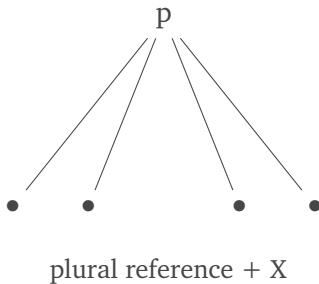


superplural reference

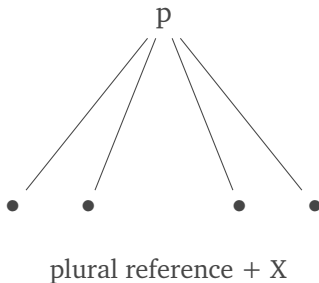
singularisation?



# Which picture?



# Which picture?



collapse?

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# Cover semantics

$$\mathcal{D}(F) = \lambda x[\forall y(y \in C_x \rightarrow F(y))]$$

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Russell and Whitehead, and Hilbert and Bernays were logicians.

$\forall y(y \in \{ \{\text{Russell}\}, \{\text{Whitehead}\}, \{\text{Hilbert}\}, \{\text{Bernays}\} \} \rightarrow$   
*Logician*(y))

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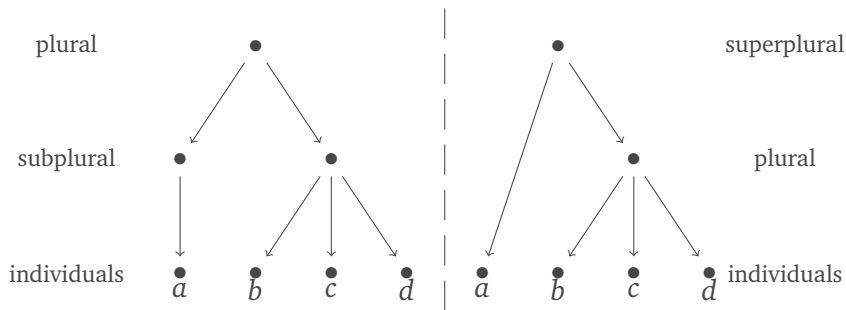
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Russell and Whitehead, and Hilbert and Bernays wrote multivolume logic books together.

$\forall y(y \in \{\{\text{Russell, Whitehead}\}, \{\text{Hilbert, Bernays}\}\} \rightarrow$   
 $\text{WroteLogicBook}(y))$



# Two hierarchies



comparing the hierarchies

# Structure

- ▶ Accounts differ in how they add structure to reference.

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- ▶ Cover semantics needs a stronger structure, leading to a subplural hierarchy.

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- ▶ Cover semantics needs a stronger structure, leading to a subplural hierarchy.
- ▶ The subplural and the superplural hierarchy are two conceptions of superplural reference.

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# Ontological commitment

A first-order sentence carries commitment to Fs just in case Fs must be counted amongst the values of the variables in order for the sentence to be true.

# Plethological commitment

A singular or plural first-order sentence carries commitment to Fs just in case Fs must be counted amongst the values of the (singular or plural) variables in order for the sentence to be true.

# Higher-level plethological commitment

An  $n$ -level first-order sentence carries commitment to  $F$ s just in case  $F$ s must be counted amongst the values of the  $n$ -level variables in order for the sentence to be true.



# The end?

# The end?

ontology vs. ideology

thank you!

# Features of pluralities

- i **Unrestricted Composition**. For any combination of individuals, there is a plurality of them.
- ii **Determinacy**. For a plurality  $P$  and any object  $a$  it is determinately true or determinately false that  $a$  is a member of  $P$ .
- iii **Extensionality**. Pluralities are identical when and only when they have the same members.
- iv **Multitude**. Unlike sets and sums, a plurality denotes several things at once.
- v **Concreteness**. A plurality is nothing over and above its members.

# Co-reference

**Co-reference.** Two plural terms of level  $n$  are co-referring iff all its pluralities of level  $0 \dots n - 1$  are co-referring, respectively.

# Geach-Kaplan sentence

(GK) Some critics admire only one another.

(GK<sub>2</sub>)  $\exists X(\exists xXx \wedge \forall x\forall y(Xx \wedge Axy \rightarrow x \neq y \wedge Xy))$

(GK<sub>s</sub>)  $\exists S(\exists x(x \in S) \wedge \forall x(x \in S \rightarrow Cx) \wedge \forall x\forall y((x \in S \wedge Axy) \rightarrow (x \neq y \wedge y \in S)))$

(GK<sub>p</sub>)  $\exists xx(\forall x(x \preccurlyeq xx \rightarrow Cx) \wedge \forall x\forall y((x \preccurlyeq xx) \wedge Axy) \rightarrow (x \neq y \wedge y \preccurlyeq xx))$

# Russellian paradox

- (R) There are some collections such that, for any  $y$ ,  $y$  is one of them just in case  $y$  is a collection which is not a constituent of itself.
- (R') There is a collection  $x$  such that, for every  $y$ ,  $y$  is a constituent of  $x$  just in case  $y$  is a collection which is not a constituent of itself.
- (R'')  $\exists x \forall y (y \leq x \leftrightarrow \neg(y \leq y))$

# Plural Cantor

For any things  $ss$ , if  $ss$  is strictly plural (i.e.  $\exists x x \prec ss$ ), then there is no (possibly multivalued) function  $f$  such that

$$\forall x((x \preccurlyeq ss \rightarrow f(x) \preccurlyeq ss) \wedge \forall xx(xx \preccurlyeq ss \rightarrow \exists y(y \preccurlyeq ss \wedge f(y) = x)))$$



# Adicity and Grade

	Fixed-grade	Multi-grade
Monadic	give a soliloquy	form a circle
Dyadic	co-author	play against each other

# Collective predication

a ‘Whitehead and Russell were logicians.’

b ‘Whitehead and Russell co-authored *Principia Mathematica*.’

A predicate  $F$  is *distributive* if it is analytic that  $F$  is true of some things iff it is true of each of them.

Otherwise it is *collective*.