Unit 3 Section 2 : Laws of Indices

There are three rules that should be used when working with indices:

When m and n are positive integers,

1. $a^m \times a^n = a^{m+n}$
2. $a^m \div a^n = a^{m-n}$ or $\dfrac{a^m}{a^n} = a^{m-n}$ $(m \geq n)$
3. $(a^m)^n = a^{m \times n}$

These three results are logical consequences of the definition of a^n , but really need a formal proof. You can 'verify' them with particular examples as below, but this is not a proof:

$$\begin{aligned} 2^7 \times 2^3 &= (2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2) \times (2 \times 2 \times 2) \\ &= 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \\ &= 2^{10} \qquad \text{(here } m = 7, n = 3 \text{ and } m + n = 10) \end{aligned}$$

or,

$$\begin{aligned} 2^7 \div 2^3 &= \frac{2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2}{2 \times 2 \times 2} \\ &= 2 \times 2 \times 2 \times 2 \\ &= 2^4 \qquad \text{(again } m = 7, n = 3 \text{ and } m - n = 4) \end{aligned}$$

Also,

$$\begin{aligned} (2^7)^3 &= 2^7 \times 2^7 \times 2^7 \\ &= 2^{21} \qquad \text{(using rule 1)} \quad \text{(again } m = 7, n = 3 \text{ and } m \times n = 21) \end{aligned}$$

The proof of the first rule is given below:

Proof

$$\begin{aligned} a^m \times a^n &= \underbrace{a \times a \times \ldots \times a}_{m \text{ of these}} \times \underbrace{a \times a \times \ldots \times a}_{n \text{ of these}} \\ &= \underbrace{a \times a \times \ldots \times a \times a \times a \times \ldots \times a}_{(m+n) \text{ of these}} \\ &= a^{m+n} \end{aligned}$$

The second and third rules can be shown to be true for all positive integers m and n in a similar way.

We can see an important result using rule 2:

$$\frac{x^n}{} = x^{n-n} = x^0$$

but $\frac{x^n}{x^n} = 1$, so

$$x^0 = 1$$

This is true for any non-zero value of x, so, for example, $3^0 = 1$, $27^0 = 1$ and $1001^0 = 1$.

Example 1

Fill in the missing numbers in each of the following expressions:

(a) $2^4 \times 2^6 = 2^{\square}$ Show me...

(b) $3^7 \times 3^9 = 3^{\square}$ Show me...

(c) $3^6 \div 3^2 = 3^{\square}$ Show me...

(d) $(10^4)^3 = 10^{\square}$ Show me...

Example 2

Simplify each of the following expressions so that it is in the form a^n, where n is a number:

(a) $a^6 \times a^7$ Show me...

(b) $\frac{a^4 \times a^2}{a^3}$ Show me...

(c) $(a^4)^3$ Show me...

Exercises

Work out the answers to the questions below and fill in the boxes. Click on the Check button to find out whether you have answered correctly. If you are right then ✔ Correct will appear and you should move on to the next question. If ✘ Try again appears then your answer is wrong. Click on ✘ Try again to clear your original answer and have another go. If you can't work out the right answer then click on Tell me to see the answer.

Question 1
Fill in the missing numbers:

(a) $2^3 \times 2^7 = 2^{10}$ ✔ Correct

(b) $3^6 \times 3^5 = 3^{11}$ ✔ Correct

(c) $3^7 \div 3^4 = 3^{3}$ Correct

(d) $8^3 \times 8^4 = 8^{7}$ Correct

(e) $(3^2)^5 = 3^{10}$ Correct

(f) $(2^3)^6 = 2^{18}$ Correct

(g) $\frac{3^6}{3^2} = 3^{4}$ Correct

(h) $\frac{4^7}{4^2} = 4^{5}$ Correct

Question 2
Fill in the missing numbers:

(a) $a^3 \times a^2 = a^{5}$ Correct

(b) $b^7 \div b^2 = b^{5}$ Correct

(c) $(b^2)^5 = b^{10}$ Correct

(d) $b^6 \times b^4 = b^{10}$ Correct

(e) $(z^3)^9 = z^{27}$ Correct

(f) $\frac{q^{16}}{q^7} = q^{9}$ Correct

Question 3
Explain why $9^4 = 3^8$.

$(3^{2})^{4} = 3^{2 \times 4} = 3^{8}$ Correct

Question 4
Calculate:

(a) $3^0 + 4^0$ 2 Correct

(b) $6^0 \times 7^0$ 1 Correct

(c) $8^0 - 3^0$ 0 Correct

(d) $6^0 + 2^0 - 4^0$ 1 Correct

Question 5
Fill in the missing numbers:

(a) $3^6 \times 3^{11} = 3^{17}$ Correct

(b) $4^6 \times 4^{\boxed{5}} = 4^{11}$ ✔ Correct

(c) $\frac{a^6}{a^{\boxed{2}}} = a^4$ ✔ Correct

(d) $(z^{\boxed{3}})^6 = z^{18}$ ✔ Correct

(e) $(a^{19})^{\boxed{5}} = a^{95}$ ✔ Correct

(f) $p^{16} \div p^{\boxed{9}} = p^7$ ✔ Correct

(g) $(p^{\boxed{5}})^8 = p^{40}$ ✔ Correct

(h) $q^{13} \div q^{\boxed{12}} = q$ ✔ Correct

Question 6
Calculate:

(a) $\frac{2^3}{2^2} + 3^0$ [3] ✔ Correct

(b) $\frac{3^4}{3^3} - 3^0$ [2] ✔ Correct

(c) $\frac{5^4}{5^2} + \frac{6^2}{6}$ [31] ✔ Correct

(d) $\frac{7^7}{7^5} - \frac{5^9}{5^7}$ [24] ✔ Correct

(e) $\frac{10^8}{10^5} - \frac{5^6}{5^3}$ [875] ✔ Correct

(f) $\frac{4^{17}}{4^{14}} - \frac{4^{13}}{4^{11}}$ [48] ✔ Correct

Question 7
Fill in the missing numbers in each of the following expressions:

(a) $8^2 = 2^{\boxed{6}}$ ✔ Correct

(b) $81^3 = 9^{\boxed{6}} = 3^{\boxed{12}}$ ✔ Correct

(c) $25^6 = 5^{\boxed{12}}$ ✔ Correct

(d) $4^7 = 2^{\boxed{14}}$ ✔ Correct

(e) $125^4 = 5^{\boxed{12}}$ ✔ Correct

(f) $1000^6 = 10^{\boxed{18}}$ ✔ Correct

(g) $81 = \boxed{3}^4$ ✔ Correct

(h) $256 = 4^{4} = 2^{8}$ ✔ Correct

Question 8
Fill in the missing numbers in each of the following expressions:

(a) $8 \times 4 = 2^{3} \times 2^{2} = 2^{5}$ ✔ Correct

(b) $25 \times 625 = 5^{2} \times 5^{4} = 5^{6}$ ✔ Correct

(c) $\frac{243}{9} = \frac{3^{5}}{3^{2}} = 3^{3}$ ✔ Correct

(d) $\frac{128}{16} = \frac{2^{7}}{2^{4}} = 2^{3}$ ✔ Correct

Question 9
Is each of the following statements true or false?

(a) $3^2 \times 2^2 = 6^4$ False ✔ Correct

(b) $5^4 \times 2^3 = 10^7$ False ✔ Correct

(c) $\frac{6^8}{2^8} = 3^8$ True ✔ Correct

(d) $\frac{10^8}{5^6} = 2^2$ False ✔ Correct

Question 10
Complete each expression:

(a) $(2^6 \times 2^3)^4 = (2^{9})^4 = 2^{36}$ ✔ Correct

(b) $\left(\frac{3^6}{3^2}\right)^5 = (3^{4})^5 = 3^{20}$ ✔ Correct

(c) $\left(\frac{2^3 \times 2^4}{2^7}\right)^4 = (2^{0})^4 = 2^{0}$ ✔ Correct

(d) $\left(\frac{3^2 \times 9}{3^3}\right)^4 = (3^{1})^4 = 3^{4}$ ✔ Correct

(e) $\left(\frac{6^2 \times 6^8}{6^3}\right)^4 = (6^{7})^4 = 6^{28}$ ✔ Correct

(f) $\left(\frac{7^8}{7^2 \times 7^3}\right)^5 = (7^{3})^5 = 7^{15}$ ✔ Correct

You have now completed Unit 3 Section 2

Your overall score for this section is 100%

Correct Answers

You answered 55 questions correctly out of the 55 questions in this section.

Incorrect Answers

There were 0 questions where you used the *Tell Me* button.

There were 0 questions with wrong answers.

There were 0 questions you didn't attempt.

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